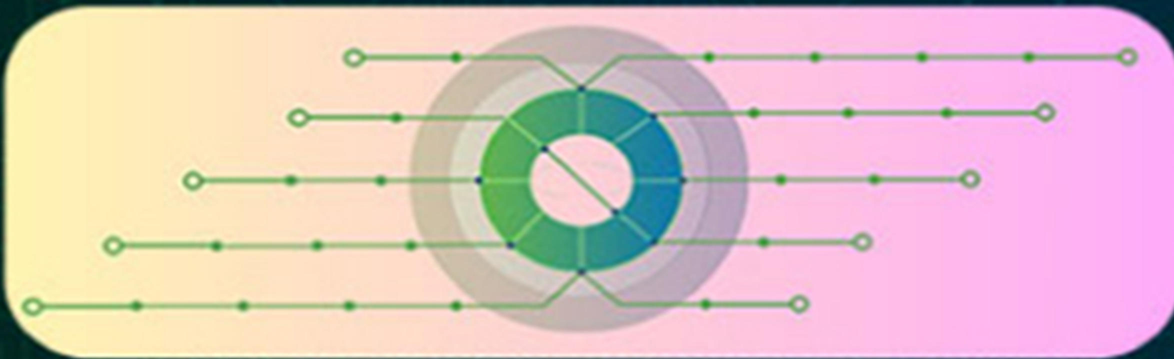

ADVANCED TUTORIALS ON QUANTUM CIRCUITS

Editors

Ayan Datta

Prabhavathi P

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Table of Contents

Introduction

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Quantum circuits, which operate on qubits rather than traditional binary bits, exploit the principles of superposition and entanglement, allowing them to perform complex computations that classical computers cannot. These circuits hold immense potential for breakthroughs in fields such as optimization, cryptography, and materials science. For example, quantum algorithms like the Quantum Approximate Optimization Algorithm (QAOA) have shown promise in solving real-world problems with greater speed and efficiency than classical methods.

QISKIT tool from IBM helps in Quantum Circuit Representation and Qubit Representation to prove different algorithms including Superposition, Entanglement and other concepts involving Quantum circuits like Samplers and Estimators.

Chapter 01

Quantum Technology: Directions and Prospects

Apoorva Patel

Centre for High Energy Physics, Indian Institute of Science,Bangalore

DOI: 10.1201/9788743807087-2

Quantum technology has its potentialities as well as its limitations. One must carefully analyse both in order to direct its evolution and figure out practical applications. Dreaming based on media hype is a disservice to the technical subject, and can destroy its future. In this context, useful directions for quantum technology development are pointed out that can be meaningfully pursued with the resources available in India.

11 **Potentialities and Limitations**

Fact 2: Quantum dynamics is highly fragile against external disturbances.

It can work well in cooperative/protected settings, but not in hostile ones. Th need to shield quantum dynamics from unwanted disturbances (including thos arising from control systems and measurements) makes quantum technolog expensive.

Fact 3: Robust classical technology continues to improve, partly because of new ways of thinking offered by developments in quantum technology.

What is described as Quantum Advantage is a moving target.

Implication: Quantum technology will therefore be attractive only in case where the advantage offered by it is sufficiently large to offset its cost. Quantur technology will be practically useful as special-purpose devices, as custor subroutines in larger applications. Hybrid classical-quantum systems, with sma quantum modules embedded in large classical peripherals, are also required for u to interpret quantum signals,

Potential and Limitations of the Computing machines was clearly the point of discussion from the days of Prof. Richard Feynmann. Potentialities emphasized that the Quantum advantage is problem dependant. Quantum Physics was invented when classical physics failed to explain/solve some problems.

The distinct signature of Quantum Physics was the Planck's constant. The Planck constant is a fundamental concept in quantum theory, where energy behaves in a discrete way. If Planck's Constant is zero there can be a qubit and Quantum Hilbert's space but there is no quantum advantage. Only if a problem needs Quantum advantage, it can be explained with this theory.

It has its own limitations such as its extreme sensitivity to external disturbances. Quantum technology can be used in larger applications where the advantage offered by it is sufficiently large to offset its cost.

12 What to Expect

Practical applications of quantum technology are expected to appear

- first in sensing and metrology,
- then in communications and simulations,
- then as feedback to foundations of quantum theory,
- and ultimately in computation.

Our Status:

Theoretical developments in the subject are way ahead of practica implementations. This is true worldwide, but much more so in India. Researc connected to either the software or the hardware of quantum technologies i constrained by both infrastructure and manpower.

It is imperative to concentrate our resources (both infrastructure and manpower and essentially build from scratch. We must go after specific areas, keeping in min our strengths and limitations.

We can narrow down our explorations and reverse engineer certain device based on others' efforts, but have to stand on our own feet by indigenou development of quantum technology.

There are lot of practical application is sensing and metrology which is already happening. There are lot of models available in quantum technology, but can it be used in communication and simulations of complex systems. They require sophisticated lab facilities.

New technologies are always studied in science and applied in technology. The feedback from one domain is used as feedback to improvise in other. It is same in quantum theory. Theoretical explanations aid practical implementation.

The use of quantum theory is ultimately in computation. Keeping a check on India’s strengths and limitations, the exploration of quantum theory depends on our efforts and indigenous development of quantum technology with policies in mind. It is clear that India needs to build its infrastructure and solutions to potential problems from scratch knowing the strengths and limitations.

15 Go after Tough Problems (1)

Fact 1: The extent of Quantum Advantage is problem dependent.

Some problems are easy (BQP), and some problems are hard (QMA).

Quantum theory was invented to explain physical phenomena that existin classical theories failed to explain. Quantum Advantage can be sizeable when suc non-classical features are at the heart of the problems to be solved. Classica strategies are sufficient in many other situations, with not much to gain fror quantum description.

Fact 2: Quantum dynamics is highly fragile against external disturbances.

It can work well in cooperative/protected settings, but not in hostile ones.

The need to shield quantum dynamics from unwanted disturbances (includin those arising from control systems and measurements) makes quantum technolog expensive.

Fact 3: Robust classical technology continues to improve, partly because of nev ways of thinking offered by developments in quantum technology.

What is described as Quantum Advantage is a moving target.

Implication: Quantum technology will therefore be attractive only in case where the advantage offered by it is sufficiently large to offset its cost.

Quantum technology will be practically useful as special-purpose devices, a custom subroutines in larger applications. Hybrid classical-quantum systems, wit small quantum modules embedded in large classical peripherals, are also require for us to interpret quantum signals.

Quantum Technology requires special purpose systems such quantum design platforms which are proprietary in nature which are capable of estimating device performance.

There is also the need to figure out things that are required with photon sources and detectors with high fidelity and low coherence and control which can be gradually improved.

Quantum inspired algorithms are classical in nature but quantum dynamics are combined to tackle dynamics of wave mode in energy transfer, tomography and tensor networks.

Go after Tough Problems (2)

- Develop quantum-safe cryptography (e.g. PQC challenges by NIST), as well a protocols for oblivious computing on a cloud platform.
  These are necessary to tackle the authentication problem for secur communications. Quantum random number generators (tunnelling, shot noise photon quadratures) are available.
- Use multi-path interferometry, superadditivity of channels and squeeze states to improve signal-to-noise ratio in communications. Astronomers hav been using these for both electromagnetic and gravitational wave detectors Feed quantum signals directly into a quantum device, without projecting ther to classical values by intervening measurements and then processing th classical values.
- Develop methods for efficient simulation of molecules and materials.
  Quantum simulators are ideally suited to closely mimic natural processes Interaction parameters can be varied widely compared to their limited value found in nature. Variational methods provide useful approximations, and nove materials and non-equilibrium reactions (relevant to molecular scale physics chemistry and biology) can be explored.
- Think about what to do in a situation where the adversaries have the quantur hardware, while we don't (and not the other way around).
  Quantum information processing is vulnerable to attacks due to its hig fragility.

Leave aside scaling. Learn some quantum physics.

There is plenty to gain from O(10)-qubit devices.

Chapter 02

Characterization of Quantum Gate Noise Using Randomized Benchmarking

Goutam Paul

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DOI: 10.1201/9788743807087-3

Benchmarking noise in quantum gates is crucial for developing dependable quantum computers, as noise directly affects gate fidelity and computational accuracy. Randomized benchmarking (RB) protocols, which evaluate performance through random gate sequences, provide a reliable way to quantify average error rates due to their scalability and robustness in averaging out complex, device-specific noise. In this talk, we will discuss several variants of randomized benchmarking protocols including an overview of our recent work in this area.

11 **Agenda**

1. Introduction
2. Introduction to Randomized Benchmarking (RB)
3. Types of Randomized Benchmarking Protocols
4. Unitarity Randomized Benchmarking (URB)

Roadmap of the presentation.

12 **The NISQ Era: An Overview**

- **NISQ (Noisy Intermediate-Scale Quantum) era:** Quantum devices wit 50–100 qubits.
- Goal: Leverage these noisy devices for problem-solving, surpassing classica computation limits.
- Future Vision: Transition towards *fault-tolerant quantum computation.*

This slide discusses NISQ (Noisy Intermediate-Scale Quantum) era and its features.

13 **Challenges in the NSIQ Era (1)**

- **Noise and Errors**: Significant obstacles to reliable quantum computation.
- Achieving **fault tolerance** requires:
  - Error-correction schemes.
  - Identification and mitigation of error sources.
- Complexity: Requires error-free “logical” qubits made from numerous nois physical qubits.

14 **Challenges in the NSIQ Era (2)**

- Noise and Errors: Significant obstacles to reliable quantum computation.
- Achieving fault tolerance requires:
  - Error-correction schemes.
  - Identification and mitigation of error sources.
- Complexity: Requires error-free ” logical” qubits made from numerous nois physical qubits.

Identifies the challenges in the NISQ era.

15 **Sources of Errors in Quantum Computation**

- SPAM Errors: State preparation and measurement inaccuracies.
- Gate Errors: Imperfections in applying quantum gates.
- Need for quantification to improve gate fidelities and understand devic performance.

Discusses State preparation and measurement error and gate errors.

16 **Why Benchmarking Is Needed?**

- Noise affects the reliability and fidelity of quantum operations.
- Benchmarking provides quantitative metrics for error characterization.
- Helps in improving error correction and fault tolerance mechanisms.

Identifies the key reasons why benchmarking of noise is needed.

17 **Types of Noise in Quantum Systems**

- Depolarizing Noise: Randomly replaces qubits with mixed states.
- Bit-flip Noise: Flips a qubit's state (0 to 1 or vice versa).
- Phase-flip Noise: Alters the phase of a qubit, causing coherence loss.
- Coherent Noise: Systematic errors that build up consistently across gates.

Discusses 4 types of quantum noise: depolarizing, bit-flip, phase-flip and coherent.

18 **Error Correction: Physical vs. Logical Qubits**

- **Logical Qubits**: Constructed from multiple physical qubits for stability.
- Estimation: 1 logical qubit requires ~ 1000–10,000 physical qubits for fau tolerance.
- Practical Challenges: Limited qubit availability in current devices
- Cross-Platform Comparisons: Efforts to standardize benchmarks across IBM Google, and IonQ devices.

Discusses logical vs. physical qubits and therefore the challenges in implementing large numbers of logical qubits.

19 **Recent Developments in Quantum Benchmarks**

- **Cross-Platform Comparisons**: Efforts to standardize benchmarks acros IBM, Google, and IonQ devices.
- **Hybrid Benchmarks**: Combining different protocols for comprehensive errc insights.
- **Machine Learning Integration**: Using ML to predict and classify nois patterns from benchmarking data.

Discusses three primary approaches: Cross-Platform Comparisons, Hybrid Benchmarks and Machine Learning Integration for noise characterization.

110 Introduction to Randomized Benchmarking (RB) Protocol

- Randomized Benchmarking (RB) is a protocol for assessing the average erro rate of quantum gates.
- It uses random gate sequences to minimize bias and averages out comple device-specific noise.
- Goal: Determine the fidelity of quantum operations and overall error rates.

Introduces randomized benchmarking via random gate sequences to minimize bias and averages out complex device-specific noise. The goal is to determine the fidelity of quantum operations and overall error rates.

111 Advantages of Randomized Benchmarking

- Scalable to larger systems.
- Averages out device-specific noise, giving consistent error estimates
- Low sensitivity to state preparation and measurement (SPAM) errors.

Lists three advantages of randomized benchmarking: scalability to larger systems, averaging out device-specific noise, and low sensitivity to state preparation and measurement (SPAM) errors.

112 **Step 1: Generate RB Sequences**

- **Random Sequence Generation:**
- For each sequence, select m random gates C^i_j from the Clifford group.
- The final element C^i_{m+1} is chosen as the inverse of all previous gates:

$$C^i_{m+1} = \left(C^i_1 \cdot C^i_2 \cdot \cdots \cdot C^i_m\right)^{-1}$$

- Ensures each sequence ideally returns to the initial state.

Discusses how to select the sequence of gates in randomized unitarity.

113 **Step 2: Execute RB Sequences (with Noise)**

- Apply each RB sequence with noise modeled as Λ^i_j (error for gate C^i_j).
- Each sequence operation can be represented as:

$$S^i_m = \prod_{j=1}^{m+1} \left(\Lambda^i_j \circ C^i_j\right)$$

- Noise per gate helps model real execution conditions in hardware.

How to model the execution of RB sequences by considering errors in each gate operation.

114 **Step 3: Get Statistics - Survival Probability**

- Measure the probability of returning to the initial state, known as the **surviva probability.**
- For each sequence, the survival probability is:

$$\mathrm{Tr}\left[E_\psi S_m^i(\rho_\psi)\right]$$

- ρ_ψ: Initial state, E_ψ: POVM (measurement operator).

115 **Step 4: Find Averaged Sequence Fidelity**

- Average survival probabilities over all random sequence realization
- The **averaged sequence fidelity** is given by:

$$F_{\mathrm{seq}}\left(m,|\psi\rangle\right) = \mathrm{Tr}[E_\psi S_K^m(\rho_\psi)]$$

- Where $S_K^m = \frac{1}{K_m}\sum_{i=1}^{K_m} S_m^i$.

116 **Step 5: Fit Fidelity Data**

- Fit the averaged sequence fidelity to the model:

$$F_{\mathrm{seq}}^{(0)}(m,\left|\psi\right\rangle) = A_0\alpha^m + B_0$$

- α: Parameter representing the average error rate.
- A_0 and B_0: Parameters related to state preparation and measurement (SPAM errors.

Discusses fitting of fidelity data with an exponentially decaying trend and as a function of four parameters: one related to state preparation errors, another related to measurement error, a third...

117 **Step 6: Plot the Results**

- Average survival probabilities over all random sequence realization
- The **averaged sequence fidelity** is given by:

$$F_{\text{seq}}(m, |\psi\rangle) = \text{Tr}[E_\psi S_K^m(\rho_\psi)]$$

- Where $S_K^m = \frac{1}{K_m} \sum_{i=1}^{K_m} S_m^i$.

How plotting gives visual representations.

118 **Summary of RB Protocol Steps**[1]

1. Website https://github.com/Qiskit/textbook/blob/maininotebooks/quantumhardware/randomized-benchmarking.ipynb⏎

- Fit the averaged sequence fidelity to the model:

$$F_{\text{seq}}^{(0)}(m, \left|\psi\right\rangle) = A_0\alpha^m + B_0$$

- α: Parameter representing the average error rate.
- A_0 and B_0: Parameters related to state preparation and measurement (SPAM) errors.

Summarizes the 6 steps of RB protocol.

119 **Standard Randomized Benchmarking (SRB)**[2]

2. J. Helsen, I. Roth, E. Onorati, A .H. Werner, and J. Eisert. General framework fc randomized benchmarking. 20 PRX Quantum, 3:020357,1a° 2022.⏎

- Applies uniformly random gate sequences to measure average gate error.
- Robust and provides general error rates across a quantum device.
- Limitations: No gate-specific error isolation.

120 **Real Randomized Benchmarking (RRB)**[3]

3. A. K. Hashagen, S. T. Flammia, D. Gross, and J. J. Wall-man. Real randomize benchmarking. Quantum, 2:85, August 2018.⏎

- RRB measures average gate infidelity for a restricted gate set.
- Unlike standard RB, RRB works with gate sets that do not form a unitary 2 design.
- Useful for specific benchmarking scenarios, especially with non-standard gat sets.

Introduces the concept of Real Randomized Benchmarking (RRB) and compares it with SRB.

121 **Direct Randomized Benchmarking (DRB)**[4]

4. Proctor et al. Direct randomized benchmarking for multiqubit devices. Physica Review Letters, 123(3), July 2019.⏎

- DRB benchmarks performance of a native gate set present in a quantur computing platform.
- Native gates are predefined instructions, optimized for the hardware's physica layer.
- DRB's advantage: evaluates platform-specific performance without requirin unitary 2-designs.
- Applicable to universal gate sets that can perform universal quant computation.

Introduces the concept of Direct Randomized Benchmarking (DRB).

122 **Interleaved Randomized Benchmarking (IRB)**[5]

5. Easwar Magesan et al, Efficient measurement of quantum gate error b interleaved randomized benchmarking. Physical Review Let-tersp I 09(i8i), Augus 2Q 12. _⏎

- Adds specific gates to random sequences to isolate their error rates.
- Advantage: Measures individual gate performance.
- Limitation: Potential bias for specific gates due to interactions with others.

123 **Introduction to Unitarity Randomized Benchmarking(URB)**[6]

6. Bas Dirkse, Jonas Helsen, and Stephanie Wehner. Efficient unitarity randomize benchmarking of few-qubit clifford gates. Physical Review A, 99(1). January 2019.⏎

- URB quantifies coherence of noise by estimating unitarity.
- Useful for identifying coherent errors that build up predictably.
- Provides deeper insights into hard-to-correct noise sources.

Introduces the concept of Unitarity Randomized Benchmarking (URB).

124 **Why Unitarity Is Important?**

- Unitarity distinguishes between coherent and incoherent errors.
- Coherent errors often require different correction strategies than incoherer noise.
- Quantifying unitarity helps guide noise mitigation techniques.

125 **Benefits of URB**

- Provides insight into coherent noise, which is crucial for high-fidelit operations.
- Quantifies the unitarity metric, differentiating coherent and incoherent noise.
- Helps in identifying systematic errors that need tailored corrections.

Lists three benefits of URB: (i) provides insight into coherent noise, which is crucial for high-fidelity operations, (ii)quantifies the unitarity metric, differentiating coherent and incoherent noise, (iii).

126 Challenges in Implementing URB

- Requires mixed-state preparation, challenging on current quantum hardware.
- Direct implementation is limited by hardware constraints.

127 Hardware Constraints in Implementing URB

- Mixed-state preparation requires quantum resources beyond current devices.
- Current devices primarily support pure states and unitary operations.

Elaborates the hardware constraints in implementing URB.

128 Comparison of Benchmarking Protocols

Protocol	Purpose	Pros	Cons
SRB	Avg. error rates	Simple, general	No specificity
RRB	Gate infidelity	Practical for few gates	Not a full 2-design
DRB	Native gates	Tailored to platforms	Limited to natives
IRB	Individual gate	Specific insights	Bias possible
URB	Coherence errors	Detects coherent noise	Needs mixed states

Gives a table of comparison between SRB, RRB, DRB, IRB and URB in terms of purpose, pros and cons.

129 **Quantum Computing Platforms and Their Archetypes**

- Quantum computing platforms vary in qubit architecture and manipulatio techniques.
- **Examples:**
  - Google Sycamore: Superconducting Xmon qubits.
  - IonQ: Trapped ions.
  - IBM Quantum (IBM-Q): Superconducting transmon qubits.
- Each archetype has unique advantages and limitations in terms of speec coherence, and gate fidelity.

Talks about several quantum computing platforms like Google Sycamore with superconducting Xmon qubits, IonQ with trapped ions, and IBM Quantum (IBM-Q) with superconducting transmon qubits.

130 **IBM Quantum and Superconducting Qubits**

- IBM-Q uses superconducting transmon qubits with free cloud access.
- Superconducting qubits are optimal for large-depth circuits due to shorter gat times.
- Drawback: Limited decoherence times compared to other qubit technologies.

Discusses advantages and disadvantages of IBM superconducting qubits.

131 **IBM's Key Quantum Performance Attributes**

- **Quality:** Fidelity and coherence of quantum operations.
- **Speed:** Execution speed of quantum gates.
- **Scale:** Number of qubits available for computation.
- These attributes are particularly relevant in NISQ, guiding benchmarks an device improvements.

Discusses three key attributes of IBM's quantum performance: quality, speed and scale.

132 **Performance Improvements in IBM Quantum Devices**

- IBM-Q devices have improved in gate fidelity and coherence over time
- Decoherence times defined by two constants:
    - **T1:** Time for a qubit to decay from excited to ground state.
    - **T2:** Time associated with phase changes in the qubit state.

Points out performance improvements in IBM quantum devices in terms of fidelity and coherence.

133 **Randomized Benchmarking Protocols for IBM-Q**

- IBM-Q devices, with fewer qubits and simpler connectivity, suit protocols lik SRB and URB.
- SRB: Measures average fidelity of random gate sequences.
- URB: Quantifies coherence in gate errors, more informative for error correctio needs.

Talks about RB protocols suitable for IBM.

134 **Attempts to Implement SRB on Various Platforms**

- SRB and its variants have been adapted to different quantum computin archetypes.
- Some platforms require specific protocol modifications due to hardwar constraints.
- Example: Harper et al. implemented RRB on logical code space, showin reduced infidelity in two-qubit gates.

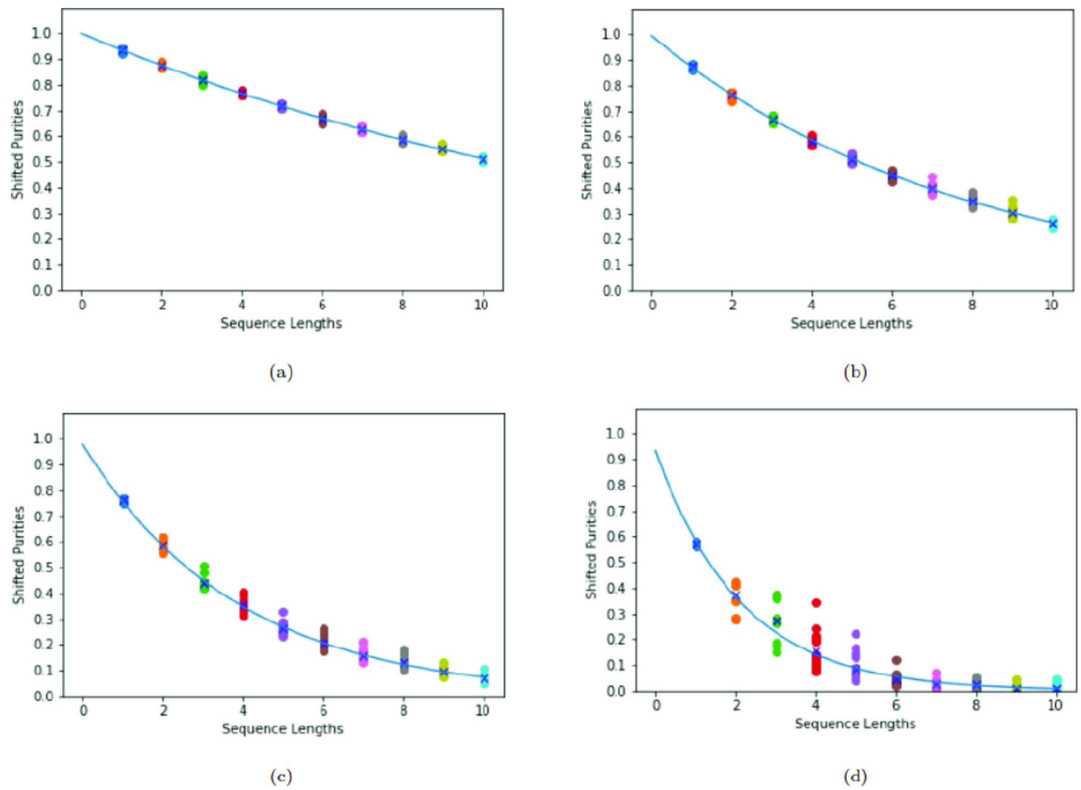
Discusses attempts to implement SRB on various platforms.

135 **Our Contributions**[7]

7. Chandrashekar, A., Das, S., & Paul, G. (2024). Characterization of Noise usin variants of Unitarity Randomized Bench-marking. arXiv preprint arXi :2410.20528.⏎

- We simulated the URB protocol using Qiskit
- URB protocol to prepare the mixed state in IBM qiskit.
    1. Single-qubit depolarising channel
    2. Single-qubit bit-flip channel
- Native gate URB (Ng-URB) in order to study the noise in the native gates.
- Using Ng-URB protocol we detect the presence of cross-talk errors which ar correlated errors caused due to non-local and entangling gates such as CNO gate.

Summarizes our contributions in the domain: URB protocol to prepare the mixed state in IBM qiskit and Native gate URB (Ng-URB) in order to study the noise in the native gates.



URB curves showing the relationship between average shifted purities and sequence lengths (in Clifford unitaries) as a result of simulations of URB single copy protocol for the bit-flip channel with different values for probability p, applied to single qubit register.

Chapter 03

Fighting Noise before Error Correction: Suppression, Mitigation, and Beyond

Ritajit Majumdar

Quantum Algorithm & Engineering Team, IBM Quantum

DOI: 10.1201/9788743807087-4

Noise is the primary hindrance of quantum computers in unleashing its full potential, and attenuating its effect is necessary for useful quantum computation. While we look forward to error correction in upcoming years, several other techniques have been developed to fight with noise in near term. In this presentation, we shall first look into the different sources of noise in current quantum devices, and discuss some error suppression and mitigation techniques for lowering their effects. We shall take a deeper dive into understanding one of the mitigation methods, namely zero noise extrapolation, and discuss the pipeline required for attaining maximal performance out of these methods[1]. We shall touch upon the results obtained by IBM[2] as evidence of utility provided by these methods in nearterm devices. Finally, we shall briefly introduce other techniques, such as operator back propagation, circuit cutting etc., which can also be used in certain scenarios for further reducing the effect of noise on quantum systems. The goal of this presentation is to enable the listeners to use the proper methods to achieve noise mitigated useful results for their research.

1 Majumdar, R., Rivero, P., Metz, F., Hasan, A., & Wang, D. S. (2023, September). Best practices for quantum error mitigation with digital zero-noise extrapolation. In 2023 IEEE International Conference on Quantum Computing and Engineering (QCE) (Vol. 1, pp. 881887). IEEE.⏎

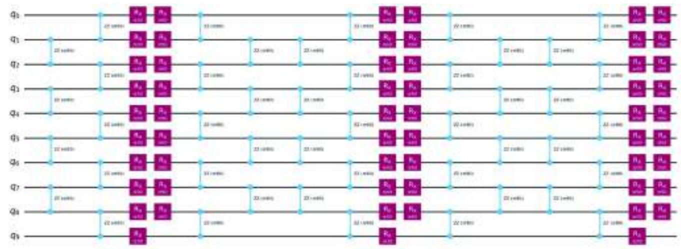
2 Kim, Y., Eddins, A., Anand, S., Wei, K. X., Van Den Berg, E., Rosenblatt, S., ... & Kandala, A. (2023). Evidence for the utility of quantum computing before fault tolerance. Nature, 618(7965), 500–505.⏎

11 Agenda

- Introduction
- Zero Noise Extrapolation
- Twirling
- Measurement error mitigation

Outline of presentation. In this talk we shall cover the various aspects of error mitigation and suppression. Don’t be alarmed by the number of points in the agenda. The goal is not to bombard you with everything. We’ll cover as much as we can, but the primary goal is to ensure that we understand whatever we cover.

12 **What Can We Do with a Quantum Computer?**

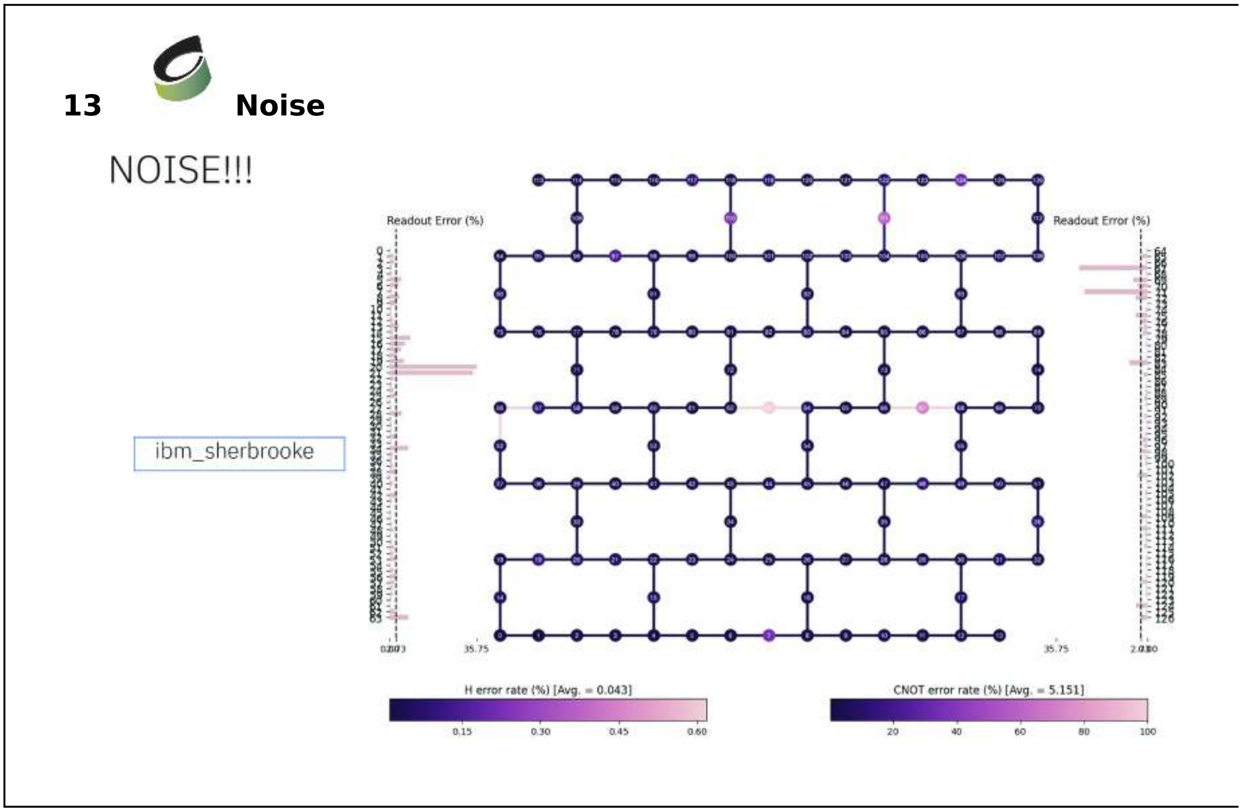


- We can obtain the probability distribution after running the circuit N times (called shots).
- We can calculate the expectation value of some observable O

- Let $|\psi\rangle$ be the outcome of the quantum circuit; we want to calculate $\langle\psi|O|\psi\rangle$.
- Write $O = \sum_P c_P P$ where P is a n-qubit Pauli; calculate $\langle\psi|P|\psi\rangle$ for each P using the quantum computer; classically calculate $\langle\psi|O|\psi\rangle = \sum_P c_P\langle\psi|P|\psi\rangle$.
- Example: $O = 0.1\, XX + 0.2\, IZ\; - 1.3\, IX$ for a 2-qubit circuit. Since $[XX, IX] = 0$, a single circuit is sufficient to calculate $\langle\psi|XX|\psi\rangle$ and $\langle\psi|IX|\psi\rangle$; a second circuit is required to calculate $\langle\psi|IZ|\psi\rangle$.

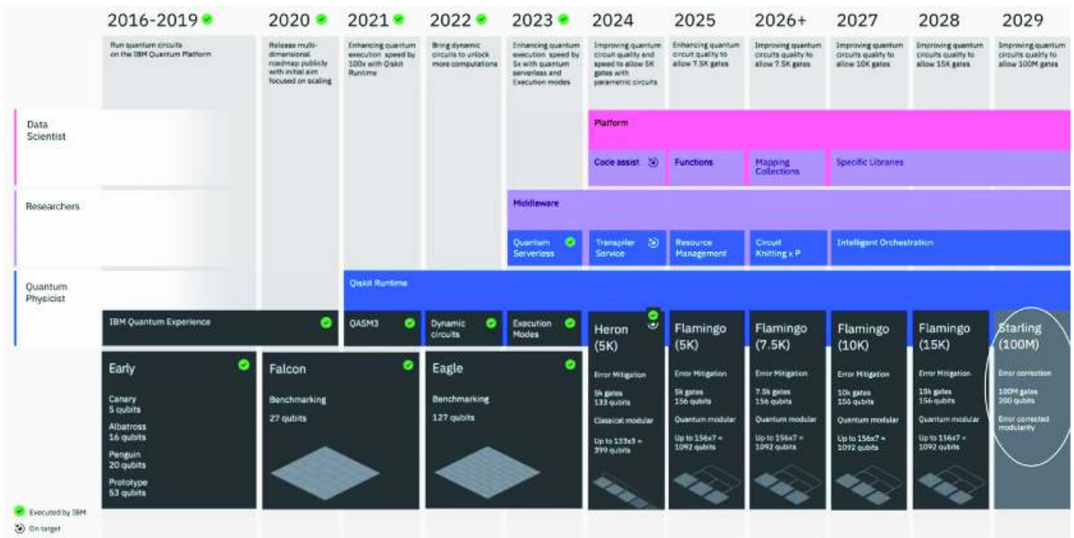
What can we do with a quantum computer? Strictly speaking, (i) we can obtain the probability distribution by running the same circuit multiple times. Since quantum algorithms are probabilistic, it is necessary to run the circuit multiple times to get the distribution. A single execution of the circuit does not provide sufficient information. And (ii) from the distribution we can calculate the expectation value of some observable. Initially it may feel like (ii) is just an extension of (i), which is true. But error mitigation is applicable only when you are interested in some observable value, and not when you need the full distribution. Hence, it is good to have the distinction between these two requirements. Tomorrow in our workshop we shall see that these two requirements correspond to two “primitives” of Qiskit – namely Sampler and Estimator.

How to calculate the expectation value of an observable? First we decompose the observable O in terms of some Pauli operators. Since Pauli operators always span the operator space, we can always do such decomposition. Next, we use the quantum computer to find the expectation value for each Pauli, and classically calculate the linear combination.



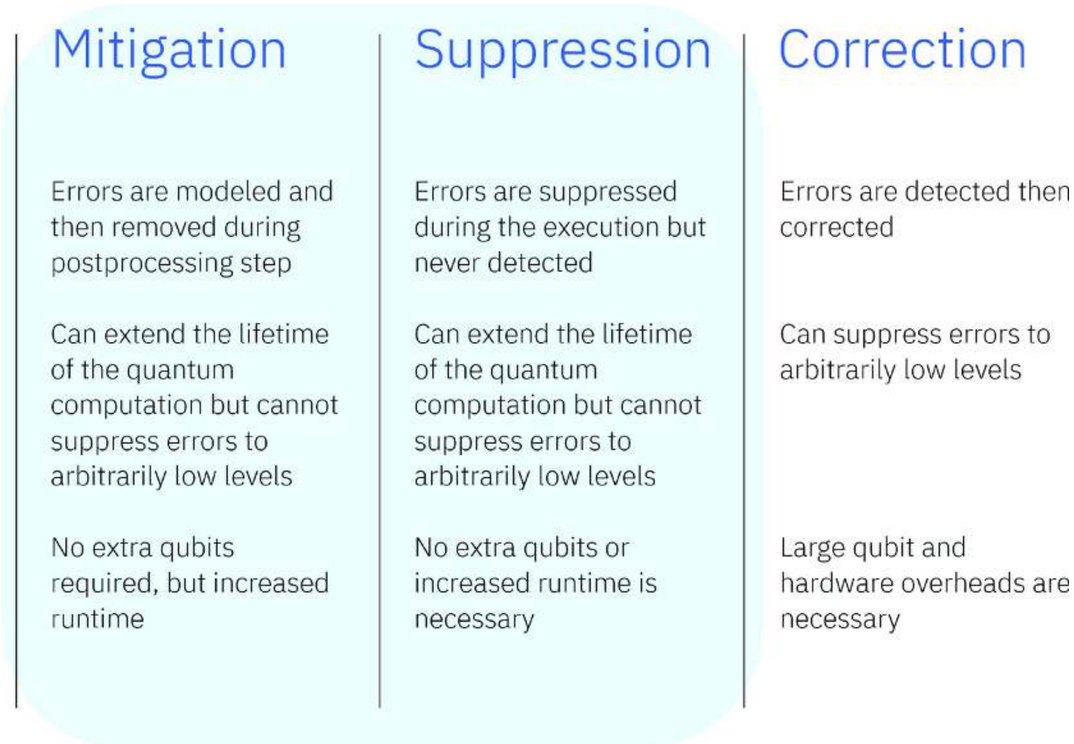
What is the primary hindrance of current quantum computers? My colleague Zlatko Minev ran an online poll, and evidently the answer was Noise! Here we show a 127 qubit IBM Quantum Eagle Processor and the associated noise. Note that the color code gives the strength of noise, lighter the colour, higher the error probability. We see different errors such as errors in 1-qubit gates (H error rate), 2-qubit gates (CNOT error rate), and readout error. We also note that error is not uniform across the topology of the hardware. Please keep this in mind since this will come handy later on.

14 **Development Roadmap**

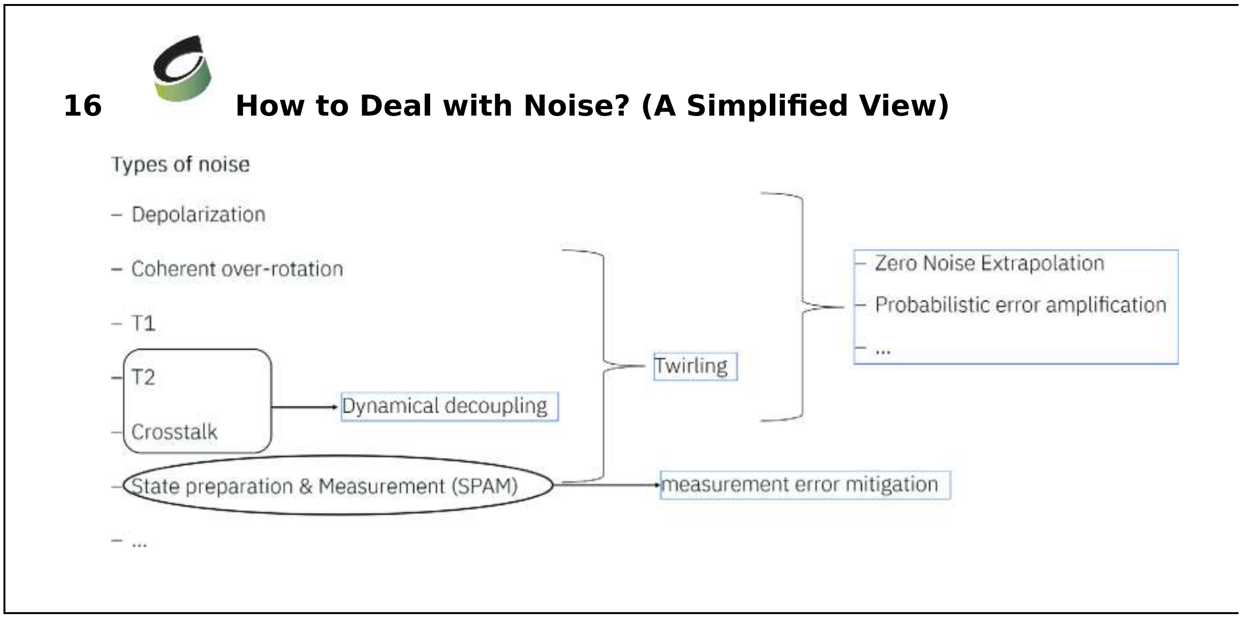


▍If there is noise, there should be some provision to correct it! In the roadmap we see that error correction is still a few years away. How do we deal with noise before that?

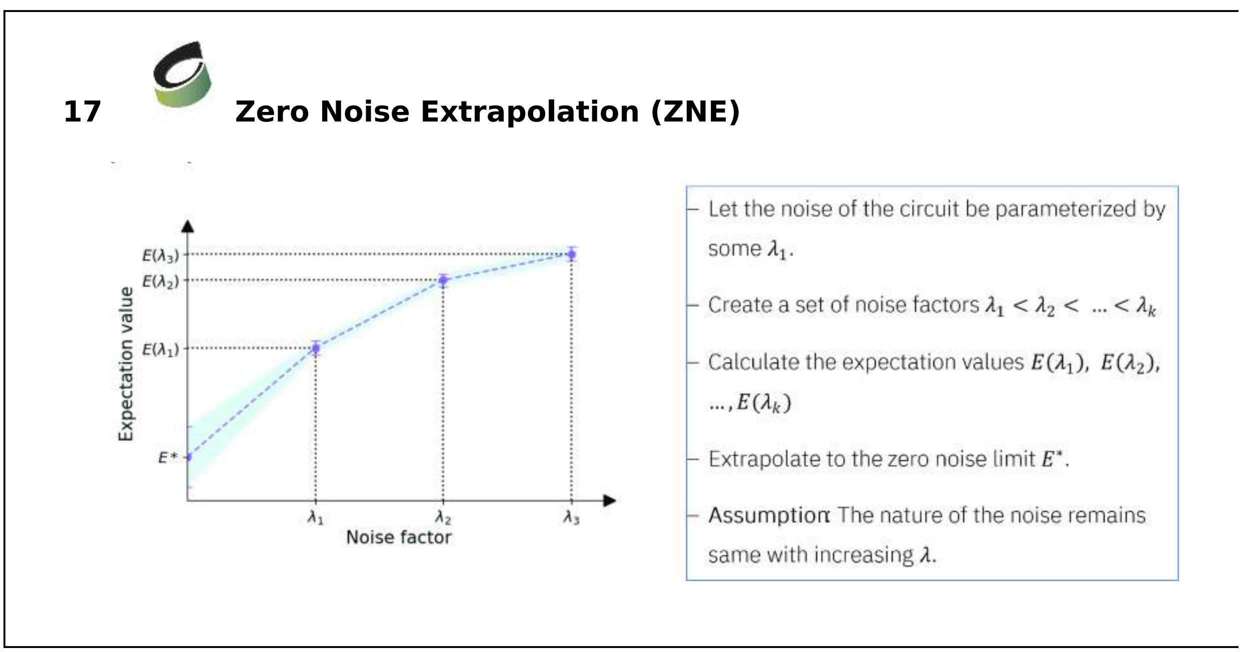
15 Different Ways to Address Quantum Errors



There are two major steps that we take – namely error mitigation and suppression. Mitigation requires some extra hardware execution of noisy circuits followed by some classical postprocessing to obtain an estimate of the noise free expectation value. Error suppression, on the other hand, attempts to minimize some sources of error. We shall see proper example of both later on.

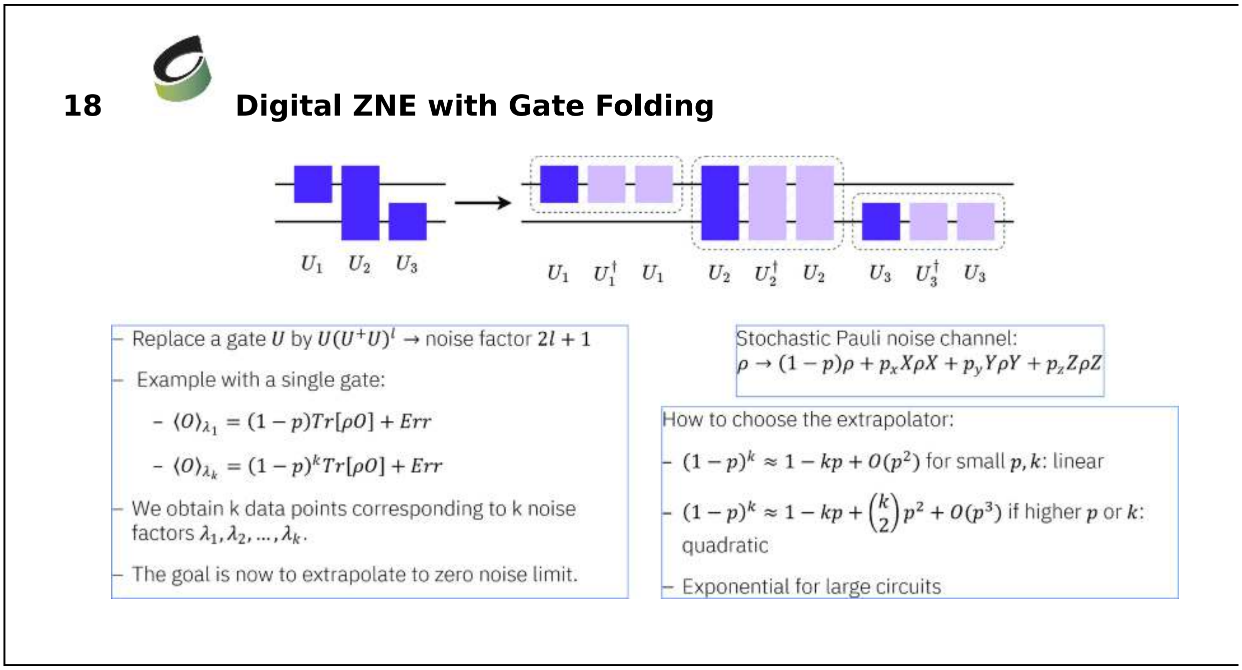


In this slide, we see some of the sources of noise. And gosh there are a lot! If there are different source and types of noise, naturally we need different techniques to deal with them. Here we represent the various techniques to deal with noise. Just looking into this slide may seem overwhelming, but let's attack them one at a time, starting from the outside. We shall start with Zero Noise Extrapolation.



here we summarize the working steps of Zero Noise Extrapolation (ZNE). The first idea is to associate some parameter \lambda with the noise of the circuit. It is usually best not to try to give a physical interpretation of \lambda, because it is more of a mathematical abstraction. The original circuit has some noise associated with it. Let us simply characterize that noise by this \lambda. We neither need to have a physical interpretation, nor an absolute value of \lambda. A relative ordering is sufficient. We simply say \lambda for the original circuit is 1. Now, we design functionally equivalent circuits, but with higher values of \lambda. How? We shall see that in the next slide. We calculate the expectation value of the observable for each \lambda. Now, we can think of this as a dataset where we have \lambda in the x-axis, and the expectation values in the y-axis. Recall that the minimum value of \lambda was 1, which was associated with the original circuit. From these datapoints, we want to fit a curve and estimate the expectation value at \lambda = 0.

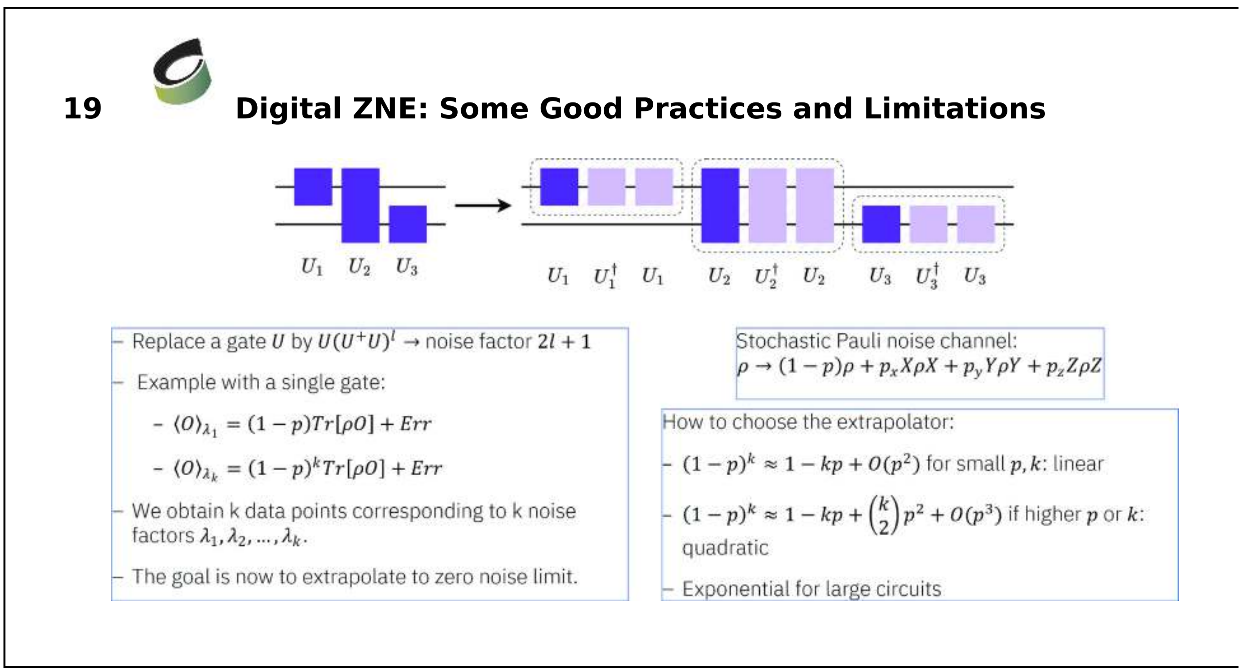
In terms of our definition, \lambda = 0 implies a circuit with no associated noise. We cannot attain it in realty, but we can estimate what the expectation value would look if we were able to attain such a scenario.



How do we create circuits with different noise parameter \lambda? One method is to stretch the pulse corresponding to each gate. This increases the noise associated with the pulse. This is called analog folding. However, this is a bit involved given that the user needs precise understanding of the nature of pulse associated with a gate. A simpler method is to fold gates. Here, we replace a gate U with U(U+U)^l. Since a quantum gate U is always unitary, U+U = I. Therefore, irrespective of the value of l, the circuit remains functionally equivalent. We can say \lambda = 2l+1, which gives a relative ordering of the noise in the system. Now when we calculate the expectation value with respect to some \lambda, we see that there is a noise-free part and some error part of the expectation value. It can be shown that extrapolation removes majority of the error part, keeping some higher order (in p, the probability) terms.

How do we choose which extrapolator to use? For depolarizing or stochastic pauli noise channel, the probability of no error is roughly (1-p)^k, where p is the probability of error in each gate, and k is the number of gates. This is an exponential series. However, when both p & k are small, the exponential series can be nicely approximated by a linear one, with the higher order terms leading to low bias. Similarly, for slightly higher p and/or k, we can approximate the exponential series by a polynomial degree-2 or higher series. This approximation curve is the extrapolator which we should use.

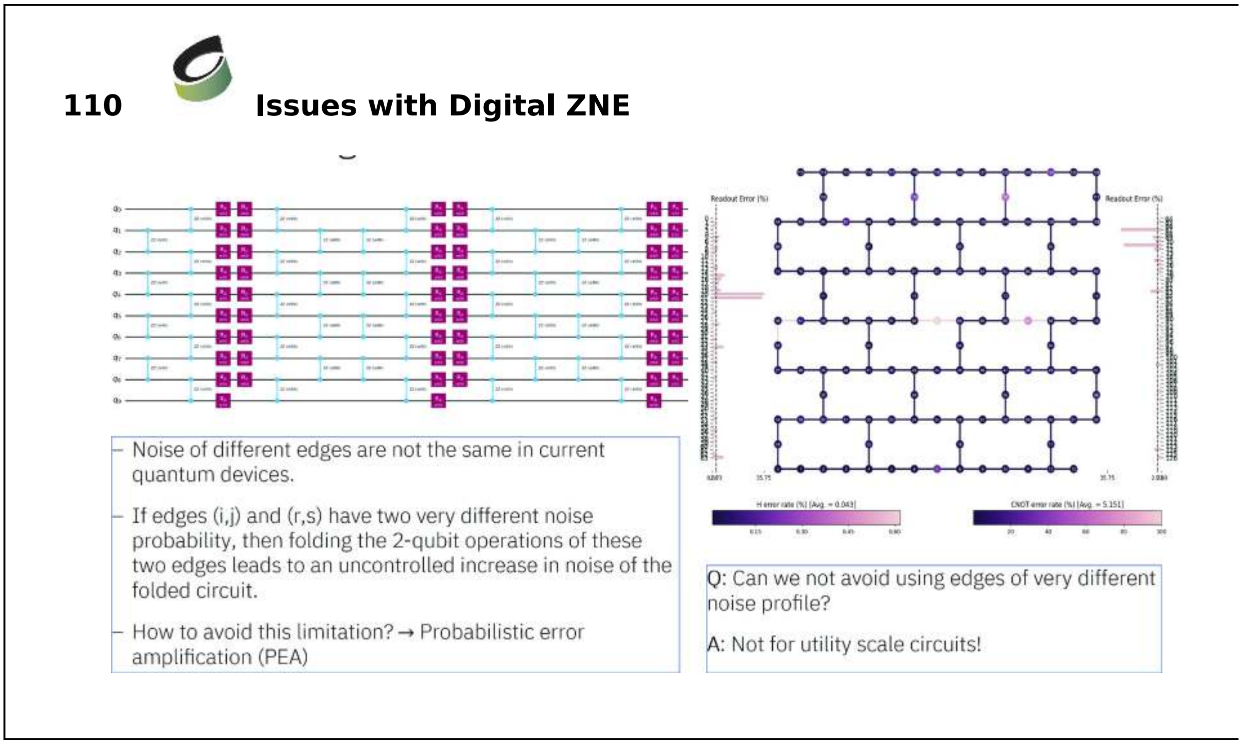
Why not always use exponential? Because in practice, exponential extrapolator often show low stability and a high standard deviation. So it is a good practice to use other extrapolators whenever feasible.



In practice, often only 2-qubit gates are folded since they are way more noisy that single qubit gates.

A general issue with folding is that it increases the depth of the circuit. If we end up with a depth 100 circuit due to folding, how to determine whether there is any viable signal from this circuit, and hence the extrapolation is faithful, or whether it is simply a garbage output, and hence the extrapolation makes no sense? Therefore, we often use partial folding when going to bigger circuits. Here, we randomly select some gates in the circuit and fold them (usually as \lambda = 3). This restricts the increase in the depth of the circuit, making ZNE useful for deeper circuits as well.

But note where ZNE fails irrespective of the size of the circuit and the nature of folding. If we assume depolarizing noise, then as the probability of error increases (due to folding), the expectation value of Z-type observables converge to 0. But if the expectation value of the original circuit itself is 0, or very close to 0, then folding does not change the value much. Hence the extrapolator is misguided, and it fails to provide a proper estimate of the noise-less scenario.



Digital ZNE assumes that all the edges of the hardware topology (i.e., the 2-qubit gates) have similar probability of error. But we see it is not true in current devices. So, when we fold, the different edges lead to different enhancement of noise. This is particularly a bane in partial folding where the random choice of gate can lead to significantly different expectation values for the folded circuits. This can be overcome by a version of ZNE called Probabilistic error amplification (PEA) where we learn the noise in the circuit, and repeat the learned noise per 2-qubit layer.

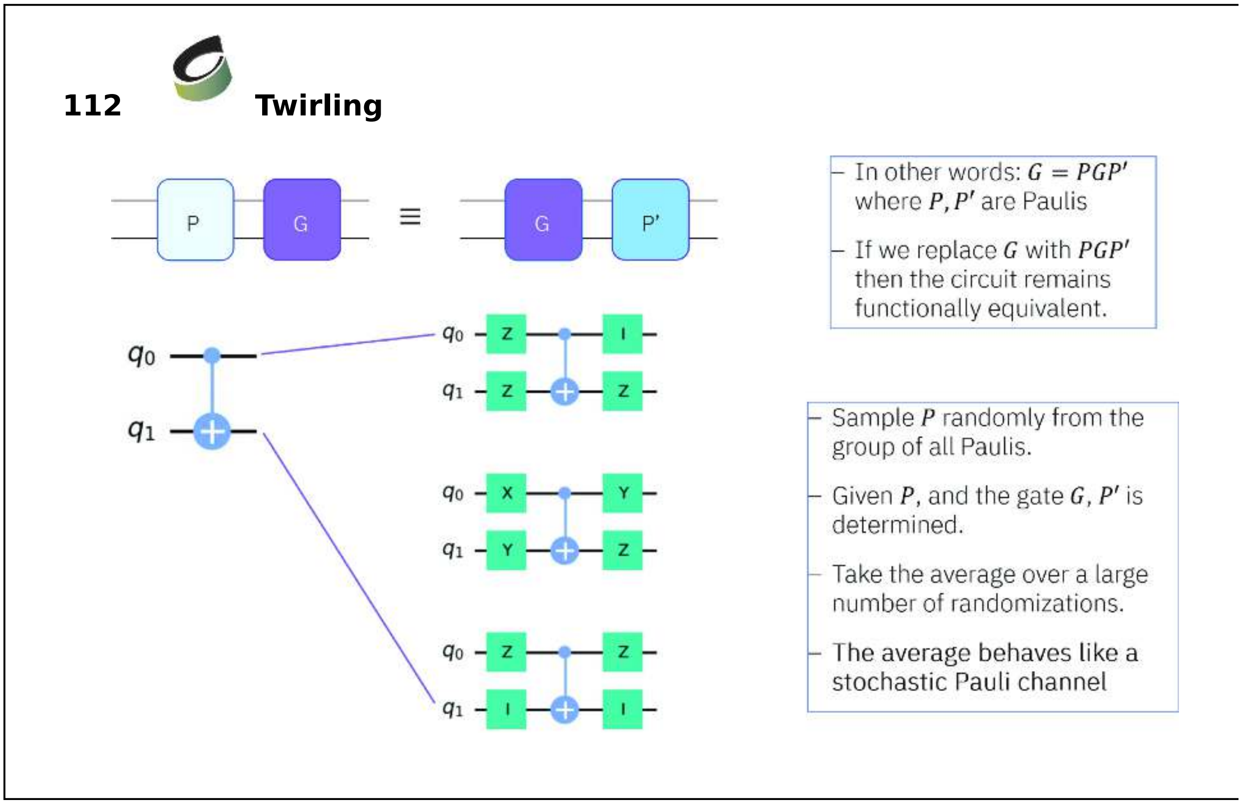
111 **But Not All Noise Models Have the Same Nature!**

- Recall one assumption for ZNEThe nature of the noise remains same wit increasing λ.
- Depolarization/Pauli noise channel, T1, T2 etc. all of them have an exponentia decay nature.
- An example of a noise model which doesn't → coherent over-rotation

A method that introduces independent, random, single-qubit gates into the logica circuit such that the effective logical circuit remains unchanged but the noise i tailored into stochastic Pauli errors (Wallman 2015)

Recall that the assumption we have been using without explicitly stating till now is that the noise model is either depolarizing or stochastic Pauli. However, this is not true. There are several other noise types that affect the quantum circuit. Some of them, such as T1, T2 noise still have an exponential decay model. So we can still consider a single exponential function with these noise models added in. But, for example, coherent over rotation doesn't. Quantum gates are unitary, and any gate corresponds to a rotation by some angle, say \theta. But due to engineering limitations, often we end up implementing \theta+\delta \theta. If you imagine now, this means initially you are diverging from the ideal operation. But if this operation is repeated many times, it starts to converge, because the operation is essentially a rotation. So the nature of this noise is a sinusoidal curve. This doesn't fit in with the exponential decay model.

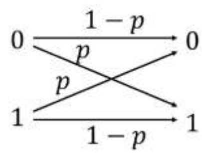
A method that is widely used is called twirling, where random gates are appended in the circuit, without changing the functionality. But when we average over many such random instances, the average noise model becomes a stochastic Pauli one.



Let G be a Clifford gate. Then we know that it maps a Pauli to another Pauli. So, PG = GP' where both P and P' are Paulis. In other words, we can write G = PGP'. So if we replace G by PGP', then the circuit doesn't change functionally. The idea of twirling is that we randomly sample P from the group of all Paulis. For each P and G, we determine P', and replace G by PGP'. We repeat this multiple times, and work with the average of these where the noise model behaves like a stochastic Pauli one. We won't go into the proof of this here.

113 **Measurement Asymmetry**

Binary symmetric channel



T_1 error

- Prepare a qubit in $|1\rangle$
- Leave it idle for time t, then measure
- $P(|1\rangle) = \exp(-\frac{t}{T_1})$

127 qubit Eagle processor

Operation	Time (in ns)
1 qubit gate	~60
2 qubit gate	~533
measurement	~1200

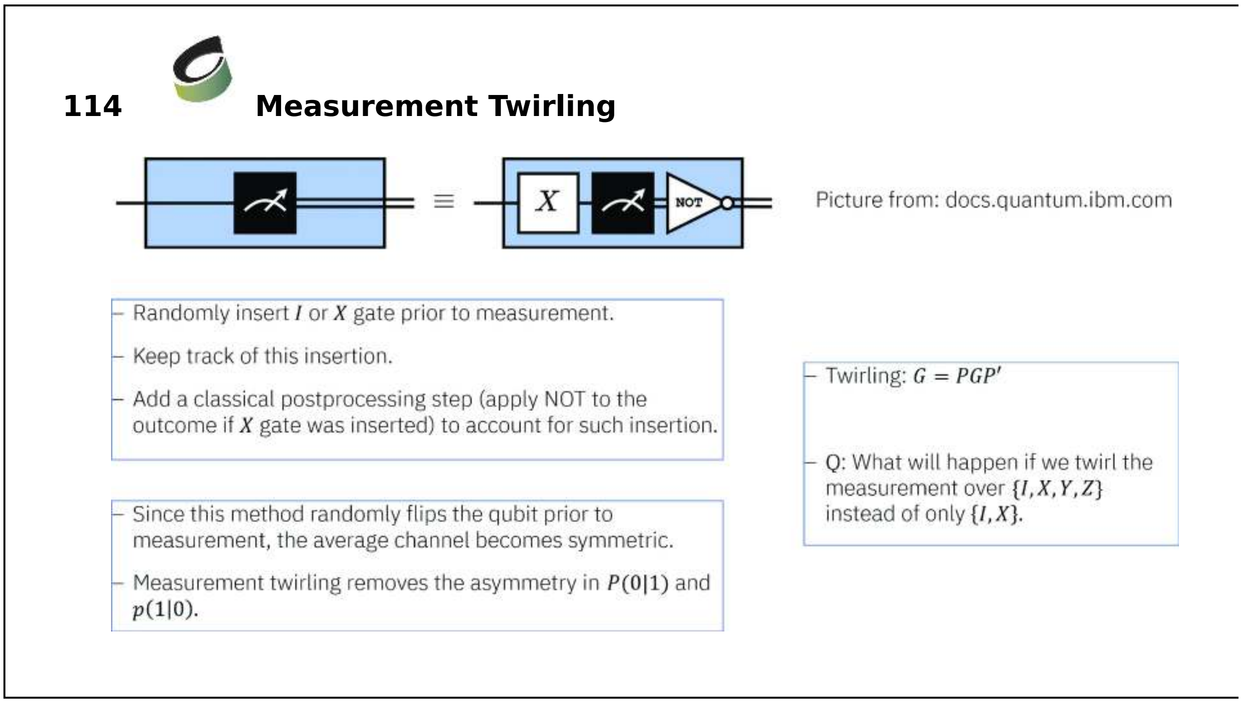
Average $T_1 = 267\mu s$; $\exp\left(-\frac{1200}{267000}\right) \approx 0.99$

- $0 \rightarrow 1$ affected only by measurement noise
- $1 \rightarrow 0$ affected by both measurement noise and decay due to T_1 error
- In general, $p(1|0) < p(0|1)$

Measurement Error Mitigation.

What we discussed previously can be termed as gate twirling. There is also a concept of measurement twirling. But before going into that, let us discuss a bit on measurement error. Measurement error can be thought of as a classical error, where if the ideal outcome is 0, with some probability we write 1, and vice versa. These probabilities are in general equal, and this model is called a binary symmetric channel. But quantum systems are also affected by T1 noise. If a qubit is prepared in the state 1 and kept idle for some time, it has a spontaneous decay towards 0. If we measure the qubit after time t, then we shall get a non-zero probability of obtaining 0 after measurement, and this probability increases exponentially with t.

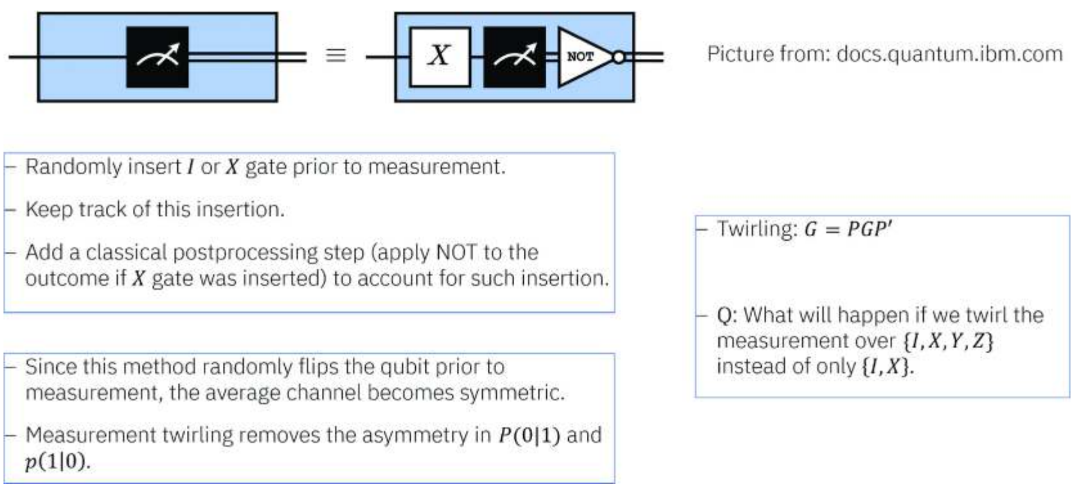
In current quantum devices, we notice that the measurement has a significantly higher time than 1- qubit or even 2-qubit gates. Naturally, T1 error is not negligible during measurement. In fact, putting in the current values, we see that we get an average probability of error in measurement due to T1 as 0.01. So, in addition to the binary symmetric nature of measurement error, p(0|1) is also influenced by T1. So essentially we end up with an asymmetric channel where p(0|1) > p(1|0).



Measurement twirling allows us to convert the asymmetric measurement channel into an average symmetric one. In measurement twirling we randomly insert X gates before measurement and keep track of it. Whenever, we have inserted an X gate, we account for it by reversing the obtained outcome by a NOT gate. See that this is like twirling. In gate twirling, we were replacing a gate G by PGP’. Here we are replacing a measurement M by XM(NOT), where the NOT is performed classically. Since this method randomly flips qubits prior to measurement, the effect of T1 error, which made the channel asymmetric, can be accounted for. Say if a qubit was to measure 1, and it would suffer from T1 noise. But roughly half of the time you are flipping the qubit, making it 0 and hence it is not susceptible to T1 noise anymore. So, when you take average, the overall asymmetry disappears. The average channel becomes symmetric.

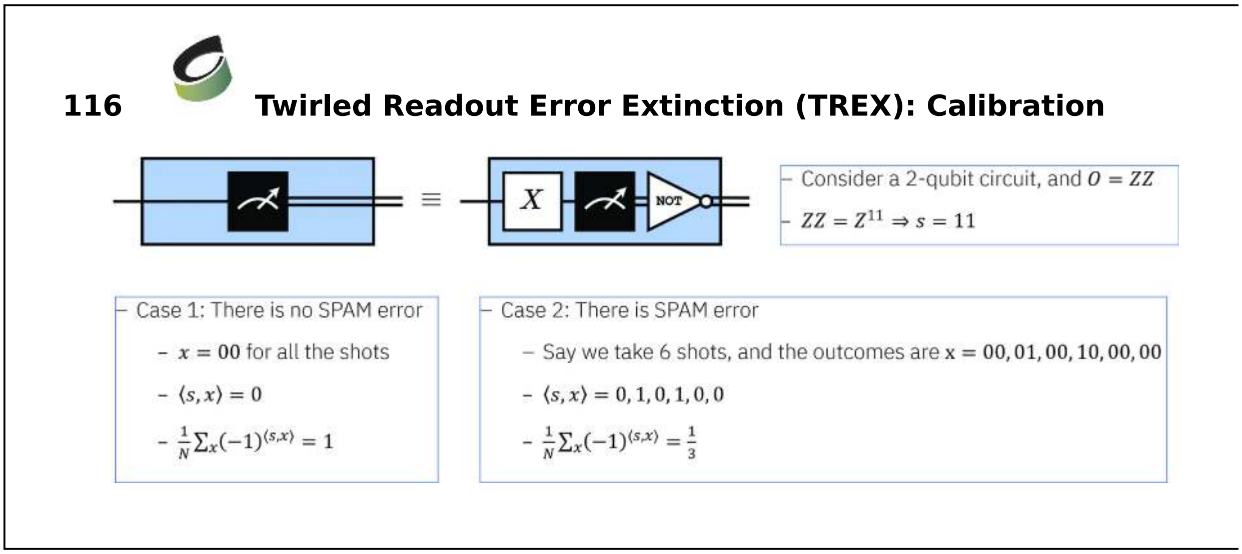
115 Twirled Readout Error Extinction (TREX)[1]

1 Van Den Berg, E., Minev, Z. K., & Temme, K. (2022). Model-free readout-erro mitigation for quantum expectation values. Physical Review A, 105(3), 032620.⏎

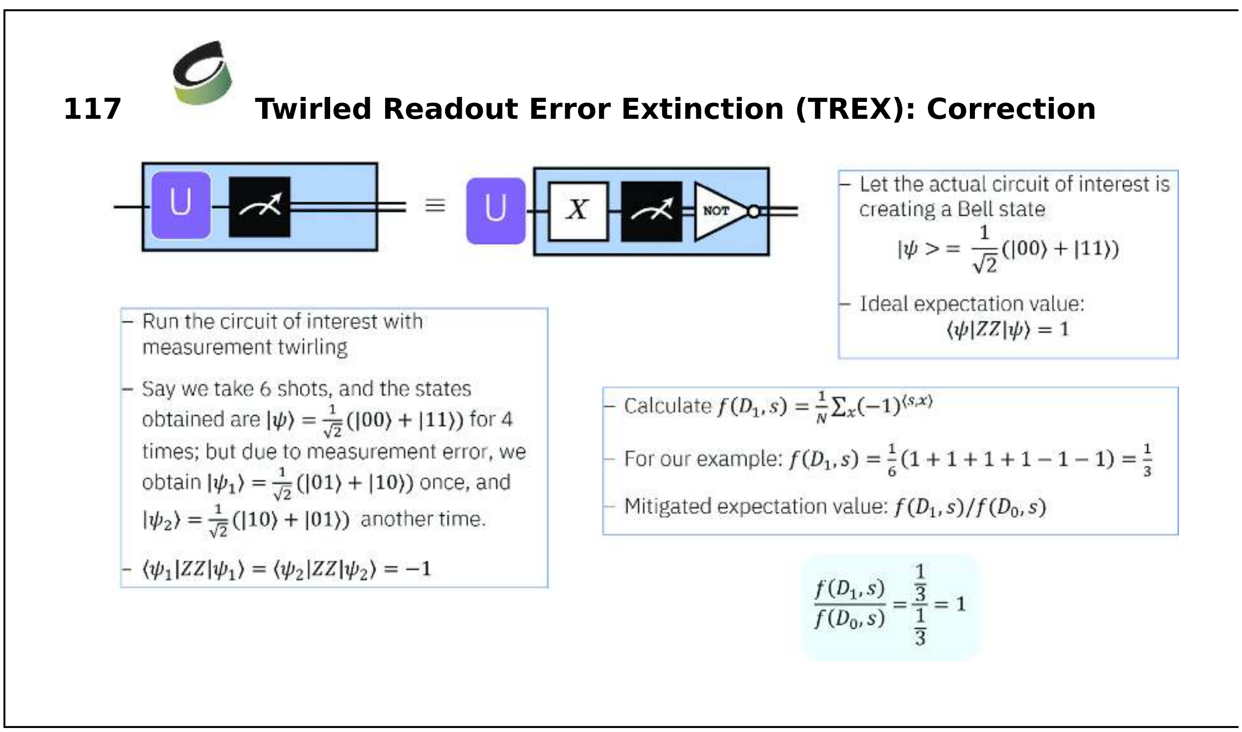


Twirling is an essential part for measurement error mitigation using the protocol TREX. Once more, we won't discuss the derivations of TREX, but will show an example to see how TREX works. TREX has a calibration step followed by a correction step. Let O be the observable that we want to measure. In the calibration step, we construct an empty n-qubit circuit initialized in |0>, perform measurement twirling, and calculate the function f.

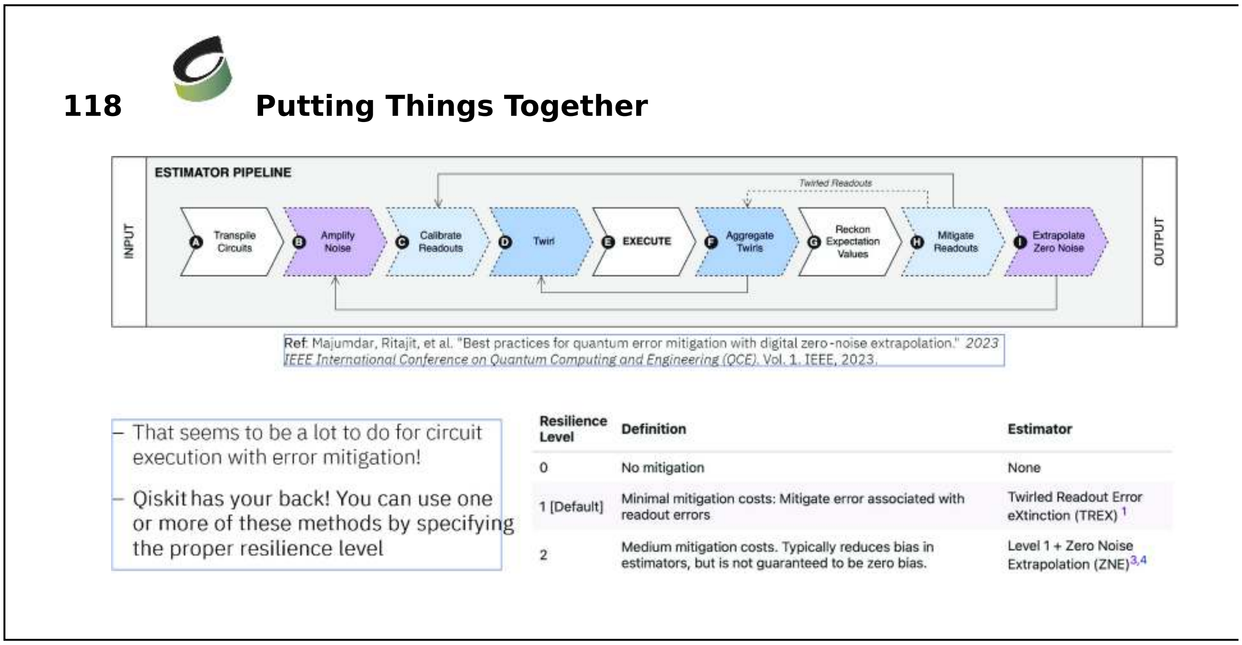
We consider an example of calibration here for a 2-qubit circuit where the observable O is ZZ.



We first show that f should always be 1 if there is no measurement error. We take an example of 6 shots, and show the calculation of f in the presence of measurement noise. Here f is non-zero.



Now we shall repeat the same experiment, but with the circuit of interest, i.e., we use measurement twirling over the circuit, say U, and calculate the same f function for it. We show an example here with a Bell state. The mitigated expectation value is the ratio of this f vs the f calculated from calibration.



Here we show the overall pipeline that we discussed for error mitigation. It is natural to feel like that is a lot of things to keep in mind when running quantum circuits. But qiskit makes it super easy. We can simply define a resilience level, and qiskit will take care of the error mitigation pipeline accordingly.

Chapter 04

Quantum and Classical Hybrid Algorithms and the Road Towards QCSC

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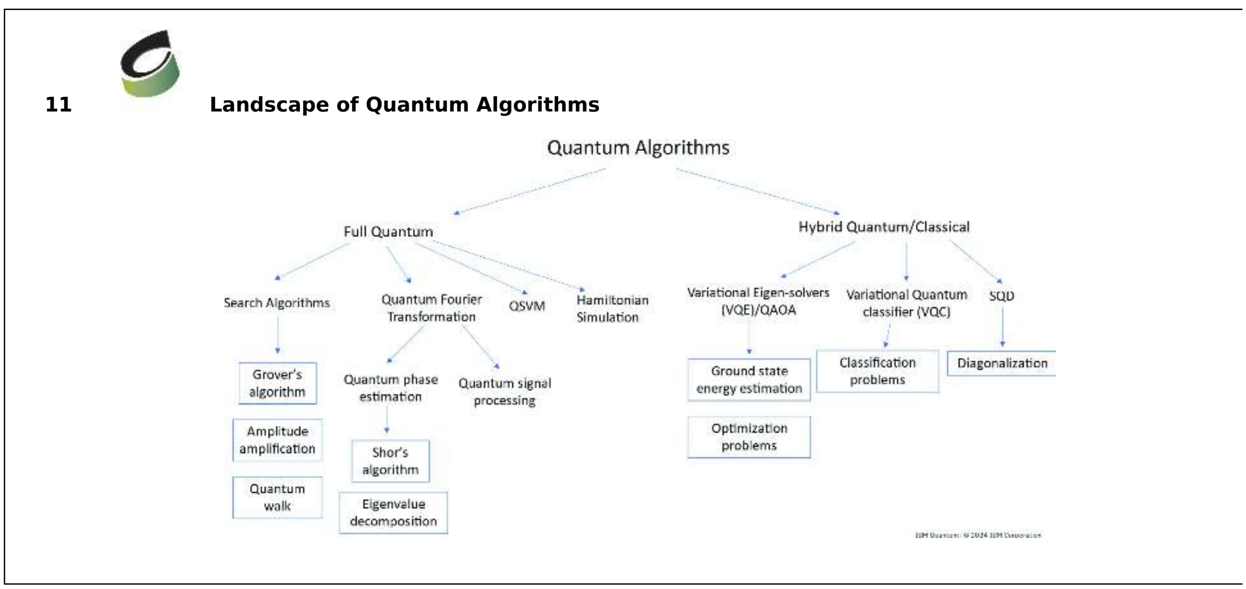
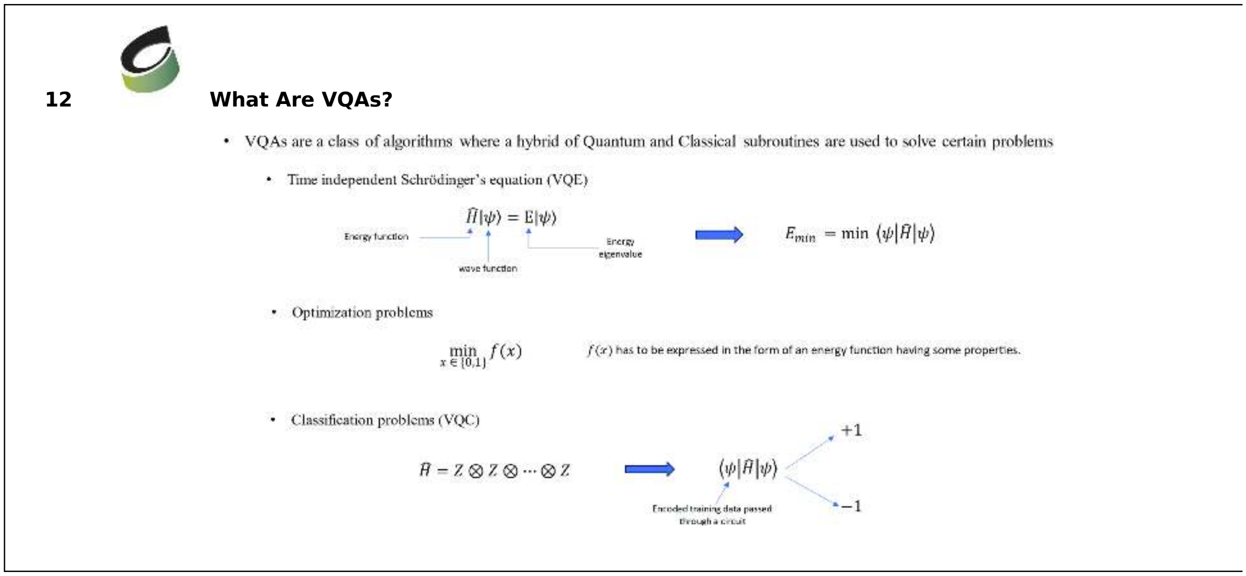
DOI: 10.1201/9788743807087-5

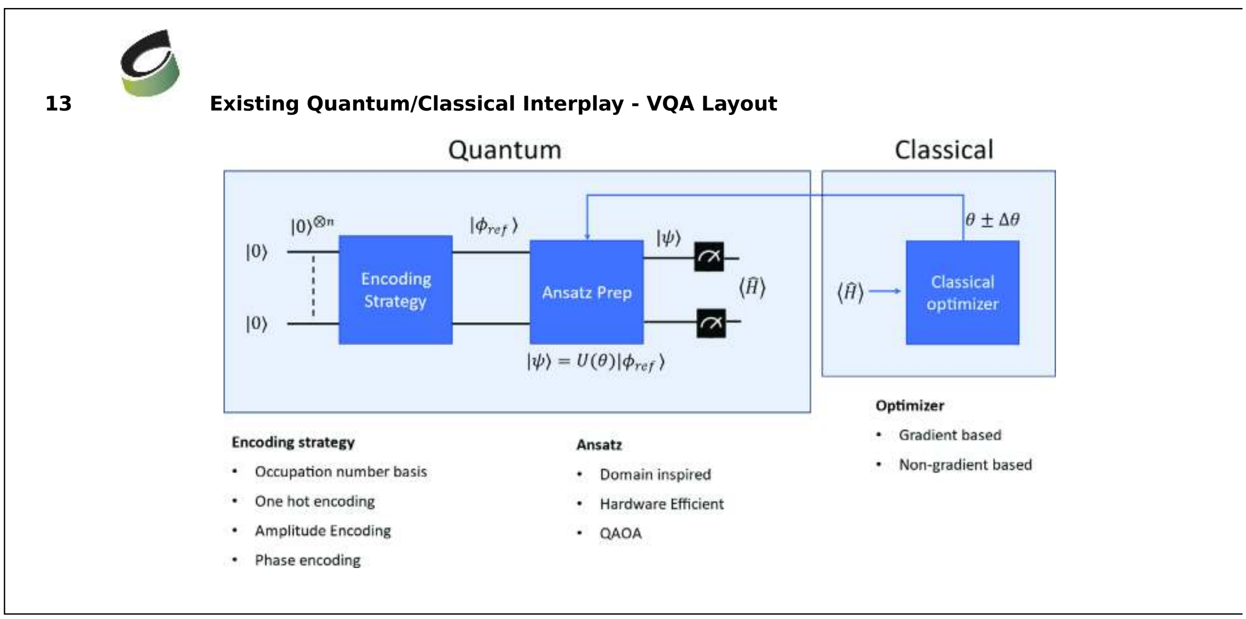
In this presentation we will first talk about the different classes of quantum algorithms currently in use. We will then discuss some hybrid (Quantum + Classical) algorithms in use today, notably the variational quantum algorithms (VQAs). The VQAs could be used in a wide range of applications. We will then discuss the limitations of these algorithms and the need for other approaches. Quantum centric supercomputing (QCSC) is an idea where Quantum algorithms in Quantum hardware work in tandem with classical algorithms in high-performance computers. We will discuss the different varieties of Quantum + HPC workloads. Finally, we will discuss a use-case where Quantum + HPC has been used to solve a chemistry problem. We will end the presentation showing IBM’s roadmap in achieving QCSC.



There are primarily two category of algorithms currently in use. The first category is of the type, we can refer to as “Full Quantum” algorithms. In this category the algorithm is executed end- to-end in a Quantum computer. The second category of algorithms fall in hybrid segment. Given the limitations of contemporary Quantum hardware, hybrid algorithms have become very popular for solving a large category of problems.



One kind of hybrid algorithms are known as variational quantum algorithms or VQAs in short. They essentially solve the time independent Schrödinger equation and tries to find out the max/min eigenvalue of a given energy function. The basic requirement is thus having a good energy function, also known as the Hamiltonian. Every application will have its own Hamiltonian. Hamiltonians need to satisfy certain matrix properties.



VQA layout consists of a Quantum part and a classical part as shown in the figure. The **A** VQA Quantum part consists of a parameterized quantum circuit that generates a bitstring. The quantum circuit is run several times and the generated bitstrings are used to calculate the expectation value of the Hamiltonian. The expectation values are provided to the classical part, which consists of an optimizer. The optimizer based on the input expectation value, updates the circuit parameters. The quantum part uses this information from the classical side to update the circuit and runs the circuit again. This cycle continues till the expectation value converges to a point. This converged value should ideally be very close to the minimum eigenvalue.

14 **Variational Principle**

We have seen

$$\langle\psi|\widehat{H}|\psi\rangle=\sum_i \lambda_i P\left(\lambda_i\right) \quad \text{for any eigenvalue} \lambda_i \quad \longrightarrow \quad \lambda_{\min} \leq \lambda_i \leq \lambda_{\max}$$

Now

$$\begin{aligned} & \sum_i \lambda_{\min} P\left(\lambda_i\right) \leq \sum_i \lambda_i P\left(\lambda_i\right) \leq \sum_i \lambda_{\max} P\left(\lambda_i\right) \\ & \lambda_{\min} \sum_i P\left(\lambda_i\right) \leq \sum_i \lambda_i P\left(\lambda_i\right) \leq \lambda_{\max} \sum_i P\left(\lambda_i\right) \quad \Longrightarrow \quad \lambda_{\min} \leq \sum_i \lambda_i P\left(\lambda_i\right) \leq \lambda_{\max} \end{aligned}$$

For any given parameterized ansatz

$$\lambda_{\min} \leq\langle\psi(\theta)|\widehat{H}| \psi(\theta)\rangle \leq \lambda_{\max}$$

The variational principle tells us that given any state, the expectation value of the Hamiltonian with respect to this state will always be bounded by the maximum and minimum eigenvalues. The output of our VQA circuit will also be bounded by the extreme eigenvalues

15 **VQA Scaling Issues : An Example**

- Finding the ground state of molecules - Fe_4S_4

Workload and estimated time for execution

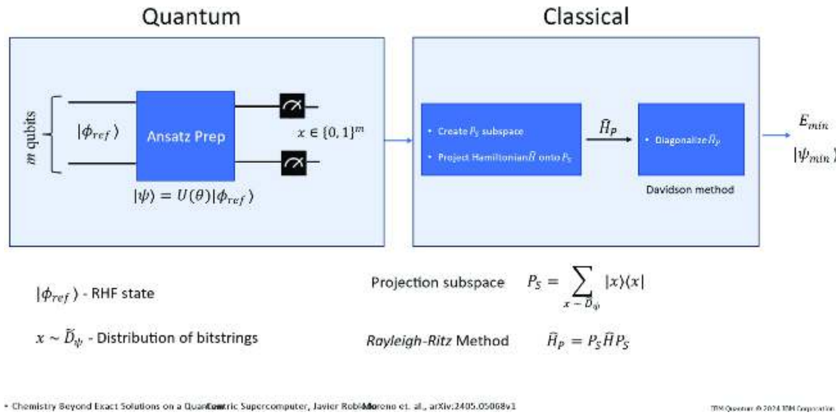
- 77 qubits with a realistic basis set
- 67 L (6.7 M) Pauli string operators
- 10^{-10} precision on each operator for milliHartree precision
- For VQE kind of algorithms, shots $= O\left(\frac{1}{\epsilon^2}\right) \approx 10^{20}$
- Runtime of approximately 10 μs/circuit ~ 30 L (3 M) years
    - Chemistry Beyond Exact Solutions on a Quantum Centric Supercomputer, Javier Roblend toreno et. al arXiv:2405.05068vl
      UM Quantum © 2024 IBM Corporation

With QCSC

- With subspace estimation at 10 μs/circuit 3.5K two-qubit gates, 10k gates overall ~ 2 hours

While VQAs may work well with smaller problems having small sized Hamiltonians, they may not work that well when the Hamiltonians have a large number of terms. It may take a very long time to arrive at a result with reasonable accuracy. However, by using techniques involving Quantum centric supercomputing this process can be speeded up.

16 **Hybrid Quantum/Classical - SQD**



This figure gives the layout of another hybrid approach, called the sample-based quantum diagonalization. Here the classical part churns out bitstrings. The bitstrings are used to created subspaces. The Hamiltonian is then projected into these subspaces and then diagonalized. Because of this projection, the dimension of the Hamiltonian reduces from a large space to a lower manageable subspace.

17 **Rayleigh-Ritz Variational Principle**

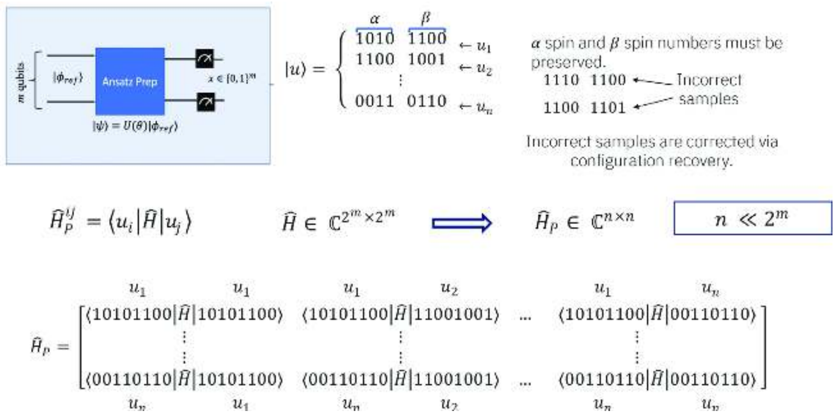
$$\min \langle\psi|\hat{H}|\psi\rangle \quad \text{s.t.} \langle\psi \mid \psi\rangle = 1 \quad |\psi\rangle = \sum_i \theta_i u_i$$

$$L = \langle\psi|\widehat{H}|\psi\rangle - \lambda(\langle\psi \mid \psi\rangle - 1) \Rightarrow L = \sum_{i,j} \theta_i^* \theta_j \langle u_i|\widehat{H}|u_j\rangle - \lambda\left(\sum_{i,j} \theta_i^* \theta_j \langle u_i \mid u_j\rangle - 1\right)$$

$$\begin{aligned} \nabla_{\theta^*}\mathscr{L} &= 0 \\ \nabla_{\lambda^*}\mathscr{L} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\widehat{H}_P \theta = \lambda S \theta \\ &\widehat{H}_P^{ij} = \left\langle u_i \middle| \widehat{H} \middle| u_j \right\rangle \\ &S^{ij} = \langle u_i \mid u_j\rangle \\ &\theta^\dagger S \theta = 1 \end{aligned} \qquad \begin{aligned} &\text{if } \langle u_i \mid u_j \\ &\widehat{H}_P \theta^* = \lambda \\ &\theta^\dagger \theta = 1 \end{aligned}$$

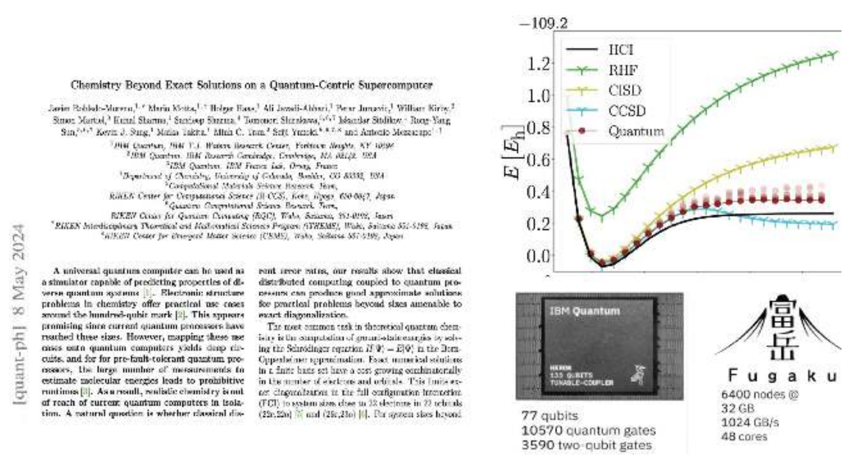
The Rayleigh-Ritz variational method gives us a way to approximate the eigenvalues of a Hamiltonian. The eigenstate is expressed as a linear combination of the bitstrings. At the optimal point, the coefficients turn out to be the components of the eigenvector corresponding to the lowest eigen-value.

18 **Sampling Based Quantum/Classical Diagonalization**



The bitstrings from the output of the quantum measurement process are used to create the subspace and project the Hamiltonian as shown in the matrix above. As shown above, the dimension of the projected subspace n could be way lesser than the dimension of the original Hilbert space, given by $2^m \times 2^m$. However, the samples could turn out to be inconsistent and may require recovery. The recovery process in this problem requires that the number of α electrons and the number of β electrons are conserved.

19 **Chemistry on QCSC**



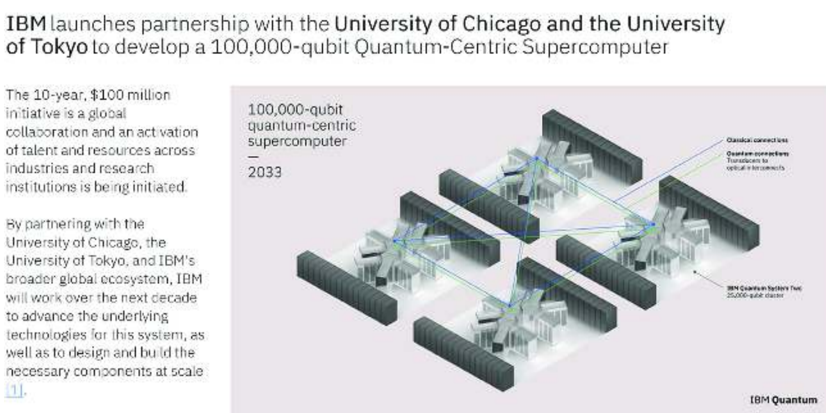
The paper referenced above has used this principle to find the ground state of large molecules, normally considered out of reach for present day Quantum hardware. The authors used IBM Heron QPUs from the Quantum side and the Fugaku supercomputer from the classical side.

110 **HPC + Quantum Workloads**

- HPC for Quantum
  - Tight integration of Quantum resources in reai-time
  - Within the coherence time of the Quantum system
  - On premises integration
- Quantum about HPC
  - Quantum requires HPC for pre- or post-processing tasks
  - Quantum tasks are independent of HPC tasks and HPC is used for support
  - Cloud based Quantum system
- Quantum in HPC
  - Quantum resources for acceleration
  - Quantum tasks tightly coupled with HPC tasks
  - On premises integration
- Yuri Alexeev, et. al., Quantumentric Supercomputing for Materials Science: A Perspective on Challenges and Futur DiradS»n2312.09733vl

HPC and Quantum could work together in the ways shown here. While HPC for Quantum and Quantum for HPC require tight integration between the classical and quantum resources, Quantum about HPC could work over cloud.

111 **100,000-qubit Quantum-Centric Supercomputer**



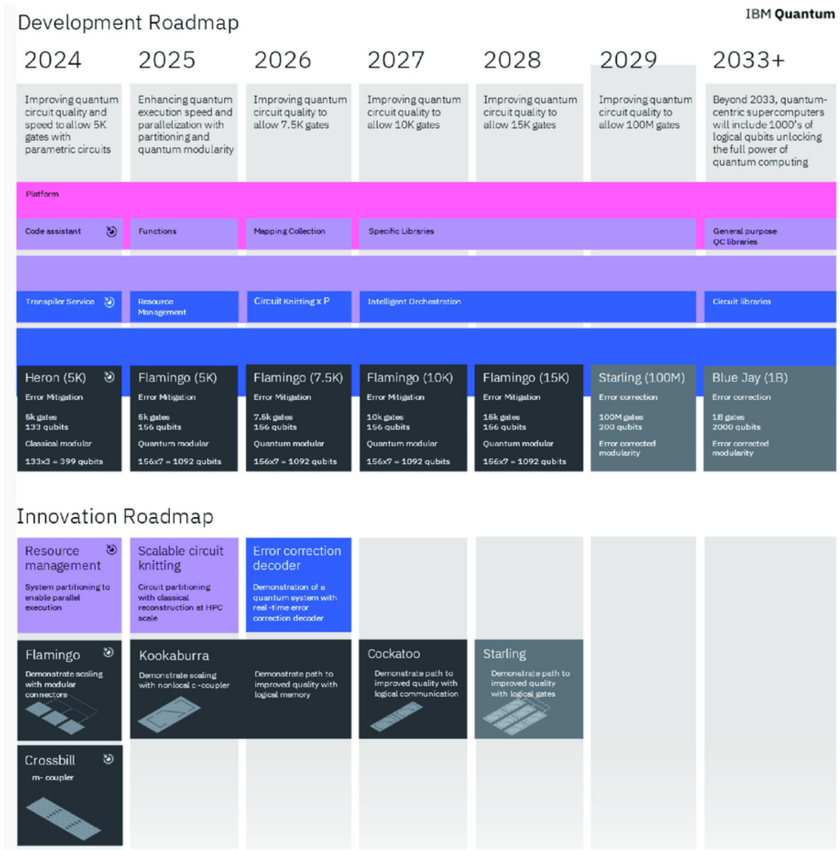
Press release: https://newsroom.ibm.com/2023-05-21-IBM-Launches-100-Million-Partnership-with-Global-Universities-to-Develop-Novel-Technologies-Towards-a-100,000-Qubit-Quantum-Centric-Supercomputer

112

Development Roadmap

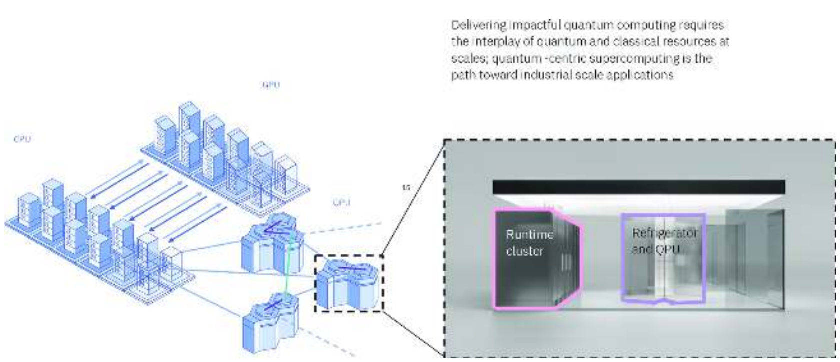
IBM’s Roadmap is given, starting from 2016 to 2033+. IBM aims to have Quantum centric supercomputing, that has 1000’s of logical qubits working in tandem with HPC resources.

113 **Let's Create the Future of Quantum-Centric Supercomputing**



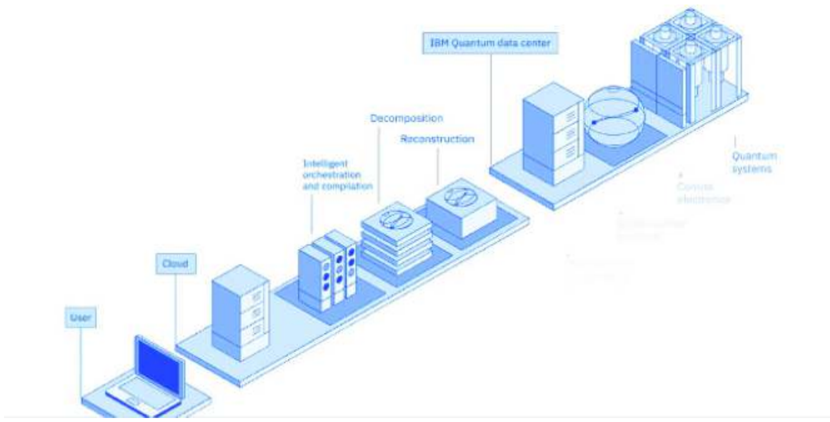
Before we can reach the full potential of QCSC, we need to innovate to have working, error-corrected qubits. This will allow us to run circuits that have gates in the millions and possibly a billion by 2033.

114 **Quantum-Centric Supercomputing**



We foresee datacenters where CPUs, GPUs and QPUs, all working together in a datacenter. The Quantum side of the datacenter will have both the dilution refrigerators housing the QPUs and some classical resources housing the runtime clusters.

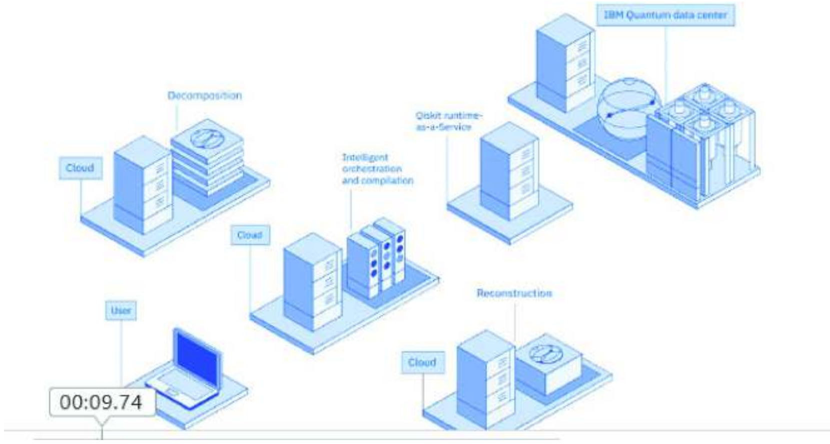
115 **An Example of How a Datacenter Might Work (1)**



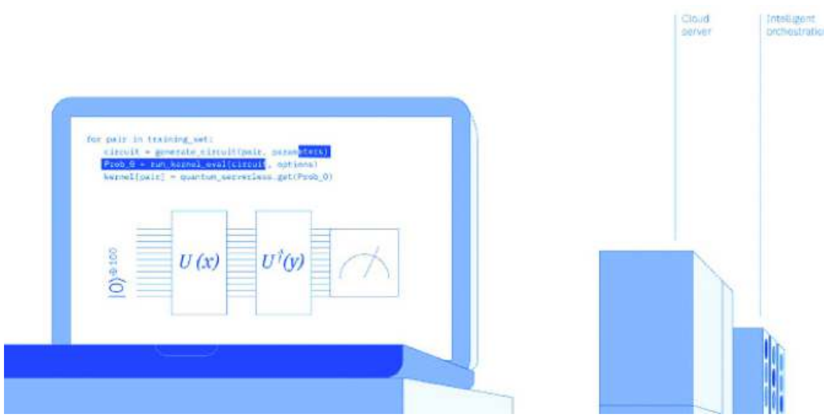
An example of how a datacenter might work.

1. First, we we need define the circuit and set up the clouds to be used
2. Then the quantum serverless tools handle the orchestration for you
3. Next, we compile the higher-level circuits and map them to physical gates
4. The circuit knitting/circuit cutting method decomposes this into smaller circuits
5. With Quantum serverless and qiskit runtime, these subcircuits can be sent in parallel and executed using the primitives. Error mitigation and suppression are applied, and the results are sent back to the qiskit-runtime-as-a service.
6. These reliable results can be combined in another cloud for the final answer

116 **An Example of How a Datacenter Might Work (2)**

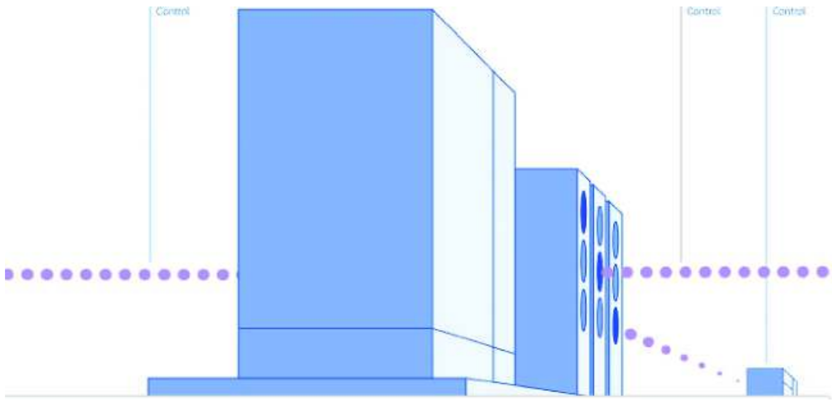


117 **An Example of How a Datacenter Might Work (3)**



Orchestration.

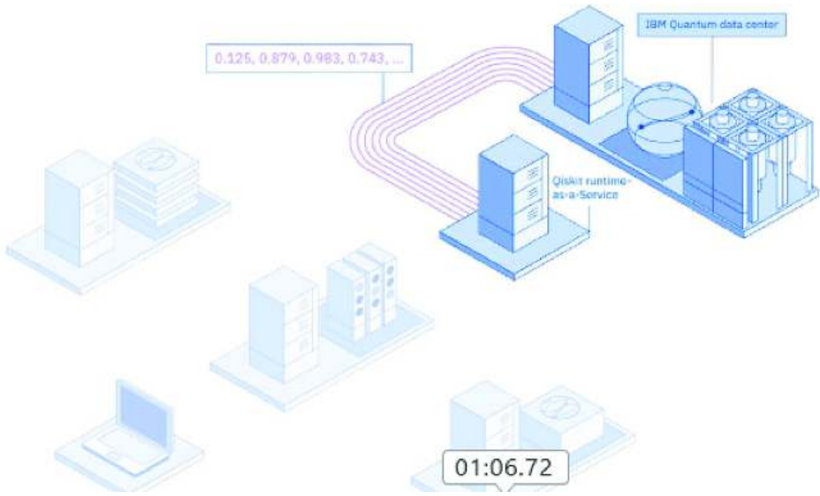
118 **An Example of How a Datacenter Might Work (4)**



119 **An Example of How a Datacenter Might Work (5)**

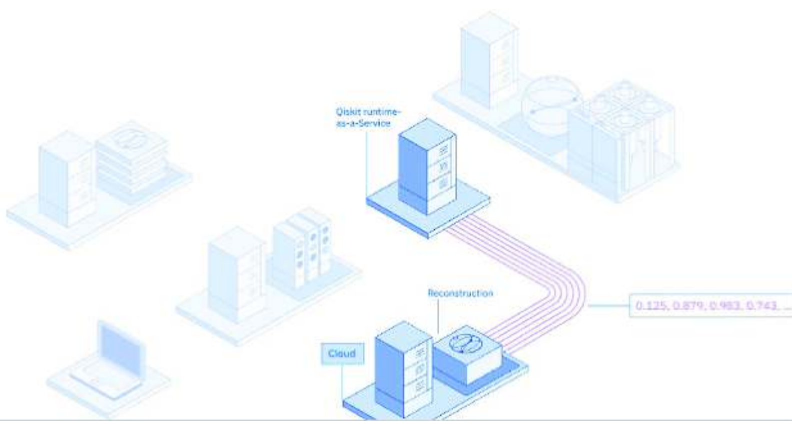
Decompose parameterized physical circuit into four subcircuits

120 **An Example of How a Datacenter Might Work (6)**



Reconstruct the results.

121 **An Example of How a Datacenter Might Work (7)**

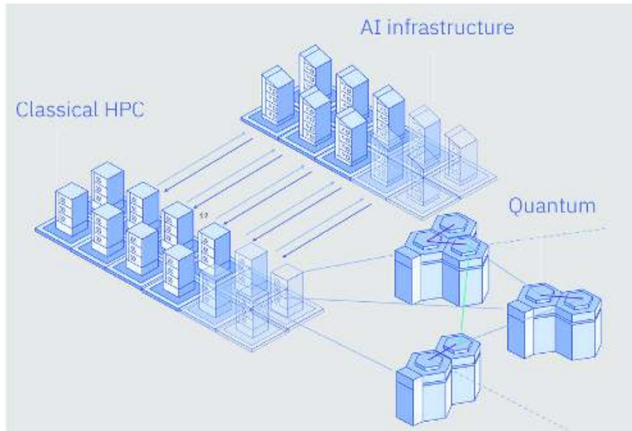


122

Quantum Is a Component in the Future of Advanced Computing

In the future, quantum will integrate with other components, including AI, to enhance the overall capability of our computational tools.

Each tool is best suited for certain types of tasks, and all will work together to solve the hardest problems that face society today.



- Quantum computing and AI are technologies that both promise to completely transform the way we do business.
- While some see quantum and AI as competing technologies, we seem them as complementary tools that will one day integrate with each other, and additional components, to enhance our overall computational capabilities.

Chapter 05

Challenges and Opportunities for Ultra- Low Power Design for Quantum Computing Applications

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This chapter will cover practical challenges for cryogenic CMOS designs for next generation quantum computing. Starting from system level, it will detail the design considerations for a non-multiplexed, semi-autonomous, transmon qubit state controller (QSC) implemented in 14nm CMOS FinFET technology. The QSC includes an augmented general-purpose digital processor that supports waveform generation and phase rotation operations combined with a low power current-mode single sideband upconversion I/Q mixer-based RF arbitrary waveform generator (AWG). Implemented in 14nm CMOS FinFET technology, the QSC generates control signals in its target 4.5GHz to 5.5 GHz frequency range, achieving an SFDR > 50dB for a signal bandwidth of 500MHz. With the controller operating in the 4K stage of a cryostat and connected to a transmon qubit in the cryostat's millikelvin stage, measured transmon T1 and T2 coherence times were 75.7µs and 73µs, respectively, in each case comparable to results achieved using conventional room temperature controls. In further tests with transmons, a qubit-limited error rate of 7.76×10-4 per Clifford gate is achieved, again comparable to results achieved using room temperature controls. The QSC’s maximum RF output power is -18 dBm, and power dissipation per qubit under active control is 23mW.

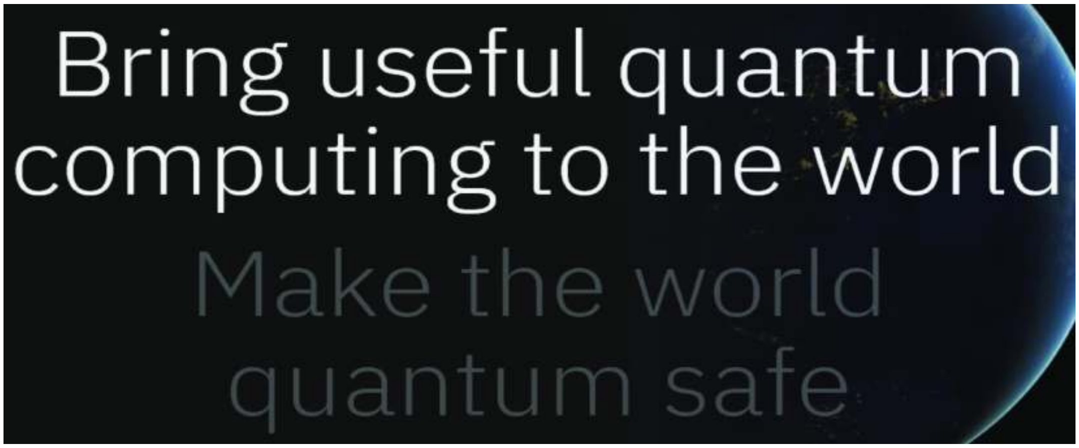
11 **Introduction (1)**

The Talk is about challenges and opportunities for ultra-low power design for quantum computing applications, specifically focusing on transmon qubits and control electronics.

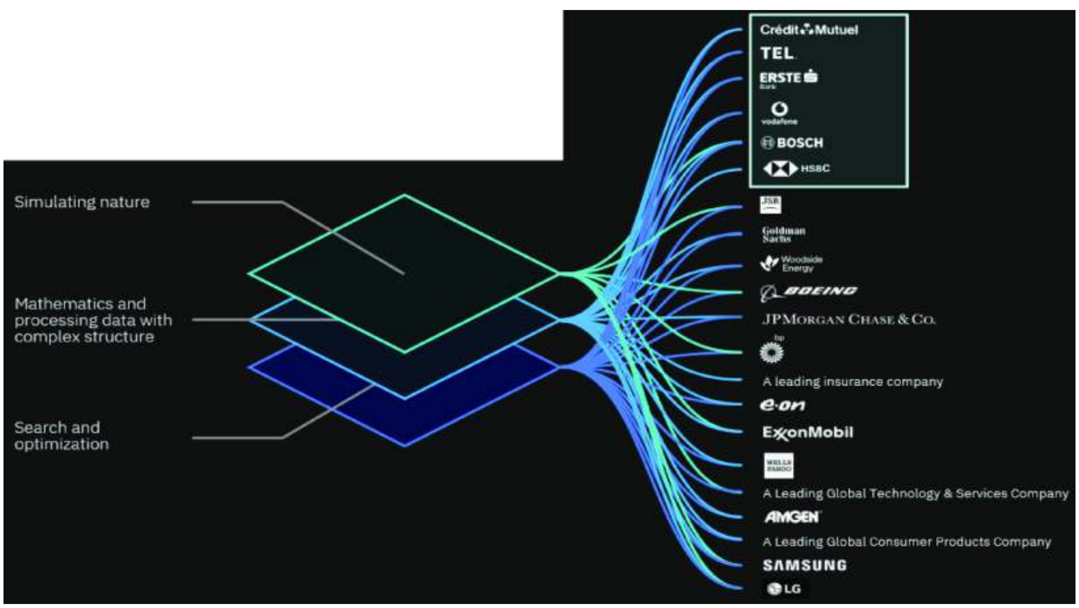
12 Introduction (2)

13 Introduction (3)

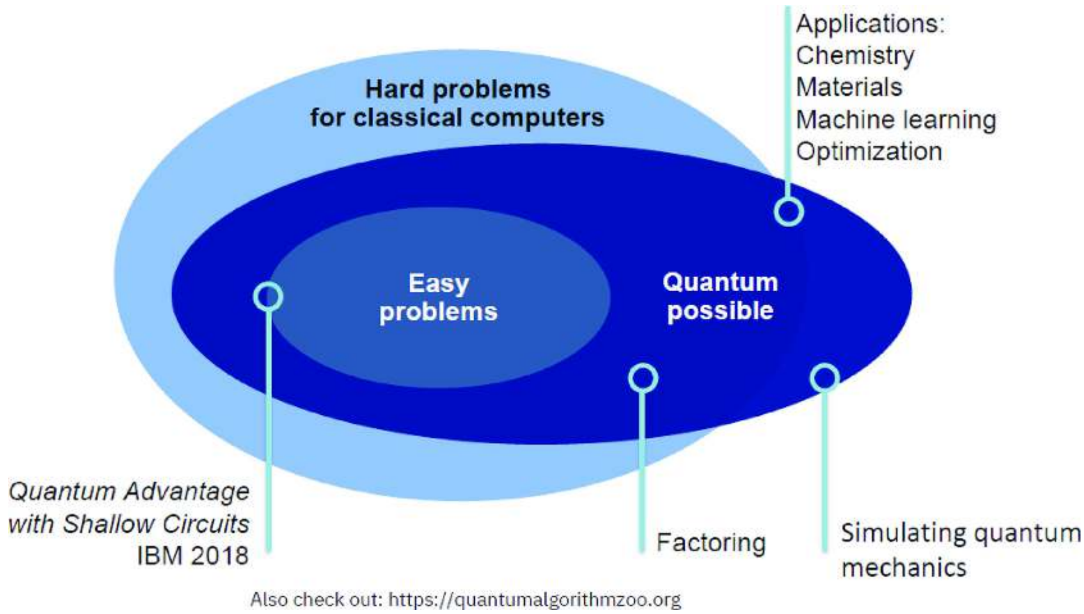
14 **Our Mission**



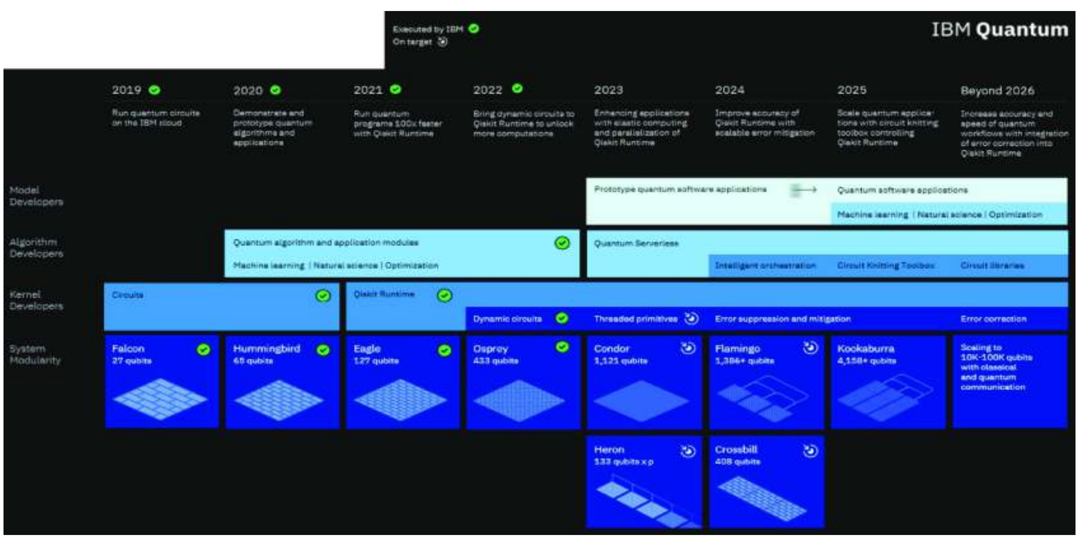
15 **Connecting Industry Clients with Quantum Computing Us Cases**



16 The Potential of Quantum Computing



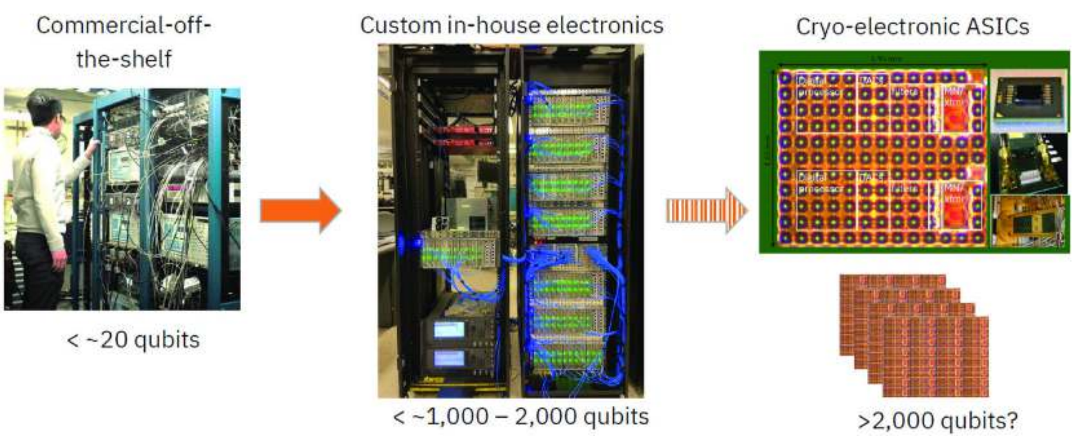
17 Development Roadmap



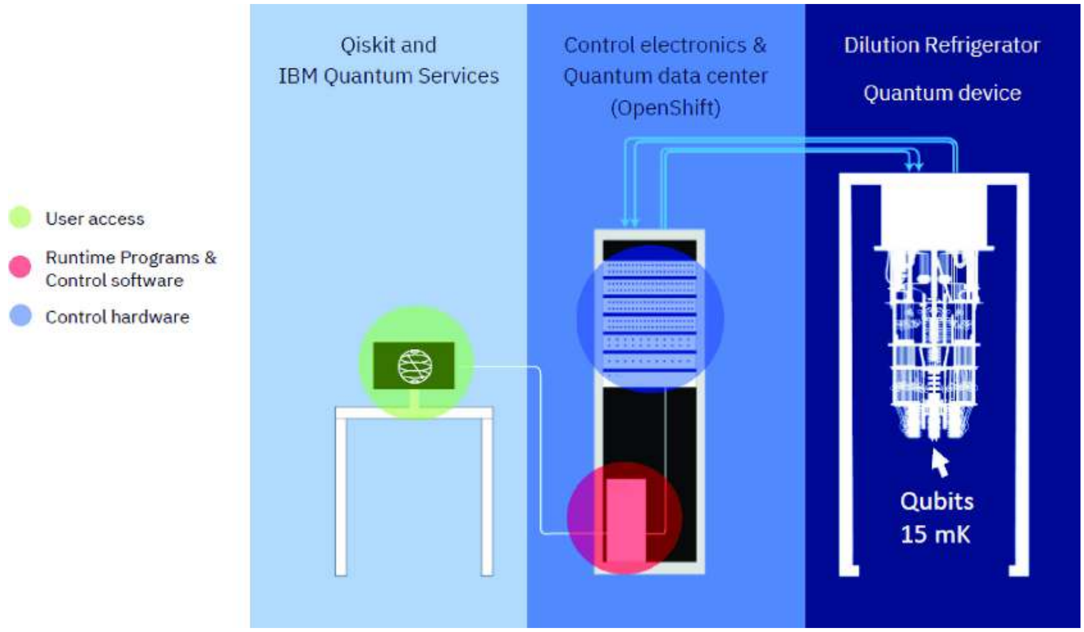
18 Outline

- Superconducting qubits
  - transmons
- Transmon control and readout
- Requirements on control electronics
  - Analog: noise, signal quality
  - Digital: QEC considerations, control flow
- System approaches to transmon control
  - RT and cryo: compare/contrast
- Cryogenic CMOS control
  - Circuit design considerations
  - CMOS qubit state controller
- Recent experimental results

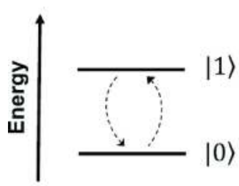
19 Evolution of Quantum Computing Control & Readout Electronics



110 Quantum System



111 The Quantum Bit (Qubit)



Two level system: $|0\rangle$ and $|1\rangle$

- State is linear superposition

$$\alpha|0\rangle + \beta|1\rangle$$

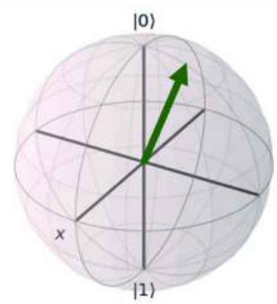
- Represent as point on surface of Bloch sphere

Multiple qubits have 2^n levels, where n = # of qubits: e.g., $|0001011\rangle$ or $|11100\rangle$

- State is linear superposition of the 2^n eigenstates
- n=2 example:

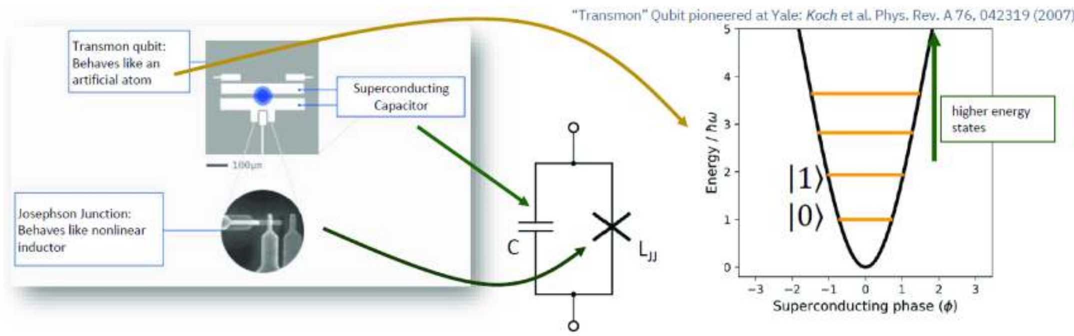
$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

One can operate on all of these coefficients at once



Bloch sphere

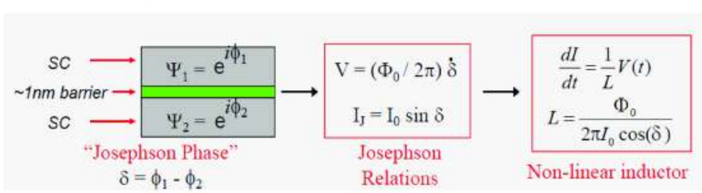
112 Transmon Qubits

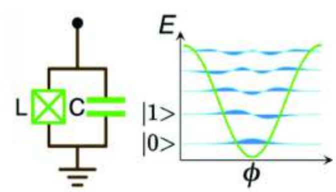


- Cryogenic operation allows us to isolate transition between two lowest levels → |0⟩ and |1⟩, whose separation defines *qubit frequency*
- Typical numbers:
  - C ~ 65 fF, I_{JJ} ~ 20 nA => L_{JJ} ~ 15 nH
  - Freq ~ 5 GHz, Q > ~10^7

113 Josephson Junction

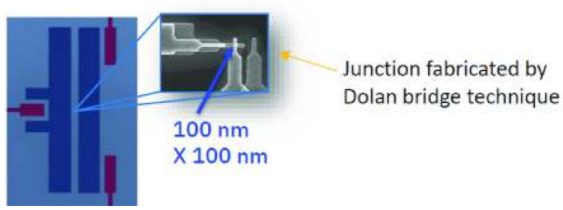
Josephson junction as a non-linear inductor





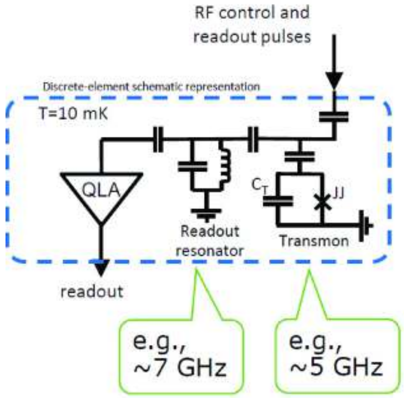
$E_{01} \approx 5$ GHz $\approx$ 240 mK

~300MHz anharmonicity

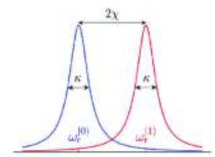


114 Coupling and Readout

- RF control signals and readout are coupled to qubits by small capacitors

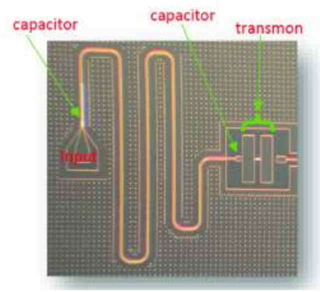


Dispersive readout



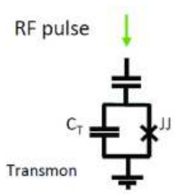
- Qubit state shifts the readout resonator frequency $\pm\chi$
- Measure the phase shift of a centered resonant tone to detect the qubit's state

Example layout of a readout approach that uses a CPW readout resonator



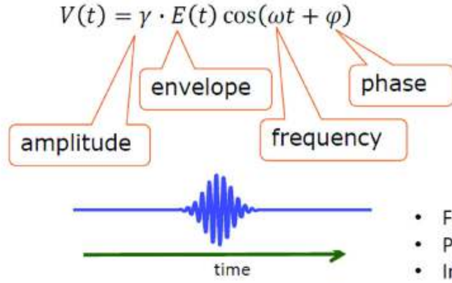
115 Single-Qubit Control Using RF Pulses

X and Y gates

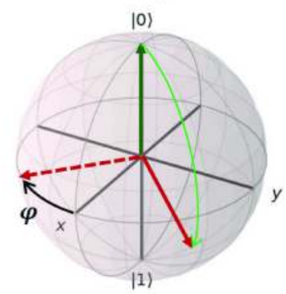


Waveform (simplified):

$$V(t) = \gamma \cdot E(t)\cos(\omega t + \varphi)$$



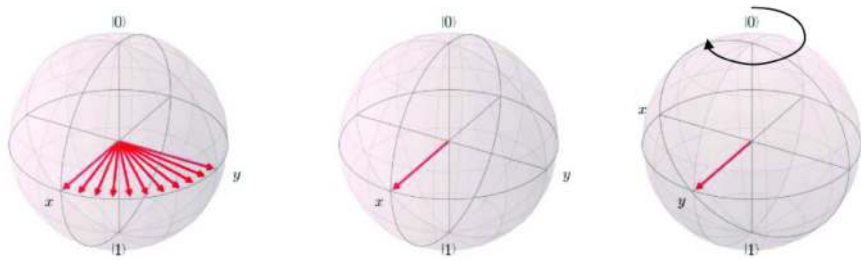
Bloch sphere



- Frequency must match that of the qubit
- Phase determines the axis of the rotation
- Integral of amplitude x envelope determines extent of the rotation

116 Single-Qubit Control: Z Gates

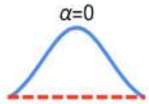
- Z gate is a rotation of the state with respect to the X/Y axes
- This rotation is equivalent to rotating the X/Y axes which is done by rotating the phase of our microwave drive. This phase is set in our classical control software and can be done with essentially perfect fidelity.



Z90

Z90 by "flipping the axes"

117 Pulse Shaping to Reduce Errors



DRAG Pulses[1]

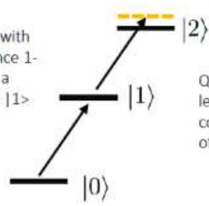
Apply a pulse to the orthogonal axis in the quadrature plane (Q). The pulse envelope is the derivative of the main Gaussian pulse (I).

What errors can we get from higher levels?

1. Phase errors

A drive resonant with 0-1 is off-resonance 1-2. This will cause a Stark shift on the |1> state

2. Leakage



Quantum information leaks outside the computational space of the qubit

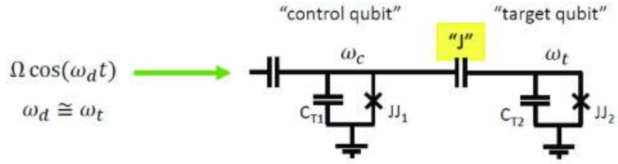
Our calibration protocols optimize DRAG for *fidelity*

For typical transmons, leakage errors are at least an order of magnitude lower than phase errors[2]

DRAG typically corrects for phase errors

[1] DRAG = Derivative Removal by Adiabatic Gate; Motzoi *et al.* *Phys. Rev. Lett.* **103**, 110501 (2009)
[2] McKay *et al. Phys. Rev. A* **96**, 022330 (2017)

118 Two-Qubit RF Gate: Cross-Resonance (CR)



J characterizes the ZZ coupling between the qubits, usually in units of frequency

This is the rate at which the phase of the $|11\rangle$ state changes relative to what is would do if there were no coupling

All-microwave entangling gate

Same electronics as for single-qubit gates

Fixed-frequency transmons

Rigetti and Devoret PRB 2010
Chow et al, PRL 2011

Challenges:

Always-on ZZ crosstalk, causing unwanted phase entanglement

Higher energy levels affect the dynamics

Strong drives and frequency crowding lead to leakage

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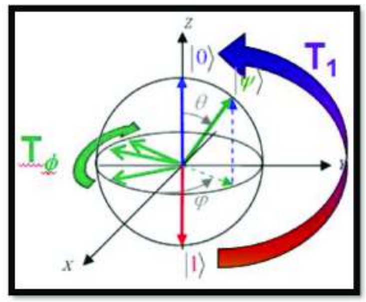
120 Qubit Decoherence

Finite qubit coherence times

- T_1: relaxation (dissipation - think resistor)
- T_φ: dephasing (randomization of f)
- T_2: parallel combination of above



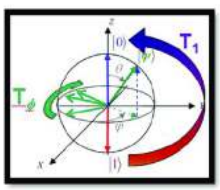
$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$

➢ **Errors unavoidable —**
Will they destroy our computation?

Yes, but there is error correction

121 Types of Qubit Errors

Decoherence

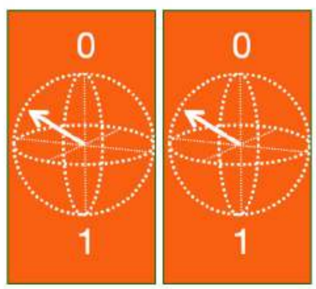


Amplitude or phase decays or becomes corrupted

Gate errors

Gate rotates around incorrect axis, or rotates with incorrect angle of rotation

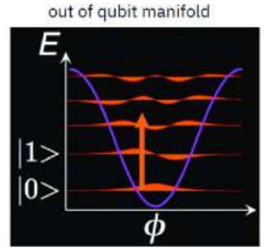
Spectator errors



Qubits not involved in a gate undergo a rotation, or they can impact qubits that are involved in a gate

Leakage

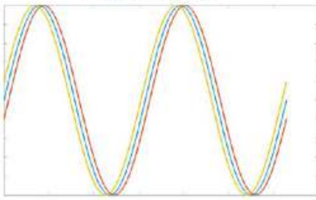
Unwanted transitions out of qubit manifold



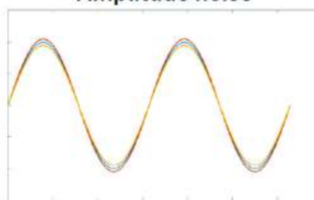
Qubit leaves computational space (this is a really bad error)

122 Sources of RF Control Errors

Phase noise

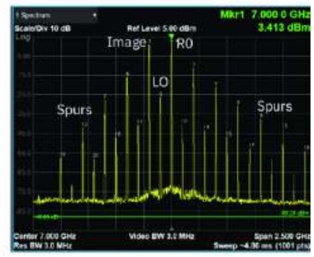


Amplitude noise



- High frequency noise
- Low frequency noise (drift)
  - Shaped pulses and refocusing can help
- Accuracy (number of bits)
  - Shaped pulses and refocusing can help
- Cross talk (spectator errors)

Mixer sidebands



- Sidebands overlap transitions (spectator and leakage errors)
- Noise away from intended microwave tone (spectator / leakage)

123 Outline

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System> approaches to transmon control

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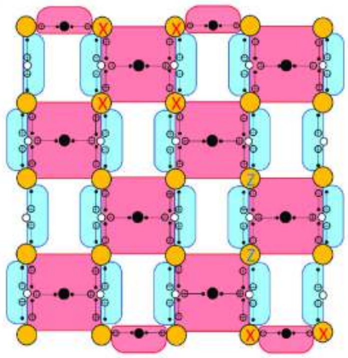
Cryogenic CMOS control

- Circult design considerations
- CMOS qubit state controller

Recent experimental results

124 Fault-Tolerant QEC

Example: d=5 heavy hexagonal code

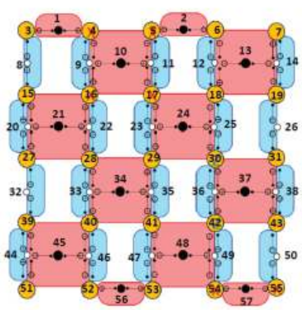


Orange = data qubits, white = flag qubits, black = ancilla qubits

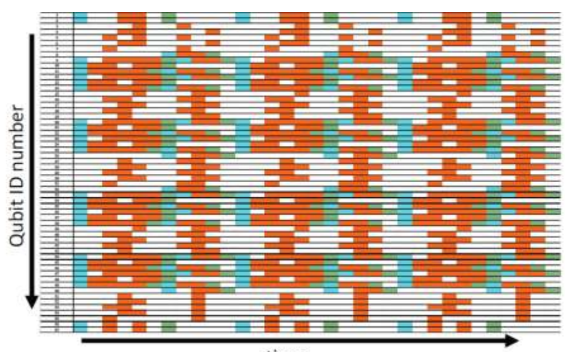
- A single logical qubit is composed from a lattice of physical qubits
- The quantum data is spread over the entire array
- Simultaneously measure the sets of error checks (red or blue)
  - Orthogonal to the data, so do not destroy it
- Compute corrections based on the measured errors

[Chamberland, Zhu, Yoder, Hertzberg, Cross, Phys. Rev. X 10, 011022 (2020)]

125 Activity Factor for QEC -> No TDM 125



d=5 heavy hexagonal code



3 QEC cycles shown

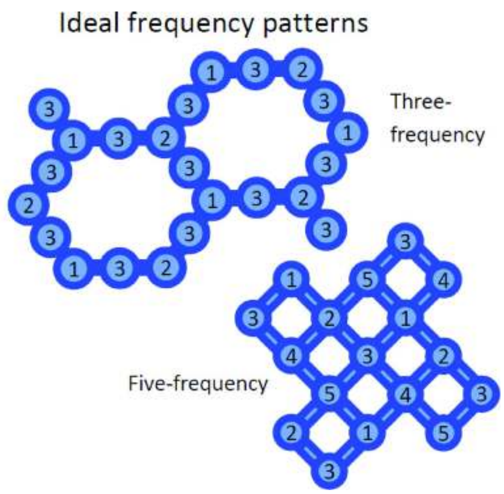
Red=logic gate
Green=readout
Cyan=initialize

>70% qubits active simultaneously => No viable TDM
Highly repetitive => use semi-autonomous controller

126 **Can We Frequency Multiplex?**

- Cross-resonance two-qubit gates
- Frequency plan => only 3 to 5 frequencies
- Each qubit must separately receive its own frequency and those of its neighbors

- **=> No multiplexing**



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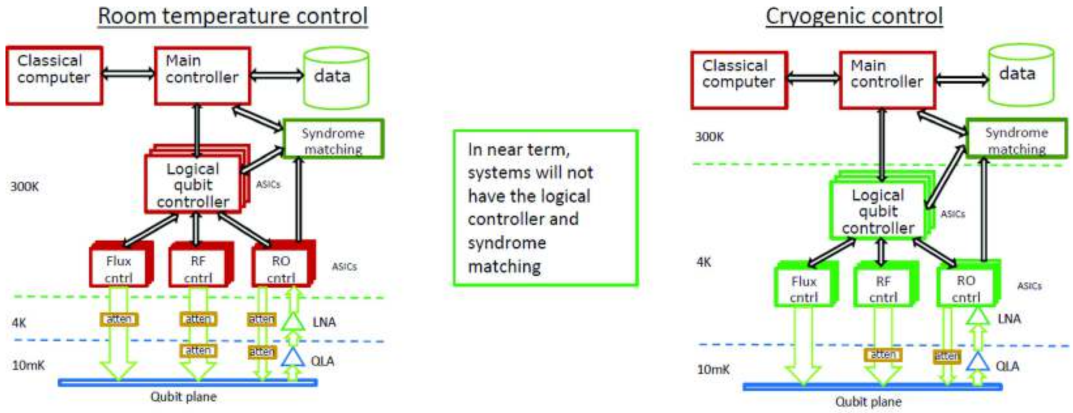
128 **Future System Block Diagram Comparison**



- All signals routed out of cryostat
- ~5n wires from RT to base, n=# of qubits
- Connectors degrade reliability, increase cost
- Easier to cool RT electronics
- High fidelity RT electronics are easier

- Fewer signals routed out of cryostat
- ~5n wires only from 4K to base , n=# of qubits
- Fewer connectors, more reliable, cheaper
- Harder to cool LT electronics
- High fidelity low-power LT electronics are difficult

129 **Large System Power/Cooling**

- For reasonable qubit density, cooling targets at 10-20 mK:
  - 1 nW/qubit heat conduction from signal wire
    - May require superconducting ribbon cables (see D. Tuckerman, et a Supercond Sci Tech 29 084007 (2016) |
  - 1 nW/qubit dissipation from signal attenuators/termination/readout QLA
  - Flux control damping/cooling: final thermalizing attenuator probabl dissipates >10 uW/line; cannot be at base T
- Cooling power estimate for ~4 K stage:
  - Each wire from RT is probably 2 1mW
  - For control electronics, ~1-10 mW/channel seems plausible
  - Lower is better!
- Need very high cooling power at ~4 K (>1 KW total for a 10 qubit system)

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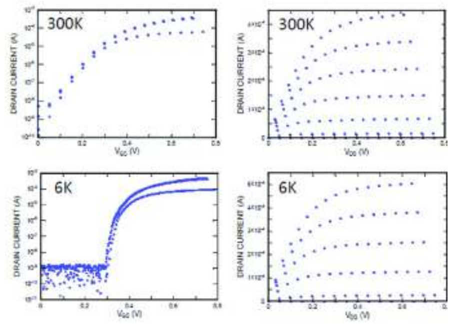
131 **Circuit Design Considerations for Cryo**

- Power: expect extremely constrained power dissipation budget due to the Carnc inefficiency of cooling to ~4K
  - Need very low power circuit designs
- FETS:
  - Expect increased sub-V_T slope, leading to greater mismatch sensitivity
    - May need more calibration adjustments
  - Expect increased V_T, leading to somewhat higher power supplies
  - Expect sub-V_T leakage currents to become negligible -> reduces power
  - Expect increased mobility, leading to higher current drive
  - Expect increased 1/f noise
    - Noise looks like V_T fluctuations being applied to steeper sub-V_T slope
- Resistances:
  - Expect wire resistance to decrease significantly
  - Expect substrate to freeze out, or at least to become more resistive

132 Measured Cryogenic Device Behaviour

- 14nm FinFET technology has been characterized
- Wire resistances:
  - Upper level wire R decreases ~10x, lower level ~2x
  - Calibrated FEOL resistors increase 1.1X
  - Substrate R increases >1000x, but substrate does not entirely freeze out
- FinFETs:
  - Sub-V_T swing decreases to ~18mV/decade, and is nearly constant for T < ~50K
  - V_T increases ~100mV, mostly due to slope
  - g_m increases, such that drive current @800mV increases ~10%
  - 1/f noise increases 3-4x, depending on bias



Example: SLVT nFET, 8 fins

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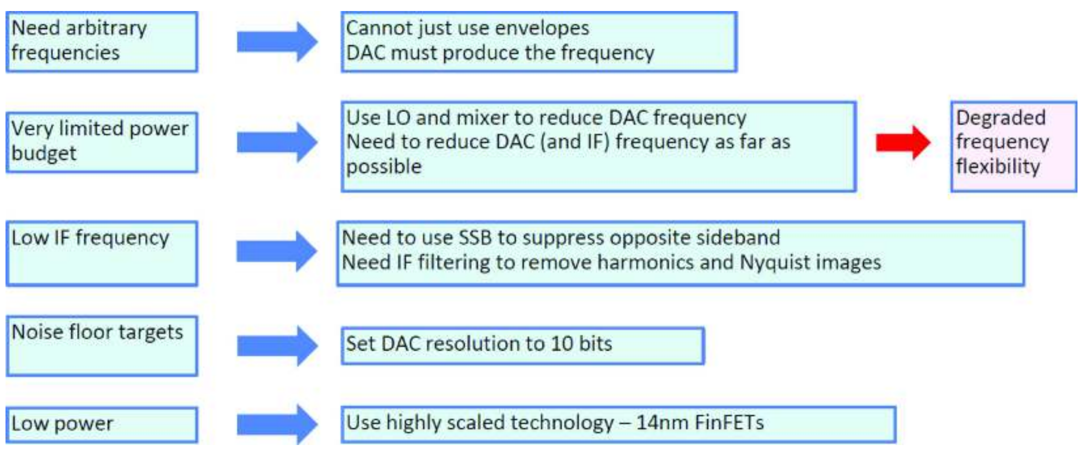
134 Qubit State Controller (QSC)

- QSC was designed and built in 14nm FinFET technology
- Approach:
  - No TDM, no FDM
  - Separate channel for each qubit
  - Semi-autonomous program control in anticipation of QEC
  - Power metric is power per qubit under active control

No multiplexing:

> Idle qubits are wasted qubits

135 Key Design Considerations for QSC

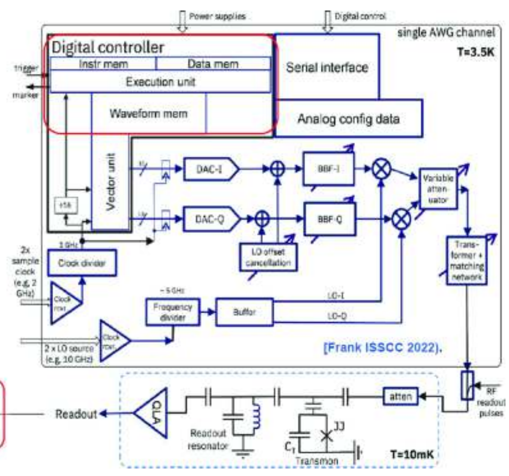


136 **High-Level Targets for this QSC**

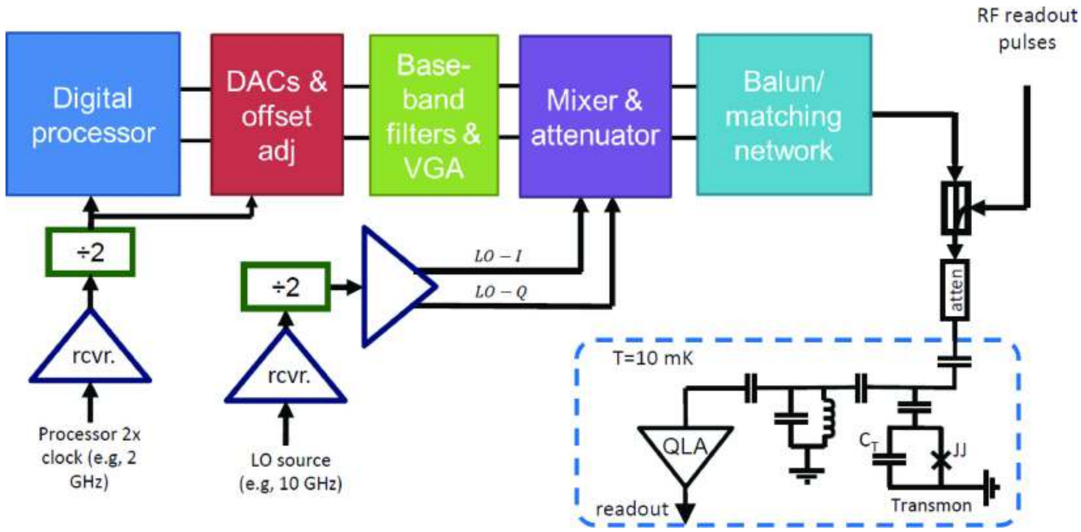
Prameter	Targets
DAC frequency	1 GHz nominal
LO frequency, f_{10}	4.85 to 6.0 GHz
RF Output, f_{out}:	4.8 to 5.5 GHz, lower sideband
Maximum output level	50mV(peak), -16 dBm
Output Amplitude Control:	40x + blanker
Noise floor	<10nV/sqrt(Hz)
Spurious Output	<-56dBm(-40dBc)
Power dissipation	<20 mW nominal
Communications interface:	Serial interface, packet-based

137 **Cryoelectronics for Quantum: Control and Readout Paths**

- Cryogenic quantum state controllers are realized as mixed-signal arbitrary waveform generators incorporating custom analog and custom processor designs; a similar structure can be used to generate tones needed for readout
- SRAM is a critical element in such custom processor designs to generate qubit control signals and required readout signals
- Cryo LNA is a key element that must be addressed in order to realize a full cryogenic readout/discrimination path
- Key considerations
  - Achieving target performance
  - Sufficient storage for waveform and other data
  - Power consumption

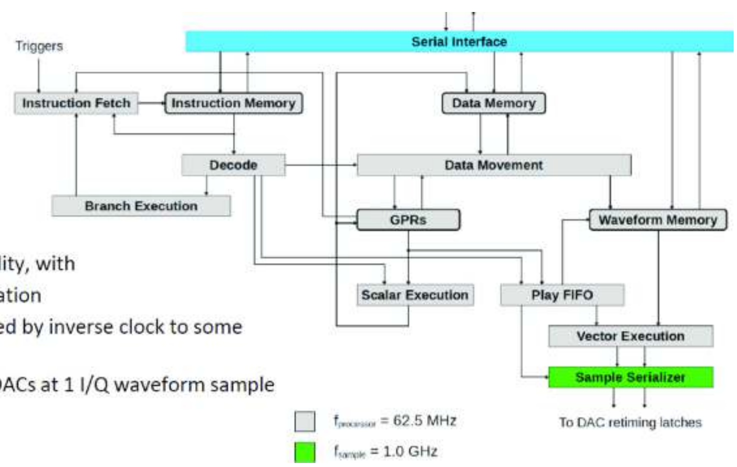


138 QSC Block Diagram



139 Digital Controller

- In-order, 32-bit fixed-point general-purpose processor with special constructs for waveform generation and phase rotations



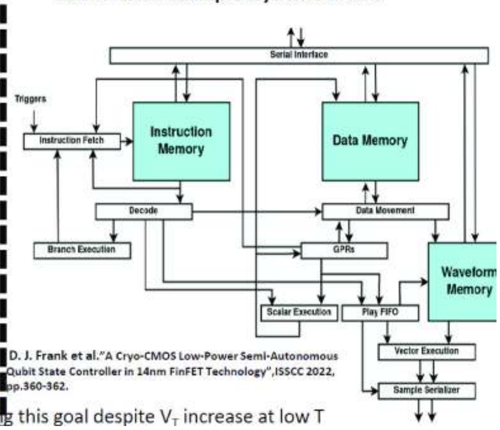
- Traditional scalar processing capability, with 16-way SIMD vector waveform generation
- Single cycle pipeline latency, enabled by inverse clock to some memory elements
- $F_{nominal}$ = 62.5 MHz, feeding 1 GHz DACs at 1 I/Q waveform sample pair per nanosecond

140 Role of Memories in Custom Processor in Demonstrate Cryogenic Quantum State Controller

- 32 kB instruction memory holds programs for waveform generation, sequencing
- 32 kB data memory serves as general-purpose scratchpad
- 20 kB waveform memory stores modulated RF waveforms
- **Memory access power is a significant proportion of overall digital power**
- **Memory requirements on V_{DD}/V_{CS} dominate overall voltage scalability**

➢ **Ultra-low-voltage, low-power, robust cryo-memory implementations are highly desirable for future scaled cryoCMOS-based quantum computing systems**

Memories in example cryoCMOS AWG



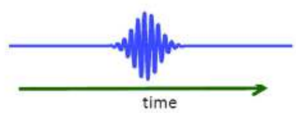
D. J. Frank et al."A Cryo-CMOS Low-Power Semi-Autonomous Qubit State Controller in 14nm FinFET Technology",ISSCC 2022, pp.360-362.

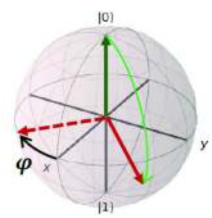
Key challenges: (1) achieving ultra-low voltage operation; (2) meeting this goal despite V_T increase at low T

141 Special Purpose Instructions

Desired waveform (simplified):

$$V_I(t) = \gamma \cdot E_I(t)\cos(\omega t + \varphi)$$
$$V_Q(t) = \gamma \cdot E_I(t)\sin(\omega t + \varphi)$$





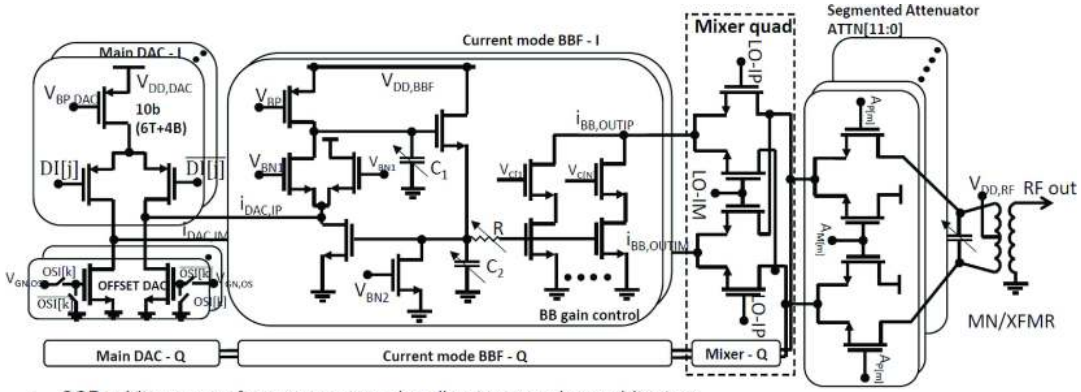
Math:

$$V_I(t) = E_I(t)\cos(\omega t)\,\gamma\cos(\varphi) - E_I(t)\sin(\omega t)\,\gamma\sin(\varphi) = \alpha U_I(t) - \beta U_Q(t)$$
$$V_Q(t) = \underbrace{E_I(t)\sin(\omega t)}_{U_Q(t)}\,\underbrace{\gamma\cos(\varphi)}_{\alpha} + \underbrace{E_I(t)\cos(\omega t)}_{U_I(t)}\,\underbrace{\gamma\sin(\varphi)}_{\beta} = \alpha U_Q(t) + \beta U_I(t)$$

Stored samples → $U_Q(t)$, $U_I(t)$

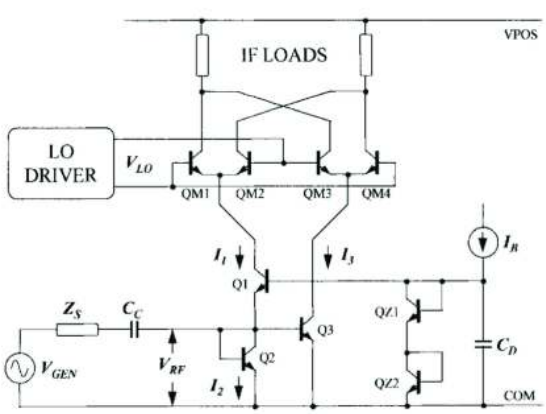
Instruction	Add Frame Phase	Compute Waveform Coeff's	Play Waveform
Purpose	Adjust phase	Scale waveforms & rotate frame	Play transformed samples
Execution	$\varphi \leftarrow \varphi + \varphi_Z$	$\alpha = \gamma\cos(\varphi)$ $\beta = \gamma\sin(\varphi)$	$V_I(t) = \alpha U_I(t) - \beta U_Q(t)$ $V_Q(t) = \alpha U_Q(t) + \beta U_I(t)$

142 Circuit Overview

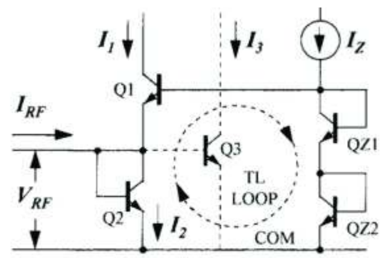


- SSB arbitrary waveform generator using direct conversion architecture
- Low power, low distortion differential current-mode architecture
- Independent amplitude controls at baseband and RF

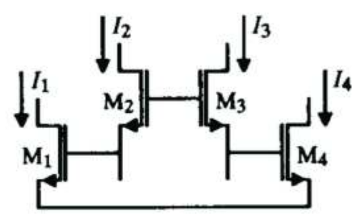
143 Current Mode Circuit Overview



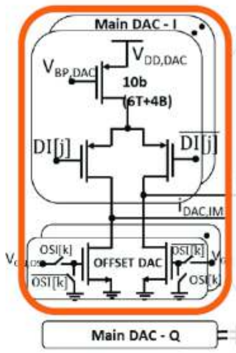
B. Gilbert, "Micromixer: A Highly Linear Variant of the Gilbert Mixer Using a Bisymmetric Class-AB Input Stage", IEEE JSSC vol 32 no. 9, pp.1412-1423, 1997



$$I_Z^2/(A_1A_2) = I_B^2/(A_{Z1}A_{Z2})$$

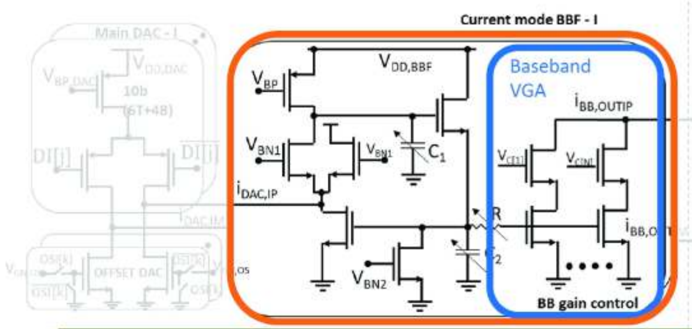


144 Digital to Analog Converter



- Current-steering I/Q DACs
  - for linearity, high speed, low power
- Segmented: 6b thermometer + 4b binary
- f_{samp}=1 to 2 GHz
- Common-mode current is subtracted
- Differential offset cancellation DAC
  - enables LO cancellation
- Filter sets common-mode output voltage
- Low VDD digital logic for lower power

145 Baseband Filter & VGA

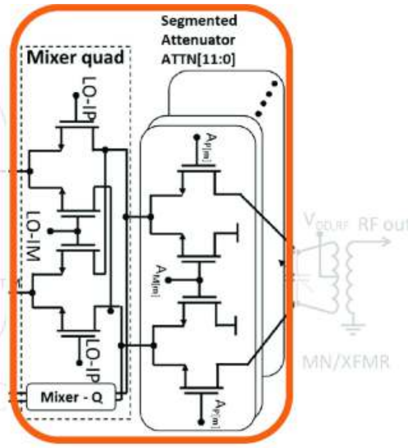


- Low-pass transfer function with 2 programmable gm/C poles
- Tunable between 100MHz and 800MHz
- Auxiliary pole outside loop (from resistors and gate C of the VGA)

- Current-mode filter: current input & current output
- VGA is programmable array of mirrors after the filter (0-14dB)

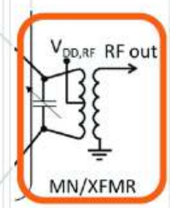
146 Mixer and Attenuator

- Gilbert cell mixer driven in current mode by the baseband VGA
- Mixer, attenuator, and VGA re-use the same bias current
- Mixer is driven with quadrature clocks at ~5GHz using CMOS waveforms
- 12 segment attenuator based on current division maintains SNR



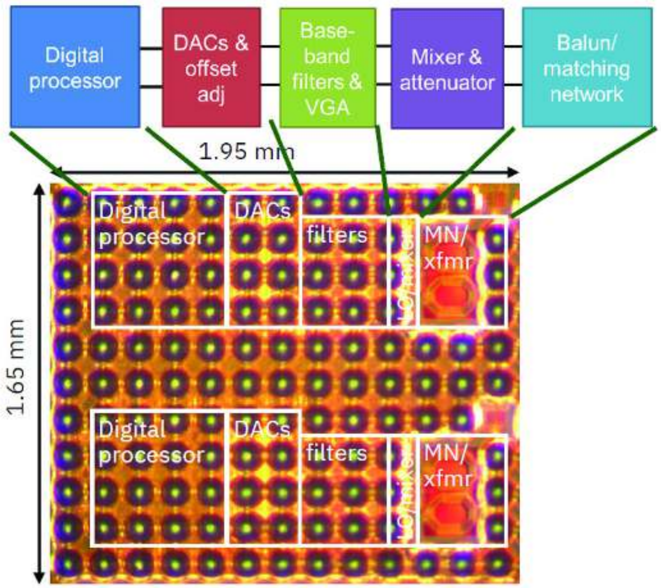
147 Tuned Balun

- Output current is scaled up using on-chip balun/transformer
    - Differential-to-single ended conversion
    - Impedance transformation to 50 Ohms
- 50 mV peak output amplitude
- Programmable tuning to operating frequency
- Programmable Q

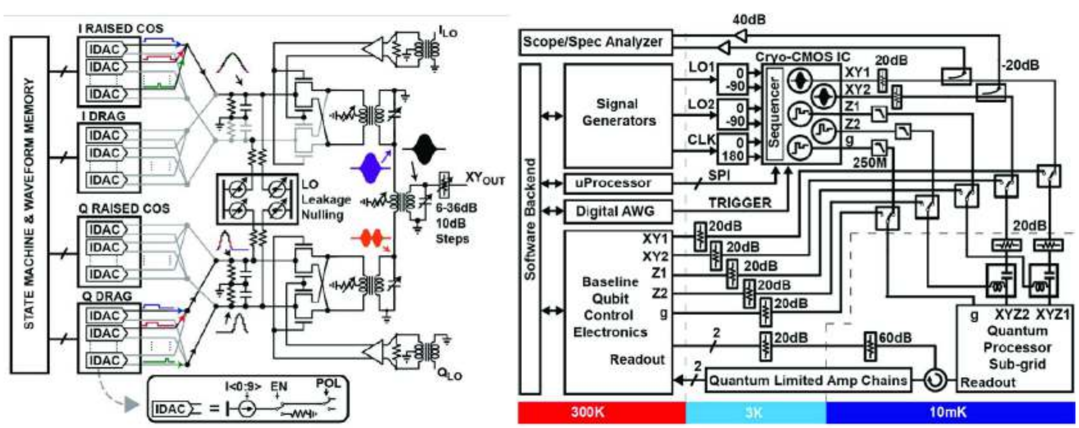


148 Tuned Balun

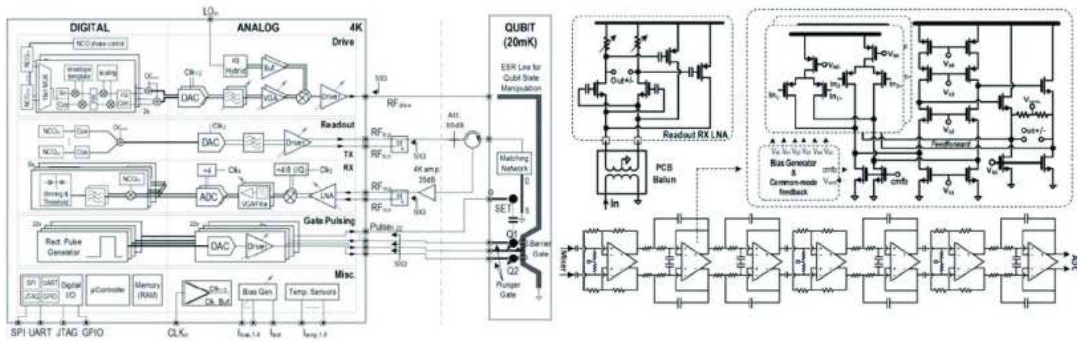
2 independent channels



149 RF-AWG Approach Reported by Google



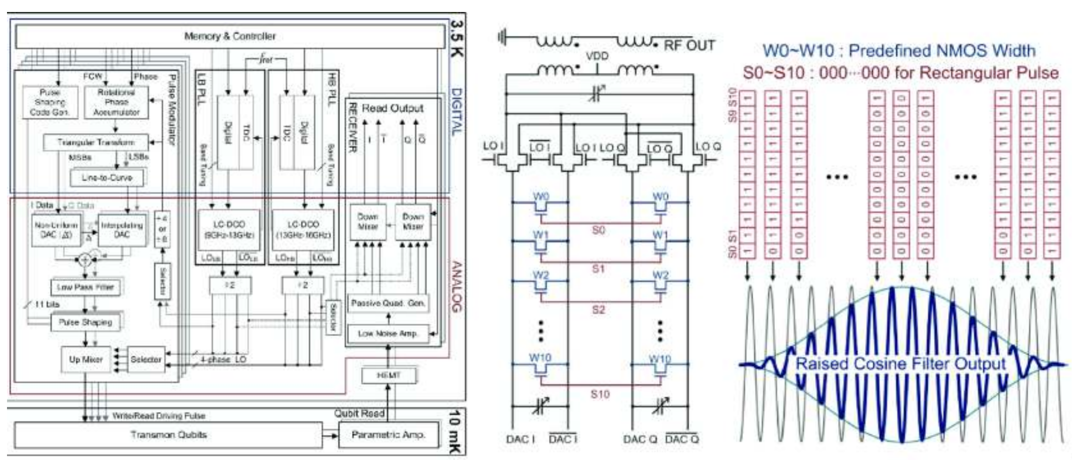
150 RF-AWG Approach Reported by Intel



(a) Qubit write controller

(b) Qubit read hardware

151 RF-AWG Approach Reported by POSTECH

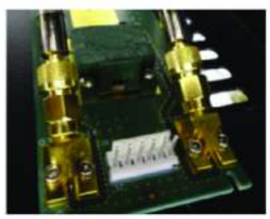


152 Outline

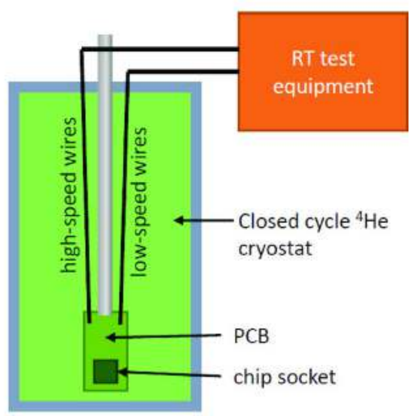
- Introduction to quantum computing
- Superconducting qubits
  - transmons
- Transmon control and readout
- Requirements on control electronics
  - Analog: noise, signal quality
  - Digital: QEC considerations, control flow
- System approaches to transmon control
  - RT and cryo: compare/contrast
- Cryogenic CMOS control
  - Circuit design considerations
  - CMOS qubit state controller
- **Recent experimental results**

153 Fabricated Chip

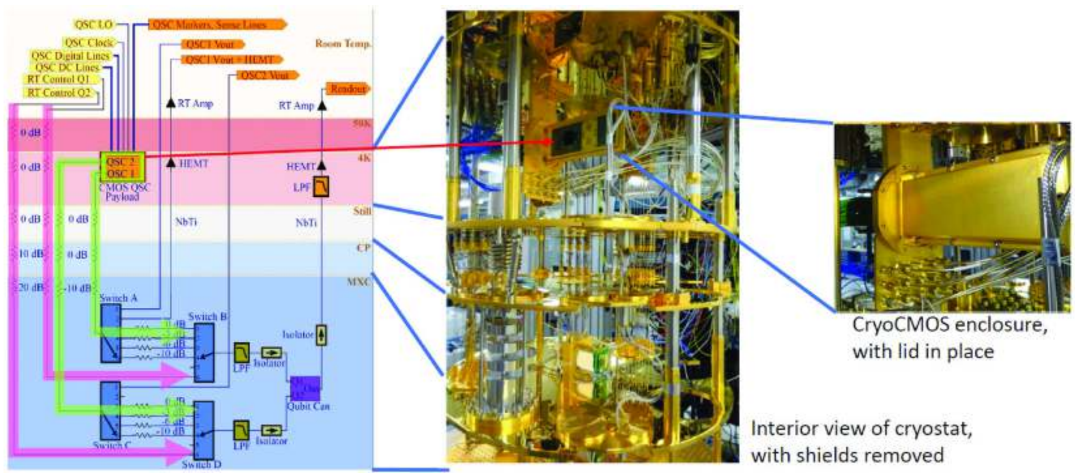

Laminate package


PCB & socket


Insert for ^{4}He cryostat



154 Dilution Refrigerator Testing



CryoCMOS enclosure, with lid in place

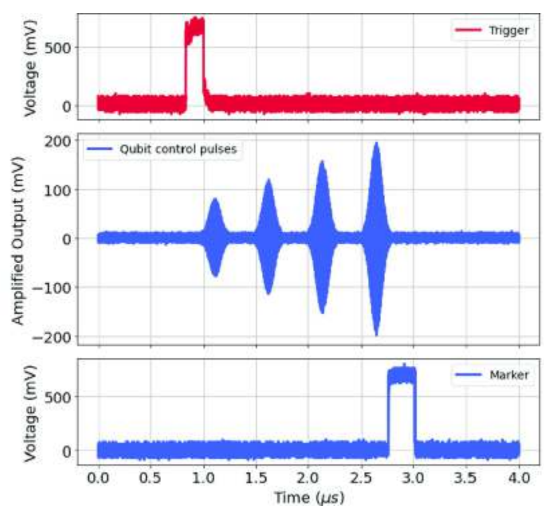
Interior view of cryostat, with shields removed

155 Qubit Control Pulses

Trigger input, to start experiment

Qubit control pulses from the QSC

Marker output, to trigger readout

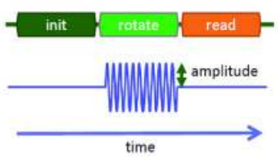


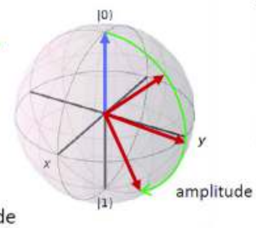
f_{samp} = 1.0 GHz
T=5K

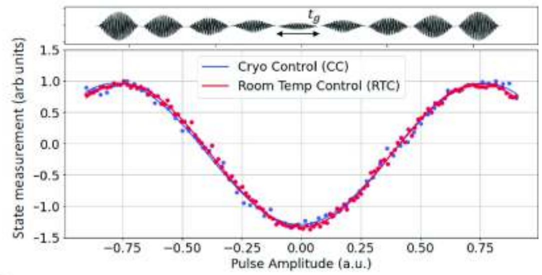
156 Rabi Oscillation

Used to calibrate the pi pulse amplitude

The angle of rotation is proportional to amplitude × duration



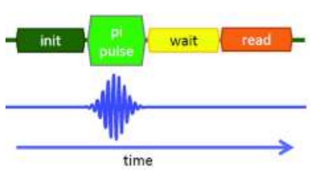


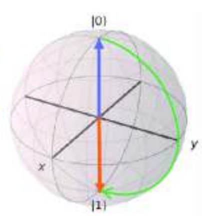


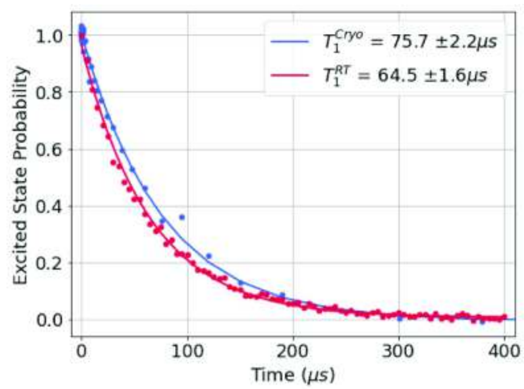
X-pulse with varying amplitude

157 Tl Measurement

Coherence time:
The time constant for the excited state of the qubit to decay back to the ground state





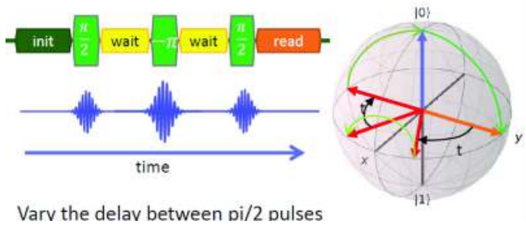


Vary the delay before reading

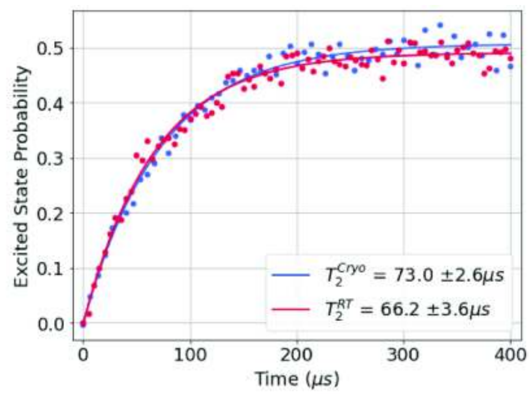
158 T2 Measurement

Phase coherence time:

The time constant for the phase of qubit to become random



Vary the delay between pi/2 pulses



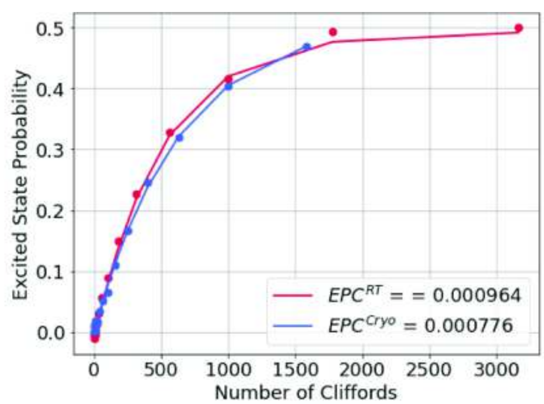
159 Randomized Benchmarking

Measures accuracy of a random sequence of *n* Clifford gates constructed to return to |0>.

Number of random seeds used: 10

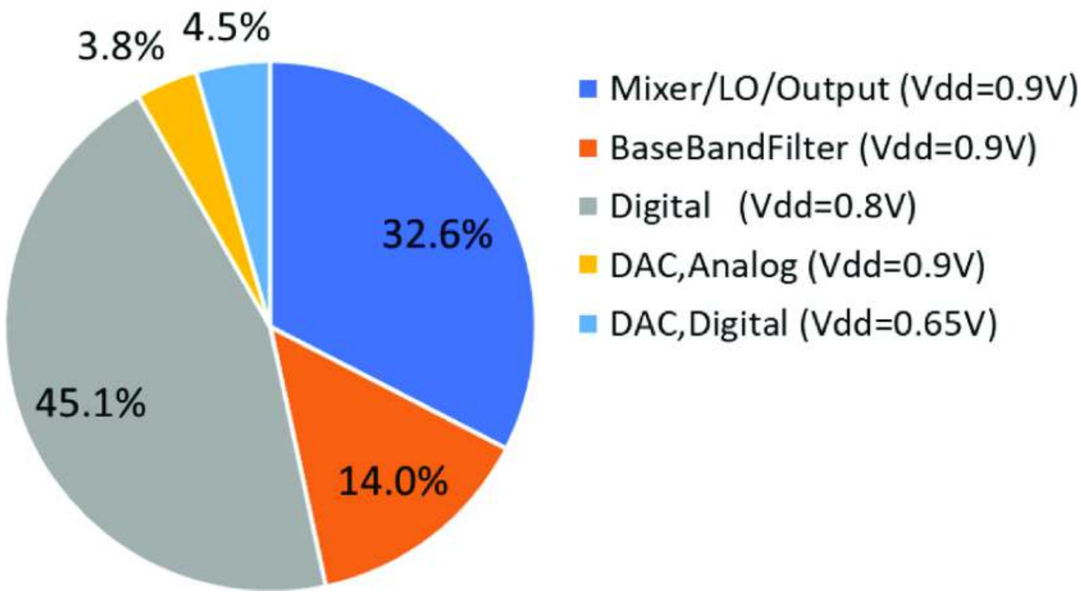
Error per Clifford gate (EPC) = 10^{-3}

(which is qubit-limited)

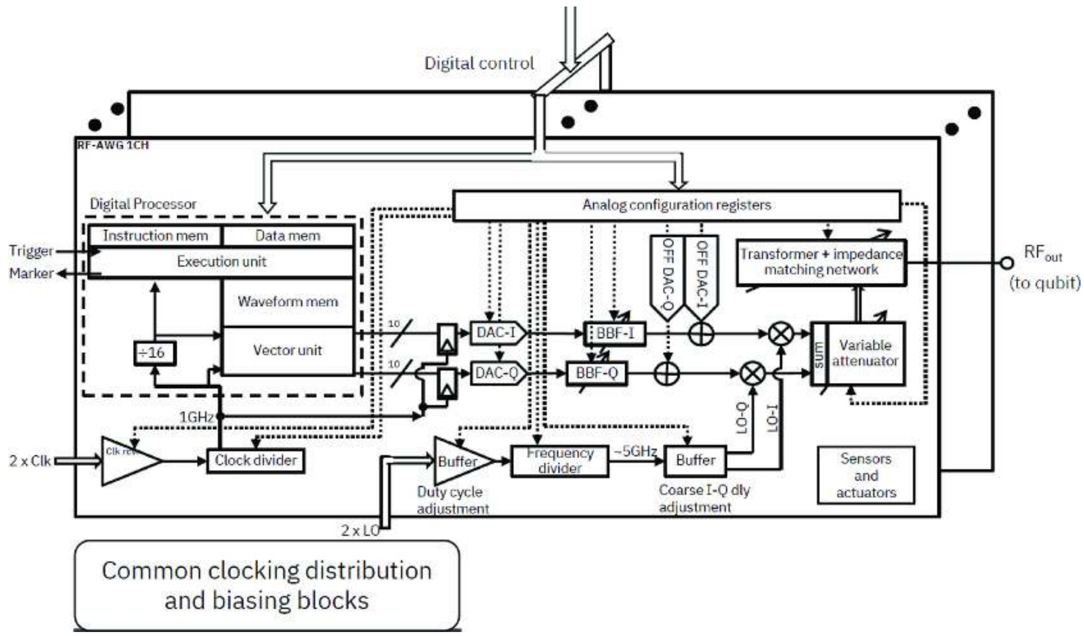


160 **Measured Power Dissipation at 4K**

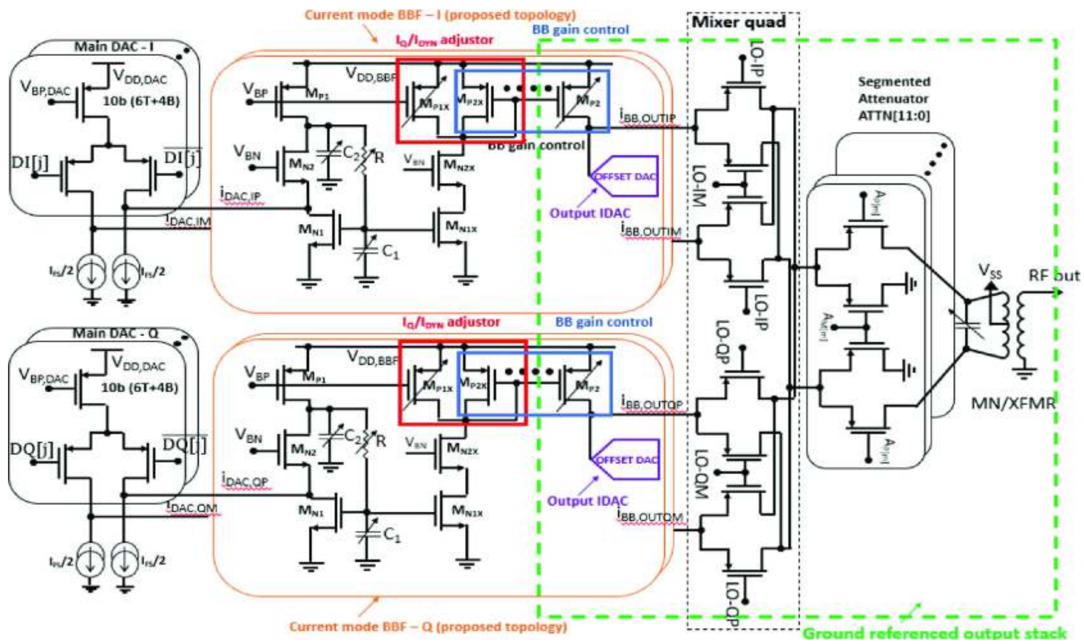
Measured Single Channel Power: 23.1mW



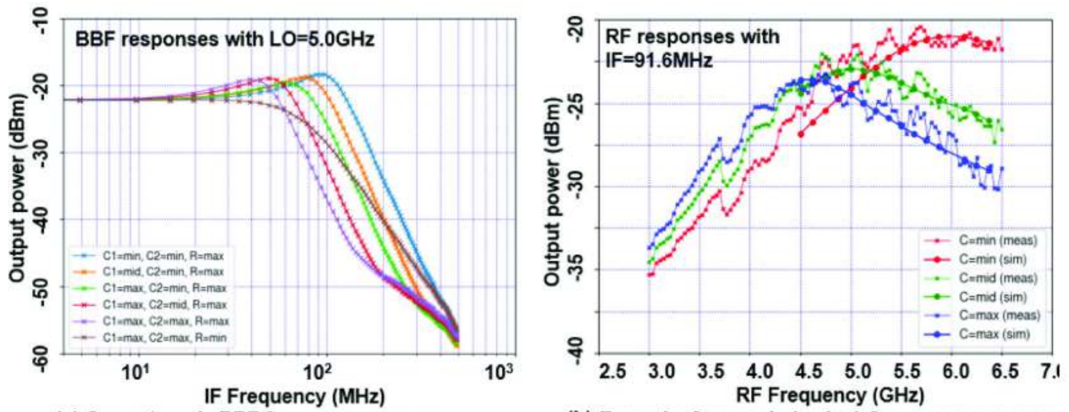
161 **Gen2 Are Itecture**



162 **Low Power Current Mode Design**



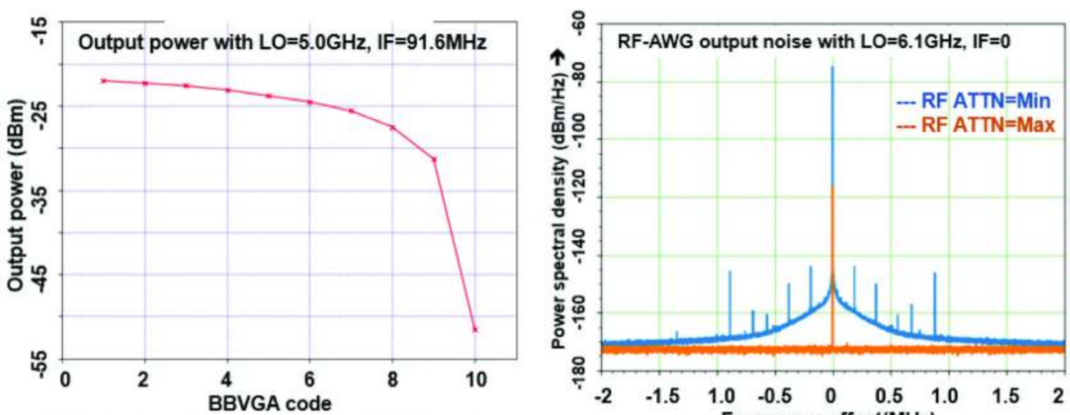
163 Current Mode BBF and RF Frequency Responses



(a) Current mode BBF frequency response

(b) Ground referenced o/p stack frequency response

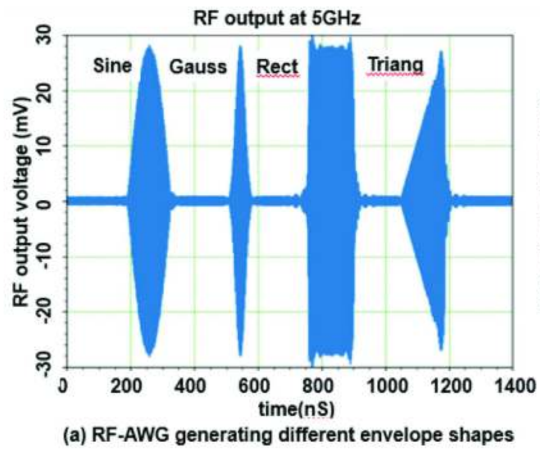
164 Output Power and Spectrum of the RF-AWG



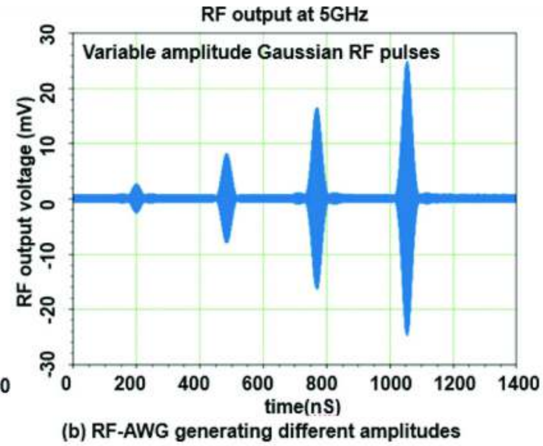
(c) Output amplitude variation vs. BBVGA code

(d) Power spectral density of output noise

165 Time Domain Output from the RF-AWG

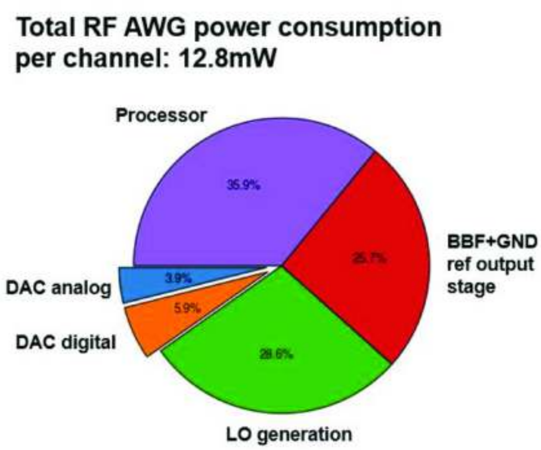


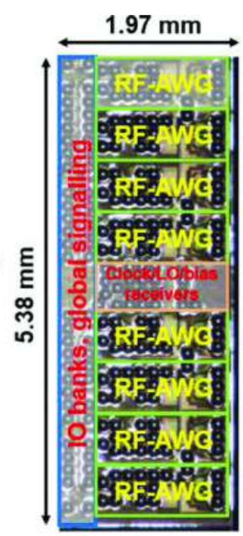
(a) RF-AWG generating different envelope shapes



(b) RF-AWG generating different amplitudes

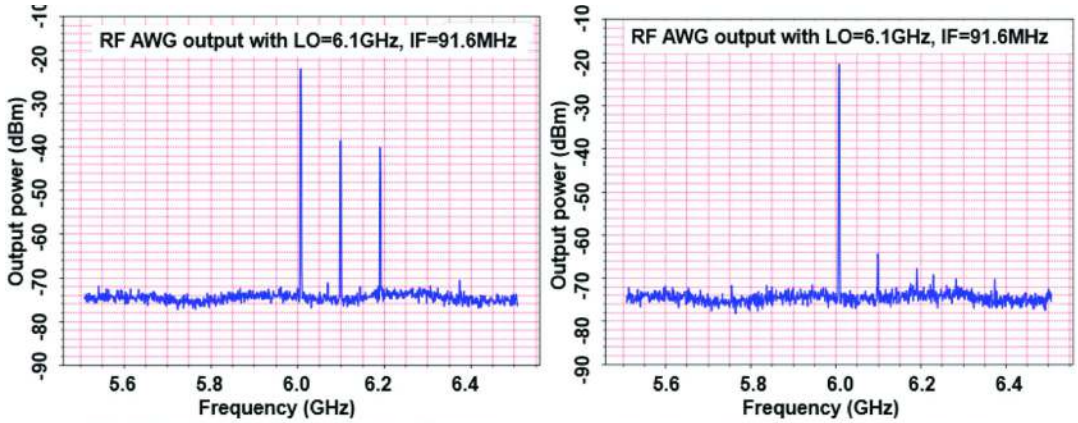
166 Power Consumption of the RF-AWG





CMOS technology provides low cost integration for multi Channel RFAWG

Spectrum Optimization for the RF-AWG



(c) RF-AWG output spectrum before calibration

(d) RF-AWG output spectrum after calibration

	This Work	[1]	[2]	[3]	[4]	[
		JSSC'22	**ISSCC'21**	**JSSC'22**	**ISSCC'21**	**ISSC**
Ambient temperature (K)	4	3.5-5	3	3.5	4	
Qubit type	Fixed frequency trans mon	Fixed frequency transmon	Flux tunable transmon	transmon.	spin	sp trar
Waveform type	arbitrary	arbitrary	Envelope only	Envelope only	arbitrary	arb
RF freq range (GHz)	4,3-7.5	4-5-5 5	4 0-8 0	4.6-6.1	11-17	2
IF frea ranae (GHz)	DC-0 12	DC-0.3	N/A	N/A	DC-0.7	DC
# channeis	8	2	2	1	4	
NCOs	updateable phase rotations + NCO option	updateable phase rotations	N/A	N/A	64	
Sideband method	SSB	SSB	Direct conversion	Direct conversion	SSB	S
DAC speed (GS/s)	1	1	1	1.5	2 5	
DAC bits	10	10	10	8	10	
Output gain/attenuation range	50x	SOx	32X	N/A	>5Gx	22
Waveform points	8K	8K	N/A	N/A	16K	4
Pulse sequence length	4K or unlim	4K or unlim	N/A	N/A	2K	

	This Work	[1]	[2]	[3]	[4]	[
		JSSC'22	ISSCC'21	JSSC'22	ISSCC'21	ISS
General purpose instruction set	yes, 36 general and 8 special	yes, 32 general and 5 special	No	No	no (special only)	(sp o
Power/qubit w/ active control (mW)	12.8	23	< 4	12.1	90 (estimated)	1
Output amplitude (mVp)	25	50	N/A	45	40	
SFDR (dB) over BW	44 over 1 GHz	40 over 500MHz	N/A	47 over 30MHz	50 over 26MHz	48 26 36 1
Chip area (mm^2)	1.32 / channels	1.61 / channel	7	5.3	-4 / channel	cha
Technology	14nm FinFET	14nm FinFET	2Snm Bulk	40nm Bulk	22nm FinFET	22 Fi

169 Summary and Future Work

- Transmons are a well-developed leading contender for qubits to be used i quantum computing
- The processes for controlling and reading out transmons involve RF an wideband signals, and can be readily implemented using CMOS electronics
- The trade-offs involved in using cryogenic electronics to control transmon suggest that cryoCMOS is a promising approach
- As an example, a cryoCMOS RF quantum state controller ASIC for quantur computing control has been described
- Experimental results demonstrate successful qubit control

Chapter 06
Qiskit
QISKIT WORKSHOP Resources and Lecture Slides with Results

Kalyan Dasgupta, Ritajit Majumdar and Jagan Natarajan
IBM Research

DOI: 10.1201/9788743807087-7

In this hands on session, we will cover the basics of Qiskit, how to create a circuit, creating superpositions, entanglements and then measure. We will learn how to get the probability distributions of the output post measurement. We will also learn a little bit about primitives (Samplers and Estimators) and how they are used in specific algorithms. In the subsequent sessions, we cover topics on optimizing and running Quantum jobs in real hardware. The third session will cover the theoretical and implementation aspects of VQE. We will start with the theory part and then will do the exercises in Qiskit. In the process we will see how the primitives come into use.

Pre-Requisites for Workshop

Link for Google Colab Project.

Python Version and Google Colab.

- Please install Python version 3.11 or above and ensure that Jupyter Notebook is installed. This setup will allow you to simply download the necessary files from GitHub tomorrow, without needing to stay logged into Google Colab continuously.
- f you prefer not to install Jupyter Notebook, you can work directly on Google Colab, though this will require you to stay logged in

throughout the session.

Prior Knowledge of the Following Topics

- Bra and KetRepresentation
- Tensor Product
- Quantum Circuit Representation
- Qubit Representation
- Gates(X, H, Z, Rx, Ry, Rz)
- Superposition
- Entanglement
- Bloch Sphere
- Bell State, Partial State, Product State

IBM Lectures for Reference

Resources

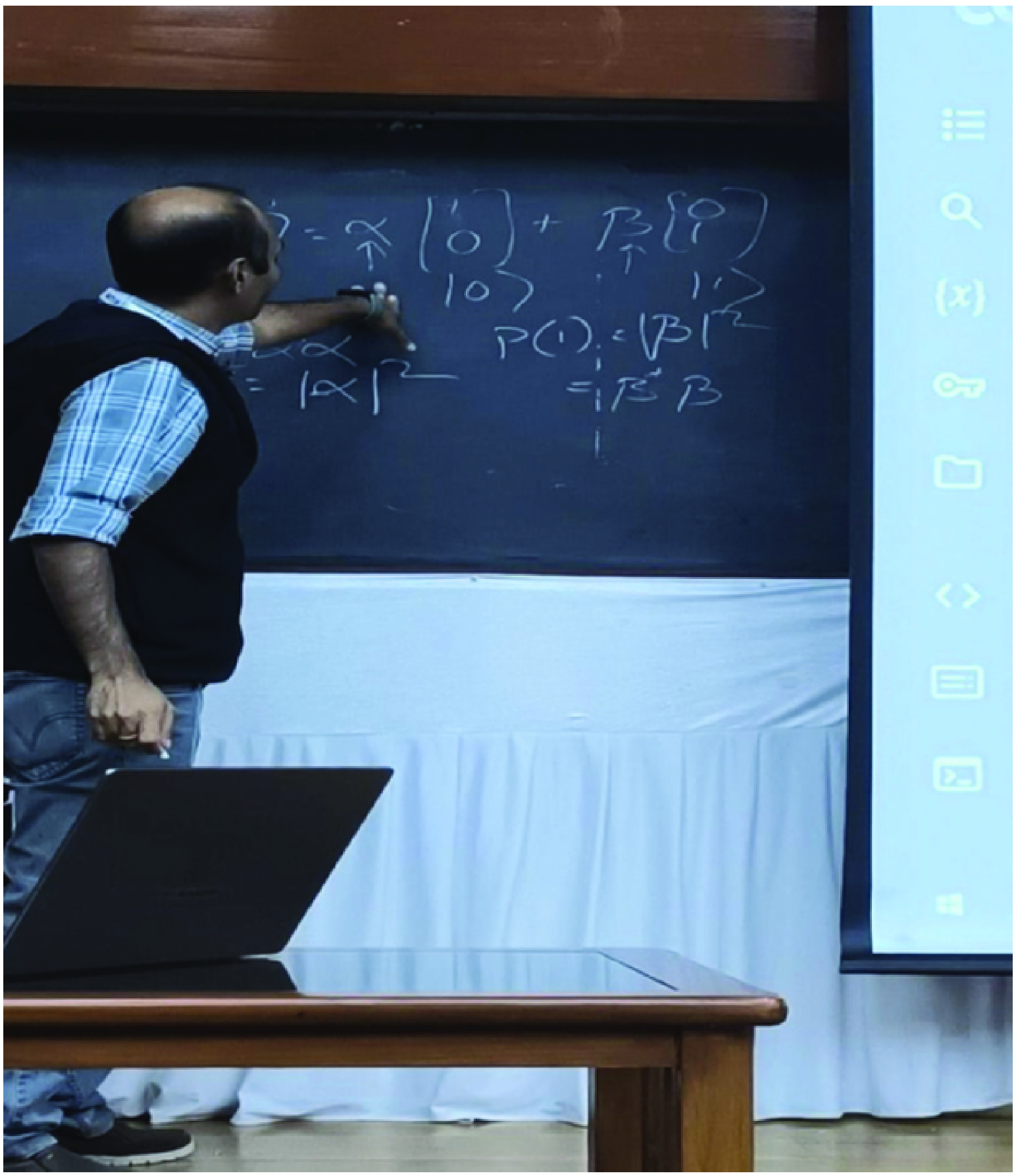
•https://learning.quantum.ibm.com/ (IBM Quantum Learning)
•https://www.youtube.com/watch?v=3-c4xJa7Flk&list=PLOFEBzvs-VvqKKMXX4vbi4EB1uaErFMSO&index=3 (Course by Prof.John Watrous)

Github Repository used for Workshop

Website: https://github.com/kaldag/CAS-Workshop

Qubit and Vector State

Vector States: We have used 2 Qubits in this example. |0> indicates Qubit state 0 represented by the corresponding vector state as shown in the image below.

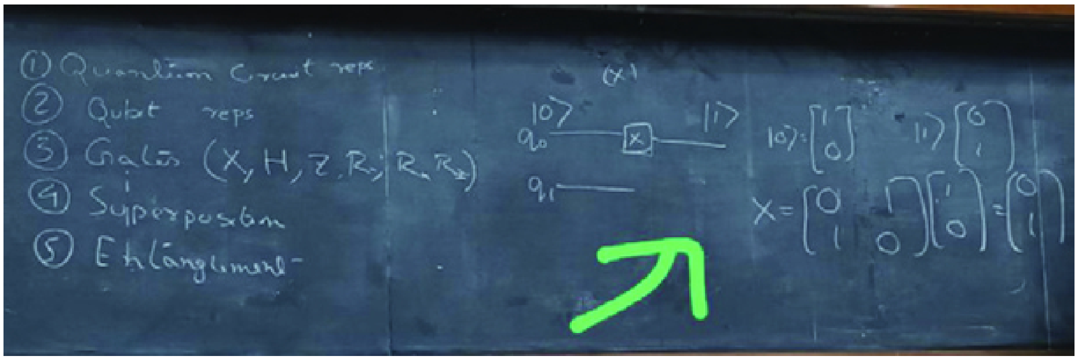


Qiskit_aer

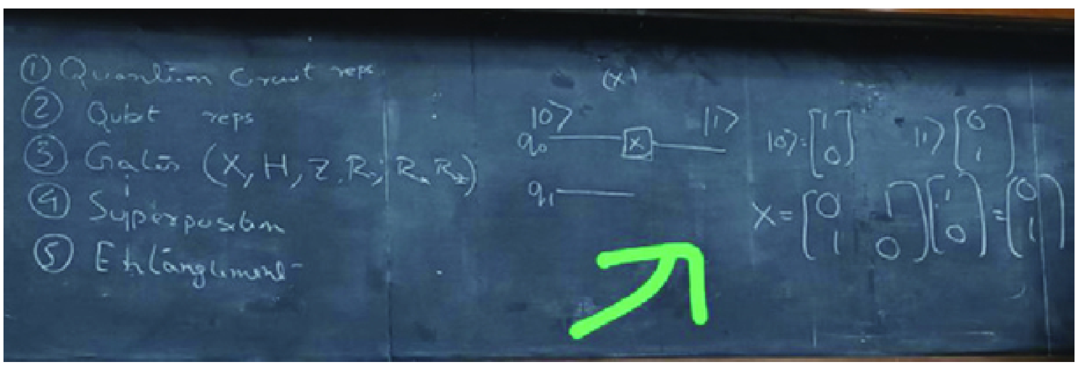
```
[ ] from qiskit_aer import Aer
```

This gives access to different simulators.

X Gate



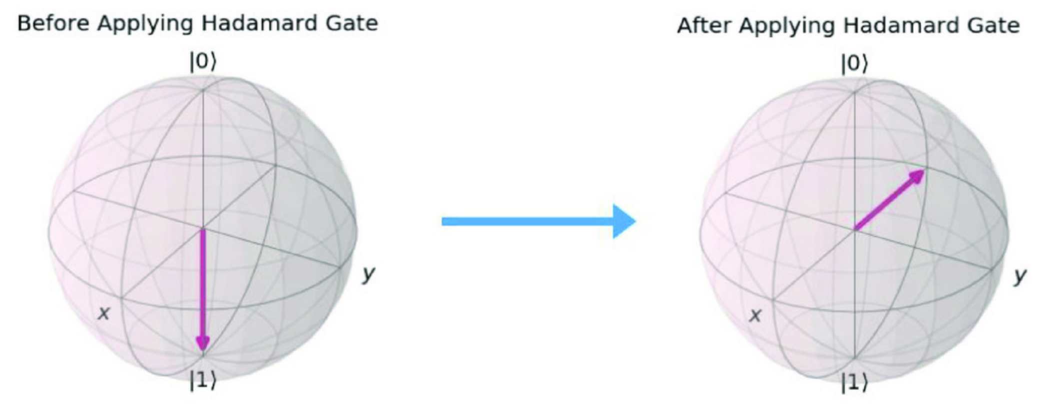
CNOT Gate



Hadamard Gate:

Hadamard Gate helps in creating the superposition state.

Hadamard Gate helps in creating the superposition state.



Bell States -> Maximally Entangled State

Bell states are specific examples of maximally entangled quantum states for two qubits. They form the foundation of quantum entanglement and are often used to demonstrate quantum phenomena like entanglement and non-locality. There are four Bell states, and each is a unique combination of two qubits that are fully entangled.

Product State -> When Measurement of One State Doesn't Affect Other State

1. A product state, or separable state, is a multi-qubit state where each qubit can be described independently of the others. In a product state, there’s no entanglement between the qubits, and each qubit can be expressed as a separate quantum state. The overall state of the system can be written as a simple product (or tensor product) of individual qubit states.
2. Product states are often the initial state in quantum computations. They have no entanglement, which means each qubit can be independently described without needing to consider correlations with other qubits.

Bell states are specific examples of maximally entangled quantum states for two qubits. They form the foundation of quantum

entanglement and are often used to demonstrate quantum phenomena like entanglement and non-locality. There are four Bell states, and each is a unique combination of two qubits that are fully entangled.

Partially Entangled States

In quantum computing, a partially entangled stateis a quantum state where two or more qubits are entangled but not maximally. This means that while the qubits exhibit some degree of correlation, they are not fully synchronized as they would be in a maximally entangled state (like a Bell state). If one qubit is measured, it gives some information about the state of the other qubit, but not complete information, as in a maximally entangled state.

Representation: A partially entangled state can be written as a superposition of basis states, but with varying amplitudes, For example, a common partially entangled state might look like this:

$$|\psi\rangle = a|00\rangle| + b|11\rangle$$

where $|a|^2$ + $|6|^2$ — 1 and *a* and *b* are not equal in magnitude. If $a = b = \frac{1}{\sqrt{2}}$, the state would be maximally entangled (a Bell state), but if |a| ≠ *b*, the entanglement is partial.

Let's go through an example of how to calculate the expectation value of a quantum observable in quantum computing.

Example: Calculating Expectation Value of an Observable

Suppose we have a quantum system in a state $|\psi\rangle$, and we want to calculate the expectation value of an observable $\hat{O}$. The expectation value is given by:

$$|O\rangle = \left\langle \psi \ \widehat{O} \ \psi \right\rangle$$

Step-by-Step Example

1. **Define the State:** Let's take a simple quantum state, say $|\psi\rangle$ which is a superposition of the $|0\rangle$ and $|1\rangle$ states. For example:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β are complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$ (normalization condition).

2. **Choose the Observable**: Let's consider a simple observable, say the Pauli Z operator, which is commonly used in quantum mechanics. The Pauli Z matrix is:

$$\widehat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This observable represents the measurement of the spin or polarization along the Z-axls.

3. **Calculate the Expectation Value:** The expectation value of $\hat{Z}$ in the state $|\psi\rangle$ is:

$$\langle Z \rangle = \left\langle \psi \ \widehat{Z} \ \psi \right\rangle$$

First, let's write $|\psi\rangle$ in vector form. Assume that $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$, which gives:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \begin{matrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{matrix}$$

Now, apply the $\hat{Z}$ operatorio $|\psi\rangle$

$$\widehat{Z}|\psi\rangle = \widehat{Z} \ \begin{matrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{matrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{matrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{matrix} = \begin{matrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{matrix}$$

Now, calculate the expectation value:

$$\langle Z \rangle = |\psi\rangle \ \widehat{Z} \ |\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{matrix}$$

This is a standard Inner product calculation:

$$\langle Z \rangle = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}} \right) = \frac{1}{2} - \frac{1}{2} = 0$$

To compute the expression $\langle 01|ZZ|01\rangle$,, we need to understand the components involved. Here, we are dealing with quantum states and the *ZZ* operator.

1. Quantum States:

- $\langle 01|$ is the bra corresponding to the state $|01\rangle$, which is the state of two qubits, with the first qubit in the $|0\rangle$ state and the second qubit in the $|1\rangle$ state.
- $|01\rangle$ is the ket corresponding to the same state.

2. The ZZ Operator:

- The ***ZZ*** operator is a two-qubit operator that acts on two qubits and is defined as:

$$ZZ = Z \otimes Z$$

where ***Z*** is the Pauli ***Z-***matrix. The Pauli ***Z-***matrix is:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

So the operator ***ZZ*** acts on the two-qubit state as follows:

$$ZZ = Z \otimes Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This means that the ***ZZ*** operator will apply a ¿T-operation to both qubits in the two-qubit state.

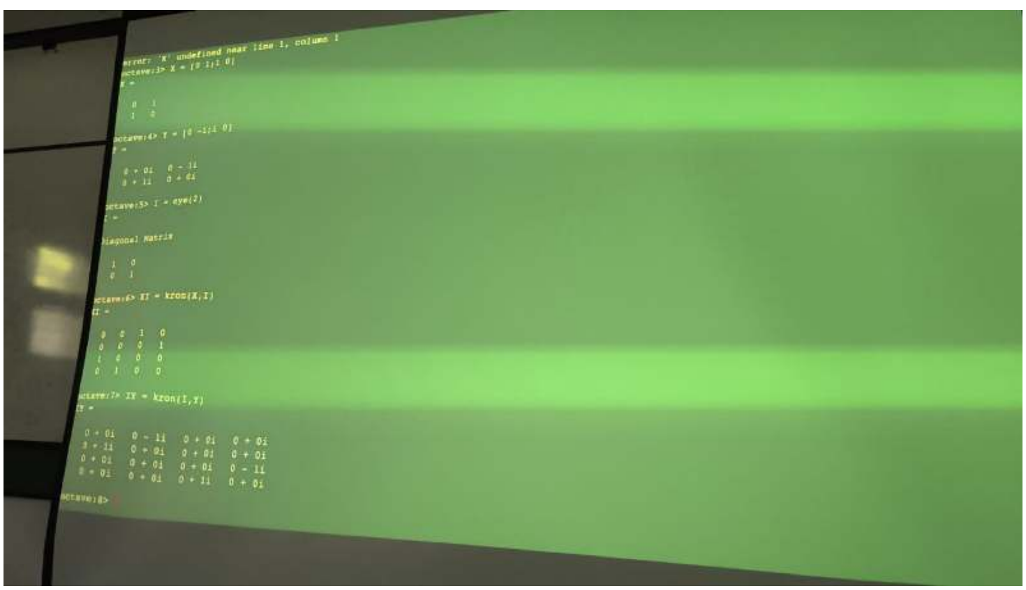
3. Apply the ZZ Operator:

Now, let's apply the ***ZZ*** operator to the state 01):

The ***ZZ*** operator will apply the Pauli Z-operator to each qubit. Here's how it acts on the state $|01\rangle$:

- For the first qubit $(|0\rangle), Z|0\rangle = |0\rangle$ (since ***Z*** leaves $|0\rangle$ unchanged).
- For the second qubit $(|1\rangle), Z|1\rangle = -|1\rangle$ (since ***Z*** appliesa phase flip to $|1\rangle$.
- So, applying ***ZZ*** to $|1\rangle$:

$$Z_1Z_1|01\rangle = (Z_2 \otimes Z_1)|01\rangle = Z|0\rangle \otimes Z|1\rangle = |0\rangle \otimes (-|1\rangle) = -|01\rangle$$



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