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Metrology and Measurement Uncertainty

Concepts and Applications

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We dedicate this new book to our beloved wives, Mariza and Marta, who are by our side and often in front of us, always encouraging us to live.

Thanks

We thank God for allowing us to be alive and healthy and for allowing us to have the proper resourcefulness to transfer our knowledge and experiences to readers of this work.

Presentation

Metrology is strategic for a nation's development and fundamental for organizations' technological and commercial growth. The specialist professional involved daily in measurement activities must know the mathematical foundations, statistical tools, techniques, practices, and operating procedures.

The International Vocabulary of Metrology—Basic and General Concepts and Associated Terms (VIM) [2] defines Metrology as the "science of measurements and its applications." The VIM complements this definition with a note: "Metrology includes all theoretical and practical aspects of measurements, whatever the measurement uncertainty and field of application."

Analyzing the definition, we need theoretical knowledge about the concepts and measurement techniques, the perception of the magnitudes of influence, and obtain consistent, practical results. Since internal and external factors influence measurement results for the measurement process, we need to estimate the measurement uncertainty associated with usage requirements.

The book's methodology for estimating measurement uncertainty follows the guidelines presented in the Guide for the Expression of Measurement Uncertainty (GUM) [4], recommended by the International Bureau of Weights and Measures (BIPM).

Throughout this book, we incorporated the conceptual definitions found in VIM and complemented them with additional clarifications when deemed necessary.

We also adopted the BIPM edition of the International System of Units (SI) [3] as an essential reference. It already incorporates the new definitions of the SI base units, which came into force on May 20, 2019.

This material is a new edition of the book published in Brazil in 2020 by *GEN—Grupo Editorial Nacional*, entitled "Metrologia e incerteza de medição: conceitos e aplicações" [1]. This new edition, which maintains the same original name, was adapted to an international metrological context and expanded. We include an additional chapter highlighting measurement uncertainty in conformity assessment processes.

In this book, we seek to present and discuss the concepts and tools in a very technical language but in a didactic, clear, and simple way. Several practical examples and solved exercises were incorporated, as well as several others to be worked on by the reader. Mid- and higher-level professionals can absorb and apply the knowledge immediately in the industry or their laboratory activities.

The first chapter analyzes the history of measurements and units of measurement, ending with the presentation of the International System of Units (SI) and associated concepts.

The second chapter addresses the basic concepts of metrology, discussing its importance and objectives. It presents the metrological structure at the international, regional, and national levels for metrology (legal and scientific) and ends with the interconnection of metrology with standardization.

In the third chapter, we present the concept of significant digits, the rounding techniques, and how to apply them in a measurement result. We highlight the main ideas and statistical tools used in metrology, such as the mean, standard deviation, variance, and the most usual probability distributions adopted in the study of metrology (uniform, triangular, normal, and t-Student).

Chapter 4 analyzes the critical metrological characteristics of measurement systems, presents the types and possible errors encountered in the measurement process, and reinforces the concepts of accuracy and precision.

The fifth and sixth chapters explore the types of uncertainty and how to evaluate their values, considering the measurement carried out both directly and indirectly. As mentioned, the methodology for estimating measurement uncertainty follows the Guide for the Expression of Measurement Uncertainty (GUM) guidelines.

We dedicate three chapters of this book to calibration and measurement uncertainty in process conformity.

We detail the metrological traceability chain and how to choose a measurement standard considering the process's tolerance. We present calibration examples for different measuring instrument types. We adjust the calibration points using a function and check the influence of this adjustment on the final uncertainty. We also perform a detailed analysis and interpretation of the calibration certificates.

However, knowing that knowledge is never too much, all criticism and suggestions that improve this book will always be welcome.

Good studies, and thank you very much.

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Preface of the Brazilian Edition

The book's publication by Alexandre Mendes and Pedro Paulo Novellino do Rosário is more than opportune. Unfortunately, in Brazil, there is a significant shortage of didactic publications dedicated to metrology intended to train professionals, whether at the middle or higher level. You can count the books offering courses or disciplines in the area on your fingers. To compound this situation, some available material is outdated and still refers to outdated concepts or definitions.

The future, getting closer every day, will require professionals with good technical and creative training. These professionals, not only those in the areas of Engineering and Technology but also in the Natural Sciences and other areas, must have a complete conceptual basis to account for performance in environments where measurements are taken, use of standards and technical regulations, understanding of the measurement process, correct expression of results and associated uncertainties.

The acronym STEAM (Science, Technology, Engineering, Arts, and Mathematics) describes the foundation of the new professions. Metrology and measurement have a key role as a transversal basis for this knowledge.

As quoted by the authors, and never emphasized enough:

> When you can measure what you are talking about and express it in numbers, you know something about it. However, when you cannot measure it and cannot express it in numbers, your knowledge is limited and unsatisfactory: it may be the beginning of knowledge, but you, in your thinking, have advanced very little towards the stage of science. (LORD KELVIN)

The Strategic Guidelines for Metrology in Brazil address and update this concern. However, fundamental metrology concepts are still clearly lacking in many areas of professional training.

Professionals not affectionate to the metrological area (such as health and environmental professionals, laboratory technicians, and industrial sectors, among others) increasingly need to deal with sophisticated and high-tech equipment and instruments in situations where measurement processes and measured quantities must be well-known, interpreted, analyzed, and treated to reflect reliable values, often with a significant impact on health, safety, and the environment.

In this context, expanding and disseminating information on metrology principles, technical barriers, conformity assessment, and standardization for the general population can provide society with technical knowledge that helps citizens know their rights and improve their quality of life (BRAZIL, 2017, p.59).

Written in direct and rigorous language but with a fluidity that facilitates reading and learning, this work is based on the discipline's fundamental compendia in their latest editions: the International Vocabulary of Metrology, the International System of Units, and the Guide to the expression of measurement uncertainty.

It is also based on the authors' enormous experience in discussion, professional performance, and teaching practice in formal, middle, and higher-level or continuous training processes. The authors have already written a book on the subject, having published several technical, didactic, and scientific works in collaboration with numerous experts.

However, in my humble opinion and to our happy surprise, the work that comes to us is a new book, with much content, updated, covering more topics, and written in a way that allows the student (at any level) to have an understanding most complete of the discipline.

Américo Tristão Bernardes

Associate Professor at the Federal University of Ouro Preto

President of the Brazilian Society of Metrology

Keywords for the Book *Metrology; Measurement uncertainty; Traceability; Metrological reliability; Probability distribution; Measuring instrument; Calibration; Calibration certificate; Measurement error; Bias; Measurement; Accuracy; Precision; Conformity assessment; Accuracy class; Measurement standard; Verification*

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About the Authors

Pedro Paulo Novellino do Rosário has a master's degree in Metrology for Industrial Quality from the Pontifical Catholic University (PUC/RJ), a postgraduate in Marketing from PUC/RJ, a degree in Electronic Engineering from the Federal University of Rio de Janeiro, and a specialization in Industrial Instrumentation Engineer by Petrobras. He served as director at the Rio de Janeiro State Industrial Development Company, responsible for attracting new industrial enterprises in the oil & gas, chemical, petrochemical, automotive, food, and drink sectors. He served as Executive Secretary of the Brazilian Society of Metrology, responsible for stimulating scientific research and technological innovation within scientific, industrial, and legal metrology. In the laboratory field, he was a consultant for calibration and testing laboratories in laboratory accreditation processes according to ISO/IEC 17025. He has developed projects and provided consulting and training in quality, metrology, and industrial and laboratory instrumentation for various institutions. In the academic field, he was a university professor in metrology, standardization, and industrial instrumentation and published books and booklets in these areas.

Alexandre Mendes has a degree in Physics from the Federal University of Rio de Janeiro, a master's in Metrology from PUC-Rio, and a PhD in Mechanical Engineering from PUC-Rio. He is a full professor of metrology at the Federal Institute of Rio de Janeiro (IFRJ). In 2000, he implemented and coordinated the IFRJ Metrology Technical Course offered by the Volta Redonda Campus today. He was General Director of the Volta Redonda campus in the period 2008–2014. He has experience in physics and metrology, focusing on uncertainty calculation, calibration and industrial instrumentation, experimental physics, physics teaching, and professional qualifications. He is the author of the books *Physics in the Park*, co-authorship with Henrique Lins Barros; *Metrology and Uncertainty of Measurement*, co-authorship with Pedro Paulo N. Rosário, and Terms and Expressions of Metrology Applicable to the Health Environment—Orientation Document—SBM (Brazilian Society of Metrology). He is Vice-President of the Brazilian Society of Metrology.

Chapter 1
The International System of Units (SI)

1.1 Units of Measure: A Brief History

Since the beginning of civilization, human beings have always had to perform measurements, even if this was done intuitively. When they began to live in a community, this need to measure and weigh increased. With the advent of commerce and the establishment of private property, measuring the land and the results of its work was necessary for sale or exchange.

The first units of measure arose from their daily use and were based on parts of the human body. In principle, they could be considered "references," that is, anyone else could verify a measure. Thus emerged measurement units such as the inch, the hand, the foot, the yard, and the step. It is obvious these "references" were not fixed, as the human body is not standardized, and measures vary from individual to individual.

The Egyptians also used the size of the cubit, one of the forearm bones, as a standard of length measurement. Again, as the cubit varied from one person to another, Pharaoh Khufu established a granite standard based on the bone length of your arm during the construction of its pyramid (about 2900 BC). This pattern, whose reproduction we see in Fig. 1.1, was called the Egyptian royal cubit.

Over time, wooden bars to facilitate transportation replaced the granite bars, but as wood was worn out, lengths equivalent to the royal cubit were recorded on the walls of the main temples. This way, people could periodically check their wooden bars or do others.

In France, in the seventeenth century, a unit of linear measurement was standardized in a two-pin iron bar at the extremes, forming a calibrator. The distance between these two pins was considered a "*toise*," the bar was spoiled on the outer wall of the Grand Châtelet, the fortification that kept the head of one of the bridges of access to Paris.

Thus, as in the case of the standard cubit, interested parties could check their measuring instruments (Fig. 1.2).

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Fig. 1.1 Reproduction of the Egyptian royal cubit. (Photo by the authors)

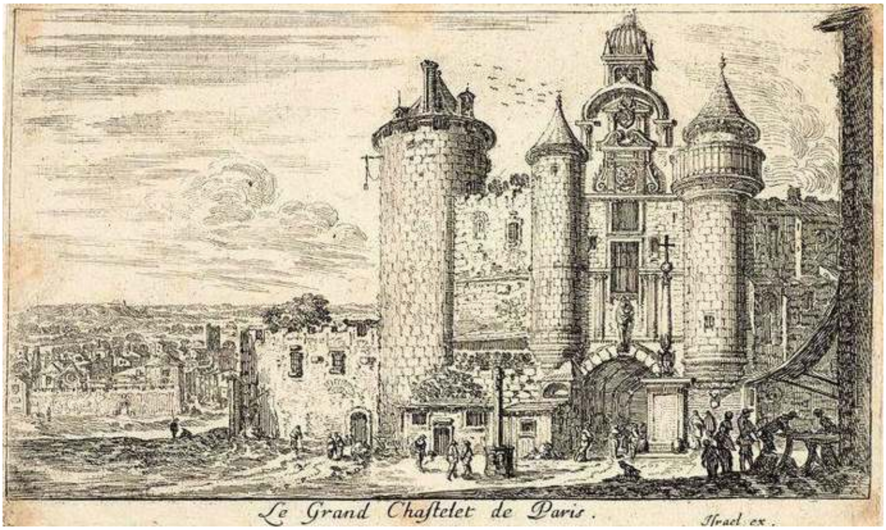


Fig. 1.2 Le Grand Châtelet—one of the oldest fortifications in Paris. (https://www.pinterest.fr/pin/319544536061326825/)

These unit systems, based on the human body, were used until the end of the eighteenth century when a revolutionary movement arose in France.

The French Revolution in 1789 resulted from the dissatisfaction of the bourgeois, composed of traders, artisans, and liberal professionals, who disagreed with King Louis XVI's absolutist domain and its privileges. They considered that a set of measures based on the anatomy of kings did not have any scientific basis, so a new measurement system that valued science should be conceived and could be adopted, with the same accuracy, around the world, and in all business transactions.

Members of the French Academy began to discuss the best way to elaborate a metric system. In 1790, Charles-Maurice de Talleyrand-Périgord presented a proposal to the National Assembly saying that the wide variety of weights and measures generated confusion and obstructed trade (Fig. 1.3).

Fig. 1.3 Talleyrand. (https://media.gettyimages.com/id/2381529/pt/foto/charles-maurice-de-talleyrand-perigord)

Knowing a Little More …

Better known as Talleyrand, he was a French politician and diplomat. It demonstrated admirable political survival capacity by holding high positions in the French revolutionary government under Napoleon during the restoration of the Bourbon monarchy and under King Louis Filipe. After 100 days of Napoleonic, he assumed the position of Chairman of the State Council, but his revolutionary past led him to be fired in September of the same year. Allied to liberals, he actively participated in the rise to the throne of Louis Filipe of Orleans. The ambassador in London had a fundamental participation in the negotiations between France and the United Kingdom, as in the creation of the Kingdom of Belgium and the signing of the covenant between France, the United Kingdom, Spain, and Portugal—the quadruple alliance. Accused of a cynical and immoral life, he claimed to serve France, not political regimes. He was one of the most controversial figures in France.

The academy wanted a standard and repelled arbitrary and uncovered definitions. The name *meter* is adopted for the basic unit of length, which came from the Greek word *metron*, which means measure. The meter was defined as a measure equivalent to one-tenth of the millionth of the distance between the North Pole and the Ecuador line throughout the meridian, which went from Dunkerque to Barcelona.

A system with multiples and submultiples was sought, and volume units were created. These were then used to form cubes with length measurements and weight units filled with distilled water.

Thus, the units of length, volume, and mass were interconnected, with the entire system deriving from a unique, universal, and invariable pattern: the meter. On March 30, 1791, the Assembly approved this measurement system. On April

Table 1.1 Measures used by England

Unit	SI equivalent
1 inch	25.4 mm
1 foot	304.8 mm
1 yard	0.9144 m
1 mile	1609 m
1 grain	64.799 mg
1 ounce	28.35 g
1 pound	453.6 g
1 ton	1016.05 kg

7, 1795, the National Convention forced the use of the metric system, adopting the names "meter," "liter," and "gram," with multiples and submultiples.

The changes were not well seen by England, which claimed to be a country whose economy was based on industry, commerce, and finance, and that abrupt changes would damage its growth, forcing it to change the dimensions of most exports and units used in pieces of machinery.

They claimed that moderate changes should be made since, with the decree of the imperial act of weights and measures, a measuring system elaborated based on Roman units was used throughout the British Empire.

Table 1.1 presents the measures most commonly used by England.

An international commission, instituted on August 8, 1870, and formed by delegates from 30 countries, proposed the establishment of an organization funded by the member countries. This commission would be tasked with defining and maintaining new standards, verifying countries' standards, and developing new instruments.

On May 20, 1875, a date known as International Metrology Day, 17 countries (Argentina, Austria-Hungary, Belgium, Brazil, Denmark, France, Germany, Italy, Peru, Portugal, Russia, Spain, Sweden-Norway, Switzerland, Ottoman Empire, United States of America, and Venezuela) created the BIPM (International Bureau of Weights and Measures) during the last session of the Metre Diplomatic Conference.

The French government made the headquarters of BIPM, 43,520 m^2, available. It is close to Paris, in the domains of the Breteuil Pavillon (Saint-Cloud Park). The members of the Meter Convention (currently 64 Member States and 36 associates) ensure that BIPM expenses are maintained (Fig. 1.4).

Knowing a Little More…

Even before the Meter Treaty (1875) definition, several scientists were already working on determining units of measure. In 1832, mathematician and scientist Carl Friedrich Gauss elaborated a system to consolidate all units into three. The velocity unit, for example, would be the combination of the distance unit (meter) with the unit of time (second), giving the unit m/s. The force unit would be the combination of the mass unit (kg) with the acceleration unit

(continued)

(m/s^2), which would give rise to kg m/s^2, also known as Newton (N). In the 1860s, James Clerk Maxwell and William Thomson (Lord Kelvin) established a basic unit system that, coupled with derived units, would compose a system of coherent units.

The first General Conference on Weights and Measures (CGPM), in 1889, adopted a materialized prototype standard in a 10% platinum–iridium bar, which is stored in BIPM to the present day.

From the subscription of the Metre Treaty, metrology advanced rapidly, and in 1921, the sixth CGPM amended the treaty. The metric system incorporated the second and ampere, being called MKSA (meter, kilogram, second, and ampere).

The 11th CGPM revised the metric system, then known as the International System of Units (SI), on October 14, 1960.

In 1983, the meter was defined as the "length of the path traveled by the light in vacuum for a time interval of 1/(299,792,458) of a second." Finally, the standard of length is no longer represented by a platinum bar and is immaterialized; that is, it contained physical greatness to describe it, being in charge of metrologists to ensure technology everywhere in the world can reproduce it.

The great discussion continued with the dematerialization of the kilogram unit, also defined as the mass of a platinum–iridium cylinder maintained at the headquarters of BIPM. For many years, until 2019, it was the only SI unit still represented by a materialized object.

Brazil is our native country, one of the 17 Meter Diplomatic Conference signatory countries. Allow us a brief history. During the reign of D. Pedro I, the units of measure followed the standards of Portugal. On June 26, 1862, D. Pedro II

Fig. 1.4 BIPM (International Bureau of Weights and Measures). (https://www.bipm.org/documents/20126/43899263/pavillon-de-breteuil-garden-september-21-sized.jpg)

promulgated Imperial Law Number 1157 and adopted the French decimal metric system throughout the country. As mentioned earlier, Brazil was one of the first nations to adopt the new system.

In 1961, the National Institute of Weights and Measures (INPM) was created, and the International System of Units became the official system through Decree 52,243 of August 30, 1963, later replaced by Decree 63,323, September 12, 1968. In 1973, the INPM was replaced by the National Institute of Metrology, Standardization and Industrial Quality (INMETRO), currently called the National Institute of Metrology, Quality and Technology.

1.2 The International System of Units (SI)

The International Vocabulary of Metrology (VIM) defines the International System of Units (SI) as follows:

> System of units, based on the International System of Quantities, their names and symbols, including a series of prefixes and their names and symbols, together with rules for their use, adopted by the General Conference on Weights and Measures (CGPM). [VIM—1.16]

To complete the reasoning, we present the definition of the International System of Quantities:

> System of quantities based on the seven base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.[VIM—1.6]

Some characteristics of SI:

- Unique units which can be reproduced and performed anywhere in the world;
- Few base units, separate and independent;
- Coherent, the combination of existing units produces other units without constants.

Knowing a Little More…
Since 1970, the BIPM has published the International System of Units in *SI Brochure* or *Brochure Sur Le SI* (in French), printed and digital versions. (source: https://www.bipm.org/en/publications/si-brochure)

The SI is the system of units in which:

- The unperturbed ground state hyperfine transition frequency of the cesium 133 atom $\Delta\nu_{Cs}$ is 9,192,631,770 Hz,
- The speed of light in vacuum c is 299,792,458 m/s,
- The Planck constant h is $6.626{,}070\ 15 \times 10^{-34}$ J s,
- The elementary charge e is $1.602{,}176\ 634 \times 10^{-19}$ C,
- The Boltzmann constant k is $1.380{,}649 \times 10^{-23}$ J/K,
- The Avogadro constant N_A is $6.022{,}140\ 76 \times 10^{23}$ mol^{-1},

- The luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} Hz, K_{cd}, is 683 lm/W,

  where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, meter, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = s^{-1}, J = kg m^2 s^{-2}, C = A s, lm = cd m^2 m^{-2} = cd sr, and W = kg m^2 s^{-3}.

The numerical values of the seven defining constants do not have uncertainty. (Source: SI brochure 9th edition)

1.2.1 Quantity

Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference.

A reference can be a measurement unit, procedure, reference material, or a combination.

The concept of 'quantity' may be generically divided into, e.g., 'physical quantity,' 'chemical quantity,' and 'biological quantity,' or base quantity and derived quantity. [VIM—1.1]

Based on the information found in VIM, the nature of quantity is a common aspect of mutually comparable quantities. The division of "quantity" according to the "nature of a quantity" is somewhat arbitrary.

Knowing a Little More…

The quantities diameter, circumference, and wavelength are usually considered of the same nature, that is, of the quantity called length.

Heat, kinetic energy, and potential energy are usually considered part of the same quantity called energy.

Quantities of the exact nature in a given system of quantities have the same dimension. However, quantities of the same dimension are not necessarily similar.

The momentum of strength and energy are not, by convention, considered of the exact nature, although they have the same dimension. The same occurs for thermal capacity and entropy, as well as for the number of entities, relative permeability, and mass fraction.

A base quantity is the greatness of a chosen subset by convention of a given system of quantities, in which no greatness of the subgroup can be expressed as a function of others. The subset mentioned in the definition is called a set of base quantities. Base quantities are considered mutually independent, as a product of powers from other base quantities cannot express base quantities.

A derived quantity is defined as a function of the base quantities of this system.

In a system of quantities with the length and mass as its base quantities, the specific mass is a derived quantity defined by the quotient of a mass by a volume (length to the cube).

1.2.2 Measurement Unit

Real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number.

NOTE 1 Measurement units are designated by conventionally assigned names and symbols.

NOTE 2 Measurement units of quantities of the same quantity dimension may be designated by the same name and symbol even when the quantities are not of the same kind. For example, joule per kelvin and J/K are respectively the name and symbol of both a measurement unit of heat capacity and a measurement unit of entropy, which are generally not considered to be quantities of the same kind. However, in some cases, special measurement unit names are restricted to be used with quantities of a specific kind only. For example, the measurement unit 'second to the power minus one' (1/s) is called hertz (Hz) when used for frequencies and becquerel (Bq) when used for activities of radionuclides.

NOTE 3 Measurement units of quantities of dimension one are numbers. In some cases, these measurement units are given special names, e.g., radian, steradian, and decibel, or are expressed by quotients such as millimole per mole equal to 10^{-3} and microgram per kilogram equal to 10^{-9}.

NOTE 4 For a given quantity, the short-term "unit" is often combined with the quantity name, such as "mass unit" or "unit of mass." [VIM—1.9]

In SI, there are two classes of measurement units: the base and the derived units.

1.2.3 Base Unit

Measurement unit that is adopted by convention for a base quantity. [VIM—1.10]

Base units are seven independent physical quantities. Table 1.2 presents the new definitions and symbols of the base units since May 20, 2019.

Knowing a Little More …

The General Conference on Weights and Measures (CGPM), in its 25th meeting in November 2014, adopted a resolution on the new revision of the International Unit System (SI), validated at the 26th meeting in 2018. In this review, the kilogram, ampere, kelvin, and mole were redefined based on the fixed numerical values of Planck constant (h), elementary load on a proton (e), Boltzmann constant (k), and Avogadro constant (N_A), respectively.

Subsequently, the seven basic SI units were defined based on seven reference constants, to be known as "SI defining constants": the hyperfine transition frequency of the cesium—second; the speed of light in the vacuum—meter; the Planck constant—kilogram; the elementary load in a proton—ampere; Boltzmann's constant—kelvin; the Avogadro constant—mol; and the luminous efficacy of a specified monochrome source—candela. This has resulted in a more straightforward and more fundamental definition of the

(continued)

whole and dismissed the last of the definitions based on a material artifact—the international prototype of the kilogram maintained in BIPM.

The main disadvantage of the old definition of the kilogram was that it referred to the mass of the artifact, which, by nature, we know is not stable. Comparisons between the official copies and the international prototype showed some disagreement over time. The drift in the mass of the international prototype, since 1889 has yet to be demonstrated, but it should undoubtedly be present. The change rate of its mass could be determined only by insufficiently high absolute experiences.

The new kilogram unit can measure with the "watt scale (or kibble scale)," an instrument that allows mechanical with electromagnetic energy to compare two separate experiences. Ampere can be measured using Ohm's law ($A = V/\Omega$) and practical achievements of V and Ω, based on Josephson and Quantum Hall effects. Kelvin can be defined by the new system using acoustic thermometry. The technique allows you to determine the speed of sound in a sphere full of gas at a fixed temperature. The mole can be performed as the accurate amount of atoms in a perfect sphere of pure-28 silicon. (Source: Adapted from https://www.bipm.org/en/publications/misses-en-pratique/)

1.2.4 Derived Unit

Measurement unit for a derived quantity. [VIM—1.11]

They are units formed by combining base units according to mathematical relations that correlate the corresponding quantities. Table 1.3 presents some examples of derived units.

1.2.5 Dimensional Analysis of the Quantities

Dimensional analysis studies the quantities and relationships between the respective measurement units of these quantities. The study of dimensional analysis becomes a powerful ally that helps us write SI and obtain some equations involving physical quantities.

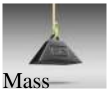
The VIM provides the following definition for quantity dimension:

Expression of the dependence of a quantity on the base quantities of a system of quantities as a product of powers of factors corresponding to the base quantities, omitting any numerical factor. [VIM—1.7]

Table 1.4 presents the symbols corresponding to the dimensions of quantities.

According to VIM—1.7, the size of a quantity Q is represented by:

Table 1.2 SI base units

Quantity	Unit	Symbol	Definition
 Length	meter	m	It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299,792,458 when expressed in the unit m s^{-1}, where the second is defined in terms of the cesium frequency $\Delta\nu_{Cs}$
 Electric current	ampere	A	It is defined by taking the fixed numerical value of the elementary charge *e* to be 1.602,176 634 × 10^{-19} when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$
 Luminous intensity	candela	cd	It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} m^{-2} s^3, where the kilogram, meter, and second are defined in terms of h, c, and $\Delta\nu_{Cs}$
 Mass	kilogram	kg	It is defined by taking the fixed numerical value of the Planck constant h to be 6.626,070 15 × 10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the meter and the second are defined in terms of c and $\Delta\nu_{Cs}$
 Amount of substance	mole	mol	One mole contains exactly 6.022,140 76 × 10^{23} elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number
 Thermodynamic temperature	kelvin	K	It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380,649 × 10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} K^{-1}, where the kilogram, meter, and second are defined in terms of h, c, and $\Delta\nu_{Cs}$
 Time	second	s	It is defined by taking the fixed numerical value of the cesium frequency $\Delta\nu_{Cs}$, the unperturbed ground state hyperfine transition frequency of the cesium 133 atom, to be 9,192,631,770 when expressed in the unit Hz, which is equal to s^{-1}

Source: SI brochure 9th edition

$$dim\ (Q) = \mathbf{L}^{\alpha}\,\mathbf{M}^{\beta}\,\mathbf{T}^{\gamma}\,\mathbf{I}^{\delta}\,\mathbf{\Theta}^{\varepsilon}\,\mathbf{N}_{\xi}\,\mathbf{J}^{\eta}$$

Where the dimensional exponents α, β, γ, δ, ε, ξ, and η can be positive, negative, or zero.

Solved Exercise 1.1

In 1851, the English physicist and mathematician George Stokes deduced a formula for the frictional force that acts in a sphere of radius R immersed in a liquid of dynamic viscosity η, which moves at speed v. The formula deduced by stakes is $F = 6\pi R\eta v$. Considering this formula, what is the dynamic viscosity unit in the Si?

Table 1.3 Examples of derived units

Quantity	Unit	Symbol	Base unit	Other SI units
Plane angle	Radian	rad	m/m	
Area	Square meter	A	m^2	
Electric field	Volt per meter	V/m	m kg s^{-3} A^{-1}	
Capacitance	Farad	F	m^{-2} kg^{-1} s^4 A^2	C/V
Electric charge	Coulomb	C	s A	
Electric conductance	Siemens	S	m^{-2} kg^{-1} s^3 A^2	A/V
Electric potential difference	Volt	V	m^2 kg s^{-3} A^{-1}	W/A
Energy, work, amount of heat	Joule	J	m^2 kg s^{-2}	N m
Luminous flux	Lumen	lm	cd	cd sr
Force	Newton	N	m kg s^{-2}	
Frequency	Hertz	Hz	s^{-1}	
Inductance	Henry	H	m^2 kg s^{-2} A^{-2}	Wb/A
Density, mass density	Kilogram per cubic meter	ρ	kg/m^3	
Power, radiant flux	Watt	W	m^2 kg s^{-3}	J/s
Pressure	Pascal	Pa	m^{-1} kg s^{-2}	N/m^2
Electric resistance	Ohm	Ω	m^2 kg s^{-3} A^{-2}	V/A
Celsius temperature	Degree Celsius[a]	°C	K	
Magnetic flux density	Tesla	T	kg s^{-2} A^{-1}	Wb/m^2
Velocity, speed	Meter per second	v	m/s	
Volume	Cubic meter	V	m^3	

Source: SI brochure 9th edition

[a]The degree Celsius is used to express Celsius temperatures. The numerical value of a temperature difference or temperature interval is the same when expressed in degrees Celsius or kelvin

Table 1.4 Dimensions of base quantities

Base quantity	Quantity symbol	Symbol for dimension
Length	l, x, r	L
Mass	m	M
Time	t	T
Electric current	I, i	I
Thermodynamic temperature	T	Θ
Amount of substance	n	N
Luminous intensity	I_v	J

Solution: Let us write the equation as a function of η. Therefore, we have:

$$\eta = \frac{F}{6\pi R v}$$

$$\dim(\eta) = \frac{\dim(F)}{\dim(R) \cdot \dim(v)}$$

$$\dim(R) = L$$

$$\dim(v) = LT^{-1}$$

The quantity force (newton) is expressed by the formula $f = m \cdot a$, where a is the acceleration of the body with mass m.

Then:

$$a = \mathrm{m/s^2} \rightarrow \dim(a) = LT^{-2}$$
$$\dim(F) = \dim(m) \cdot \dim(a)$$
$$\dim(F) = MLT^{-2}$$

Substituting in the dim(η) equation, we have:

$$\dim(\eta) = \frac{MLT^{-2}}{L \cdot LT^{-1}} = ML^{-1}T^{-1}$$

Thus, the unit of η is kg $\mathrm{m^{-1}}$ $\mathrm{s^{-1}}$. However, as the pascal unit (Pa) is kg $\mathrm{m^{-1}}$ $\mathrm{s^{-2}}$, we can represent the dynamic viscosity unit in SI as pascal second (Pa s).

Solved Exercise 1.2

We have the equation $P = v^2 k$, where v is velocity. Since P is pressure, k must be.

(a) Mass
(b) Density
(c) Mass flow
(d) Weight

Solution: Let us write the equation as a function of k.

$$k = \frac{P}{v^2}$$

Doing a dimensional analysis of k:

$$\dim(k) = \frac{\dim(P)}{\dim(v^2)}$$

As pressure is force/area, we have:

$$\dim(P) = \frac{\dim(\text{force})}{\dim(\text{area})}$$

$$\dim(P) = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

The dimension of v^2 is:

$$\dim\left(v^2\right) = L^2T^{-2}$$

Therefore, the dimension of k is:

$$\dim(k) = \frac{\dim(P)}{\dim(v^2)} = \frac{ML^{-1}T^{-2}}{L^2T^{-2}} = ML^{-3}$$

In SI, this represents, $Un(k) = \frac{\text{kg}}{\text{m}^3}$, the density unit. Therefore, the correct answer is (b).

1.2.6 Decimal Multiples and Submultiples

Multiples and submultiples were defined in the SI, with the names and symbols given in Table 1.5.

Except for the prefixes da (deca), h (hecto), and k (kilo), all multiple prefix symbols are written with capital letters, and all submultiple symbols are written with lowercase letters. All prefix names are written with lowercase letters, except at the beginning of a sentence.

Table 1.5 Decimal multiples and submultiples

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{30}	ronna	R	10^{-1}	deci	d
10^{27}	quetta	Q	10^{-2}	centi	c
10^{24}	yotta	Y	10^{-3}	milli	m
10^{21}	zetta	Z	10^{-6}	micro	μ
10^{18}	exa	E	10^{-9}	nano	n
10^{15}	peta	P	10^{-12}	pico	p
10^{12}	tera	T	10^{-15}	femto	f
10^{9}	giga	G	10^{-18}	atto	a
10^{6}	mega	M	10^{-21}	zepto	z
10^{3}	kilo	k	10^{-24}	yocto	y
10^{2}	hecto	h	10^{-27}	ronto	r
10^{1}	deca	da	10^{-30}	quecto	q

Source: SI brochure 9th edition

Although the multiples da (deca) and h (hecto) and submultiple d (deci) are not foreseen, their use is not shared, and it is recommended to express in k (kilo), m (milli), or μ (micro).

1.2.7 SI Units and Symbols Writing Rules

The writing rules of symbols and units were initially proposed by the 9th CGPM in 1948. They were then adopted by ISO/TC 12 (ISO 31, quantities and units). Some rules are presented below.

1. The symbols are expressed with lowercase letters and in Roman characters.

   Example: meter (m) second (s)
   The exceptions are the Greek letter Ω (unit of electrical resistance) and the liter unit, which can also be written with L.
   Note: The liter is not an SI unit, but its use is accepted.

2. If the unit's name is a proper name, the first letter of the symbol is capitalized, but it is written with a lowercase letter.

   Example: pascal (Pa) kelvin (K)
   The spelling of °C is degree Celsius, as the degree unit begins with a lowercase letter. Celsius is an adjective that starts with a capital letter because it is a proper name.

3. The symbols of the units have no plural and are not followed by points.

   Example: 10 kg 500 m 25 s

4. When dividing one unit by another, use an inclined bar, horizontal trace, or negative power.

   Example: km/h $\frac{\text{km}}{\text{h}}$ km h^{-1}

5. To avoid ambiguities, use only one inclined bar, parentheses, or negative powers.

   Example: m/s^2 or m s^{-2} and never m/s/s

6. The multiplication of the symbols of the units must be indicated by a space or a point centered at half height (·).

   Example: newton meter → N m or N•m

7. The tonic accent does not fall on the prefix but on the unit.

   Example: micro**me**ter kilo**me**ter

8. In writing a unit composed of the multiplication of unit names, a space or a hyphen should be used to separate the names from the units.

Example: Pa s → pascal second or pascal-second.

9. Time measurements:

   Correct: 5 h 14 min   3 h 30 min 15 s   2 h
   Wrong: 5:14 h   3 h 30′15′   3:30:15 h
   Note: The hour (h) and the minute (min) are not SI units, but their use is accepted.

10. The numerical value precedes the unit, and there is always a space between the number and the unit. Thus, since the value of a quantity is the product of a number by a unit, the space is considered a sign of multiplication.

    Example: 124.6 mm   45.9 °C   50 kg
    Exception for this rule is the symbols of the units of grade (°), minute (′), and second (″) of the flat angle (units outside of SI), for which there is no space between the numerical value and the symbol of the unit. Example: 45° 25′ 6″

11. When a multiple or submultiple prefix is used, it is part of the unit and precedes the symbol of unity without space between the prefix symbol and the unit symbol.

    Correct: 124.6 mm (numerical value/space/prefix of the unit/unit)
    Wrong: 124.6 m m

12. Do not mix the name with the symbol.

    Correct: kilometer per hour or km/h
    Wrong: km/hour or kilometer/h

1.2.8 Non-SI Units Accepted for Use with the SI

BIPM recognizes the need to use widely used units, although they are not part of the SI. Table 1.6 presents some of these units.

1.3 Proposed Exercises

1.3.1 What is the symbol of the quantity length in SI?

(a) mts
(b) m
(c) KM
(d) km

Table 1.6 Non-SI units that are accepted

Quantity	Name	Symbol	Value in SI units
Time	Minute	min	60 s
	Hour	h	3600 s
	Day	d	86,400 s
Plane and phase angle	Degree	°	$\pi/180$ rad
	Minute	′	$\pi/10{,}800$ rad
	Second	″	$\pi/648{,}000$ rad
Volume	Liter	l or L	1 $dm^3 = 10^{-3}$ m^3
Mass	Tonne	T	1000 kg
	Dalton	Da	$1.660{,}539{,}040\ (20) \times 10^{-27}$ kg
Energy	Electronvolt	eV	The kinetic energy acquired by an electron in passing through a potential difference of one volt in a vacuum ($1.602{,}176{,}634 \times 10^{-19}$ J)
Pressure	Bar	bar	0.1 MPa = 100 kPa
	Millimeter of mercury	mmHg	133.322 Pa
Area	Hectare	ha	10^4 m^2

Source: SI brochure 9th edition

1.3.2 What is the symbol of the quantity time in SI?

(a) s
(b) sec
(c) h
(d) hs

1.3.3 What is the symbol of the quantity of electric current in SI?

(a) A
(b) a
(c) Amp
(d) Ap

1.3.4 What is the symbol of the quantity velocity in SI?

(a) mts/s
(b) m/sec
(c) km/hr
(d) m/s

1.3.5 What is the symbol of the quantity voltage in SI?

(a) T
(b) VA
(c) V
(d) VT

1.3.6 What is the value of 1 μm in power of ten?

(a) 10^3 m
(b) 10^6 m
(c) 10^{-6} m
(d) 10^{-3} m

1.3.7 The unit of force in SI is:

(a) dyna
(b) newton
(c) kilogram-force
(d) kilogram

1.3.8 The unit of pressure in SI is:

(a) pascal
(b) psi
(c) kilogram-force
(d) bar

1.3.9 The symbol of quantity temperature in SI is:

(a) K
(b) °F
(c) °K
(d) C

1.3.10 Mark the correct writing.

(a) 18 hrs
(b) 3 mts
(c) 10 hs
(d) 9 L

1.3.11 Mark the correct writing.

(a) 18 h
(b) 4 KM/H
(c) 10 mts
(d) 9 Kg

1.3.12 What is the value of 1 MHz in power of ten?

(a) 10^6 Hz
(b) 10^{-6} Hz
(c) 10^{-3} Hz
(d) 10^{-9} Hz

1.3.13 What is the value of 1 ns in power of ten?

(a) 10^3 s
(b) 10^6 s
(c) 10^{-9} s
(d) 10^9 s

1.3.14 Check the option that only has base units of the SI.

(a) meter, second, degree Celsius
(b) meter, hour, degree Celsius
(c) kilometer, second, kelvin
(d) meter, ampere, kelvin

1.3.15 Check the option that only has derived units of the SI.

(a) meter, second, degree Celsius
(b) joule, hour, degree Celsius
(c) joule, newton, volt
(d) meter, ampere, kelvin

1.3.16 The base units of the SI include:

(a) second, meter, candela, newton
(b) second, meter, candela, kelvin
(c) second, meter, kelvin, joule
(d) second, mole, joule, ampere
(e) second, mole, ampere, pascal

1.3.17 Check the option that contains a pressure value written adequately in units of the SI.

(a) 200 MPA
(b) 200 MPa
(c) 200 Mpa
(d) 200 mpa
(e) 200 mPA

1.3.18 What is the unit for pressure?

(a) pascal
(b) mol
(c) candela
(d) kelvin

1.3.19 What is the unit for volume?

(a) kilogram
(b) cubic meter
(c) square meter
(d) mol

1.3.20 The symbol m^3/s represents:

(a) density
(b) volume
(c) flow
(d) velocity

1.3.21 The symbol of electric resistance is:

(a) B
(b) Ω
(c) μ
(d) Σ

1.3.22 What is the unit for the electric charge?

(a) joule
(b) coulomb
(c) volt
(d) farad

1.3.23 The base units of SI are:

(a) sec, °C, PA, kg, A
(b) km, kg, K, mol, A
(c) m, K, s, A, kg
(d) s, m, cd, bar, °C

1.3.24 The time interval of 2.4 min is equivalent to the SI:

(a) 24 seconds
(b) 124 seconds
(c) 144 seconds
(d) 160 seconds
(e) 240 seconds

1.3.25 In equation $x = k\frac{v^n}{a}$
x represents a distance, v represents velocity, a represents acceleration, and k represents a dimensionless constant. What should be the value of exponent n so that the expression is physically correct?

1.3.26 In the SI, the units of electrical potential difference, electric field, work, and capacitance are, respectively:

(a) W, N/C, F, J
(b) V, N/C, J, C
(c) V, V/m, J, F
(d) W, V/m, F, J
(e) W, V/m, J, F

1.3.27 The physical intensity (I) of sound is the ratio between the amount of energy (E) that crosses a unit of area (S) perpendicular to the direction of propagation of sound in the unit of time (t). In the SI, what is the unit of I?

1.3.28 Mathematically, expressing any physical quantity according to other physical quantities through the dimensional formula is possible. Using the dimensional symbols of the fundamental quantities of SI, determine the dimensional formula of *power* quantity.

(a) MLT^{-1}
(b) $ML^{-2}T^{-3}$
(c) $M^{-1}L^{3}T^{-2}$
(d) $ML^{2}T^{-3}$
(e) MLT^{-2}

1.3.29 In the analysis of specific movements, it is reasonable to suppose that the frictional force is proportional to the square of moving particle speed. Analytically, $f = kv^2$. What is the unit of the proportionality constant k in the SI?

Chapter 2
Knowing Metrology and Its Structure

2.1 Metrology: Introduction

Metrology supports a universal agreement for units of measure, that is, the standardization of values. For this to happen, there must be an international and national metrological structure to ensure that the measuring instruments are maintained and applied properly and correctly in daily operational and business transactions. This standardization of units of measure is of great commercial importance for nations and companies.

For example, car manufacturing has several parts suppliers, each with its own production system and measurement instruments. However, all parts should fit perfectly into the car assembly. Imagine a wheeled supplier manufacturing and measuring the holes of the fixing screws with a slightly smaller diameter than those manufactured and measured by the screw supplier; the wheels could not be used.

As we saw in the previous chapter, using different units of measurement for the same quantity conflicts with the standardization of language established in the International System of Units (SI). However, some British colonization countries still employ other units of measure, such as the inch, foot, pound, yard, and mile.

This use can lead to severe misconceptions and disastrous consequences for society. Let us consider some real cases as examples.

Case 2.1: Vasa, the Swedish Warship (Fig. 2.1)

Swedish warship Vasa wrecked in 1628 on its inaugural trip, less than two kilometers from the coast, causing the death of 30 crew members. At the time, armed with 64 bronze cannons, it was considered the most powerful ship in the world. The archeologists who studied him after he was lifted from the bottom of the sea in 1961 said he was thicker to the stubborn than the stubby. One reason may be that the workers used different systems of measurements, as archeologists found four

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_2

Fig. 2.1 ‘Vasa ship’ can be visited today at the Vasa Museum in Stockholm, Sweden. (https://schweden-tipp.de/wp-content/uploads/2016/08/Vasa-Museum-Gem%C3%A4lde-Francis-Smitheman.jpg)

Fig. 2.2 Air Canada: Boeing 767–200. (Photo @ Robert Pearson)

rulers used in the construction: two were marked in Swedish feet, which were 12 inches. In contrast, the others were Amsterdam feet with 11 inches.

(Text adapted from https://en.wikipedia.org/wiki/Vasa_(ship))

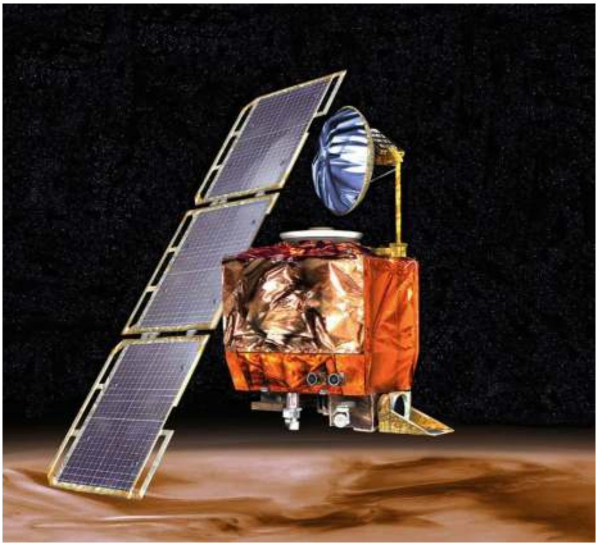


Fig. 2.3 Artist's conception of the Mars Climate Orbiter. (Source: NASA/JPL/Corby Waste—Wikimedia Commons)

Case 2.2: Boeing 767–200 from Air Canada, Known as Gimli Glider (Fig. 2.2)

In 1983, Canada adapted its measurement system from English to the International Unit System. At a land stop, an Air Canada Boeing 767–200 had problems with the fuel control device. The maintenance team then used the manual measuring ruler to define and complete the volume of kerosene in the plane tanks. However, that aircraft was the first of the fleet that used fuel control in SI, but the track technicians based on the fuel density of 1.77 pounds per liter (English system), while in SI, this value was 0.80 kilogram per liter. Because of this confusion, the plane was fueled with less than half of the volume of kerosene necessary to make the route between the cities of Montreal and Edmonton (would need 22,300 kilograms of fuel and received 22,300 pounds, approximately 10,115 kilograms). The result of the "metrological failure" was a dry crash in the middle of the way and at an altitude of 12,500 m. The aircraft landed planning safely at the Gimli Industrial Aero Park in Manitoba.

(Text adapted from https://en.wikipedia.org/wiki/Gimli_Glider)

Case 2.3: Spaceship Mars Climate Orbiter (Fig. 2.3)

In 1999, the spaceship Mars Climate Orbiter deviated from the original route when entering Mars' atmosphere, because its trajectory was erroneously calculated using two measurement systems: SI and the English system. This caused NASA to lose US$ 300 million, as it caused the loss of spacecraft. The explanation is that the spaceship was not disintegrated but had a propeller destroyed as it entered the planet's atmosphere. Attempts to replace it in the correct orbit and to prevent Mars from giving space were unsuccessful. NASA director Carl Pilcher told Science News

Fig. 2.4 Laufenburg Bridge. (https://news.bbcimg.co.uk/media/images/75025000/jpg/_75025152_laufenburg_ap624.jpg)

magazine that the fact they did not identify the "metrological failure" during the route was a grave mistake made by the mission officials.

(Text adapted from http://goo.gl/tRfBeJ)

Case 2.4: Laufenburg Bridge (Fig. 2.4)

Sea level varies from place to place, and countries use different reference points. Britain, for example, measures the height from the sea level in Cornwall, and France does it from the sea level in Marseille. Germany measures concerning the North Sea, while Switzerland, like France, opts for the Mediterranean. In 2003, this generated a problem in Laufenburg. This village is on the border between Germany and Switzerland, because, as the two halves of a bridge approached each other during the construction, instead of being "at the same height from sea level," one side was 54 centimeters above the other. The German side had to be relegated to complete the bridge.

(Source: https://marine-digital.com/article_bridge_between_germany_and_switzerland)

The Laufenburg Bridge was not motivated by using different systems but by adopting a different "reference." However, the situations experienced by Air Canada and NASA would not have happened, if there were no flaws in "metrological communication" because, in principle, there were no errors in the calibration of the measuring instruments.

Calibrating is essential, but harmonizing the concepts and measurement units enables the correct interpretation of information and specific decision-making in a globalized market.

2.2 Importance of Measuring

Measuring is part of our daily lives:

- When looking at the display of a clock, we see the result of the time measurement (hour, minute, and second);
- When buying a heavy product on a scale, we have the mass measurement (kilogram, gram);
- When supplying the car at the gas station, we noticed the (liter) volume measurement of fuel;
- When we received the electricity bill from our residence, we could see the electricity consumption measured in that period (kW).

Anyway, we are constantly witnessing and experiencing the results of measurements.

Measuring is a process that involves the existence of:

- A phenomenon (either a quantity or a substance) we want to know;
- A measuring instrument (or a set of instruments) calibrated, preferably;
- A unit of measure (kg, m, °C, etc.);
- An individual trained to perform the act of measuring and correctly interpreting the result.

Two great scientists highlighted the importance of measuring many years ago (Figs. 2.5 and 2.6).

> Measure what is measurable, and make measurable what is not.
> (https://mathshistory.st-andrews.ac.uk/Biographies/Galileo/quotations/)

> When you measure what you are speaking about and express it in numbers, you know something about it, but when you cannot, your knowledge about it is meager and unsatisfactory.
> (https://mathshistory.st-andrews.ac.uk/Biographies/Thomson/quotations/)

Fig. 2.5 Galileo Galilei. (https://tse4.mm.bing.net/th?id=OIP.5WKAICf1vtt9p2OtlS3wNQHaHa&pid=Api&P=0&h=180)

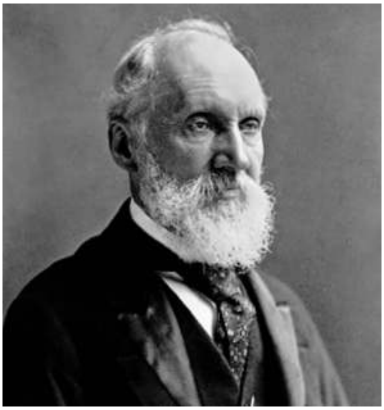


Fig. 2.6 William Thomson (Lord Kelvin). (https://res.cloudinary.com/dk-find-out/image/upload/q_80,w_1920,f_auto/A-Corbis-IH190312_l5pagc.jpg)

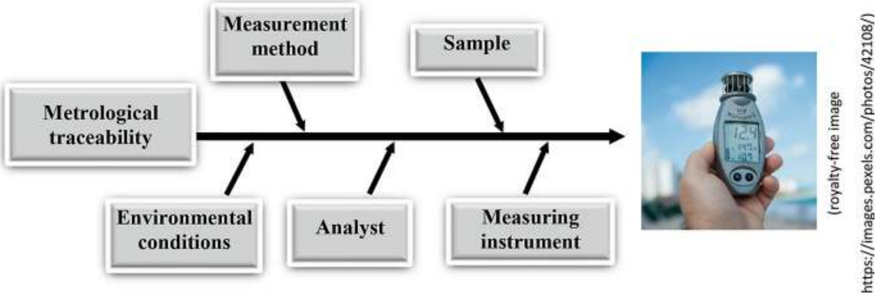


Fig. 2.7 Metrological agents

2.3 Measurement Objective

Decisions must be made based on information in any field of activity. In the scientific and technological area, such information is generally the result of measurements directly or indirectly related to the object under study.

Measurement is the "process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity."

Measurements can be influenced by different metrological agents, such as (i) the measurement method, (ii) the sample, (iii) the analyst, (iv) the measurement instrument, (v) the environmental conditions, and (vi) the traceability of the measurement instruments and standards. Thus, we understand the measure as the "result of the measurement process," in this sense, its quality depends on how such a process is managed.

Figure 2.7 presents the different metrological agents that influence the measurement result.

Let us discuss each metrological agent briefly.

2.3.1 Measurement Method

Here is a definition of measurement method: "*generic description of a logical organization of operations used in a measurement*" [VIM—2.5].

The measurement method should ideally be contained in a technical standard; however, it may also be present in an operational procedure, a work instruction, a flowchart, or any other internal organization document.

It should be noted that the measurement method should be developed by experts in the subject and used by professionals with knowledge and training in the techniques defined by the process.

Example 2.1

The standard CEN-EN 837-1 Pressure Gauges—Part 1: Bourdon Tube Pressure Gauges—Dimensions, Metrology, Requirements, and Testing determines the conditions for calibrating the Bourdon-type gauge. Among the various requirements, it defines, for example, the minimum number of calibration points as a function of the accuracy class (Table 2.1).

Accuracy class: Class of measuring instruments or measuring systems that meet stated metrological requirements that are intended to keep measurement errors or instrumental measurement uncertainties within specified limits under specified operating conditions.

NOTE 1 An accuracy class is usually denoted by a number or symbol adopted by convention.

NOTE 2 Accuracy class applies to material measures. [VIM—4.25].

Knowing a Little More... (Fig. 2.8)

Eugène Bourdon was born in 1808 in Paris, France. He began his career as a watchmaker and later as an engineer. In 1849, Bourdon invented the meter that bears his name. This gauge can measure up to 6800 atmospheres. This invention also helped decrease the number of steam engines because, before Bourdon, measuring this amount of pressure was almost impossible. He died in 1884, but his Bourdon tube pressure gauge is still used today.

Table 2.1 Number of calibration points of a pressure gauge according to CEN-EN 837-1

Accuracy class	Number of calibration points
0.1; 0.25; 0.6	10
1; 1.6; 2.5	5

Fig. 2.8 Eugène Bourdon. (Photo: public domain)

Fig. 2.9 Sample of screws to determine the thread step. (https://images.pexels.com/photos/259988/)

2.3.2 Sample

A sample is a representative part of a whole that allows the results to be attributed to the original set once they have been evaluated, analyzed, and measured.

Example 2.2
Let us check the thread step of a lot of 10,000 screws (Fig. 2.9).

One option would be to measure all 10,000 screws. However, it would be a time-consuming and costly process. Thus, a viable alternative is to randomly choose a certain number of production parts as a sample, using criteria established in a technical standard, for example. The average step measurements of the selected pieces are considered a reasonable estimate for the total screws.

We must be careful when selecting and using a sample to represent the set; otherwise, we can assign wrong values due to improper choice or handling of the sample.

Some primary care should be observed in the choice and definition of the sample:

- Apply statistical methods to determine the sample size since it should represent the whole;
- Make the random selection of the sample and ensure that it belongs to the same manufacturing batch. For example, an excellent way to determine the ambient temperature of a laboratory is to measure temperature in various locations, not just one place;
- Ensure that measurements are performed under conditions defined in standards, methods, or technical procedures. Example: using CEN-EN 837-1 again, it represents that the temperature of the calibration site must be comprised between (20 ± 2) °C;
- Avoid contamination that may modify the physical or chemical characteristics of the sample;
- Check, where applicable, the validity of the sample.

2.3.3 Analyst

The analyst, human factor, and central element of the measurement process (Fig. 2.10) need:

- Know the measurement method;
- Know how to evaluate environmental conditions and decide on whether or not to measure measurements;
- Be able to select the sample to be evaluated adequately;
- Be trained for the correct use of the instruments that make up the measurement system;
- Register and correctly interpret the result of the measurements.

Fig. 2.10 Ultrasonography examination measurement system. (https://images.pexels.com/photos/7089400/)

2.3.4 Environmental Conditions

Initially, we need to define *influence quantity*:

> *Quantity that, in a direct measurement, does not affect the quantity that is measured, but affects the relation between the indication and the measurement result.*
>
> *Example: Temperature of a micrometer used for measuring the length of a rod, but not the temperature of the rod itself, which can enter into the definition of the measurand.* [VIM—2.52].

Influence quantities can usually not be avoided but should be monitored and controlled to minimize their effects on the measurement result.

Thus, what we call environmental conditions are the influences of these environmental factors, such as temperature, humidity, dust, vibration, fluctuation in the power supply, electric or magnetic noise, lighting, or other factors in a place where measurements will be realized.

Fig. 2.11 Digital thermohygrometer (temperature and relative humidity). (https://pixabay.com/photos/time-clock-humidity-air-hygrometer-2353382/)

Example 2.3
To measure the concentration of a particular active ingredient that enters a medicine's composition, the laboratory temperature must be maintained at (22.0 ± 0.5) °C and relative humidity at (50 ± 5) %.

Under ideal conditions, an air conditioner system should control temperature and moisture.

A thermohygrometer (Fig. 2.11) must measure these conditions to enable the analyst to take action if these variables leave control.

We must interrupt measurements or correct their results when any anomaly arises, either in temperature or moisture (Fig. 2.11).

2.3.5 Measuring Instrument

A measuring instrument is a *"device used for making measurements, alone or in conjunction with one or more supplementary devices."* [VIM—3.1].

Examples 2.4
Some measuring instruments (Figs. 2.12, 2.13, 2.14, 2.15, and 2.16).

Measurement indicates or controls a process, monitors an alarm, or investigates a physical, chemical, or biological phenomenon. In simple monitoring, measurement

Fig. 2.12 Vernier caliper. (https://pixabay.com/photos/vernier-caliper-measuring-instrument-452987/)

Fig. 2.13 Micrometer. (https://pixabay.com/photos/micrometer-measure-measuring-tool-505350/)

Fig. 2.14 Multimeter. (https://pixabay.com/photos/multimeter-ohm-meter-voltmeter-523153/)

Fig. 2.15 Bourdon gauge. (https://pixabay.com/photos/pressure-manometer-measure-up-1646350/)

Fig. 2.16 Analog scale. (https://pixabay.com/photos/libra-kitchen-scale-1638996/)

systems indicate the instant or accumulated value of the quantity to be measured. Examples are automobile speedometers and odometers, clinical thermometers, and pressure gauges.

Fig. 2.17 Odometer

Knowing a Little More... (Fig. 2.17)
(https://pixabay.com/photos/speedometer-mileage-speed-car-498748/)

The function of the odometer is to measure the distances traveled by the vehicle. The cars have a digital odometer, which works by electrical pulses a sensor generates on its axis. Each turn sends a pulse to the electronics center, generating a signal for the panel to scan the information. However, many vehicles in circulation still have a mechanical odometer. This system consists of a wire rope connected to the gearbox, the speedometer clock, and a gear game hidden behind the vehicle panel. In the gearbox, the cable is connected to a gear that moves according to the axle turns. Consequently, the wire rope also turns around, moving an endless gear installed near the panel. This piece triggers a gear game that pushes the marker.

A control system uses a transducer and controller to maintain a quantity or process within specific values. According to the definition, a transducer is a "device used in measurement, which provides a quantity of output, which has a specified relationship with an input quantity."

In this situation, the quantity is measured, its value is compared to a reference value, and a correction action is taken to maintain the quantity close to the reference value.

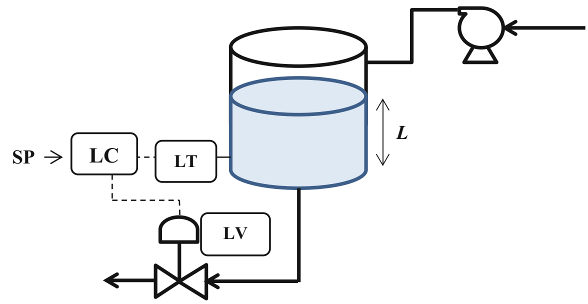


Fig. 2.18 Level control system

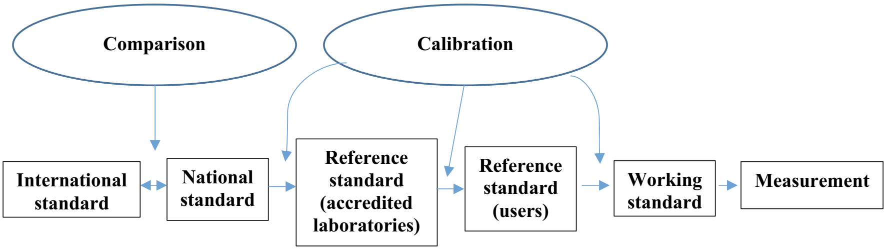


Fig. 2.19 Metrological traceability

Knowing a Little More...

See the Level control system of a reservoir, as shown in Fig. 2.18.

The transducer (LT) sends to the controller (LC) an electrical signal proportional to the level (L) variation in the water tank. The controller compares this sign of the process variable (level) to a reference (set point) value (SP) and, depending on the magnitude of this difference, sends a correction signal to the control valve (LV), so that it reduces (or increases) the liquid flow rate to keep the level stable within the reservoir.

An alarm system operates on warning, sound, or visual warning equipment after an unwanted or dangerous situation (e.g., a fire alarm).

The alarm system can also operate with safety systems to maintain equipment integrity, especially for people.

When investigating a phenomenon, an example is measuring the “hole in the ozone layer” in the earthly atmosphere to determine its consequences for life on the planet.

2.3.6 *Metrological Traceability*

The definition of *metrological traceability* is (Fig. 2.19):

> *Property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty*. [VIM—2.41].

The calibrated standards and instruments with guaranteed traceability transfer accuracy to measurements, enabling an adequate estimate of the final measurement uncertainty. Thus, before performing and using a measurement's result as relevant information for any decision, it is necessary to analyze the measurement process to know all sources of influence associated with metrological agents.

Once these sources of influence are identified, the measurement process must be active to yield quality measures, that is, metrological reliability.

Thus, the uncertainty derived from each metrological agent influences the final uncertainty of the measurement process. In the measurement chapters (Chaps. 5 and 6), we will address the concept of uncertainty and the methodology for its estimate, considering the variables of influence.

2.4 Metrological Reliability

Generically, reliability is the capacity or probability of a system to perform a function and maintain its operation under specific conditions, correctly, as provided in the project, during a predetermined period, under routine circumstances, as well as in hostile and unexpected circumstances.

Thus, metrological reliability is the ability of a measurement system to convey certainty and confidence in the results obtained. Without metrological proof, there is no way to guarantee the reliability of control data of characteristics that determine the quality of the product.

Analyzing the environment on the consumer side, the existing metrological system should enable users' access to compliance verification mechanisms of the products offered. From the results of the measurements performed by manufacturers and verified by the controlling agencies, consumers may trust that industrialized products have been adequately measured (e.g., weight, volume, chemical composition, concentration, etc.) and released for commercialization.

2.5 Metrology Areas of Expertise

We can separate metrology into two significant areas of activity: Legal metrology and Scientific and industrial metrology.

2.5.1 Legal Metrology

It is the area of metrology closest to the ordinary citizen. Its primary function is to protect products and services that involve and need measurement.

The International Organization of Legal Metrology (OIML) defines it as "the application of legal requirements for measures and measuring instruments."

Metrological regulations based on the OIML guidelines establish technical requirements, metrological control, use, and marking requirements, and the units of measure that measuring instrument manufacturers and users must meet.

In addition to commercial activities, measuring instruments used in official activities, in the medical area, in the manufacture of medicines, and the fields of occupational, environmental, and radiation protection are subjected to metrological control. In these cases, control assumes particular importance in the face of the dangerous adverse effects of wrong results on human health.

Table 2.2 shows the measuring instrument categories included in the OIML Certification System and the corresponding OIML Recommendations.

The OIML Certification System (OIML-CS) is a system for issuing, registering, and using OIML certificates and their associated OIML-type evaluation/test reports for types of measuring instruments (including families of measuring instruments, modules, or families of modules) based on the requirements of OIML Recommendations.

2.5.2 Scientific and Industrial Metrology

Scientific metrology is linked to scientific research and methodologies of the highest metrological quality. It deals with measurement standards and laboratory instruments.

As they unfold, these actions also include industry measurement systems (industrial metrology), which control production processes and ensure the quality of products and services offered to the market.

2.6 International Metrological Structure

The international structure of each of the two significant areas of Metrology (legal and scientific) is very similar.

2.6.1 Legal Metrology

International Organization of Legal Metrology (OIML)

It is an intergovernmental treaty that, among other activities, develops regulations, rules, and documents for use by the legal and industry authorities.

Table 2.2 Measuring instrument category and OIML Recommendation

Measuring instrument category	Recommendation number
Taximeters	R 21
Material measures of length	R 35
Active electrical energy meters	R 46
Water meters	R 49
Continuous totalizers	R 50
Automatic catchweighers	R 51
Sound level meters	R 58
Moisture meters for cereal grains and oilseeds	R 59
Load cells	R 60
Automatic gravimetric filling instruments	R 61
Heat meters	R 75
Non-automatic weighing instruments	R 76
Cryogenic liquids	R 81
Level gauges for stationary storage tanks	R 85
Integrating-averaging sound level meters	R 88
Focimeters	R 93
Vehicle exhaust emissions	R 99
Sound calibrators	R 102
Pure-tone audiometers	R 104
Automatic rail-weighbridges	R 106
Discontinuous totalizers	R 107
Pressure balances	R 110
Weights	R 111
Liquids other than water	R 117
Speech audiometry	R 122
Evidential breath analyzers	R 126
Ergometers for foot crank work	R 128
Multidimensional measuring instruments	R 129
Liquid-in-glass thermometers	R 133
Weighing road vehicles in motion	R 134
Areas of leather	R 136
Gas meters	R 137
Compressed gaseous fuel systems for vehicles	R 139
Continuous measurement of SO2 in stationary source emissions	R 143
Continuous measurement of CO, NOx in stationary source emissions	R 144
Ophthalmic instruments—Impression and applanation tonometers	R 145
Protein measuring instruments for cereal grains and oilseeds	R 146
Non-invasive non-automated sphygmomanometers	R 148
Non-invasive automated sphygmomanometers	R 149
Continuous totalizing automatic weighing instruments of the arched chute type	R 150

Fig. 2.20 OIML

Knowing a Little More... (Fig. 2.20)

Since 1955, OIML has launched the foundations for a world metrology system.

> *The mission of the OIML is to enable economies to put in place effective legal metrology infrastructures that are mutually compatible and internationally recognized, for all areas for which governments take responsibility, such as those which facilitate trade, establish confidence and harmonize the level of consumer protection worldwide.*

Available in: https://www.oiml.org/en

International Conference of Legal Metrology
OIML's maximum decision-making body. The conference, which takes place every four years, comprises representatives of member countries, countries that come together as observers, and associations of international institutions. Its purpose is to define the general policy and promote the implementation of OIML metrological guidelines.

International Committee on Legal Metrology (CIML)
It is the organization's functional decision body. Approve the annual BIML work plan and adopt OIML recommendations, documents, and publications.

International Bureau of Legal Metrology
It is OIML's secretariat and headquarters. The bureau organizes conference and commission meetings, executes conference decisions and commissions, and disseminates and distributes the organization's publications (Fig. 2.21).

2.6.2 Scientific Metrology

General Conference of Weights and Measures (CGPM)
The CGPM is made up of member state delegates and associate observers. Among its attributions is the discussion and analysis of the necessary provisions to ensure the propagation and improvement of the SI.

Fig. 2.21 BIML. (https://www.oiml.org/en/ressources/biml/sacre_coeur.jpg)

Fig. 2.22 BIPM

Bureau
International des
Poids et
Mesures

International Committee on Weights and Measures (CIPM)
Composed of 18 member countries, it acts as an international scientific authority, and its main task is to promote world uniformity in units of measure through direct action or the presentation of resolution projects to the CGPM.

International Bureau of Weights and Measures (BIPM) (Fig. 2.22)

An intergovernmental organization established by the Meter Convention in 1875 aims to ensure and promote the global comparability of measurements, including the supply of an international unit system (SI) and the International Reference Time Scale (UTC) for scientific research and innovation.

2.7 Regional Metrological Structure

2.7.1 EURAMET—The European Association of National Institutes of Metrology

The mission is to develop and disseminate an integrated, profitable, and competitive measurement infrastructure for Europe, always considering the needs of the industry, companies, and governments. With its services, EURAMET supports members in meeting national requirements and establishing a balanced European measurement infrastructure. Improving the benefits of metrology for society is one of the highest priorities for EURAMET and its members.

The CIPM MRA is very important for achieving the objectives. The International Committee of Weights and Measures (CIPM) sponsored the creation of a mutual recognition scheme (CIPM MRA) to promote and formalize the technical competence of its national metrology institutes and designated signatories.

Regional Metrology Organization
EURAMET is the European Regional Metrology Organization (RMO). It coordinates the cooperation of the European National Metrology Institutes (NMI) in fields such as Metrology Research, measurement traceability to SI units, international recognition of national measurement standards, and calibration and measurement capabilities (CMC). Through the transfer of knowledge and cooperation between the members, EURAMET facilitates the development of national metrology infrastructure.

European Research Programs in Metrology
EURAMET is responsible for the elaboration and execution of the European Metrology Research Program (EMRP) and the European Metrology Program for Innovation and Research (EMPIR), which is designed to promote collaboration between the National Institutes of Metrology (INM) European and the partners of the industry or the academic world.

Goals
Commitment of key stakeholders

EURAMET must understand and prioritize investment in European measurement infrastructure to address companies' and governments' present and future priorities. To achieve this, EURAMET will strengthen your links and influence with key users of the measurement infrastructure.

The objective is to develop critical associations, understand interested parties' needs, increase the work's impact, and anticipate market trends and needs based on prospective analysis.

Increase influence with European political leaders and national governments

EURAMET's responsibilities include supporting policy formulation, mainly when measurement is essential in establishing and implementing the policy. Measures are an essential component of many European directives. EURAMET members actively support the implementation of many CE directives through measurement and monitoring work.

Further, develop cooperation in I&D

In recent years, EURAMET has worked successfully with the European Commission and many national governments of member countries to develop the European Metrology Research Program (EMPR) and the European Metrology Program for Innovation and Research (EMPIR).

The programs have initiated more than 100 joint research projects, and many more are to come. The objective is to continue developing Europe's metrology capacity to face world challenges.

Give a high value to members and associates
The objective is:

- Understand the actual needs and visions of the members and have an inclusive approach to all the needs of the members;
- Support all members and associates in achieving their objectives, taking into account the existing diversity but in balance with general European needs;
- Increase the scope of cooperation and exchange of resources/facilities for mutual benefit and convergent development;
- Stimulate the development of a stable national framework for metrology through the proper participation of the critical actors in metrology and support for excellence in metrology as a driving force.

Support quality infrastructure in Europe and internationally
The objective is:

- Improve the efficiency and efficacy of CIPM/MRA;
- Influence the Joint Committee of the Regional Metrology Organization in close cooperation with other RMOs to optimize CIPM/MRA processes and governance;
- Strengthen cooperation with European Cooperation for Accreditation (EA) in areas of common interest associated with accreditation;
- Work with the EA to implement technical aid projects in accession and outside European countries.

European Metrology Networks
Close collaboration in the science of measurement with a new sustainable structure

EURAMET and its members envision a world-leading metrology capacity based on high-quality scientific research and an effective and inclusive infrastructure that meets the rapid needs and progress of the end users. The European Metrology Networks (EMN) help achieve this goal.

There are currently 12 EMNs: Advanced Manufacturing, Clean Energy, Climate and Ocean Observation, Energy Gases, Laboratory Medicine, Mathematics and Statistics, Pollution Monitoring, Quantum Technologies, Radiation Protection, Safe and Sustainable Food, Smart Electricity Grids, and Smart Specializations in Northern Europe.

The EMNs will analyze the needs of European and world metrologies and address them in a coordinated manner. Next, the members of the EMN will formulate common metrology strategies that include aspects such as research, infrastructure, knowledge transfer, and services. The members will commit to contributing to the EMN and helping to establish sustainable structures strategically planned from the beginning.

By providing a single point of contact to obtain information, support regulation and standardization, promote best practices, and establish a longer-term integral infrastructure, the EMNs aim to create and disseminate knowledge, obtain international leadership and recognition, and foster collaboration throughout the scientific community of the measurement.

(Text adapted from https://www.euramet.org)

2.7.2 Inter-American Metrology System (SIM)

The Inter-American Metrology System (SIM), instituted in 1979, resulted from a broad agreement between the national metrology organizations involving 34 nations. Its mission is to promote and support an integrated measurement infrastructure in the Americas, which allows each National Institute of Measurement to encourage innovation, competitiveness, trade, consumer security, and sustainable development, effectively participating in the international metrology community.

Organized in five subregions (Noramet, Carimet, Camet, Andimet, and Suramet), it has a governor board structured by a coordinator of each subregion, a technical committee, a professional development committee, and an integrated representation that provides access to SIM in a worldwide agreement to compare standards at the highest metrology level.

SIM is committed to implementing a global measurement system in the Americas that will ensure the confidence of all users. Working to establish a robust regional measuring system, SIM is essential to the development of a free trade area in the Americas.

In the context of established cooperation, measures taken by the member countries will help to achieve:

- Establishment of national and regional measurement systems.
- Establishment of a hierarchy of the national standards of each country and binding on regional and international standards.
- Establishment of equivalence between national measurement standards and calibration certificates issued by national metrology laboratories.
- Comparability of the results obtained in measurement processes performed in laboratories within the system.
- Training of technical and scientific personnel.
- Distribution of technical and scientific documentation.
- Binding with international standards maintained by the International Bureau of Weights and Measures (BIPM).
- Straight cooperation with the BIPM, the OIML, and other international organizations interested in laboratory accreditation (ILAC) and with technology and measurement patterns (IMEKO), research and development (universities and organizations P&D), oriented to promote competitiveness, promote more equitable business transactions and support essential development in health, safety, sustainable industrial development, and environmental protection.

(Text adapted from https://sim-metrologia.org/)

2.7.3 *Other Regional Metrological Structures*

The Asia Pacific Metrology Programme (APMP)

The APMP is a grouping of national metrology institutes (NMIs) from the Asia-Pacific region engaged in improving regional metrological capability by sharing expertise and exchanging technical services among Member laboratories. The APMP is one of the six Regional Metrology Organizations (RMOs) that implement the CIPM MRA for the worldwide mutual recognition of measurement standards and calibration and measurement certificates. It is also one of the Specialist Regional Bodies (SRBs) working with Asia-Pacific Economic Cooperation (APEC) to facilitate developing and implementing standards and conformance infrastructures that address APEC goals.

(Text adapted from https://apmpweb.org/)

Euro-Asian Cooperation of National Metrological Institutions (COOMET)

COOMET is an organization for the Euro-Asian cooperation of National Metrology Institutions (from the countries of Central and Eastern Europe, Asia, and nearby countries). It is open to the National Metrology Institutions of countries from other regions to join it. Its mission is to raise the level of metrology development, support, and expansion of an integrated measurement infrastructure for countries in the Euro-Asian region and other interested countries that makes it possible for every national metrological institute to promote innovation, competitiveness, trade, consumer safety, sustainability and ensuring international recognition. Main areas of cooperation: Measurement standards of physical quantities; Legal metrology; Quality management systems; Information and training; Innovative research in metrology.

(Text adapted from https://www.coomet.net/)

Southern African Development Community Cooperation in Measurement Traceability (SADCMET)

The SADC Cooperation in Measurement Traceability coordinates metrology activities and services in the region to provide regional calibration and testing services, including regulatory bodies, with readily available traceability to the SI units of measurement through legally defined and regionally and internationally recognized national measurement standards. Its primary objectives are to (i) Promote closer collaboration among its members in their work on measurement standards within the present decentralized regional metrology structure; (ii) Improve existing national measurement standards and facilities and make them accessible to all members; (iii) Ensure that new national measurement standards and facilities developed in the context of SADCMET collaborations are accessible to all members; (iv) Contribute to the formulation of and participate in intra- and interregional systems to maintain the continued traceability of the national measurement standards of the SADC member states to the SI units of measurement; (v) Encourage the harmonization of legislation relating to national measurement standards.

(Text adapted from http://www.sadcmet.org/SitePages/Home.aspx)

2.8 Metrological Structure in Brazil

2.8.1 National System of Metrology, Standardization, and Industrial Quality (SINMETRO)

SINMETRO was instituted by Law Number 5966 of December 11, 1973, and is tasked with managing the technological services infrastructure in metrology (legal, scientific, and industrial), standardization, industrial quality, and conformity assessment.

Compose the SINMETRO:

- National Council of Metrology, Standardization and Industrial Quality (CONMETRO) and its technical committees.
- National Institute of Metrology, Quality and Technology (INMETRO).
- Brazilian Association of Technical Standards (ABNT).
- Certification bodies for quality systems, environmental management, products, and personnel; Inspection organisms; Training bodies; Proficiency testing bodies.
- Accredited calibration and testing laboratories.
- State Institutes of Weights and Measures (IPEM) and.
- State metrological networks.

SINMETRO Areas

Legal Metrology

The activities of Legal Metrology in Brazil are before the law that instituted the SINMETRO. In the 1930s, there was already a "Metrology Law," and metrological control began, in fact, with the creation of the National Institute of Weights and Measures (INPM) in 1961, replaced in 1973 by INMETRO, which incorporated its activities.

As stated earlier, legal metrology is one of the largest consumer protection systems. INMETRO coordinates the Brazilian Network of Legal Metrology and Quality (RBMLQ-I), which comprises the States' Weight and Measures Institutes (IPEM).

Scientific and Industrial Metrology

Scientific and industrial metrology promotes competitiveness and stimulates an environment favorable to the country's scientific and industrial development. It is also essential to technological innovation. INMETRO coordinates this process, is responsible for the fundamental metrological quantities with reliability equal to that of the countries of the first world, and transfers measurement standards to the society.

Testing and Calibrations

Responsibility for test activities (used for product certification) and calibrations (from standards and industrial instruments) within the SINMETRO are the laboratories that make up the Brazilian Testing Laboratories Network (RBLE) and the Brazilian Calibration Network (RBC). They are laboratories accredited by INMETRO and can be public, private, mixed, national, or foreign.

Standardization and Technical Regulation

The Brazilian Association of Technical Standards (ABNT) has this responsibility in the SINMETRO and the authority to accredit sectoral standardization bodies for performing these tasks. ABNT is a non-governmental organization that is maintained with the contribution of the federal government and its associates. It represents Brazil in international standardization forums (ISO and IEC) and regional forums (COPANT and MERCOSUR). Conformity assessment and accreditation activities are based on ISO/IEC standards and guides.

Accreditation

Accompanying the international trend in the sense that there is only one accrediting organism per country within the scope of the SINMETRO, the only accrediting body is INMETRO. ABNT Standards and Guides, COPANT, AMN (MERCOSUR), IAF, ILAC, and IAAC guidelines establish the accreditation criteria adopted in the SINMETRO. INMETRO, therefore, believes in certification organisms (for quality systems, environmental management, products, and personnel), inspection, training, proficiency testing (which provides more excellent reliability to RBC and RBLE), calibration, and testing laboratories.

Knowing a Little More...

International institutions related to standardization, regulation, and accreditation activities:

International Organization for Standardization (ISO) (www.iso.org)

It is an independent, non-governmental organization endorsed by 171 national standardization bodies. Through its members, this entity gathers experts to share knowledge and, based on consensus, voluntarily develops relevant international standards that support innovation and provide solutions to global challenges.

International Electrotechnical Commission (IEC) (www.iec.ch)

It is the world leader organization that prepares and publishes international standards for all electrical, electronic, and related technologies. Industry, commerce, government, testing and research laboratory experts, universities, and consumer groups participate in IEC standardization work.

Pan American Standards Commission (COPANT) (www.copant.org)

It is a non-profit civil association composed of the national standardization bodies of the Americas. It is the reference for technical standardization and conformity assessment for the countries of the Americas, their members, and their international peers.

Mercosur Standardization Association (AMN) (www.amn.org.br)

It is a non-profit civil association and non-governmental organization and the only organism responsible for voluntary normalization within Mercosur. It is composed of the Argentine Institute of Standardization and Certification (IRAM), the Brazilian Association of Technical Standards (ABNT), the

(continued)

Uruguay Institute of Technical Standards (UNIT), and the National Institute of Technology, Standardization and Metrology (INTN—Paraguay).

International Accreditation Forum (IAF) (www.iaf.nu)

The World Association of Accreditation Bodies and other bodies are interested in conformity assessment in management systems, products, services, personnel, and similar conformity assessment programs. Its principal function is to develop a single worldwide conformity assessment program to reduce risk for companies and their customers and ensure that accredited certificates can be reliable.

Inter-American Accreditation Cooperation (IAAC) (www.iaac.org.mx)

It is an association whose mission is to promote cooperation between accreditation bodies and stakeholders of the Americas, aiming at developing conformity assessment structures to improve products, processes, and services. It was created in 1996 in Uruguay and incorporated in 2001 as a civil association according to Mexican law. It is not for profit and works based on the cooperation of its members and stakeholders. It obtains adhesion fees, voluntary contributions from its members, and donations based on regional organizations projects, particularly the Organization of American States (OAS) and the Physikalisch Technische Bundesanstalt (PTB) from Germany.

2.8.2 Brazilian Laboratory Structure

National Institute of Metrology, Standardization and Technology (INMETRO)
Their skills and attributions in the area of metrology are:

- Ensuring the standardization, maintenance, and dissemination of fundamental units of the international system (SI).
- Tracking the measurement units to international standards and spreading them to industries.
- Establishing the methodologies for the comparison of measurement standards, instruments, and materialized measures.
- Tracking the reference standards of the laboratories accredited to national standards.
- Acting in the area of legal metrology and supporting standardization and industrial quality activities.
- Accrediting laboratories and establishing value ranges and measurement uncertainty.

INMETRO's laboratories are home to technical divisions in acoustics and vibrations, electricity, mechanics, optics, thermal, and chemistry. They are responsible for:

- Standardizing the units of the International System of Units.
- Ensuring the traceability of national standards to BIPM standards or comparing them to national standards of other countries by key comparisons coordinated by the BIPM.
- Ensuring the traceability of the reference standards of the accredited laboratories to national standards.
- Performing calibration of standards and measurement instruments as well as specific tests.

Designated Laboratories by INMETRO

- *National Ionizing Radiation Metrology Laboratory (LNMRI) of the Institute of Radioprotection and Dosimetry of the National Nuclear Energy Commission (IRD/CNEN)*

LNMRI, since 1989, has been designated by INMETRO to work in the area of ionizing radiation. Before that, in 1976, the laboratory joined the *Secondary Standard Dosimetry Laboratory—SSDL* network of the International Atomic Energy Agency (IAEA) to ensure the quality of radiotherapy measurements worldwide.

LNMRI aims to develop, maintain, and disseminate national ionizing radiation and radioactivity standards. In addition, it provides calibration services and standards and develops necessary research in scientific metrology support for national nuclear technological development. It maintains radioactive standards and measurement systems for calibrating monitors, dosimeters, and radioactive sources. It is responsible for the custody and maintenance of the Brazilian standard of neutron fluency and for developing metrological techniques to standardize new radionuclides.

(Text adapted from http://www.ird.gov.br).

- *Division Hour Service (DSHO) of the National Observatory (ON)*

DSHO, whose activities began at the Imperial Observatory of Rio de Janeiro, created on October 15, 1827, by Emperor Dom Pedro I, obeys the established international conventions and is in charge of generating, conserving, and disseminating the Brazilian legal time (HLB) to the entire national territory, with different levels of accuracy and reliability, according to Brazilian law, besides promoting research and development in the field of time and frequency metrology.

Since 1983, INMETRO has accredited the *time service* to perform time and frequency calibrations, gaining the function of Time and Frequency Primary Laboratory. Thus, DSHO is responsible for the national time and frequency standards that underlie Brazilian metrological traceability. Internationally, BIPM is the body that defines the traceability of national and HLB standards.

All signs generated and transmitted are referenced to national time and frequency metrological standards, interreferenced by four cesium and one rubidium clock. The frequencies of these signals have an accuracy of 0.5×10^{-12}, equivalent to an error of 2.5×10^{-6} Hz at a frequency of 5 MHz. There is a permanent reference to the coordinated universal time generated by BIPM.

(Text adapted from http://pcdsh01.on.br).

Laboratories Accredited by INMETRO
INMETRO grants accreditation based on the standard ISO/IEC 17025: 2017, according to the guidelines established by the International Laboratory Accreditation Cooperation (ILAC) and the Good Practices Codes (GPC) from the Organization for Economic Co-Operation and Development (OECD).

Accreditation is allowed to any laboratory that provides calibration or testing service, independently or linked to an organization, public or private, national or foreign, despite its size or area of expertise.

2.9 Technical Standards and Metrology

A technical standard establishes quality, performance, and safety requirements for providing something, its use, or its final destination. It also stipulates procedures, standardizes shapes, dimensions, types, and uses, fixes classifications or terminologies and glossaries, and defines how to measure and determine characteristics, such as test methods.

Technical standards apply to products, services, processes, and management systems in the most diverse fields. In general, the customer establishes the technical standard to supply the good or service he wants to acquire. This can be done explicitly when the customer clearly defines the applicable standard or expects that the rules in use will be followed in the market where it operates.

Important
We can say that there is no metrology without technical standards!

2.9.1 The ISO 9001:2015 and the Metrology

ISO 9001:2015—Quality Management Systems—Requirements [11] specifies requirements for a management system that can be used for internal application by organizations, certification, or contractual purposes. Focusing on the metrological issue, there is a specific technical requirement in the standard, *7.1.5.2 Measurement Traceability*, which establishes the following:

> *When measurement traceability is a requirement or is considered by the organization to be an essential part of providing confidence in the validity of measurement results, measuring equipment shall be:*
>
> (a) *calibrated or verified, or both, at specified intervals, or before use, against measurement standards traceable to international or national measurement standards; when no such*

standards exist, the basis used for calibration or verification shall be retained as documented information;

(b) *identified to determine their status;*
(c) *safeguarded from adjustments, damage or deterioration that would invalidate the calibration status and subsequent measurement results;*

The organization shall determine if the validity of previous measurement results has been adversely affected when measuring equipment is found unfit for its intended purpose and shall take appropriate action as necessary.

Knowing a Little More...

ISO 9001 requirement 7.1.5.2 requires the measuring instruments to be verified, calibrated, or both. According to VIM—2.44, **verification** means providing *objective evidence that a given item fulfills specified requirements.*

EXAMPLE 1 Confirmation that a given reference material, as claimed, is homogeneous for the quantity value and measurement procedure concerned, down to a measurement portion having a mass of 10 mg.

EXAMPLE 2 Confirmation that performance properties or legal requirements of a measuring system are achieved.

EXAMPLE 3 Confirmation that a target measurement uncertainty can be met.

NOTE 1 When applicable, measurement uncertainty should be taken into consideration.

NOTE 2 The item may be, e.g. a process, measurement procedure, material, compound, or measuring system.

NOTE 3 The specified requirements may be, e.g. that a manufacturer's specifications are met.

NOTE 4 Verification in legal metrology, as defined in VIML, and in conformity assessment in general, pertains to the examination and marking and/or issuing of a verification certificate for a measuring system.

NOTE 5 Verification should not be confused with calibration. Not every verification is a validation.

NOTE 6 In chemistry, verification of the identity of the entity involved, or of activity, requires a description of the structure or properties of that entity or activity.

In addition to this requirement, we see the need for metrology in others, particularly in *7.1.4 Environment for the operation of processes*, which defines that *"the organization must determine, provide and maintain a necessary environment for the operation of its processes and to achieve the conformity of products and services."*

The requirement also adds that an appropriate environment may include human and physical factors (e.g., temperature, heat, humidity, illumination, ventilation, and noise).

Moreover, how do we measure these physical factors? Answer: Metrology.

2.9.2 The ISO/IEC 17025:2017 and the Metrology

The ISO/IEC 17025:2017 General requirements for the competence of testing and calibration laboratories [12] comprise testing and calibrations performed using standardized methods, non-normalized methods, and methods developed by the laboratory. Figure 2.23 summarizes the various normative requirements in which metrology is strongly present.
Facilities and environmental conditions—req. 6.3

- Monitor, control, and register environmental conditions
- Facilities and environmental conditions cannot adversely affect the validity of the results

Equipment—req. 6.4

- Laboratory must have all measuring instruments, standards, and reference materials required to perform their activities.
- Equipment capable of achieving the accuracy and measurement uncertainty required.
- Equipment must be calibrated.
- The laboratory must have a calibration program.
- Indicate calibration status.
- Ensure operation and calibration of an instrument that has come out of direct control of the laboratory.
- Intermediate checks performed according to procedure.
- Instruments protected against adjustments that invalidate results.

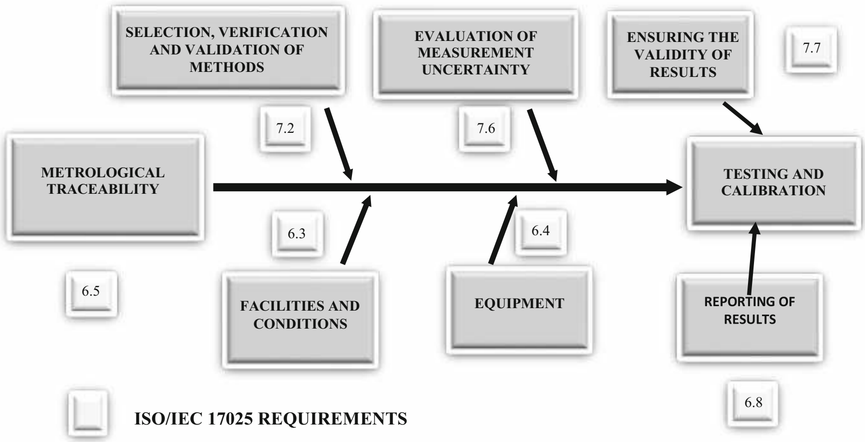


Fig. 2.23 Metrology in the ISO/IEC 17025:2017 requirements

Metrological traceability—req. 6.5

- Laboratory should establish and maintain metrological traceability of their measurement results.
- Calibrations and measurements traceable to the SI.
- Program and procedure for calibrating reference patterns by traceable organisms; Used only for calibration; calibrated before and after adjustments.
- Reference materials: traceable to SI units or already certified reference materials.

Selection, verification, and validation of methods—req. 7.2

- Use appropriate methods and procedures to evaluate the measurement uncertainty.
- Method validation includes, among other techniques, the calibration or evaluation of the trend and accuracy using standards or reference materials.

Evaluation of measurement uncertainty—req. 7.6

- Identification of contribution sources for measurement uncertainties.
- Calibration: assessment of measurement uncertainty for all calibrations.
- Testing: assessment of measurement uncertainty or a method based on the method.

Ensuring the validity of results—req. 7.7

- Use of certified reference materials.
- Interlaboratory comparison program or proficiency tests.
- Intermediate checks on measurement equipment.
- Replicated testing or calibrations.
- Retesting or recalibration of retained items.

Reporting of results—req. 7.8

We fully dedicate Chap. 9 of this book to discuss the importance of this requirement.

2.9.3 Laboratory Accreditation

Calibration Laboratories
The authorized laboratories to perform calibration services gather technical skills and abilities linked to industries, universities, and technological institutes and adopt standards traceable to national or international metrological references, establishing a relationship with the units of the International System of Units (SI).

Testing Laboratories
Like calibration, these laboratories gather skills and technical capacities associated with industries, universities, and technological institutes. They are trained to carry

out tests and performance tests on products with mandatory or voluntary certification. The traceability of the measures is guaranteed by calibration of standards in proven laboratories or directly in the laboratories of the National Metrology Laboratory.

2.9.4 ILAC—International Laboratory Accreditation Cooperation (www.ilac.org)

ILAC is the global association for the accreditation of laboratories, inspection bodies, proficiency testing providers, and reference material producers, with a membership consisting of accreditation bodies and stakeholder organizations worldwide. It is a representative organization that is involved with:

- *the development of accreditation practices and procedures,*
- *the promotion of accreditation as a trade facilitation tool,*
- *supporting the provision of local and national services,*
- *the assistance in developing accreditation systems,*
- *the recognition of competent testing (including medical) and calibration laboratories, inspection bodies, proficiency testing providers, and reference material producers around the world.*

ILAC actively cooperates with other relevant international organizations to pursue these aims. ILAC facilitates trade and supports regulators by operating a worldwide mutual recognition arrangement—the ILAC Arrangement—among Accreditation Bodies (ABs). The data and test results issued by laboratories and inspection bodies, collectively known as Conformity Assessment Bodies (CABs), accredited by ILAC Accreditation Body members are accepted globally via *this Arrangement. Thereby, technical barriers to trade, such as the re-testing of products each time they enter a new economy, are reduced to realize the free-trade goal of "accredited once, accepted everywhere." In addition, accreditation reduces risk for business and its customers by assuring accredited CABs are competent to carry out their work within their scope of accreditation. Further, the results from accredited facilities are used extensively by regulators for the public benefit in providing services that promote an unpolluted environment, safe food, clean water, energy, health, and social care services.* (Text obtained from ILAC-P14:09/2020—ILAC Policy for Measurement Uncertainty in Calibration)

ILAC first started as a conference, held on 24-28 October 1977 in Copenhagen, Denmark, to develop international cooperation for facilitating trade by promoting the acceptance of accredited test and calibration results. In 1996, ILAC became a formal cooperation with a charter to establish a network of mutual recognition agreements among accreditation bodies. In 2000, the 36 ILAC Full Members, consisting of laboratory accreditation bodies from 28 economies worldwide, signed the ILAC Mutual Recognition Arrangement (ILAC MRA) in Washington, DC, to promote the acceptance of technical test and calibration data for exported goods. The ILAC MRA for calibration and testing laboratories came into effect on 31 January 2001. The ILAC MRA was then extended in October 2012 to include the accreditation of inspection bodies. In May 2019, it was further extended to include the accreditation of proficiency testing providers and in May 2020 for the accreditation of reference material producers. (Text obtained from https://ilac.org/about-ilac/).

The ILAC Mutual Recognition Arrangement (ILAC MRA) provides the significant technical underpinning to the calibration, testing, medical testing, and inspection results, provision of proficiency testing programs and production of the reference materials of the accredited *conformity assessment bodies that in turn delivers confidence in the acceptance of services and results. The ILAC MRA supports the provision of local or national services, such as providing safe food and clean drinking water, providing energy, delivering health and social care, or maintaining an unpolluted environment. In addition, the ILAC MRA enhances the acceptance of products across national borders. Technical trade barriers are reduced by removing the need for additional calibration, testing, medical testing, and inspection of imports and exports. In this way, the ILAC MRA promotes international trade, and the free-trade goal of "accredited once, accepted everywhere" can be realized.* (Text obtained from https://ilac.org/ilac-mra-and-signatories/).

On the ILAC website, you can search (https://ilac.org/signatory-search/) for accreditation bodies of the various countries to verify the activities of calibration and testing (ISO/IEC 17025), medical testing (ISO 15189), inspection (ISO/IEC 17020), proficiency testing providers (ISO/IEC 17043), and reference material producers (ISO 17034). Use this directory to find an accreditation body in the economy where you require the calibrations, testing, or inspections to be carried out.

2.9.5 The ISO 10012:2003 and the Metrology

The ISO 10012:2003 Measurement management systems—Requirements for measurement processes and measuring equipment[1] [13]—provide guidelines for measuring process management and metrological evidence of measuring instruments used to support and demonstrate conformity with metrological requirements.

The standard declares that an effective management system ensures the instruments and measurement processes are suitable for their intended use. It also points out that the management system should manage the risk that these measuring instruments and methods can produce incorrect results that affect the quality of an organization's products.

ISO 10012: 2003 is an "essentially metrological" standard, and all its requirements deal with important subjects. However, we will highlight requirement 7—metrological confirmation and realization of measurement processes—which presents a series of exciting guidelines, some of which we present to follow.

Knowing a Little More...
The following scheme represents a management model of a measurement process, and the numbers in Fig. 2.24 refer to ISO 10012: 2003 requirements.

- Recalibration of a measuring instrument is unnecessary if it is within a valid calibration situation. The metrological proof procedure may include mechanisms

[1]The ISO 10012: 2003 standard treats measuring instrument as a measurement equipment. As we did not find this term in VIM, we always replace for instrument.

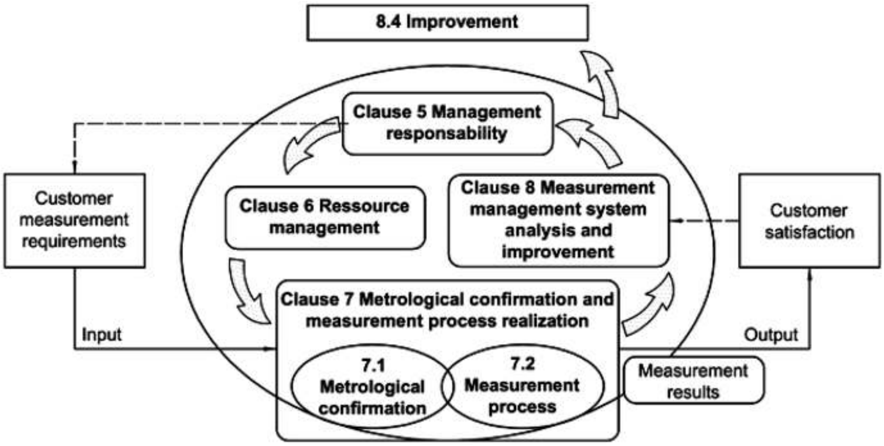


Fig. 2.24 Measurement process management. (Source: ISO 10012:2003)

to verify that the uncertainties and measurement errors are within the permissible limits.

- Examples of metrological characteristics of instruments: range, trend, repeatability, stability, hysteresis, effects of influence quantities, error, and reading resolution.
- Calibration history, technology, and knowledge advances can be used to determine metrological proof intervals. Statistical process control techniques may help analyze calibration intervals.
- The calibration results should be recorded, so that the traceability of all measurements can be demonstrated and that calibration results can be reproduced under conditions close to the original conditions.
- A measurement process may require data correction, for example, due to environmental conditions.
- When specifying the measurement process, it may be necessary to determine which measurements are required, what methods to use, what instruments should be employed, and which skills and qualifications of the team will perform the measurements.
- It is recommended that the impact of the quantities of influence on the measurement process be quantified.
- The performance characteristics required for the intended use of the measurement process must be identified and quantified. These characteristics are measurement uncertainty, stability, repeatability, reproducibility, maximum permissible error, and user skill level.
- It is recommended that the measurement uncertainty consider the uncertainty of the calibration of the measurement instrument.
- Traceability is usually achieved through reliable calibration laboratories that can be traced to national measurement standards. A laboratory that meets the requirements of ISO/IEC 17025: 2017 can be considered trustworthy.

Still, within requirement *7—Metrological confirmation and realization of measurement processes*, it is essential to highlight the issue of records of the metrological evidence process. These records should include:

- Single description and identification of the instrument: type, model, serial number, manufacturer, etc.
- Date that metrological evidence was performed.
- Evidence results.
- Interval of the following evidence.
- Identification of the procedure (or method, norm, instruction, etc.) of evidence.
- Maximum acceptable or permissible errors.
- Relevant environmental conditions and declaration on necessary corrections.
- Uncertainties involved in calibration.
- Provide details of any intervention (maintenance, adjustment, modification) in the measuring instrument.
- Limitations of use.
- Identification of those who performed the metrological evidence.
- Identification of those who are responsible for any correction of information recorded.
- Single identification of the report or calibration certificate.
- Traceability of measurement results.
- Metrological requirements for intended use.
- Result of calibration performed after, and where required, before any intervention in the measuring instrument.

The standard states that the retention time of metrological evidence records depends on several factors, such as customer requirements, statutory or regulatory requirements, and the manufacturer's civil liability. Records related to measurement standards may need to be kept indefinitely.

2.9.6 Technical Standard and Technical Regulation

2.9.6.1 Technical Standard

ISO (International Organization for Standardization) defines a technical standard as "a document established by consensus and approved by a recognized organism, which provides minimum rules, guidelines or characteristics for activities or their results, aiming at obtaining a great degree of sorting in a given context."

It should be highlighted that technical standards are established by consensus among those interested and approved by a recognized organism. They are also developed for the benefit and cooperation of all interested parties and, in particular, to promote the optimal global economy, taking into account functional conditions and safety requirements.

(a) Use of the Standards

Standards are used as a reference for conformity evaluation, such as certification, calibration, or testing.

In addition to intending that the product follow a particular standard, the customer often wants conformity with this standard to be demonstrated through conformity evaluation procedures. Sometimes, these procedures, particularly certification, are legally required for some markets (compulsory certification established by the government to commercialize products and services). In others, although there is no legal obligation, current practices in this market make it indispensable to use specific conformity procedures, usually certification.

The legal order generally considers that the rules in force in the market should be followed unless the client explicitly establishes another rule. Thus, when a company intends to introduce its product or service into a particular market, it should seek to know the rules that apply there and fit.

(b) Voluntarity of the Standards

The standards are voluntary and not mandatory by law, and it is possible to provide a product or service that does not follow the applicable standard in the particular market. However, in several countries, they are mandatory, at least in some areas.

On the other hand, providing a product that does not follow the applicable standard in the market implies additional efforts to introduce it to this market. These include convincingly demonstrating that the product meets customer needs and ensuring that issues like exchanging components and inputs will not represent an additional impediment or difficulty. From a legal point of view, when the applicable standard does not follow, the supplier has additional responsibilities for using the product.

Frequently, a standard refers to other standards necessary for its application. Standards may also be required to comply with technical regulations or compulsory certification.

(c) International, Regional, and National Standards

International

These are the technical standards established by an international standardization organism for application in all countries, for example, the rules published by ISO (International Organization for Standardization), IEC (International Electrotechnical Commission), or ITU (International Telecommunication Union).

Regional

These are the technical standards established by a regional standardization organism for application in countries that belong to this region, such as the rules published by CEN (European Committee for Standardization), CENELEC (European Committee for Electrotechnical Standardization), or COPANT (Pan American Standards Commission).

National

These are technical standards established by a national standardization organism for application in a given country. For example, in Brazil, Brazilian standards (NBR) are prepared by ABNT (Brazilian Association of Technical Standards), in Germany by DIN (Deutsches Institut Für Normung), in England by BSI (British Standards Institute), and in the United States of America by ANSI (American National Standards Institute).

2.9.6.2 Technical Regulation

A technical regulation is a document a legal authority adopts to do so. It contains mandatory rules and establishes technical requirements, either directly, by reference to technical standards, or by incorporating their content, in whole or in part. In general, technical regulations aim to ensure aspects related to health, safety, environment, consumer protection, and fair competition. Compliance with a technical regulation is mandatory, and non-compliance is illegible and punishable by the corresponding punishment.

Sometimes, a technical regulation establishes the technical rules and requirements for a product, process, or service and can also establish procedures for assessing compliance with regulation, including compulsory certification.

Technical Regulations and International Trade
All countries issue technical regulations. Thus, when it is intended to export a product for a particular market, it is essential to know if the product or service to be exported is subject to a technical regulation in that country in particular.

The WTO Trade Technical Barriers Agreement establishes a series of principles to eliminate unnecessary obstacles to trade, particularly technical barriers related to standards, technical regulations, and compliance assessment procedures that can make it difficult to access products to markets. One of the essential points of the agreement is the understanding that the standards prepared by international standardization bodies (ISO or IEC) constitute the reference for global trade. The agreement stipulates that, whenever possible, governments must adopt technical regulations based on international standards, considering that technical barriers do not constitute those who follow these rules.

Knowing a Little More...
Whenever a government decides to adopt a technical regulation that does not follow an international standard, it should formally notify the other members of the WTO at least 60 days in advance, a justification presented. Other WTO members may request clarification and submit comments and suggestions to the proposed regulation. This information is conveyed by the so-called "Inquiry Points," which are organizations designated by each of the WTO members responsible for notifications of the regulation to be adopted by that country and for receiving notifications made by other countries. Brazil's Inquiry Point is INMETRO.

Fig. 2.25 VIM

2.10 International Vocabulary of Metrology (VIM)

The following text reproduces the introduction of the Bilingual, English, and French editions issued by BIPM and highlights the document's importance (Fig. 2.25).

> *In general, a vocabulary is a "terminological dictionary which contains designations and definitions from one or more specific subject fields" (ISO 1087-1:2000, 3.7.2). The present Vocabulary pertains to metrology, the "science of measurement and its application." It also covers the basic principles governing quantities and units. The field of quantities and units could be treated in many different ways. Clause 1 of this Vocabulary is one such treatment and is based on the principles laid down in the various parts of ISO 31, Quantities and units, currently being replaced by ISO 80000 and IEC 80000 series Quantities and units, and in the SI Brochure, The International System of Units (published by the BIPM).*
>
> *The second edition of the International vocabulary of basic and general terms in metrology (VIM) was published in 1993. The need to cover measurements in chemistry and laboratory medicine for the first time, as well as to incorporate concepts such as those that relate to metrological traceability, measurement uncertainty, and nominal properties, led to this third edition. Its title is now International vocabulary of metrology — Basic and general concepts and associated terms (VIM), to emphasize the primary role of concepts in developing a vocabulary.*

In this Vocabulary, it is taken for granted that there is no fundamental difference in the basic principles of measurement in physics, chemistry, laboratory medicine, biology, or engineering. Furthermore, an attempt has been made to meet the conceptual needs of measurement in fields such as biochemistry, food science, forensic science, and molecular biology.

Several concepts that appeared in the second edition of the VIM do not appear in this third edition because they are no longer considered to be basic or general. For example, the concept 'response time,' used in describing the temporal behavior of a measuring system, is not included. For concepts related to measurement devices that are not covered by this third edition of the VIM, the reader should consult other vocabularies such as IEC 60050, International Electrotechnical Vocabulary, IEV. For concepts concerned with quality management, mutual recognition arrangements about metrology, or legal metrology, the reader is referred to documents given in the bibliography.

The development of this third edition of the VIM has raised some fundamental questions about different current philosophies and descriptions of measurement, as will be summarized below. These differences sometimes lead to difficulties in developing definitions that could be used across the different descriptions. No preference is given in this third edition to any of the particular approaches.

The change in the treatment of measurement uncertainty from an Error Approach (sometimes called Traditional Approach or True Value Approach) to an Uncertainty Approach necessitated a reconsideration of some of the related concepts appearing in the second edition of the VIM. The objective of measurement in the Error Approach is to determine an estimate of the true value that is as close as possible to that single true value. The deviation from the true value is composed of random and systematic errors. The two kinds of errors, assumed to be always distinguishable, have to be treated differently. No rule can be derived on how they combine to form the total error of any given measurement result, usually taken as the estimate. Usually, only an upper limit of the absolute value of the total error is estimated, sometimes loosely named "uncertainty."

In the CIPM Recommendation INC-1 (1980) on the Statement of Uncertainties, it is suggested that the components of measurement uncertainty should be grouped into two categories, Type A and Type B, according to whether they were evaluated by statistical methods or otherwise, and that they be combined to yield a variance according to the rules of mathematical probability theory by also treating the Type B components in terms of variances. The resulting standard deviation is an expression of a measurement uncertainty. A view of the Uncertainty Approach was detailed in the Guide to the expression of uncertainty in measurement (GUM) (1993, corrected and reprinted in 1995), which focused on the mathematical treatment of measurement uncertainty through an explicit measurement model under the assumption that the measurand can be characterized by an essentially unique value. Moreover, in the GUM as well as in IEC documents, guidance is provided on the Uncertainty Approach in the case of a single reading of a calibrated instrument, a situation normally met in industrial metrology.

The objective of measurement in the Uncertainty Approach is not to determine a true value as closely as possible. Rather, it is assumed that the information from measurement only permits the assignment of an interval of reasonable values to the measurand, based on the assumption that no mistakes have been made in performing the measurement. Additional relevant information may reduce the range of the interval of values that can reasonably be attributed to the measurand. However, even the most refined measurement cannot reduce the interval to a single value because of the finite amount of detail in the definition of a measurand.

The definitional uncertainty, therefore, sets a minimum limit to any measurement uncertainty. The interval can be represented by one of its values, called a "measured quantity value."

In the GUM, the definitional uncertainty is considered to be negligible concerning the other components of measurement uncertainty. The objective of measurement is to establish a probability that this essentially unique value lies within an interval of measured quantity values based on the information available from measurement.

The IEC scenario focuses on measurements with single readings, permitting the investigation of whether quantities vary in time by demonstrating whether measurement results are compatible. The IEC view also allows non-negligible definitional uncertainties. The validity of the measurement results is highly dependent on the metrological properties of the instrument, as demonstrated by its calibration. The interval of values offered to describe the measurand is the interval of values of measurement standards that would have given the same indications.

In the GUM, the concept of true value is kept for describing the objective of measurement, but the adjective "true" is considered to be redundant. The IEC does not use the concept to describe this objective. In this Vocabulary, the concept and term are retained because of common usage and the importance of the concept.

[VIM—introduction].

2.11 Proposed Exercises

2.11.1 Analyze the following statement: "It is not necessary to calibrate a brand-new instrument from a reputable and traditional manufacturer in the market because the manufacturer guarantees its traceability." Do you agree or disagree? Justify your answer.
2.11.2 What is metrology?
2.11.3 Present some differences between scientific metrology and legal metrology.
2.11.4 What is the function of legal metrology in our society?
2.11.5 According to ISO 10012:2003, present five items should be included in the records of metrological evidence processes.
2.11.6 What do you mean by "influence quantity"?
2.11.7 What is the leading world organism of legal metrology?
2.11.8 What does it mean for a laboratory to be accredited?
2.11.9 What is the difference between technical standards and technical regulation?
2.11.10 What is the importance of the international vocabulary of metrology (VIM)?

Chapter 3
Statistics Applied to Metrology

3.1 Significant Digits of a Measure

The result of a calculation using all the calculator's display digits implies that it is accurate for all digits, a fact that, in practice, is rarely possible (the number of digits can be increased considerably using computers).

When we use the measurement results from calculations, we must consider that the numbers used have only a limited number of significant digits, because the concepts of uncertainty, accuracy, resolution, and conversion of units are involved.

Suppose measurement 13.403 m indicates the most likely value of a quantity, and the maximum variation in the measurement series to calculate this value is 0.04 m. As this variation can be more or less, we must express the result of the measurement as follows:

$$(13.403 \pm 0.04)\ \mathrm{m}$$

Analyzing the result, we note that the second decimal digit of the most likely value is uncertain. Therefore, it is unnecessary to write the third decimal, since the previous one is already uncertain.

The measurement result must be expressed as (13.40 ± 0.04) m.

Of the considerations made, we can establish the concept of significant digits of a measure.

Attention!
The significant digits of a measure are the digits considered correct, from the first different from zero, plus the latter, which is regarded as the doubtful significant digit.

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_3

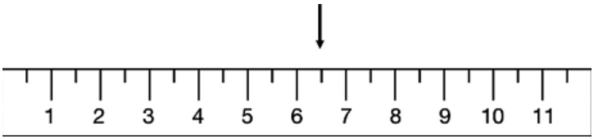


Fig. 3.1 Ruler graduated in centimeters

In the case presented, measure 13.40 m has four significant digits: 1, 3, and 4 are considered correct significant digits, and zero is considered a doubtful significant digit.

In every measurement, the last estimated digit will always be doubtful. This is either because we will always doubt this value—after all, we must exhibit it—or because the digital instrument "estimated" it for us.

Let us look at the following figure (Fig. 3.1).

The ruler is graduated in centimeters. If we look at the position of the arrow, the value is 6.5 cm. Note that the digit five would be doubtful of the measurement. This is because we cannot affirm that the arrow position is 6.5 cm. If the ruler had a lower division of 0.1 cm, we could read 6.4 cm, 6.5 cm, or 6.6 cm, or if it had subdivisions of 0.01 cm, we could read 6.48 cm, 6.49 cm, 6.50 cm, or 6.51 cm. Even so, the digits 8, 9, 0, and 1 would be the doubtful. This is why a measurement will always have a doubtful digit.

In the chapters on measurement uncertainty (Chaps. 5 and 6), we will see this question in more detail and study how this reading limitation will imply the appearance of a source of measurement uncertainty: the uncertainty of reading resolution.

Example 3.1 Measurements and the Numbers of Significant Digits

(a) 23.50 m: four significant digits
(b) 0.0043 m: two significant digits
(c) 67 °C: two significant digits
(d) 127 V: three significant digits.

We must be careful when zero numbers are at the end of the numbers. If the "zeros" are written correctly to correspond to significant numbers, 36.00 has four significant digits, and 36.0 has three. In these two cases, zeros are necessary to define the accuracy of the measurement.

To decrease ambiguities, we must observe the following rules on "zeros":

Rule 1: Zeros are insignificant if situated to the left of the first significant digit.

Example: 0.023 kg (two significant digits).

The zeros on the left of digit two only express that the measurement result is less than the unit (1 kg).

Rule 2: Right zeros should only be written when guaranteed significant.

Example: 0.12300 mm (five significant digits).

When a number ends in zeros on the right, these zeros may not necessarily be significant. For example, 50,600 calories may have three, four, or five significant digits. Ambiguity can be avoided by using exponential or "scientific" standard notation.

If the number of significant digits is three, four, or five, we could write 50,600 calories, such as: 5.06×10^4 calories (three significant digits)

5.060×10^4 calories (four significant digits)
5.0600×10^4 calories (five significant digits)

When writing a number in scientific notation, the number of significant digits is indicated by the number of numerical digits in the term "digits," as shown in the examples.

Important
The power of ten is not considered a significant digit.

3.1.1 Number Rounding

When the measure has more significant numerals than you need, we should keep only those necessary and abandon the others.

For example, measurement 34.527 m has five significant digits. If we have to express it with only three, we should write 34.5 m. If we need four, we write 34.53 m.

In the latter case, we observed that the digit of the second decimal house went from 2 to 3.

Here is the reason. If we had used 34.52 m, we would have made a mistake, for lack of it, equal to $(34.527 - 34.52)$ m $= 0.007$ m.

Using 34.53 m, we made a minor error, by excess, of:

$$(34.53\text{–}34.527)\ \text{m} = 0.003\ \text{m}.$$

When we round a number, we must keep in mind the following rules:

(a) The last digit of a number should always be added from a unit if the discarded digit is bigger than five.

Examples of rounding to three significant digits:

$$134.7\ \text{m} = 135\ \text{m}$$

$$0.03432\ \text{mm} = 0.0343\ \text{mm}$$

(b) If the discarded digit is equal to five, if there are any other numerals different from zero after the discarded five, the last retained digit will be plus a unit.

Examples of rounding to three significant digits:

$$14.751\,°\mathrm{C} = 14.8\ °\mathrm{C} \quad 0.0346501\ \mathrm{km} = 0.0347\ \mathrm{km}$$

(c) If the discarded digit is five, if there are only zero or no other digit after five, the last retained digit will be added to a unit only if it is odd.
Examples of rounding to three significant digits:

$$4.8350\ \mathrm{N} = 4.84\ \mathrm{N}$$
$$34.25\ °\mathrm{C} = 34.2\ °\mathrm{C}$$

3.1.2 *Operations with Significant Digits*

We must act as follows in mathematical operations for the result of operations containing significant digits only.

3.1.2.1 Addition and Subtraction

We usually add or subtract, and the operation result must have the same number of decimal digits in the portion with *the smallest number of decimal digits.*

Example: Give the result of the sum (85.45 m + 5.6 m + 98.523 m) with the correct number of significant digits.

Solution: Add the numbers and provide the result with the decimal digits in the portion with the fewest digits.

$$\begin{array}{r} 85.45 \\ 5.6 \\ +\ 98.523 \\ \text{-----------} \\ 189.573 \end{array}$$

As the portion with fewer decimal digits is 5.6 (one decimal), adopting the sum and subtraction rule present will result in **189.6 m**.

3.1.2.2 Multiplication and Division

We usually multiply or divide, and the result must have the same *number of significant digits as the portion with the smallest number of significant digits*.

Example: Give the result of the division of 89.1 m^2 by 5.4690 m, with the correct number of significant digits.

Solution: Divide and provide the result with the number of significant digits of the portion with the smallest number of significant digits.

$$\frac{89.1\ \mathrm{m}^2}{5.4690\ \mathrm{m}} = 16.29182\ \mathrm{m}$$

Adopting the rule of multiplication and division, as 89.1 has only three significant digits, the result of the division will be **16.3 m.**

3.1.2.3 Square Root

The square root of a number with n significant digits can have significant digits at most n and at least $n - 1$.

Example: $\sqrt{25.5}\,\mathrm{km}$

Since 25.5 km has three significant digits, we can represent the result as 5.05 or 5.0. The amount of significance used will depend on the accuracy of the calculation.

Example: $\left(\sqrt{25.5} + 4.8\right)\ \mathrm{km} = (5.0 + 4.8)\ \mathrm{km} = 9.8\ \mathrm{km}.$

$$\left(\sqrt{25.5} + 4.81\right)\ \mathrm{km} = (5.05 + 4.81)\ \mathrm{km} = 9.86\ \mathrm{km}.$$

3.1.3 Mixed Operations

When using a calculator, if you work all the long calculations without writing the intermediate results, you may be unable to tell if an error has been made. In addition, even if you realize there were any errors, you may be unable to say where it is. In an extensive calculation involving mixed operations, as many digits should be performed as possible in the entire set of calculations, and then the result should be appropriately rounded.

For example: (5.00/1.235) m + 3.000 m + (6.35/4.0) m = (4.04858 + 3.000 + 1.5875) m = 8.630829 m.

The first division should result in three significant digits. The last division should result in two significant digits. The three added numbers should result in a number with one digit after the decimal home. Thus, the correct rounded result must be **8.6 m**. The last operation (division) limits this result's accuracy.

Important

In an extensive calculation involving mixed operations, as many digits as possible should be performed in the entire set of calculations, and then the result should be appropriately rounded.

Most modern calculators allow you to load the results of intermediate calculations on the display by performing complex calculations. In doing so, it is possible to maintain the results of each calculation step without inserting the intermediate results (a practice that perhaps encourages rounding too early). This way, you can altogether avoid truncation errors introduced by intermediate rounding.

Using all digits in the result can be critical for many mathematical operations in statistics. Rounding intermediate results by calculating squares can seriously compromise their accuracy.

3.2 Concepts of Statistics Applied to Metrology

Here is a question: Why must we know the statistical tools to work in metrology?

It is well known that every measure performed has an associated measurement uncertainty, which, depending on the type and quality of the instrument or system used, can be small or large compared to the measurement's result.

Thus, we can say that MR = X ± U, where.

MR = measurement result.

X = measurement value (or the mean measurement) performed and.

U = uncertainty of the measurement.

In VIM—2.26, we find the following definition for measurement uncertainty: *"non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used."* The VIM also cites that *"the parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability."*

We realize, then, that measurement uncertainty is an evaluated parameter through some statistical tools. As the result of measurement will always have an associated doubt (called the uncertainty of measurement), what is sought is to estimate the values of measure and uncertainty in the best possible way.

Measurement uncertainty will always exist and will never be eliminated, since the actual value of greatness is estimated (in practice, the value of the standard is used as a reference value). However, it is possible to define the limits within which the value of a measurement is found by considering a specific probability value using statistical techniques and analyses.

Experimental measurements are carried out based on random experiments, and random experiment means that which is influenced by non-controlled, random variables. Thus, measurements are random experiments and can be repeated indefinitely, and once repeated, we will probably obtain different results.

Random experiments are associated with a sample space or population. Population N is defined as all elements available for evaluation. This population can be a finite number (e.g., the total residents of a building) or infinite (e.g., the set of natural numbers). A sample n represents a portion of the population, but it should be chosen to present the characteristics and properly represent this origin population.

After this measurement process is completed, a specific value will represent an estimate of the measurement result. From the result of this sample and by attributing a certain degree of confidence, one can analyze the behavior of the measuring instrument as a whole. Conclusions obtained based on a sample or amount of data streamline the measurement process and reduce costs.

Suppose we need to determine the density of a solid (ρ). It is known that the density is the relationship between the mass of the body (m) and its volume (V), given by the expression:

$$\rho = \frac{m}{V} \tag{3.1}$$

Then, when measuring the mass of the body with the aid of a scale, we have the variable mass M as a random component since its value can be affected by the position in which we place the body on the scale plate, besides the characteristic of the scale itself not to reproduce the measured values repeatedly.

The same is valid for measuring the part's volume. It is affected by the temperature variation where its measurement is performed and by the instability of the measurement instruments.

This is why statistical analysis is fundamental to metrology. It enables data description from central trend measures, dispersal measures, and probability distribution, followed by the analysis and interpretation of the obtained results.

Before we discuss the concept of uncertainty of measurement and present its calculation methodology, it is necessary to introduce some basic statistics foundations and tools.

3.2.1 Random Variable, Random Experiment, and Sample Space

When we experiment with a measurement, we are subject to results that can be influenced by variables we do not control. For example, the variation in ambient temperature can influence the length of a metal part (dilation), or the relative humidity of the air can affect the mass of a moisture-absorbing substance. Finally, random variables in every measurement process can interfere or may interfere with the measurement result.

Thus, such experiments have random variables, regardless of being careful with the experiment, and we cannot avoid these influence variables. Our goal, then, will be to understand, quantify, and model these types of variations that we often find in measurements.

We can define a random experiment as anyone who provides different results, even taking all the precautions to perform the measurement procedure similarly. The set of all possible results of a random experiment is called the sample space of the experiment.

These random variables can be divided into two types: discreet and continuous.

3.2.1.1 Discreet Random Variable (Fig. 3.2)

One dice has x values (1, 2, 3, 4, 5, 6). The variable is discreet; it cannot assume intermediate values. When we launch a dice, we will not find, for example, values between 1 and 2 or between 4 and 5. A discreet random variable is a variable with a finite number of values. Other examples are the number of wrinkles in a car, the number of oranges in a basket, and the number of parts manufactured in one day.

3.2.1.2 Continuous Random Variable

For example, the room temperature of a laboratory (T) measured over a week is considered a continuous random variable, because it can take any value throughout the day and week. Therefore, a continuous random variable assumes infinite values. Examples include temperature measurement, pressure measurement, and electric current measurement.

Note that the typical variables of interest in metrology are continuous. The most commonly used continuous probability distributions in metrology are:

- Uniform or rectangular distribution.
- Triangular distribution.
- Normal or Gaussian distribution.
- Student's t distribution.

This chapter will study these probability distributions and their main characteristics.

Fig. 3.2 A dice

3.2.2 *Distribution of Measured Data*

3.2.2.1 Not Grouped Data

For example, the CEN-EN 837–1 standard states that the type Bourdon manometer's calibration temperature should be between (20 ± 2) °C. Here, we have 80 measurements performed over one day, and the results are found in Table 3.1.

Question: Are all measurements within the tolerance range established by the standard?

Answer: Yes! After analyzing all 80 data points, it is clear that the lowest value was 18 °C, and the largest was 22 ° C.

Let us ask other questions:

- What was the value that prevailed in the measurement set?
- Did measurements vary a lot or little?
- Have you ever thought that instead of 80 measurements, we had 8000?
- Do you agree that there should be a more appropriate way to dispose of this data to facilitate answers?

3.2.2.2 Grouped Data

The immediate way is to group the data in a list. The list consists of grouping this data increasingly or decreasingly. For the temperature example, we will put it in ascending order of values (Table 3.2).

The list makes it easy to answer if the results are within tolerance. Just look at the first (18 °C) and the last (22 °C), but it is not so immediate to answer the other questions!

Table 3.1 Temperature measurements

19	21	20	21	19	20	19	19
22	20	18	18	20	22	21	21
18	21	19	21	18	19	19	22
21	19	22	22	20	20	22	22
21	18	19	18	18	20	18	21
20	19	20	21	22	20	18	21
19	18	22	21	18	21	19	19
19	19	19	20	22	18	21	21
18	19	18	21	22	22	21	21
20	21	19	22	18	21	21	21

Table 3.2 List of temperature measurements

18	18	18	18	18	18	18	18
18	18	18	18	18	18	18	19
19	19	19	19	19	19	19	19
19	19	19	19	19	19	19	19
19	20	20	20	20	20	20	20
20	20	20	20	20	21	21	21
21	21	21	21	21	21	21	21
21	21	21	21	21	21	21	21
21	21	21	22	22	22	22	22
22	22	22	22	22	22	22	22

Table 3.3 Simple absolute frequency (f_a)

Value °C	f_a
18	15
19	18
20	12
21	22
22	13
Σ (sum)	80

Let us then group the data by value. This means ordering similar with similar, that is, grouping equal values into classes by providing them in a table called a frequency distribution.

There are four types of frequencies:

- *Simple absolute frequency* (f_a)—corresponds to the number of occurrences of a value within a class.

Table 3.3 shows that the predominant value was 21 ° C, which appeared 22 times!

- *Accumulated absolute frequency* (F_a)—corresponds to the sum of the absolute frequencies of the current class with the sum of the frequencies immediately before it.

By this table it is observed that more than half of the data (45 of 80) are between 18 and 20 °C! (Table 3.4)

- *Simple relative frequency* (f_r)—is the ratio between the absolute simple frequency of the class and the sample size (n).

Table 3.4 Accumulated absolute frequency (F_a)

Value °C	f_a	F_a
18	15	15
19	18	33
20	12	45
21	22	67
22	13	80

Table 3.5 Simple relative frequency (f_r)

Value °C	f_a	f_r
18	15	18.75%
19	18	22.5%
20	12	15%
21	22	27.5%
22	13	16.25%
Σ	80	100%

Table 3.6 Accumulated relative frequency (F_r)

Value °C	f_a	f_r	F_r
18	15	18.75%	18.75%
19	18	22.5%	41.25%
20	12	15%	56.25%
21	22	27.5%	83.75%
22	13	16.25%	100%

$$f_r = \frac{fa}{n}$$

Table 3.5 shows that the values of 19 and 21 °C together represent 50% of the values. The remaining 50% are distributed by 18, 20, and 22 °C.

- *Accumulated relative frequency (F_r)*—corresponds to the sum of the relative frequencies of the current class with the sum of the immediately preceding relative frequencies (Table 3.6).

More than half of the data (56.25%) is between 18 and 20 °C!

3.2.2.3 Histogram

A histogram is a bar graph that shows a frequency distribution, that is, the table where we present the data collected due to the frequency of its occurrence. The base of a rectangle represents a class of the frequency table. The height of the bar is proportional to the frequency value contained in the class. The horizontal scale of the graph is quantitative. The vertical scale indicates the absolute or relative frequency.

This data must be divided into class intervals. An efficient method for determining the number of class intervals consists of obtaining the square root of the number

Table 3.7 Oven temperature

Temperature (°C)					
49.59	49.60	49.63	49.64	49.66	49.68
49.59	49.61	49.63	49.65	49.67	49.68
49.59	49.62	49.63	49.65	49.67	49.68
49.59	49.62	49.64	49.65	49.67	49.69
49.60	49.62	49.64	49.66	49.67	49.69
49.60	49.62	49.64	49.66	49.67	49.69
49.60	49.62	49.64	49.66	49.67	49.69
49.60	49.62	49.64	49.66	49.67	49.70
49.60	49.62	49.64	49.66	49.67	49.70
49.60	49.63	49.64	49.66	49.68	49.70

of data collected. The number of class intervals will be approximately equal to the value of this root. Class intervals should be equal wide to increase graphic information in frequency distribution.

Solved Exercise 3.1

Consider Table 3.7 with 60 temperature values of a stabilized thermometer calibration oven around 50.00 °C.

When the variation in oven temperature reaches its stability, it generates uncertainty in the thermometer calibration, called oven stability uncertainty.

Based on these measurement results, make your histogram.

Solution: Let us build the histogram using five steps.

Step 1: Determination of the measurement interval's range R. The range measures the dispersion between the minimum and maximum distribution values, not considering the intermediate values.

$$R = X_{max} - X_{min} \tag{3.2}$$

One feature of the range is that even though the number of measurements increases, it does not decrease (may even increase). In this example, the range is determined by:

$$R = (49.70–49.59)\,^\circ C = 0.11\,^\circ C$$

Step 2: Class number *C*.

We determine the class *C* number by calculating the square root of the number of measurements performed *n*, therefore:

$$C = \sqrt{n} = \sqrt{60} = 7.746 \tag{3.3}$$

Thus, our histogram's class number will be 7 or 8, depending on the size of each class.

Step 3: Width of classes L.

To determine the width of class L, we must divide the range R by the class number chosen, C.

$$L = \frac{R}{C} \tag{3.4}$$

Choosing $C = 8$:

$$L = \frac{0.11}{8} = 0.013$$

Choosing $C = 7$:

$$L = \frac{0.11}{7} = 0.016$$

Note that both L values do not provide numbers with the same decimal places as the measured data. It would be interesting if the width L provided us with values such as 0.01 °C or 0.02 °C.

A proper technique in building histograms is to increase a small range R, so that we begin to count the frequency of incidence of our values just before and shortly after the beginning.

For example:

$$R = (49.71\text{–}49.58)\ \ {}^{\circ}\mathrm{C} = 0.13\ \ {}^{\circ}\mathrm{C}$$

Thus, the new value of L will be:

$$L = \frac{0.13}{8} = 0.01625\,{}^{\circ}C$$

$$L = \frac{0.13}{7} = 0.01857\,{}^{\circ}C$$

So let us round out to $L = 0.02$ °C.

Step 4: Counting by class—frequency.

This is the penultimate stage, where we build a table relating the classes and their frequency of occurrence. In this example, we have (Table 3.8):

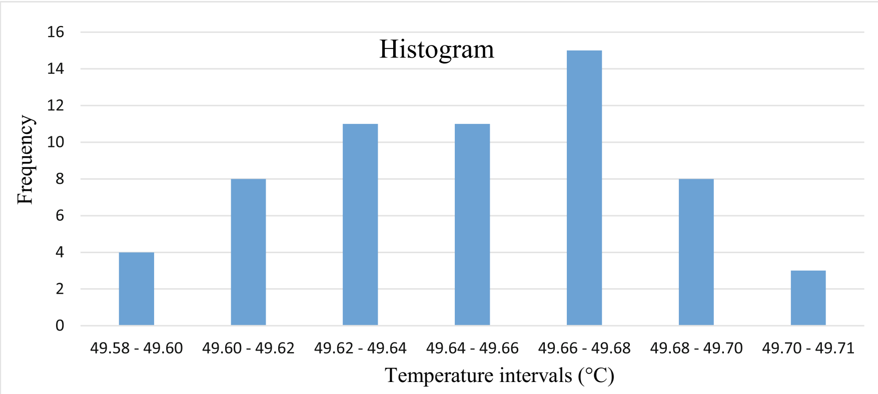
Note that each class interval is "closed" (≤) at the beginning and open (<) at the end. This is important so that the end value of the interval is not counted more than once.

Step 5: Histogram graph.

At this stage, we selected the class interval and their frequency and set up a bar chart, for example, using Microsoft® Excel software.

Table 3.8 Class interval and the frequency

Class interval	Frequency
49.58 °C ≤ x < 49.60 °C	4
49.60 °C ≤ x < 49.62 °C	8
49.62 °C ≤ x < 49.64 °C	11
49.64 °C ≤ x < 49.66 °C	11
49.66 °C ≤ x < 49.68 °C	15
49.68 °C ≤ x < 49.70 °C	8
49.70 °C ≤ x < 49.71 °C	3



Graph 3.1 Stabilized oven temperature distribution histogram

The same Excel also automatically makes the histogram by selecting the data and clicking on the histogram in the data tab within the data analysis icon (Graph 3.1).

Note that the mean oven temperature is worth 49.64 °C and is in the middle of the frequency distribution in the histogram.

This is a feature of most statistical distributions, as we will see later in this chapter.

3.2.3 Probability Density Function (PDF)

One probability density function $f(x)$ describes the distribution of a random variable x. If a value x is very likely to occur within an interval [a, b], its PDF $f(x)$ will be significant in this interval. The PDF $f(x)$ then describes the probabilities associated with a random variable.

The PDF $f(x)$ may be discreet or continuous, depending on whether the variable x is discrete or continuous.

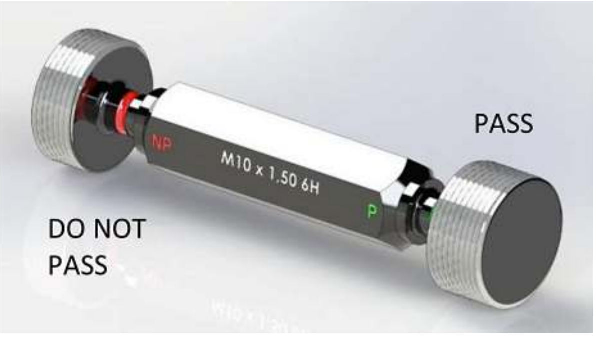


Fig. 3.3 Standard caliber. (https://www.calibratools.com.br)

3.2.3.1 Discrete Density Function

A discrete PDF describes the behavior of the variables that provide integer and finite values.

Example 1: It is very common to use measurement devices called calibers, which inform whether one piece conforms or not. Variable X then has only two values: pass/do not pass or go/do not go (Fig. 3.3).

The discrete PDF that characterizes the example well is the *Binomial Distribution*.

A binomial distribution is adequate when the results of a random variable are grouped into only two classes or categories. These categories must be mutually exclusive. For example, a manufactured product may be perfect or defective, an answer may be correct or wrong, and a telephone call is local or long distance.

Even continuous variables can be divided into two categories. For example, a car's speed may be below or above the legal limit on a road. These categories are usually called *success* or *failure*.

Application conditions:

- They are made in independent repetitions of the experiment, that is, the result of repetition is not influenced by others.
- The probability of success p and failure $(1 - p)$ remain constant in all repetitions.

The following expression gives the binomial model:

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad x = 0, 1, \ldots, n \tag{3.5}$$

$$\text{Mean} \rightarrow \mu = np \tag{3.6}$$

$$\text{Standard deviation} \quad \rightarrow \quad \sigma = \sqrt{np(1-p)} \tag{3.7}$$

The binomial model is usually used in quality control when sampling a large population. In these applications, x represents the number of defective observations in a n-sample.

Table 3.9 Discreet uniform PDF

Value of X	Probability $p(X)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Another interesting statistic is the defective fraction of a sample:

$$\widehat{p} = \frac{x}{n} \tag{3.8}$$

$$\sigma_{\widehat{p}}^2 = \frac{p(1-p)}{n} \tag{3.9}$$

Example 2: Consider a dice whose values of X are (1, 2, 3, 4, 5, 6). When launching it, the probability of obtaining any of the values of X is $p(X) = 1/6$.

Thus, the PDF $p(x)$ of this discreet variable is (Table 3.9):

The PDF that characterizes the example well is the Discreet Uniform.

The following equations define the discreet uniform distribution:

$$P(X = x) = \frac{1}{k}, \quad x_1, \quad x_2, \quad \ldots, \quad x_k \tag{3.10}$$

$$\text{Mean} \rightarrow \mu = \frac{\sum_1^k x_i}{k} \tag{3.11}$$

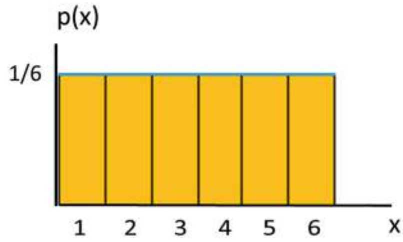
$$\text{Standard deviation} \quad \rightarrow \quad \sigma = \sqrt{\frac{\sum_1^k (x_i - \mu)^2}{k}} \tag{3.12}$$

In the case of launching the dice, we will have:

$$P(X = x) = \frac{1}{6} \qquad x = 1, 2, 3, 4, 5, 6$$

$$\mu = \frac{\sum_1^k x_i}{k} = 3.5 \qquad \sigma = \sqrt{\frac{\sum_1^k (x_i - \mu)^2}{k}} = \sqrt{\frac{\sum_1^k (x_i - 3, 5)^2}{6}} = 1.71$$

Graph 3.2 Discreet uniform PDF



3.2.3.2 Continuous Density Function

For a continuous random variable x, a PDF $f(x)$ is such a function that:

$$f(x) \geq 0$$

$$f(x) = \int_{-\infty}^{+\infty} f(x)\, dx = 1$$

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

In Graph 3.3, the area under $f(x)$ represents the probability of x assuming a value between a and b (Graph 3.3).

$$\mu = \frac{\sum_1^k x_i}{k} = 3,5 \qquad \sigma = \sqrt{\frac{\sum_1^k (x_i - \mu)^2}{k}} = \sqrt{\frac{\sum_1^k (x_i - 3,5)^2}{6}} = 1,71$$

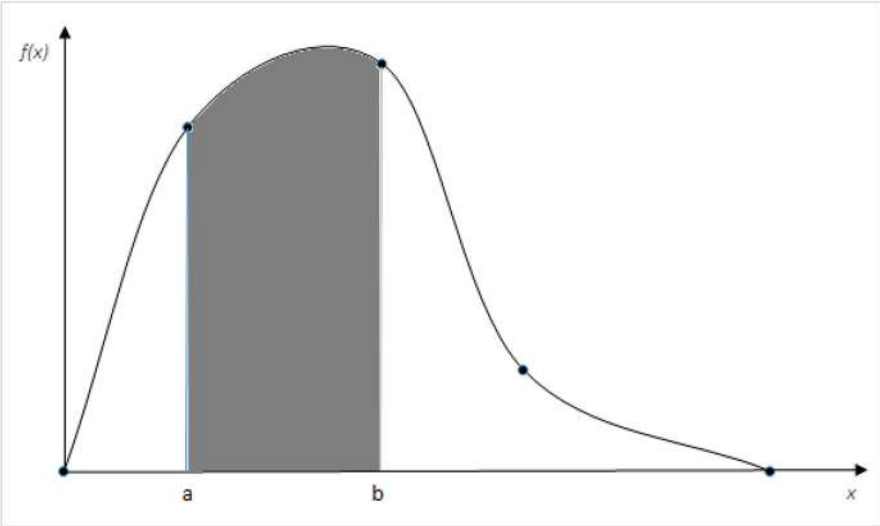
For a discrete random variable X, the sum of the discrete distribution $P(x)$ values between the boundaries $-\infty$ and $+\infty$ always results in one.

Remembering the example of the dice, the sum of $P(x)$ will be:

$$P(-\infty \ \leq X \leq +\infty \ ) = P(1 \leq X \leq 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

3.2.4 *Mean and Standard Deviation of a Probability Distribution*

Probability distributions are characterized by their mean value and their standard deviation. When the mean in question is the population's mean, we designate the Greek letter μ. When it comes to the mean of a sample, we designate by $\bar{x}$.



Graph 3.3 Probability $P\ (a \leq X \leq b)$

The mean (μ), or expected value $E(X)$, is the best estimate of a measurement and is defined by the equation:

$$\mu = E(X) = \int x.f(x)dx \tag{3.13}$$

where X is a random variable, and $f(x)$ is a PDF.

Although the mean sample or the mean population is valid, it is important to know how dispersed the data around the mean is. The variable that measures the dispersion of these data around the mean is called *standard deviation*, and its square is known as *variance*.

The variance of X is denoted by σ^2 or $V(X)$ and defined by the expression:

$$\sigma^2 = V(X) = E\left(\left((x - E(x)\right)^2\right) = \int (\mu - x)^2 f(x)dx \tag{3.14}$$

3.2.5 Distributions of Probabilities More Adopted in Metrology

We mention, in Sect. 3.2.1, that the most commonly used probability distributions in metrology are uniform or rectangular, triangular, normal or Gaussian, and Student's t.

Now, let us study the main features of these distributions and their applications in metrology.

3.2.5.1 Rectangular or Uniform Distribution

We will face a uniform or rectangular distribution when the probability distribution is constant in a defined interval (Graph 3.4).

$$f(x) = \frac{1}{b-a}; \qquad a \le x \le b \tag{3.15}$$

$$f(x) = 0 \qquad x < a \text{ ou } x > b.$$

The mean of a uniform continuous random variable x is defined by Eq. (3.13): $\mu = E(X) = \int x.f(x)dx$.

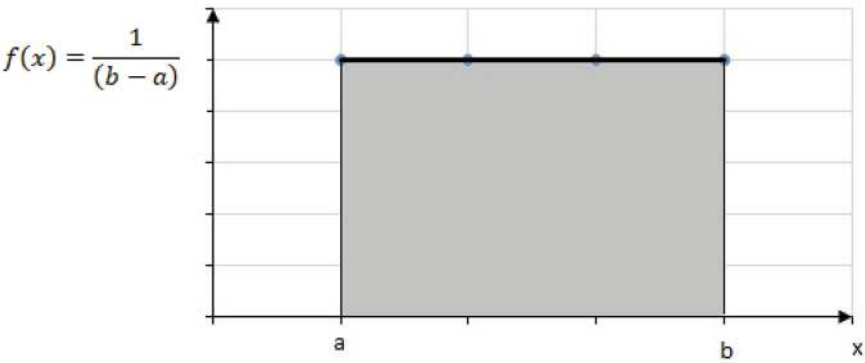
Integrating within the limits between a and b and adopting $f(x) = \frac{1}{(b-a)}$, we have:

$$\int_a^b \frac{x}{b-a}dx = \frac{x^2}{2(b-a)}\Bigg|_a^b$$

$$\mu = \bar{x} = \frac{a+b}{2} \tag{3.16}$$

Using the variance definition by Eq. (3.14), we have:

$$\sigma^2 = \int (\mu - x)^2 f(x)dx$$



Graph 3.4 Uniform or rectangular distribution

$$\sigma^2 = \int_a^b \frac{\left(x - \left(\frac{a+b}{2}\right)\right)^2}{b-a} dx$$

$$\sigma^2 = \left. \frac{\left(x - \frac{a+b}{2}\right)^2}{3(b-a)} \right|_a^b = \frac{(b-a)^2}{12} \tag{3.17}$$

Since standard deviation is the square root of variance, so we have:

$$\sigma = s(x) = \frac{b-a}{\sqrt{12}} = \frac{b-a}{2\sqrt{3}} \tag{3.18}$$

We adopted the expression $s(x)$ for the sample standard deviation and the expression $\sigma(x)$ for the population standard deviation. In the case of a uniform distribution, the same equation gives the result.

Solved Exercise 3.2
Suppose the value of the mass of an object is 25.9 g and that the digital scale used for this measurement has a reading resolution of 0.1 g. This means that the scale reads increments of 0.1 g in 0.1 g. Considering the existing algorithm in the digital scale, responsible for digitizing the indicated values, the "true value" of the mass will be comprised between interval 25.85 g and 25.95 g. Values such as 25.96 g or larger shall be rounded by the instrument to 26.0 g, just as values such as 25.84 g or smaller to 25.8 g.

Based on this information, the mean and standard deviation of this distribution will be determined.

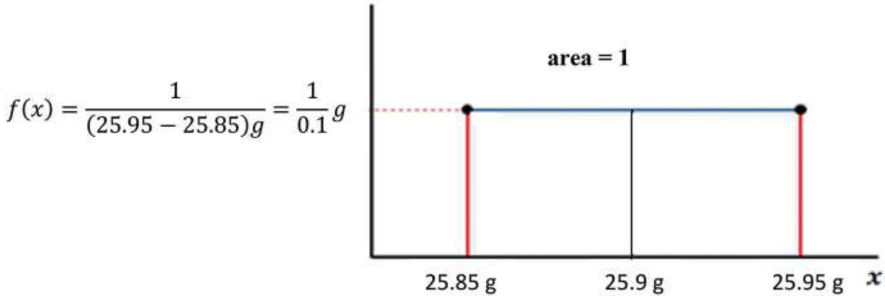
Solution
Considering that the scale has a reading limitation of 0.1 g (resolution), we know that every time the instrument indicates 25.9 g, we will doubt the "true value" of the mass in question caused by its limitation of resolution. Considering that the probability that the "true value" is between 25.85 g and 25.95 g is the same within this interval, it is reasonable to adopt a statistical distribution that reflects this behavior, that is, rectangular or uniform distribution. In Graph 3.5, we have:

Note the area under the graph is 1, as expected. Thus, the mean will be:

$$\bar{x} = \frac{a+b}{2} = \frac{25.85 + 25.95}{2} g = 25.9\ g$$

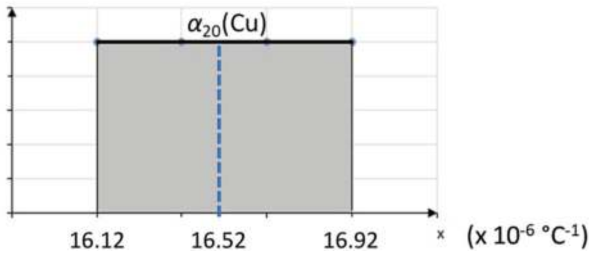
The standard deviation is calculated by Eq. (3.18):

$$s(x) = \frac{b-a}{\sqrt{12}} = \frac{25.95 - 25.85}{\sqrt{12}} g = 0.0288675\ g$$



Graph 3.5 Statistical distribution of Solved Exercise 3.2

Graph 3.6 Statistical distribution of Solved Exercise 3.3



We will see further that this result is considered the uncertainty of reading resolution of the instruments with rectangular distribution.

Solved Exercise 3.3 (Source ISO GUM 2008)
A manual provides the value of the linear thermal expansion coefficient of pure copper at 20 °C [α_{20}(Cu)] as 16.52 × 10^{-6} °C^{-1} and establishes that the error in this value should not exceed +0.40 × 10^{-6} °C^{-1}.

Based on this limited information, it is not absurd to suppose that the value of α_{20}(Cu) will be distributed with equal probability in the interval of 16.12 × 10^{-6} °C^{-1} to 16.92 × 10^{-6} °C^{-1}, and it is doubtful that α_{20}(Cu) is out of it (Graph 3.6).

The standard deviation of this symmetrical rectangular distribution of possible values of α_{20}(Cu) is:

$$s(\alpha 20) = \frac{(16.92 - 16.12)x10^{-6}\,{}^{\circ}C^{-1}}{2\sqrt{3}} = \frac{0.80x10^{-6}\,{}^{\circ}C^{-1}}{2\sqrt{3}} = 0.23x10^{-6}\ \ {}^{\circ}C^{-1}$$

Standard deviation from a uniform distribution, adopted as a dispersion measure for the copper thermal expansion coefficient variation, is a reasonable estimate of standard uncertainty.

3.2.5.2 Symmetrical Triangular Distribution

When the probability distribution is more prominent in the central part, at a defined interval, and decays linearly at the ends, we will face a triangular distribution.

In many cases, it is more realistic to expect values near the boundaries to be less likely than those near the midpoint. Replacing the rectangular distribution with a symmetrical triangular distribution is reasonable (Graph 3.7).

$$f(x) = \begin{cases} 0 & \text{para } x < a \\ \dfrac{4(x-a)}{(b-a)^2} & \text{para } a \le x \le \dfrac{a+b}{2} \\ \dfrac{4(b-x)}{(b-a)^2} & \text{para } \dfrac{a+b}{2} \le x \le b \\ 0 & \text{para } x > b \end{cases} \tag{3.19}$$

For triangular distribution with the mean $\overline{x}$ at the center of interval a, b, we have:

$$\mu = \overline{x} = \frac{a+b}{2} \tag{3.20}$$

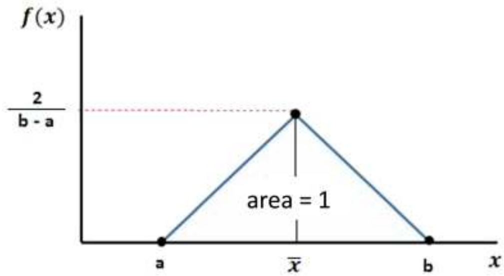
and the standard deviation is:

$$\sigma(x) = s(x) = \frac{b-a}{\sqrt{24}} = \frac{b-a}{2\sqrt{6}} \tag{3.21}$$

We adopted the expression $s(x)$ for the sample standard deviation and the expression $\sigma(x)$ for the population standard deviation. The same equation gives the standard deviation in a triangular distribution.

Solved Exercise 3.4

Suppose we calibrate a pressure gauge with a measurement interval (0 to 40) bar and a resolution of 1 bar using a comparative pump and fix the calibration points on the object gauge at 10 bar, 20 bar, 30 bar, and 40 bar (Fig. 3.4).



Graph 3.7 Symmetrical triangular distribution

Fig. 3.4 Pressure gauge calibration. (Photo by the authors)

These values are fixed to present a greater probability of occurrence than any other. For example, for a point 30 bar, the "true value" of the pressure will be understood at 29.5 bar at 30.5 bar. Values such as 30.5 bar (or larger) will be rounded to 31 bar, just as values such as 29.4 bar (or smaller) to 29 bar. Consider the probability that the "true value" is higher at point 30 bar than at any other point, because we set this value. Based on this information, determine this distribution's mean and standard deviation at point 30 bar.

Solution

Considering that the "true value" probability is higher at point 30 bar than at any other point because we fix this value, it is reasonable to adopt a statistical distribution that reflects this behavior, that is, the triangular distribution.

In Graph 3.8, we have:

Let us calculate the mean and the standard deviation:

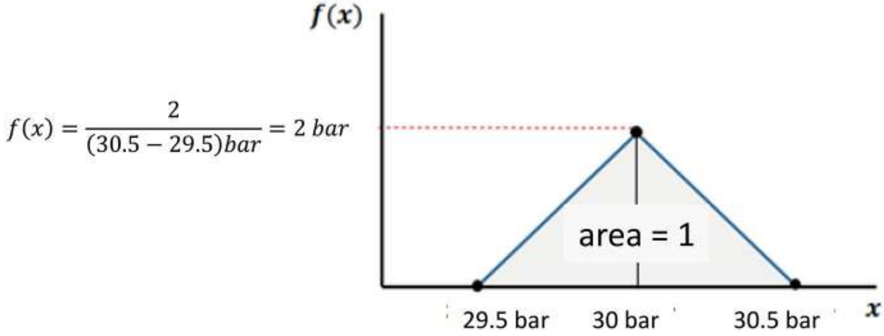
$$\overline{x} = \frac{a+b}{2} = \frac{29.5 + 30.5}{2}\,\text{bar} = 30.0\ \text{bar}$$

$$s(x) = \frac{b-a}{\sqrt{24}} = \frac{30.5 - 29.5}{\sqrt{24}}\,\text{bar} = 0.04167\ \text{bar}$$

The standard deviation is considered the uncertainty of the reading resolution of the instruments with triangular distribution.

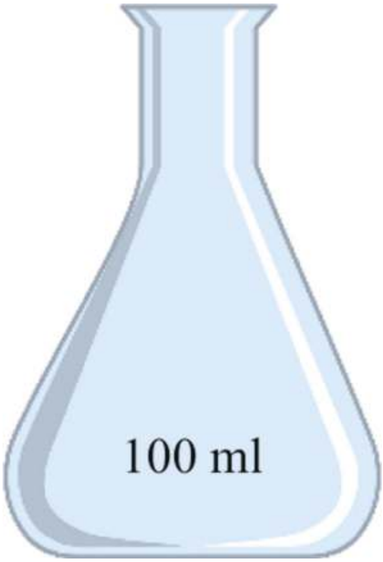
Solved Exercise 3.5: Based on Eurachem Guide [5] (Fig. 3.5)

A solution's total volume (V) is measured by filling a 100 ml balloon. The balloon manufacturer informs that it has a volume of (100 ± 0.1) ml, measured at 20 °C.



Graph 3.8 Statistical distribution of Solved Exercise 3.4

Fig. 3.5 100 ml balloon



Considering the little information available, the Eurachem Guide considers it more realistic to expect the values close to the limits to be less likely than near the midpoint.

Therefore, it recommends assuming a triangular distribution for this source of input, ranging from 99.9 ml to 100.1 ml, with an expected value of 100.0 ml.

$$s_V = \frac{100.1 - 99.9}{\sqrt{24}}\,\text{mL} = 0.04\ \text{mL}$$

3.2.5.3 Asymmetrical Triangular Distribution

In an asymmetrical triangular distribution, the value with the highest probability is the mode of the dataset, which is not equal to the average. So, when the probability distribution is higher in *mode*, in a defined interval, and decays linearly at the ends, we will be facing an asymmetrical triangular distribution (Graph 3.9).

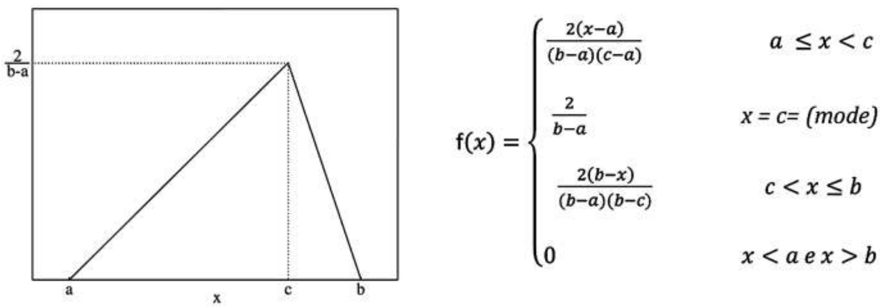
$$\mathrm{f}(x)=\begin{cases}\dfrac{2(x-a)}{(b-a)\,(c-a)} & a\leq x<c\\ \dfrac{2}{b-a} & x=c=(\text{mode})\\ \dfrac{2(b-x)}{(b-a)\,(b-c)} & c<x\leq b\\ 0 & x<a\ \ e\ \ x>b\end{cases} \tag{3.22}$$

The mean and standard deviation expressions for this distribution are:

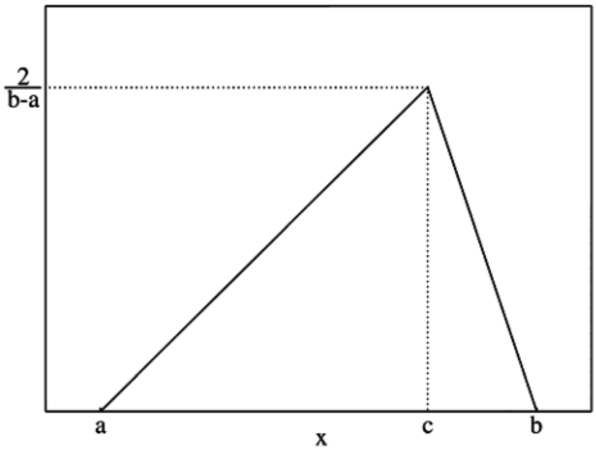
$$\mu=\frac{a+b+c}{3} \tag{3.23}$$

$$\sigma=\sqrt{\frac{a^2+b^2+c^2-ab-ac-bc}{18}} \tag{3.24}$$

The probabilities accumulated for this distribution are:



Graph 3.9 Asymmetrical triangular distribution



Graph 3.10 Asymmetrical triangular distribution of Solved Exercise 3.6. $a = 10$ °C. $b = 30$ °C. $c = 25$ °C

$$\mathrm{P}(x) = \begin{cases} \dfrac{(x-a)^2}{(b-c)\,(c-a)} & a \le x < c \\ \dfrac{c-a}{b-a} & x = c \\ 1 - \dfrac{(b-x)^2}{(b-a)\,(b-c)} & c < x \le b \end{cases} \tag{3.25}$$

Solved Exercise 3.6

Consider that in a set of measurements, the lowest value found was 10 °C, the largest was 30 °C, and the mode was 25 °C. What is the probability that a new measurement is less than 20 °C? What is the probability of being greater than 26 °C?

Solution (Graph 3.10)

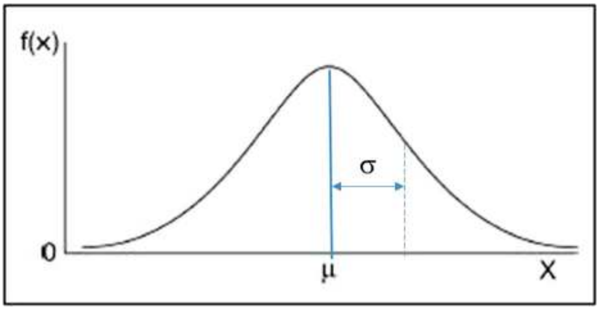
$$P(x < 20) = \frac{(x-a)^2}{(b-a)(c-a)} = \frac{(20-10)^2}{(30-10)(25-10)} = 0.33 \rightarrow 33\%$$

$$P(x > 26) = 1 - \frac{(b-x)^2}{(b-a)(b-c)} = 1 - \frac{(30-26)^2}{(30-10)(30-25)} = 0.84 \rightarrow 84\%$$

3.2.5.4 Normal or Gaussian Distribution

Normal or Gaussian distribution is undoubtedly the most important PDF. Several variables behave according to a Gaussian distribution.

Graph 3.11 Normal or Gaussian distribution



Gauss based the errors theory on postulates. One concern is that *"the most likely value of quantities, measured several times, is the arithmetic mean of the measures found, provided they deserve the same confidence."*

Graph 3.11 represents a Gaussian or normal probability distribution. It has the classic form of a bell where the center is the mean μ, and the width of its base represents the dispersion of values σ around the average.

The normal distribution has a PDF defined by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}; \; -\infty \; < x < \infty \tag{3.24}$$

where σ corresponds to the population standard deviation and has the equation:

$$\sigma = \sqrt{\frac{\sum_{1}^{n} (x-\mu)^2}{n}} \tag{3.25}$$

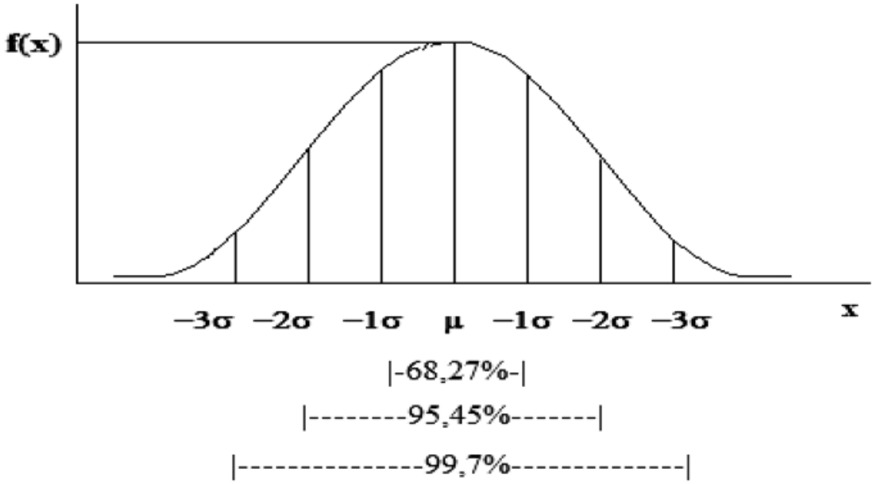
μ is the population mean, and the equation is:

$$\mu = \frac{\sum_{1}^{n} x_i}{n} \tag{3.26}$$

The mean and standard deviation are fundamental characteristics of any statistical distribution. The mean indicates the most likely value, and the standard deviation is the scattering of these values around the mean.

Suppose you have to measure the length of an object with a simple ruler and write down the result. Ask others who repeat the measurement, without each one, to know about the results obtained by others, and write down all the results. You will observe that the measurements differ. Repeat the measurement ten times, and you will probably find some different results. This fact is called dispersion of measurement.

As its name implies, the dispersion of the values found in a measurement statistically evaluates the degree of spreading these values around the mean. The higher the dispersion, the more away values around the mean distribution are found.



Graph 3.12 Probabilities associated with standard deviations in a normal distribution

In a normal distribution, 68.27% of the results will be dispersed around the mean for a standard deviation (1σ), 95.45% for two standard deviations (2σ), and 99.7% for three standard deviations (3σ). The intervals mentioned are shown in Graph 3.12.

We rarely know the entire population in metrology, because we do not perform infinite measurements. In this case, the standard deviation of the sample (s) is adopted, calculated by the equation:

$$s = \sqrt{\frac{\sum_{1}^{n} (\overline{x} - x)^2}{n - 1}} \tag{3.27}$$

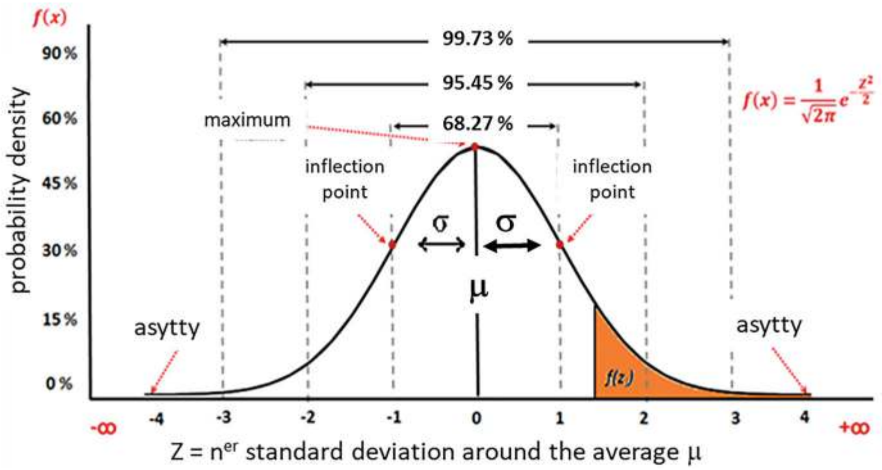
where $\overline{x}$ is the sample mean, and n is the sample size.

For example, the measures deserve the same confidence if performed by the same observer using the same instrument and method. A question arises: What is the convenient number of measurements to realize?

This number varies from case to case, but in practice, an interval of three to ten measurements is adopted. Below three measurements, errors may not be well represented, and above ten, the measurement process can become costly.

The sample variance s^2 is the square of the sample standard deviation, given by the expression:

$$s^2 = \frac{\sum_{1}^{n} (\overline{x} - x)^2}{n - 1} \tag{3.28}$$



Graph 3.13 Probabilities associated with standard deviations in a standardized normal distribution

The variance is used in calculating measurement uncertainty, because it is a variable that can be combined linearly, that is, we can add the variances of different distributions, not the standard deviations.

Example: Considering the variance of a sample equal to 3 and another sample equal to 4, determine the variance resulting from this sum and its standard deviation.

Solution: $s^2 = s_1^2 + s_2^2 = 3 + 4 = 7$

The resulting standard deviation will be: $s = \sqrt{s^2} = \sqrt{7} = 2.646$

which is different from the direct sum of the deviations.

$$\sqrt{3} + \sqrt{4} = 1.73 + 2 = 3.73 \neq \sqrt{7}$$

A normal distribution's mean and standard deviation can assume any values. A simplified mathematical model solves the complexity of the normal distribution, creating a *standardized normal distribution*.

Any variable can be transformed into variable Z, whose value is the difference between variable x and the mean, divided by standard deviation.

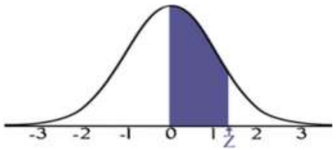
$$Z = \frac{x_i - \mu}{\sigma} \tag{3.29}$$

In a standardized normal distribution, the average (μ) assumes zero value, and the standard deviation (σ) assumes a value of one. Thus, the probability distribution function assumes the following conformation, as shown in Graph 3.13.

The total area under the curve corresponds to 100%. Each half has 50% of the total area. After transforming the normal curve into the standard normal curve, it assumes the same form as normal distribution, with average $\mu = 0$ and standard deviation $\sigma = 1$.

The probabilities of standardized normal distribution can be obtained in Table 3.10.

Table 3.10 Standard normal table (z). (https://www.aplustopper.com/wp-content/uploads/2017/04/Normal-Distribution-6-840x1024.png)

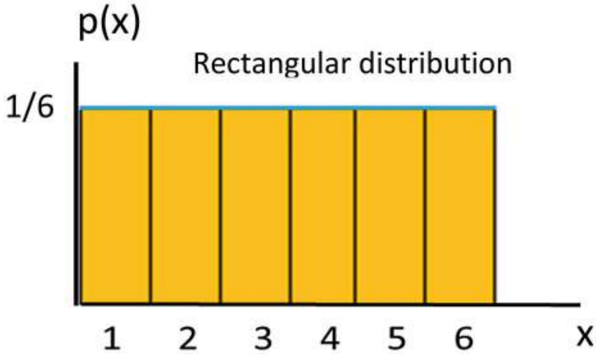


Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

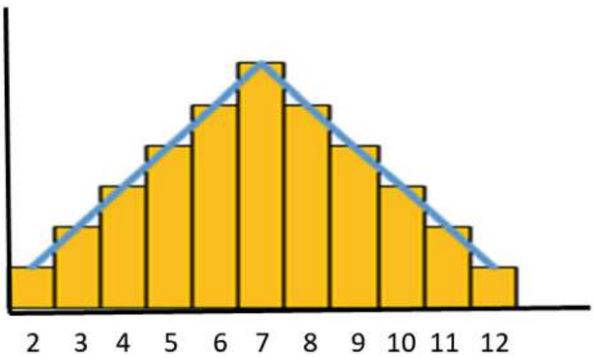
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633

1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4060	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

https://www.aplustopper.com/wp-content/uploads/2017/04/Normal-Distribution-6-840x1024.png



Graph 3.14 Rectangular distribution



Graph 3.15 Triangular distribution

In statistics, there is a theorem that is applied in metrology. It is known as the *Central Limit Theorem*. [GUM—G2]

It says: *"The more random variables are combined, even though they have different statistical distributions, the closer to a normal distribution will be the result of this combination of variables."*

Let us exemplify this theorem.

Consider the launch of a dice. The probability distribution for this event is a rectangular distribution (Graph 3.14).

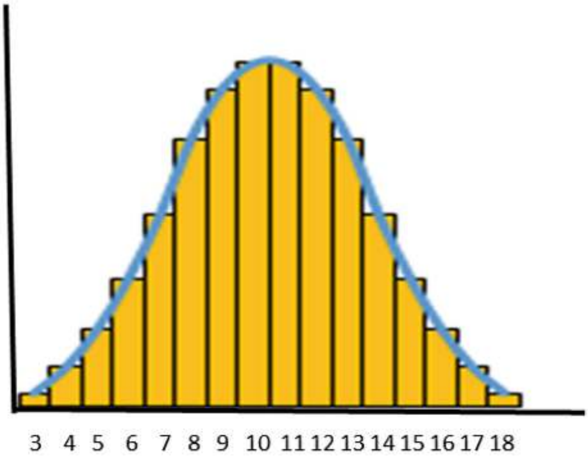
Let us now consider the launch of two dice. The probability distribution of this event approaches a triangular distribution (Graph 3.15).

Let us now consider the launch of three dice. The probability distribution of this event approaches a normal distribution (Graph 3.16).

For this reason, in metrology, we treat the final result of combining the various sources of uncertainty as a normal distribution, even though these sources have different statistical distributions. The sum of these influences results in a normal distribution behavior.

Consider the uncertainty calculation in the calibration of an analog pressure gauge. In it, we find several sources of uncertainty that will be estimated (the best

Graph 3.16 Normal distribution



estimate of the various sources of measurement uncertainties is the standard deviation of each source). Sources can come from multiple distributions, such as:

- Variation of pressure gauge reading and discharge (hysteresis)—uniform distribution.
- Variation of measurements performed by the pressure gauge (repeatability uncertainty)—Student's t distribution.
- Influence of standard measurement uncertainty used in the calibration of the gauge—normal distribution.
- Influence of object gauge resolution when we "set" its value in the calibration at a defined point—triangular distribution.

The final uncertainty, from the various sources of uncertainty mentioned above, according to the central limit theorem, will be a normal distribution.

One consequence of the central limit theorem is the fact that if we remove several samples of size n and calculate their averages $(\overline{x1}, \overline{x2}, \ldots, \overline{xp})$, where p is the number of samples and n the sample size, we will have for the standard deviation of the mean the expression given by the equation:

$$s(\overline{x}) = \frac{s}{\sqrt{n}} \tag{3.30}$$

where n is the number of measurements.

The standard deviation of the mean is of great importance in metrology because without having to do infinite measurements, we can estimate the standard deviation between the means of various samples of the same population. ISO GUM considers the standard deviation of the mean to be Type A uncertainty, if the sample belongs to the same population. Otherwise, Type A uncertainty will be equal to the standard deviation of the sample (s).

Fig. 3.6 Johann Carl Friedrich Gauss (1777–1855)

The higher the number of measurements of the same measure, the closer their values will behave as a normal distribution. Infinite measurements will have a normal distribution.

Important

The standard deviation from the mean represents the dispersion between the averages of the samples belonging to the same population.

Knowing a Little More... (Fig. 3.6)

(https://cdn.britannica.com/27/190027-050-A9A35298/Carl-Friedrich-Gauss-engraving.jpg)

He was one of the biggest names in the contemporary era of mathematics, having made significant contributions to astronomy and physics. Coming from a humble peasant family with illiterate parents, Gauss had already shown ease with numbers from the early years of life, even before he was literate. At the age of seven, challenged by his teacher to sum up the digits of 1 to 100, he reached the response of 5050 in a few seconds, stating the hitherto unknown formula of arithmetic progression. Although with strong resistance from his father, Gauss followed his studies, which he had been encouraged and funded since his youth by Buttner, director of the school where he studied, and Carl Wilhelm Ferdinand, Duke of Braunschweig. Impressed with Gauss's potential, the Duke funded his course at the University of Göttingen.

He elaborated on the minimum square method and also worked on the theory of numbers, the theory of elliptical functions, electromagnetism, and gravitation, among other topics. He reached a remarkable reputation in Europe and became a university professor, having written several works. If it had not been for the influence of Buttner and Duque Ferdinand on Gauss' trajectory, perhaps the genius theorems and laws of mathematics would not have come to light as they were stated. Gauss is known as the prince of mathematics.

3.2.5.5 *Student's T* Distribution

When we perform a small number of measurements, less than 30, we realize that even when the sample belongs to a normal distribution, its histogram does not take the shape of a bell, typical of this distribution.

To visualize this feature, we randomly generate in Excel 1000 values belonging to a normal distribution of population average $\mu = 2.00$ and standard deviation of the population $\sigma = 0.40$.

From this population (let us consider that the data generated are large enough to be viewed as the population), we remove sample sizes $n = 5$, $n = 20$, $n = 100$, and $n = 1000$. Our goal is to build histograms of different samples and observe their behavior.

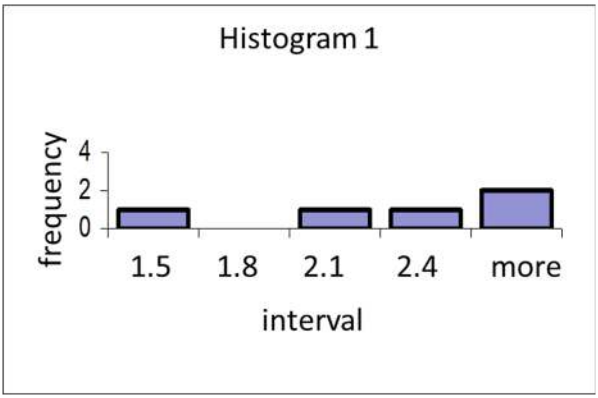
3.2.5.6 Sample Analysis with $N = 5$ (Graph 3.17)

The result found was $\bar{x} = 2.09$ and $s = 0.43$

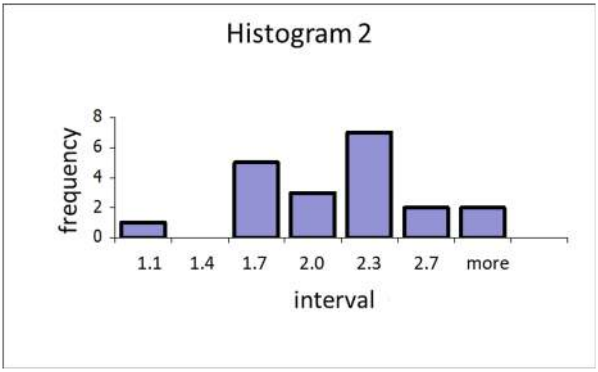
Interval	Frequency
1.5	1
1.8	0
2.1	1
2.4	1
More	2

Note that although the values are removed from a normal distribution with a mean of 2.00 and a standard deviation of 0.40, the mean of the five values is worth 2.09 and a standard deviation of 0.43. This is because we only get a mean of 2.00 and a deviation of 0.40, when we have all the values that generated the normal curve (infinite values).

Graph 3.17 Data and histogram to $n = 5$



Graph 3.18 Data and histogram to $n = 20$



Another essential feature is that the histogram containing the five values does not look like a normal distribution (bell form). This will only happen as the sample number approaches the population number.

3.2.5.7 Sample Analysis with $N = 20$ (Graph 3.18)

The result found was $\bar{x} = 1.97$ and $s = 0.44$

Interval	Frequency
1.1	1
1.4	0
1.7	5
2.0	3
2.3	7
2.7	2
More	2

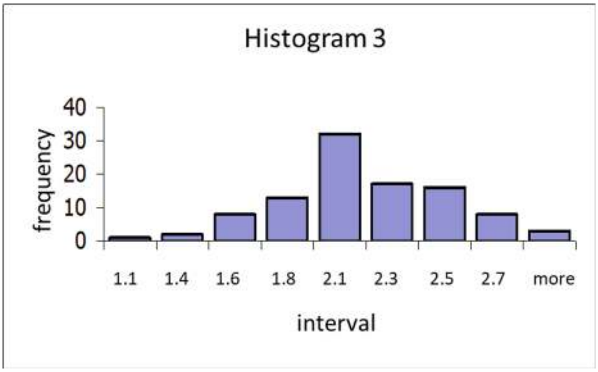
3.2.5.8 Sample Analysis with $N = 100$ (Graph 3.19)

The result found was $\bar{x} = 2.05$ and $s = 0.36$

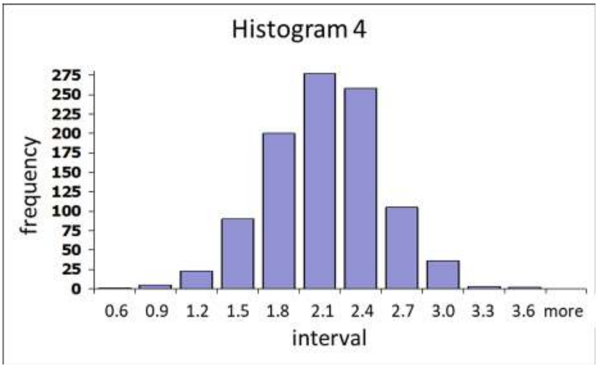
Interval	Frequency
1.1	1
1.4	2
1.6	8
1.8	13
2.1	32
2.3	17

(continued)

Graph 3.19 Data and histogram to $n = 100$



Graph 3.20 Data and histogram to $n = 1000$



Interval	Frequency
2.5	16
2.7	8
More	3

3.2.5.9 Sample Analysis with $N = 1000$ (Graph 3.20)

The result found was $\bar{x} = 2.00$ and $s = 0.40$

Interval	Frequency
0.6	1
0.9	5
1.2	23
1.5	90
1.8	200
2.1	277

(continued)

Interval	Frequency
2.4	258
2.7	105
3.0	36
3.3	3
3.6	2
More	0

We realize that the distribution tends to form a normal distribution as the number of measurements increases. In practice, if n is 30, we can consider the approximation with the normal curve.

As demonstrated, many high measurements ($n \geq 30$) are required to obtain a distribution close to normal. As it is not always feasible to perform 30 measurements of the same measurement, we must apply a correction factor, bringing the distribution of small values closer to normal.

This factor, known as a *Student's t factor* for statistics and coverage factor k for metrology, is a function of sample size n, or the number of degrees of freedom, and probability p. In metrology, it was normalized to consider the probability of 95.45% for calculating measurement uncertainty.

The chemist and mathematician William Gosset, who signed his work with the pseudonym Student, developed the Student's t factor, or coverage factor k. Around the late nineteenth century, William Gosset developed the Student's t distribution. The basic idea was to correct the factors that would multiply standard deviations for small measurements. As we saw in Graph 3.13 for a standard deviation ($\pm 2\sigma$), we have a probability of 95.45% to find the measurements scattered around the mean. This is true for infinite measurements. As in practice, we do three, four, and five measurements; it is necessary to multiply the standard deviation by a factor greater than two.

Table 3.11 presents the coverage factor for various probabilities. This table can be built in Excel© using the function INV.T.BC. You must choose the degree of freedom you want and the probability coverage, remembering that the probability used should be 100% less than the desired probability (level of significance). For example, if we wish 95.45% probability, we must insert the value of 0.0455 (4.55%) in the probability field.

Considering the previous example, where the mean population is $\mu = 2.00$, and its standard deviation is $\sigma = 0.40$ for $n = 1000$ values, we can check, for various values of n (5; 20; 100 ...), which the mean population will always be understood in the interval:

$$\bar{x} \pm k \cdot s(\bar{x}) \tag{3.31}$$

k is the coverage factor, and $s(\bar{x})$ is the standard deviation of the mean.

Analyzing Table 3.12 and Graph 3.21, we can see that the higher the number of measurements, the lower the interval where we will find, with a defined probability, the mean of the population.

Table 3.11 Student's t distribution table

Sample size (n) and degrees of freedom (ν)	n	$\nu = (n\text{-}1)$	**Confidence level or probability coverage**								
			50%	68.27%	70%	80%	90%	95%	95.45%	99%	99.73%
			Level of significance (α)								
			0.50	0.3173	0.30	0.20	0.10	0.05	0.0455	0.01	0.0027
	2	1	1.00	1.84	1.96	3.08	6.31	12.71	13.97	63.66	235.8
	3	2	0.82	1.32	1.39	1.89	2.92	4.30	4.53	9.92	19.21
	4	3	0.76	1.20	1.25	1.64	2.35	3.18	3.31	5.84	9.22
	5	4	0.74	1.14	1.19	1.53	2.13	2.78	2.87	4.60	6.62
	6	5	0.73	1.11	1.16	1.48	2.02	2.57	2.65	4.03	5.51
	7	6	0.72	1.09	1.13	1.44	1.94	2.45	2.52	3.71	4.90
	8	7	0.71	1.08	1.12	1.41	1.89	2.36	2.43	3.50	4.53
	9	8	0.71	1.07	1.11	1.40	1.86	2.31	2.37	3.36	4.28
	10	9	0.70	1.06	1.10	1.38	1.83	2.26	2.32	3.25	4.09
	11	10	0.70	1.05	1.09	1.37	1.81	2.23	2.28	3.17	3.96
	12	11	0.70	1.05	1.09	1.36	1.80	2.20	2.25	3.11	3.85
	13	12	0.69	1.04	1.08	1.35	1.77	2.16	2.23	2.98	3.69
	14	13	0.69	1.04	1.08	1.35	1.77	2.16	2.21	3.01	3.69
	15	14	0.69	1.04	1.08	1.35	1.76	2.14	2.20	2.98	3.64
	16	15	0.69	1.03	1.07	1.34	1.75	2.13	2.18	2.95	3.59
	17	16	0.69	1.03	1.07	1.34	1.75	2.12	2.17	2.92	3.54
	18	17	0.69	1.03	1.07	1.33	1.74	2.11	2.16	2.90	3.51
	19	18	0.69	1.03	1.07	1.33	1.73	2.10	2.15	2.88	3.48
	20	19	0.69	1.03	1.07	1.33	1.73	2.09	2.14	2.86	3.45
	21	20	0.69	1.03	1.06	1.33	1.72	2.09	2.13	2.85	3.42
	22	21	0.69	1.02	1.06	1.32	1.72	2.08	2.13	2.83	3.40
	23	22	0.69	1.02	1.06	1.32	1.72	2.07	2.12	2.82	3.38

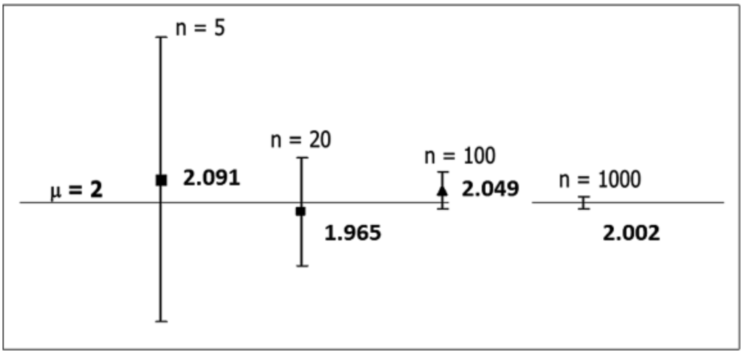
(continued)

Table 3.11 (continued)

24	23	0.69	1.02	1.06	1.32	1.71	2.07	2.11	2.81	3.36
25	24	0.68	1.02	1.06	1.32	1.71	2.06	2.11	2.80	3.34
26	25	0.68	1.02	1.06	1.32	1.71	2.06	2.11	2.79	3.33
27	26	0.68	1.02	1.06	1.31	1.71	2.06	2.10	2.78	3.32
28	27	0.68	1.02	1.06	1.31	1.70	2.05	2.10	2.77	3.30
29	28	0.68	1.02	1.06	1.31	1.70	2.05	2.09	2.76	3.29
30	29	0.68	1.02	1.06	1.31	1.70	2.05	2.09	2.76	3.28
∞	∞	0.67	1.00	1.04	1.28	1.64	1.96	2.00	2.58	3.00

Table 3.12 Data interval to 95.45% probability

n	$\bar{x}$	s	$s(\bar{x})$	k	$\bar{x} \pm ks(\bar{x})$ 95.45%
5	2.09	0.43	0.19	2.869	(2.09 ± 0.55)
20	1.97	0.44	0.10	2.140	(1.97 ± 0.21)
100	2.05	0.36	0.04	2.026	(2.05 ± 0.08)



Graph 3.21 Dispersion around the mean with 95.45% probability

Table 3.13 Resistance measurements

	R (Ω)
1	199.8
2	200.0
3	200.1
4	200.4
5	199.5
6	200.0
7	200.5
8	199.9

Note: In measurements, we call the mean population its true value, which we cannot determine in practice, since we cannot measure infinite times.

Solved Exercise 3.7

Eight electrical resistance measurements were made in resistor R, and the following values were found (Table 3.13).

Considering the distribution of this sample as belonging to a normal distribution, determine the following:

(a) The mean.
(b) The sample standard deviation.
(c) The standard deviation of the mean.

(d) The interval in which we have a 95.45% probability of finding the mean of the measurements.

Solution

(a) Mean

$$\mu = \frac{\sum_1^n x_i}{n} = 200.0\ \ \Omega.$$

(b) Standard deviation

$$s = \sqrt{\frac{\sum_1^n (\overline{x} - x)^2}{n-1}} = 0.3196\ \ \Omega.$$

(c) Standard deviation of the mean

$$s(\overline{x}) = \frac{s}{\sqrt{n}} = \frac{0.3196}{\sqrt{8}} = 0.112995\ \ \Omega$$

(d) To find the interval in which we have 95.45% of all measured values, we must determine the Student's t distribution value, equivalent to the coverage factor k found in the calibration or testing certificates.

To this, we must verify in Table 3.2 the corresponding value of k for the degree of freedom, $\nu = n - 1$. In our case, $\nu = 8 - 1 = 7 \rightarrow k$ will be 2.43 to 95.45% probability.

Thus, the interval will be given by Eq. 3.31:

$$\overline{x} \pm ks(\overline{x}) \ \ \rightarrow \ \ 200.0 \pm 2.43 \times 0.112995 = 200.0 \pm 0.281922$$
$$(200.0 \pm 0.3)\ \ \Omega$$

This result informs that we have a 95.45% probability of making eight more measurements, and the new mean is between 199.7 and 200.3 Ω.

Fig. 3.7 William Gosset. (Photo: public domain)

Knowing a Little More... (Fig. 3.7)

William Gosset (1876–1937).

The older son of Agnes Sealy Vidal and Colonel Frederic Gosset, he was educated in Winchester. In the New College Oxford, where he studied chemistry and mathematics, he obtained a first-class diploma in both sciences, being graduated in mathematics (1897) and chemistry (1899). Gosset obtained a post as a chemist at the Guinness Brewery in Dublin (Ireland) in 1899. Working at the brewery, he did important work in statistics. In 1905, he studied at the University College laboratory in London. He developed works in Poisson limit, mean sample distribution, standard deviation, and correlation coefficient. Later, he published three critical works on his accomplishments during the year he was in the laboratory.

Many people are familiar with the name Student but not with the Gosset. William Gosset signed with the pseudonym Student, which explains why his name can be less known than his significant statistics results. He invented the t-test to manipulate small samples for quality beer manufacturing control. Gosset discovered the form of t-distribution by combining mathematical and empirical work with random numbers, an initial application of the Monte Carlo method.

From 1922, he slowly built a small statistics department at the brewery, directing it until 1934. In late 1935, Gosset left Ireland to take over the new Guinness brewery in London. Despite the hard work involved in this venture, he continued to publish statistics articles. He died in 1937.

3.3 Proposed Exercises

3.3.1 Round correctly to one decimal digit.

(a) 34.450 m
(b) 23.852 m
(c) 8.351 m
(d) 19.7489 m
(e) 43.4501 m
(f) 43.852 m
(g) 52.3511 m
(h) 66.7205 m.

3.3.2 Check the number of significant digits in the following measurements:

(a) 1.320 m
(b) 0.050 kg
(c) 0.0001 km
(d) 9642 m^2.

3.3.3 Round correctly to three significant digits.

(a) 478.9 m
(b) 642.5 kg
(c) 123.4 L
(d) 56.150 cm.

3.3.4 Perform the following operations and present the result with the number of correct significant digits:

(a) 52.69 m + 36.8 m
(b) 68.487 m × 0.12 m

$$\sqrt{47.8\ \text{m}^2} - 1.36\ \text{m}$$

3.3.5 Round for one significant digit.

(a) 3682
(b) 0.00245
(c) 0.00058763
(d) 0.000030456.

Table 3.14 Voltage values

Voltage (V)					
2.01	2.15	2.40	2.56	2.75	2.91
2.01	2.17	2.42	2.59	2.76	2.91
2.01	2.19	2.44	2.63	2.77	2.93
2.05	2.20	2.44	2.63	2.80	2.93
2.09	2.25	2.44	2.64	2.80	2.93
2.10	2.26	2.45	2.65	2.80	2.94
2.10	2.26	2.52	2.70	2.81	2.95
2.11	2.27	2.53	2.71	2.84	2.96
2.13	2.33	2.55	2.74	2.84	2.98
2.14	2.34	2.55	2.74	2.86	2.99

3.3.6 Perform the following operations and present the result with the number of correct significant digits:

(a) $37.76 + 3.907 + 226.4$
(b) $319.15 - 32.614$
(c) $104.630 + 27.08362 + 0.61$
(d) $125 - 0.23 + 4.109$
(e) 2.02×2.5
(f) $600.0 / 5.2302$
(g) 0.0032×273
(h) $(5.5)^3$
(i) $0.556 \times (40 - 32.5)$
(j) 45×3.00
(k) What is the mean value of the five time measurements in seconds?

0.1707 s	0.1713 s	0.1720 s	0.1704 s	0.1715 s

(l) $3.00 \times 10^5 - 1.5 \times 10^2$.

3.3.7 Consider the 60 Voltage (Table 3.14) values belonging to a uniform distribution, make the histogram, and determine the mean and standard deviation.

3.3.8 Table 3.15 represents the temperature measurements, in Celsius degree, of a laboratory over a morning. Make the histogram of these values considering a normal distribution. Also, the mean and the standard deviation should be determined.

Table 3.15 Temperature measurements

Temperature (°C)					
23.6	23.8	24.0	24.0	24.1	24.2
23.6	23.8	24.0	24.0	24.1	24.2
23.7	23.8	24.0	24.0	24.1	24.2
23.7	23.8	24.0	24.0	24.2	24.2
23.7	23.8	24.0	24.1	24.2	24.3
23.7	23.9	24.0	24.1	24.2	24.3
23.7	23.9	24.0	24.1	24.2	24.3
23.7	23.9	24.0	24.1	24.2	24.3
23.8	23.9	24.0	24.1	24.2	24.5
23.8	24.0	24.0	24.1	24.2	24.6

Table 3.16 Voltage values

Voltage (V)					
128.42	128.62	128.69	128.75	128.80	128.84
128.49	128.63	128.69	128.76	128.80	128.87
128.49	128.63	128.71	128.76	128.80	128.88
128.56	128.65	128.72	128.77	128.81	128.89
128.57	128.65	128.72	128.77	128.82	128.90
128.58	128.66	128.73	128.77	128.83	128.91
128.59	128.66	128.74	128.78	128.83	128.93
128.59	128.66	128.74	128.79	128.83	128.94
128.60	128.67	128.75	128.80	128.83	129.01
128.61	128.69	128.75	128.80	128.83	129.11

3.3.9 A standard block manufacturer manual for calibration of calipers and micrometers provides the value of the blocks' linear thermal expansion coefficient (α) as 11.5×10^{-6} °C^{-1}. It also informs that the maximum variation of the linear expansion coefficient is $\pm 0.2 \times 10^{-6}$ °C^{-1}. Based on this information and considering that the linear thermal expansion coefficient (α) is distributed with equal probability, determine the standard deviation from the probability distribution of the linear thermal expansion coefficient (α).

3.3.10 Table 3.16 presents 60 voltage values obtained from an electrical outlet of the metrology laboratory. Based on these values, do what you are asked.

(a) A histogram of these values. Adopt seven classes to build it better.
(b) Determine the sample standard deviation.
(c) Determine the interval with a 95.45% probability of finding a measurement between the 60 measurements.
(d) Check how many values are within the interval determined in item © and make sure these values correspond to 95.45% of the measured values.
(e) Determine the interval with 95.45% probability where we can find the mean of 60 measurements.

Table 3.17 Scales values

Scale 1 (kg)	15.00	14.80	15.20	14.90	15.10	14.70
Scale 2 (kg)	14.60	14.70	15.40	15.30	14.90	14.90

Table 3.18 pH values

pH			
7.24	7.20	7.23	7.25
7.24	7.21	7.26	7.24
7.24	7.24	7.24	7.24

3.3.11 Consider the following samples (Table 3.17) taken from weighing lots of two distinct scales:

(a) Calculate the mean of the two samples.
(b) Calculate the standard deviation of the two samples.
(c) Based on previous items, which scale has the highest dispersion of measurements?
(d) Determine, for each scale, the interval where we have a 95.45% probability of finding the mean of the measurements.

3.3.12 Consider that the pH monitoring of a substance over one day has a triangular distribution. Based on the 12 values measured (Table 3.18) throughout this day, calculate:

(a) the mean,
(b) the standard deviation.

3.3.13 A metrology technician measured the internal temperature of a greenhouse, finding that the mean of the eight measurements performed was 48.9 °C and the standard deviation equal to 0.6 °C. Considering the measured values belonging to a Student's t distribution, determine the probability of the following measurement being between:

(a) 48.3 and 49.5 °C
(b) 47.4 and 50.4 °C
(c) 46.2 and 51.6 °C.

Chapter 4
Measuring Systems

4.1 Measurement: Forms of Realization

Measurements can be carried out in two ways: direct and indirect. In this section, we will address these two modalities and their particularities.

4.1.1 Direct Measurement

Direct measurement occurs when only a quantity is involved in the process, and the instrument is used directly to obtain the desired measurement result.

Some examples of direct measurements:

- Diameter measurement of a cylinder with a Vernier caliper
- Weighing an object with a scale
- Measurement of the electric current of a circuit with an ammeter
- Pressure indication using a Bourdon-type pressure gauge.

4.1.2 Indirect Measurement

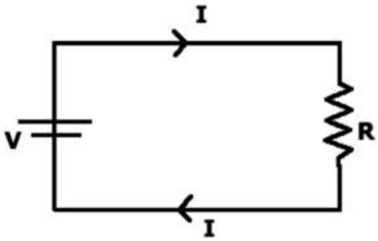
It occurs when measurements involve one or more related quantities through a mathematical equation.

Examples:

- Determination of the area (A) of a rectangular terrain measuring the length of each of its sides L1 and L2. We adopted the expression: $A = L1 \times L2$.

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_4

Fig. 4.1 Electric circuit



- Determination of the electric current (I) of a simple circuit, measuring resistance (R) and the electrical potential difference (V). We adopted the expression: $I = V/R$ (Fig. 4.1).

Each measurement method has different metrological characteristics. The proper choice of measurement (direct or indirect) enables the closest result to the desired.

For example, we can measure the density (ρ) of a liquid using a float densimeter (direct method), or we can, by the indirect method, measure the mass (m) and the volume of the liquid (v) and apply the relationship $\rho = m/v$.

Knowing a Little More... (Fig. 4.2)

The float densimeter is an instrument to measure the density of liquids. Among its utilities is to determine the properties of liquids by inspecting their density, especially when liquids are mixtures of substances. Thus, we can see if the composition of the mix is expected or not from the expected value for the density of the mixture. There are several ways to set up this apparatus, but the most common is a long closed glass tube at both ends, broader at its bottom, and narrower at its top. We must immerse the whole instrument in a container filled with the liquid from which the density is desired until it fluctuates freely. The principle of buoyancy (which is the force that makes the bodies float), revealed by Archimedes, is the basis of the densimeter.

In direct measurement, we use only one instrument, densimeter, whereas in indirect measurement, we need a balance and a glass of known volume. Regarding density value, both methods should have similar results, but the final uncertainty of each process may be significantly different.

Thus, the choice of method should evaluate the existence of error and the uncertainty of measuring the result.

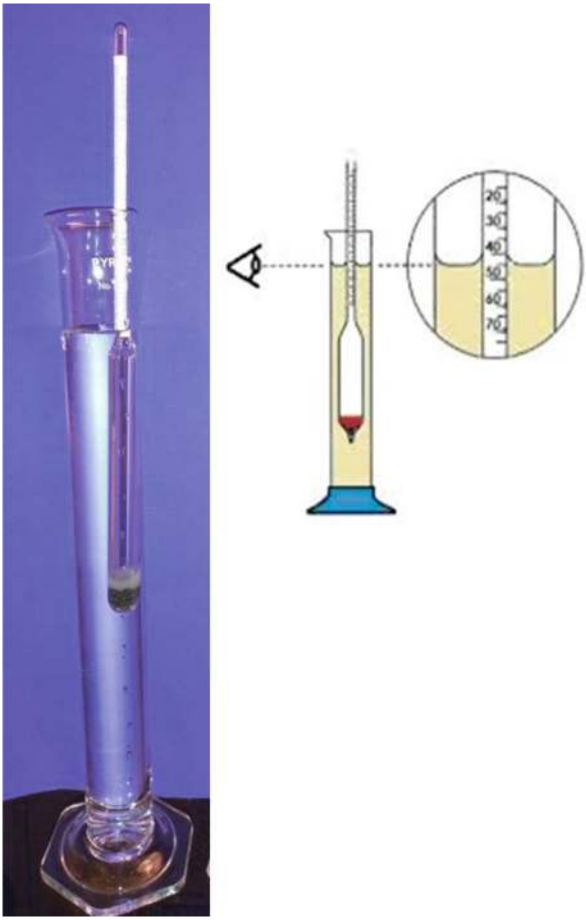


Fig. 4.2 Float densimeter. (https://http2.mlstatic.com)

4.2 Metrological Characteristics of Measurement Systems

Measurement systems have several metrological characteristics described in the International Vocabulary of Metrology. In this section, we will highlight the most usual.

4.2.1 Indication Interval

According to VIM—4.3, the indication interval is:

> *Set of quantity values bounded by extreme possible indications.*
>
> *NOTE 1 An indication interval is usually stated in terms of its smallest and greatest quantity values, for example, "99 V to 201 V."*
>
> *NOTE 2 In some fields, the term is "range of indications"*

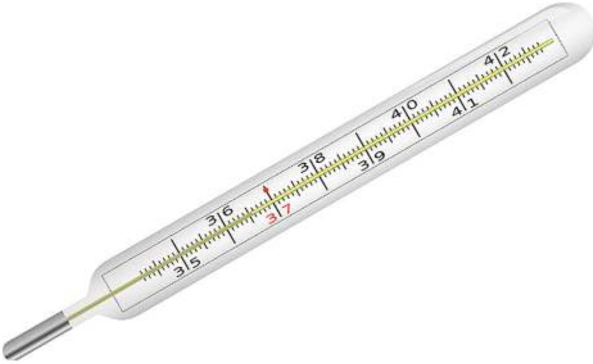


Fig. 4.3 Clinical thermometer. (https://pixabay.com/vectors/clinical-thermometer-fever-153666)

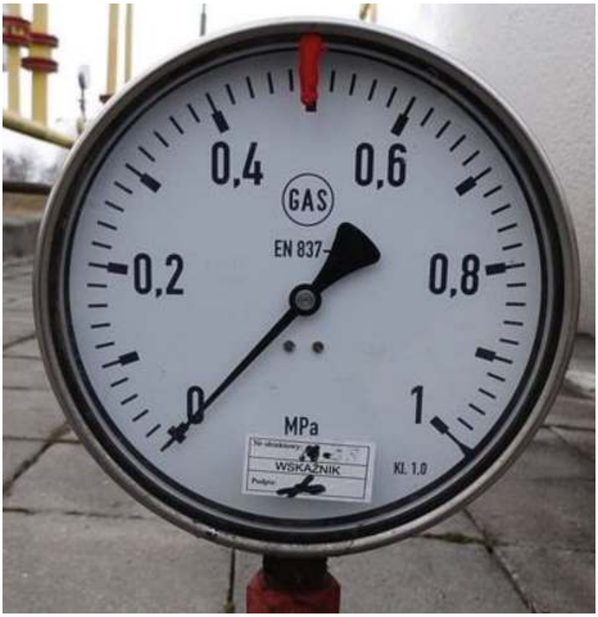


Fig. 4.4 Manometer. (https://pixabay.com/photos/manometer-oil-mine-extraction-863210/)

Examples:

(a) Clinical thermometer: indication interval (35 to 42) °C (Fig. 4.3).

(b) Pressure gauge: indication interval (0 to 1) MPa (Fig. 4.4).

Fig. 4.5 Digital multimeter. (https://pixabay.com/vectors/device-electric-electronics-measure-1296017/)

4.2.2 Measuring Interval (Working Interval)

The VIM—4.7 is defined as:

> *Set of values of quantities of the same kind that can be measured by a given measuring instrument or measuring system with specified instrumental measurement uncertainty under defined conditions.*
>
> *NOTE 1 In some fields, the term is "measuring range" or "measurement range"*
>
> *NOTE 2 The lower limit of a measuring interval should not be confused with the detection limit*

The measurement interval is lower or, at most, equal to the indication interval and can be obtained in manuals, technical standards, or calibration reports.

Example 4.1

The 3 ½ digit digital multimeter (Fig. 4.5) measures continuous electrical voltage with an indication interval of (0 to 1000) V. However, this interval is subdivided into the following measurement intervals: (0 to 200) mV; (0 to 2000) mV; (0 to 20) V; (0 to 200) V; and (0 to 1000) V.

Fig. 4.6 Manometer: vacuum and positive pressure. (https://pixabay.com/photos/pressure-meter-engineering-gauge-2113401/)

4.2.3 Range of a Nominal Indication Interval

The VIM—4.5 defines the range as follows:

> *Absolute value of the difference between the extreme quantity values of a nominal indication interval.*
>
> *EXAMPLE: For a nominal indication interval of −10 V to +10 V, the range of the nominal indication interval is 20 V.*
>
> *NOTE Range of a nominal indication interval is sometimes termed "span of a nominal interval."*

Example 4.2
Note that the working interval of the gauge shown in Fig. 4.6 is (−100 to 500) kPa, but its measurement range is:

$$\text{Range} = [500 - (-100)]\ \text{kPa} = 600\ \text{kPa}$$

4.2.4 Division of Scale (Not in VIM)

It is the difference between the scale values corresponding to two successive marks. The unit scheduled on the scale expresses the value of a division, whatever the unit of the measurement (Fig. 4.7).

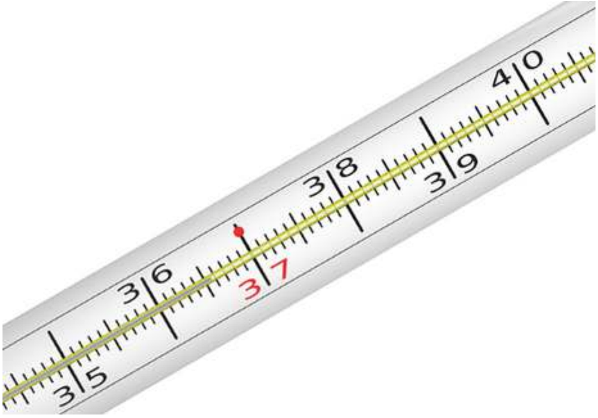


Fig. 4.7 Clinical thermometer with division of scale equal to 0.1 °C. (https://pixabay.com/vectors/clinical-thermometer-fever-153666)

Fig. 4.8 Manometer with division of scale equal to 10 kPa and resolution of 5 kPa. (https://pixabay.com/photos/pressure-meter-engineering-gauge-2113401/)

4.2.5 Resolution of a Displaying Device

The VIM—4.15 defines it as the *“smallest difference between displayed indications that can be meaningfully distinguished.”*

The operator should evaluate the reading resolution in systems with analog dials (Fig. 4.8).

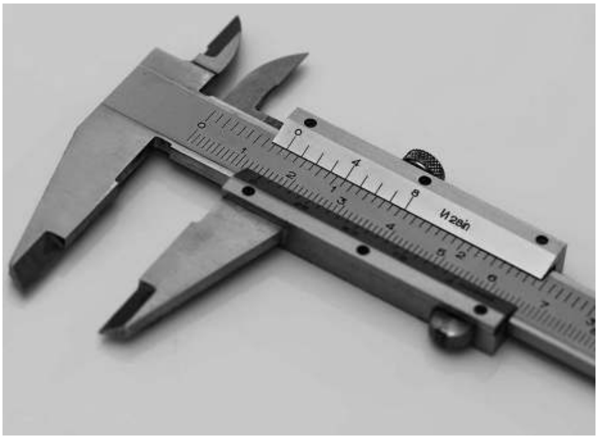


Fig. 4.9 Analog Vernier caliper with resolution of 0.0125 inch. (https://pixabay.com/photos/vernier-caliper-measuring-instrument-452987/)

Figure 4.8 shows a gauge (vacuum and positive pressure) with a 10 kPa scale division. To determine the instrument resolution, we must answer the following question: What is the lowest reading value I can achieve?

Answer: If the pointer is between two consecutive strokes and can read, we can consider a resolution of 5 kPa. Otherwise, we must consider the resolution equal to the value of the division, 10 kPa. In this example, we can admit a resolution of 5 kPa and the indicated value of -15 kPa.

Defining this gauge's reading resolution as 2.5 kPa would be difficult. This would only be possible if we could "with the naked eye" divide the value of a division into four parts!

The resolution of a display device will always be the slightest difference between indications that can be significantly perceived. That is the lowest value that can safely be read in a measurement.

We should not assume that the reading resolution is lower than it is. We must recognize the instrument's sensitivity for proper choice.

Important

1. Resolution will always be the slightest difference between indications that can be significantly perceived and will never be less than the instrument's sensitivity.
2. Resolution on a digital display device will be the lowest variation of this dial, that is, its digital increase.

Example 4.3

A caliper's resolution is calculated as the ratio between the value of a division on the fixed scale and the number of nonius, or Vernier, divisions (Fig. 4.9).

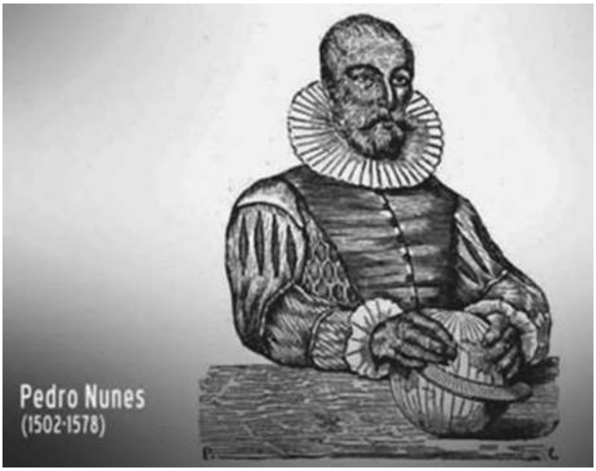


Fig. 4.10 João Pedro Nunes. (http://ensina.rtp.pt/site-uploads/2017/05/pedro-nunes-667x376.jpg)

A caliper with 0.1 division inches on a fixed scale and a nonius with eight divisions have a resolution of 0.0125 inches. Even using a magnifying glass and expanding the scale view, we continue with a resolution of 0.0125 inches.

Knowing a Little More... (Fig. 4.10)

Where does nonius come from?

This measurement device was one of the inventions of this Portuguese, born in Alcácer do Sal. It has worked in numerous areas, such as moral, metaphysical, and logical philosophy, since its formation in medicine in 1525. He became a cosmographer in 1529 by King D. João III and, in 1544, began to teach at the University of Coimbra. Nonius served to measure grade fractions in two nautical instruments at height, the astrolabe and the quadrant. Pierre Vernier perfected the base concept of this instrument, allowing its broad diffusion in the eighteenth century (Fig. 4.11)—Source: Adapted from pt.wikipedia.org.

Born in Ornans, France, this geometry and manufacturer of scientific instruments learned mathematics and science from his father, a lawyer and engineer of the Spain Government Chancellery. He acted as an engineer in the fortifications of various cities. His work in cartography resulted in the creation of numerous instruments, such as the Vernier caliper (1631), similar to the nonius of João Pedro Nunes, to measure the length accurately, using two graduated scales that slid in parallel, one of which provides exact subdivisions of a division of the other scale. His most famous publication, *La Construction, l'usage, et les propriétés du quadrant nouveau de mathématiques* (1631),

(continued)

Fig. 4.11 Pierre Vernier (1580–1637). (https://alchetron.com/cdn/pierre-vernier-7b642191-a050-4c17-8bd2-7dea18d3bb1-resize-750.jpeg)

Knowing a Little More... (Fig. 4.10) (continued)
describes his invention of a senior board and a method to determine the angles of a triangle with its known sides.

Source: Adapted from https://www.biografias.es/famosos/pierre-vernier.html.

4.2.6 Sensitivity of a Measuring System

According to VIM—4.12, the sensitivity of a measuring system is:

Quotient of the change in an indication of a measuring system and the corresponding change in a value of a quantity being measured.

NOTE 1 Sensitivity of a measuring system can depend on the value of the quantity being measured

NOTE 2 The change considered in the value of a quantity being measured must be large compared with the resolution

Example 4.4

(a) A Pt-100 type platinum resistance thermometer has a sensitivity of 0.38 Ω/°C, that is, each one °C stimulus in temperature causes a variation in the electrical resistance of the Pt-100 of 0.38 ohm.

(b) An electrode’s sensitivity for pH measurement shall be 59.16 mV/pH, that is, one pH variation in the substance should generate 59.16 mV of electrode output variation.

(c) A type K thermocouple must have a sensitivity of 39.5 mV/°C, and a type J thermocouple must have a 50.4 mV/°C.

4.2.7 Stability of a Measuring Instrument

According to VIM—4.19, it is:

Property of a measuring instrument, whereby its metrological properties remain constant in time

NOTE: Stability may be quantified in several ways.

EXAMPLE 1 In terms of the duration of a time interval over which a metrological property changes by a stated amount

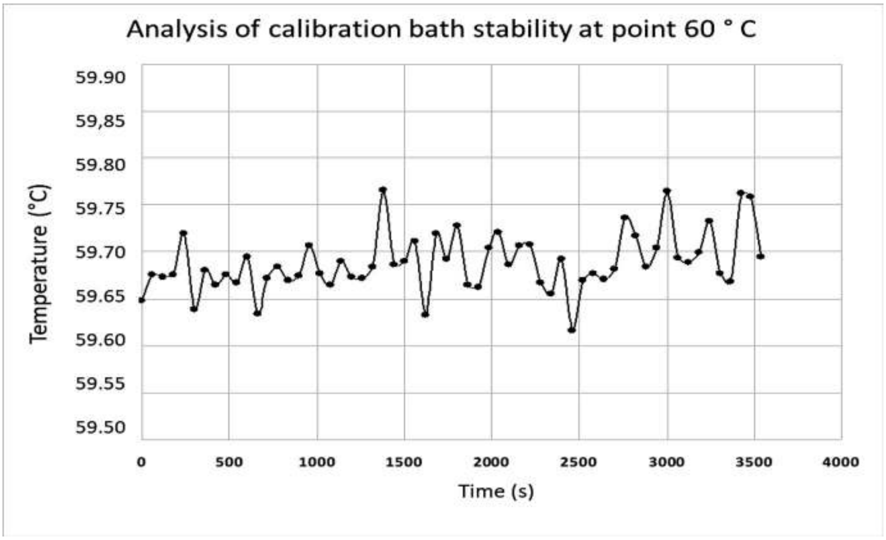
EXAMPLE 2 In terms of the change of a property over a stated time interval

In Table 4.1, we present an example of the analysis of the stability of the temperature of a liquid calibration bath at a point close to 60 °C. This bath has a measurement range between 50 °C and 300 °C.

The stability of a bath is the variation of its temperature at a given point after the bath goes into thermal equilibrium. Since no instrument is stable, this investigation will verify how much the calibration bath temperature oscillates after it is fixed at a given value. In this example, the desired value is 60 °C.

Table 4.1 Calibration bath temperature values at point 60 °C

Time (s)	Temperature (°C)	Time (s)	Temperature (°C)	Time (s)	Temperature (°C)
0	59.65	1201	59.67	2402	59.69
60	59.68	1261	59.67	2462	*59.62*
120	59.67	1321	59.68	2522	59.67
180	59.68	1381	*59.77*	2582	59.68
240	59.72	1441	59.69	2642	59.67
300	59.64	1501	59.69	2702	59.68
360	59.68	1561	59.71	2762	59.74
420	59.67	1621	59.63	2822	59.72
480	59.68	1681	59.72	2882	59.68
540	59.67	1741	59.69	2942	59.70
600	59.69	1801	59.73	3002	59.76
661	59.63	1861	59.66	3062	59.69
721	59.67	1921	59.66	3122	59.69
781	59.68	1981	59.70	3182	59.70
841	59.67	2042	59.72	3242	59.73
901	59.67	2102	59.69	3303	59.68
961	59.71	2162	59.71	3363	59.67
1021	59.68	2222	59.71	3423	59.76
1081	59.67	2282	59.67	3483	59.76
1141	59.69	2342	59.66	3543	59.69



Graph 4.1 Analysis of calibration bath stability at point 60 °C

For this analysis, we collected the bath temperature values for one hour at time intervals of one minute. We use a resistance thermometer (Pt-100 four wires) connected to a 6 ½ digit multimeter.

Note that after one hour of analysis, the highest temperature value is 59.77 °C, and the smallest is 59.62 °C. We can conclude that the stability (*E*) of the calibration bath, after the thermal equilibrium at point 60 °C, was:

$$E = (59.77\text{–}59.62)\ \ {}^\circ\mathrm{C} = 0.15\ \ {}^\circ\mathrm{C}$$

Graph 4.1 demonstrates the temperature variation of the calibration bath when it is stable at around 60 °C.

4.3 Errors in Measurement Systems

When we calibrate a measurement instrument, we set a comparison between the values obtained by the calibration instrument and the values provided by the standard. Some metrological characteristics obtained in this comparison are the errors and trends of the instruments. In this section, we will address these definitions and their applications.

4.3.1 Measurement Error

The VIM—2.16 defines measurement error as follows:

Measured quantity value minus a reference quantity value.

NOTE 1 The concept of 'measurement error' can be used in both

(a) *when there is a single reference quantity value to refer to, which occurs if a calibration is made by means of a measurement standard with a measured quantity value having a negligible measurement uncertainty or if a conventional quantity value is given, in which case the measurement error is known, and,*

(b) *if a measurand is supposed to be represented by a unique true quantity value or a set of true quantity values of negligible range, in which case the measurement error is not known.*

NOTE 2 Measurement error should not be confused with production error or mistake

Then, the error is:

$$E = X–Rv \tag{4.1}$$

where E = measurement error; X = measured value; and Rv = reference value.

Usually, the reference value is attributed to the value of the standard.

Mathematically, the measurement error can be positive or negative. A positive error denotes that the instrument measurement is greater than the reference value, and a negative error denotes that the measurement is less than the reference value.

Important

When we do more than one measurement at the same point and get different values for error, we adopt the largest of these values as the measurement error.

Solved Exercise 4.1

Four voltage measurements were performed using a voltmeter. The values found were 127.5 V, 127.6 V, 127.5 V, and 127.4 V. Knowing that the reference value is 127.68 V, determine the voltmeter measurement error.

Solution

The measurement error is given by Eq. (4.1). Therefore, we will have:

$$E1 = (127.5–127.68)\ \text{V} = -0.18\ \text{V}$$

$$E2 = (127.6–127.68)\ \text{V} = -0.08\ \text{V}$$

$$E3 = (127.5–127.68)\ \text{V} = -0.18\ \text{V}$$

$$E4 = (127.4–127.68)\ \text{V} = -0.28\ \text{V}$$

Since the voltmeter has four error values, we will adopt the value of the most significant measurement error (in absolute terms).

$$E = -\ 0.28\ \text{V} = -\ 0.3\ \text{V}$$

Important
The measurement error result will be −0.3 V, since we must round it to the same number of decimal digits as the instrument reading from which we are determining the measurement error.

4.3.2 Instrumental Bias and Correction

4.3.2.1 Instrumental Bias

The definition of VIM—4.20 for instrumental bias is *"average of replicate indications minus a reference quantity value."*

We should not confuse instrumental bias with measurement error. The instrumental bias determines the instrument's average measurement error.

$$B = \overline{X} - R_v \tag{4.2}$$

B = instrumental bias; $-X$ = mean of measurements; and R_v = reference value.

Solved Exercise 4.2
Determining the instrumental bias of Solved Exercise 4.1, we have (Table 4.2):

$$B = (127.5 - 127.68)\ \text{V} = -0.18\ \text{V}$$
$$B = -0.2\ \text{V}$$

Important
The result of the bias will be −0.2 V, since we must round the result to the same number of decimal digits as the instrument reading from which we are determining the instrumental bias.

Table 4.2 Measurements and mean

Measurements (V)	Mean (V)	Reference value (V)
127.5	127.5	127.68
127.6		
127.5		
127.4		

4.3.2.2 Correction

According to VIM—2.53, we have the following definition for correction:

Compensation for an estimated systematic effect.
NOTE 1 See GUM:1995, 3.2.3, for an explanation of 'systematic effect'
NOTE 2 The compensation can take different forms, such as an addend or a factor, or can be deduced from a table

Correction is equal to the bias with a changed signal and must be added to the value of indications to compensate for the systematic effect.

In Solved Exercise 4.2, the correction would be +0.2 V, and the value of the corrected voltmeter measurement would be (127.5 + 0.2) V = 127.7 V.

4.3.3 *Instrumental Drift*

The VIM—4.21 defines instrumental drift as:

Continuous or incremental change over time in indication due to changes in metrological properties of a measuring instrument.
NOTE: Instrumental drift is related neither to a change in a quantity being measured nor to a change of any recognized influence quantity.

It is widespread for an instrument of measurement to vary its metrological properties, such as measurement uncertainty and measurement error, over time. For this reason, we must verify the periodicity of these variations and perform calibrations in the measuring instruments at smaller intervals than their instrumental drift.

To verify the stability of a measurement instrument, we analyze your calibration certificate over two or more consecutive calibrations. We keep the calibration certificates from one period to another (usually from year to year) and compare their uncertainties, trends, and measurement errors.

Solved Exercise 4.3
An analytical scale, class I, with a resolution of 0.1 mg, was calibrated, and the table data for its calibration certificate were obtained (Table 4.3).

Table 4.3 Calibration results

Calibration results					
Indication (g)	Standard (g)	Object (g)	Bias (mg)	Uncertainty (mg)	*k*
20	20.000011	20.0000	0.0	0.2	2.01
40	40.000028	40.0000	0.0	0.3	2.00
70	70.000021	70.0003	0.3	0.3	2.02
100	100.000010	100.0001	0.1	0.3	2.01
120	120.000021	120.0001	0.1	0.4	2.01
150	150.000020	150.0001	0.1	0.4	2.00
220	220.000041	220.0003	0.3	0.5	2.00

Table 4.4 Data from the certificate

Indication (g)	Standard (g)	Object (g)	Bias (mg)	Uncertainty (mg)	*k*
20	20.000005	20.0000	0.0	0.2	2.00
40	40.000022	40.0000	0.0	0.4	2.01
70	70.000051	70.0005	0.4	0.4	2.01
100	100.000006	100.0001	0.1	0.4	2.00
120	120.000018	120.0001	0.1	0.5	2.02
150	150.000014	150.0001	0.1	0.5	2.02
220	220.000011	220.0004	0.4	0.5	2.02

Table 4.5 Instrumental Drift

Indication (g)	Bias (mg) (year 1)	Bias (mg) (year 2)	Drift (mg)
20	0.0	0.0	0.0
40	0.0	0.0	0.0
70	0.3	0.4	0.1
100	0.1	0.1	0.0
120	0.1	0.1	0.0
150	0.1	0.1	0.0
220	0.3	0.4	0.1

A year later, it was calibrated again. Table 4.4 shows data from the certificate. Determine the instrumental drift from the scale one year to the next.

Solution

We must subtract the trend values between two consecutive years to determine the instrumental drift from the balance from one year to another. See Table 4.5.

4.3.4 Maximum Permissible Measurement Error

The VIM—4.26 definition is:

Extreme value of measurement error, with respect to a known reference quantity value, permitted by specifications or regulations for a given measurement, measuring instrument, or measuring system.

NOTE 1 Usually, the term "maximum permissible errors" or "limits of error" is used where there are two extreme values

NOTE 2 The term "tolerance" should not be used to designate 'maximum permissible error'

Example 4.5
The standard CEN EN 837-1 Pressure Gauges—Part 1: Bourdon Tube Pressure Gauges—Dimensions, Metrology, Requirements, and Testing defines the following maximum permissible errors for analog manometer concerning its measurement range, such as:

- Class 0.1—maximum error of 0.1 %.
- Class 0.25—maximum error of 0.25 %.
- Class 0.6—maximum error of 0.6 %.
- Class 1—maximum error of 1.0 %.
- Class 1.6—maximum error of 1.6 %.
- Class 2.5—maximum error of 2.5 %.

4.3.5 Hysteresis (Not in VIM)

Hysteresis (H) is the most significant difference, in absolute value, of the charge (C) values (measurement made when applying an increasing signal in value) and (D) discharge (measurement made when applying a decreasing signal in value) of a measurement instrument.

$$H = |C - D| \tag{4.4}$$

Hysteresis is a typical phenomenon in mechanical instruments, with a source of error, especially clearances and deformations associated with friction. Examples of instruments that may present hysteresis errors are scales, dynamometers, and analog gauges.

Solved Exercise 4.4
When calibrating a pressure gauge, we determine its hysteresis by charging (increasing pressure) and discharging (decreasing pressure). Table 4.6 shows the result of one calibration cycle. Determine the gauge's hysteresis at each point.

Solution
Table 4.7 shows how we can determine hysteresis at each point from the previous data by subtracting the charge and discharge values (in absolute value).

The pressure gauge hysteresis will be at its highest value: 0.2 bar.

Table 4.6 Result of one calibration cycle

Value read in gauge (bar)	Charge read in standard (bar)	Discharge read in standard (bar)
10	9.9	10.0
20	19.9	20.1
30	30.0	30.0
40	40.2	40.1
50	50.3	50.1

Table 4.7 Hysteresis

Value read in gauge (bar)	Charge read in standard (bar)	Discharge read in standard (bar)	Hysteresis (bar)
10	9.9	10.0	0.1
20	19.9	20.1	0.2
30	30.0	30.0	0.0
40	40.2	40.1	0.1
50	50.3	50.1	0.2

4.3.6 *Measurement Accuracy and Precision*

These concepts may be the metrological characteristics with the most application mistakes. It is common for people to change the definition of accuracy with that of precision. It became common sense to call a precise measurement when referring to an exact measurement.

This section will define these terms and check their correct application.

4.3.6.1 Measurement Accuracy

The VIM—2.13 presents the following definition of measurement accuracy:

Closeness of agreement between a measured quantity value and a true quantity value of a measurand.

NOTE 1 The concept 'measurement accuracy' is not a quantity and is not given a numerical quantity value. A measurement is said to be more accurate when it offers a smaller measurement error

NOTE 2 The term "measurement accuracy" should not be used for measurement trueness and the term "measurement precision" should not be used for 'measurement accuracy', which, however, is related to both these concepts

NOTE 3 'Measurement accuracy' is sometimes understood as the closeness of agreement between measured quantity values that are being attributed to the measurand.

The true value would be obtained by a perfect measurement (which does not exist), being, by nature, indeterminate. Since the true value is indeterminate, it is used the *conventional quantity value* [VIM—2.12]:

Quantity value attributed by agreement to a quantity for a given purpose.

EXAMPLE 1 Standard acceleration of free fall (formerly called "standard acceleration due to gravity"), $g_n = 9.806\ 65\ m{\cdot}s^2$

EXAMPLE 2 Conventional quantity value of the Josephson constant, $K_{J\text{-}90} = 483{,}597.9\ GHz\ V^{-1}$

EXAMPLE 3 Conventional quantity value of a given mass standard, $m = 100.003\ 47\ g$

NOTE 1 The term "conventional true quantity value" is sometimes used for this concept, but its use is discouraged

NOTE 2 Sometimes a conventional quantity value is an estimate of a true quantity value

NOTE 3 A conventional quantity value is generally accepted as being associated with a suitably small measurement uncertainty, which might be zero

Or the *reference quantity value* [VIM—5.18]:

Quantity value used as a basis for comparison with values of quantities of the same kind.

NOTE 1 A reference quantity value can be a true quantity value of a measurand, in which case it is unknown, or a conventional quantity value, in which case it is known

NOTE 2 A reference quantity value with associated measurement uncertainty is usually provided with reference to

(a) *a material, e.g., a certified reference material,*
(b) *a device, e.g., a stabilized laser,*
(c) *a reference measurement procedure,*
(d) *a comparison of measurement standards.*

Thus, considering the value of a measurement standard such as the "conventional value," the instrument's accuracy is related to its ability to present the measurement results as close as possible to the value of this standard.

Measurement accuracy is not a quantity and is not given a numerical quantity value. A measurement is said to be more accurate when it offers a smaller measurement error. [VIM—2.13 NOTE 1]

4.3.6.2 Measurement Precision

The definition in VIM—2.15 for measurement precision is:

Closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.

NOTE 1 Measurement precision is usually expressed numerically by measures of imprecision, such as standard deviation, variance, or coefficient of variation under the specified conditions of measurement

NOTE 2 The 'specified conditions' can be, for example, repeatability conditions of measurement, intermediate precision conditions of measurement, or reproducibility conditions of measurement (see ISO 5725-1:1994)

NOTE 3 Measurement precision is used to define measurement repeatability, intermediate measurement precision, and measurement reproducibility

NOTE 4 Sometimes "measurement precision" is erroneously used to mean measurement accuracy

4.3.7 Measurement Precision × Measurement Accuracy

The following example is a "classic of metrology," but we consider it the simplest and fastest way to visually convey the concepts of precision and accuracy.

Consider four people (A, B, C, and D) who shoot ten times at the same distance as the target. The results of the shots are shown in Fig. 4.12.

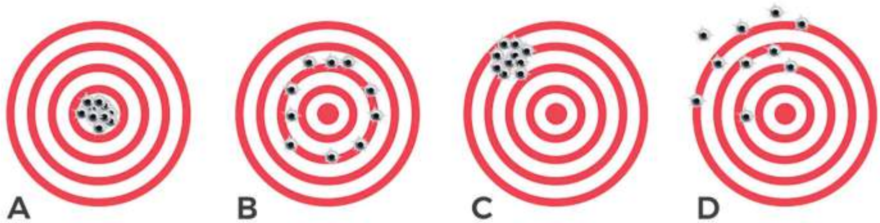


Fig. 4.12 Precision × Accuracy

Table 4.8 Precision × Accuracy

Shooter	Accuracy	Precision
A	High	High
B	Good	Low
C	Low	High
D	Low	Low

Shooter A hit almost every shot in the center of the target, demonstrating good accuracy (distance from the average shot from the center of the target) and reasonable accuracy (low shot dispersal).

Shooter B showed a very large spreading around the center of the target, but the shots are approximately equidistant in the center. The scattering of the shots stems directly from their low precision when analyzed individually. Still, when we observe the average position of the shots, which coincides approximately with the position of the center of the target, this reflects good accuracy.

Shooter C's shots are concentrated, with low dispersion, but away from the center of the target. This indicates low accuracy and high precision.

Shooter D, besides presenting a vast spreading, failed to make the "center" of the shots near the center of the target. This shooter has low accuracy and precision. Table 4.8 presents a summary of this analysis:

Shooter A is the ideal. Comparing B, C, and D, we can consider shooter C the best because although none of the shooter's shots hit the center of the target, its spread is very small (high precision). If the shooter's target C is corrected, he will get a condition close to that of A, which we can never get with B and D.

Important

Accuracy is not as critical as precision, since calibration can determine and correct it. Precision is also determined by calibration but cannot be corrected. It can be proved that its influence on the mean value is reduced in the proportion of $1/\sqrt{n}$, in which n is the number of repetitions of the measurement considered in the mean calculation.

4.3.8 Accuracy Class

The definition of accuracy class is as follows:

Class of measuring instruments or measuring systems that meet stated metrological requirements that are intended to keep measurement errors or instrumental measurement uncertainties within specified limits under specified operating conditions.

NOTE 1 An accuracy class is usually denoted by a number or symbol adopted by convention

NOTE 2 Accuracy class applies to material measures. [VIM—4.25]

Example 4.6

(a) According to Mercosur standard NM 215: 2000, a Class 1 standard block may have a variation in its length L (in mm) of $\pm$ (0.05 + 0.5 × 10^{-6} L) (m) /year).

(b) According to the recommendations of OIML, the standard masses used in the calibration of scales are classified in the accuracy classes E1, E2, F1, F2, M1, and M2. A mass of 100 mg, for example, presents, by accuracy class, the following maximum permissible errors (Table 4.9):

4.4 Repeatability and Reproducibility

The measurement result is intrinsically linked to these definitions. We can only compare results that meet the conditions of repeatability or reproducibility.

4.4.1 Repeatability Condition of Measurement

According to VIM—2.20, the repeatability condition of measurement is:

Condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions, and same location, and replicate measurements on the same or similar objects over a short period of time.

NOTE 1 A condition of measurement is a repeatability condition only with respect to a specified set of repeatability conditions

NOTE 2 In chemistry, the term "intra-serial precision condition of measurement" is sometimes used to designate this concept

Table 4.9 Maximum permissible error values for the mass of 100 mg

Accuracy class	Maximum error
Class E1	±0.005 mg
Class E2	±0.015 mg
Class F1	±0.05 mg
Class F2	±0.15 mg
Class M1	±0.5 mg
Class M2	±1.5 mg

4.4.2 Measurement Repeatability

The VIM—2.21 presents the following definition of repeatability of measurement: *"measurement precision under a set of repeatability conditions of measurement."*

4.4.3 Reproducibility Condition of Measurement

Again, the VIM—2.24 presents the following definition for the reproducibility condition of measurement:

> *Condition of measurement, out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects.*
> *NOTE 1 The different measuring systems may use different measurement procedures*
> *NOTE 2 A specification should give the conditions changed and unchanged, to the extent practical*

Measurements in reproducibility conditions are very common in exports, as it is not possible for the same operator in the same place, following the same measurement system, to accompany the product.

4.4.4 Measurement Reproducibility

According to VIM—2.25, measurement reproducibility is *"measurement precision under reproducibility conditions of measurement."*

In the case of exports, measurement reproducibility will verify the variability of measurements between places or countries. This variability must be within criteria previously established in the contract.

4.5 Proposed Exercises

4.5.1 According to the pressure gauge shown in Fig. 4.13, answer:

(a) What is the division of the scale?
(b) Which reading resolution would you adopt?
(c) How would you write the result of reading the gauge?

4.5.2 According to the thermometer shown in Fig. 4.14, answer:

(a) What is the division of the scale?
(b) Which reading resolution would you adopt?
(c) How would you write the result of reading the thermometer?

Fig. 4.13 Pressure gauge. (https://pixabay.com/vectors/manometer-measure-pressure-40253/)

Fig. 4.14 Thermometer. (https://pixabay.com/photos/celsius-degree-equipment-industrial-16047/)

4.5.3 What is the gauge's range from (-1 to 10) bar?
4.5.4 What is reading resolution?

(a) Lower division of an instrument.
(b) Lower difference between indications of a dial device that can be significantly perceived.
(c) Greater difference between indications of a dial device that can be significantly perceived.
(d) Lower difference between indications of a display device that cannot be significantly perceived.

4.5.5 What is repeatability?

(a) Aptitude of a measurement instrument to provide very close indications in repeated applications of the same measure under different conditions.
(b) Aptitude of a measuring instrument to provide very close indications in repeated applications of the same measuring under the same measurement conditions.
(c) Aptitude of a measuring instrument to provide very dispersed indications in repeated applications of the same measuring under the same measurement conditions.
(d) Aptitude of a measuring instrument to provide very close uncertainties in repeated applications of the same measure under different measurement conditions.

4.5.6 A pressure gauge, with measurement interval (0.0 to 200.0) bar, has the following characteristics:

- Resolution: 0.4 bar.
- Maximum error: 0.8 bar.
- Hysteresis error: 1.2 bar.

(a) Determine, in relative terms, the maximum error and the hysteresis as a function of the measurement range of the gauge.
(b) Determine, in relative terms, the maximum error and the hysteresis as a function of the value indicated when the measured value is 65.0 bar.

4.5.7 A resistor was measured with a standard multimeter, and the value obtained was (15.977 ± 0.008) Ω. This resistor was used to calibrate another multimeter, and the following indications (Table 4.10) were obtained (all in Ω).
Determine:

(a) The value of the mean of the indications.
(b) The bias of the instrument.
(c) The measurement error.

Table 4.10 Multimeter values

Measurements	1	2	3	4	5	6	7	8	9	10
Resistor (**Ω**)	15.97	15.96	15.96	15.95	15.95	15.97	15.98	15.97	15.98	15.98

4.5.8 In the calibration of a mercury glass liquid thermometer, we found for the standard value (Vr) 20.0 °C, and for the thermometer, the values 20.0 °C, 21.0 °C, 20.0 °C, 21.0 °C.

Determine:

(a) The bias of the thermometer.
(b) The measuring error of the thermometer.

4.5.9 A pressure gauge with a bias of 1 psi (psi is an English pressure unit and means *pounds per square inch*. 1 psi = 689,476 kPa) made a pressure measurement, finding 45 psi. What is your pressure value corrected?

4.5.10 Figure 4.15 represents five shots fired by a shooter. Which of the alternatives best qualifies this shooter?

(a) Low accuracy and low precision.
(b) Low accuracy and high precision.
(c) High accuracy and low precision.
(d) High accuracy and high precision.

4.5.11 Figure 4.16 represents three arrows thrown by one person. Which of the alternatives best qualifies this person?

(a) Low accuracy and low precision.
(b) Low accuracy and high precision.
(c) High accuracy and low precision.
(d) High accuracy and high precision.

4.5.12 Figure 4.17 represents three arrows thrown by one person. Which of the alternatives best qualifies this person?

(a) Low accuracy and low precision.
(b) Low accuracy and high precision.
(c) High accuracy and low precision.
(d) High accuracy and high precision.

4.5.13 What is measurement error?

(a) Value of the indication of an instrument plus the reference value of the input quantity.
(b) Reference value of the input quantity minus the value of the indication of an instrument.
(c) Uncertainty of the indication of an instrument minus the reference value of the input quantity.
(d) Indication value of an instrument minus the reference value of the input quantity.

Fig. 4.15 Shots at a target (https://pixabay.com/vectors/tiro-target-butt-shot-gun-bullet-160574/)

Fig. 4.16 Arrows at a target (https://pixabay.com/photos/dart-sports-goal-dart-board-arrow-3910686/)

4.5.14 Observe Fig. 4.18 of a thermometer and present the information requested for both scales (°C—left side and °F—right side).

(a) The scale division.
(b) The resolution.
(c) Indication value.

Fig. 4.17 Three arrows at a target (https://pixabay.com/photos/darts-goal-target-direct-hit-arrow-2349468/)

4.5.15. Observe Fig. 4.19 of the instrument and present the information requested.

(a) The scale division.
(b) The resolution.
(c) The measurement range.
(d) Indication value when the red pointer is on number 5.
(e) The value of the maximum permissible error.

4.5.16 A standard resistor, whose value is (10,000 ± 0.005) Ω, was measured with two multimeters under the same repeatability conditions. The results are shown in Table 4.11.

(a) What is the most precise multimeter? Justify your answer.
(b) What is the most accurate multimeter? Justify your answer.

4.5.17 A digital scale with 0.001 g resolution was calibrated using a set standard mass class E2. The result of calibration is in Table 4.12. Based on this information, answer what is asked.

(a) At what point is the scale most accurate? Justify.
(b) At what point is the scale most inaccurate? Justify.
(c) When we measured three times the value of a mass M, in this scale, we find the following: 5.003 g; 5.004 g; 5.005 g. Determine the corrected mean value of mass M.

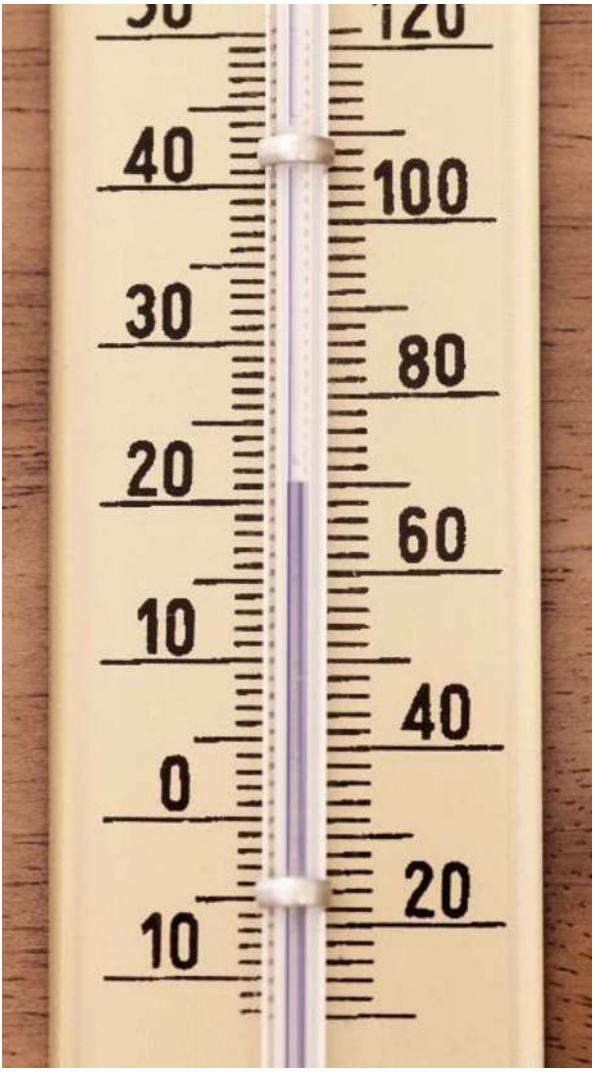


Fig. 4.18 Thermometer with ℃ and °F scales (https://pixabay.com/photos/thermometer-pay-scale-fluid-level-1176354/)

Fig. 4.19 Analog voltmeter (https://pixabay.com/photos/instrument-voltage-volt-meter-217276/)

Table 4.11 Multimeters values

Multimeter 1 (**Ω**)	10.02	10.03	10.04
Multimeter 2 (**Ω**)	10.02	10.04	10.06

Table 4.12 Scale calibration

Point	Nominal value (g)	Standard (g)	Object (g)	Bias (g)
1	1	1.000004	1.003	0.003
2	2	2.000007	2.004	0.004
3	5	5.000009	5.002	0.002
4	10	10.000005	9.999	−0.001
5	20	20.000017	20.000	0.000
6	50	50.000010	49.998	−0.002

4.5.18 Consider a calibration of the thermometer of Exercise 4.5.2. A standard thermometer was used to calibrate at point 50 °C, whose certificate correction to point 50 °C is −0.3 °C. Three measurements of the standard were made, obtaining a mean of 50.2 °C and, for the thermometer, a mean of 50 °C. Based on this information, determine:

(a) The standard temperature value at point 50 °C.
(b) The bias of the thermometer at point 50 °C.
(c) The correction to be applied to the thermometer at point 50 °C.

Chapter 5
Evaluation of Uncertainty in Direct Measurements

5.1 Concept of Measurement Uncertainty

Direct measurements are obtained by directly reading a measurement instrument that measures the same quantity.

In a direct measurement, the result is obtained by comparing the value read by the measuring instrument with the desired quantity. The measurement is done directly, without the use of mathematical equations.

Examples of direct measurements:

- Temperature measurement with a glass liquid thermometer (Fig. 5.1);
- Length measurement with a metric tape;
- Mass measurement with a scale;
- Pressure measurement with a gauge;
- Measurement of a thickness gear with a Vernier caliper (Fig. 5.2).

A measurement's result will always have a doubt associated with it, which we consider measurement uncertainty. What is sought in a measurement with metrological reliability is to estimate the measurement's results and associated uncertainty in the most reliable way possible.

Measurement uncertainty will always exist and never be eliminated since, as we have previously presented, the true quantity value is also estimated.

It is possible, however, to define the limits within which the value of a measurement with a certain associated probability is found.

The **measurement uncertainty** is defined by VIM—2.26 as:

- *Non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used.*
- *NOTE 1 Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty.*

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_5

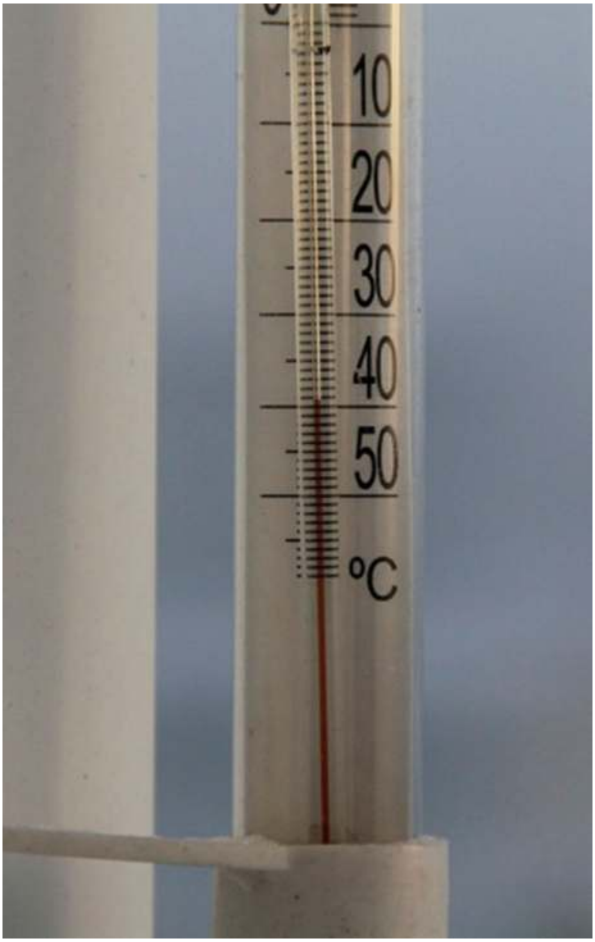


Fig. 5.1 Measurement with thermometer. https://pixabay.com/photos/thermometer-temperature-equipment-5947734/

Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

- *NOTE 2 The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.*
- *NOTE 3 Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by Type A evaluation of measurement uncertainty from the statistical distribution of the quantity values from a series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.*
- *NOTE 4 In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to*

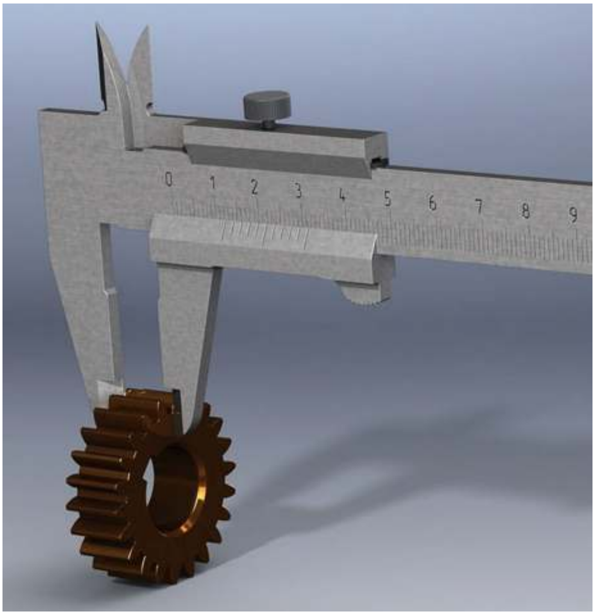


Fig. 5.2 Measurement with Vernier caliper. https://pixabay.com/illustrations/caliper-gear-measurement-1121746/

the measurand. A modification of this value results in a modification of the associated uncertainty.

The result of a measurement is an estimate of the measurement value, and thus, the presentation of the result is only complete when accompanied by an amount declaring its uncertainty.

As an example, we have read the room temperature in a laboratory. Suppose the value is $T_{laboratory} = 21.0\ °C$. The thermometer that made this measurement had a measurement uncertainty of 0.5 °C. Therefore, the result of the measurement will be:

$$T_{laboratory} = (21.0 \pm 0.5)\quad °C.$$

We note that the ambient temperature in the laboratory ranges between 20.5 °C and 21.5 °C. This means that the true value of room temperature in the laboratory is understood within this measurement interval with a given probability; that is, there is a probability of performing a new measurement of this temperature and finding the value understood in this measurement interval.

Important
Since measurement uncertainty is a probabilistic value and thus estimated, we can never be certain of a measurement's result.

In metrology, we usually adopt a confidence level of 95.45 % probability (remember that, in a normal distribution, 95.45 % probability represents two standard deviations). Therefore, when we say that the laboratory temperature is (21.0 ± 0.5) °C, we are saying that the true value of ambient temperature in the laboratory has a 95.45 % probability of being understood in this interval.

The uncertainty of a measurement's result is usually influenced by various components, which can be grouped into two categories according to the method characteristics used to estimate their numerical values: (1) uncertainties Type A and (2) uncertainties Type B, which will be detailed below.

5.2 Types of Measurement Uncertainties

Various sources of uncertainty exist in a measurement. Therefore, we should estimate these uncertainties and minimize their influences so that the measurement outcome is known in a smaller interval.

5.2.1 Type A Evaluation of Measurement Uncertainty

According to VIM—2.28, it consists of evaluating *"a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions."*

Type A uncertainties can, therefore, be characterized by experimental standard deviations. In metrology, the best estimate of a quantity that varies randomly is the arithmetic mean x of n measurements made. The estimated variance (s^2) or the estimated standard deviation (s) characterizes the variability of the measured values, that is, the dispersal around the mean value.

The best estimate of the variance of the mean is the experimental mean-variance $s^2(\bar{x})$, whose expression is:

$$s^2(\bar{x}) = \frac{s^2}{n} \tag{5.1}$$

The experimental standard deviation of the medium $\bar{x}$ serves to qualify how much the mean value $\bar{x}$ represents the quantity to be measured. The better this estimate, the greater the number of repetitions made in the measurement.

Important
The equation $s(\bar{x}) = \frac{s}{\sqrt{n}}$ determines Type A uncertainty measurement or the repeatability measurement uncertainty.

For several reasons, especially economic ones, the number of repetitions of a measurement is reduced, usually ranging from three to ten.

5.2.2 *Type B Evaluation of Measurement Uncertainty*

According to VIM—2.29:

Evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty.
EXAMPLES Evaluation based on information

- *associated with authoritative published quantity values,*
- *associated with the quantity value of a certified reference material,*
- *obtained from a calibration certificate,*
- *about drift,*
- *obtained from the accuracy class of a verified measuring instrument,*
- *obtained from limits deduced through personal experience.*

Type B uncertainties can be characterized by standard deviations estimated by distributions of probabilities assumed or based on experience or other observations. Accessory and external information to the measurement process—obtained from previous measurements of similar measurements, experience or knowledge of measuring instrument behavior, manufacturer data, data provided by calibration certificates, and instruction manual references—allows you to determine the uncertainties of this type.

Examples of Type B uncertainty:

- Temperature gradient during measurement;
- Difference of ambient temperature concerning the stipulated reference temperature;
- Indicator reading resolution;
- Stability of the power supply;
- Parallax error;
- Uncertainty of the measurement standard;
- Drift from the standard;
- Geometric errors;
- Mechanical deformations;
- Hysteresis error.

In the evaluation of Type B uncertainty, it is necessary to consider and include, when pertinent, at least those originating from the following sources:

(a) The uncertainty associated with the reference standard and any instability in its value or indication (standard subject to instrumental drift or temporal instability).

(b) The instability associated with measurement equipment or calibration, for example, aged connectors, and any instability in their value or indication (equipment subject to instrumental drift).
(c) The uncertainty associated with the equipment (measurand) to be measured or calibrated, such as the value of its resolution or any instability during calibration.
(d) The uncertainty associated with the calibration (or measurement) procedure.
(e) The uncertainty associated with the effect of environmental conditions on one or more of the previous items.

Comments:

1. Whenever possible, measuring errors or instrumental bias should be corrected.
2. A careful analysis should always be done when adding Type B uncertainties, so that there is no repetition and a given source of uncertainty is not considered more than once.

The document *Guide to the expression of uncertainty in measurement (GUM)* regarding Type B uncertainties states that:

> The proper use of the pool of available information for a Type B evaluation of standard uncertainty calls for insight based on experience and general knowledge and is a skill that can be learned with practice. It should be recognized that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation, especially in a measurement situation where a Type A evaluation is based on a comparatively small number of statistically independent observations.

Knowing a Little More...
JCGM 100:2008 GUM 1995 with minor corrections

Evaluation of measurement data—Guide to the expression of uncertainty in measurement

The following text reproduces part of the original document's Preliminary and Scope.

"This Guide establishes general rules for evaluating and expressing uncertainty in measurement that are intended to be applicable to a broad spectrum of measurements. The basis of the Guide is Recommendation 1 (CI-1981) of the Comité International des Poids et Mesures (CIPM) and Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties. The Working Group was convened by the Bureau International des Poids et Mesures (BIPM) in response to a request of the CIPM. The ClPM Recommendation is the only recommendation concerning the expression of uncertainty in measurement adopted by an intergovernmental organization.

This Guide was prepared by a joint working group consisting of experts nominated by the BIPM, the International Electrotechnical Commission (IEC), the International Organization for Standardization (ISO), and the International Organization of Legal Metrology (OIML).

(continued)

1 Scope

1.1 This Guide establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields — from the shop floor to fundamental research. Therefore, the principles of this Guide are intended to apply to a broad spectrum of measurements, including those required for:

- *maintaining quality control and quality assurance in production;*
- *complying with and enforcing laws and regulations;*
- *conducting basic research and applied research and development in science and engineering;*
- *calibrating standards and instruments and performing tests throughout a national measurement system in order to achieve traceability to national standards;*
- *developing, maintaining, and comparing international and national physical reference standards, including reference materials.*

1.2 This Guide is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity — the measurand — that can be characterized by an essentially unique value. If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence.

1.3 This Guide is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. Because a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data, the term "result of a measurement" as used in this Guide should be interpreted in this broader context.

1.4 This Guide provides general rules for evaluating and expressing uncertainty in measurement rather than detailed, technology-specific instructions. Further, it does not discuss how the uncertainty of a particular measurement result, once evaluated, may be used for different purposes, for example, to conclude the compatibility of that result with other similar results, to establish tolerance limits in a manufacturing process, or to decide if a certain course of action may be safely undertaken. It may therefore be necessary to develop particular standards based on this Guide that deal with the problems peculiar to specific fields of measurement or with the various uses of quantitative expressions of uncertainty. These standards may be simplified versions of this Guide but should include the detail that is appropriate to the level of accuracy and complexity of the measurements and uses addressed."

You can get the full document on the BIPM website:
<https://www.bipm.org/en/committees/jc/jcgm/publications>.

5.3 Evaluations of More Frequent Type B Uncertainties

As we have seen, the estimate of Type A uncertainty is obtained by calculating the standard deviation of the mean measurement. The estimate of Type B uncertainties already has several origins. Next, we present the primary sources of Type B uncertainty and how to calculate them.

5.3.1 Estimation of the Uncertainty of Reading Resolution

It is essential to evaluate the contribution of reading resolution in estimating measurement uncertainty, as it is widespread to find a low dispersion of the values obtained in a measurement process, which characterizes Type A uncertainty as being "zero." In this case, depending on the value of the resolution and the type of probability distribution adopted, this uncertainty may be one of the largest or the most significant contribution to final uncertainty.

In a measurement process, we can come across two situations.

Situation 1: Measurement where we "seek" the value of the desired quantity; that is, we do not know a priori what the value is.

Example 5.1 Reading Obtained ON A Digital Scale

Suppose the value of the mass of an object is 25.9 g and that the digital scale used for this measurement has a resolution of 0.1 g. This means that the lowest value read by the scale is 0.1 g.

Considering the algorithm in the digital scale responsible for digitizing the values indicated, the "true value" of the mass will be comprised between the interval [25.85 and 25.949 …] g. Values such as 25.95 g or larger should be rounded by the instrument to 26.0 g, just as values such as 25.84 g or smaller to 25.8 g.

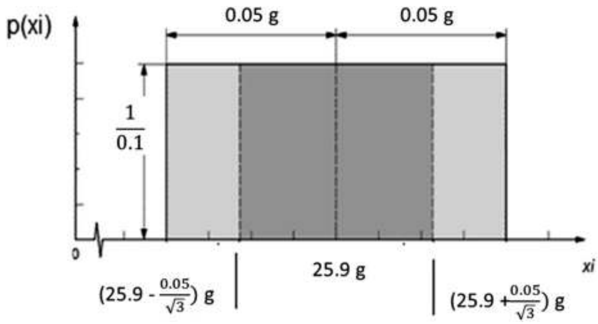
Therefore, every time the scale indicates 25.9 g, we will doubt the "true value" of the mass caused by its resolution limitation. Considering that the probability that the "true value" is understood between [25.85 and 25.949 …] g is the same within this interval, it is reasonable to adopt a statistical distribution that reflects this behavior, that is, rectangular or uniform distribution. Graph 5.1 shows:

Note that the uncertainty of reading resolution will be the standard deviation of the rectangular distribution, that is:

$$u_{res} = \frac{R}{\sqrt{12}} \tag{5.2}$$

Where R is the resolution adopted.

Graph 5.1 Uncertainty of reading resolution (uniform distribution)



$$u_{res} = \frac{0.01}{\sqrt{12}} g = 0.029\ g$$

Example 5.2 Reading Obtained in a Bimetallic Thermometer
Figure 5.3 presents the dial of a bimetallic thermometer with a scale of 0 to 120 °C, with a division of 2 °C. Observing the figure, we realize the possibility of dividing the division in half "with the naked eye." Thus, we will adopt a reading resolution equal to ½ of the division of the bimetallic thermometer (1 °C). The value read will then be 20 °C, which may be understood in the interval 19.5 to 20.5 °C with the same probability (uniform distribution).

(https://pixabay.com/photos/celsius-degree-equipment-industrial-16047/)

The uncertainty of reading resolution will be the standard deviation of the rectangular distribution with R = 1 °C:

$$u_{res} = \frac{1}{\sqrt{12}} {}^\circ C = 0.29\,{}^\circ C$$

Situation 2: Measurement where we "fix" the desired value.

When we set the desired value, we know a priori the most likely value of measuring, so it makes sense to attribute a greater probability to this value. In this case, we can consider that the triangular distribution best represents the probability distribution of reading resolution.

Example 5.3 Pressure Gauge Calibration
Suppose we calibrated a pressure gauge (Fig. 5.4) with a measurement interval 0 to 40 bar and resolution of 1 bar when using a comparative pump and fixed the calibration points on the object at 10 bar, 20 bar, 30 bar, 40 bar, and 50 bar.

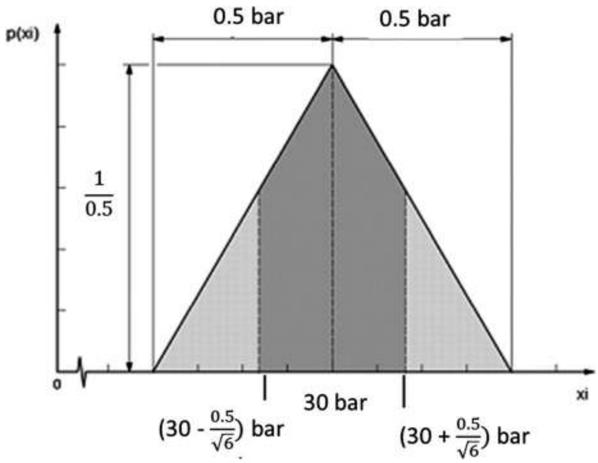
These values were fixed primarily to present a greater probability of occurrence than any other did.

For point 30 bar, for example, the "true value" of the pressure will be in the interval [29.5 to 30.49 …] bar. Values such as 30.6 bar or larger will be rounded to 31 bar, just as values such as 29.4 bar or smaller will be rounded to 29 bar.

Fig. 5.3 Bimetallic thermometer

Fig. 5.4 Calibration of a pressure gauge. (Photo by the authors)

Graph 5.2 Uncertainty of pressure gauge resolution (triangular distribution)



Considering that the probability of "true value" is higher at point 30 bar than at any other point, because we fix this value, it is reasonable to adopt a statistical distribution that reflects this behavior, that is, the triangular distribution.

In Graph 5.2, we have:

The uncertainty of reading resolution will be the standard deviation of the triangular distribution with R = 1 bar.

$$u_{res} = \frac{R}{\sqrt{24}} = \frac{1}{\sqrt{24}} bar = 0.20\, bar \tag{5.3}$$

Given the limitations of the digitization algorithm, the digital instrument will adopt a uniform probability in rounding readings regardless of whether or not we fix the desired value.

For the analog instrument, the probability of the measurement may be triangular or uniform, depending on whether or not we fix the reading value.

5.3.2 Reading Resolution Adopted by the Calibration Laboratory

Let us look at the following situation: using a calibrated instrument to perform a measurement.

Some questions

1. Given that the laboratory that calibrated our instrument has incorporated the uncertainty of reading resolution in the estimate of final uncertainty, we must, in our measurements, consider the reading resolution of this calibrated standard as

one of the components, as "it is considered" in the uncertainty declared on the certificate?
2. When considering the resolution, are we not repeating the same source of uncertainty twice?
3. In the case of a digital standard, the value of the resolution adopted by the calibration laboratory is known as it will be equal to the digital increase.
4. In the case of an analog standard, a question arises: Was the value of reading resolution adopted by the laboratory the same as what we will adopt?
5. Is it possible to place a magnifying glass on the analog instrument reading scale, thus reducing its measurement uncertainty, since we are reducing its reading resolution as long as it does not make it lower than the instrument sensitivity? This is a usual procedure in laboratories, but the end user of the instrument should be informed.

Answering the questions.
If the user can repeat, in reading with the instrument that came from calibration, the resolution adopted by the laboratory during calibration, the contribution of the standard resolution should not be considered in the final uncertainty of the measurement. Otherwise, this portion should be considered in the final estimate.

For this reason, the calibration laboratories must provide the instrument's calibration certificate with the resolution adopted in calibration. Thus, we will know the value of the resolution adopted and can repeat it at the time of measurement with this instrument.

Using a magnifying glass to read analog instruments is allowed and healthy. However, we should not determine the resolution of a measuring instrument with a magnifying glass. It will lower the resolution than what can be discerned with the "naked eye," and the user usually reads with the "naked eye."

5.3.3 Hysteresis Uncertainty Estimate

In Chap. 4, we saw that hysteresis is the biggest difference between a measurement instrument's charge and discharge values. Scales, comparator clocks, gauges, among others, most commonly present hysteresis errors.

To estimate the uncertainty of hysteresis, we calculate the instrument's hysteresis (H) at the point and adopt a uniform or rectangular probability distribution.

$$u_{\text{hysteresis}} = \frac{H}{\sqrt{12}} \tag{5.4}$$

Solved Exercise 5.1.
A Bourdon-type gauge, whose measurement range is 0 to 20 kgf/cm^2, was calibrated by comparison with a standard gauge. The values found are in Table 5.1.

Table 5.1 Bourdon gauge calibration

Object (kgf/cm^2)	Standard (kgf/cm^2)			
	Charge 1	Discharge 1	Charge 2	Discharge 2
5	5.0	5.2	5.0	5.2
12	12.2	11.9	11.6	11.8
20	20.1	20.2	20.4	20.0

Table 5.2 Hysteresis

Object (kgf/cm^2)	Standard (kgf/cm^2)		
	H_1	H_2	H
5	$\|5.0–5.2\| = 0.2$	$\|5.0–5.2\| = 0.2$	0.2
12	$\|12.2–11.9\| = 0.3$	$\|11.6–11.8\| = 0.2$	0.3
20	$\|20.1–20.2\| = 0.1$	$\|20.4–20.0\| = 0.4$	0.4

Table 5.3 Hysteresis uncertainty

Object (kgf/cm^2)	Standard (kgf/cm^2)	
	H	Hysteresis uncertainty
5	0.2	$\frac{\mathbf{0.2}}{\sqrt{\mathbf{12}}} = \mathbf{0.058}$
12	0.3	$\frac{\mathbf{0.3}}{\sqrt{\mathbf{12}}} = \mathbf{0.087}$
20	0.4	$\frac{\mathbf{0.4}}{\sqrt{\mathbf{12}}} = \mathbf{0.12}$

Calculate:

(a) Hysteresis at each point.
(b) The uncertainty of hysteresis at each point.
(c) Hysteresis and uncertainty of gauge hysteresis.

Solution:

(a) Knowing that hysteresis is the most significant difference between charge and discharge, we can determine hysteresis at each point as follows (Table 5.2):

(b) Adopting a uniform distribution for hysteresis uncertainty, we have (Table 5.3):

(c) The gauge hysteresis will be the highest value:

$$H = 0.4 \ \text{kgf/cm}^2$$

$$u_{\text{hysteresis}} = 0.12 \ \text{kgf/cm}^2$$

Knowing a Little More…
In calibrating the gauge, we fix the object's pointer directly to the desired pressure and verify the variation of its pressure using the standard pressure gauge. Thus, when we fix the pressure at 5 kgf/cm^2, for example, we realize the variation of the object gauge in the standard. This is why, at point 5 kgf/cm^2, we have the standard marking (5.0, 5.2, 5.0, and 5.2) kgf/cm^2. This variation is not caused by the standard but by the calibration object gauge.

5.3.4 Evaluation of the Uncertainty of the Standard Instrument

A source of Type B uncertainty always exists in the calibration of measurement instruments: the uncertainty from the standard instrument. In Chap. 2, when we define calibration, we highlight that calibrating confronts the values measured by the standard instrument with the calibration instrument (object). Therefore, the object instrument's uncertainty inherits the standard instrument's uncertainty.

To determine the uncertainty of the standard instrument, check this value in the standard instrument calibration certificate.

5.4 Standard Measurement Uncertainty

According to VIM—2.30, we have: *"measurement uncertainty expressed as a standard deviation."*

We must express all components of uncertainty (u_i) of Types A and B corresponding to a standard deviation. For this, we need to evaluate the probability distribution applied to the uncertainty: normal distribution, rectangular or uniform distribution, triangular distribution, etc.

5.5 Combined Standard Measurement Uncertainty

According to VIM—2.31, we have: *"standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model."*

The equation can briefly determine combined standard uncertainty (u_c):

$$\boldsymbol{u_c} = \sqrt{\boldsymbol{u_A^2} + \boldsymbol{u_B^2}} \tag{5.5}$$

Where u_A are all Type A uncertainties, and u_B are all Type B uncertainties.

5.6 Effective Degrees of Freedom

The definition of ISO GUM says the following for the degree of freedom: *"In general, the number of terms in a sum minus the number of constraints on the terms of the sum."*

When more than 30 measurements of the same measurement are performed, we know, through statistics, that these results are very close to a normal distribution; if a smaller number of measurements is used, we must bring this distribution closer to a normal distribution by applying the t-distribution correction factor. However, to establish this correction factor, it is necessary to determine the number of effective degrees of freedom of the distribution.

When various sources of uncertainty are considered to estimate combined standard uncertainty (u_C), the number of effective degrees of freedom resulting from combined uncertainty has to be calculated from information from each source of uncertainty. Therefore, it is recommended to use the Welch-Satterthwaite equation to estimate the number of effective degrees of freedom:

$$\frac{\boldsymbol{u_c^4}}{\upsilon_{\boldsymbol{eff}}} = \frac{\boldsymbol{u_1^4}}{\upsilon_{\boldsymbol{1}}} + \frac{\boldsymbol{u_2^4}}{\upsilon_{\boldsymbol{2}}} + \ldots + \frac{\boldsymbol{u_i^4}}{\upsilon_{\boldsymbol{i}}} \tag{5.6}$$

Where u_C is the combined standard measurement uncertainty; $u_1, u_2, \ldots, u_i$ are the standard measurement uncertainties of each source of uncertainty (Type A and Type B uncertainty); $\nu_1, \nu_2, \nu_3 \ldots, \upsilon_i$ are the degrees of freedom of each i source of uncertainty, and $\upsilon_{\boldsymbol{eff}}$ is the number of effective degrees of freedom associated with combined standard uncertainty.

Eq. (5.6) can be reordered and presented by:

$$\upsilon_{eff} = \frac{u_c^4}{\sum \frac{u_i^4}{\upsilon_i}} \tag{5.7}$$

Important
The degree of freedom associated with the uncertainty of repeatability (Type A) equals $n - 1$, where n is the number of measurements.

(continued)

In evaluating the degree of freedom of Type B standard uncertainty from a probability distribution a priori, for example, a uniform or triangular distribution, it is implicitly supposed that the value of uncertainty resulting from such evaluation is known precisely. This implies that the degree of freedom associated with this uncertainty will be infinite.

5.7 Coverage Factor

VIM—2.38 defines it as the *"number larger than one by which a combined standard measurement uncertainty is multiplied to obtain an expanded measurement uncertainty. NOTE: A coverage factor is usually symbolized k"*.

The coverage factor $\boldsymbol{k}$ should always be declared so that the standard uncertainty of quantity measurement can be recovered to calculate the combined standard uncertainty of other measurement results that eventually depend on this quantity.

This factor k will be obtained from the determination of the number of effective degrees of freedom ($\boldsymbol{\nu}_{\mathbf{eff}}$) and using the t-distribution, in which the value of t will be the coverage factor k.

The value of $\boldsymbol{\nu}_{\mathbf{eff}}$ obtained by Eqs. (5.6) or (5.7) is usually not an integer. From the effective degree of freedom, the coverage factor k can be obtained from Excel, using the INV.T.BC function, or in the *Student's t-table*.

When we use the calculated value of the $\boldsymbol{\nu}_{\mathbf{eff}}$ in the t-table, we should always approach it to the immediately lower integer. For example, if the computed value is $\boldsymbol{\nu}_{\mathbf{eff}} = 10.46$, we must enter the table with $\boldsymbol{\nu}_{\mathbf{eff}} = 10$ and obtain $k = 2.28$. This will be the value used for the coverage factor k.

Important
By using the calculated value of the effective degree of freedom ($\boldsymbol{\nu}_{\mathbf{eff}}$) in the Student's t-table, approach it to the immediately lower integer. This will ensure a more prominent coverage factor and, thus, more significant expanded uncertainty.

5.8 Expanded Measurement Uncertainty

According to VIM—2.35, expanded measurement uncertainty is defined as:

Product of a combined standard measurement uncertainty and a factor larger than the number one.

NOTE 1 The factor depends upon the type of probability distribution of the output quantity in a measurement model and on the selected coverage probability.

NOTE 2 The term "factor" in this definition refers to a coverage factor.

NOTE 3 Expanded measurement uncertainty is termed "overall uncertainty" in paragraph 5 of Recommendation INC-1 (1980) (see the GUM) and simply "uncertainty" in IEC documents.

The expanded uncertainty $\boldsymbol{U}$ is the product of the combined standard uncertainty $\boldsymbol{u_C}$ and the coverage factor $\boldsymbol{k}$:

$$U = k.u_c \tag{5.8}$$

Multiplication of combined standard uncertainty by a constant does not provide additional information. It is just a way of representing the final uncertainty associated with a coverage probability. In calibration and industrial measurements, it is expected to adopt the probability of 95.45 %, which would correspond, in a normal distribution, to a coverage factor equal to two.

Important
We must always combine standard uncertainties with one standard deviation. Therefore, when we use the uncertainty of measuring from a calibration certificate, we must divide it by the scope factor k since the uncertainties declared in a calibration certificate are expanded to 95.45 %.

5.9 Presentation of the Measurement Result

According to the document ILAC-P14:09/2020—ILAC Policy for Measurement Uncertainty in Calibration, section 5.3: "*The numerical value of the expanded uncertainty shall be given to, at most, **two significant digits**. Where the measurement result has been rounded, that rounding shall be applied when all calculations have been completed; resultant values may then be rounded for presentation. For the process of rounding, the usual rules for rounding of numbers shall be used, subject to the guidance on rounding provided,* i.e., *in Sect. 7 of the GUM.*"

5.10 Sources of Measurement Uncertainty

Next, we will present some sources of frequent measurement uncertainties in various areas of metrology.

5.10.1 Dimensional Metrology

- Measurement uncertainty of the standard: this information is in the standard calibration certificate.
- Temperature effect: the temperature difference between measuring, the standard, and the temperature of the calibration laboratory should be considered. As a rule, the ambient temperature of the calibration laboratory should be 20.0 °C. This effect is more significant for considerable lengths and in cases where the measurand is of a material different from the standard. Although it is possible to correct these errors, residual uncertainties of the uncertainty of the coefficients of dilation and uncertainty in the calibration of the thermometer will always remain.
- Elastic deformation at the point of contact: is critical in the most exact measurements and in the cases involving different materials. Its magnitude is a function of the measurement force and the nature of the contact between driving and measurand. Although it is possible to correct the results of these errors, the uncertainty of this correction should be considered due to the uncertainty of the applied force and the physical properties of the components in contact.
- Cosine error: misalignment between the measurand or standard and the measurement axis. Residual errors will often persist by the assumption that reference surfaces are exempt from geometric errors.
- Geometric error: planning or spherical errors, parallelism or perpendicularity of the support surface, measurand, or standard cylindrical error.
- Doubt in reading: uncertainty in the resolution of the instrument.
- Stability of the standard, or measurand, as a function of time.

In this area, in general, the following instrument calibration intervals are used (Table 5.4):

5.10.2 Thermal Metrology

- Measurement uncertainty of the standard: this information is in the standard calibration certificate.
- Electrical equipment/instruments used as support: standard resistors uncertainty, multimeters, power supplies, thermal baths, etc.

Table 5.4 Calibration interval

Instrument	Calibration interval—months
Measuring tape	6
Vernier caliper	12
Micrometer	12
Planger dial gauge	12
Slip gauges (standard block)	12

Table 5.5 Calibration interval

Instrument	Calibration intervals—months
Glass thermometer	6 a 12
Resistance thermometer (Pt-100)	12
Thermocouple	12
Bimetallic thermometer	12

- Doubt in reading: uncertainty in the resolution of the instrument.
- Partial immersion in glass thermometers: the part of the column of the immersion thermometer outside the medium provides a difference in the temperature indication.
- Effect of resistance thermometers self-heating: the sensor is heated by the current circulating.
- Parasitic electrical uncertainties: uncertainties of electrical origin resulting from static electricity at contact terminals. We can estimate this $\frac{2\mu V}{\sqrt{3}}$ value primarily when we calibrate thermocouples.
- Drifts of standards and electrical instruments.

In this area, in general, the following instrument calibration intervals are used (Table 5.5):

5.10.3 Mass Metrology

- Measurement uncertainty of the mass standard: this information is in the standard calibration certificate.
- Drift of the masses as a function of time: change of standard mass measurement error as a function of time, depending on the surface finish and the quality of manufacture, material type, handling, atmospheric corrosion, etc. Without this information, we replace it with the maximum permissible error.
- Environmental conditions: temperature gradients, humidity, static electricity.
- Doubt in reading: uncertainty in the resolution of the instrument.
- Air buoyancy: the density of the air can be determined from the measurement of atmospheric pressure, temperature, and relative humidity. Even when the density is corrected, uncertainties of pressure, temperature, and humidity measurements will be present.
- Measurement process: the quality of the scale influences the result of the measurement and, therefore, we must know its characteristics:
    - Repeatability of measurements;
    - Linearity;
    - Eccentricity of the load, especially when more than one mass is placed on the plate;
    - Influence of magnetic fields;

Table 5.6 Calibration interval

Instrument	Calibration interval—months
Standard mass	24 to 48
Precision scales	12 to 36
Analytical scales	12

Table 5.7 Calibration interval

Instrument	Calibration interval—months
Digital multimeter	12
Oscilloscope	12 to 36
Resistive decade	24 to 48

- Effects of temperature;
- Lever arm's length.

In this area, in general, a calibration interval of the instruments is used as follows (Table 5.6):

5.10.4 *Electric Metrology*

- Uncertainty of electrical reference standards: this information is in the standard calibration certificate.
- Different environmental conditions from the recommended.
- Measurement system stability: as a function of time and conditions of use.
- Doubt in reading: uncertainty in the resolution of the instrument.
- Impedance of cables, terminals, and instruments: parasitic electrical uncertainties resulting from static electricity at contact terminals. Its value is estimated in $\frac{2\mu V}{\sqrt{3}}$.
- Layout of instruments and standards during calibration: current leaks, electromagnetic fields, grounding.

In this area, in general, a calibration interval of the instruments is used as follows (Table 5.7):

5.10.5 *Pressure Metrology*

- Measurement uncertainty of the standard: This information is in the standard calibration certificate.
- Drift of the masses as a function of time: change of standard mass measurement error as a function of time, depending on the surface finish and the quality of manufacture, material type, handling, atmospheric corrosion, etc. Without this information, we replace it with the maximum permissible error.
- Doubt in reading: uncertainty in the resolution of the instrument.

- Hysteresis.
- Different environmental conditions from the recommended.
- Drifts of standards and electrical instruments.
- Impedance of cables, terminals, and instruments: parasitic electrical uncertainties resulting from static electricity at contact terminals. Its value is estimated in $\frac{2\mu V}{\sqrt{3}}$.

5.10.6 Analytical Metrology

- *Sampling: where in-house or field sampling forms part of the specified procedure, and effects such as random variations between different samples and any potential for bias in the sampling procedure form components of uncertainty affecting the final result.*
- *Storage Conditions: where test items are stored for any period prior to analysis, the storage conditions may affect the results. The duration of storage, as well as conditions during storage, should, therefore, be considered uncertainty sources.*
- *Instrument effects: may include, for example, the limits of accuracy on the calibration of an analytical balance; a temperature controller that may maintain a mean temperature that differs (within specification) from its indicated set point; an auto-analyzer that could be subject to carryover effects.*
- *Reagent purity: the concentration of a volumetric solution will not be known precisely even if the parent material has been assayed, since some uncertainty related to the assaying procedure remains. Many organic dyestuffs, for instance, are not 100 % pure and can contain isomers and inorganic salts. The purity of such substances is usually stated by manufacturers as being not less than a specified level. Any assumptions about the degree of purity will introduce an element of uncertainty.*
- *Assumed stoichiometry: where an analytical process is assumed to follow a particular reaction stoichiometry, it may be necessary to allow for departures from the expected stoichiometry, or incomplete reaction or side reactions.*
- *Measurement conditions: for example, volumetric glassware may be used at an ambient temperature different from that at which it was calibrated. Gross temperature effects should be corrected, but any uncertainty in the temperature of liquid and glass should be considered. Similarly, humidity may be important where materials are sensitive to possible changes in humidity.*
- *Sample effects: the recovery of an analyte from a complex matrix, or an instrument response, may be affected by composition of the matrix. Analyte speciation may further compound this effect. The stability of a sample/analyte may change during analysis because of a changing thermal regime or photolytic effect. When a 'spike' is used to estimate recovery, the recovery of the analyte from the sample may differ from the recovery of the spike, introducing an uncertainty that needs to be evaluated.*

Table 5.8 Voltage measurements

Measurements	Results (V)
1	1.22
2	1.22
3	1.24
4	1.22
5	1.20

- *Computational effects: selection of the calibration model,* e.g. *using a straight line calibration on a curved response, leads to poorer fit and higher uncertainty. Truncation and round-off can lead to inaccuracies in the final result. Since these are rarely predictable, an uncertainty allowance may be necessary.*
- *Blank Correction: there will be an uncertainty on both the value and the appropriateness of the blank correction. This is particularly important in trace analysis.*
- *Operator effects: possibility of reading a meter or scale consistently high or low. Possibility of making a slightly different interpretation of the method.*

Source: EURACHEM/CITAC Guide CG 4—Quantifying Uncertainty in Analytical Measurement.

Solved Exercise 5.2: Three or more measurements with a calibrated instrument.

With a digital multimeter, we performed five voltage measurements in a circuit. The results were (Table 5.8):

Consider the uncertainty of the multimeter obtained in the calibration certificate is 0.02 V, for a probability of 95.45 % and k = 2.23, with the instrumental bias of +0.02 V.

Determine:

(a) The Type A measurement uncertainty.

Type A uncertainty is calculated by standard deviation from the mean of five measurements.

$$s = 0.01412\ V \rightarrow u_A = \frac{s}{\sqrt{n}} = \frac{0.01412\ V}{\sqrt{5}} = 0.0063245\ V$$

Note: As the result of uncertainty is partial, it is not to round it up. We will leave to make the rounding when declaring expanded uncertainty.

(b) The Type B multimeter uncertainty.

We must divide the declared uncertainty in its Calibration Certificate by the coverage factor *k*. Thus, its measurement uncertainty after division will be standardized uncertainty with a standard deviation.

$$u_B = \frac{U_{mult}}{k} = \frac{0.02\ V}{2.23} = 0.008969\ V$$

(c) Combined uncertainty of measurement

$$u_c = \sqrt{u_A^2 + u_B^2} = \sqrt{0.0063245^2 + 0.008969^2} = 0.010975\ V$$

(d) Expanded uncertainty for a probability of 95.45 %.

Expanded uncertainty is determined by multiplying the combined uncertainty by the factor k. To determine k, it is necessary to calculate the effective degree of freedom of the combination of these uncertainties (Type A and Type B) and then consult the t-table.

The degree of freedom of Type A uncertainty is: $\upsilon_A = n - 1 = 5 - 1 = 4$.

The degree of freedom to the Type B multimeter equals 12 (from the t-table for $k = 2.23$).

$$\text{Then } \upsilon_{eff} = \frac{u_c^4}{\sum \frac{u_i^4}{\upsilon_i}} = \frac{0.010975^4}{\frac{0.0063245^4}{4} + \frac{0.008969^4}{12}} = 15.44 \rightarrow k = 2.18$$

$$\mathrm{U} = \mathrm{k.u_c} = 2.18 \text{ x } 0.010975\ \mathrm{V} = 0.02393\ \ \mathrm{V}$$

(e) The metrological correct result of the expanded uncertainty.

In addition to being unable to declare uncertainty with more than two significant digits, in this particular case, the multimeter measurement uncertainty cannot go beyond the second decimal digit (one significant digit) since the multimeter can only read to the second decimal digit.

$$U_{\text{mult}} = 0.02\ \ \mathrm{V} \quad k = 2.18, \ \ \text{with a probability of}\ \ 95.45\ \ \%.$$

(f) The corrected voltage value.

Mean = 1.22 V.
Instrumental bias + 0.02 V.
Corrected value = (1.22–0.02) V = 1.20 V.

(g) Measurement result (MR)

$$\mathrm{MR} = (1.20 \pm 0.02)\ \mathrm{V}$$

Solved Exercise 5.3: Only one measurement with a calibrated instrument.

Whenever possible, we must perform at least three measurements. This allows us to evaluate Type A uncertainty, that is, the repeatability of the measuring. If the measurement is stable with very low or no variation in instrument resolution, we can proceed according to the following exercise.

Exercise: A glass liquid thermometer measures fuel oil temperature with a measurement uncertainty of 0.1 °C (for k = 2 and 95.45 %) and instrumental bias of − 0.2 °C. The value found was 28.4 °C. Determine the measurement result.
Solution: Corrected measurement = (28.4 + 0.2) °C = 28.6 °C

$$\mathrm{MR} = (28.6 \pm 0.1)\quad {}^\circ\mathrm{C}$$

Note that it was impossible to calculate the uncertainty of the repeatability of the measurement. Thus, the declared measurement instrument in its calibration certificate only inherited the final uncertainty.

5.11 Proposed Exercises

5.11.1 A car speedometer ranges from (0 to 200) km/h. The uncertainty at any point is 2 km/h

(a) What is the uncertainty at 100 km/h?
(b) What is the percentage uncertainty compared to 100 km/h?
(c) What is the percentage uncertainty to 50 km/h?
(d) What is the percentage uncertainty compared to 5 km/h?
(e) At what point is the lowest percentage uncertainty?

5.11.2 Mike measures his brother's height and finds 176.35 cm, with an uncertainty of 0.21 cm.

(a) Round out and write the height of Mike's brother with one significant digit concerning his uncertainty.
(b) Give the same answer in meters.

5.11.3 Martha uses a timer to measure the period of a pendulum. The results are (Table 5.9):

(a) What is the mean value of the period?
(b) What is the standard deviation of the mean?

Table 5.9 Period measurements

Measurements	Period (s)
1	0.63
2	0.64
3	0.65
4	0.63
5	0.65

Table 5.10 Time measurements

Measurements	Fall time (s)
1	0.45
2	0.42
3	0.41
4	0.48
5	0.44

Table 5.11 Diameter measurements

Measurements	Diameter (mm)
1	256.90
2	257.05
3	256.95
4	257.00

(c) What is the best estimate of measurement uncertainty?
(d) Express its result, considering only Type A uncertainty as the only source of measurement uncertainty.

5.11.4 The results of five measures of the fall time of a body, performed by a digital timer, were (Table 5.10)

Considering that the uncertainty of the timer is 0.02 s to k = 2 and 95.45 % of probability, calculate:

(a) The number of observations n.
(b) The mean of observations.
(c) The standard deviation of the mean.
(d) The expanded uncertainty of body fall measurement.
(e) The expanded uncertainty despising the uncertainty of the timer.

5.11.5 To determine the diameter of an axis, a mechanic used a Vernier caliper with an uncertainty of 0.05 mm (k = 2 and 95.45 %) and a resolution of 0.05 mm. Four measurements were performed, and the values found for the diameter were (Table 5.11)

What is the diameter value and its measurement uncertainty?

5.11.6 The length measurement of a piece with a "true value" of 10.1538 mm was performed by a micrometer with a resolution of 0.001 mm and measurement uncertainty equal to 0.002 mm, with $k = 2.23$ to 95.45 %. Determine

(a) The bias of the micrometer.
(b) The Type A uncertainty for the set of measurements.
(c) The combined uncertainty and its degree of freedom.
(d) The expanded uncertainty of measuring the length of the piece (Table 5.12).

5.11.7 Using a digital scale with a resolution equal to 0.1 g, the mass of metal was measured four times, finding the following values (Table 5.13)

Table 5.12 Length measurements

Measurements	Length (mm)
1	10.158
2	10.157
3	10.159
4	10.155
5	10.153
6	10.156
7	10.154
8	10.156
9	10.155
10	10.157

Table 5.13 Mass measurements

Measurements	Mass (g)
1	23.5
2	23.5
3	23.6
4	23.8

Table 5.14 Temperature measurements

Measurements	Temperature (°C)
1	80.5
2	80.5
3	81.0
4	81.0
5	80.0

Considering that scale measurement uncertainty is double its resolution (to k = 2.00 and 95.45 %):

(a) What is the Type A uncertainty of this measurement?
(b) What is the bias, knowing that the "true value" of the metal mass is 23.60 g?
(c) What is the expanded uncertainty of this measurement?

5.11.8 A metrology student declared the measuring uncertainty of the density as follows

$$\rho = (1.003 \pm 0.0235) \ \ g/mL$$

What is the error in the statement of this measurement?

5.11.9 Consider a bimetallic thermometer with a resolution of 0.5 °C used to measure the temperature of mineral oil contained in a tank. Five measurements were made, obtaining the following values (Table 5.14)

Knowing that the bimetallic thermometer used in this control has an uncertainty of 0.6 °C (k = 2.87; 95.45 %), calculate:

Table 5.15 Mass measurements

Measurements	Mass (g)
1	100.0034
2	100.0038
3	100.0032

Table 5.16 Mass measurements

Measurements	Mass (g)
1	12.0004
2	12.0006
3	12.0006

(a) The mean of measurements.
(b) The repeatability uncertainty.
(c) The standardized uncertainty of the bimetallic thermometer.
(d) The combined uncertainty of this measurement.
(e) The effective degree of freedom of the measurement.
(f) The coverage factor is 95.45 %.
(g) The expanded uncertainty to 95.45 %.
(h) What source of uncertainty has the most significant influence on the process?

5.11.10 Consider measuring a mass, presented in Table 5.15, using an analytical scale performed in a laboratory at the point for 100 g. The scale bias at point 100 g is declared in the calibration certificate −0.0050 g. The uncertainty declared in the calibration certificate is 0.0008 g (k = 2.00; 95.45 %).

Based on this information, determine:

(a) The mean of measurements.
(b) The repeatability uncertainty.
(c) The expanded uncertainty to 95.45 %.

5.11.11 Consider a mass measurement M on a scale. The scale measurement correction at this point is −1.5 mg, with an uncertainty of 0.3 mg (k = 2.11; 95.45 %). Three mass measurements were made to obtain the values in Table 5.16. Based on this information, determine:

(a) The mean of measurement.
(b) The repeatability uncertainty.
(c) The bias of the scale.
(d) The expanded uncertainty to 95.45 % with its respective factor k and the degrees of freedom.

5.11.12 What is the false alternative regarding the measurement uncertainty?

(a) It is a non-negative parameter that characterizes the dispersion of the values assigned to the measurement.

(b) Combined standard uncertainty is obtained using uncertainties in the form of an individual standard deviation associated with input quantities.
(c) Expanded measurement uncertainty is the sum of all Type A and Type B uncertainty.
(d) The probability of coverage refers to the chance that a proper set of measurement values is contained in a specified coverage interval.

Chapter 6
Evaluation of the Uncertainty in Indirect Measurements

6.1 Uncertainty Propagation Law

Consider a greatness W described by the function $W = f(a, b, c \ldots)$, where $a, b, c, \ldots$ are statistically independent[1] variables. If the most likely values for these quantities are $\overline{a}, \overline{b}, \overline{c}, \ldots,$ the most likely value for W will be $W = f\left(\overline{a}, \overline{b}, \overline{c}, \ldots\right)$. Expanding the W function in the Taylor series, we will have:

$$\begin{aligned} W_i \approx W\left(\overline{a}, \overline{b}, \overline{c} \ldots\right) + \frac{\partial W}{\partial a}(a_i - \overline{a}) + \frac{\partial W}{\partial b}\left(b_i - \overline{b}\right) + \frac{\partial W}{\partial c}(c_i - \overline{c}) + \ldots \\ + \frac{1}{2}\frac{\partial^2 W}{\partial a^2}(a_i - \overline{a})^2 + + \frac{1}{2}\frac{\partial^2 W}{\partial b^2}\left(b_i - \overline{b}\right)^2 + \frac{1}{2}\frac{\partial^2 W}{\partial c^2}(c_i - \overline{c})^2 + \ldots \end{aligned} \tag{6.1}$$

We will make an approximation disregarding the quadratic terms when $(a_i - \overline{a}),\ \left(b_i - \overline{b}\right),\ (c_i - \overline{c})\ldots$ are of the order of greatness of the standard deviation $\sigma_a,\ \sigma_b,\ \sigma_c,\ \ldots$ Then, $\sigma_a = (a_i - \overline{a})$ and $\frac{1}{2}\ \frac{\partial^2 W}{\partial a^2}(a_i - \overline{a})^2 \approx 0$.

This condition applies to all other variables.

Therefore, we have:

$$W_i - \overline{W\ \left(\overline{a}, \overline{b}, \overline{c} \ldots\right)} = \Delta W_i = \left[\frac{\partial W}{\partial a} \cdot \sigma_a + \frac{\partial W}{\partial b} \cdot \sigma_b + \frac{\partial W}{\partial c} \cdot \sigma_c + \ldots\right] \tag{6.2}$$

The term $\frac{\partial W}{\partial a}$ represents the partial derivative of W concerning the variable a, $a = \overline{a}$, calculated in which all other variables were kept constant.

[1] The variables are considered statistically independent when the variation of one does not influence the variation of the other, that is, all behave in a detached manner. Statistically, these variables have a correlation coefficient equal to zero.

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_6

W variance can be obtained by:

$$s_W^2 = \frac{1}{N-1}\sum_{i=1}^{N}\Delta W_i^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left[\frac{\partial W}{\partial a}\cdot\Delta a_i + \frac{\partial W}{\partial b}\cdot\Delta b_i + \frac{\partial W}{\partial c}\cdot\Delta c_i + \ldots.\right]^2 =$$

$$s_W^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left[\left(\frac{\partial W}{\partial a}\cdot\Delta a_i\right)^2 + \left(\frac{\partial W}{\partial b}\cdot\Delta b_i\right)^2 + \left(\frac{\partial W}{\partial c}\cdot\Delta c_i\right)\right.$$

$$\left. + \ldots\right] \quad + \frac{1}{N-1}\sum_{i=1}^{N}\left[2\left(\frac{\partial W}{\partial a}\cdot\Delta a_i\cdot\frac{\partial W}{\partial b}\cdot\Delta b_i\right) + 2\left(\frac{\partial W}{\partial a}\cdot\Delta a_i\cdot\frac{\partial W}{\partial c}\cdot\Delta c_i\right)\right.$$

$$\left. +2\left(\frac{\partial W}{\partial b}\cdot\Delta b_i\cdot\frac{\partial W}{\partial c}\cdot\Delta c_i\right) + \ldots\right]$$

As variables *a, b, c,* ... are statistically independent, there is no correlation between their deviations. Δa_i, Δb_i, $\Delta c_i \ldots$ and, consequently, all types of greatness, as

$$\frac{\partial W}{\partial a}\cdot\Delta a_i\cdot\frac{\partial W}{\partial b}\cdot\Delta b_i$$

has the same probability to be both positive and negative. Thus, for a large number N of measures, the second term of the sum is nullified, resulting in:

$$s_W^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left[\left(\frac{\partial W}{\partial a}\right)^2(\Delta a_i)^2 + \left(\frac{\partial W}{\partial b}\right)^2(\Delta b_i)^2 + \left(\frac{\partial W}{\partial c}\right)^2(\Delta c_i)^2 + \ldots\right]$$

We can rewrite it as:

$$s_W^2 = \left(\frac{\partial W}{\partial a}\right)^2\cdot\frac{1}{N-1}\sum_{i=1}^{N}(\Delta a_i)^2 + \left(\frac{\partial W}{\partial b}\right)^2\cdot\frac{1}{N-1}\sum_{i=1}^{N}(\Delta b_i)^2 + \left(\frac{\partial W}{\partial c}\right)^2\cdot\frac{1}{N-1}$$
$$\times\sum_{i=1}^{N}(\Delta c_i)^2$$

$$s_W^2 = \left(\frac{\partial W}{\partial a}\right)^2\cdot\sigma_a^2 + \left(\frac{\partial W}{\partial b}\right)^2\cdot\sigma_b^2 + \left(\frac{\partial W}{\partial c}\right)^2\cdot\sigma_c^2 + \ldots$$

Considering the uncertainties of variables *a, b, c,* ... as their standard deviations, we can rewrite the previous equation:

Fig. 6.1 Brook Taylor (1685 – 1731, England)

$$u_W^2 = \left(\frac{\partial W}{\partial a}\right)^2 \cdot u_a^2 + \left(\frac{\partial W}{\partial b}\right)^2 \cdot u_b^2 + \left(\frac{\partial W}{\partial c}\right)^2 \cdot u_c^2 + \ldots \tag{6.3}$$

Equation (6.3) is the uncertainties propagation equation of any function W (a, b, c,...), in which variables a, b, c ... are independent.

Knowing a Little More ... (Fig. 6.1)
https://images.fineartamerica.com/images-medium-large/1-brook-taylor-1685-1731-granger.jpg

Brook Taylor (1685 – 1731, England) came from a relatively wealthy family: his father, John Taylor, although disciplining, was interested in painting and music and taught his son. Thus, Brook was later able to apply his mathematical knowledge in these two areas. Born in a family of possessions, it was possible to have private teachers. Brook was educated at home (having acquired a good base in classics and mathematics) before entering Cambridge in 1703. There, Taylor improved his mathematical knowledge, graduating in 1709. However, a year earlier (1708), he had already written his first relevant mathematics work, although his publication occurred only in 1714. In 1712, Taylor was elected to the Royal Society and appointed to a commission created to decide who the inventor of the calculation was: Newton or Leibniz.

Several personal tragedies marked his career, such as his marriage to Brydges of Wallington in 1721, which suffered opposition from John Taylor due to social class differences. Thus, the father-and-son relationship was broken until 1723, when Brook's wife died in childbirth along with their

(continued)

Knowing a Little More ... (Fig. 6.1) (continued)
son. After this loss, Brook returned to live with his father. In 1725, he married again, with his father's approval. The chosen one was Sabetta Sawbridge of Olantigh. In 1729, after his father's death, Brook inherited the property. However, personal tragedies continued to torment him when his second wife died in childbirth. The child, named Elizabeth, managed to survive. Taylor lived a few more years (died at age 46), but his mathematical deeds are surprising and probably not deepened due to personal factors (disappointments, fragile health). It gave rise to a new branch in mathematics called "Calculus of finite differences," parts integration, and the series known as Taylor's expansion.

6.2 When Variables Are Statistically Dependent

Statistically, dependent variables behave in a linked manner; that is, the variation of one influences the variation of another. These variables have a different correlation coefficient from zero.

In the presence of statistically dependent variables, measurement uncertainty should consider the correlation coefficient (r) between variables. The correlation coefficient may vary between [−1, +1], being zero when the variables are independent, and their expression is given by:

$$r = \frac{s_{ab}}{s_a s_b} \tag{6.4}$$

$$r(a, b) = \frac{\sum_{i=1}^{n}(a_i - \overline{a})\left(b_i - \overline{b}\right)}{\sqrt{\sum (a_i - \overline{a})^2}.\sqrt{\sum \left(b_i - \overline{b}\right)^2}} \tag{6.5}$$

Where s_a and s_b are the standard deviation of a and b variables, and s_{ab} is the standard deviation of the correlation between the variables given by Eq. (6.5).

If the variables are dependent on each other, the uncertainty propagation equation will be:

$$u_W^2 = \left(\frac{\partial W}{\partial a}\right)^2 u_a^2 + \left(\frac{\partial W}{\partial b}\right)^2 u_b^2 + \left(\frac{\partial W}{\partial c}\right)^2 u_c^2 + \ldots + 2\left(\frac{\partial W}{\partial a}\right) \times \left(\frac{\partial W}{\partial b}\right) r(a, b) u_a u_b + 2\left(\frac{\partial W}{\partial a}\right)\left(\frac{\partial W}{\partial c}\right) r(a, c) u_a u_c + \ldots \tag{6.6}$$

where $r(a, b)$, $r(a, c)$, $r(b, c)$, ... are the correlation coefficients between the variables (a, b, c...).

Table 6.1 Relative uncertainties of some functions

Case 1	$F = x \cdot y$	$u_F = F \cdot \sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2}$
Case 2	$F = \frac{x}{y}$	$u_F = F \cdot \sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2}$
Case 3	$F = x^m$	$u_F = F\left(m\frac{u_x}{x}\right)$
Case 4	$F = x^P + y^q$	$u_F = \quad F\sqrt{\left(p\frac{u_x}{x}\right)^2 + \left(q\frac{u_y}{y}\right)^2}$
Case 5	$F = \log x$	$u_F = 0,43429 \cdot \left(\frac{u_x}{x}\right)$
Case 6	$F = \ln x$	$u_F = \left(\frac{u_x}{x}\right)$
Case 7	$F = e^x$	$u_F = F \cdot u_x$
Case 8	$F = 10^x$	$u_F = F \cdot (2.3026 \cdot u_x)$

Note: u_F is the combined uncertainty of the function F

6.3 Method of Relative Uncertainties[2]

If we do not want to calculate the partial derivative of a function or this knowledge has not yet been addressed by the reader, follow, in Table 6.1, a propagation relationship of uncertainty in which the relative uncertainties are used in some mathematical functions.

The relationship between the measurement uncertainty of a variable x and the value of this variable determines relative measurement uncertainty.

The variables must be statistically independent.

$$u_{\text{relative}} = \frac{u_x}{x} \tag{6.7}$$

Solved Exercise 6.1
Deduces the formula of relative uncertainty to case 1: $F = x \cdot y$.

Solution

- The partial derivative of function F concerning x: $\frac{\partial F}{\partial x} = y$.
- The partial derivative of function F concerning y: $\frac{\partial F}{\partial y} = x$.

$$u_F^2 = \left(\frac{\partial F}{\partial x}\right)^2 {u^2}_x + \left(\frac{\partial F}{\partial y}\right)^2 {u^2}_y$$

$$u_F^2 = (y)^2 {u^2}_x + (x)^2 {u^2}_y$$

[2]This method can be employed when derivation techniques are not known. It can be applied to studies on the calculation of measurement uncertainties at the high school/technical level.

$$u_F^2 = y^2 {u^2}_x + x^2 {u^2}_y$$

- Raising to the square both members of the equation $F = x \cdot y$

$$(F)^2 = (x{\cdot}y)^2 = F^2 = x^2{\cdot}y^2$$

- Dividing each member of the equation by $F^2 = x^2 \cdot y^2$

$$\frac{u_F^2}{x^2{\cdot}y^2} = \left(\frac{y^2}{x^2{\cdot}y^2}\right){u^2}_x + \left(\frac{x^2}{x^2{\cdot}y^2}\right){u^2}_y$$

- But: $F^2 = x^2 \cdot y^2$:

$$\frac{u_F^2}{F^2} = \left(\frac{1}{x^2}\right){u^2}_x + \left(\frac{1}{y^2}\right){u^2}_y \qquad \left(\frac{u_F}{F}\right)^2 = \left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2$$

$$u_F = F\sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2}$$

Solved Exercise 6.2
Deduces the formula of relative uncertainty to case 2: $F = \frac{x}{y}$.

Solution

- The partial derivative of function F concerning x: $\frac{\partial F}{\partial x} = \frac{1}{y}$.
- The partial derivative of function F concerning y: $\frac{\partial F}{\partial y} = -\frac{x}{y^2}$.

$$u_F^2 = \left(\frac{1}{y}\right)^2 {u^2}_x + \left(-\frac{x}{y^2}\right)^2 {u^2}_y$$

$$u_F^2 = \left(\frac{1}{y^2}\right){u^2}_x + \left(\frac{x^2}{y^4}\right){u^2}_y$$

- Raising to the square both members of the equation: $F = \frac{x}{y}$

$$F^2 = \frac{x^2}{y^2}$$

- Dividing each member of the equation by $F^2 = \frac{x^2}{y^2}$

$$\frac{u_F^2}{\frac{x^2}{y^2}} = \left(\frac{\frac{1}{y^2}}{\frac{x^2}{y^2}}\right) u^2{}_x + \left(\frac{\frac{x^2}{y^4}}{\frac{x^2}{y^2}}\right) u^2{}_y$$

But: $F^2 = \frac{x^2}{y^2}$,

$$\frac{u_F^2}{F^2} = \left(\frac{\frac{1}{y^2}}{\frac{x^2}{y^2}}\right) u^2{}_x + \left(\frac{\frac{x^2}{y^4}}{\frac{x^2}{y^2}}\right) u^2{}_y \qquad \left(\frac{u_F}{F}\right)^2 = \left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2$$

$$u_F = F\sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2}$$

Solved Exercise 6.3

A circuit was assembled to determine the electrical resistance R, in which the value of the electric current (i) that passes through the resistor and its voltage (V) were measured. The variables V and i are statistically independent (different instruments measured them).

The following values were found with $k = 2$ and 95.45%:

$$V = (15.0 \pm 0.1)\ \text{V}$$

$$i = (0.286 \pm 0.003)\ \text{A}$$

Determine:

(a) The value of the electrical resistance R, with the correct number of significant digits.
(b) The uncertainty of electrical resistance by the derivative method.
(c) The uncertainty of electrical resistance by the relative uncertainties method.

Solution

(a) $R = V/i = 15.0 / 0.286 = 52.44755\ \Omega \rightarrow R = 52.4\ \Omega$.
(b) In this situation, we can use Eq. (6.3):

$$u_R^2 = \left(\frac{\partial R}{\partial V} \cdot u_V\right)^2 + \left(\frac{\partial R}{\partial i} \cdot u_i\right)^2$$

$$\frac{\partial R}{\partial V} = \frac{1}{i} \qquad \frac{\partial R}{\partial i} = \frac{-V}{i^2} \qquad u_R = \sqrt{\left(\frac{1}{i} u_V\right)^2 + \left(\frac{-V}{i^2} u_i\right)^2}$$

The standard uncertainty of V and i:

$$u_V = \frac{U_V}{k} = \frac{0.1}{2} = 0.05\ V$$

$$u_i = \frac{U_i}{k} = \frac{0.003}{2} = 0.0015\ A$$

Then:

$$u_R = \sqrt{(0.05/0.286)^2 + \left(\frac{-15}{0.286^2}0.0015\right)^2} = 0.32593\ \Omega$$

Since the coverage factors of both voltage (V) and electric current (i) are equal to two, we will have an infinite degree of freedom and, consequently, a coverage factor for the electrical resistance (R) equal to two.[3] Thus, expanded uncertainty will be:

$$U_R = k\ .\ u_R = 2.00 \times 0.32593 = 0.6519\ \Omega$$

Since we cannot declare uncertainties with more than two significant digits, we will have to round the result to one decimal digit, being compatible with the measured electrical resistance value of 52.4 Ω. The uncertainty of R will be:

$$U_R = 0.7\ \Omega$$

The result will be:

$$R = (52.4 \pm 0.7)\ \Omega$$

(c) Relative uncertainties method.

$$\left(\frac{u_R}{R}\right)^2 = \left(\frac{u_V}{V}\right)^2 + \left(\frac{u_i}{i}\right)^2$$

$$\left(\frac{u_R}{52.4}\right)^2 = \left(\frac{0.05}{15.0}\right)^2 + \left(\frac{0.0015}{0.286}\right)^2 = 0.00003862$$

$$u_R = 52.4\ \sqrt{0.00003862} = 0.3256$$

$$k = 2.00$$

[3] In the next section, we will study how to determine the coverage factor, when the components that contribute to uncertainty have a different degree of freedom from infinity.

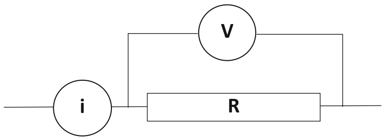


Fig. 6.2 Measurement circuit

$$U_R = k.u_R = 2.00 \times 0.3256 = 0.6513 \ \Omega$$
$$U_R = 0.7 \ \Omega$$

The result will be: $R = (52.4 \pm 0.7)\ \Omega$

6.4 Evaluation of the Effective Degree of Freedom for Relative Uncertainties

In Chap. 5, we saw that we must use the Welch-Satterthwaite Eq. (5.6) or Eq. (5.7) to determine the effective degree of freedom.

In the case of indirect measurements, we must apply the uncertainties relative to Eq. (5.6), since the quantities involved have different units. In this way, the Welch-Satterthwaite equation is like this:

$$\frac{u_{Rc}^4}{\upsilon_{\text{eff}}} = \frac{u_{R1}^4}{\upsilon_1} + \frac{u_{R2}^4}{\upsilon_2} + \ldots + \frac{u_{Ri}^4}{\upsilon_i} \tag{6.8}$$

- u_{Rc} is the combined relative uncertainty of the quantity that we need to determine the degree of freedom.
- u_{R1}, u_{R2}, u_{R3}, ..., u_{Ri} are the standard relative uncertainties of each *i* source (Type A and B uncertainties).
- υ_1, υ_2, υ_3, υ_i … are the degrees of freedom of each *i* source of uncertainties;
- υ_{eff} is the effective degree of freedom associated with standard combined relative uncertainty.

Solved Exercise 6.4

A cylindrical metal bar (Fig. 6.3) has a diameter $d = (2.50 \pm 0.01)$ cm, $k = 2.37$, and 95.45%; length $L = (30.48 \pm 0.01)$ cm, $k = 2.28$, and 95.45%; mass $M = (1158.0 \pm 0.1)$ g, $k = 2.23$, and 95.45%. Whereas the equation calculates the volume of a cylinder:

Fig. 6.3 Cylindrical bar

$$V = \frac{\pi d^2 L}{4},$$

Determine:

(a) The density of the metal bar.
(b) The expanded uncertainty of metal bar density to 95.45% metrological reliability.

Solution
(a) The density:

$$\rho = \frac{M}{V}$$

$$\rho = \frac{M}{\frac{\pi\ d^2 L}{4}}$$

$$\rho = \frac{4M}{\pi\ d^2 L}$$

$$\rho = \frac{4\ x\ 1\ 158}{3.1416\ x\ 2.50^2 x 30.38} = 7.7397 \rightarrow \rho = 7.74\ \mathrm{g/cm^3}$$

(b) Considering the variables statistically independent, we can use Eq. (6.3):

$$u_\rho^2 = \left(\frac{\partial \rho}{\partial M}\right)^2 \cdot u_M^2 + \left(\frac{\partial \rho}{\partial d}\right)^2 \cdot u_d^2 + \left(\frac{\partial \rho}{\partial L}\right)^2 \cdot u_L^2$$

$$\frac{\partial \rho}{\partial M} = \frac{4}{\pi d^2 L} = 0.006684/\mathrm{cm^3}$$

$$\frac{\partial \rho}{\partial d} = \frac{-8M}{\pi d^3 L} = -6.19174\ \mathrm{g/cm^4}$$

$$\frac{\partial \rho}{\partial L} = \frac{-4M}{\pi d^2 L^2} = -0.25393\ \mathrm{g/cm^4}$$

$$u_M^2 = \left(\frac{0.1}{2.23}\right)^2 = 0.002011$$

$$u_d^2 = \left(\frac{0.01}{2.37}\right)^2 = 0.0000178$$

$$u_L^2 = \left(\frac{0.01}{2.28}\right)^2 = 0.00001924$$

$$u_\rho = \sqrt{0.006684^2 x 0.002011 + (-6.19174)^2 x 0.0000178 + (-0.25393)^2 x 0.00001924}$$

$$u_\rho = 0.026151 \text{g/cm}^3$$

Note: We could also have calculated density uncertainty using the relative uncertainties method.

$$\rho = \frac{4M}{\pi d^2 L} = \frac{4}{\pi} M d^{-2} L^{-1}$$

$$u_\rho = \rho \sqrt{\left(\frac{u_M}{M}\right)^2 + \left(-2\frac{u_d}{d}\right)^2 + \left(\frac{-u_L}{L}\right)^2} = 0.026151 \text{ g/cm}^3$$

Degree of freedom, using Eq. (6.8), and the coverage factor:

$$\frac{u_{RC}^4}{\upsilon_{ef}} = \frac{u_{R1}^4}{\upsilon_1} + \frac{u_{R2}^4}{\upsilon_2} + \ldots + \frac{u_{Ri}^4}{\upsilon_i}$$

$$\frac{\left(\frac{u_\rho}{\rho}\right)^4}{\upsilon_{ef}} = \frac{\left(\frac{u_M}{M}\right)^4}{\upsilon_M} + \frac{\left(\frac{u_d}{d}\right)^4}{\upsilon_d} + \frac{\left(\frac{u_L}{L}\right)^4}{\upsilon_L}$$

Student's t-table

$$M: \quad k = 2.23 \rightarrow \upsilon_M = 12$$

$$d: k = 2.37 \rightarrow \upsilon_d = 8$$

$$L: k = 2.28 \rightarrow \upsilon_L = 10$$

$$\frac{\left(\frac{0.026151}{7,74}\right)^4}{v_{eff}} = \frac{\left(\frac{0.1/2.23}{1\ 158}\right)^4}{12} + \frac{\left(\frac{0.01/2.37}{2.50}\right)^4}{8} + \frac{\left(\frac{0.01/2.28}{30.48}\right)^4}{10}$$

$$\upsilon_{eff} = 128.4717$$

Using the Excel© function INV.T.BC (0.0455; 128.4717) → $k = 2.02$

$$U = k \ . \ u_\rho = 2.02\ x\ 0.026151 = 0.0528 \text{ g/cm}^3$$

$$U = 0.05 \text{ g/cm}^3$$

The result will be: $\rho = (7.74 \pm 0.05)$ g/cm^3; $k = 2.02$ and 95.45 %.

6.5 Sensitivity Coefficient

Considering a function W (a, b, c, …), in which a, b, c, … are its variables, as described in Sect. 6.1, we have Eq. (6.3) below.

$$u_W^2 = \left(\frac{\partial W}{\partial a}\right)^2 \cdot u_a^2 + \left(\frac{\partial W}{\partial b}\right)^2 \cdot u_b^2 + \left(\frac{\partial W}{\partial c}\right)^2 \cdot u_c^2 + \ldots$$

This equation describes the propagation of uncertainties of a function W.

Partial derivatives $\left(\frac{\partial W}{\partial a}\right)$ $\left(\frac{\partial W}{\partial b}\right)$ $\left(\frac{\partial W}{\partial c}\right)$ … describe the variation of function W concerning each variable a, b, c, …

Important
In metrology, these partial derivatives are called sensitivity coefficient (SC_i) and describe how each input a, b, c,... influences the output value W.

Experimentally, if all variables remain constant and only one, for example, the variable a, changes, we can verify the variation of W. In Solved Exercise 6.4, we saw that the partial derivative method allows us to determine the function's sensitivity coefficient concerning each variable.

The sensitivity coefficient

$$\frac{\partial \rho}{\partial M} = \frac{4}{\pi d^2 L}$$

indicates the variation of cylinder density as a function of the variation of its mass (M).

The same happens with the expression

$$\frac{\partial \rho}{\partial d} = \frac{-8M}{\pi d^3 L}$$

which indicates the change of cylinder density by variation of its diameter (d), and the expression

$$\frac{\partial \rho}{\partial L} = \frac{-4M}{\pi d^2 L^2}$$

indicates the variation of cylinder density as a function of the variation of its length (L).

Knowledge of the sensitivity coefficient is essential to know how much a variable influences the result of an indirect measurement, minimizing its influence and, thus, its measurement uncertainty.

In Solved Exercise 6.4, we saw that the largest sensitivity coefficient (in absolute value) of the three-cylinder variables in question—diameter (d), mass (M), and length (L)—is the sensitivity coefficient of cylinder density concerning its diameter:

$$\frac{\partial \rho}{\partial M} = \frac{4}{\pi d^2 L} = 0.006684/\mathrm{cm}^3$$

$$\frac{\partial \rho}{\partial d} = \frac{-8M}{\pi d^3 L} = -6.19174\ \mathrm{g/cm}^4$$

$$\frac{\partial \rho}{\partial L} = \frac{-4M}{\pi d^2 L^2} = -0.25393\ \mathrm{g/cm}^4$$

This fact indicates that we should be more concerned with diameter measurement uncertainty (higher absolute value of SC_i), if we want to minimize its influence on the result of cylinder density.

6.5.1 Sensitivity Coefficient Transforming Uncertainties

The sensitivity coefficient is also applicable, when we want to transform a measurement uncertainty that presents itself into a quantity to another quantity. This case is widespread when we want to measure a quantity, and the instrument used provides a sign in another quantity. We can cite sensors (thermoresistors and thermocouples) and pressure and temperature transducers.

Take as an example the PT-100 platinum resistance thermometer (RTD), which is widely used in industry and thermometry laboratories, because it has low uncertainty and good accuracy. It measures temperature through the variation of its platinum resistance, which, at zero °C, has a value close to 100 ohms.

The Callendar–Van Dusen equation describes the relationship between resistance (R) and temperature (T) of platinum resistance thermometers.

$$R(T) = R_0 \left(1 + AT + BT^2 - 100CT^3 + CT^4\right) \tag{6.9}$$

where $R(T)$ is the resistance value at the desired temperature T, and R_0 is the value of platinum resistance at 0 °C. A, B, and C are the RTD coefficients, with typical values for an industrial platinum resistance thermometer such as:

$$A = 3.9083 \times 10^{-3}/\,^\circ\mathrm{C}; \quad B = -5.775 \times 10^{-7}/\,^\circ\mathrm{C}^2;$$

$$C = 0 \ \ (T \leq 650^\circ C); \quad C = -4.183 \times 10^{-12}/^\circ C^4 \ \ (T > 650^\circ C)$$

Table 6.2 PT-100 sensitivity coefficient as a function of its temperature

Temperature (°C)	SC_T (Ω/°C)	Uncertainty (°C)
0	0.390830	3.8×10^{-3}
10	0.389675	3.8×10^{-3}
20	0.388520	3.9×10^{-3}
30	0.387365	3.9×10^{-3}
40	0.386210	3.9×10^{-3}
50	0.385055	3.9×10^{-3}
60	0.383900	3.9×10^{-3}
70	0.382745	3.9×10^{-3}
80	0.381590	3.9×10^{-3}
90	0.380435	3.9×10^{-3}
100	0.379280	4.0×10^{-3}

Solved Exercise 6.5

Let us consider that we will use a multimeter with a measuring uncertainty of 0.003 Ω to read the resistance $R(T)$. How will we transform this measurement uncertainty and the quantity of electrical resistance for the temperature quantity, ° C unit?

Solution

To solve this problem, we will determine the PT-100 sensitivity coefficient (SC_T) to obtain the relationship between the electrical resistance and the temperature quantities.

The PT-100 sensitivity coefficient is provided by deriving Eq. (6.9) in the function of T. Then:

$$SC_T = \frac{\partial R(T)}{\partial T} = R_0\left(A + 2BT - 300CT^2 + 4CT^3\right)$$

For each temperature value, we will use an SC_T value. Table 6.2 presents temperature and uncertainty values up to 100 °C considering:

$$R_0 = 100\ \ \Omega; \quad A = 3.9083 \times 10^{-3}/{}^\circ\mathrm{C}; \quad B = -5.775 \times 10^{-7}/{}^\circ\mathrm{C}^2; \quad C = 0$$

$$u_{\mathrm{mult}\,{}^\circ c} = \frac{u_{\mathrm{mult}\ \Omega}}{SC_T}$$

Knowing a Little More ...

Platinum resistance thermometer

This thermometer works based on ohmic resistance variation as a function of temperature. The sensor element is commonly made of platinum with a high

(continued)

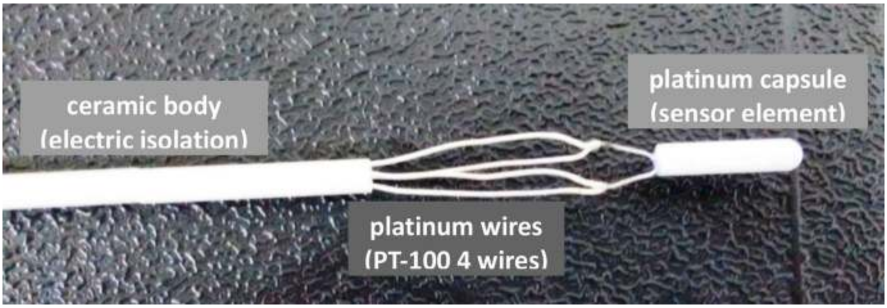


Fig. 6.4 Four-wire platinum resistance thermometer. (Photo by the authors)

degree of purity and encapsulated in ceramic or glass bulbs (Fig. 6.4). Several types of resistance thermometers, from the standard thermometer to the more robust industrial thermometer, may have uncertainties in the tenth of the degree. The most common platinum types are those with a resistance of 25 ohms, 100 ohms, 500 ohms, or 1000 ohms at the ice point (0 °C) (Fig. 6.4).

A two-, three-, or four-wire resistance bulb connection is used according to the type of instrument and the desired accuracy in the measurement. The four-wire thermometers are the most exact and are called half standard. The PT-25 is considered standard, being the most accurate and uncertain of 0.001 Ω. Its main constructive characteristics are:

(i) The sensor element is made of platinum with a purity better than 99.999 %;
(ii) Great thermometer stability and measurement accuracy, with uncertainty values of (0.0006 to 0.01) °C.

6.6 Proposed Exercises

6.6.1 The expression gives the density of a sphere:

$$\rho = \frac{M}{V} = \frac{6M}{\pi D^3}$$

Consider $M = (1000 \pm 1)$ g, $D = (8.000 \pm 0.002)$ cm, and $k = 2.00$ and 95.45 %. Determine the density and the measure uncertainty using:

(a) The partial derivatives.
(b) The relative uncertainties.

6.6.2 Consider a square with side L and area $A = L^2$. The uncertainty of area A was calculated in two ways.

- Mode 1

$$u_A = \sqrt{\left(\frac{\partial A}{\partial L}\right)^2 u^2(L)}$$
$$u_A = 2L \cdot\ u(L)$$

- Mode 2.

Consider that the square now has an $L1$ side and another $L2$. Thus, the area $A = L1 \cdot L2$

The area uncertainty can be calculated by:

$$u_A = \sqrt{\left(\frac{\partial A}{\partial L1} u_{L1}\right)^2 + \left(\frac{\partial A}{\partial L2} u_{L2}\right)^2} = \sqrt{L2^2 u_{L1}^2 + L1^2 u_{L2}^2}$$

But if $L1 = L2 = L \rightarrow u_{L1} = u_{L2} = u_L$

$$u_A = \sqrt{2L^2 u_L^2} = \sqrt{2}\, L u_L$$

Why was the result for mode one different from that of mode 2? Where is the error in the solution?

6.6.3 A bearing factory tests the uniformity of the spheres' diameter, weighing them. The percentage uncertainty of the mass is 1.00 %. If all spheres have the same density, with relative uncertainty equal to 1.20 %, what is the uncertainty in the diameter of a 1.000 cm sphere?

6.6.4 The thickness of a 200-page book is (3.0 ± 0.1) cm. Determine:

(a) The absolute uncertainty of the book's thickness
(b) The relative uncertainty of the book's thickness
(c) The percentage uncertainty of the book's thickness
(d) The thickness of a single sheet of the book
(e) The percentage uncertainty of item (d)

6.6.5 A rectangular block of wood has length L = (10.0 ± 0.1) cm, width W = (5.0 ± 0.1) cm, height H = (2.0 ± 0.1) cm, and mass M = (50.0 ± 0.1) g. All uncertainties are declared with k = 2.00 and 95.45 %.

Determine:

(a) The density of the rectangular wood block.
(b) The uncertainty of the density with all sources of uncertainty considered.
(c) The uncertainty of the density neglects all sources of uncertainty except that of more significant relative uncertainty.
(d) Compare the results of items (b) and (c) and declare your conclusions.

6.6.6 The expression determines the volume of a sphere:

$$V = \frac{\pi d^3}{6}$$

Considering its diameter as $d = (1.00 \pm 0.01)$ cm, with $k = 2.00$ and 95.45 %, determine:

(a) The volume V.
(b) The percentage uncertainty of d.
(c) The uncertainty of volume by the derivative method.
(d) The uncertainty of volume by the method of relative uncertainties.

6.6.7 The expression determines the frequency of a circuit:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where L is the inductance and C capacitance. If the percentage uncertainty of L is known at 5 % and the percentage uncertainty of C at 20 %, determine the value of the percentage uncertainty of frequency f.

6.6.8 The free fall of a body obeys the equation:

$$y = \frac{gt^2}{2}$$

where g is the acceleration of local gravity, and y is the height of the fall. If $y = (1.000 \pm 0.001)$ m with $k = 2.43$ and 95.45 % and $t = (0.45 \pm 0.01)$ s with $k = 2.23$ and 95.45 %, calculate:

(a) Relative uncertainty of y.
(b) Relative uncertainty of t.
(c) The value of g.
(d) Gravity measurement uncertainty to 95.45 %.
(e) Could you neglect any source of uncertainty, or is it necessary to complete an uncertainty analysis?

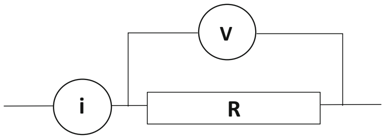


Fig. 6.5 Power Dissipated by a resistor

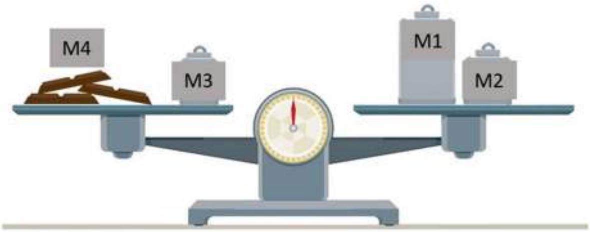


Fig. 6.6 Mass measurement. (Made by the authors)

6.6.9 Knowing that the electrical power (P) dissipated by a resistor can be calculated by the following expressions (Fig. 6.5).

(a) $P = V.I.$
(b) $P = R.I^2.$
(c) $P = V^2/R.$

Evaluate the best way to measure power P on resistor R, which has the lowest measurement uncertainty.

Data:

$R = (10.0 \pm 0.1)\ \Omega$, $k = 2.43$ and 95.45 %
$I = (10.0 \pm 0.1)$ A, $k = 2.23$ and 95.45 %
$V = (100 \pm 1)$ V, $k = 2.21$ and 95.45 %

6.6.10 A chemist measured the mass (M_4) of a product using the following scale (Fig. 6.6):

Data:

$M_1 = (128.0 \pm 0.2)$ g
$M_2 = (56.4 \pm 0.4)$ g
$M_3 = (39.7 \pm 0.7)$ g

Considering the scale in equilibrium, the uncertainties declared with $k = 2.00$ and 95.45 %, calculate the value of mass $M4$ and its measurement uncertainty.

6.6.11 To calculate a car's consumption ($C =$ km/L) on a trip, the car tank was filled, and the odometer was zeroed. In a particular stretch of the route, the car was

Table 6.3 Blocks composition

Number of blocks	Composition
112	One block: 1.0005 mm Nine blocks: (1.001 to 1.009) mm (step 0.001 mm) 49 blocks: (1.01 to 1.49) mm (step 0.01 mm) 49 blocks: (0.5 to 24.5) mm (step 0.5 mm) Four blocks: (25 to 100) mm (step 25 mm)

Table 6.4 Blocks uncertainty

Block size (mm)	Uncertainty (μm)
≤ 10	0.20
>10 and ≤ 25	0.30
>25 and ≤ 50	0.40
>50 and ≤ 75	0.50
>75 and ≤ 100	0.60

replenished with (38.0 ± 0.2) L of gasoline, filling its tank. Arriving at the final destination, the automobile was again replenished with (42.8 ± 0.1) L, completing its tank. The total distance traveled indicated by the odometer was (834.5 ± 2.5) km. What was fuel consumption by km/L and its measurement uncertainty?

Adopt $k = 2.00$ and 95.45 % for declared uncertainties.

6.6.12 We have a set of standard blocks with the following characteristics (Table 6.3):

Let us consider the uncertainty for the blocks, as shown in Table 6.4. All with $k = 2.00$ and 95.45 %:

We need to calibrate a micrometer at point 72.467 mm.

(a) Which blocks should be used as standard in this calibration to obtain the lowest measurement uncertainty?
(b) What is the value of this uncertainty?

6.6.13 A standard resistor, with a nominal value of 100, was measured by checking the voltage (V) and the electric current (i) that passed through it. A voltmeter and a calibrated ammeter were used for this measurement. The voltage and current measurement sequence results, the mathematical model that defines the measurement, and information related to the equipment used to measure the resistor are described in Table 6.5.

The mathematical model that defines the measurand (R): $\overline{R} = \frac{\overline{V}}{\overline{i}}$

$\overline{V}$ is the mean voltage, and $\overline{i}$ is the mean electric current.

Metrological characteristics of the voltmeter and ammeter used in the measurement (Table 6.6).

Based on this information, answer the following:

Table 6.5 Voltage and current measurements

Measurement	Voltage (V)	Electric current (A)
1	199.9	1.99
2	200.2	2.02
3	200.1	1.98
4	199.9	1.99
5	199.9	1.99
6	200.0	2.00
7	200.0	2.00
8	199.9	1.99
9	200.0	1.99

Table 6.6 Voltmeter and ammeter characteristics

Voltmeter		Ammeter	
Bias (V)	+0.1	Bias (A)	−0.04
Uncertainty (V) ($k = 2.00$ and 95.45 %)	0.2	Uncertainty (A) ($k = 2.00$ and 95.45 %)	0.02

(a) The value of the resistor
(b) The resistor bias
(c) The correction of the value of the resistor
(d) The expanded uncertainty

Chapter 7
Industrial Calibration

7.1 Calibration Concept

According to VIM—2.39, calibration differs from adjustment and verification. See the calibration definition:

Operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication.

NOTE 1 A calibration may be expressed by a statement, calibration function, calibration diagram, calibration curve, or calibration table. In some cases, it may consist of an additive or multiplicative correction of the indication with associated measurement uncertainty.

NOTE 2 Calibration should not be confused with adjustment of a measuring system, often mistakenly called "self-calibration," nor with verification of calibration.

NOTE 3 Often, the first step alone in the above definition is perceived as being calibration.

Note that, according to VIM, calibrating is confronting the metrological behavior of a measuring instrument with a reference standard, which can be a standard measurement instrument, a standard measurement system, a materialized measure, or a certified reference material.

Common sense considers calibrating how to fix equipment. For this reason, many professionals misunderstand that it is unnecessary to calibrate new equipment by thinking it is in perfect condition. The equipment may be in perfect condition, but we do not know its metrological characteristics, such as measurement error, measurement uncertainty, instrumental bias, and hysteresis.

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_7

7.2 Calibration × Verification

VIM—2.44 defines verification as the "*provision of objective evidence that a given item fulfills specified requirements.*"

These specified requirements can be:

- The manufacturer's specifications;
- The measurement instrument hysteresis or linearity;
- The instrument measuring error compared to its specification or technical standard.

Example 7.1 Maximum Permissible Error for Resistance Thermometer
The maximum permissible error for class A and class B resistance thermometers, according to DIN-IEC 751/85 standard, is worth:

Class A: $(0.15 + 0.002\ T)$ °C.
Class B: $(0.30 + 0.005\ T)$ °C.

Where T is the value of the measurement temperature.

Example 7.2 Maximum Permissible Error for Materialized Measure
The maximum permissible error for a materialized measure, for example, a standard mass, depending on its class, according to the International Recommendation OIML R-111-1, is presented in Table 7.1.

Attention!
Do not confuse verification with calibration. In calibration, the measuring uncertainty of the object must be determined, but this is not necessary for verification. A calibration may cover a verification, but the opposite is not valid. In this sense, a calibration becomes a more complex procedure than a verification.

7.3 Measurement Standard

According to VIM—5.1, the measurement standard is the realization of the definition of a given quantity, with a *stated quantity value and associated measurement uncertainty, used as a reference.*

EXAMPLE 1 1 kg mass measurement standard with an associated standard measurement uncertainty of 3 μg.

EXAMPLE 2 100 Ω measurement standard resistor with an associated standard measurement uncertainty of 1 μΩ.

EXAMPLE 3 Caesium frequency standard with a relative standard measurement uncertainty of 2×10^{-15}.

Table 7.1 Maximum permissible error for standard masses in mg

Nominal value	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
5000 kg			25,000	80,000	250,000	500,000	800,000	1,600,000	2,500,000
2000 kg			10,000	30,000	100,000	200,000	300,000	600,000	1,000,000
1000 kg		1600	5000	16,000	50,000	100,000	160,000	300,000	500,000
500 kg		800	2500	8000	25,000	50,000	80,000	160,000	250,000
200 kg		300	1000	3000	10,000	20,000	30,000	60,000	100,000
100 kg		160	500	1600	5000	10,000	16,000	30,000	50,000
50 kg	25	80	250	800	2500	5000	8000	16,000	25,000
20 kg	10	30	100	300	1000		3000		10,000
10 kg	5.0	16	50	160	500		1600		5000
5 kg	2.5	8.0	25	80	250		800		2500
2 kg	1.0	3.0	10	30	100		300		1000
1 kg	0.5	1.6	5	16	50		160		500
500 g	0.25	0.8	2.5	8.0	25		80		250
200 g	0.10	0.30	1.0	3.0	10		30		100
100 g	0.05	0.16	0.5	1.6	5		16		50
50 g	0.030	0.10	0.30	1.0	3.0		10		30
20 g	0.025	0.080	0.25	0.8	2.5		8		25
10 g	0.020	0.060	0.20	0.6	2		6		20
5 g	0.016	0.050	0.16	0.5	1.6		5		16
2 g	0.012	0.040	0.12	0.4	1.2		4		12
1 g	0.010	0.030	0.10	0.3	1.0		3		10
500 mg	0.008	0.025	0.08	0.25	0.8		2.5		
200 mg	0.006	0.020	0.06	0.20	0.6		2.0		
100 mg	0.005	0.016	0.05	0.16	0.5		1.6		
50 mg	0.004	0.012	0.04	0.12	0.4				

(continued)

Table 7.1 (continued)

Nominal value	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
20 mg	0.003	0.008	0.03	0.10	0.3				
10 mg	0.003	0.008	0.025	0.08	0.25				
5 mg	0.003	0.006	0.020	0.06	0.20				
2 mg	0.003	0.006	0.020	0.06	0.20				
1 mg	0.003	0.006	0.020	0.06	0.20				

EXAMPLE 4 Standard buffer solution with a pH of 7.072 with an associated standard measurement uncertainty of 0.006.

EXAMPLE 5 Set of reference solutions of cortisol in human serum having a certified quantity value with measurement uncertainty for each solution.

EXAMPLE 6: Reference material providing quantity values with measurement uncertainties for the mass concentration of each of ten different proteins.

NOTE 1 A "realization of the definition of a given quantity" can be provided by a measuring system, a material measure, or a reference material.

NOTE 2 A measurement standard is frequently used as a reference in establishing measured quantity values and associated measurement uncertainties for other quantities of the same kind, thereby establishing metrological traceability through calibration of other measurement standards, measuring instruments, or measuring systems.

NOTE 3 The term "realization" is used here in the most general meaning. It denotes three procedures of "realization." The first one consists in the physical realization of the measurement unit from its definition and is realization sensu stricto. The second, termed "reproduction," consists not in realizing the measurement unit from its definition but in setting up a highly reproducible measurement standard based on a physical phenomenon, as it happens, e.g., in the case of the use of frequency-stabilized lasers to establish a measurement standard for the metre, of the Josephson effect for the volt or of the quantum Hall effect for the ohm. The third procedure consists of adopting a material measure as a measurement standard. It occurs in the case of the measurement standard of 1 kg.

NOTE 4 A standard measurement uncertainty associated with a measurement standard is always a component of the combined standard measurement uncertainty (see GUM:1995, 2.3.4) in a measurement result obtained using the measurement standard.

Frequently, this component is small compared with other components of the combined standard measurement uncertainty.

NOTE 5 Quantity value and measurement uncertainty must be determined at the time when the measurement standard is used.

NOTE 6 Several quantities of the same kind or different kinds may be realized in one device which is commonly also called a measurement standard.

NOTE 7 The word "embodiment" is sometimes used in the English language instead of "realization."

NOTE 8 In science and technology, the English word "standard" is used with at least two different meanings: as a specification, technical recommendation, or similar normative document (in French "norme") and as a measurement standard (in French "étalon"). This Vocabulary is concerned solely with the second meaning.

NOTE 9 The term "measurement standard" is sometimes used to denote other metrological tools, e.g., 'software measurement standard' (see ISO 5436-2).

A measurement standard is presented as a system, a materialized measure, or a certified reference material. Regardless of how they present themselves,

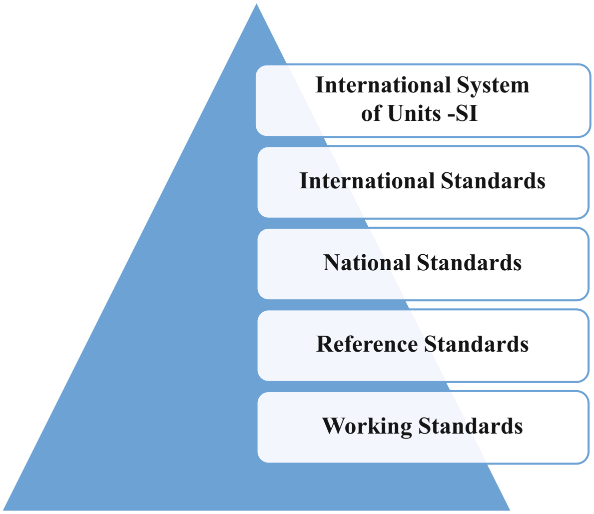


Fig. 7.1 Hierarchy of measurement standards

measurement standards often have a small measurement uncertainty, contributing little or almost nothing to the uncertainty after a calibration process. Their measurement uncertainty should be combined with the other measurement uncertainties involved in the calibration process.

A measurement standard serves as a reference for calibrating other hierarchically inferior standards in terms of accuracy and measurement uncertainty. Figure 7.1 presents the various types of measurement standards in the decreasing sense of their metrological hierarchy.

The International System of Units (SI) was presented and discussed in Chap. 1.

7.3.1 International Measurement Standard

In the VIM—5.10, the definition of an **international measurement standard** is a *measurement standard recognized by signatories to an international agreement and intended to serve worldwide.*

EXAMPLE 1 The international prototype of the kilogram. (Authors note: At the time of the 3rd edition of VIM, 2012, this was still the definition of the international mass standard—kilogram. From 2019, the new definition is: "It is defined by taking the fixed numerical value of the Planck constant h to be $6.626070\ 15 \times 10^{-34}$ when expressed in the unit $J\ s$, which is equal to $kg\ m^2\ s^{-1}$, where the meter and the second are defined in terms of c and $\Delta\nu_{Cs}$.")

EXAMPLE 2 Chorionic gonadotrophin, World Health Organization (WHO) 4th international standard 1999, 75/589, 650 International Units per ampoule.

EXAMPLE 3 VSMOW2 (Vienna Standard Mean Ocean Water) distributed by the International Atomic Energy Agency (IAEA) for differential stable isotope amount-of-substance ratio measurements.

Knowing a Little More ...
The standard kilogram

King Louis XVI of France summoned a group of sages to elaborate a new measurement system, establishing the foundations for the "decimal metric system," which evolved into modern SI. The original idea of the King's commission (which included remarkable, like Lavoisier) was to create a mass unit that, by definition, would be the mass of a liter of water at the ice point (i.e., essentially 1 kg). The definition should be incorporated into a standard mass prototype. Given that the masses being measured at that time were much lower than the kilo, they decided that the unit of mass would be the "gram." However, since a gram standard is difficult to use as well as to handle (too small), the new definition should be incorporated into a prototype of a kilogram. The republican government's decision probably had political motivation, after all, these same people condemned Lavoisier to the guillotine. Anyway, it remains to regret that a base unit has a "prefix" in the name. The kilogram prototype has been conserved in the BIPM since 1889, when it was sanctioned by the First General Conference on Weights and Measures. It is cylindrically, with a diameter and height of about 39 mm, and made of a 90% platinum alloy and 10% iridium. In November 2018, at the General Conference on Weights and Measures, the highest revision of the International Unit System (SI) was made since 1960. In this conference, four basic units of measurement were redefined: kilogram, ampere, kelvin, and mol. The purpose of the change was to relate these units to fundamental, not arbitrary constants, as has been done so far.

7.3.2 National Measurement Standard

In the VIM—5.3, the definition of a **national measurement standard** is a *measurement standard recognized by national authority to serve in a state or economy as the basis for assigning quantity values to other measurement standards for the kind of quantity concerned.*

Therefore, these standards are devices maintained by organizations and national laboratories worldwide. They represent the fundamental and derived quantities and are calibrated independently through absolute measurements.

INMETRO is responsible for maintaining national standards in Brazil both for those existing in their laboratories and those in the designated laboratories.

7.3.3 Reference Measurement Standard

In VIM—5.6, a **reference measurement standard** is a *measurement standard designated for the calibration of other measurement standards for quantities of a given kind in a given organization or at a given location.*

These standards should not be employed for daily measurement work and, preferably, have to be kept under specific conditions of temperature and humidity. They are used for the calibration or verification of working standards.

7.3.4 Working Measurement Standard

The definition of a **working measurement standard** in VIM—5.7 is a standard that is *routinely used to calibrate or verify measuring instruments or systems.*

NOTE 1 A working measurement standard is usually calibrated with respect to a reference measurement standard.

NOTE 2 In relation to verification, the terms "check standard" or "control standard" are also sometimes used.

7.4 Certified Reference Material (CRM)

Certified reference material is a *reference material, accompanied by documentation issued by an authoritative body and providing one or more specified property values with associated uncertainties and traceabilities, using valid procedures.*

EXAMPLE Human serum with assigned quantity value for the concentration of cholesterol and associated measurement uncertainty stated in an accompanying certificate, used as a calibrator or measurement trueness control material.

NOTE 1 'Documentation' is given in the form of a 'certificate' (see ISO Guide 31: 2000).

NOTE 2 Procedures for the production and certification of certified reference materials are given, e.g.*, in ISO Guide 34* (actually ISO 17034) *and ISO Guide 35.*

NOTE 3 In this definition, "uncertainty" covers both 'measurement uncertainty' and 'uncertainty associated with the value of a nominal property,' such as for identity and sequence. "Traceability" covers both 'metrological traceability of a quantity value' and 'traceability of a nominal property value.'

NOTE 4 Specified quantity values of certified reference materials require metrological traceability with associated measurement uncertainty.

NOTE 5 ISO/REMCO has an analogous definition, but uses the modifiers "metrological" and "metrologically" to refer to both quantity and nominal property. [VIM—5.14].

A reference material can be a pure substance or a gas, liquid, or solid mixture. Examples of reference materials are water used in viscometer calibration, sapphire used in calorimetry heat capacity calibration, and solutions used in chemical analysis.

A certificate of analysis with one or more property or physical characteristic values always accompanies a certified reference material. These materials are certified by procedures that establish the traceability for the exact obtaining of the unit in which the property values are expressed. Each certified value is accompanied by uncertainty to an established confidence level.

The following text, related to the preparation of a CRM, was obtained from the website <https://en.wikipedia.org/wiki/Certified_reference_materials>.

Sample preparation.

Detailed sample preparation depends on the type of material. Pure standards are most likely prepared by chemical synthesis and purification and characterized by determining remaining impurities.

Natural matrix CRMs contain an analyte or analytes in a natural sample (for example, lead in fish tissue). They are typically produced by homogenizing a naturally occurring material and measuring each analyte. Due to the difficulty in production and value assignment, they are usually made by national or transnational metrology institutes like NIST (USA), BAM (Germany), KRISS (Korea), and EC JRC (European Commission Joint Research Centre).

Natural materials are rarely homogeneous on the scale of grams so production of a solid natural matrix reference material typically involves processing to a fine powder or paste. Homogenization can have adverse effects, for example on proteins, so producers must take care not to over-process materials.

The stability of a certified reference material is also essential, so a range of strategies may be used to prepare a reference material that is more stable than the natural material it is prepared from. For example, stabilizing agents such as antioxidants or antimicrobial agents may be added to prevent degradation, liquids containing certified concentrations of trace metals may have their pH adjusted to keep metals in solution, and clinical reference materials may be freeze-dried for long-term storage if they can be reconstituted successfully.

7.5 Selection of the Measurement Sstandard

For the value of a measurement standard or standard measurement system to be accepted as a reference value, its measurement accuracy and uncertainty must be lower than those of the measurement system to be calibrated. Therefore, it is possible to imagine that the smaller your measurement uncertainty, technically, the better the standard, but the more expensive it will be as well.

We must always seek a technical and economic balance, keeping in mind that the final uncertainty (U_F) will be the combination of the uncertainty of the measurement system that we want to calibrate (U_{CMS}) with the uncertainty of the standard measurement system (U_{SMS}), that is,

$$\boldsymbol{U_F = \sqrt{U_{CMS}^2 + U_{SMS}^2}} \tag{7.1}$$

The lower the uncertainty of the *SMS* compared to *CMS*, the lower its influence on the final result. Let us evaluate U_F for some U_{SMS} values compared to U_{CMS}. If *SMS* uncertainty were infinitely inferior to *CMS* uncertainty, we would have the uncertainty U_F equal to *CMS* uncertainty; the influence of *SMS* measurement uncertainty would tend to be zero. Let us look at the following simulations:

(a) Considering $U_{SMS} = U_{CMS}$, we have:

$$\boldsymbol{U_F = \sqrt{U_{CMS}^2 + U_{SMS}^2}} = \sqrt{2.U_{CMS}^2} = 1.41\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 41% of the final uncertainty, U_F.

(b) Considering $U_{\mathrm{SMS}} = 1/2\ U_{\mathrm{CMS}}$, we have:

$$\boldsymbol{U_F = \sqrt{U_{CMS}^2 + U_{SMS}^2}} = \sqrt{(1 + 1/4)\ U_{CMS}^2} = 1.12\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 12% of the final uncertainty, U_F.

(c) Considering $U_{\mathrm{SMS}} = 1/3\ U_{\mathrm{CMS}}$, we have:

$$\boldsymbol{U_F = \sqrt{U_{CMS}^2 + U_{SMS}^2}} = \sqrt{(1 + 1/9)\ U_{CMS}^2} = 1.054\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 5.4% of the final uncertainty, U_F.

(d) Considering $U_{\mathrm{SMS}} = 1/4\ U_{\mathrm{CMS}}$, we have:

$$\boldsymbol{U_F = \sqrt{U_{CMS}^2 + U_{SMS}^2}} = \sqrt{(1 + 1/16)\ U_{CMS}^2} = 1.032\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 3.2% of the final uncertainty, U_F.

(e) Considering $U_{\mathrm{SMS}} = 1/5\ U_{\mathrm{CMS}}$, we have:

$$\boldsymbol{U_F} = \sqrt{\boldsymbol{U_{CMS}^2} + \boldsymbol{U_{SMS}^2}} = \sqrt{(1 + 1/25)\ U_{CMS}^2} = 1.02\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 2% of the final uncertainty, U_F.

(f) Considering $U_{\mathrm{SMS}} = 1/10\ U_{\mathrm{CMS}}$, we have:

$$\boldsymbol{U_F} = \sqrt{\boldsymbol{U_{CMS}^2} + \boldsymbol{U_{SMS}^2}} = \sqrt{(1 + 1/100)\ U_{CMS}^2} = 1.005\ U_{CMS}$$

We observe that the influence of *SMS* uncertainty is approximately 0.5 % of the final uncertainty, U_F.

Suppose we adopt a standard with measurement uncertainty equal to or less than one-tenth of the expected uncertainty for the *CMS*. In that case, *SMS* will not contribute to the uncertainty, with *SMS* passing unnoticed by the *CMS*.

Important
In practice, if we adopt *SMS* with measurement uncertainty equal to or less than ¼ of the *CMS*, we will have an excellent condition.

7.6 Solved Exercises of Measurement Instrument Calibration

We can use a technical standard or even an orientation guide whenever we want to calibrate a measurement instrument. For example, if we wish to calibrate thermocouples, we can use the guide *Calibration of Thermocouples—EURAMET cg-8*.

If we wish to calibrate manometers, we can use the *Guidelines on the Calibration of Electromechanical and Mechanical Manometers—EURAMET Calibration Guide No. 17*.

Thus, we are led to believe that the entire calibration process, including the choice of standards, the assembly of the experiment, and the calculation of the measurement uncertainty, will be found in all technical standards. Deceit! The part of the measurement uncertainty calculation is not usually provided in the technical calibration standards, so we are at a "dead end."

Important
Calibration technical standards usually do not contain information on calculating measurement uncertainty. Therefore, you are responsible for performing them!

To solve this problem, interested parties must take courses in measurement and calibration uncertainty in their field of interest (temperature, pressure, electricity, etc.).

Here are some examples of measuring uncertainty calculation.

Solved Exercise 7.1: Glass Liquid Thermometer (GLT) Calibration.

A GLT, with 0.5 °C resolution, is calibrated against a 0.1 °C resolution standard. The calibration bath has ±0.04 °C stability.[1] The standard thermometer calibration certificate is shown in Fig. 7.2. Determine the thermometer uncertainty and its bias to 20 °C, 40 °C, and 100 °C (Table 7.2).

Solution:
I. Point 20 °C

(a) Standard and object mean value

$$\overline{x}_{std} = 20.0\,^{\circ}\mathrm{C} \rightarrow bias\ correction \rightarrow \overline{x}_{std} = (20.0 - 0.2)\,^{\circ}\mathrm{C} = 19.8\,^{\circ}\mathrm{C}$$

Note that the standard indication has been corrected because, as seen in the certificate, it has a bias of 0.2 °C at point 20 °C.

$$\overline{x}_{obj} = 20.5\,^{\circ}\mathrm{C}$$

(b) Object bias

$$B = (20.5 - 19.8)\,^{\circ}C = 0.7\,^{\circ}\mathrm{C}$$

(c) Type A uncertainty—object repeatability

$$u_{A-obj} = \frac{s}{\sqrt{n}} = 0$$

(d) Standard uncertainty from the calibration certificate.

This data is extracted from the standard thermometer calibration certificate. As the certificate always informs the expanded measurement uncertainty (95.45 %) and

[1] Stability is defined as fluctuation of calibration bath temperature after reaching the thermal equilibrium.

PP&AM
Calibration Laboratory

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CALIBRATION CERTIFICATE N^{er} 1234 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: High Temperature	Class: NA
Description: Glass thermometer	Resolution: 0.1 °C
Model: Partial immersion	Range: (0 – 100) °C
Serial number: 123321123321	

Method and procedure used

Calibration made by direct comparison, as described in the SOP 001 procedure - Standard Operating Procedure for glass thermometers.

Traceability

Description	TAG	Model	Manufacturer	Certificate	Serial
Standard thermometer	Pt – 107	Pt-100 Four wires	Ohms	107/24	ABC123

Calibration results

Indication °C	Standard °C	Object °C	Bias °C	Uncertainty °C	*k*	Degree of freedom
0	0.00	0,1	0,1	0.2	2.37	8
10	10.00	10.0	0.0	0.2	2.05	47
20	20.00	20.2	0.2	0.2	2.00	Infinite
30	30.00	30.0	0.0	0.3	2.05	47
40	40.00	40.0	0.0	0.3	2.02	102
50	50.00	50.1	0.1	0.3	2.11	23
60	60.00	60.1	0.1	0.4	2.06	40
70	70.00	70.2	0.2	0.5	2.07	35
80	80.00	80.0	0.0	0.5	2.06	40
90	90.00	90.1	0.1	0.5	2.02	102
100	100.00	100.2	0.2	0.6	2.00	Infinite

Environmental data	**Temperature** °C	(20.6 $\pm$ 0.5)	**Humidity** %	(56 $\pm$ 5)	**Pressure** hPa	(1 018 $\pm$ 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other, even if it is similar. The partial reproduction of this document is not allowed. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%.

Calibration date: 3/6/2024
Emission date: 3/6/2024

Galileo Galilei
Metrologist technician

Lord Kelvin
Authorized firmer

Page 1/1

Fig. 7.2 Standard thermometer calibration certificate

we need to combine it with the other uncertainties in standard form (a standard deviation), we must divide it by the *k* factor, informed in the instrument's calibration certificate standard.

Table 7.2 Result of GLT calibration

Nominal value (°C)	Standard (°C)	Object (°C)
20	20.0	20.5
	20.0	20.5
	20.0	20.5
	20.0	20.5
40	40.1	40.5
	40.1	40.5
	40.1	40.5
	40.1	40.5
100	99.8	100.5
	99.8	100.5
	99.8	100.5
	99.8	100.0

$$u_{\text{certif}} = \frac{0.2}{2} = 0.1\,^\circ\text{C}$$

(e) Bath stability uncertainty.

After stabilization, the variation in bath temperature follows a uniform probability distribution. In this case, as stability is provided as ±0.04 °C, the interval of bathing temperature variation has already been divided by two, and thus, applying the uniform distribution, divide by $\sqrt{3}$.

$$u_{\text{stability}} = \frac{0.04}{\sqrt{3}} = 0.0231\,^\circ\text{C}$$

(f) GLT resolution uncertainty.

We will adopt a uniform probability distribution, since the probability of finding a reading value varies evenly.

$$u_{\text{GLT res}} = \frac{0.5}{\sqrt{12}} = 0.14434\,^\circ\text{C}$$

(g) Standard resolution uncertainty.

We will adopt a uniform probability distribution, since the probability of finding a reading value varies evenly.

$$u_{\text{std res}} = \frac{0.1}{\sqrt{12}} = 0.0289\,^\circ\text{C}$$

Table 7.3 Metrological characteristics

Metrological characteristics	(°C)
Object mean	40.5
Corrected standard mean	40.1
Bias	0.4
Type A uncertainty—Repeatability	0.0
Standard uncertainty from certificate	0.1485
Stability uncertainty	0.0231
GLT resolution uncertainty	0.1443376
Standard resolution	0.0289
Combined uncertainty	0.2104
Effective degree of freedom	120
Coverage factor, k	2.02
Expanded uncertainty, 95.45 %	0.4

(h) Combined uncertainty

$$u_c = \sqrt{u_A^2 + u_{cert}^2 + u_{stab}^2 + u_{GLT\ res}^2 + u_{std\ res}^2} = 0.1795\,^\circ\mathrm{C}$$

(i) Effective degree of freedom

$$v_{eff} = \frac{u_c^4}{\frac{u_A^4}{4-1} + \frac{u_{cert}^4}{v} + \frac{u_{stab}^4}{\infty} + \frac{u_{GLT\ res}^4}{\infty} + \frac{u_{std\ res}^4}{\infty}} = \infty$$

(j) Coverage factor, k

$$v_{eff} = \infty \quad \rightarrow \quad k = 2.00$$

(k) Expanded uncertainty 95.45%

$$U = k.u_c = 0.359\,^\circ\mathrm{C}$$

We must round the expanded uncertainty to a decimal digit, since the object's GLT has a resolution of 0.5 °C. Thus, the result will be: $U = 0.4$ °C.

For the other calibration points, the calculation methodology is the same. Let us present only the tables with the final results.

II. Point 40 °C (Table 7.3)

III. Point 100 °C (Tables 7.4 and 7.5)

Solved Exercise 7.2: Bourdon Gauge Calibration

Table 7.4 Metrological characteristics

Metrological characteristics	(°C)
Object mean	100.5[a]
Corrected standard mean	99.6
Bias	0.9
Type A uncertainty—Repeatability	0.125
Standard uncertainty from certificate	0.3
Stability uncertainty	0.0231
GLT resolution uncertainty	0.1443376
Standard resolution	0.0289
Combined uncertainty	0.3575
Effective degree of freedom	168
Coverage factor, k	2.01
Expanded uncertainty, 95.45 %	0.7

[a] Since the GLT object only reads from 0.5 to 0.5 °C, we should round the result to 100.5 °C.

Table 7.5 Result of GLT calibration

Standard (°C)	Object (°C)	Bias (°C)	Uncertainty (°C)	k	υ
19.8	20.5	+ 0.7	0.4	2.00	∞
40.1	40.5	+ 0.4	0.4	2.02	120
99.6	100.5	+ 0.9	0.7	2.01	168

Table 7.6 Bourdon gauge calibration result

Object (kgf/cm^2)	Standard (kgf/cm^2)			
	Charge 1	Discharge 1	Charge 2	Discharge 2
5.0	5.50	5.50	5.50	5.25
15.0	16.25	15.75	15.50	15.50
25.0	26.00	25.50	25.50	26.00
35.0	36.25	36.00	35.50	36.00
40.0	41.00	41.00	41.00	41.00

A Bourdon gauge (object), class 2.5 %, with a measurement range 0 to 40 kgf/cm^2, was calibrated against a standard gauge class 0.6 %. Consider the resolution of the calibration gauge of 0.5 kgf/cm^2. The standard gauge resolution is 0.05 kgf/cm^2 (Table 7.6).

Calibration Instructions The pressure value on the object gauge is fixed, and the reading of charge and discharge pressure is performed on the standard gauge (see the standard certificate in Fig. 7.3) for hysteresis, error, and bias.

Solution Before we start the uncertainty calculations of this exercise, we will correct the standard gauge measurement values (Table 7.4). Correction implies eliminating the error or measurement bias at each point measured by the standard

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CALIBRATION CERTIFICATE N^{er} 4321 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: Bourdon Pressure Co.	Class: 0.6
Description: analog pressure gauge	Resolution: 0.05 kgf/cm²
Model: BouP 12	Range: (0 – 60) kgf/cm²
Serial number: 123321123321	

Method and procedure used

Calibration made by direct comparison, as described in the POE 002 procedure - Standard Operating Procedure for Pressure Gauge.

Traceability

Description	**TAG**	**Model**	**Manufacturer**	**Certificate**	**Serial**
Thermohygrometer	TH – 10	Digital	THMetro	207/24	ABC123
Barometer	BA-20	Analog	BA	215/24	XYZ00
Standard pressure gauge	MP-99	Digital	MPM	789/24	GFD873

Calibration results

Indication kPa	Indication kgf/cm²	Standard kgf/cm²	Object kgf/cm²	Bias kgf/cm²	Uncertainty kgf/cm²	*K*	Degree of freedom
0	0	0.000	0.00	0.00	0.05	2.00	Infinite
588	6	6.100	6.00	-0.10	0.05	2.00	Infinite
1 177	12	12.000	12.00	0.00	0.05	2.00	Infinite
2 354	24	24.005	24.00	0.00	0.06	2.10	27
2 942	30	30.005	30.00	0.00	0.06	2.10	27
3 530	36	36.100	36.00	-0.10	0.06	2.15	18
4 119	42	42.100	42.00	-0.10	0.07	2.15	18
4 707	48	48.150	48.00	-0.15	0.07	2.15	18
5296	54	54.250	54.00	-0.25	0.08	2.20	14
5 884	60	59.700	60.00	0.30	0.08	2.20	102

Environmental data	**Temperature** °C	(20.6 $\pm$ 0.5)	**Humidity** %	(56 $\pm$ 5)	**Pressure** hPa	(1 018 $\pm$ 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other, even if it is similar. The partial reproduction of this document is not allowed. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%.

Calibration date: 6/3/2024
Emission date: 6/3/2024

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Metrologist technician

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Fig. 7.3 Standard Bourdon gauge calibration certificate

Table 7.7 Corrected values of the standard

Object (kgf/cm^2)	Standard (kgf/cm^2)			
	Charge 1	Discharge 1	Charge 2	Discharge 2
5.0	5.60	5.60	5.60	5.35
15.0	16.25	15.75	15.50	15.50
25.0	26.00	25.50	25.50	26.00
35.0	36.35	36.10	35.60	36.10
40.0	41.10	41.10	41.10	41.10

instrument. To do so, consult the error or bias in the standard instrument calibration certificate (Fig. 7.3) (Table 7.7).

Point 5 kgf/cm^2

(a) Measurement error.

$$E = X - Vr = 5.0 - 5.60 = -0.6 \text{ kgf/cm}^2$$

To determine the gauge measurement error, we must subtract from the value read by the calibration gauge (object) the standard's value farther from the object's value. That is, the standard value will generate the most significant measurement error.

(b) Type A uncertainty

$$u_A = \frac{s}{\sqrt{n}} = \frac{0.125}{\sqrt{4}} = 0.0625 \text{ kgf/cm}^2$$

Note that once we fix the value in the object, the standard will feel its repeatability.

(c) Uncertainty from certificate

$$u_{\text{cert}} = 0.05/2 = 0.025 \text{ kgf/cm}^2$$

(d) Hysteresis uncertainty

$$u_{hys} = \frac{H}{\sqrt{12}} = \frac{5.60 - 5.35}{\sqrt{12}} = 0.0722 \text{ kgf/cm}^2$$

(e) Object resolution uncertainty.

We will adopt a triangular probability distribution, since the probability of finding a reading value at the center point of the distribution is greater than at the ends.

$$u_{obj\ res} = \frac{0.5}{\sqrt{24}} = 0.10206\ \text{kgf/cm}^2$$

(f) Standard resolution uncertainty

$$u_{std\ res} = \frac{0.05}{\sqrt{12}} = 0.0144\ \text{kgf/cm}^2$$

(g) Combined uncertainty

$$u_c = \sqrt{0.0625^2 + 0.025^2 + 0.0722^2 + 0.1026^2 + 0.0144^2} = 0.143\ \text{kgf/cm}^2$$

(h) Effective degree of freedom

$$v_{eff} = \frac{u_c^4}{\frac{u_A^4}{4-1} + \frac{u_{cert}^4}{v} + \frac{u_{hyst}^4}{\infty} + \frac{u_{obj\ res}^4}{\infty} + \frac{u_{std\ res}^4}{\infty}} = 82$$

(i) Coverage factor, k

$$\text{Excel}^{\text{®}}: \text{INV.T.BC}\ (0.0455; 82) \ \rightarrow \ k = 2.03$$

(j) Expanded uncertainty

$$U = 2.03 \times 0.143 = 0.3\ \text{kgf/cm}^2$$

II. Point 15 kgf/cm² (Table 7.8).

III. Point 25 kgf/cm² (Table 7.9)

IV. Point 35 kgf/cm² (Table 7.10)

V. Point 40 kgf/cm² (Table 7.11)

Object accuracy class: **2.5 %** (Table 7.12)

At points 15, 25, and 40, the object error exceeded the class limit error (2.5 %). This shows that this gauge needs to be adjusted (Table 7.13).

Table 7.8 Metrological characteristics

Metrological characteristics	(kgf/cm^2)
Object	15.0
Error	−1.2
Type A uncertainty—Repeatability	0.1768
Standard uncertainty from certificate	0.0286[a]
Hysteresis uncertainty	0.14434
Object resolution uncertainty	0.10206
Standard resolution	0.0144
Combined uncertainty	0.2523
Effective degree of freedom	12
Coverage factor, *k*	2.23
Expanded uncertainty, 95.45 %	0.6

[a]As the point is between (12 and 24) kgf/cm^2 and the respective uncertainties between (0.05 and 0.06) kgf/cm^2, we must adopt the more significant measurement uncertainty, in this case, 0.06 kgf/cm^2. The reason is to make the most conservative decision possible, adopting the most significant uncertainty in the interval.

Table 7.9 Metrological characteristics

Metrological characteristics	(kgf/cm^2)
Object	25.0
Error	−1.0
Type A uncertainty—Repeatability	0.14434
Standard uncertainty from certificate	0.0286
Hysteresis uncertainty	0.14434
Object resolution uncertainty	0.10206
Standard resolution	0.0144
Combined uncertainty	0.2307
Effective degree of freedom	19.6
Coverage factor, *k*	2.14
Expanded uncertainty, 95.45 %	0.5

Table 7.10 Metrological characteristics

Metrological characteristics	(kgf/cm^2)
Object	35.0
Error	−1.4
Type A uncertainty—Repeatability	0.1573
Standard uncertainty from certificate	0.0279
Hysteresis uncertainty	0.14434
Object resolution uncertainty	0.10206
Standard resolution	0.0144
Combined uncertainty	0.2389
Effective degree of freedom	16
Coverage factor, *k*	2.19
Expanded uncertainty, 95.45 %	0.5

Table 7.11 Metrological characteristics

Metrological characteristics	(kgf/cm^2)
Object	40.0
Error	−1.1
Type A uncertainty—Repeatability	0.0
Standard uncertainty from certificate	0.03256
Hysteresis uncertainty	0.0
Object resolution uncertainty	0.10206
Standard resolution	0.0144
Combined uncertainty	0.1086
Effective degree of freedom	2200
Coverage factor, *k*	2.00
Expanded uncertainty, 95.45 %	0.2

Table 7.12 Error

Point (kgf/cm^2)	Error (kgf/cm^2)	Error (%)
5.0	−0.6	1.5
15.0	−1.2	3.0
25.0	−1.0	2.5
35.0	−1.4	3.2
40.0	−1.1	2.8

Table 7.13 Result of Bourdon gauge calibration

Object kgf/cm^2	Error kgf/cm^2	Uncertainty kgf/cm^2	k	υ
5.0	−0.6	0.3	2.03	82
15.0	−1.2	0.6	2.23	12
25.0	−1.0	0.5	2.14	19
35.0	−1.4	0.5	2.19	16
40.0	−1.1	0.2	2.00	2200

Solved Exercise 7.3: Digital Voltmeter Calibration

Calibration conditions:

- Object: digital voltmeter
- Resolution: 0.01 mV
- Range: (0 to 200) mV

Parasite uncertainty[2] $= \frac{2\mu V}{\sqrt{3}}$.

Calibration Instructions using a voltage source, fix the value in the voltmeter in calibration (object) and read the standard voltmeter (Table 7.14).

Before we start the uncertainty calculations of this exercise, we will correct the standard voltmeter measurement values Table 7.5). Correction implies eliminating the error or measurement bias at each point measured by the standard instrument. Therefore, consult the error or bias in the standard instrument calibration certificate (Fig. 7.4) (Table 7.15).

[2] Uncertainty from static electricity at voltmeter connection terminals.

Table 7.14 Calibration results

Object (mV)	Standard (mV)			
	1	2	3	4
40.00	40.110	40.150	40.160	40.120
80.00	80.120	80.160	80.140	80.130
120.00	120.150	120.170	120.190	120.190
160.00	160.230	160.180	160.170	160.180
200.00	200.210	200.230	200.260	200.270

Solution:

Point 40.00 mV

(a) Error

$$E = x - Vr = 40.00 - 40.160 = -0.16 \text{ mV}$$

(b) Type A uncertainty

$$u_A = \frac{s}{\sqrt{n}} = \frac{0.0238}{\sqrt{4}} = 0.0119 \text{ mV}$$

Note that once we fix the value in the object, the standard will feel its repeatability.

(c) Uncertainty from certificate

$$u_{\text{cert}} = 0.002/2 = 0.001 \ \text{ mV}$$

(d) Parasite uncertainty

$$u_{par} = \frac{2\mu V}{\sqrt{3}} = 0.0011547 \text{ mV}$$

(e) Object resolution

$$u_{obj\ res} = \frac{0.01}{\sqrt{12}} = 0.002887 \text{ mV}$$

We will adopt a uniform probability distribution even when setting the reading in the object, since the voltmeter is digital. The probability of finding a reading value varies evenly.

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CALIBRATION CERTIFICATE N^{er} 1324 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: Voltmeters Co.	Class: NA
Description: digital voltmeter	Resolution: 0.001 mV
Model: VTVM-1	Range: (0 – 200) mV
Serial number: 123321123321	

Method and procedure used

Calibration made by direct comparison, as described in the POE 003 procedure - Standard Operating Procedure for Voltmeters.

Traceability

Description	TAG	Model	Manufacturer	Certificate	Serial
Thermohygrometer	TH – 10	Digital	THMetro	207/24	ABC123
Barometer	BA-20	Analog	BA	215/24	XYZ00
Standard voltmeter	VP-99	Digital	VPM	089/24	FKL777

Calibration results

Indication mV	Standard mV	Object mV	Bias mV	Uncertainty mV	*k*	Degree of freedom
0	0.0000	0.000	0.000	0.002	2.00	Infinite
40	40.0005	40.001	0.000	0.002	2.00	Infinite
80	80.0000	80.003	0.003	0.002	2.00	Infinite
120	120.0005	120.005	0.004	0.002	2.00	Infinite
160	159.9995	160.005	0.005	0.002	2.00	Infinite
200	199.9995	200.005	0.005	0.002	2.00	Infinite
Environmental data	Temperature °C	(20.6 ± 0.5)	Humidity %	(56 ± 5)	Pressure hPa	(1 018 ± 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other, even if it is similar. The partial reproduction of this document is not allowed. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%.

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Emission date: 6/8/2024

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Fig. 7.4 Standard voltmeter calibration certificate

(f) Standard resolution

$$u_{std\ res} = \frac{0.001}{\sqrt{12}} = 0.0002887 \text{ mV}$$

(g) Combined uncertainty

Table 7.15 Corrected values of the standard

Object (mV)	Standard (mV)			
	1	2	3	4
40.00	40.110	40.150	40.160	40.120
80.00	80.120	80.160	80.140	80.130
120.00	120.147	120.167	120.187	120.187
160.00	160.226	160.176	160.166	160.176
200.00	200.205	200.225	200.255	200.265

Table 7.16 Metrological characteristics

Metrological characteristics	(mV)
Object	80.00
Error	−0.16
Type A uncertainty—Repeatability	0.0085
Standard uncertainty from certificate	0.001
Parasite uncertainty	0.0011547
Object resolution uncertainty	0.002887
Standard resolution	0.0002887
Combined uncertainty	0.009111
Effective degree of freedom	4
Coverage factor, k	3.31
Expanded uncertainty, 95.45 %	0.03

$$u_c = \sqrt{0.0119^2 + 0.001^2 + 0.0011547^2 + 0.002887^2 + 0.0002887^2} = 0.0123 \text{ mV}$$

(h) Effective degree of freedom

$$v_{eff} = \frac{u_c^4}{\frac{u_A^4}{4-1} + \frac{u_{cert}^4}{v} + \frac{u_{par}^4}{\infty} + \frac{u_{obj\ res}^4}{\infty} + \frac{u_{std\ res}^4}{\infty}} = 3.5$$

(i) Coverage factor, k

$$\text{Excel}^{\text{®}}: \ \text{INV.T.BC}(0.0455; 3.5) \ \rightarrow \ k = 3.31$$

(j) Expanded uncertainty

$$U = 3.31 \times 0.0123 = 0.04 \ \text{mV}$$

We will round the uncertainty for two decimal digits (object voltmeter resolution).

II. Point 80.00 mV (Table 7.16)

Table 7.17 Metrological characteristics

Metrological characteristics	(mV)
Object	120.00
Error	−0.19
Type A uncertainty—Repeatability	0.0096
Standard uncertainty from certificate	0.001
Parasite uncertainty	0.0011547
Object resolution uncertainty	0.002887
Standard resolution	0.0002887
Combined uncertainty	0.010145
Effective degree of freedom	3.7
Coverage factor, k	3.31
Expanded uncertainty, 95.45 %	0.03

Table 7.18 Metrological characteristics

Metrological characteristics	(mV)
Object	160.00
Error	−0.18
Type A uncertainty—Repeatability	0.0135
Standard uncertainty from certificate	0.001
Parasite uncertainty	0.0011547
Object resolution uncertainty	0.002887
Standard resolution	0.0002887
Combined uncertainty	0.0139
Effective degree of freedom	3.4
Coverage factor, k	3.31
Expanded uncertainty, 95.45 %	0.05

III. Point 120.00 mV (Table 7.17)

IV. Point 160.00 m V (Table 7.18)

V. Point 200.00 mV (Table 7.19)

Relative Error: Voltmeters are also classified by their relative error so that, so we can calculate their accuracy class (Tables 7.20 and 7.21).

Table 7.19 Metrological characteristics

Metrological characteristics	(mV)
Object	200.00
Error	−0.26
Type A uncertainty—Repeatability	0.0138
Standard uncertainty from certificate	0.001
Parasite uncertainty	0.0011547
Object resolution uncertainty	0.002887
Standard resolution	0.0002887
Combined uncertainty	0.0142
Effective degree of freedom	3.4
Coverage factor, *k*	3.31
Expanded uncertainty, 95.45 %	0.05

Table 7.20 Relative error

Point (mV)	Error (mV)	Error (%)
40.00	−0.16	0.08
80.00	−0.16	0.08
120.00	−0.19	0.10
160.00	−0.18	0.09
200.00	−0.26	0.13

Table 7.21 Result of calibration

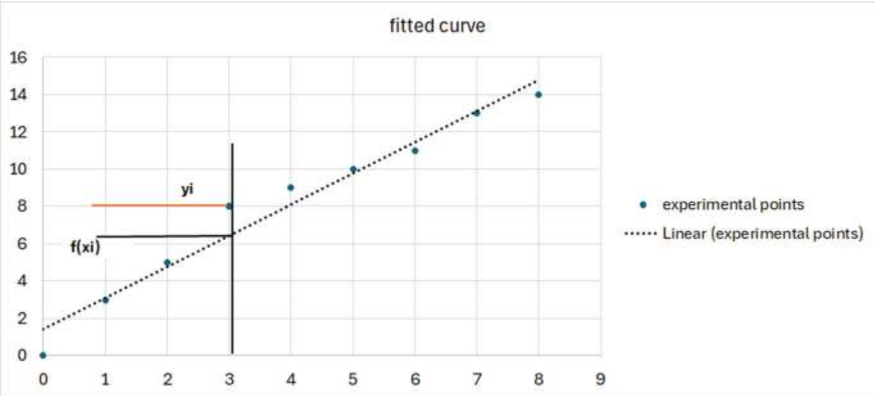
Object mV	Error mV	Uncertainty mV	*k*	υ
40.00	−0.16	0.04	3.31	3.5
80.00	−0.16	0.03	3.31	4
120.00	−0.19	0.03	3.31	3.7
160.00	−0.18	0.05	3.31	3.4
200.00	−0.26	0.05	3.31	3.4

7.7 Measurement Uncertainty in Fitting a Function

Most of the time, a phenomenon or a physical, chemical, or mechanical process is represented by an experimental dataset. In these cases, it can be extremely interesting to "represent" this dataset by a defined mathematical function. This approach procedure is known as fitting or a function regression, and one of the techniques used is the **Least-Squares Method** (LSM).

The literature widely discusses the fitting of experimental points by the LSM, and we usually adopt software that makes these adjustments, such as Microsoft Excel©. For this reason, we do not intend to address the demonstrations of the equations that allow the determination of both the fitting function and its uncertainties.

We are interested in presenting the technique for calculating the measure uncertainty of a function by the LSM. We apply this method whenever we want to describe experimental data behavior—for example, the results in a calibration certificate—through a mathematical equation.



Graph 7.1 The adjustment of experimental points

This method consists of adjusting the dataset to a function that minimizes the experimental variance of the set, that is, we must reduce the difference:

$$f(x_i) - y_i$$

Where $f(x_i)$ is the value of the fitted function for point x_i, and y_i is the experimental value obtained for point XI, as shown in Graph 7.1.

As the method minimizes the difference but does not eliminate it, we will always have to fit a function, whether it is the first (linear), of the second degree (parable), or any other order, to obtain a measurement uncertainty related to both coefficients of this function as those of its value on the y axis.

7.7.1 *Measurement Uncertainty of* Y

When we fit an experimental curve, for example, when we make a calibration curve relating to the *y-axis*, the value of the standard, and the *x-axis*, the value of the calibrated object, we generate a function $f(x_i)$ with measurement uncertainty associated with fitting, since no adjustment is perfect.

According to GUM 2008—Guide to the expression of uncertainty in measurement (Annex H.3.2 Least-Squares Fitting), the variance s^2 is a measure of the overall uncertainty, and the equation that determines the fitting uncertainty of the values found on the axis y is as follows:

$$u_{\text{fitting}} = \sqrt{s^2} = \sqrt{\frac{1}{n-p}\sum_1^n [f(x_i) - y_i]^2} \tag{7.2}$$

Where $f(x_i)$ is the value of the fitting function for point x_i; y_i is the experimental value obtained for point x_i; p is the number of parameters to be fitted; n is the number of experimental points; and $(n - p)$ is the fitting degree of freedom.

7.7.2 Fitting Uncertainty

Considering the experimental points (x, y) obtained through calibration and using a standard instrument, the total uncertainty of variable y will be the combination of the uncertainty of the calibrated object (u_{object}) and the fitting uncertainty (u_{fitting}) through the equation:

$$u_y = \sqrt{u_{\text{fitting}}^2 + u_{\text{object}}^2} \tag{7.3}$$

Fitting uncertainty and the calibrated object uncertainty should always be combined in a standardized form.

Solved Exercise 7.4: Calibration Graph—Fitting the Experimental Data.

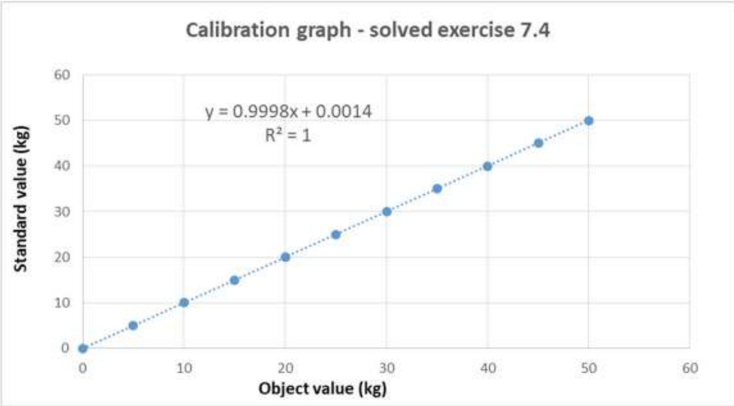
It should be a digital scale with a measurement range of 0 to 50.00 kg and a resolution of 0.01 kg. Table 7.22 shows the calibration result.

We know the uncertainties at each point of the scale. Suppose we now want to obtain an equation that describes the behavior of the scale at any point within the calibration interval 0 to 50 kg. Make the calibration curve of this scale and determine the final uncertainty considering the fitting made by a first-degree function.

Solution Adjusting the scale calibration points by a line of type $y(x) = ax + b$, we will have Graph 7.2:

Table 7.22 Calibration result

Standard (kg)	Object (kg)	Bias (kg)	Uncertainty (kg)	k
0.000	0.02	0.02	0.01	2.00
5.000	4.97	− 0.03	0.01	2.04
10.000	10.02	0.02	0.01	2.04
15.000	14.96	− 0.04	0.01	2.08
20.000	20.02	0.02	0.01	2.09
25.000	24.98	− 0.02	0.01	2.09
30.000	30.05	0.05	0.02	2.09
35.000	35.01	0.01	0.02	2.05
40.000	39.99	− 0.01	0.02	2.06
45.000	45.02	0.02	0.03	2.09
50.000	49.99	− 0.01	0.03	2.09



Graph 7.2 Scale calibration curve of Solved Exercise 7.4

Table 7.23 Solved Exercise 7.4: y and $f(x_i)$ values

Object (x_i) kg	Standard (y_i) kg	$f(x_i)$ kg	$[f(x_i) - y_i]^2$
0.02	0.000	0.021396	0.000457789
4.97	5.000	4.970406	0.000875805
10.02	10.000	10.0194	0.000376205
14.96	15.000	14.95841	0.001729894
20.02	20.000	20.0174	0.000302621
24.98	25.000	24.9764	0.000556771
30.05	30.000	30.04539	0.002060252
35.01	35.000	35.0044	1.93424E-05
39.99	40.000	39.9834	0.000275494
45.02	45.000	45.0124	0.000153661
49.99	50.000	49.9814	0.000345886
		Σ	0.007153719
		u_{fitting}	0.028193221

The calibration curve in Graph 7.2 shows the fitting equation and the value of R^2 (correlation coefficient). This coefficient demonstrates the quality of the adjustment (the closer to one, the better the function adjustment) (Table 7.23).

(a) Fitting uncertainty.

Using Eq. 7.2:

$$u_{fit} = \sqrt{\frac{0.007153719}{11 - 2}} = 0.028193221$$

$n = 11$ and $p = 2$ (coefficients a and b of the fitted equation).

(b) Combined uncertainty (Eq. 7.3).

$$u_c = \sqrt{u_{obj}^2 + u_{fit}^2} = \sqrt{\left(\frac{0.03}{2.09}\right)^2 + 0.028193221^2} = 0.031637\ kg$$

The combined uncertainty was calculated by adding the certificate's most significant uncertainty (0.03 kg), divided by the respective coverage factor ($k = 2.09$), to the fitting uncertainty (0.028193221 kg).

(c) Expanded uncertainty

$$k = 2.09 \rightarrow v_{obj} = 29$$

$$v_{eff} = \frac{u_c^4}{\frac{u_{fit}^4}{n-p} + \frac{u_{obj}^4}{v_{obj}}} = \frac{0.031637^4}{\frac{0.028193^4}{9} + \frac{0.014354^4}{29}} = 14 \rightarrow k = 2.21$$

$$U = 2.21 \quad \times 0.031637\ \text{kg}$$
$$U = 0.07\ \text{kg}$$

Note that uncertainty after fitting increased, but it gave us the convenience of not having to correct the scale bias at each point or calibrate at more points (besides the 11 presented). Just use equation ***y = 0.9998 x + 0.0014*** and adopt the uncertainty of **0.07 kg** for all points.

Attention!

1. we can only perform interpolations, never extrapolations. That is, we can only adopt the fitted equation for points within the calibration range performed. In solved exercise 7.4, this represents values between 0 and 50 kg.
2. it is necessary to format the fitted equation with many decimal places for a lower fitting uncertainty value. This way, the value of $f(x_i)$ will be closer to the value of y_i.

Solved Exercise 7.5: Temperature Transmitter Calibration.

A temperature transmitter with a nominal range of (0 to 100) °C/ (4.00 to 20.00) mA is calibrated with a standard mercury thermometer with measurement uncertainty equal to 0.05 °C ($k = 2.00$ and 95.45 %). In calibration were used a thermal

Table 7.24 Temperature transmitter calibration

Points	Transmitter electric current (mA)	Standard temperature (°C)
1	4.00	0.00
	4.00	0.00
	4.00	0.00
2	8.82	30.30
	8.83	30.30
	8.83	30.30
3	12.12	50.70
	12.12	50.70
	12.12	50.70
4	15.22	70.30
	15.22	70.30
	15.22	70.30
5	18.30	90.00
	18.30	90.00
	18.30	90.00
6	20.00	100.00
	20.00	100.00
	20.00	100.00

bath, a power supply, and a 3½ digit multimeter with measurement uncertainty equal to 0.8 % of the value read +0.01 mA, $k = 2.00$, and 95.45 %.

Table 7.9 shows the temperature transmitter calibration. Knowing that the thermal bath used has a stability of ±0.05 °C, determine the transmitter measurement uncertainty (Table 7.24).

Solution:
I—Point 0 °C

(a) Standard and object mean

$$\bar{x}_{\text{standard}} = 0,00\,^\circ\text{C}$$

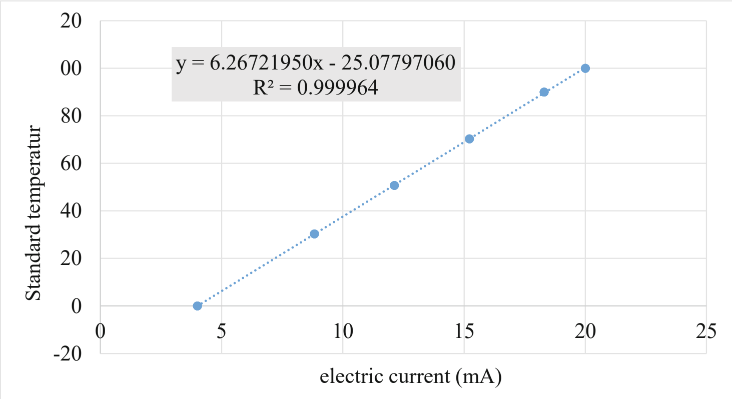
$$\bar{x}_{\text{transmitter}} = 4.00\ \text{mA}$$

(b) Type A uncertainty

$$u_{A\,-\text{object}} = 0$$

$$u_{A\,-\text{standard}} = 0$$

(c) Type B uncertainty (standard certificate).



Graph 7.3 Temperature transmitter calibration curve

$$u_{\text{standard}} = \frac{0.05}{2.00} = 0.025\,^\circ\text{C}$$

(d) Multimeter uncertainty (Type B).

$$u_{\text{multimeter}} = \frac{0.008\ x\ 4.00 + 0.01}{2.00} = \frac{0.042}{2.00} = 0.021\ \text{mA}$$

(e) Bath stability uncertainty

$$u_{\text{bath}} = \frac{0.05}{\sqrt{3}} = 0.0288675\,^\circ\text{C}$$

As we can see, we have uncertainties in °C and mA units from different quantities: temperature and electric current, respectively. How can we transform the uncertainties from mA to °C?

We want the transmitter to measure temperature, so we should have the uncertainty in °C.

The solution to this problem is to discover a function related to °C and then find the transmitter sensitivity coefficient. Graph 7.3, temperature versus electric current, generated in Excel© software, will give us the desired function.

$$\boldsymbol{t} = \mathbf{6.26721950}\ \boldsymbol{i}\ -\ \mathbf{25.07797060} \tag{7.4}$$

t is the temperature (°C), and ***i*** is the transmitter electric current (mA).

Derivating Eq. (7.4), we have the sensitivity coefficient of the temperature transmitter:

Table 7.25 Fitting uncertainty

Mean electric current x_i (mA)	Mean temperature y_i (°C)	$f(x_i)$	$[f(x_i) - y_i]^2$
4.00	0.00	−0.00909	8.26754E−05
8.83	30.30	30.24069	0.003518049
12.12	50.70	50.88073	0.032663239
15.22	70.30	70.30911	8.29956E−05
18.30	90.00	89.61215	0.150430531
20.00	100.00	100.2664	0.070979297
		Σ	0.257756787
		u_{fit}	0.253848768

$$\frac{\partial t}{\partial i} = 6.26721950 \ \frac{^\circ\mathrm{C}}{\mathrm{mA}}$$

Important
Using the sensitivity coefficient, we can transform mA uncertainty values into °C Measurement results in mA can only be converted into °C using the equation $t = 6.26721950$ i − 25.07797060.

(f) Multimeter uncertainty (°C).

$$u_{\text{mult}} = 0.021 \text{ mA}$$

$$u_{\text{mult}} = 0.021 \text{ mA} \times 6.26721950\,^\circ\mathrm{C/mA}$$

$$u_{\text{mult}} = 0.131611609\,^\circ\mathrm{C}$$

(g) Fitting uncertainty.

We need to use Eq. 7.2. The fitting uncertainty will be the same for all calibrated points (Table 7.25).

$$u_{\text{fit}} = 0.253848768\,^\circ\mathrm{C}$$

(h) Combined uncertainty.

$$u_c = \sqrt{0.025^2 + 0.131611609^2 + 0.0288675^2 + 0.253848768^2}$$

Table 7.26 Point 0.0 °C

Metrological characteristics	Results
Mean electric current (mA)	4.00
Mean temperature standard (°C)	0.00
Object temperature (°C)	0.00
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.00
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.131611609
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.28847287
Effective degree of freedom, $\boldsymbol{\upsilon}_{\text{eff}}$	6
Coverage factor, k	2.52
Expanded uncertainty, 95.45 % (°C)	0.7

$$u_c = 0.28847287\,°\mathrm{C}$$

(i) Effective degree of freedom.

$$\frac{u_c^4}{\nu_{\text{eff}}} = \frac{u_{\text{std}}^4}{\infty} + \frac{u_{\text{mult}}^4}{\infty} + \frac{u_{fit}{}^4}{n-p} + \frac{u_{\text{bath}}{}^4}{\infty}$$

$$\frac{0.28847287^4}{\nu_{\text{eff}}} = \frac{0.253848768^4}{6-2}$$

$$\nu_{\text{eff}} = \frac{0.28847287^4}{\left(\frac{0.253848768^4}{4}\right)}$$

$$\nu_{\text{eff}} = 6$$

(j) Coverage factor k.

t-Student table, for 95.45% and $\nu_{\text{eff}} = 6$ $k = 2.52$

(k) Expanded uncertainty.

$$U = k \times u_c = 2.52 \times 0.28847287$$

$$U = 0.726951632\,°\mathrm{C}$$

$$U = 0.7\,°\mathrm{C}$$

(l) Instrumental bias.

We are using Eq. 7.4, the instrumental bias will be the difference between the transmitter and standard values (Table 7.26).

Table 7.27 Point 30.0 °C

Metrological characteristics	Results
Mean electric current (mA)	8.83
Mean temperature standard (°C)	30.30
Object temperature (°C)	30.2
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.208886425
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.252612816
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.416346373
Effective degree of freedom, $\boldsymbol{\upsilon}_{\text{eff}}$	15
Coverage factor, k	2.18
Expanded uncertainty, 95.45 % (°C)	0.9

Table 7.28 Point 50.0 °C

Metrological characteristics	Results
Mean electric current (mA)	12.12
Mean temperature standard (°C)	50.70
Object temperature (°C)	50.9
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.00
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.335170898
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.422181313
Effective degree of freedom, $\boldsymbol{\upsilon}_{\text{eff}}$	30
Coverage factor, k	2.09
Expanded uncertainty, 95.45 % (°C)	0.9

$$B = \overline{x}_{\text{obj}} - \overline{x}_{\text{std}}$$

$$B = (6.26721950 \cdot 4.00 - 25.07797060) - 0.00$$

$$B = -0.00909 - 0.00$$

$$B = 0.0\,^{\circ}\text{C}$$

II. Point 30.0 °C (Table 7.27)

III. Point 50.0 °C (Table 7.28)

Table 7.29 Point 70.0 °C

Metrological characteristics	Results
Mean electric current (mA)	15.22
Mean temperature standard (°C)	70.30
Object temperature (°C)	70.3
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.00
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.41288442
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.486180084
Effective degree of freedom, $\boldsymbol{\upsilon}_{\text{eff}}$	53
Coverage factor, k	2.05
Expanded uncertainty, 95.45 % (°C)	1.0

Table 7.30 Point 90.0 °C

Metrological characteristics	Results
Mean electric current (mA)	18.30
Mean temperature standard (°C)	90.00
Object temperature (°C)	89.6
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.00
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.490096564
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.553255973
Effective degree of freedom, $\boldsymbol{\upsilon}_{\text{eff}}$	90
Coverage factor, k	2.03
Expanded uncertainty, 95.45 % (°C)	1.1

IV. Point 70.0 °C (Table 7.29)

V. Point 90.0 °C (Table 7.30)

VI. Point 100.0 °C (Tables 7.31 and 7.32)

7.8 Proposed Exercises

7.8.1 The calibration of a temperature sensor (PT-100 to 3 wires) against a temperature standard presented the values contained in Table 7.33

Table 7.31 Point 100.0 °C

Metrological characteristics	Results
Mean electric current (mA)	20.00
Mean temperature standard (°C)	100.00
Object temperature (°C)	100.3
Sensitivity coefficient (°C/mA)	6.26721950
Type A uncertainty—Object repeatability (°C)	0.00
Uncertainty from the standard certificate (°C)	0.025
Multimeter uncertainty (°C)	0.532713657
Bath stability uncertainty (°C)	0.0288675
Fitting uncertainty (°C)	0.253848768
Combined uncertainty (°C)	0.591338625
Effective degree of freedom, $\boldsymbol{\upsilon}_{\mathrm{eff}}$	118
Coverage factor, k	2.02
Expanded uncertainty, 95.45 % (°C)	1.2

Table 7.32 Temperature transmitter calibration

Mean electric current (mA)	Mean temperature standard (°C)	Object temperature (°C)	Bias (°C)	Uncertainty (°C)	k	υ_{eff}
4.00	0.00	0.0	0.0	0.7	2.52	6
8.83	30.30	30.2	−0.1	0.9	2.18	15
12.12	50.70	50.9	−0.2	0.9	2.09	30
15.22	70.30	70.3	0.0	1.0	2.05	53
18.30	90.00	89.6	−0.4	1.1	2.03	90
20.00	100.00	100.3	0.3	1.2	2.02	118

Table 7.33 Temperature sensor calibration

Temperature Standard (°C)	Resistance $R(t)$ Ω
0.00	99.99
25.00	109.74
50.00	119.40
75.00	128.99
100.00	138.50
125.00	147.95
150.00	157.32
175.00	166.63
200.00	175.86

Considering the multimeter uncertainty is 0.02 Ω ($k = 2.00$ and 95.45 %), the stability of the calibration bath is ±0.02 °C, the repeatability uncertainty (Type A uncertainty) is equal to zero, and the standard thermometer uncertainty is 0.02 °C ($k = 2.00$ and 95.45 %), determine:

Table 7.34 Scale calibration

Standard mass (kg)	Scale lectures (kg)
0.00	0.2; 0.3; 0.3
10.00	10.2; 10.4; 10.4
15.00	14.9; 14.9; 14.7
20.00	20.2; 20.0; 20.3

Table 7.35 GLT calibration

Measurements	Standard (°C)	Object (°C)
1	10.1	10.5
	10.1	10.5
	10.1	10.0
2	20.0	19.5
	20.0	19.5
	20.0	19.5
3	50.2	50.0
	50.2	50.0
	50.2	50.0

(a) The fitting equation, knowing that a platinum resistance thermometer behaves second to eq. $R(t) = R(0)\ [1 + At + B\ t^2]$, where $R(0)$ is the resistance of the Pt-100 to 0 °C, t is the temperature, $R(t)$ is the electrical resistance at the desired temperature, and A and B its coefficients.
(b) The fitting uncertainty.
(c) The Pt-100 expanded uncertainty.

7.8.2 A digital scale with a resolution of 0.1 kg was calibrated against a standard mass set. The calibration result is in Table 7.34 (three measurements were performed at each point).

Considering the standard masses uncertainty is 0.02 kg (for 95.45 % and $k = 2.09$), answer:

(a) What is the Type A uncertainty for each scale calibration point?
(b) What is the expanded uncertainty for each scale calibration point, considering reading resolution, repeatability of measurements, and the standard mass as the uncertainty sources?
(c) Build the graph "standard mass value × scale reading." Find the fitting equation $y = ax + b$ and determine the fitting uncertainty.
(d) Build a table with expanded uncertainty values and bias for points of 0 to 20 kg in intervals of 1 kg.

7.8.3 A glass liquid thermometer (GLT) has an uncertainty of 0.2 °C. What is the highest value of standard uncertainty so that its influence on final uncertainty is not greater than 2.5 %?

7.8.4 A GLT with a resolution of 0.5 °C is calibrated against a standard with a resolution of 0.1 °C (Table 7.35).

Table 7.36 Pressure gauge calibration

Object	Standard (bar)			
bar	Charge 1	Discharge 1	Charge 2	Discharge 2
6.0	6.4	6.5	6.4	6.5
10.0	10.5	10.4	10.5	10.4
24.0	24.3	24.2	24.3	24.2
30.0	30.3	30.4	30.3	30.4
40.0	40.5	40.4	40.5	40.4

Table 7.37 Voltmeter calibration

	Standard (mV)			
Object (mV)	V1	V2	V3	V4
40.00	40.110	40.150	40.160	40.120
80.00	80.120	80.160	80.140	80.130
120.00	120.150	120.170	120.190	120.190
160.00	160.230	160.180	160.170	160.180
200.00	200.210	200.230	200.260	200.270

The calibration bath stability is ±0.08 °C. The standard thermometer certificate is in Solved Exercise 7.1 (Fig. 7.2).

Determine:

(a) The Type A uncertainty.
(b) The bath stability uncertainty.
(c) The object resolution uncertainty.
(d) The bias at each point.
(e) The linear calibration curve of the object thermometer and the equation that relates the standard values (y) and the object (x) thermometer values.
(f) The uncertainty of the linear fitting of this thermometer.
(g) The thermometer uncertainty considering the fitting.

7.8.5 A Bourdon-type pressure gauge (object), with measurement range 0 to 40 bar and 0.5 bar resolution, was calibrated against a standard gauge that has a measurement uncertainty of 0.1 bar ($k = 2.00\%$ and 95.45 %) and resolution of 0.1 bar. Table 7.36 presents the result of the object gauge calibration.

Determine:

(a) The object gauge hysteresis at each point.
(b) The gauge relative error at each point.
(c) The gauge uncertainty at each point.
(d) The linear calibration curve of the object gauge and the equation that relates the standard values (y) and the values of the object (x) pressure gauge.
(e) The uncertainty of the linear fitting of this gauge.
(f) The uncertainty of the gauge considering the fitting.

7.8.6 A digital voltmeter, with a resolution of 0.01 mV, was calibrated at an interval of 0 to 200 mV against a standard voltmeter. Table 7.37 presents the result of the object voltmeter calibration. Consider the parasite voltage uncertainty equal to $2\mu V/\sqrt{3}$.

See the standard voltmeter certificate in Solved Exercise 7.3 (Fig. 7.4).
Determine:

(a) The object error at each point.
(b) The voltmeter relative error at each point.
(c) The voltmeter uncertainty at each point.
(d) The linear calibration curve of the voltmeter and the equation that relates the standard values (y) and the values of the object (x).
(e) The uncertainty of the linear fitting of this voltmeter.
(f) The uncertainty of the voltmeter considering the fitting.

Chapter 8
Measurement Uncertainty in Conformity Assessment

8.1 Statement of Conformity and Decision Rules

Statements of conformity and decision rules are important issues to discuss, especially after the last edition of ISO/IEC 17025: 2017, which made the requirements more rigorous for these issues.

Based on document ILAC-G8:09/2019—Guidelines on Decision Rules and Statements of Conformity, we can establish the following definitions:

Statement of conformity: is an expression that describes the state of conformity or non-conformity with a specification, standard, or requirement.

Decision rule*: a rule that describes how measurement uncertainty will be accounted for when stating conformity with a specified requirement*. (ISO/IEC 17025:2017 clause 3.7)

As we can see, we need to adopt a decision rule to declare a product, process, or measurement standard "conforming."

The ISO/IEC 17025 standard was first published in 1999. Since then, statements of conformity based on specifications or standards have become increasingly required, following the evolution of documentation on the concept of rule decisions used for such statements.

The current standard cites various requirements for the statement of conformity; however, no "unique" rule can address all conformity statements throughout the scope of testing or calibration.

Additionally, professionals (auditors and audited) have doubts about understanding and writing. How do we obey, that is, what laboratories need to apply to meet the customer's requirements and the ISO/IEC 17025 standard?

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_8

8.2 Conformity Assessment

Conformity assessment[1] is performed in tests, inspections, and calibrations to ensure the compliance of products, materials, services, and systems regarding requirements defined by standards, regulations, and legal frameworks, being adopted to establish confidence for consumers and safety and quality of life.

Evaluating conformity is essential in the global economy, because it implies accepting or rejecting items that impact risk analysis, business decisions, financial costs, and sometimes reputation costs.

The result of a measurement can be used to decide whether a variable of interest complies with a specific requirement, and this variable may be, for example:

(a) **The value of a standard 200 g mass of class E2.**

According to the OIML R 111-1 Edition 2004 recommendation, a standard mass of 200 g of class E2 should have a maximum permissible error of 0.3 mg. This is to say that the mass in question must have a value between [199.9997 to 200.0003] g.

(b) **The indication error of a digital voltmeter.**

The value indicated in the standard 999 V.
The value indicated in the voltmeter 1000 V.
Error = 1 V.

(c) **The pH value of a solution.**

Example: pH = 7.474.

These variables of interest have values usually within tolerance limits, called tolerance intervals. If the variable's actual value is within the tolerance range, it is considered "conforming"; otherwise, it is considered "non-conforming."

In practical situations, to perform conformity assessment (e.g., conformity with geometric tolerances), are necessary objective criteria called "decision rules" (which consider a probability of occurrence), which define a "conformity zone" and an "acceptance zone" (which are the results plus measurement uncertainty).

A traditional approach to a decision rule involves comparing a single limit (or limit interval) with the result of a single measurement.

Currently, the probabilistic approach to measurement, which introduces uncertainty as a parameter that expresses measurement variability, significantly affects the decision-making process. See Fig. 8.1 for example.

Figure 8.1 presents four possible measurement results and their uncertainties within a tolerance interval to which we must apply a decision rule. We can undoubtedly say that case (a) is "conforming" and case (d) is "non-conforming"; however, in

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[1] According to ISO/IEC 17000: 2004, conformity assessment is any activity performed to determine, directly or indirectly, whether a product, process, system, person, or body meets relevant standards and complies with the specified requirements.

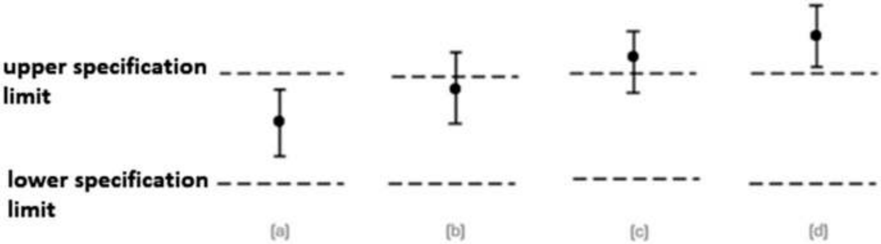


Fig. 8.1 Representation of four different measurement results within a tolerance interval. (Source: the authors)

cases (b) and (c), we have an indefinite situation that needs a formal criterion based on an expected confidence interval, to be used to decide on your compliance.

A decision rule always carries a "risk," and the declarants of conformity are responsible for and directly control this risk, as they establish both the decision-making criteria and the rules to be applied.

Together with you, we will develop the general concepts and procedures for assessing conformity based on measurement results, recognizing the central role of decision-making uncertainty in approving or disapproving the product or measurement process.

Therefore, to better understand the content, it is essential to know some terms and definitions widely used in the area, described in the JCGM 106:2012—Evaluation of measurement data—The role of measurement uncertainty in conformity assessment.

(a) ***Tolerance Limit (TL) (Specification Limit):*** *specified upper or lower bound of permissible values of a property.*

Example: The temperature of a laboratory should be maintained between (20 ± 2) °C. The lower TL is 18 °C and the upper TL is 22 °C

(b) ***Tolerance Interval:*** *interval of permissible values of a property.*

NOTE 1 Unless otherwise stated in a specification, the tolerance limits belong to the tolerance interval.
NOTE 2 The term "tolerance interval" as used in conformity assessment has a different meaning from the same term as it is used in statistics.
NOTE 3 A tolerance interval is called a "specification zone" in ASME B89.7.3.1: 2001.

Example: The temperature of a laboratory should be kept within a tolerance interval (20 ± 2) °C.

(c) ***Tolerance:*** *specified tolerance difference between upper and lower tolerance limits.*

Example: The temperature of a laboratory should be maintained between 20 ± 2 ° C. The tolerance is 4 °C (18 °C − 22 °C).

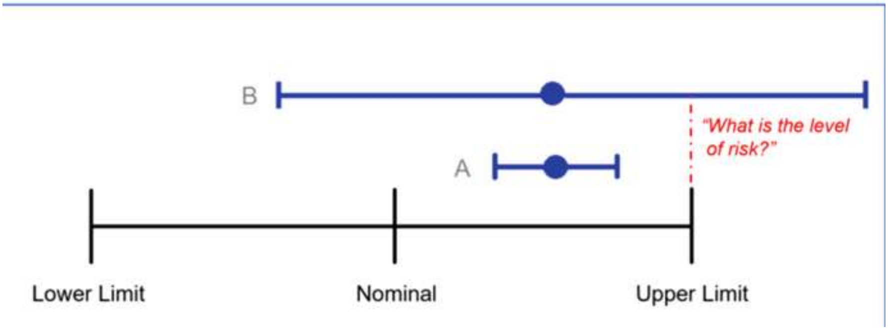


Fig. 8.2 Measurement decision risk. (Source: ILAC-G8:09/2019)

8.3 Uncertainty of Measurement and Risk of Decision

When performing a measurement, there are two situations when stating conformity: inside or outside tolerance regarding the manufacturer's specifications, approved or disapproved in a specific requirement.

In real life, as much as the instrument is used, however accurate and precise, we know that all measured value has an associated measurement uncertainty (U).

Figure 8.2 shows two measurements of the same measurand, but with different measurement uncertainties (the central point is the measurement value, and the horizontal bar shows the measurement uncertainty).

Note that the measurement result (measurement value + measurement uncertainty) in case A is entirely within the tolerance limit. In case B, which has significantly greater measurement uncertainty, the risk of accepting a false result exists. What is the risk level of a false result?

Further, we will discuss the types of errors, risks, and possible decision rules in detail, but now we will anticipate some considerations.

A binary decision rule exists when the result is limited to two options (approved/disapproved, right/wrong, passes/does not pass), and a non-binary decision rule exists when, of course, we have several alternatives to express the result (approved, disapproved, conditional approval, conditional failure).

Analyzing Fig. 8.1, where measurements (a) and (b) are considered approved, and the values (c) and (d) failed in a binary statement with a simple acceptance rule, statements of conformity may be reported as:

- *Pass* (a) and (b): The measured value (central point) is within the specification limits.
- *Fail* (c) and (d): The measured value is outside the specification limits.

In this case, the expanded uncertainty of measurement U was ignored, and the decision was made only based on the measurement value.

Note that even cases (a) and (b), considered approved, have a percentage risk of providing values outside tolerance limits. This percentage is much lower in case

(a) than in case (b), but it exists. Remember that measurement uncertainty is calculated for a probability of 95.45 % and that we have a 4.55 % probability that the declared value is outside the measurement interval.

Thus, it is necessary to study the decision errors assumed when accepting or rejecting a measurement result against specification limits (tolerance intervals). Next, we will analyze these mistakes.

8.4 False Positive and False Negative

When performing a conformity assessment, there are probabilities related to two types of incorrect decisions:

(a) Accept an incorrect result—*false positive.*
(b) Reject a correct result—*false negative.*

Figure 8.3 has four measurement results with their respective uncertainties.

Accepting or rejecting an item when the measured value of your property is close to the tolerance limit may result in an incorrect decision and undesirable consequences.

Analyzing situation (b) in Fig. 8.3, we see that the measured value is below the tolerance limit, but the true value is above, configuring a false positive situation. That is, a situation where we believe the product is within the specification, but it is not. The situation (c) is opposite. We have the value measured above the tolerance limit but below the true value. In this case, we have a false negative situation. That is, we believe that the measurement does not meet the specifications but meets.

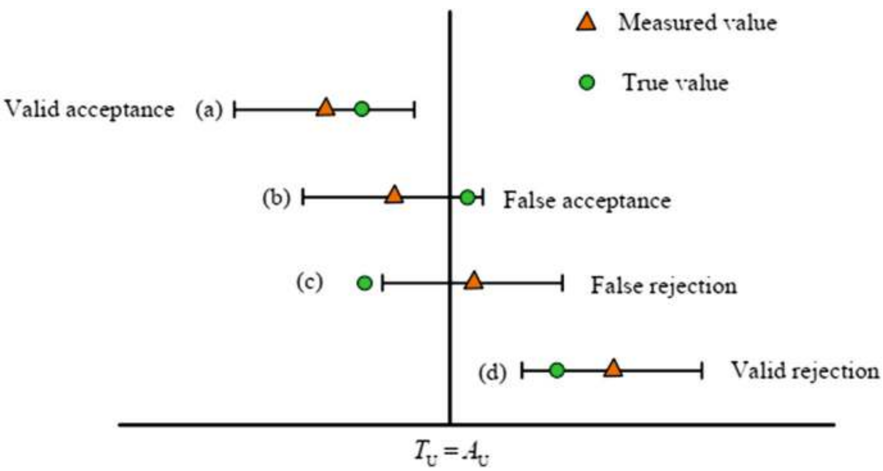


Fig. 8.3 T_U is the upper tolerance limit, and A_U is the upper acceptance limit. There were four measurement results, with their respective uncertainties at 95%. (Source: JCGM 106:2012)

We have consumer risk in the case of a false positive (type II error or β error). In the case of a false negative, we already have producer/supplier risk (type I error or α error).

For measurements, the probability of not accepting an item as shown in Fig. 8.3, (b) or rejecting an item as shown in Fig. 8.3, (c) can reach 50 %. This would happen, for example, if the value measurement of a property were very close to the tolerance limit. In this case, about 50 % of the probability of the measurement result would be on both sides of the tolerance limit, so whether the item was accepted or rejected, there would be a 50 % chance of an incorrect decision.

Any of these odds can be reduced, but at the cost of increasing the other, choosing acceptance limits, which removes the result from tolerance limits.

This is a compliance decision strategy called Guard Band.

8.5 Decision Rules and Guard Band

When measurement is very close to tolerance limits or when uncertainty is significant, an acceptance criterion only considering the upper and lower tolerance limits (simple acceptance) can lead to a high risk of an incorrect decision. Often, more confidence is necessary in accepting or rejecting an analyzed item. For these situations, we can use a guard band.

The guard band is a protection created to remove (reject) values near tolerance limits. Acceptance and rejection zones can be determined, as shown in Fig. 8.4.

(a) **High confidence in correct rejection.**

Only measurements that exceed the guard band placed after the tolerance limit will be rejected in this case.

(b) **High confidence in correct acceptance.**

Only measurements inferior to the guard band placed before the tolerance limit will be accepted in this case.

The region between the upper tolerance limit and the higher acceptance limit is called the *guard band*, reducing the risk of an incorrect decision.

The use of guard bands provides a straightforward way to define decision rules; choosing the size of the guard band defines an acceptance zone that can be used for decision-making. The guard band is generally defined as the expanded uncertainty of measurement (U). It can also be described as zero. This is called simple acceptance or "shared risk."

Figure 8.5 shows the acceptance and rejection zones. The guard band has been chosen so that, for a sample that is in accordance, there is a high probability that the measurement is within the specification limits; this is high confidence in correct acceptance.

Figure 8.5 shows the relative positions of specification limits and acceptance and rejection zones for high acceptance confidence.

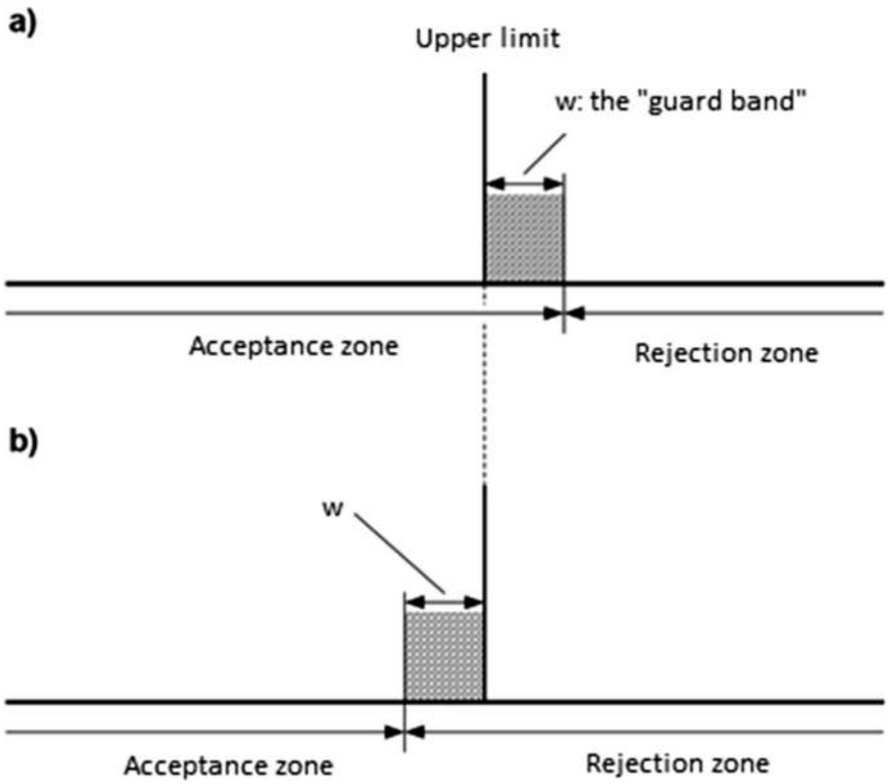
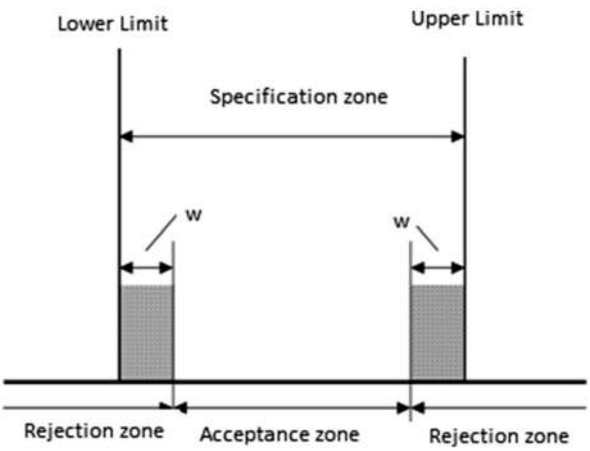


Fig. 8.4 The relative positions of the acceptance and rejection zones for (**a**) high confidence in correct rejection and (**b**) high confidence in correct acceptance. The interval w is called the guard band. The upper end of the acceptance zone is the acceptance limit. (Source: EURACHEM/CITAC Guide—Use of uncertainty information in compliance assessment—Second Edition 2021)

Fig. 8.5 Relative positions of the specification limits and the acceptance and rejection zones, allowing for high confidence in correct acceptance. (Source: EURACHEM/CITAC Guide—Use of Uncertainty Information in Compliance Assessment—Second Edition 2021)



The decision to accept an item as in accordance or reject it as not as, according to the specification, is based on a measured value of a property of the item about a declared decision rule that specifies the role of measurement uncertainty in the formulation of acceptance criteria. An interval of measured property values that results in acceptance of the item is called the acceptance zone (see Fig. 8.5), defined by one or two acceptance limits (see Fig. 8.4).

The limits of acceptance and the corresponding decision rules are chosen to manage the undesirable consequences of incorrect decisions. Several widely used decision rules are simple to implement. They can be applied when knowledge of a property of interest is summarized in terms of a better estimate and corresponding coverage interval. Two of these decision rules are described below.

8.5.1 Decision Ruler Based on Simple Acceptance

An important and widely used decision rule is simple acceptance or shared risk. In this rule, the producer and user (consumer) agree, implicitly or explicitly, to accept as (and reject otherwise) an item whose property has a value measured within the tolerance range. With the alternative name of "shared risk," the producer and the user share the consequences of incorrect decisions.

In practice, to maintain the chances of incorrect decisions at acceptable levels for both the producer and the user, there is usually a requirement that measurement uncertainty is considered sufficient for the intended purpose.

One approach to such consideration is to require, given an estimated measured quantity, which expanded uncertainty U for a probability of scope, for example, of 95.45 %, must satisfy:

$$U \leq U_{\max}$$

Where $U_{\max}$ is a mutually agreed expanded uncertainty, this approach is explained by the following situation.

Legal Metrology's decision rule based on simple acceptance has been used to verify instrument measurement.

Consider an instrument that must have an indication error in the interval [$-E_{\max}$; + $E_{\max}$]. The instrument is accepted under the specified requirement, if it meets the following criteria:

(a) Analyzing the measuring instrument calibration certificate, its measurement error E will be accepted if it satisfies the condition: $E \leq E_{max}$.
(b) Expanded uncertainty will be accepted if it is less than 1/3 of the maximum error.

$$U \leq U_{max} = E_{max}/3$$

8.5.2 Decision Rules Based on Guard Bands

The difference between a tolerance limit (T_U) and a corresponding acceptance limit (A_U) defines the width of a guard band (w).

$$w = T_U - A_U. \tag{8.1}$$

When we consider that a decision rule is protected, that is, it has a small probability of generating false positives or false negatives, we adopt $w > 0$. (Fig. 8.6).

8.6 Unilateral Tolerance Interval with the Normal Curve

The probability of a measurement (product, component, etc.) being in compliance depends on the knowledge of the measuring X and its respective probability density function (PDF) $P(X)$. In most cases, it is reasonable to characterize *the* knowledge X by a normal distribution, and thus, we can calculate its probability.

If the production distribution is normal and a normal distribution also characterizes the measurement system, then the distribution function $P(X)$ will also be normal.

More generally, if the probability function is characterized by a normal distribution and the previous information is insufficient, then the posterior PDF (post-measured) will be approximately normal. In this case, $P(X)$ may be adequately approached by a normal distribution with the mean $\bar{x}$ and the standard deviation by standard uncertainty $u(x)$, calculated according to ISO Gum criteria.

Assuming, then, that the PDF $P(X)$ for the measurement X is (i.e., well approximated by) a normal distribution specified by a mean $\bar{x}$ and a standard uncertainty $u(x)$, we will have:

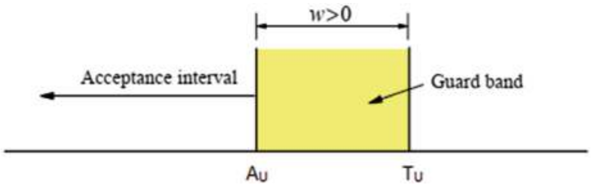


Fig. 8.6 Guard band-based decision rule. An upper A_U acceptance limit within an upper T_U tolerance limit defines an acceptance range that reduces the probability of false acceptance of a non-conforming item (consumer risk). By convention, the length parameter w associated with a guard band is considered positive: $w = T_U - A_U > 0$. (Source: JGCM 106:2012)

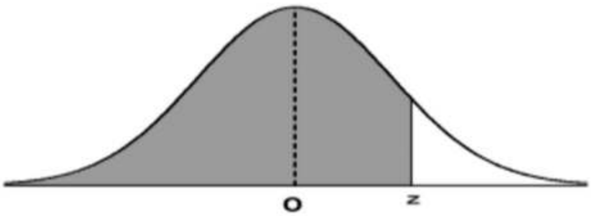


Fig. 8.7 Standard normal accumulated probability

$$p(x) = \frac{1}{u\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\overline{x}}{u}\right)^2} = \varphi\left(x; \overline{x}; u^2\right) \tag{8.2}$$

The probability of *X* being in the interval [a, b], with a normal PDF *P*(*X*), will be:

$$\mathrm{P}(a \leq X \leq b) = \varphi\left(\frac{b-\overline{x}}{u}\right) - \varphi\left(\frac{a-\overline{x}}{u}\right) \tag{8.3}$$

The probability *P*(*X*) can be found in a standard normal accumulated table or through MS EXCEL© software, function DIST.NORM.N (*x*, mean, standard deviation, cumulative).

Figure 8.7 and Table 8.1 show the standard normal accumulated probability of z from 0.00 to 3.99.

Next, we will apply Eq. (8.3) to estimate the probability of obtaining a value accepted as valid for a given specification from a measurement result.

8.6.1 Examples of Probability Estimation in Simple Acceptance

(a) **Single upper tolerance limit—T_U.**

As we saw at the beginning of this chapter, a measurement process's tolerance is the maximum variation admitted by the process variables. This tolerance range is the limit within which the parameters of interest must be located.

In some situations, we do not work with a tolerance range but with a single tolerance value. Given a single upper tolerance limit (T_U) and an estimated y measurement with standard measurement uncertainty $u(y)$, a decision rule must define a probability of compliance (*Pc*) assuming a false negative (products in compliance are incorrectly rejected).

The expression to be tested is:

$$Pc = P(y \leq T_U) = \varphi\left(\frac{T_U - y}{u}\right) \tag{8.4}$$

Table 8.1 Standard normal accumulated distribution

z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

(continued)

Table 8.1 (continued)

z	0	1	2	3	4	5	6	7	8	9
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Example 8.1: Upper Tolerance Limit—$T_{U.}$ Consider a measurement estimate $y = 2.7$ mm with an uncertainty u $(y) = 0.4$ mm (for $k = 2.00$ and 95.45% metrological reliability). A single upper limit of tolerance $T_U = 3.0$ mm and a probability for 95% compliance was established, thus assuming a false negative (supplier risk—type I) of 5%.

Based on this experimental result (2.7 mm), on the defined tolerance limit (3.0 mm), on standardized uncertainty $u = 0.2$ mm (U/K), and assuming a Gaussian PDF, the decision rule will be to accept that the hypothesis of the measured value $y \leq 3.0$ mm is equal to or greater than 95% (0.95). In statistical language, we have:

$$H_0: \quad P \quad (y \leq 3.0 \quad \text{mm}) \geq 0.95 \text{ is true.}$$

To estimate the probability related to the given example, the probability of compliance (Pc) needs to be calculated using the general expression for the Gaussian probability density (PDF) function.

$$Pc = P(y \leq T_U) = \varphi\left(\frac{T_U - y}{u}\right) = \varphi\left(\frac{3.0 - 2.7}{0.2}\right) = \varphi(1, 5) \approx 0.933 \ (93.3\%) < 0.95$$

Therefore, hypothesis H_0 is false, and the decision is **non-conforming**.

Conclusion If the measurement is 2.7 mm with standard uncertainty $u = 0.2$ mm, the probability that the accepted value is 93.3 %. As we want a probability greater than or equal to 95 %, the result of 2.7 mm will be rejected.

Now, Let Us Look at Another Question!

Keeping the initial conditions, that is, tolerance limit defined at 3.0 mm, standardized uncertainty $u = 0.2$ mm, decision rule H_0: P ($y \leq 3.0$ mm) ≥ 0.95, we ask:

What should be the highest value of y to have conformity?

We need to identify the value of Y that meets the equation:

$$P_c = P(y \leq T_U) = \varphi\left(\frac{T_U - y}{u}\right) = \varphi\left(\frac{3.0 - y}{0.2}\right) \geq 0.95$$

In Table 8.1, we see that for $z = 1.65$, we have $p = 0.9505 > 0.95$. Then: $z = 1.65 = \frac{3.0 - y}{0.2} \rightarrow y = 2.67\ mm$

Conclusion we will have a **conforming** when y $\leq$ 2.67 mm.

Note that in this example, we need to use a measuring instrument with expanded uncertainty equal to 0.4 mm, and the measurement error is corrected at the time of reading. What should be the instrument's resolution that will meet these characteristics?

To detect the upper limit value of 2.67 mm, it must have a resolution of 0.01 mm. This instrument can be a digital caliper with 0.01 mm resolution and measurement error added to measurement uncertainty of 0.4 mm, both are read in the caliper's calibration certificate.

In industrial processes, it is usual to add the measuring error of the instrument with its uncertainty stated in the calibration certificate. This sum, error (or bias) (in absolute value) more measurement uncertainty is a way for industries not having to correct the measurement. On the other hand, it increases measurement uncertainty.

We must, whenever possible, correct measurement and eliminate the error of measuring or instrumental bias. However, for various reasons, we find industrial processes where measurement error (E) or instrumental bias (B) is added to uncertainty, thus generating what is usually called maximum uncertainty ($U_{\max}$) and, in some cases, maximum permissible error (MPE).

$$U_{max} = |E \ or \ B| + U$$

Analyze, now, this new provocation!

If we want to increase the cutting value from 2.67 mm to 2.99 mm, what measurement uncertainty should we adopt for this measurement, maintaining the probability of acceptance of 95%?

$$z = 1.65 = \frac{3.0 - 2.99}{u} \rightarrow u = 0.006 \ mm$$

Note that in this case, we must have an expanded measurement uncertainty of 0.012 mm and use a micrometer with a resolution of 0.001 mm.

(b) **Single lower tolerance limit—T_L**

Similarly, given a single lower tolerance limit (T_L) and an estimated y measurement with standard measurement uncertainty $u(y)$, a decision rule should define a probability of compliance (Pc) assuming a false positive (supplier error—type I error).

Expression for test:

$$P_c = P(y \geq T_L) = 1 - P(y \leq T_L) = 1 - \varphi\left(\frac{T_L - y}{u}\right)$$

$$P(y \geq T_L) = \varphi\left(\frac{y - T_L}{u}\right) \tag{8.5}$$

Example 8.2: Single Lower Tolerance Limit—T_L Consider a measurement estimate $y = 0.012$ g with an uncertainty $U(y) = 0.002$ g (for $k = 2.00$ and 95.45 % metrological probability). A single lower tolerance limit $T_L = 0.010$ g was defined, and a probability for conformity was 0.99 (99 %), thus assuming a risk of false positive (supplier error) of 0.01 (1%).

With the experimental result (0.012 g), the tolerance limit (0.010 g), and assuming a Gaussian PDF, the decision rule will be to accept that the hypothesis

H_0: P ($y \geq 0.010$ g) ≥ 0.99 is true.

To estimate the probability related to the given example, the probability of compliance (Pc) needs to be calculated using the overall expression for Gaussian PDF:

$$P_c = \varphi\left(\frac{y - T_L}{u}\right) = \varphi\left(\frac{0.012 - 0.010}{0.001}\right) = \varphi(2.0) \approx 0.977\ (97.7\%) < 0.99$$

Then, H_0 is false, and the decision is **non-conforming**.

Conclusion If the measurement is 0.012 g with standard uncertainty $u = 0.001$ g, the probability that the accepted value is 97.7 %. As we want a probability greater than or equal to 99 %, the result of 0.012 g will be rejected.

If the conformity probability were redefined to 95 %, the decision rule would be to accept hypothesis H_0: P ($y \geq 0.010$ g) ≥ 0.95 as true.

Using the results obtained:

$$P_c = \varphi\left(\frac{y - T_L}{u}\right) = \varphi\left(\frac{0.012 - 0.010}{0.001}\right) = \varphi(2.0) \approx 0.977\ (97.7\%) > 0.95$$

Then, H_0 is true, and the decision is confirmed.

Conclusion If the measurement is 0.012 g with standard uncertainty $u = 0.001$ g, the probability that the accepted value is 97.7 %. As we want a probability greater than or equal to 95 %, the result of 0.012 g will be accepted.

(c) **General approach with unique tolerance limits**

As we have seen earlier, for both the probability of compliance (Pc) for an upper tolerance limit (T_U) and for a lower tolerance limit (T_L), we must define a decision rule for a probability of compliance (Pc) for type I error (α error).

Type I error occurs when we reject a product in accordance, that is, we adopt a safety margin. This is why we say that type I error is the wrong decision for the supplier, as it rejects a conforming product.

Remembering:

$$z = \frac{(y - T_L)}{u}, \quad \text{lower limit} \tag{8.6}$$

And

$$z = \frac{(T_U - y)}{u}, \quad \text{upper limit} \tag{8.7}$$

where

Table 8.2 z values for Pc values (probability)

Pc	z
0.80	0.84
0.90	1.28
0.95	1.65
0.99	2.33
0.999	3.09

y—is the value of the measured quantity and we want to analyze.
u—is the value of standardized uncertainty of measurement.
Pc—is the probability that the product, specification, or variable analyzed follows the specification (Table 8.2).

Note that if $z \geq 1.64$, we will have the probability of 95 % or more of having an approved specification.

Example 8.3 (Source: JGCM 106:2012)
The rupture V_b voltage of a Zener diode is measured by producing a better estimate $v_b = -5.47$ V with standard uncertainty $u = 0.05$ V. The diode specification requires $V_b \leq -5.40$ V, which is an upper limit of the voltage. What is the probability of this diode conforming to the specification?

Using the Eq. (8.7): $z = \frac{[-5.40-(-5.47)]}{0.05}$

$$z = 1.40$$

$Pc = \Phi\ (1.40) = 0.9192$. There is a 92% probability that the diode conforms to the specification.

Example 8.4 (Source: JGCM 106:2012)
A metal container is tested destructively using pressurized water to measure its resistance to rupture B. The measurement produces a better estimate $b = 509.7$ kPa, with standard uncertainty associated $u = 8.6$ kPa. The container's specification requires $B \geq 490$ kPa, the lower limit of the rupture pressure.

Using the Eq. (8.6):

$$\mathrm{z} = \frac{(509.7 - 490)}{8.6} = 2.29$$

$Pc = \Phi\ (2.29) = 0.989$. There is a 98.9 % probability of the container conforming.

(d) **Bilateral tolerance interval with the normal curve**

As seen earlier, measuring Y obeys a normal distribution. The estimate y is in the interval of tolerance. Using Eqs. 8.6 and 8.7, we have:

$$P_c = \Phi\left(\frac{T_U - y}{u}\right) - \Phi\left(\frac{T_L - y}{u}\right) \tag{8.8}$$

Knowing the upper and lower tolerance limits T_U and T_L of a measurement process, how do we know if a measurement result has a certain probability of being within the tolerance limits?

Equation 8.8 allows us to answer this question. Let us look at the following example.

Example 8.5 (Source: JGCM 106:2012)
A SAE Grade 40 engine oil needs to have a kinematic viscosity Y to 100 °C not less than 12.5 mm^2/s and not greater than 16.3 mm^2/s. The kinematic viscosity of the sample at 100 °C has a value of y = 13.6 mm^2/s and a standard uncertainty of u = 1.8 mm^2/s. What is the probability that the engine oil conforms to the specification?

Solution: Adopting Eq. 8.8, we have:

$$P_c = \Phi\left(\frac{16.3 - 13.6}{1.8}\right) - \Phi\left(\frac{12.5 - 13.6}{1.8}\right)$$

$$P_c = \Phi\left(\frac{2.7}{1.8}\right) - \Phi\left(\frac{-1.1}{1.8}\right)$$

$$P_c = \Phi(1.5) - \Phi(-0.6)$$

$$P_c = 0.9332 - 0.2743$$

$$P_c = 0.9332 - 0.2743$$

$$P_c = 0.6589$$

The probability of the engine oil sample as specified is 65.89 %. We obtain the probability of φ (1.5) by associating the value of z = 1.5 with its respective probability value (see Table 8.1), which gives us a value of 0.9332. Already the value φ (−0.6), we get the complement of φ (0.6), which is (1–0.7257 = 0.2743) (Fig. 8.8).

Example 8.6
Consider a measurement estimate y = 23.5 kN with a standard uncertainty u-(y) = 0.5 kN, a tolerance range of [22 kN, 25 kN], and a 95 % conformity specification, thus assuming a type I error of 5 %.

With the experimental result and the interval of tolerance, assuming a Gaussian, the decision rule will be accepted if hypothesis H_0: Pc (22 ≤ Y ≤ 25) ≥ 0.95 is true.

To estimate probabilities related to the given example, the probability of compliance (Pc) needs to be calculated using Eq. 8.8.

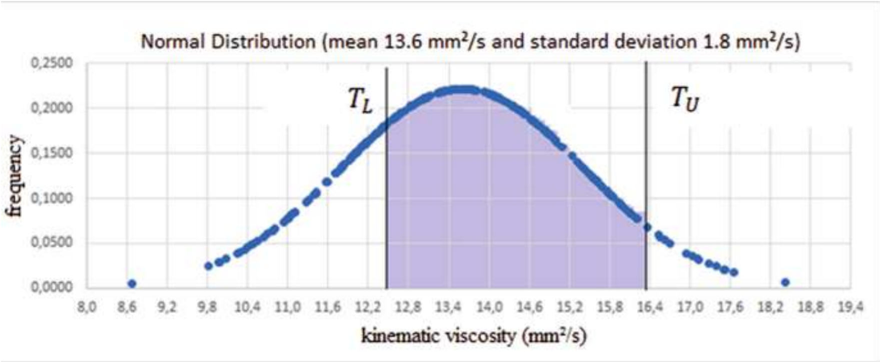


Fig. 8.8 Normal distribution refers to the kinematic viscosity measurement in example 8.5. Note that the hatched area comprises a 65.89 % probability that the measurement (13.6 ± 1.8) mm^2/s complies with the product specifications

Table 8.3 Probabilities calculated at some points *y* of Example 8.6

y (kN)	$\Phi 1\left(\frac{T_U - y}{u}\right)$	$\Phi 2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
22.0	6.00	0.00	1.00	0.50	50%
22.5	5.00	−1.00	1.00	0.16	84%
22.8	4.40	−1.60	1.00	0.05	95%
23.0	4.00	−2.00	1.00	0.02	98%
23.5	3.00	−3.00	1.00	0.00	100%
24.0	2.00	−4.00	0.98	0.00	98%
24.2	1.60	−4.40	0.95	0.00	95%
24.5	1.00	−5.00	0.84	0.00	84%
25.0	0.00	−6.00	0.50	0.00	50%

$$P_c = \Phi\left(\frac{T_U - y}{u}\right) - \Phi\left(\frac{T_L - y}{u}\right)$$

$$P_c = \Phi\left(\frac{25 - 23.5}{0.5}\right) - \Phi\left(\frac{22 - 23.5}{0.5}\right)$$

$$P_c = \Phi(3) - \Phi(-3)$$

$$P_c = 0.9987 - 0.0013$$

$$P_c = 0.9974 = 99.7\%$$

As 99.7 % > 95 %, H_0 is true, and the decision conforms.

Table 8.3 represents the probabilities found for some *y* values measured. The *Pc* column is likely to see the measurement result within tolerance limits.

Analyzing the values obtained in Table 8.3, we find that if the acceptance criterion adopted is *Pc* = 95 %, the values equal to or less than 22.8 kN and equal to or higher than 24.2 kN would fail.

The question is: Is it possible to determine minimum and maximum values within the tolerance range, where all values found will comply with the criterion adopted?

To answer this question, the concept of a guard band was created.

8.6.2 Examples of Probability Estimate Using Guard Bands

Example 8.7

Suppose that in a line of rubber sandals production, we must control the mass of the essential raw material: rubber. Consider that the specification of the rubber mass required for a sandal is (8.0 ± 0.5) g. The scale used to control rubber weighing has an acceptance criterion (error + expanded uncertainty of the scale calibration certificate) less than or equal to 0.1 g and a resolution of 0.01 g. Based on this information, determine the lower and higher values where we have a probability of 95 % acceptance (risk to the 5 % producer).

Consider:

- $T_U = 8.5$ g.
- $T_L = 7.5$ g.
- $u = 0.05$ g ($U/2 = 0.1/2$).[2]

(a) Determining the upper acceptance limit to 95 %

$$Z_U = \left(\frac{T_U - y}{u}\right) \tag{8.9}$$

For 95 %, consulting Table 8.1, we have $z = 1.65$, replacing Eq. 8.9.

$$\left(\frac{8.5 - y}{0.05}\right) = 1.65 \rightarrow y = 8.42\ g$$

(b) Determining the lower acceptance limit to 95 %

$$Z_L = \left(\frac{y - T_L}{u}\right) \tag{8.10}$$

$$1.65 = \left(\frac{y - 7.5}{0.05}\right)$$

$$y = 7.58\ g$$

[2]Consider the coverage factor $k = 2$; 95.45 % of probability.

Table 8.4 Probabilities calculated in some possible values of the mass m of Example 8.7 The *Pc* column is likely to find the measurement result within tolerance limits

m(g)	$\Phi1\left(\frac{T_U-y}{u}\right)$	$\Phi2\left(\frac{T_L-y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	20.00	0.00	1.00	0.50	50%
7.58	18.40	−1.60	1.00	0.05	95%
7.60	18.00	−2.00	1.00	0.02	98%
7.70	16.00	−4.00	1.00	0.00	100%
7.80	14.00	−6.00	1.00	0.00	100%
7.90	12.00	−8.00	1.00	0.00	100%
8.00	10.00	−10.00	1.00	0.00	100%
8.10	8.00	−12.00	1.00	0.00	100%
8.20	6.00	−14.00	1.00	0.00	100%
8.30	4.00	−16.00	1.00	0.00	100%
8.40	2.00	−18.00	0.98	0.00	98%
8.42	1.60	−18.40	0.95	0.00	95%
8.50	0.00	−20.00	0.50	0.00	50%

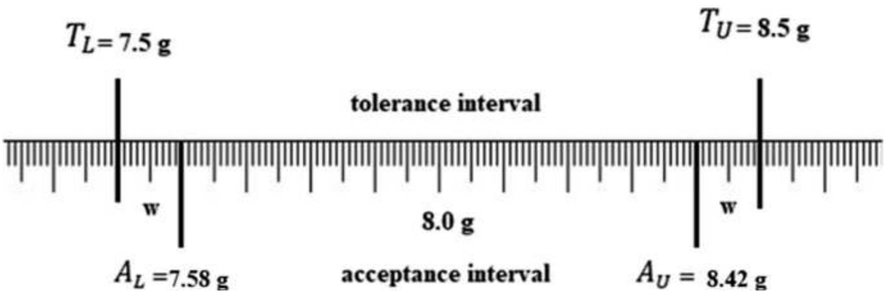


Fig. 8.9 Acceptance interval of Example 8.7 with guard band $w = 0.1$ g

Table 8.4 presents some possible mass (m) values of the rubber in question and its respective probabilities to be between the tolerance limits.

The values above 7.58 g and below 8.42 g are, respectively, called the lower acceptance limit (A_L) and higher acceptance limit (A_U) for a probability of 95 % (decision rule). Thus, we can establish:

$A_L = 7.58$ g; $A_U = 8.42$ g; Guard band (w) = 0.1 g (95 %)[3]

If we adopt the values 7.60 g and 8.40 g for acceptance limits, we could even consider a 98% decision rule for values accepted within the tolerance range. However, this value is not usual, and we maintain the criterion of 95 % (Fig. 8.9).

[3]This guard band can be adopted as a criterion for accepting the scales used in the control of rubber mass measuring used in the production of sandals. The error added to the uncertainty of the scale, obtained in its calibration certificate, cannot be greater than 0.1 g.

8.7 Risks Inherent in Decision-Making

In many cases, decision-making may have severe and harmful consequences and may lead to legal, criminal, accusatory, and even health damage.

Next, we will examine how to manage the risks inherent in making decisions based on safe and reliable measurements.

8.7.1 Protected Rejection—Categorical Decision

Protected rejection occurs when the measurement's result will result in a categorical decision, often with legal consequences, causing legal or high risk in decision-making. Let us look at the following example.

Example 8.8 (Source: JGCM 106:2012)
In the application of road law, the road police use devices such as radars to measure car velocity. A decision to issue a speed fine must be made with a high degree of confidence that the speed limit has been exceeded.

Using a radar, speed measurements in the field can be performed with a relative standard uncertainty of 2% within the interval of (50 to 150) km/h. In this interval, the measured velocity v is characterized by a normal distribution with a standard deviation of 0.02 v.

Under these conditions, one may ask: For a limit speed of 100 km/h (the maximum speed at which the driver does not take a fine), what should be the measured velocity by the radar to ensure, with a 99.9 % probability, that the driver exceeded the limit speed of 100 km/h?

This mathematical problem is equivalent to calculating a probability of conformity for a higher unilateral tolerance interval.

Note that z is given by Eq. (8.9) and that the probability of desired trust is 99.9 %, $z = 3.09$ (Table 8.1). Then we have:

$$z = \frac{(T_U - y)}{u}$$

$$3.08 = \frac{(T_U - 100)}{2}$$

$$\mathrm{T_U} = 106.16\ \mathrm{km/h}$$

The interval [100 km/h ≤ v ≤ 107 km/h] is a guard band that ensures a probability of at least 99.9 %. If the measured speed is over 107 km/h, the police will have 99.98 % confidence that the driver was above 100 km/h, and he will fine the violator.

Table 8.5 TUR = 5, without guard band

m(g)	$\Phi 1\left(\frac{T_U - y}{u}\right)$	$\Phi 2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	20.00	0.00	1.00	0.50	50%
7.58	18.40	−1.60	1.00	0.05	95%
7.60	18.00	−2.00	1.00	0.02	98%
7.70	16.00	−4.00	1.00	0.00	100%
7.80	14.00	−6.00	1.00	0.00	100%
7.90	12.00	−8.00	1.00	0.00	100%
8.00	10.00	−10.00	1.00	0.00	100%
8.10	8.00	−12.00	1.00	0.00	100%
8.20	6.00	−14.00	1.00	0.00	100%
8.30	4.00	−16.00	1.00	0.00	100%
8.40	2.00	−18.00	0.98	0.00	98%
8.42	1.60	−18.40	0.95	0.00	95%
8.50	0.00	−20.00	0.50	0.00	50%

8.7.2 Binary Decision Rule Applied to the Conformity Assessment Without Guard Band

Remembering, a binary decision rule exists when the result is limited to two options (approved or disapproved) and a non-binary decision rule when various terms can express the result (approved, conditional approval, conditional disapproval, disapproved).

We will start this theme by analyzing the situation in which we have a binary decision without using the guard band. In a binary statement where the simple acceptance rule is adopted ($w = 0$), we have the situations:

- Approved—the measured value is between the tolerance limits.
- Disapproved—the measured value is outside the tolerance limits.

Let us look at the following example.

Example 8.9
Analyzing the rubber mass specification of Example 8.7, we present the following questions:

1. What should be the uncertainty of the scale that will measure the value of the rubber used in the production of sandals?
2. Should this uncertainty be related to the tolerance of the process?

To answer these two questions, we must adopt a decision rule. Initially, we will adopt a decision rule without a guard band; that is, we are sharing the risk with the consumer of sandals. Knowing that the specification of the mass of rubber used in the production is (8.0 ± 0.5) g, we have $T_L = 7.5$ g and $T_U = 8.5$ g. If we adopt an expanded uncertainty of the scale according to Example 8.7, that is, $U = 0.1$ g (for 95.45 %), we will have (Table 8.5):

Table 8.6 *TUR* = 2, without guard band

m(g)	$\Phi 1\left(\frac{T_U - y}{u}\right)$	$\Phi 2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	8.00	0.00	1.00	0.50	50%
7.58	7.36	−0.64	1.00	0.26	74%
7.60	7.20	−0.80	1.00	0.21	79%
7.70	6.40	−1.60	1.00	0.05	95%
7.80	5.60	−2.40	1.00	0.01	99%
7.90	4.80	−3.20	1.00	0.00	100%
8.00	4.00	−4.00	1.00	0.00	100%
8.10	3.20	−4.80	1.00	0.00	100%
8.20	2.40	−5.60	0.99	0.00	99%
8.30	1.60	−6.40	0.95	0.00	95%
8.40	0.80	−7.20	0.79	0.00	79%
8.42	0.64	−7.36	0.74	0.00	74%
8.50	0.00	−8.00	0.50	0.00	50%

- Standardized uncertainty $u = 0.1/2 = 0.05$ g.
- Relationship between tolerance and expanded measurement uncertainty (*TUR*—Test Uncertainty Ratio)[4]

$$TUR = \frac{TL}{U} = \frac{0.5}{0.1} = 5$$

Note that if we accept the measurement of $m = 8.50$ g, we will have the shared decision with the customer since 8.50 g gives a probability of acceptance of 50 %. Even adopting a *TUR* equal to 5, as we do not have a guard band, we can have a low acceptance probability in some values within the tolerance range.

Adopting a *TUR* equal to 5 implies a scale with $U_{\max}$ or *PME* ($U + E$) equal to 0.1 g, for $k = 2.00$ and 95.45 %,

What if we adopt a *TUR* equal to 2? This would allow a balance with $PME = 0.25$ g. Let us look at the results (Table 8.6).

$$TUR = \frac{TL}{U}$$

$$U = \frac{T}{TUR} = \frac{0.5}{2} = 0.25$$

With *TUR* equal to 2, we have the same mass value; for example, $m = 8.42$ g, a probability of 74% acceptance, and with *TUR* equal to 5, this probability becomes 95%. If we want a probability of 95 %, this value for a *TUR* equal to 5 is equal to $m = 8.42$ g, and for a *TUR* equal to 2, it is equal to or less than $m = 8.30$ g.

[4] TL = tolerance limit = ½ tolerance

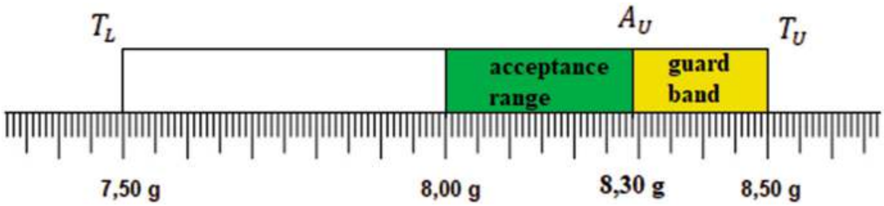


Fig. 8.10 *TUR* = 2, and point with 95% of acceptance (8.30 g)

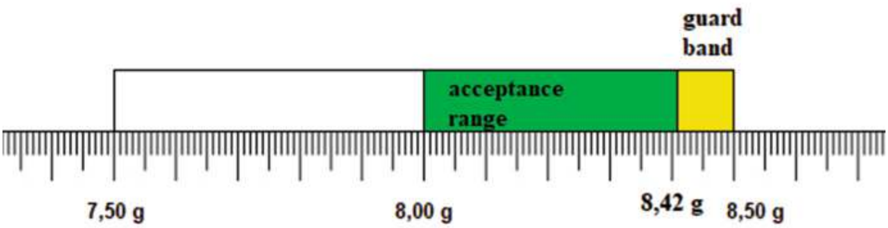


Fig. 8.11 *TUR* = 5, and point with 95 % of acceptance (8.42 g)

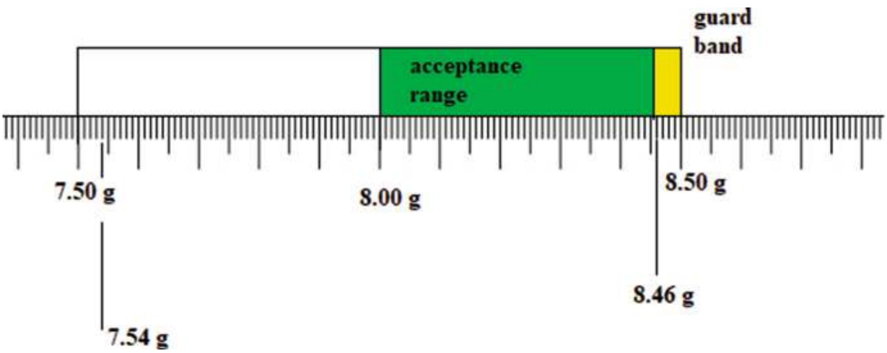


Fig. 8.12 *TUR* = 10, and point with 95% of acceptance (8.46 g)

Conclusion When the larger *TUR* will be the acceptance range, more measured values will be accepted in this interval.

Note Figs. 8.10 and 8.11, and notice that the acceptance range increased by increasing *TUR* for the same probability of acceptance.

The price of a larger *TUR* is to invest in the measuring instrument, in this case, the scale. For a *TUR* equal to 2, we have $U = 0.25$ g, and for a *TUR* equal to 5, $U = 0.1$ g.

TUR values range from 3 to 10, since *TUR* equal to 1 is to have an acceptance probability only for the central value, in our example, 8.00 g. *TUR* equal to 2 leaves a minimal acceptance range (Fig. 8.12) (Table 8.7).

Table 8.7 *TUR* = 10. Values for acceptance limit at 95 % equal to 7.54 g and 8.46 g

m(g)	$\Phi1\left(\frac{T_U - y}{u}\right)$	$\Phi2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	40.00	0.00	1.00	0.50	50%
7.54	38.40	−1.60	1.00	0.05	95%
7.60	36.00	−4.00	1.00	0.00	100%
7.70	32.00	−8.00	1.00	0.00	100%
7.80	28.00	−12.00	1.00	0.00	100%
7.90	24.00	−16.00	1.00	0.00	100%
8.00	20.00	−20.00	1.00	0.00	100%
8.10	16.00	−24.00	1.00	0.00	100%
8.20	12.00	−28.00	1.00	0.00	100%
8.30	8.00	−32.00	1.00	0.00	100%
8.40	4.00	−36.00	1.00	0.00	100%
8.46	1.60	−38.40	0.95	0.00	95%
8.50	0.00	−40.00	0.50	0.00	50%

Note how the guard band reduces and consequently increases the acceptance interval when the *TUR* increases to 10. This increases the chances of product acceptance but dramatically reduces the acceptance criterion and, consequently, the uncertainty of the measuring instrument.

8.8 Binary Decision Rule Applied to the Conformity Assessment with a Guard Band

When we adopt a binary decision rule in the conformity assessment, the measurement result is accepted if the measured value is within the acceptance interval. A value measured outside the acceptance interval leads to the item's rejection.

Using guard bands provides a way to limit the probability of making a decision incorrectly based on measurement information summarized by a probability interval.

The probabilities evaluated depend on two factors: (i) the measurement system and (ii) the production process.

If the measurement system were perfectly accurate, all decision-making would be correct, and the risks would be null. An increase in measurement uncertainty means an increase in the probability of an incorrect decision, and the probability is higher when the measured values are close to the tolerance limits.

Risks also depend on the nature of the production process. If the process rarely produces an item whose properties of interest are close to tolerance limits,[5] there is less opportunity to make incorrect decisions. On the other hand, if a process

[5] Process under control.

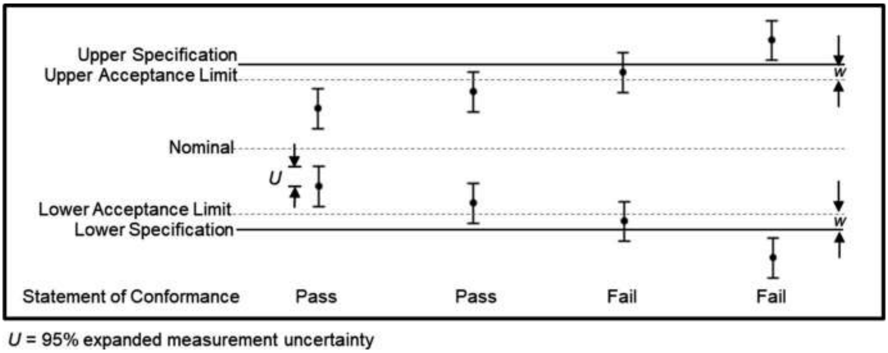


Fig. 8.13 Graphical representation of a binary statement with a guard band. (Source: ILAC G8:09/2019)

produces items with properties that are probably close to tolerance[6] limits, the uncertainties associated with measurements are placed in the game.

An out-of-control process, with a significant dispersion of measured values, produces values near tolerance limits, significantly increasing the probability of wrong decision-making.

The guard band's boundaries for a confidence level of $(1 - \alpha)$ can be obtained, considering a symmetrical error of $\alpha/2$ on each PDF tail.

Statements of conformity are reported as:

- Pass—acceptance is based on the acceptance range; the measurement result is between the acceptance limits.
- Fail—rejection based on the guard band if the measurement result exceeds acceptance limits (Fig. 8.13).

The adoption of a guard band gives protection to risk, according to the size of the band. We adopted the relationship $w = r{\cdot}U$, where r is the multiplicative factor to the guard band. Let us look at some typical guard bands and their respective r.

8.8.1 Guard Band with r = 1 (w = U*)—ILAC G8:2009 Rule Decision*

In this case, the guard band has the same value as expanded uncertainty. Applying the guard band in the example of sandal production, for a *TUR* equal to five, we have (Table 8.8):

[6]Process out of control.

Table 8.8 *TUR* = 5, guard band $w = U$

m(g)	$\Phi 1\left(\frac{T_U - y}{u}\right)$	$\Phi 2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	20.00	0.00	1.00	0.50	50.00%
7.54	19.20	−0.80	1.00	0.21	78.81%
7.60	18.00	−2.00	1.00	0.02	97.72%
7.70	16.00	−4.00	1.00	0.00	100.00%
7.80	14.00	−6.00	1.00	0.00	100.00%
7.90	12.00	−8.00	1.00	0.00	100.00%
8.00	10.00	−10.00	1.00	0.00	100.00%
8.10	8.00	−12.00	1.00	0.00	100.00%
8.20	6.00	−14.00	1.00	0.00	100.00%
8.30	4.00	−16.00	1.00	0.00	100.00%
8.40	2.00	−18.00	0.98	0.00	97.72%
8.46	0.80	−19.20	0.79	0.00	78.81%
8.50	0.00	−20.00	0.50	0.00	50.00%

Adopting $r = 1$, we will always have the acceptance limits with a probability of 97.72 % acceptance at risk of false positives less than 2.5 %.

Thus, for example, at 8.7, we will have a lower acceptance limit equal to 7.60 g and an upper acceptance limit of 8.40 g with a probability of false positive (consumer risk) below 2.5 %.

8.8.2 *Guard Band with r = 0.83 (w = 0.83* U*)—ISO 14253-1:2017 Rule Decision*

In this case, the protection band has a value of 0.83 of expanded uncertainty. We will apply the guard band to the sandal production example for a *TUR* equal to five. With the guard band $w = 0.83\ U$, the risk of false positives is less than 5 % for any value of $TUR > 1$ adopted.

Note that the $r \times U$ relationship is independent of the adopted *TUR* since when we change the *TUR*, we must change the uncertainty of measurement U and, with that, the size of the guard band w.

Thus, for example, at 8.7, we will have a lower acceptance limit equal to 7.58 g and an upper acceptance limit of 8.42 g, with a probability of false positive (consumer risk) below 5 %. The guard band will be equal to $w = 0.83 \times 0.1 = 0.083$ g (Table 8.9).

$$A_U = 8.50–0.083 = 8.471 = 8.42\ \text{g}$$

$$A_L = 7.50 + 0.083 = 7.583 = 7.58\ \text{g}$$

Table 8.9 TUR = 5, guard band $w = 0.83\ U$

m(g)	$\Phi1\left(\frac{T_U - y}{u}\right)$	$\Phi2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	20.00	0.00	1.00	0.50	50.00%
7.58	18.34	−1.66	1.00	0.05	95.15%
7.60	18.00	−2.00	1.00	0.02	97.72%
7.70	16.00	−4.00	1.00	0.00	100.00%
7.80	14.00	−6.00	1.00	0.00	100.00%
7.90	12.00	−8.00	1.00	0.00	100.00%
8.00	10.00	−10.00	1.00	0.00	100.00%
8.10	8.00	−12.00	1.00	0.00	100.00%
8.20	6.00	−14.00	1.00	0.00	100.00%
8.30	4.00	−16.00	1.00	0.00	100.00%
8.40	2.00	−18.00	0.98	0.00	97.72%
8.42	1.66	−18.34	0.95	0.00	95.15%
8.50	0.00	−20.00	0.50	0.00	50.00%

But if we consider, for example, a *TUR* equal to two, we will have $U = 0.25$ and $w = 0.25 \times 0.83 = 0.23$.

$$\mathrm{A_U} = 8.50 - 0.23 = 8.27\ \mathrm{g}$$

$$\mathrm{A_L} = 7.50 + 0.23 = 7.73\ \mathrm{g}$$

With the reduction of *TUR*, measurement uncertainty increases. This generates a lower acceptance interval, which may imply more rejected measurements.

8.8.3 Guard Band with r = 1.5 (w = 1.5 U)—Three Sigma Rule Decision

In this case, the protection band is 1.5 times expanded uncertainty. Applying the guard band in the example of sandal production, for a *TUR* equal to five, we have (Table 8.10):

With the guard band $w = 1.5$ U, we have, for any value of $TUR > 1$ adopted, a risk of false positive less than 0.16 % (100 % − 99.87 % = 0.13 %).

Thus, for example, at 8.7, we will have a lower acceptance limit equal to 7.65 g and an upper acceptance limit of 8.35 g, with a probability of false positives (consumer risk) below 0.16 %. The guard band will be equal to $w = 1.5 \times 0.1 = 0.15$ g.

Table 8.10 *TUR* = 5, guard band w = 1.5 *U*

m(g)	$\Phi 1\left(\frac{T_U - y}{u}\right)$	$\Phi 2\left(\frac{T_L - y}{u}\right)$	$Pc1$	$Pc2$	Pc
7.50	20.00	0.00	1.00	0.50	50.00%
7.60	18.00	−2.00	1.00	0.02	97.72%
7.65	17.00	−3.00	1.00	0.00	99.87%
7.70	16.00	−4.00	1.00	0.00	100.00%
7.80	14.00	−6.00	1.00	0.00	100.00%
7.90	12.00	−8.00	1.00	0.00	100.00%
8.00	10.00	−10.00	1.00	0.00	100.00%
8.10	8.00	−12.00	1.00	0.00	100.00%
8.20	6.00	−14.00	1.00	0.00	100.00%
8.30	4.00	−16.00	1.00	0.00	100.00%
8.35	3.00	−17.00	1.00	0.00	99.87%
8.40	2.00	−18.00	0.98	0.00	97.72%
8.50	0.00	−20.00	0.50	0.00	50.00%

Table 8.11 Probability of false positive (FP) for any *TUR* > 1

Decision rule	Guard band w	Risk
Six Sigma	3 *U*	<1 ppm FP
Three Sigma	1.5 *U*	<0.16 % FP
ILAC G8:2009	*U*	<2.5 % FP
ISO 14253-1:2017	0.83 *U*	<5 % FP
Simple acceptance	0	<50 % FP
Defined by the customer	$r \times U$	Customers can define r multiple arbitraries to apply as a guard band

8.8.4 Guard Band with r = 3 (w = 3 U)—Six Sigma Rule Decision

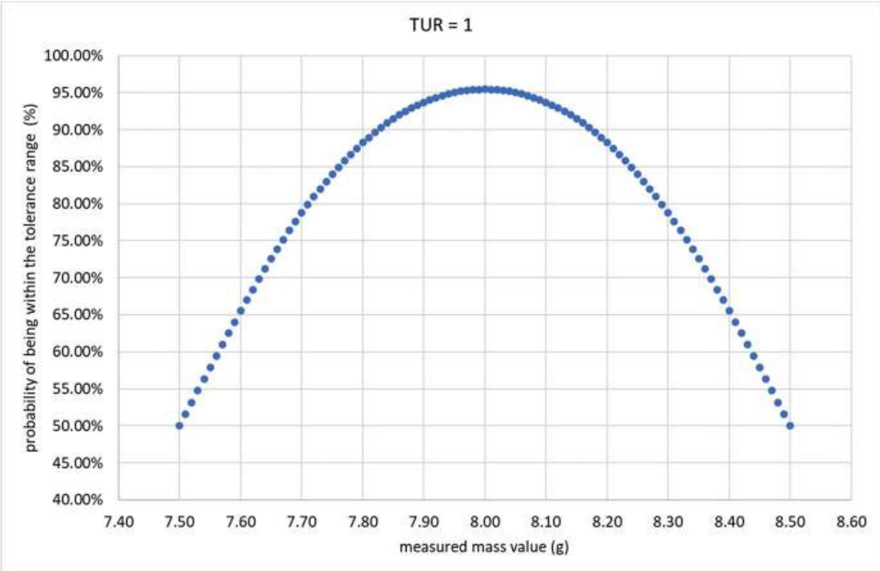
In this case, the protection band is three times expanded uncertainty. With the guard band $w = 3\ U$, we have, for any value of *TUR* > 1 adopted, a risk of false positive less than 1 ppm (0.0001 %).

Thus, at 8.7, we will have a lower acceptance limit equal to 7.80 g (7.50 + 0.3) and an upper acceptance limit of 8.20 g (8.50 − 0.3).

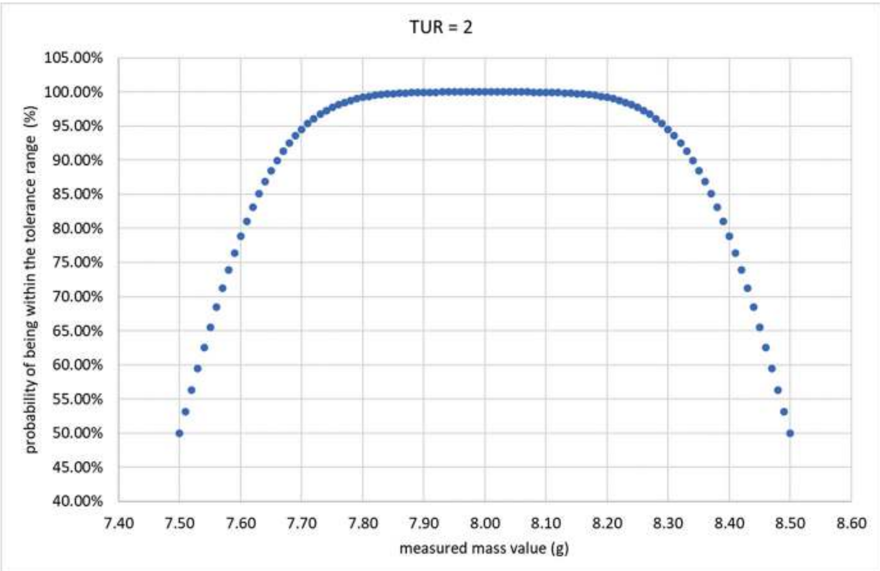
Table 8.11, copied from ILAC G8:09/2019 Guidelines on Decision Rules and Statements of Conformity, summarizes some of the guard band values presented in this chapter.

As we can observe in Graphs 8.1, 8.2, 8.3, 8.4 and 8.5, mass values between the tolerance limits increase with the highest probability when the *TUR* increases. This entails greater security in the results obtained.

Note that when *TUR* equals 1, a few values are approved for a given probability, for example, 95 %. For this reason, we should not adopt *TUR* equal to 1. When *TUR* equals 2, the values with the highest probability of occurrence within tolerance limits

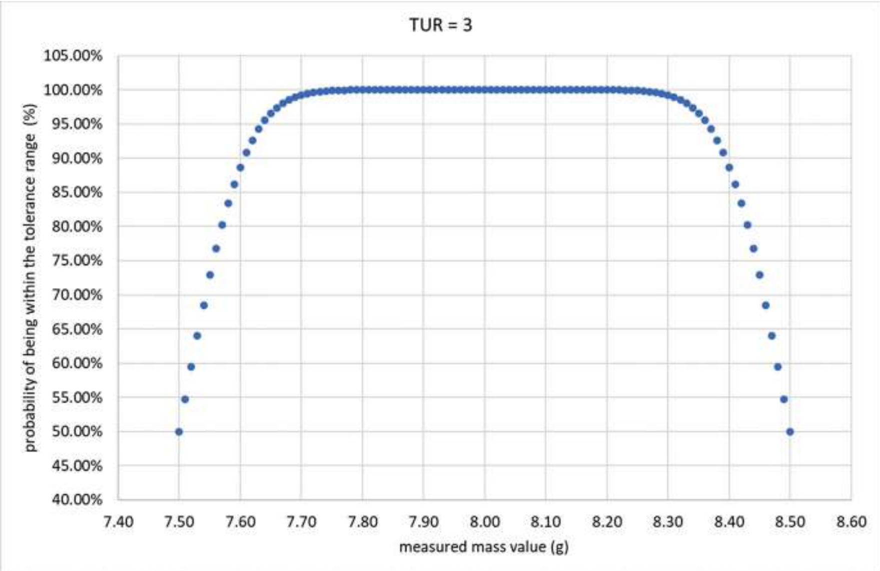


Graph 8.1 Probability × measured mass, for example 8.7 with *TUR* equal to one

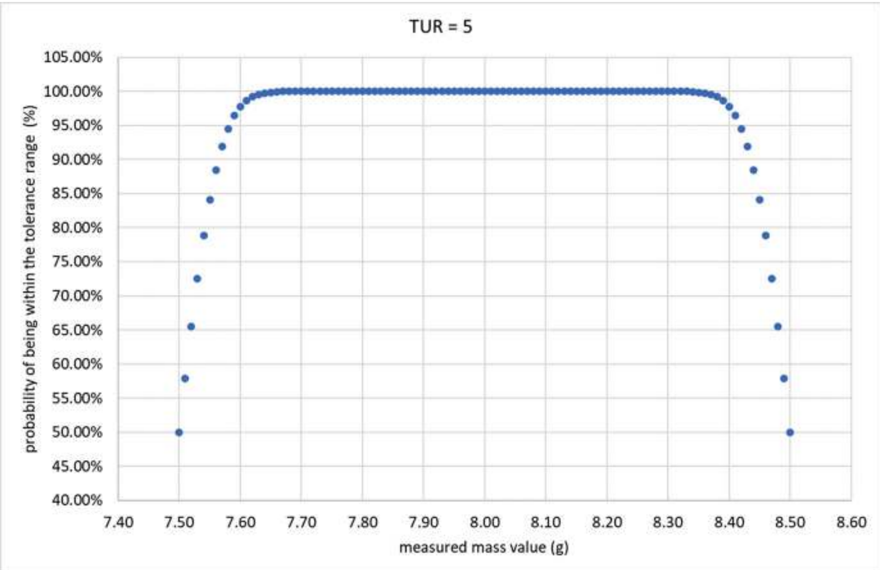


Graph 8.2 Probability × measured mass, for example 8.7 with *TUR* equal to two

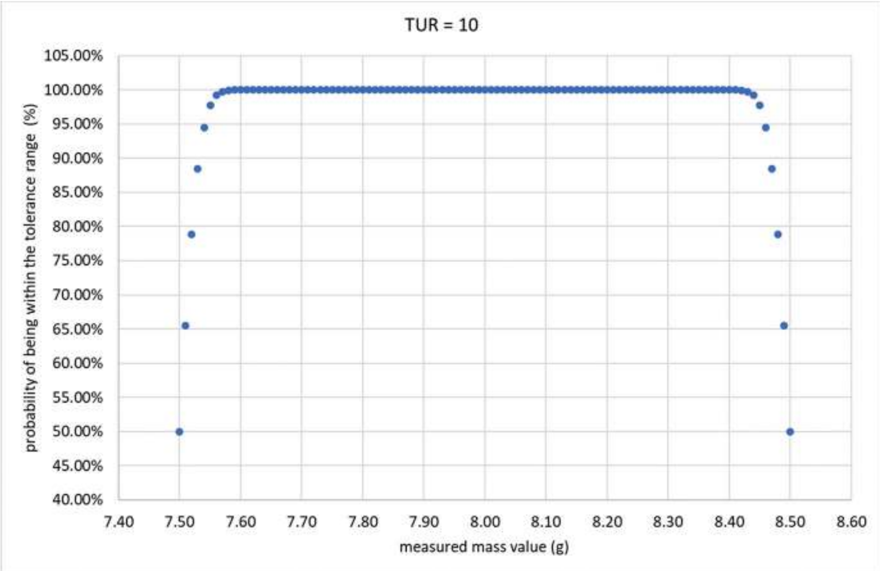
increase slightly. However, when *TUR* is equal to or greater than 3, we have a significant improvement, being its point of excellence when it approaches *TUR* equal to 10.



Graph 8.3 probability × measured mass, for example 8.7 with *TUR* equal to three



Graph 8.4 Probability × measured mass, for example 8.7 with *TUR* equal to five



Graph 8.5 Probability × measured mass, for example 8.7 with *TUR* equal to 10

8.9 Proposed Exercises

8.9.1 A "specific risk" is a probability that an accepted item is not in compliance or a rejected item is in accordance. Considering this definition, tick the correct alternative.

(a) The "specific consumer risk" is the probability of accepting an item according to, and the "specific risk of the producer" is the probability of rejecting an item not according to.
(b) The "specific consumer risk" is the probability of accepting an item not according to, and the "specific risk of the producer" is the probability of rejecting an item according to.
(c) The "specific risk of the producer" is the probability of accepting an item not according to, and the "specific consumer risk" is the probability of rejecting an item according to.
(d) The "specific consumer risk" and the "specific risk of the producer" will always be the same.

8.9.2. Check the alternative that best defines a binary decision rule.

(a) The result is limited to two options: approved/disapproved.
(b) We have two options; however, a guard band will always exist.
(c) When there is no possibility of using a non-binary decision rule.

(d) When the result has two options: approved/disapproved and conditional approved/conditional disapproved.

8.9.3. A gas container cylinder has a rupture pressure of 900 bar (lower tolerance limit). In a destructive test, the cylinder rupture pressure was 910 bar. What is the probability that the cylinder will meet the specifications? Consider the expanded measurement uncertainty U of the test equal to 10 bar ($k = 2.00$ and 95.45 %).

8.9.4. Consider the temperature measurement of an industrial process, with standard measurement uncertainty $u(y) = 0.5$ °C. The process temperature tolerance interval is 30.0 to 34.0 °C. Within this tolerance interval, determine the probability of finding the values of 34.0 °C, 32.0 °C, and 33.0 °C.

8.9.5. Check the alternative below that best defines the guard band and its use.

(a) Guard bands provide a straightforward way to define decision rules; by choosing the size of the guard band, we will have 100% product approval.
(b) The use of guard bands provides a particularly simple way to define decision rules; by choosing the size of the guard band, an acceptance zone with 95.45 % of the final product is defined.
(c) Guard bands provide a particularly simple way to define decision rules; by choosing the size of the guard band, an acceptance zone can be defined.
(d) Guard bands provide a particularly simple way to define decision rules; by choosing the size of the guard band, a rejection zone can be defined.

8.9.6. AS500 diesel oil should be a fuel with a maximum sulfur content of 500 mg/kg. What should be the maximum sulfur content found in an AS500 diesel sample so we have a 99 % probability of being out of the specification? Consider the expanded measurement uncertainty of the test $U = 4$ mg/kg ($k = 2.00$ and 95.45 %).

8.9.7. Check the alternative that best defines *TUR* and its application.

(a) *TUR* is the relationship between process tolerance and measurement uncertainty associated with the instrument that measures the quantity being evaluated in the process. For a given tolerance, increasing *TUR* means choosing instruments with more significant measurement uncertainty and, consequently, a higher acceptance interval.
(b) *TUR* is the relationship between process measurement uncertainty and tolerance associated with the instrument that measures the quantity being evaluated in the process. For a given tolerance, increasing *TUR* means choosing instruments with minor measurement uncertainty and, consequently, a minor acceptance interval.
(c) *TUR* is the relationship between process tolerance and measurement uncertainty associated with the instrument that measures the quantity being evaluated in the process. For a given tolerance, increasing *TUR* means choosing instruments with more significant measurement uncertainty and, consequently, a minor acceptance interval.

(d) *TUR* is the relationship between process tolerance and measurement uncertainty associated with the instrument that measures the quantity being evaluated in the process. For a given tolerance, increasing *TUR* means choosing instruments with minor measurement uncertainty and, consequently, a greater acceptance interval.

8.9.8. Consider the minimum value of the gasoline density at 20 °C as 715 kg/m^3. What should be the density value collected at a gas station so that we have a 99.9 % probability that it is adulterated (density below specification—a new lower tolerance limit)? Consider the expanded uncertainty of the analysis as 5 kg/m^3 to $k = 2.0$ and 95.45 %.

8.9.9. Consider a food industry that needs to monitor the temperature of a cooking process. The process temperature should be between 80 °C and 86 °C, not pass this interval. Based on this information, answer:

(a) What is the tolerance interval of this process?
(b) What should be the expanded uncertainty of the thermometer used in the temperature measurement if we adopt a $TUR = 3$?

8.9.10. Consider a honey processing industry that needs to transport honey in ducts to automate the flood. As we know, honey is a viscous product that is difficult to flow at ambient temperature. To facilitate the flow of the product, the piping is heated from 44.0 °C to 50.0 °C and should not pass this interval. Based on this information, answer:

(a) What are this process's upper and lower limits of tolerance?
(b) Adopting a *TUR* equal to 5 will define lower and higher acceptance limits for this process. Adopt a binary decision rule 3 sigma ($w = 1.5$ u).

Chapter 9
Critical Analysis of Calibration Certificate

9.1 Introduction

Sometimes, a result may fall within or outside the limit of a specification, but uncertainty can overlap with the limit, as shown in Fig. 9.1.

In Fig. 9.1, we see four cases of how a measurement result and its uncertainty may be within the limits of a specification or tolerance.

In case (a), both the result and uncertainty fall within the specified limits. This is classified as a "**compliance**."

In case (d), neither the result nor any part of the uncertainty range falls into the tolerance range. This is classified as "**non-compliance**."

In cases (b) and (c), measurements and their respective uncertainties are neither entirely within nor outside the boundaries. For this reason, these results are inconclusive and are in the **range of doubt**.

When conclusions are extracted from measurement results, measurement uncertainty should not be neglected. This is particularly important when measurements are used to verify that the result is within process tolerance or specification. Thus, we need to define an acceptance criterion for the instrument used.

This is the central theme of this chapter.

9.2 Calibration Certificate

In the International Vocabulary of Metrology (VIM—2.39), we found in Note 1 of the calibration definition: "*A calibration may be expressed by a statement, calibration function, calibration diagram, calibration curve, or calibration table. In some cases, it may consist of an additive or multiplicative correction of the indication with associated measurement uncertainty*."

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9_9

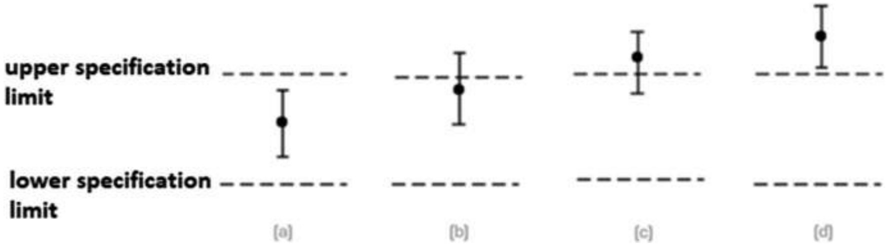


Fig. 9.1 Representation of four different measurement results within a tolerance interval. (Source: the authors)

We can consider that the document called Calibration Certificate, issued by the laboratory that performed the service, encompasses some or all expressions of calibration (declaration, function, diagram, curve, or table).

The certificate is an essential technical record and contains important information on the instrument calibration process. Through this reported information, it is possible to evaluate the conformity[1] of the measuring instrument.

The standard ISO/IEC 17025: 2017—General Requirements for the Competence of Test and Calibration Laboratories determines that the calibration results contained in a certificate must be presented clearly, objectively, and accurately, following the specific instructions of the calibration method. The certificate should also include all the information necessary for the correct interpretation of the results.

9.3 Calibration Certificate and ISO/IEC 17025

In Chap. 2, we present some management standards (ISO 9001, ISO/IEC 17025, and ISO 10012) that all highlight the importance of calibration of measurement instruments. However, none of them have a model or pattern for elaborating and presenting a calibration certificate.

ISO/IEC 17025, although it does not define the model, establishes in requirement 7.8—Results Report—what minimum information required should be included in a calibration certificate, namely:

(a) Title (e.g., calibration certificate).
(b) Name and address of the laboratory.
(c) Place of activities, including when carried out at the client's facilities or outside the laboratory's permanent facilities.

[1] According to ISO/IEC 17000: 2004, conformity assessment is any activity performed to determine, directly or indirectly, whether a product, process, system, person, or body meets relevant standards and complies with the specified requirements.

(d) Univocal identification that all components are recognized as part of a complete report and an identification of the end of the document.
(e) Client identification.
(f) Presentation of the method used in calibration.
(g) Identification of the calibrated instrument.
(h) Date of calibration.
(i) Date of issuance of the certificate.
(j) Declaration that the results apply only to the calibrated instrument.
(k) Presentation of calibration results, with their respective units of measure.
(l) Name, function, and identification of the authorized person to issue the certificate.
(m) Declaration that the certificate should only be reproduced entirely.
(n) Environmental conditions from which calibration was performed.
(o) The uncertainties of measurement.
(p) Traceability of measurements.
(q) If there is any adjustment to the instrument, the results must be reported before and after the adjustments.
(r) There should be no recommendation on the next calibration date unless the customer has previously agreed.

In Fig. 9.2, we present an example of a fictitious glass liquid thermometer calibration certificate, issued by the PP & AM Calibration Laboratory. The letters highlighted signal the necessary minimum information, as established by ISO/IEC 17025. We have not included in the certificate the information related to the letters "*q*" (adjustment) and "*r*" (new calibration recommendation).

9.4 Interpretation of Metrological Requirements in Calibration Certificates

We reiterate that every measurement instrument important to the production process should be calibrated. This allows us to know the associated errors and uncertainties. The calibration certificate provides this information.

However, a certificate does not guarantee that the instrument meets the intended requirements for its application in the measurement process, that is, a calibrated instrument is not necessarily fit for use.

From the information found in the certificate, it is necessary to evaluate this content to validate the use of the instrument.

Solved Exercise 9.1: Analytical Scale

Consider the analytical scale calibration certificate (Fig. 9.3). What should be analyzed in this certificate to evaluate the scale conformity?

PP&AM *Calibration Laboratory*	***METROLOGY & UNCERTAINTY OF MEASUREMENT*** Address: 10012, Uncertainty Propagation Street E-mail: m&um@uncertainty.com Phone: 00 11 22 33 44 55 b,c

a **CALIBRATION CERTIFICATE Ner 1324 / 2024**

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street e
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: High Temperature
Description: Glass thermometer g
Model: Partial immersion
Serial number: 123321123321

Class: NA
Resolution: 0.1 °C
Range: (0 – 100) °C

Method and procedure used f

Calibration made by direct comparison, as described in the SOP 001 procedure - Standard Operating Procedure for glass thermometers.

Traceability p

Description	TAG	Model	Manufacturer	Certificate	Serial
Standard thermometer	Pt – 107	Pt-100 4 wires	Ohms	107/24	ABC123

Calibration results k

Indication °C	Standard °C	Object °C	Bias °C	Uncertainty °C o	*k*	Degree of freedom
0	0.00	0.1	0.1	0.2	2.37	8
10	10.00	10.0	0.0	0.2	2.05	47
20	20.00	20.2	0.2	0.2	2.00	Infinite
30	30.00	30.0	0.0	0.3	2.05	47
40	40.00	40.0	0.0	0.3	2.02	102
50	50.00	50.1	0.1	0.3	2.11	23
60	60.00	60.1	0.1	0.4	2.06	40
70	70.00	70.2	0.2	0.5	2.07	35
80	80.00	80.0	0.0	0.5	2.06	40
90	90.00	90.1	0.1	0.5	2.02	102
100	100.00	100.2	0.2	0.6	2.00	Infinite

Environmental data	Temperature °C n	(20.6 ± 0.5)	Humidity %	(56 ± 5)	Pressure hPa	(1 018 ± 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other even if it is similar. It is not allowed the partial reproduction of this document. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%. j m

Calibration date: 3/6/2024
Emission date: 3/6/2024 h, i

Galileu Galilei
Metrologist technician

l

Lord Kelvin
Authorized firmer

d Page 1/1

Fig. 9.2 Example of a calibration certificate with the minimum information required by ISO/IEC 17025: 2017

PP&AM *Calibration Laboratory*	***METROLOGY & UNCERTAINTY OF MEASUREMENT*** Address: 10012, Uncertainty Propagation Street E-mail: m&um@uncertainty.com Phone: 00 11 22 33 44 55

CALIBRATION CERTIFICATE N^{er} 98765 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: Digital Scales Co.	Class: I
Description: analytical scale	Resolution: 0.0001 g
Model: DAS-1	Range: (0.1 – 200) g
Serial number: 11111111111	Verification scale interval ***e***: 0.001 g

Method and procedure used

Calibration made by direct comparison, as described in the POE 004 procedure - Standard Operating Procedure for Analytical Scales.

Traceability

Description	TAG	Model	Manufacturer	Certificate	Serial
Set of standard weights	JMP-01	E2	XYZW	M2227/23	ABC123

Calibration results

Indication g	Standard g	Object g	Bias mg	Uncertainty mg	*k*	Degree of freedom
20	19.999560	20.0004	0.8	0.8	2.03	84
50	50.000270	50.0009	0.6	0.9	2.02	126
60	60.000150	60.0010	0.8	0.9	2.02	126
100	100.000540	100.0014	0.9	1.1	2.01	251
150	150.000000	149.9999	-0.1	1.4	2.00	Infinite
200	200.001180	200.0045	3.3	1.6	2.00	Infinite

Environmental data	**Temperature** °C	(20.6 ± 0.5)	**Humidity** %	(56 ± 5)	**Pressure** hPa	(1 018 ± 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other even if it is similar. It is not allowed the partial reproduction of this document. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%.

Calibration date: 6/8/2024
Emission date: 6/8/2024

Galileu Galilei
Metrologist technician

Lord Kelvin
Authorized firmer

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Fig. 9.3 Analytical scale calibration certificate

Solution:
First: Maximum Permissible Error (MPE).

The OIML R 76–1 Edition 2006 document—Non-Automatic Weighing Instruments, Part 1: Metrological and Technical Requirements—Tests, establishes for scales the value of the maximum permissible error on initial and in-service verification, applying increasing and decreasing loads according to the instrument accuracy class, as given in Table 9.1. The MPE in-service shall be twice the MPE on initial verification.

Table 9.1 Maximum permissible error for scales: OIML R 76-1

Maximum permissible errors in initial verification	For loads, m, expressed in verification scale intervals, *e*			
	Class I	Class II	Class III	Class IV
±0.5 *e*	$0 \leq m \leq 50{,}000$	$0 \leq m \leq 5000$	$0 \leq m \leq 500$	$0 \leq m \leq 50$
±1.0 *e*	$50{,}000 < m \leq 200{,}000$	$5000 < m \leq 20{,}000$	$500 < m \leq 2000$	$50 < m \leq 200$
±1.5 *e*	$200{,}000 < m$	$20{,}000 < m \leq 100{,}000$	$2000 < m \leq 10{,}000$	$200 < m \leq 1000$

Table 9.2 Maximum permissible error for scale

	Measurement range	
Class I	$0 \leq m \leq 50$ g ($0 \leq m \leq 50{,}000 \times 0.001$ g)	50 g $< m \leq 200$ g ($50 < m \leq 200{,}000 \times 0.001$ g)
Maximum permissible errors (initial verification)	± 0.5 mg (0.5 × 0.001)	± 1.0 mg (1 × 0.001)
Maximum permissible errors (in-service)	± 1.0 mg	± 2.0 mg

In the case of the scale under analysis, we can observe in the calibration certificate that the instrument is class I and the reading resolution is 0.0001 g and $e = 0.001$ g.

According Table 9.2 the maximum permissible error (initial and in-service) per measuring range will be:.

In the calibration certificate, we find the scale measurement bias. Comparing the measurement bias values indicated on the certificate with the maximum allowable error table by range, we can conclude that:

- $m \leq 50$ g: scale bias (0.8 mg) is lower than MPE in-service (1.0 mg) ⇨ APPROVED.
- 50 g $< m \leq$ 150 g: scale bias (0.9 mg) is lower than MPE in-service (2.0 mg) ⇨ APPROVED.
- 150 g $< m \leq$ 200 g: scale bias (3.3 mg) is higher than MPE in-service (2.0 mg) ⇨ DISAPPROVED!

Conclusion The scale needs to be adjusted to reduce its bias at the end of the range, but if the adjustment is not performed, it should only be used in measurements between 0 and 150 g.

Second: Final Uncertainty

We consider whether to weigh a particular substance in the laboratory using our calibrated scale. We also think that the mass value of this product must be understood in the interval of (50.0000 ± 0.0100) g, that is, it must be between 49.9900 g (T_L—the lower limit of tolerance) and 50.0100 g (T_U—the upper tolerance limit).

The decision rule to be adopted is to consider a false positive of 2.5 % or less.

According to Table 8.11 (Chap. 8), if we use a guard band equal to expanded uncertainty, we will have an acceptance interval where the probability of a value is not equal to or less than 2.5%. The scale sheet certificate states that the measurement uncertainty value (U) of 50 g is 0.9 mg (Fig. 9.4).

So, $T_L = 49.9900$ g and $T_U = 50.0100$ g. The control limits (A_L and A_U) are:

$$A_L = T_L + U = 49.9900 + 0.0009 = 49.9909 \text{ g}$$

$$A_U = T_U - U = 50.0100 - 0.0009 = 50.0091 \text{ g}$$

According to the decision rule adopted, the mass result will always be acceptable if it falls within the acceptance interval of 49.9909 g to 50.0091 g.

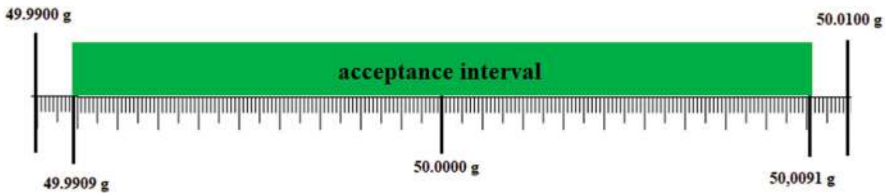

Fig. 9.4 Acceptance interval—Solved Exercise 9.1

> **Attention**
> We must correct the mass reading value by eliminating its bias.

Note that the acceptance interval was large, showing that we have many possible mass values . This is because the *TUR* adopted for this scale is high. Let us see:

$$TUR = \frac{Tolerance/2}{Uncertainty}$$

$$TUR = \frac{0.01}{0.0009} = 11.1$$

> **Knowing a Little More ...**
> **Scale verification**
>
> Among the calibration intervals, the instrument's user should verify the balance. The OIML R 76-1 (3.7.1 weights) document defines that the maximum permissible error of the default mass used in the scale verification or calibration shall be 1/3 of the scale maximum permissible error at the point considered. *"In principle, the standard weights or standard masses used for the type examination or verification of an instrument shall meet the metrological requirements of OIML R 111. They shall not have an error greater than 1/3 of the maximum permissible error of the instrument for the applied load. If they belong to class E2 or better, their uncertainty (rather than their error) is allowed to be not greater than 1/3 of the maximum permissible error of the instrument for the applied load, provided that the actual conventional mass and the estimated long-term stability are taken into account."*
>
> The choice of mass classes for scale calibration or verification should be compatible with the scale's maximum permissible error.
>
> Table 9.2 Shows the maximum permissible error by measurement range for our scale of solved exercise 9.1.
>
> Then, up to 50 g:

(continued)

$$MPE_{mass} \leq \frac{MPE_{balance}}{3}$$

$$MPE_{mass} \leq \frac{1\ mg}{3} = 0.333\ mg$$

Over 50 g:

$$MPE_{mass} \leq \frac{2\ mg}{3} = 0.666\ mg$$

Referring to Table 7.1 (Chap. 7):

- Up to 50 g—Class F_1 mass.
- Between 50 g and 200 g—Class E_2 mass.

Suppose the bias presented in the mass calibration certificate to be used in the scale verification is greater than the maximum permissible error of the class. In that case, the mass changes to a lower class.

9.5 Acceptance Criterion (AC) of a Measurement Instrument

To date, we have discussed the analysis of a calibration certificate by verifying if it is fit for use and meets the requirements of technical regulation, as well as calculating the upper (A_U) and lower (A_L) control limits.

In this section, we will analyze a calibration certificate by verifying whether the measurement instrument is approved for use in an industrial measurement process.

The first step is to know the acceptance criterion adopted by the industry. The acceptance criterion considers the maximum uncertainty of the instrument used to verify the product's conformity with its specification. Since the acceptance criterion is a fraction of tolerance, it will always be less than the process tolerance.

To determine the AC, we must adopt the following relationship:

$$AC = \frac{TL}{TUR} \tag{9.1}$$

TUR ranges from 3 to 10, and TL is the process tolerance interval.

In Solved Exercise 9.1, the tolerance interval (TL) is equal to 0.01 g for measurement at point 50 g.

If we adopt:

(a) $TUR = 10 \rightarrow AC = 0.001$ g $\rightarrow$ $A_L = 49.9900 + 0.001 = 49.9910$ g and. $A_U = 50.0100–0.001 = 50.0090$ g.

Acceptance interval = [49.9910 to 50.0090] g.

(b) $TUR = 5 \rightarrow AC = 0.002$ g $\rightarrow$ $A_L = 49.9900 + 0.002 = 49.9920$ g and.
$A_U = 50.0100–0.002 = 50.0080$ g.

Acceptance interval = [49.9920 to 50.0080] g.

(c) $TUR = 3 \rightarrow AC = 0.0033$ g $\rightarrow$ $A_L = 49.9900 + 0.0033 = 49.9933$ g and.
$A_U = 50.0100–0.0033 = 50.0067$ g.

Acceptance interval = [49.9933 to 50.0067] g.

We can observe that the larger the *TUR*, the closer the acceptance limits will be to the limits of specification or tolerance, which is very good but can also cause problems. Let us see.

In item (a), *AC* for 50 g was 0.001 g. Is this scale approved or disapproved for use?

If we perform the bias correction for 50 g when we measure, we will compare the *AC* with the uncertainty of point measurement, which is worth 0.0009 g (see certificate Fig. 9.3).

U (0.0009 g) ≤ A.C (0.001 g)—Approved!

However, if we do not perform the bias correction, we must add the bias module with expanded uncertainty at the desired point, in this case, 50 g. This we call **maximum uncertainty**.

$$U_{\text{maximum}} = |B| ou\ |E| + U \tag{9.2}$$

$$U_{\text{maximum}} = 0.0006 + 0.0009 = 0.0015 \text{ g}$$

In this case, the scale is Disapproved!

$$U_{\text{maximum}} \ \ (0.0015 \text{ g}) \geq \text{AC} \ \ (0.001 \text{ g})$$

Attention
We must correct the error or bias at the measurement point whenever possible. Thus, we eliminate systematic error and only have random error, the measurement uncertainty.

Now, analyzing item (c), the *AC* for point 50 g was 0.0033 g. Is the scale approved for this use?

If, when we measure, we perform the bias for point 50 g, we will compare the *AC* with the uncertainty of point measurement, which is worth 0.0009 g (see certificate Fig. 9.3).

U (0.0009 g) ≤ AC (0.0033 g) — Approved!

But, if we do not perform the bias correction, we will have:

$$U_{\text{maximum}} = 0.0006 + 0.0009 = 0.0015 \text{ g}$$

And the scale will remain approved.

$$U_{\text{maximum}} \ \ (0.0015 \text{ g}) \leq AC \ \ (0.0033 \text{ g})$$

The advantage of *TUR* being large and, consequently, the small *AC* is that we will be close to the specification limit and thus reject a few parts (products). The significant disadvantage is that we will disapprove of many measuring instruments or cannot find one that satisfies this requirement.

So far, we have analyzed the criterion of accepting an instrument for a desired point. Still, the usual is to explore the instrument as a whole for a particular measurement process.

Let us continue with the example of the Solved Exercise 9.1; however, now, no longer for a single point, but analyzing the scale as a whole.

We want to adopt an acceptance criterion for the scale, whose certificate is presented in Fig. 9.3. Let us consider that the tolerance interval of the mass measurement process is ±0.01 g. What acceptance criteria will we adopt for this process?

To answer this question, we must analyze the measurement process performed and verify that the measured values of the mass, for any point in your measurement range, are under control if we do not have significant variability in the results obtained from mass measurement.

Processes with high variability and large dispersions generate high uncertainties, which can easily lead to values outside or close to tolerance limits, which should be avoided.

To decide how many parts we will divide the tolerance interval (*TL*) of our process, we must take into account some factors:

(a) Is the process under control? That is, the values of the variables under measurement have little or no variation. If so, we can divide by a small *TUR*, for example, 3 or 4. If not, we should divide by a large *TUR*, for example, 8 or 10.
(b) Can the instrument we choose to control the process meet our AC? Is your error or bias added to your measurement uncertainty inferior or equal to the *AC*? A good choice is associated with the resolution of the instrument. It is known, from experience, that the uncertainty of measuring an instrument in perfect condition usually has its value guided by its resolution. We must always choose an instrument that has better resolution than process tolerance. Typically, resolution is ten times less than process tolerance.
(c) Often, an instrument of measurement has its maximum permissible error (MPE) defined by standard or technical regulation. We cannot choose an instrument whose *AC* is greater than the MPE.

Consider that the scale measurement of the Solved Exercise 9.1 is under control, with little or no variability. So, let us divide the process tolerance interval by four.

Table 9.3 Calibration results

Calibration results						
Indication g	Standard g	Object g	Bias mg	Uncertainty mg	*k*	Degree of freedom
20	19.999560	20.0004	0.8	0.8	2.03	84
50	50.000270	50.0009	0.6	0.9	2.02	126
60	60.000150	60.0010	0.8	0.9	2.02	126
100	100.000540	100.0014	0.9	1.1	2.01	251
150	150.000000	149.9999	-0.1	1.4	2.00	Infinite
200	200.001180	200.0045	3.3	1.6	2.00	Infinite

$$AC = \frac{0.01}{4} = 0.0025\ g$$

If we adopt this acceptance criterion, will the balance of Solved Exercise 9.1 be approved for use in this measurement process? How will we analyze the balance for the entire measurement range we have? (Table 9.3)

The highest uncertainty of scale measurement is 1.6 mg (0.0016 g). If we correct the bias, this measurement uncertainty will be enough to compare with the adopted *AC*.

$$U\ \ (0.0016\ \text{g}) \leq AC\ \ (0.0025\ \text{g})\ \text{ - Approved!}$$

But if we do not perform the reading correction, we will have:

$$U_{\text{maximum}}\ \ (0.0016\ \text{g} + 0.0033\ \ \text{g}) \geq AC\ \ (0.0025\ \text{g})\ \text{ -Disapproved!}$$

In addition to defining an acceptance criterion, we must define a decision rule.

Attention!
Evaluating the results of a calibration without defining acceptance criteria is not very useful. The acceptance criterion determines whether a measurement instrument is approved for the required use.

To define the acceptance criteria, we must take into account at least the following factors:

- Measurement variability—We want processes under control;
- Maximum permissible error by the method—We do not wish to *AC* superior to the MPE;
- Accuracy required by the method;
- Maximum uncertainty accepted for measurements.

Table 9.4 Calibration certificate

Calibration certificate—Class A					
Nominal value mL	Measured value mL	Error mL	Uncertainty mL	Coverage factor (95.45 %)	Degree of freedom
500	500.145	0.145	0.008	2.00	∞

Table 9.5 MPE for volumetric balloon

Capacity (mL)	MPE (mL)	
	Class A	Class B
5	0.02	0.04
10	0.02	0.04
25	0.03	0.06
50	0.05	0.10
100	0.08	0.16
250	0.10	0.20
500	0.12	0.24
1000	0.20	0.40
2000	0.30	0.60
4000	0.50	1.00

Solved Exercise 9.2: Volumetric Balloon (Laboratory Glassware).
Analyze the data from the volumetric calibration certificate comparing with ASTM E 288, and consider the volumetric balloon *AC* as 0.05 mL (Table 9.4).

Solution
(a) MPE.

According to ASTM E 288, we have the following MPEs for volumetric balloons (Table 9.5):

In Table 9.5, the maximum error for a volume of 500 mL and class A is 0.12 mL. As the error presented in the calibration was 0.145 mL, the volumetric balloon can no longer be considered class A and moves to class B (maximum error of 0.24 mL).

(b) Maximum uncertainty.

$$U_{\text{maximum}} = |E| + |U|$$

$$U_{\text{maximum}} = (0.145 + 0.008)\ mL = 0.153\ mL$$

$$U_{\text{maximum}} = 0.15\ \text{ mL} > AC = 0.05\ \text{mL} \quad \rightarrow \quad \text{Disapproved}$$

Knowing a Little More ...
Laboratory glassware calibration

Glassware used in laboratories is classified according to the maximum acceptable errors in class A and class B. Class B is calibrated with errors that usually comprise twice the error allowed for class A. Beyond this classification, they are also known as:

- *TD* (to deliver) glassware—Indicate the volume raised or transferred by glassware (e.g., pipette and burette).
- *TC* (to contain) glassware—Indicate the volume contained by glassware (e.g., volumetric balloon).

According to the ASTM E542 standard, glassware may remain indefinitely calibrated if it is not subjected to extreme conditions, such as temperatures above 150 °C or contact with fluoride acid, heated phosphoric acid, or heated strong bases.

Practical recommendation: Calibrate every 5 years of use or when the surface indicates wear.

Equipment required for the calibration of laboratory glassware (Table 9.6):

Glassware verification

Among the calibration intervals, the glassware verification is performed at least once a year at the reference temperature of 20 °C (water and laboratory environment) and using distilled water. Table 9.7 relates the glassware capacity to the scale used in the verification.

Volumetric calibration is performed using distilled water, with its density taken into account. The equation used is:

$$V_{20^\circ C} = \frac{m}{\rho}$$

The mass $\boldsymbol{m}$ is the difference between the mass of full and empty glassware.

The water density can be corrected, as per Table 9.8, if the water temperature is not 20 ° C.

Glassware MPE (Tables 9.9, 9.10, 9.11, and 9.12)

Table 9.6 Instruments used in the glassware calibration

Quantity	Equipment	Measurement range	Minimum resolution
Ambient and water temperature	Thermometer	(15 ± 5) °C	0.1 °C
Pipette and burette flow time	Timer	15 min	1 s
Relative humidity	Hygrometer	(50 ± 30) %	1 %
Atmospheric pressure	Barometer	(1000 ± 100) hPa	1 hPa

Table 9.7 Glassware capacity × scale resolution

Capacity	Resolution (mg)
$1\ \mu L \leq V_{20\ °C} \leq 10\ \mu L$	0.001
$10\ \mu L \leq V_{20\ °C} \leq 100\ \mu L$	0.01
$100\ \mu L \leq V_{20\ °C} \leq 1000\ \mu L$	0.1
$1\ mL \leq V_{20\ °C} \leq 10\ mL$	0.1
$10\ mL \leq V_{20\ °C} \leq 200\ mL$	1
$200\ mL \leq V_{20\ °C} \leq 1000\ mL$	10

Table 9.8 Water density × temperature

Temperature (°C)	Density (g/cm^3)
18	0.99860
19	0.99840
20	0.99820
21	0.99799
22	0.99777

Table 9.9 Pipettes with piston (ISO 8655-2:2002)

Capacity (μL)	MPE (μL)
1	0.05
10	0.12
100	0.8
1000	8
10,000	60

Table 9.10 Burettes (ASTM E287)

Capacity (mL)	MPE (mL) ±	
	Class A	Class B
10	0.02	0.04
25	0.03	0.06
50	0.05	0.10
100	0.10	0.20

Table 9.11 Volumetric pipettes (ASTM E969)

Capacity (mL)	MPE (mL) ±	
	Class A	Class B
0.5	0.006	0.012
1	0.006	0.012
2	0.006	0.012
3	0.01	0.02
4	0.01	0.02
5	0.01	0.02
6	0.01	0.03
7	0.01	0.03
8	0.02	0.04
9	0.02	0.04
10	0.02	0.04
15	0.03	0.06
20	0.03	0.06
25	0.03	0.06
30	0.03	0.06

Table 9.12 Cylinders and beakers (ASTM E1272)

Capacity (mL)	MPE (mL) ±	
	Class A	Class B
5	0.05	0.10
10	0.10	0.20
25	0.17	0.34
50	0.25	0.50
100	0.50	1.00
250	1.00	2.00
500	2.00	4.00
1000	3.00	6.00
2000	6.00	12.00
4000	14.50	29.00

Solved Exercise 9.3: Elaboration of the Acceptance Criterion and Analysis of the Thermometer Calibration Certificate Used in the Process.

Consider a honey processing industry that needs to transport honey in ducts to automate the flood. As we know, honey is a viscous product that is difficult to flow at ambient temperature. To facilitate the flow of the product, the piping is heated between 44 ° C and 50 ° C and should not pass this interval. Adopt, as a decision rule, a false positive less than 2.5 %. Reply:

(a) What should be the mean process control temperature?
(b) What is the tolerance interval of this process?
(c) Considering the process is under control, what should the *AC* for the thermometers used to control this temperature be?

(d) In the market, is there a thermometer to serve the established *AC*? Which one? What should its resolution be?
(e) How should this process be controlled for the statement decision rule?
(f) How should this process be controlled if we consider simple acceptance?
(g) Considering the calibration certificate of one of the thermometers adopted in the measurement process, such as shown in Fig. 9.6, answer if the thermometer is approved concerning the *AC* adopted in item *c*.

Solution:

(a) Mean temperature = 47 °C.
(b) Tolerance interval—*TL* = ± 3 °C.
(c) Considering the process under control, we can use a *TUR* = 3.

$$AC = \frac{\frac{(50-44)}{2}}{3} = \frac{3}{3} = 1\,^{\circ}\mathrm{C}$$

(d) Yes. Resistance thermometer (PT-100 3 wires) or type *K* thermocouple, both with resolution 0.1 ° C. Ensuring that the instrument's resolution is ten times lower than *AC* is always essential.
(e) In Table 8.11, we have that the guard band used for a probability of false positive 2.5 % is equal to the *AC* (Fig. 9.5).

As the acceptance limit is between (45 and 49) °C, we must inform the temperature controller coupled to the thermometers that when the temperature reaches close to 45 °C, the thermal blanket that surrounds the metallic ducts should be turned on. When the temperature reaches 49 °C, it should be turned off. We will always have a safety margin, the 1 °C guard band, at risk of measurement outside the specification [44–50] °C less than 2.5 %.

(f) If we adopt a simple acceptance as a decision rule, we will have:

In this case, acceptance limits will equal tolerance limits, and we risk a false positive of less than 50%, especially when we approach tolerance limits.

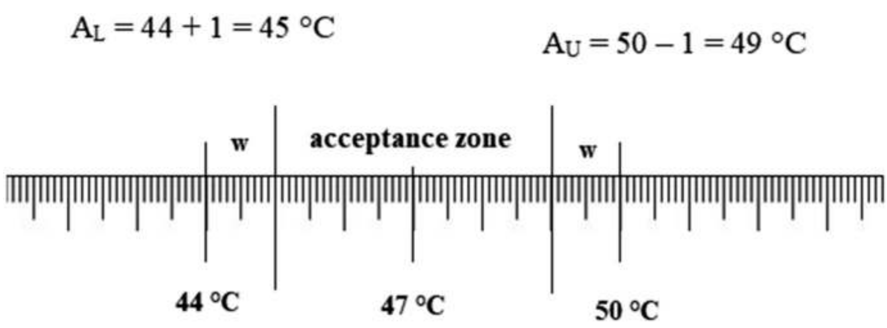

Fig. 9.5 Acceptance zone

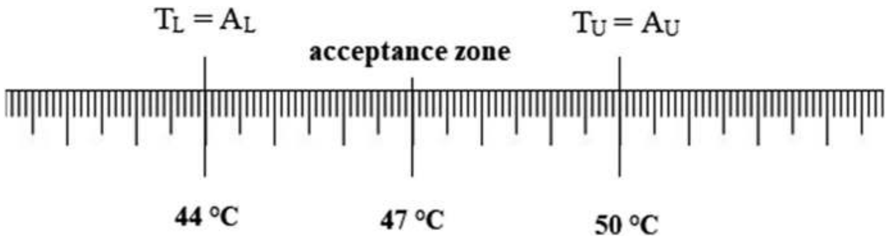

Fig. 9.6 Acceptance zone

(g) Note that the thermometer calibration certificate of Fig. 9.6 has its maximum uncertainty (U_{maximum}) in the use range (44–50) °C of 0.4 °C (Fig. 9.7).

$$B = 0.1\,^\circ C; U = 0.3\,^\circ C$$

$$0.1 + 0.3 = 0.4\,^\circ C$$

$$U\text{maximum}\ (0.4\,^\circ C) < AC\ (1\,^\circ C)$$

Thermometer approved for use in the process!

9.6 Proposed Exercises

9.5.1 Regarding the Calibration Certificate, check (*R*) for the correct statements and (*W*) for the wrong ones (Table 9.13).

9.5.2 Table 9.14 shows the calibration result of a Bourdon-type gauge, class 1.0, with a measurement range between 0 and 60 bar and 0.5 bar resolution.

Based Table 9.14, ensure the object gauge can continue to be used as class 1.0.

9.5.3 Consider the calibration certificate presented. According to ISO/IEC 17025: 2017, what information is needed and missing? (Fig. 9.8)

9.5.4 A 25 mL burette, class B, was calibrated, and the data obtained were: measured volume = 25.05 mL; uncertainty = 0.01 mL. Based on this information, ensure the burette can continue to be used as class B and meets an AC of 0.1 mL.

9.5.5 The pulley of an engine must have a width of (25.000 ± 0.012) mm and a diameter of (960.0 ± 1.5) mm. To measure the width, a micrometer with a maximum uncertainty of 0.002 mm and a digital tape with a maximum uncertainty of 0.5 mm for the diameter. What should be the control limits of the manufacture of this pulley considering these measuring instruments? Adopt a 95 % decision rule.

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CALIBRATION CERTIFICATE N^{er} 1234 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: High Temperature	Class: NA
Description: Glass thermometer	Resolution: 0.1 °C
Model: Partial immersion	Range: (0 – 100) °C
Serial number: 123321123321	

Method and procedure used

Calibration made by direct comparison, as described in the SOP 001 procedure - Standard Operating Procedure for glass thermometers.

Traceability

Description	TAG	Model	Manufacturer	Certificate	Serial
Standard thermometer	Pt – 107	Pt-100 4 wires	Ohms	107/24	ABC123

Calibration results

Indication °C	Standard °C	Object °C	Bias °C	Uncertainty °C	k	Degree of freedom
0	0.00	0,1	0,1	0.2	2.37	8
10	10.00	10.0	0.0	0.2	2.05	47
20	20.00	20.2	0.2	0.2	2.00	Infinite
30	30.00	30.0	0.0	0.3	2.05	47
40	40.00	40.0	0.0	0.3	2.02	102
50	50.00	50.1	0.1	0.3	2.11	23
60	60.00	60.1	0.1	0.4	2.06	40
70	70.00	70.2	0.2	0.5	2.07	35
80	80.00	80.0	0.0	0.5	2.06	40
90	90.00	90.1	0.1	0.5	2.02	102
100	100.00	100.2	0.2	0.6	2.00	Infinite

Environmental data	**Temperature** °C	(20.6 ± 0.5)	**Humidity** %	(56 ± 5)	**Pressure** hPa	(1 018 ± 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

These results refer exclusively to the object described in this document in the specified conditions, not extending to any other even if it is similar. It is not allowed the partial reproduction of this document. Expanded uncertainty (U) reported corresponds to a coverage probability of 95.45%.

Calibration date: 3/6/2024
Emission date: 3/6/2024

Galileu Galilei	Lord Kelvin
Metrologist technician	Authorized firmer

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Fig. 9.7 Thermometer calibration certificate for Solved Exercise 9.3

Table 9.13 Calibration certificate

( )	The calibration certificate is a technical record that enables the user to measure instrument conformity assessment.
( )	The laboratory that performs the measuring instrument calibration will never make any recommendation on the periodicity of calibration.
( )	The calibration laboratory does not usually adjust the measurement instrument; however, results should be reported before and after adjustment if this is done.
( )	The certificate must present calibration results in the units of measure used and declare the measurement uncertainty, scope factor, and confidence level used.
( )	The environmental conditions of the calibration site only need to be declared if they affect the calibration result.

Table 9.14 Gauge calibration result

Object (bar)	Standard (bar)			
	Charge 1	Discharge 1	Charge 2	Discharge 2
5.0	5.00	5.20	5.25	5.25
15.0	15.25	15.55	15.00	15.50
25.0	25.00	25.55	25.50	25.55
35.0	35.25	35.00	35.50	35.25
45.0	44.55	45.05	45.00	45.50
55.0	56.00	56.00	55.55	55.50
60.0	60.00	60.00	60.00	60.00

9.5.6 A micrometer was adopted for quality control in serial production of a length part (15.00 ± 0.05) mm. Considering the acceptance criterion (AC) as 0.017 mm (1/3 of tolerance), determine the upper control limits (A_U) and the lower control limit (A_L) of the part. Adopt the false positive decision rule of less than 2.5 %.

9.5.7 How can we prove whether a measurement instrument meets the desired acceptance criterion?

(a) Buying the instrument indicated by the manufacturer.
(b) Calibrating the instrument in a competent laboratory and analyzing its certificate.
(c) Performing checks with a standard in the company itself.
(d) Authorizing its use only by qualified personnel.

9.5.8 What is the main objective of calibrating a measurement instrument?

(a) Know the errors and uncertainty of measurement at each calibrated point and correct, if necessary, the instrument's readings.
(b) Meet management standards applied to measurement instruments.
(c) Obtain the calibration certificate to guarantee the measurement instrument is in perfect condition.
(d) Make adjustments by minimizing their measurement errors.

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CALIBRATION CERTIFICATE N^{er} 98765 / 2024

Customer Information

Company:	PPN&AM Metrology
Address:	17025, Measurement Error Street
E-mail:	ppn&am@ppn&am.com
Phone:	00 34 91 01 02 03

Calibrated object information

Manufacturer: Digital Scales Co.	Class: I
Description: analytical scale	Resolution: 0.0001 g
Model: DAS-1	Range: (0.1 – 200) g
Serial number: 11111111111	Verification scale interval **e**: 0.001 g

Traceability

Description	TAG	Model	Manufacturer	Certificate	Serial
Set of standard weights	JMP-01	E2	XYZW	M2227/23	ABC123

Calibration results

Indication g	Standard g	Object g	Bias mg	Uncertainty mg	*k*	Degree of freedom
20	19.999560	20.0004	0.8	0.8	2.03	84
50	50.000270	50.0009	0.6	0.9	2.02	126
60	60.000150	60.0010	0.8	0.9	2.02	126
100	100.000540	100.0014	0.9	1.1	2.01	251
150	150.000000	149.9999	-0.1	1.4	2.00	Infinite
200	200.001180	200.0045	3.3	1.6	2.00	Infinite

Environmental data	**Temperature °C**	(20.6 ± 0.5)	**Humidity %**	(56 ± 5)	**Pressure hPa**	(1 018 ± 1)

Environment: (x) Stable ( ) Unstable (x) Acclimatized

Emission date: 6/8/2024

Galileu Galilei
Metrologist technician

Lord Kelvin
Authorized firmer

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Fig. 9.8 Thermometer calibration certificate for solved exercise

9.5.9 If the calibration certificate of a measurement instrument has a measurement error above expected, however, the instrument is not with its functionality affected, which CANNOT be done:

(a) Ask the laboratory to adjust to reduce its measurement error, and then make a new calibration.
(b) Use the instrument without any caveat, remembering to calibrate again in the defined period.
(c) Create mathematical corrections for errors and apply them to their use.
(d) Remove the measuring instrument.

9.5.10 To know if a measuring instrument is adapting to the intended use:

(a) The measuring instrument should be appropriately functioning and calibrated periodically.
(b) Periodic checks must be performed.
(c) Acceptance criteria should be established and sent to calibration, and the results should be evaluated.
(d) Acceptance criteria must be established, and the equipment must be sent for calibration.

9.5.11 What does a calibration of a measurement instrument consist of?

(a) In comparing the values obtained by it in the face of standards, a certificate with the values of their errors and measurement uncertainty is obtained.
(b) In comparing the values obtained by it in the face of standards by obtaining a certificate with only the values of their measurement errors.
(c) In the maintenance of the measuring instrument.
(d) In its periodic adjustment.

9.5.12 To evaluate a calibration certificate, it must be established primarily before:

(a) The periodicity of the calibration of the measurement instruments.
(b) The acceptance criteria for the results.
(c) The laboratory that will calibrate.
(d) The purchase of a new measurement instrument.

9.5.13 Must any measurement instrument be calibrated?

(a) Yes, measurement instruments cannot be used without being calibrated.
(b) Yes, otherwise, it leads to non-conformities in audits.
(c) No, the need for its measurements in the face of the process requirements should be evaluated.
(d) No, we need to calibrate after some failure is found.

9.5.14 When analyzing a calibration certificate, what alternative represents the primary information we should evaluate?

(a) If the laboratory that calibrated the measuring instrument is accredited.
(b) If the error results and measurement uncertainty are presented.
(c) If the authorized signatory signed the Calibration Certificate.
(d) The traceability of the standards used is contained in the certificate.

9.5.15 The calibration results of a measurement instrument did not meet the acceptance criteria. What should be done?

(a) Discard the measuring instrument.
(b) Evaluate whether error adjustment or the use of the measuring instrument in other bands is possible.
(c) Perform a new calibration until the expected value is obtained.
(d) Modify the acceptance criteria, so that it can be accepted.

Proposed Exercises—Answers and Solutions

Chapter 1

1.3.1 (b)	1.3.2 (a)
1.3.3 (a)	1.3.4 (d)
1.3.5 (c)	1.3.6 (c)
1.3.7 (b)	1.3.8 (a)
1.3.9 (a)	1.3.10 (d)
1.3.11 (a)	1.3.12 (a)
1.3.13 (c)	1.3.14 (d)
1.3.15 (c)	1.3.16 (b)
1.3.17 (b)	1.3.18 (a)
1.3.19 (b)	1.3.20 (c)
1.3.21 (b)	1.3.22 (b)
1.3.23 (c)	1.3.24 (c)

1.3.25 $n = 2$

$$\dim(x) = \frac{\dim(v^n)}{\dim(a)}$$
$$\dim(x) = L$$
$$\dim(v) = LT^{-1} \therefore \dim(v^n) = \left(LT^{-1}\right)^n$$
$$\dim(a) = LT^{-2}$$
$$L = \frac{\left(LT^{-1}\right)^n}{LT^{-2}}$$

P. P. Novellino do Rosario, A. Mendes, *Metrology and Measurement Uncertainty*,
https://doi.org/10.1007/978-3-031-82303-9

1.3.26 (c)
1.3.27 kg s^{-3}

$$I = \frac{E}{S \cdot t}$$

Unit of E: m^2 kg s^{-2}
Unit of S: m^2
Unit of t: s
Unit of I: $\frac{m^2\ kg\ s^{-2}}{m^2\ s} = kg\ s^{-3}$

1.3.28 (d)
1.3.29 kg m^{-1}

$$k = \frac{f}{v^2}$$

Unit of f: m kg s^{-2}
Unit of v^2: m^2 s^{-2}
Unit of k: $\frac{m\,kg\,s^{-2}}{m^2\,s^{-2}} = kg\,m^{-1}$

Chapter 2

2.11.1 I'm afraid I have to disagree. Traceability is only guaranteed when the instrument, even new, is calibrated with recognized standards and accepted nationally or internationally.
2.11.2 Metrology is the science of measurement and its applications and includes all theoretical and practical aspects of measurement, whatever the uncertainty of measurement and the field of application.
2.11.3

Legal Metrology
It is the area of metrology closest to the ordinary citizen, whose primary function is to protect products and services that involve and need some measurement. It is defined by the International Organization of Legal Metrology (OIML) as: "the application of legal requirements for measurement and instruments." Metrological regulations based on the OIML guidelines establish the technical requirements, metrological control, use, and marking requirements, as well as the requirements of the units of measure that must be met by manufacturers and by users of the measuring instruments.

In addition to commercial activities, measuring instruments used in official activities, medical areas, medicine manufacture, occupational, environmental protection, and radiation are subject to metrological control. In these cases, control assumes special importance in the face of the dangerous negative effects that wrong results can cause human health.

Scientific Metrology

Scientific and industrial metrology promotes competitiveness and provides an environment favorable to the country's scientific and industrial development. It is also essential to technological innovation. BIPM coordinates the process, is responsible for the basic metrological quantities with reliability equal to that of the countries of the first world, and transfers measurement standards to the society.

2.11.4 This is the area of metrology closest to the common citizen, whose primary function is to ensure the protection of products and services that involve and require some measurement. It is defined by the International Organization of Legal Metrology (OIML) as: "the application of legal requirements for measurement and instruments". Metrological regulations based on OIML guidelines establish the technical, metrological control, usage and marking requirements, as well as the requirements for units of measurement that must be met by manufacturers and users of measuring instruments.

2.11.5

- Single description and identification of the instrument: type, model, serial number, manufacturer, etc.
- The date that metrological evidence was performed.
- Evidence results.
- Interval of the next evidence.
- Identification of the procedure (or method, norm, instruction, etc.) of evidence.
- Maximum acceptable or permissible errors.
- Relevant environmental conditions and declaration on necessary corrections.
- Uncertainties involved in calibration.
- Provide details of any intervention (maintenance, adjustment, modification) in the measuring instrument.
- Use limitations.
- Identification of those who performed the metrological evidence.
- Identification of those responsible for any corrections of information recorded.
- Single identification of the report or calibration certificate.
- Traceability of measurement results.
- Metrological requirements for intended use.
- The calibration result was performed after, and where required, before any intervention in the measuring instrument.

2.11.6 Quantity that does not affect the quantity effectively measured, but affects the relationship between the indication and the result of the measurement.

2.11.7 The International Organization of Legal Metrology (OIML)

2.11.8 It means that the laboratory's competence to perform calibrations or tests was recognized by an accrediting organism based on the ISO/IEC 17025:2017 standard, according to the guidelines established by the International Laboratory Accreditation Cooperation (ILAC) and the Good Practices Codes (GPC) from the Organization for Economic Co-Operation and Development (OECD).

2.11.9

A technical standard is a document established by consensus and approved by a recognized organism that provides minimum rules, guidelines, or characteristics for activities or their results, aiming to obtain a great degree of sorting in a given context. The technical standard is voluntary, that is, not mandatory by law.

A technical regulation is a document adopted by an authority with legal power that contains mandatory rules and establishes technical requirements, either directly by reference to technical standards or by incorporating their content, in whole or in part. In general, technical regulations aim to ensure aspects related to health, safety, environment, consumer protection, and fair competition. Compliance with a technical regulation is mandatory, and non-compliance with the corresponding punishment is illegal.

2.11.10 VIM is a document that seeks international harmonization of the terminologies and definitions used in metrology and instrumentation.

Chapter 3

3.3.1

(a) 34.4 m	(b) 23.9 m
(c) 8.4 m	(d) 19.7 m
(e) 43.5 m	(f) 43.9 m
(g) 52.4 m	(h) 66.7 m

3.3.2

(a) 4	(b) 2
(c) 1	(d) 4

3.3.3

(a) 479 m	(b) 642 kg
(c) 123 L	(d) 56.2 cm

3.3.4

(a) 89.5 m
(b) 8.2 m^2
(c) 5.55 m

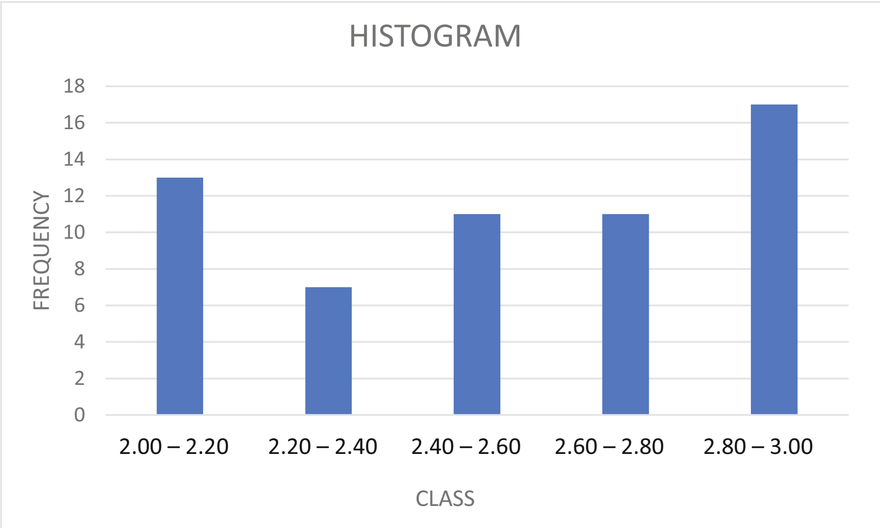
3.3.5

(a) 4×10^3	(b) 0.002
(c) 0.0006	(d) 0.00003

3.3.6

3.3.7

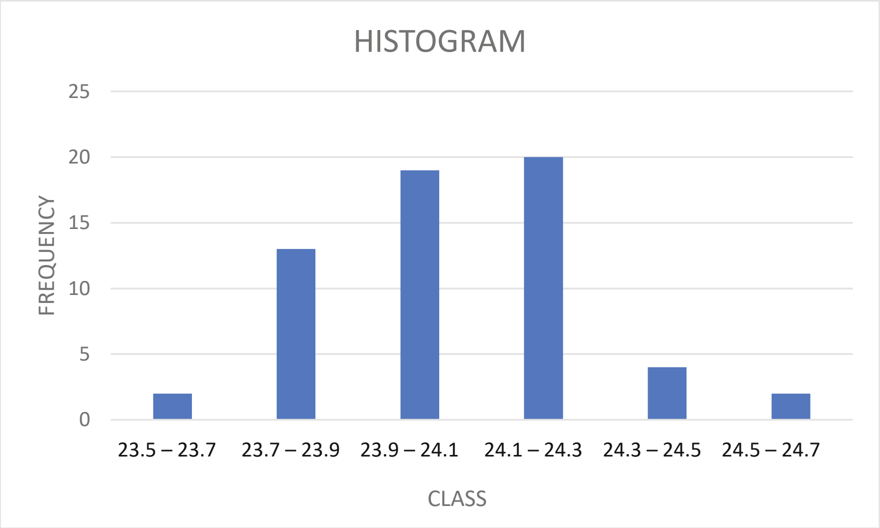
(a) 268.1	(b) 286.54
(c) 132.32	(d) 129
(e) 5.0	(f) 114.7
(g) 0.87	(h) 1.7×10^2
(i) 4.2	(j) 1.4×10^2
(k) 0.1712 s	
(l) $3.0 \times 10^5 - 1.5 \times 10^2 = 3.0 \times 10^5$ (the result must have the same number of decimal digits of the portion that has the smallest number of decimal digits)	



$$\bar{x} = \frac{a+b}{2} = \frac{(2.99 + 2.01)\mathrm{V}}{2} = 2.5\ \mathrm{V}$$

$$s = \frac{b-a}{\sqrt{12}} = \frac{(2.99 - 2.01)\mathrm{V}}{\sqrt{12}} = 0.283\ \mathrm{V}$$

3.3.8



$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} = 24.0 \quad {}^\circ\mathrm{C}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (\overline{x} - x_i)^2}{n-1}} = 0.212 \quad {}^\circ\mathrm{C}$$

3.3.9 $\alpha = (11.5 \times 10^{-6} \pm 0.2 \times 10^{-6})\ {}^\circ\mathrm{C}^{-1}$

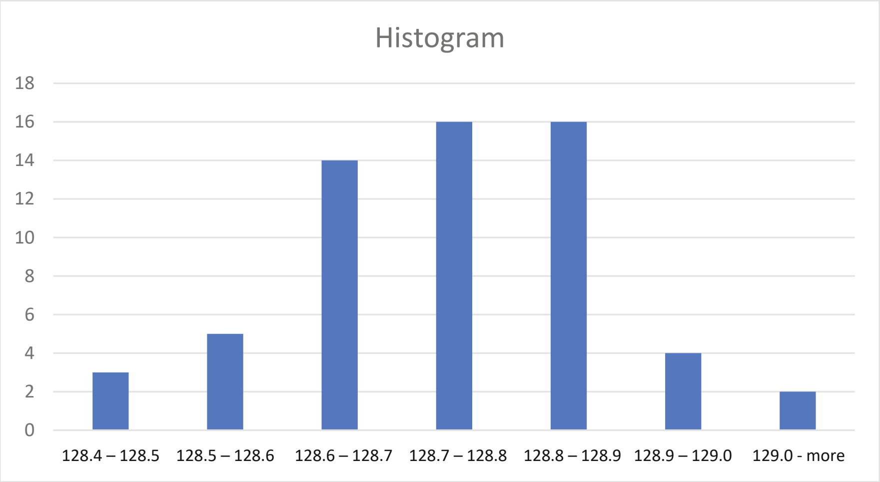
Uniform distribution

$$a = 11.3 \times 10^{-6} \quad {}^\circ\mathrm{C}^{-1}$$

$$b = 11.7 \times 10^{-6} \quad {}^\circ\mathrm{C}^{-1}$$

$$s = \frac{b-a}{\sqrt{12}} = 0.1 \times 10^{-6}\ {}^\circ\mathrm{C}^{-1}$$

3.3.10



(b) The histogram shows a distribution that tends to be normal.

$$s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n-1}} = 0.1282885 \ \ \text{V}$$

(c) To determine the probability interval of 95.45%, we must calculate the degree of freedom and the factor k. With 60 measurements, we have a degree of freedom of 59 ($n-1$). Consulting the t-Student table, we have $k = 2.043$. Therefore, to 95.45%, we have 2.042 × 0.1282885 = 0.262096 V. Note that here, we do not calculate the standard deviation of the mean, since we want to determine the interval in which we have the probability of 95.45% of finding a measurement within the 60 performed.

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} = 128.74 \ \ \text{V}$$

$$\text{Interval} = (128.74 \pm 0.26) \ \ \text{V} \quad [128.48 \ \ \text{V}; \quad 129.00 \ \ \text{V}]$$

(d) Within the interval, we have 57 measurements totaling 95.00% of the measured values.

(e) As we now wish for the interval where we will find the mean of measurements, we will work with the concept of standard deviation from the mean.

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0.1282885 \text{ V}}{\sqrt{60}} = 0.016562 \text{ V}$$

For $n = 60$, we have the degree of freedom = 59, and $k = 2.043$. The interval will be:

$$k\, s_{\overline{x}} = 2.043\ x\ 0.016562\ \text{V} = 0.033836\ \text{V}$$

Then, with 95.45% probability, we will find the mean of 60 measurements in the interval (128.74 ± 0.03) V.

3.3.11

(a) Scale 1: $\mu = \frac{\sum_{i=1}^{n} x_i}{n} = 14.95\ \text{kg}$
Scale 2: $\mu = \frac{\sum_{i=1}^{n} x_i}{n} = 14.97\ \text{kg}$
(b) Scale 1: $s = \sqrt{\frac{\sum_{i=1}^{n} (\overline{x} - x_i)^2}{n-1}} = 0.187083\ \text{kg}$
Scale 2: $s = \sqrt{\frac{\sum_{i=1}^{n} (\overline{x} - x_i)^2}{n-1}} = 0.320416\ \text{kg}$
(c) Scale 2: $s = \sqrt{\frac{\sum_{i=1}^{n} (\overline{x} - x_i)^2}{n-1}} = 0,320416\ \text{kg}$
(d) Interval with 95.45%: $\overline{x} \pm k\ s(\overline{x})$
t-Student table → $n = 6$, $k = 2.649$
Scale 1: $s(\overline{x}) = \frac{0.187083\ \text{kg}}{\sqrt{6}} = 0.076376\ \text{kg}$
Scale 2: $s(\overline{x}) = \frac{0.320416\ \text{kg}}{\sqrt{6}} = 0.130809\ \text{kg}$
Scale 1: (14.95 ± 0.20) kg
Scale 2: (14.97 ± 0.35) kg

3.3.12

(a) 7.23 pH
(b) 0.012247

3.3.13

(a) Considering that eight measurements imply a degree of freedom equal to 7 and consulting the t-Student table, we find that for the degree of freedom 7, we have $k = 1.077$, and we will have $U = k \cdot s = 0.642$ °C, so the interval will be (48.3 to 49.5) °C—68.27 %.
(b) To 95.45 %, we have $k = 2.429$, implying to $k \cdot s = 1.4574$ °C, so the interval will be (47.4 to 50.4) °C.
(c) To 99.7 %, we have $k = 4.442$, implying to $k \cdot s = 2.6652$ °C, so the interval will be (46.2 to 51.6) °C.

Chapter 4

4.5.1

(a) 0.1 bar
(b) 0.05 bar
(c) 0.30 bar

4.5.2

(a) 2 °C
(b) 1 °C
(c) 20 °C

4.5.3 11 bar
4.5.4 (b)
4.5.5 (b)
4.5.6

(a) Maximum error = 0.8/200 = 0.4%; Hysteresis = 1.2/200 = 0.6 %
(b) Maximum error = 0.8/65 = 1.23%; Hysteresis = 1.2/65 = 1.85 %

4.5.7

(a) $\overline{x} = \frac{\sum x_i}{10} = 15.97\ \Omega$
(b) $B = 15.97 - 15.977 = -0.007\ \Omega = -0.01\ \Omega$
(c) $E = (15.95 - 15.977)\ \Omega = -0.027\ \Omega = -0.03\ \Omega$

4.5.8

(a) $B = (20.5 - 20.0)$ °C = 0.5 °C
(b) $E = (21.0 - 20.0)$ °C = 1 °C

4.5.9 C = (45 – 1) psi = 44 psi
4.5.10 (a)
4.5.11 (b)
4.5.12 (d)
4.5.13 (d)
4.5.14

(a) 1 °C; 2 °F
(b) 1 °C; 2 °F
(c) 21 °C; 70 °F

4.5.15

(a) 0.5 V
(b) 0.25 V
(c) (0 a 15) V
(d) 2% × 15 V = 0.3 V

4.5.16

(a) The precision of an instrument is related to little or no dispersion of the measured values. This can be found in the standard deviation of measurements. The standard deviation of multimeter one is 0.01 Ω, and multimeter two is 0.02 Ω. Therefore, multimeter 1 is more precise.
(b) The accuracy of a measurement instrument is related to its lower error; multimeter one is more accurate since its error is 0.03 Ω, and the multimeter two error equals 0.04 Ω.

4.5.17

(a) Point 5, the bias is zero.
(b) Point 2, the highest bias, 0.004 g.
(c) Mean = 5.004 g
Bias in this point = 0.002 g
Correct value = (5.004 – 0.002) g = 5.002 g

4.5.18

(a) Correction of the standard = (50.2 + 0.3) °C = 50.5 °C
(b) Mean of the object = 50 °C
Bias of the object = (50 – 50.5) °C = – 0.5 °C = 0 °C (the resolution of the object is 1 °C)
(c) Correction = 0 °C.

Chapter 5

5.11.1

(a) 2 km/h
(b) 2 %
(c) 4 %
(d) 40 %
(e) The point of 200 km/h = 1 %.

5.11.2

(a) (176.4 $\pm$ 0.2) cm
(b) (1.764 $\pm$ 0.002) m

5.11.3

(a) 0.64 s
(b) $s_{\overline{x}} = \frac{s}{\sqrt{5}} = 0.004472$ s
(c) The standard deviation of the mean.
(d) $v_{\text{eff}} = 5 - 1 = 4 \rightarrow k = 2.869$

$$U = u_A \ . \ k = 0.004472 \times 2.869 = 0.0128 \text{ s} = 0.01 \text{ s}$$

5.11.4

(a) 5
(b) 0.44 s
(c) $s_{\overline{x}} = \frac{s}{\sqrt{5}} = 0.012247$ s
(d)

$$u_c = \sqrt{u_A^2 + u_{\text{timer}}^2} = \sqrt{0.012247^2 + 0.01^2} = 0.015811 \, s$$

$$v_{\text{eff}} = \frac{0.015811^4}{\frac{0.012247^4}{4} + \frac{0.01^4}{\infty}} = 11 \rightarrow k = 2.26$$

$$U = 2.26 \times 0.015811 \text{ s} = 0.04 \text{ s}$$

(e) $u_c = u_A = 0.012247$ s

$$v_{\text{eff}} = n - 1 = 4 \rightarrow k = 2.87 \quad U = 2.87 \times 0.012247 \text{ s} = 0.04 \text{ s}$$

5.11.5

The mean of the measurements is 256.98 mm, but as the resolution is 0.05 mm, we have to round to 257.00 mm.

Uncertainty of repeatability: $u_A = \frac{s}{\sqrt{n}} = \frac{0.06455}{2} = 0.032275$ mm
Combined uncertainty:

$$u_c = \sqrt{u_A^2 + u_{\text{inst}}^2} = \sqrt{0.0032275^2 + 0.025^2} = 0.040825 \text{ mm}$$

Degree of freedom and expanded uncertainty:

$$v_{\mathrm{eff}} = \frac{u_c^4}{\frac{u_A^4}{3}} = 7.68 \rightarrow k = 2.43 \rightarrow U = 2.43 \times 0.040825\ \mathrm{mm} = 0.10\ \mathrm{mm}$$

$$d = (257.00 \pm 0.10)\ \ \mathrm{mm}$$

5.11.6

(a) $\bar{x} = 10.156\ \mathrm{mm} \rightarrow B = \bar{x} - TV = (10.156 - 10.1538)\mathrm{mm} = 0.002\ \mathrm{mm}$
(b) $u_A = \frac{s}{\sqrt{10}} = \frac{0.001826}{\sqrt{10}} = 0.000577\ \mathrm{mm}$
(c) $u_c = \sqrt{u_A^2 + u_{\mathrm{micro}}^2} = \sqrt{0.000577^2 + (0.002/2.23)^2} = 0.001067\ \mathrm{mm}$

$$v_{\mathrm{eff}} = \frac{u_c^4}{\frac{u_A^4}{9} + \frac{u_{\mathrm{micro}}^4}{12}} = 19.53$$

(d) $k = 2.14 \ \ \rightarrow \ U = 2.14 \times 0.001067\,\mathrm{mm} = 0.00228\ \ \mathrm{mm} = 0.002\,\mathrm{mm}$

5.11.7

(a) 0.070711 g
(b) 0 g
(c) 0.3 g, $k = 2.10$ for 95.45 %

5.11.8 The expanded uncertainty was declared with more than two significant digits, and this is not allowed according to ILAC-P14:09/2020. Uncertainty should have been declared as 0.024 g/ml.

5.11.9

(a) Mean = 80.6 °C = 80.5 °C (the thermometer resolution is 0.5 °C)
(b) $u_{\mathrm{repeat}} = \frac{s}{\sqrt{n}} = \frac{0.41833}{\sqrt{5}} = 0.1871\,^{\circ}\mathrm{C}$
(c) $u_{\mathrm{thermo}} = \frac{U}{k} = \frac{0.6}{2.87} = 0.2091\,^{\circ}\mathrm{C}$
(d) $u_c = \sqrt{u_{\mathrm{repeat}}^2 + u_{\mathrm{thermo}}^2} = 0.2805\,^{\circ}\mathrm{C}$
(e) Degree of freedom
(f) k factor

$$v_{\mathrm{eff}} = \frac{0.2805^4}{\frac{0.1871^4}{4} + \frac{0.2091^4}{4}} = 7.9 \rightarrow k = 2.43$$

(g) $U = 2.43 \times 0.2805\ ^{\circ}\ \mathrm{C} = 0.7$ °C
(h) The greatest source of uncertainty is the uncertainty of bimetallic thermometer measurement.

5.11.10

(a) $\bar{x}_{\text{corrected}} = \bar{x} - B = 100.0035 - (-0.0050) = 100.0085$ g
(b) $u_{\text{repeat}} = 0.000176383$ g
(c) $u_c = \sqrt{u_{\text{repeat}}^2 + u_{\text{scale}}^2} = \sqrt{0.000176383^2 + 0.0004^2} = 0.000437163$ g

$$v_{\text{eff}} = \frac{u_c^4}{\frac{u_A^4}{2}} = 75,5 \rightarrow k = 2.03 \rightarrow U = 2.03 \times 0.000437163 \text{ g} = 0.0009 \text{ g}$$

5.11.11

(a) $\bar{x}_{\text{corrected}} = \bar{x} - B = 12.005 - 0.0015 = 11.9990$ g
(b) $u_{\text{repeat}} = 0.00006667$ g
(c) (c) B = 0.0015 g
(d) $u_c = \sqrt{u_{\text{repeat}}^2 + u_{\text{scale}}^2} = \sqrt{0.00006667^2 + 0.00014218^2} = 0.000157034$ g

$$v_{\text{eff}} = \frac{u_c^4}{\frac{u_A^4}{2} + \frac{u_{\text{scale}}^4}{25}} = 17,93 \rightarrow k = 2.16 \rightarrow U = 2.16 \times 0.000157034 \text{ g} = 0.0003 \text{ g}$$

5.11.12 (c)

Chapter 6

6.6.1

$$M = (1\ 000 \pm 1)\ \text{g} \quad k = 2.00 \text{ and } 95.45\ \%. \quad u_M = 1\,\text{g}/2 = 0.5\ \text{g}$$

$$D = (8.000 \pm 0.002)\ \text{cm} \quad k = 2.00 \text{ and } 95.45\%. \quad \text{u}_\text{D} = 0.002\ \text{cm}/2 = 0.001\ \text{cm}$$

$$\rho = \frac{M}{V} = \frac{6M}{\pi D^3} = \frac{6 \times 1\ 000}{3.1416 \times 8^3} = 3.730\ \text{g/cm}^3$$

(a)

$$u_\rho = \sqrt{\left(\frac{\partial\rho}{\partial M} u_M\right)^2 + \left(\frac{\partial\rho}{\partial D} u_D\right)^2}$$

$$u_\rho = \sqrt{\left(\frac{6}{\pi D^3} u_M\right)^2 + \left(\frac{-18M}{\pi D^4} u_D\right)^2} = 0.00233\ \text{g/cm}^3$$

$$U = k.\ u_\rho = 2 \times 0.00233 \frac{\text{g}}{\text{cm}^3} = 0.005\ \text{g/cm}^3$$

(b)

$$u_\rho = \rho \sqrt{\left(\frac{u_M}{M}\right)^2 + \left(\frac{3u_D}{D}\right)^2}$$

$$u_\rho = 3.730 \sqrt{\left(\frac{0.5}{1\,000}\right)^2 + \left(\frac{3 \times 0.001}{8}\right)^2} = 0.00233\ \text{g/cm}^3$$

$$U = k.\ u_\rho = 2 \times 0.00233 \frac{\text{g}}{\text{cm}^3} = 0.005\ \text{g/cm}^3$$

6.6.2 The result of mode 2 was different, because the $L1$ and $L2$ variables are statistically dependent. Thus, measurement uncertainty should consider the correlation coefficient (r) between the variables.

$$u_A^2 = \left(\frac{\partial A}{\partial L1}\right)^2 u_{L1}^2 + \left(\frac{\partial A}{\partial L2}\right)^2 u_{L2}^2 + 2\left(\frac{\partial A}{\partial L1}\right)\left(\frac{\partial A}{\partial L2}\right) r(L1, L2) u_{L1} u_{L2}$$

$$r(L1, L2) = 1;\quad L1 = L2 = L \ \rightarrow\ u_{L1} = u_{L2} = u_L$$

$$u_A = \sqrt{L2^2 u_{L1}^2 + L1^2 u_{L2}^2 + 2.L1.L2 u_{L1} u_{L2}}$$

$$u_A = \sqrt{4L^2 u_L^2} = 2L u_L$$

6.6.3

$$\left(\frac{u_\rho}{\rho}\right)^2 = \left(\frac{u_m}{m}\right)^2 + \left(\frac{3u_d}{d}\right)^2$$

$$\left(u_{\text{rel density}}\right)^2 = \left(u_{\text{rel mass}}\right)^2 + \left(\frac{3u_d}{d}\right)^2$$

$$(0.012)^2 = (0.01)^2 + \left(\frac{3u_d}{1.000}\right)^2$$

$$u_d = 0.002 \ \text{cm} = 0.2\%$$

6.6.4

(a) 0.1 cm
(b) 0.1/3 = 0.033
(c) 3.33 %
(d) 3/200 = 0.015 cm
(e) 3.33%, the same as the book

6.6.5

(a) $L = (10.0 \pm 0.1)$ cm, $W = (5.0 \pm 0.1)$ cm, $H = (2.0 \pm 0.1)$ cm, $M = (50.0 \pm 0.1)$ g. All uncertainties are declared with $k = 2.00$ and 95.45 %.

$$\rho = \frac{m}{V} = \frac{m}{LWH} = \frac{50}{10.5.2} = 0.50 \text{ g/cm}^3$$

(b)

$$u_\rho = \rho\sqrt{\left(\frac{u_m}{m}\right)^2 + \left(\frac{u_L}{L}\right)^2 + \left(\frac{u_W}{W}\right)^2 + \left(\frac{u_H}{H}\right)^2}$$

$$u_\rho = 0.50\sqrt{\left(\frac{0.05}{50}\right)^2 + \left(\frac{0.05}{10}\right)^2 + \left(\frac{0.05}{5}\right)^2 + \left(\frac{0.05}{2}\right)^2} = 0.0137 \text{ g/cm}^3$$

$$U = 2 \times 0.0137 = 0.03 \,\text{g/cm}^3$$

(c) The variable that has the highest relative uncertainty is the height variable of block H.

$$u_\rho = 0.50\sqrt{\left(\frac{0.05}{2}\right)^2} = 0.0125 \text{ g/cm}^3$$

$$U = 2 \times 0.0125 = 0.02 \,\text{g/cm}^3$$

Note that variable H preponderantly influences the wooden block density measurement uncertainty. Thus, to reduce the final uncertainty of the specific mass, we must improve the measurement of this H variable.

6.6.6

$d = (1.00 \pm 0.01)$ cm, with $k = 2.00$ and 95.45 %

(a) $V = \frac{\pi d^3}{6} = 0.524\ \text{cm}^3$

(b) 0.01/1 × 100 % = 1 %

(c)

$$u_V = \frac{\partial V}{\partial d} u_d = \frac{\pi d^2}{2} u_d = 0.0079\ \text{cm}^3 \rightarrow U = 2 \times 0.0079 = 0.016\ \text{g/cm}^3$$

$$u_V = V.\frac{3u_d}{d} = 0.524 \times \frac{0.015}{1} = 0.0079 \rightarrow U = 0.016\ \text{g/cm}^3$$

6.6.7

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{(LC)^{-1/2}}{2\pi}$$

$$\left(\frac{u_f}{f}\right)^2 = \left(\frac{-u_L.1/2}{L}\right)^2 + \left(\frac{-u_C.1/2}{C}\right)^2$$

$$\left(\frac{u_f}{f}\right)^2 = \left(-\frac{1}{2} \times 0.05\right)^2 + \left(-\frac{1}{2} \times 0.2\right)^2 = 0.010625$$

$$\frac{u_f}{f} = 0.0103 \rightarrow u_{\%f} = 10.3\%$$

6.6.8

$$y = (1.000 \pm 0.001)\,\text{m}; \quad k = 2.43 \ \text{ and } \ 95.45\%$$

$$t = (0.45 + 0.01)\,\text{s}; \quad k = 2.23 \ \text{ and } \ 95.45\%$$

(a) $\frac{U_y}{y} = \frac{0.001}{1} = 0.1\%$

(b) $\frac{U_t}{t} = \frac{0.01}{0.45} = 2.2\%$

(c) $g = \frac{2y}{t^2} = 9.9\ \text{m/s}^2$

(d) $g = \frac{2y}{t^2} = 2yt^{-2}$

$$u_g = g\sqrt{\left(\frac{u_y}{y}\right)^2 + \left(\frac{-2u_t}{t}\right)^2}$$

$$u_g = 9.9\sqrt{\left(\frac{0.001/2.43}{1}\right)^2 + \left(\frac{-2\times 0.01/2.23}{0.45}\right)^2} = 0.197\ \mathrm{m/s^2}$$

$$y : k = 2.43 \ \rightarrow \ \upsilon = 7 \quad u_y = 0.001/2.43 = 0.000411\,\mathrm{m}$$

$$t : k = 2.23 \ \rightarrow \ \upsilon = 12 \quad u_t = 0.01/2.23 = 0.0045\,\mathrm{s}$$

$$v_{\mathrm{eff}} = \frac{\left({}^{0.197}/_{9.9}\right)^4}{\frac{\left({}^{0.000411}/_{1}\right)^4}{7} + \frac{\left({}^{0.0045}/_{0.45}\right)^4}{12}} = 190{,}35 \rightarrow k = 2.01$$

$$U = 2.01 \times 0.197 = 0.4\ \ \mathrm{m/s^2}$$

(e) It is possible to neglect the uncertainty of height y (0.1%), since its relative uncertainty is minimal compared to the relative uncertainty of time t (2.2%).

$$u_g = 9.9\sqrt{\left(\frac{-2\times 0.01/2.23}{0.45}\right)^2} = 0.197\ \mathrm{m/s^2}$$

6.6.9

$$R = (10.0 \pm 0.1)\ \ \Omega,\ \ k = 2.43\ \text{ and }\ 95.45\%$$

$$I = (10.0 \pm 0.1)\ \ \mathrm{A},\ \ k = 2.23\ \text{and}\ \ 95.45\%$$

$$V = (100 \pm 1)\ \ \mathrm{V},\ \ k = 2.21\ \text{and}\ \ 95.45\%$$

$$uR = 0.1/2.43 = 0.04115 \quad \upsilon = 7$$

$$uI = 0.1/2.23 = 0.04484 \quad \upsilon = 12$$

$$uV = 1/2.21 = 0.4525 \quad \upsilon = 13$$

(a) P = V.I = 1 000 W = 1.00 kW

$$u_P = P\ \sqrt{\left(\frac{u_V}{V}\right)^2 + \left(\frac{u_I}{I}\right)^2} = 6.3705\ \ \mathrm{W}$$

$$v_{\mathrm{eff}} = \frac{\left(\frac{u_P}{P}\right)^4}{\left(\frac{u_V}{V}\right)^4/13 + \left(\frac{u_I}{I}\right)^4/12} = 24.98 \rightarrow k = 2.11$$

$$U = 2.11 \times 6.3705 = 13.44\,\mathrm{W} = 0.01\,\mathrm{kW}$$

(b) P = R.I^2 = 1 000 W = 1.00 kW

$$u_P = P\ \sqrt{\left(\frac{u_R}{R}\right)^2 + \left(2\frac{u_I}{I}\right)^2} = 9.868\,\mathrm{W}$$

$$V_{ef} = \frac{\left(\frac{u_P}{P}\right)^4}{\left(\frac{u_R}{R}\right)^4/7 + \left(\frac{u_I}{I}\right)^4/12} = 126.98 \quad \rightarrow k = 2.02$$

$$U = 2.02 \times 9.868 = 19.99\,\mathrm{W} = 0.02\,\mathrm{kW}$$

(c) $P = \mathrm{V}^2/R = 1\,000$ W $= 1.00$ kW

$$u_P = P\ \sqrt{\left(\frac{u_R}{R}\right)^2 + \left(2\frac{u_V}{V}\right)^2} = 9.942\,\mathrm{W}$$

$$V_{ef} = \frac{\left(\frac{u_P}{P}\right)^4}{\left(\frac{u_R}{R}\right)^4/7 + \left(\frac{u_V}{V}\right)^4/13} = 133.41 \quad \rightarrow k = 2.02$$

$$U = 2.02 \times 9.942 = 20.07\,\mathrm{W} = 0.02\,\mathrm{kW}$$

Conclusion: The letter (a) is the measurement method that provides the lowest uncertainty for electric power. Its value is half of the uncertainty provided by alternatives (b) and (c).

6.6.10

$$M_1 = (128.0 \pm 0.2)\ \mathrm{g}$$

$$M_2 = (56.4 \pm 0.4)\ \mathrm{g}$$

$$M_3 = (39.7 \pm 0.7)\ \mathrm{g}$$

$$M4 = M1 + M2 – M3 = 144.7\,\mathrm{g}$$

$$u_{M1} = U/k = 0.2/2 = 0.1\,\mathrm{g}$$

$$u_{M2} = U/k = 0.4/2 = 0.2\,\mathrm{g}$$

$$u_{M3} = U/k = 0.7/2 = 0.35\,\mathrm{g}$$

$$u_{M_4} = \sqrt{u_{M_1}^2 + u_{M_2}^2 + u_{M_3}^2} = \sqrt{0.1^2 + 0.2^2 + 0.35^2} = 0.42\,\mathrm{g}$$

$$U = 2 \times 0.42 = 0.8\,\mathrm{g}$$

6.6.11
Liters $= V = 80.8$ L
Distance $= S = 834.5$ km
C $= S/V = 10.3$ km/L

$$u_C = C\ \sqrt{\left(\frac{u_S}{S}\right)^2 + \left(\frac{u_V}{V}\right)^2} \quad u_V = \sqrt{u_{V1}^2 + u_{V2}^2}$$

$$u_C = C\sqrt{\left(\frac{u_S}{S}\right)^2 + \left(\frac{u_V}{V}\right)^2} = D\sqrt{\left(\frac{u_S}{S}\right)^2 + \left(\frac{u_{V1}^2 + u_{V2}^2}{V^2}\right)} = 0.02106\,\text{km/L}$$

$$U = 2 \times 0.02106 = 0.042\,\text{km/L}$$

$$C = \left(10.3 \pm 4.2 \times 10^{-2}\right)\ \text{km/L}$$

6.6.12
(a) 72.467 mm = 50 mm + 20 mm + 1.46 mm + 1.007 mm
(b)

$$U_{72.467\ \text{mm}} = \sqrt{U_{50\ \text{mm}}^2 + U_{20\ \text{mm}}^2 + U_{1.46\ \text{mm}}^2 + U_{1.007\ \text{mm}}^2}$$

u V1 =	0,1	L
u V2 =	0,05	L
u S =	1,25	km

$$U_{72.467\ \text{mm}} = \sqrt{0.0004^2 + 0.0003^2 + 0.0002^2 + 0.0002^2} = 0.0006\ \text{mm} = 0.6\ \mu\text{m}$$

6.6.13
(a) $\overline{V} = 200.0\ V \quad \bar{i} = 1.99\ A$

Voltmeter		Ammeter	
Bias (V)	+0.1	Bias (A)	−0.04
Uncertainty (V)	0.2	Uncertainty (A)	0.02
(k = 2.00; 95.45%)		(k = 2.00; 95.45%)	

$$\overline{V}_{\text{cor}} = 199.9\ V \quad \bar{i}_{\text{cor}} = 2.03\ \text{A}$$

$$R = \frac{\overline{V}_{\text{cor}}}{\bar{i}_{\text{cor}}} = 98.3\ \Omega$$

(b) $B = R - R_{\text{nom}} = 98.3 - 100 = -1.7\ \Omega$
(c) $C = -B = 1.7\ \Omega$
(d)

$$u_R = R\sqrt{\left(\frac{u_V}{V}\right)^2 + \left(\frac{u_i}{i}\right)^2}$$

$$u_V^2 = u_{AV}^2 + u_{BV}^2 \qquad u_i^2 = u_{Ai}^2 + u_{Bi}^2$$

$$u_R = R\sqrt{\frac{u_{AV}^2 + u_{BV}^2}{V^2} + \frac{u_{Ai}^2 + u_{Bi}^2}{i^2}} = 98.3\ \sqrt{\frac{0.035^2 + 0.1^2}{199.9^2} + \frac{0.0038^2 + 0.01^2}{2.03^2}}$$
$$= 0.52\ \Omega$$

$$u_V = \sqrt{0.035^2 + 0.1^2} = 0.106\ \mathrm{V}$$

$$u_i = \sqrt{0.0038^2 + 0.01^2} = 0.0107\ \mathrm{A}$$

$$v_{\text{eff}} = \frac{\left(\frac{u_R}{R}\right)^4}{\frac{\left(\frac{u_V}{V}\right)^4}{v_V} + \frac{\left(\frac{u_i}{i}\right)^4}{v_i}}$$

$$v_V = \frac{u_V^4}{\frac{uA_V^4}{8}} = \frac{0.106^4}{\frac{0.035^4}{8}} = 673$$

$$v_i = \frac{u_i^4}{\frac{uA_i^4}{8}} = \frac{0.0107^4}{\frac{0.0038^4}{8}} = 503$$

$$v_{\text{eff}} = \frac{\left(\frac{u_R}{R}\right)^4}{\frac{\left(\frac{u_V}{V}\right)^4}{v_V} + \frac{\left(\frac{u_i}{i}\right)^4}{v_i}} = \frac{\left(\frac{0.52}{98.3}\right)^4}{\frac{\left(\frac{0.106}{199.9}\right)^4}{673} + \frac{\left(\frac{0.0107}{2.03}\right)^4}{503}} = 510 \rightarrow k = 2.00$$

$$U = 2.00 \ \times\ 0.52 = 1.04\ \ \Omega$$

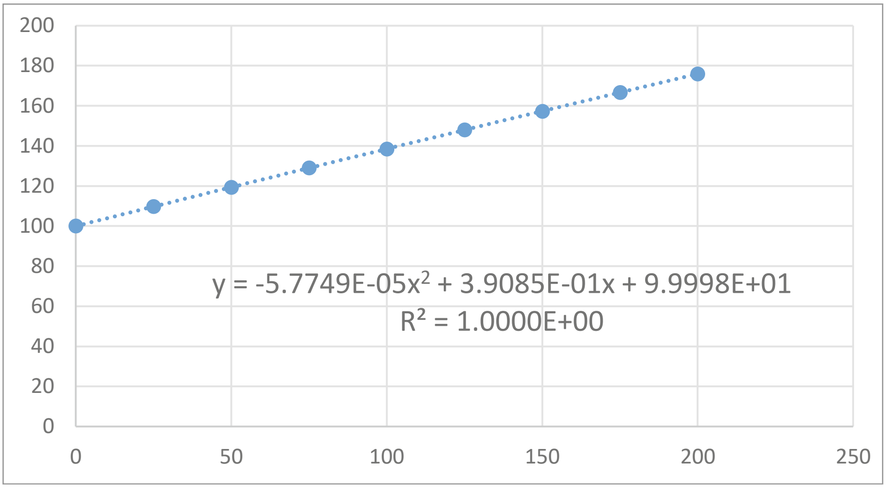
$$R = (98.3 \pm 1.0)\ \ \Omega \quad k = 2.00$$

	Voltage (V)	Electric current (A)
	199.9	1.99
	200.2	2.02
	200.1	1.98
	199.9	1.99
	199.9	1.99
	200.0	2.00
	200.0	2.00
	199.9	1.99
	200.0	1.99
$s(V)$; $s(i)$	0.105	0.0011
$uA(V)$; $uA(i)$	0.105/3 = 0.035	0.0011/3 = 0.0038

Chapter 7

7.8.1

(a)



$$R(t) = R(0)\ \left[1 + At + B\ t^2\right] = 99.998\ \left(1 + 3.91\text{E-}3\,t – 5.78\text{E-}7\ t^2\right)$$

(b)

(x) Temp. Std. (°C)	(y) Ω	$R(t)$ Ω	$[y – R(t)]^2$
0.00	99.99	99.9980	6.4E-05
25.00	109.74	109.7332	4.6828E-05
50.00	119.40	119.3961	1.4996E-05
75.00	128.99	128.9869	9.5365E-06
100.00	138.50	138.5055	3.036E-05
125.00	147.95	147.9519	3.6936E-06
150.00	157.32	157.3261	3.7792E-05
175.00	166.63	166.6282	3.2874E-06
200.00	175.86	175.8580	3.8416E-06
		Σ	0.000214336
		u_{fit}	0.005977

(c) $\frac{\partial R}{\partial t} = R0\ (A + 2Bt)\,\frac{\Omega}{°\mathrm{C}} = 99.998\ \left(3.91\ \times\ 10^{-3} - 2\ \times\ 5.78\ \times\ 10^{-7}t\right)\ \frac{\Omega}{°\mathrm{C}}$

$$t = 200\,^\circ\mathrm{C} \rightarrow \frac{\partial R}{\partial t} = 99.998\left(3.91 \times 10^{-3} - 2 \times 5.78 \times 10^{-7} \times 200\right)\frac{\Omega}{^\circ\mathrm{C}}$$
$$= 0.36775\,\frac{\Omega}{^\circ\mathrm{C}}$$

$$u_{\mathrm{mult}\,^\circ\mathrm{C}} = \frac{u_{\mathrm{mult}\,\Omega}}{\partial R/\partial t} = \frac{0.01}{0.36775}\,^\circ\mathrm{C} = 0.02719\,^\circ\mathrm{C}$$

$$u_{\mathrm{bath}} = \frac{0.02\,^\circ\mathrm{C}}{\sqrt{3}} = 0.0155\,^\circ\mathrm{C}$$

$$u_{\mathrm{std}} = 0.01\,^\circ\mathrm{C}$$

$$u_{\mathrm{comb}} = \sqrt{0.005977^2 + 0.02719^2 + 0.0155^2 + 0.01^2}\,^\circ\mathrm{C} = 0.0318\,^\circ\mathrm{C}$$

$$U = k\,u_{\mathrm{comb}} = 2 \times 0.0318\,^\circ\mathrm{C} = 0.06\,^\circ\mathrm{C}$$

7.8.2

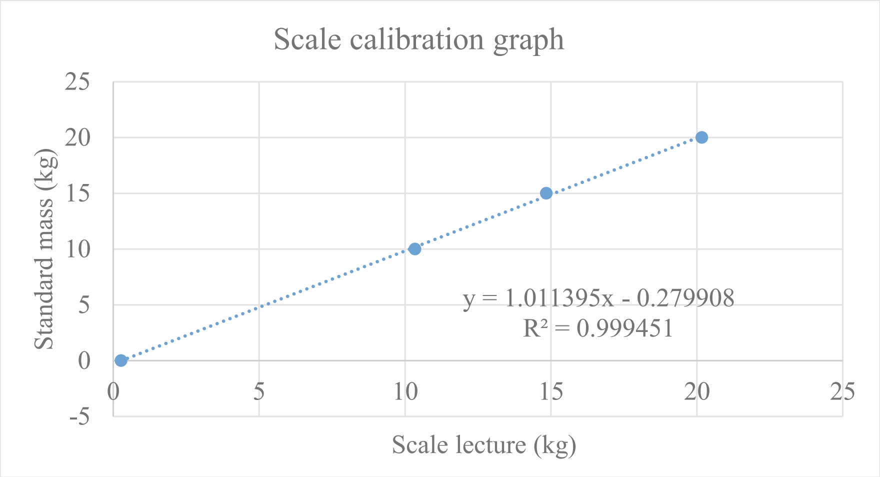
(a)

Scale (kg)			$u_{\mathrm{type\ A}} = \frac{s}{\sqrt{n}}$
0.2	0.3	0.3	0.0333
10.2	10.4	10.4	0.0667
14.9	14.9	14.7	0.0667
20.2	20.0	20.3	0.0882

(b)

$u_{\mathrm{type\ A}}$	u_{resol}	u_{mass}	u_{comb}	ν_{eff}	k	U (kg)
0.0333333	0.028868	0.009569	0.045122	6.7	2.5	0.1
0.0666667	0.028868	0.009569	0.073276	2.9	4.5	0.3
0.0666667	0.028868	0.009569	0.073276	2.9	4.5	0.3
0.0881917	0.028868	0.009569	0.093288	2.5	4.5	0.4

(c)



Scale (mean)	Std mass	$f(x)$	$[y-f(x)]^2$
0.3	0	−0.0101933	0.000104
10.3	10	10.1712333	0.029321
14.8	15	14.7225333	0.076988
20.2	20	20.1166667	0.013611
		Σ	0.120024
		u_{fit}	0.244973

$$u_{\mathrm{fitt}} = 0.244973\,\mathrm{kg}$$

(d)

Scale (kg)	Fitting eq. (kg)	Bias (kg)	u_{comb} (kg)	υ_{eff}	k	U (kg)
1.0	0.7	0.3	0.2621	2.84	4.5	1.2
2.0	1.7	0.3				
3.0	2.8	0.2				
4.0	3.8	0.2				
5.0	4.8	0.2				
6.0	5.8	0.2				
7.0	6.8	0.2				
8.0	7.8	0.2				
9.0	8.8	0.2				
10.0	9.8	0.2				
11.0	10.8	0.2				
12.0	11.9	0.1				
13.0	12.9	0.1				

(continued)

(continued)

Scale (kg)	Fitting eq. (kg)	Bias (kg)	u_{comb} (kg)	υ_{eff}	k	U (kg)
14.0	13.9	0.1				
15.0	14.9	0.1				
16.0	15.9	0.1				
17.0	16.9	0.1				
18.0	17.9	0.1				
19.0	18.9	0.1				
20.0	19.9	0.1				

7.8.3

$$U_{\text{GLT}} = 0.2 \quad ^\circ\text{C}$$

$$U_F = \sqrt{U_{\text{GLT}}^2 + U_{\text{SMS}}^2} \leq 1.025\ U_{\text{GLT}}$$

$$U_{\text{GLT}}^2 + U_{\text{SMS}}^2 \leq 1.050625\ U_{\text{GLT}}^2 \rightarrow U_{\text{SMS}}^2 \leq 0.050625\ U_{\text{GLT}}^2 \leq 0.002025\,^\circ\text{C}$$

$$U_{\text{SMS}} \leq 0.045 \quad ^\circ\text{C}$$

7.8.4

Measurements	Standard (°C)[a]	Object (°C)
1	10.1	10.5
	10.1	10.5
	10.1	10.0
2	19.8	19.5
	19.8	19.5
	19.8	19.5
3	50.1	50.0
	50.1	50.0
	50.1	50.0

[a]Values are corrected by the bias found in the calibration certificate

(a)

Measurements	u Type A std (°C)	u Type A object (°C)
1	0	0.1667
2	0	0
3	0	0

(b) $u_{\text{bath}} = \frac{0.08\,^\circ\text{C}}{\sqrt{3}} = 0.04619\,^\circ\text{C}$

(c) $u_{\text{res}} = \frac{0.5\,^\circ\text{C}}{\sqrt{12}} = 0.14434\,^\circ\text{C}$

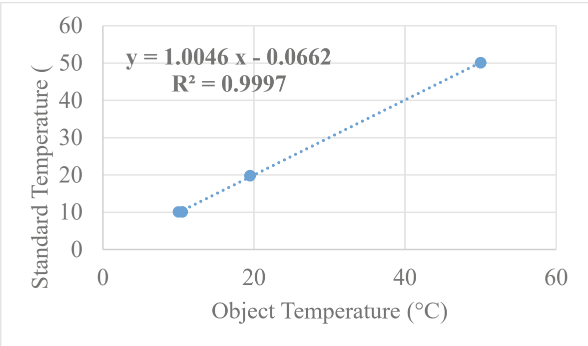
(d)

Standard (°C)	Object (°C)	Bias (°C)
10.1	10.5[a]	0.4
19.8	19.5	−0.3
50.1	50.0	−0.1

[a]The mean object is 10.33 °C, but its resolution is 0.5 °C. We should round to 10.5 °C

(e)

Object (x)	Standard (y)
10.5	10.1
10.5	10.1
10.0	10.1
19.5	19.8
19.5	19.8
19.5	19.8
50.0	50.1
50.0	50.1
50.0	50.1



(f)

Object	Std	$f(x)$	$[y-f(x)]^2$
10.5	10.1	10.4821	0.14600041
10.5	10.1	10.4821	0.14600041
10.0	10.1	9.9798	0.01444804
19.5	19.8	19.5235	0.07645225
19.5	19.8	19.5235	0.07645225
19.5	19.8	19.5235	0.07645225
50.0	50.1	50.1638	0.00407044
50.0	50.1	50.1638	0.00407044
50.0	50.1	50.1638	0.00407044
		Σ	0.54801693
		u_{fit}	0.279800166

(g)

Object (mean) (°C)	Std (mean) (°C)	u_A	u_{fitt}	$u_{obj\ res}$	$u_{std\ res}$	u_{bath}	u_{std}	u_{comb}	ν_{eff}	k	U (°C)
10.5	10.1	0.1667	0.2798	0.14434	0.02887	0.04619	0.09756	0.3733	15.4	2.18	0.8
19.5	19.8	0	0.2798	0.14434	0.02887	0.04619	0.1	0.3348	14.3	2.20	0.7
50.0	50.1	0	0.2798	0.14434	0.02887	0.04619	0.14218	0.3497	16.8	2.17	0.8

7.8.5

(a)

Object	Standard (bar)				Hysteresis
bar	Charge 1	Discharge 1	Charge 2	Discharge 2	bar
6.0	6.4	6.5	6.4	6.5	0.10
10.0	10.5	10.4	10.5	10.4	0.10
24.0	24.3	24.2	24.3	24.2	0.10
30.0	30.3	30.4	30.3	30.4	0.10
40.0	40.5	40.4	40.5	40.4	0.10

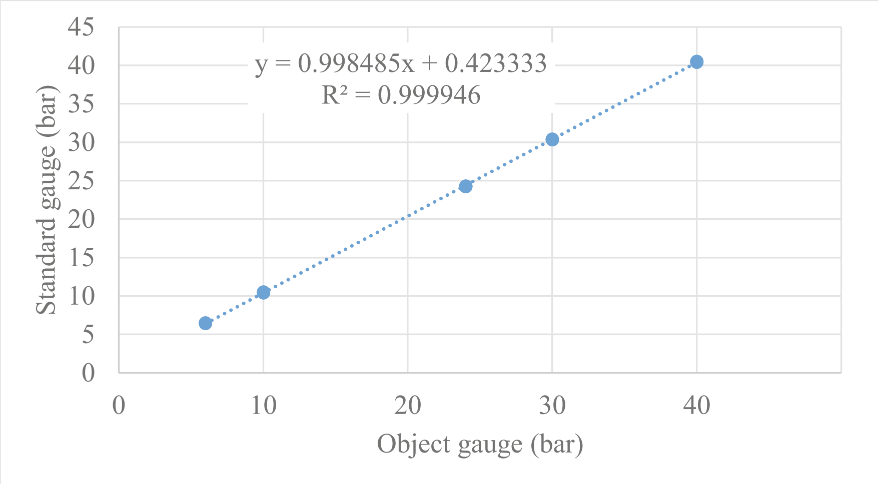
(b)

Object	Standard (bar)					
bar	Charge 1	Discharge 1	Charge 2	Discharge 2	Error (bar)	Error (%)
6.0	6.4	6.5	6.4	6.5	−0.5	1.25%
10.0	10.5	10.4	10.5	10.4	−0.5	1.25%
24.0	24.3	24.2	24.3	24.2	−0.3	0.75%
30.0	30.3	30.4	30.3	30.4	−0.4	1.00%
40.0	40.5	40.4	40.5	40.4	−0.5	1.25%

(c)

Object	Standard (bar)				u	u	u	u	u			U
bar	Charge 1	Discharge 1	Charge 2	Discharge 2	Type A	Obj. resol	Std. resol	std	comb	ν_{eff}	k	bar
6.0	6.4	6.5	6.4	6.5	0.0289	0.102	0.0289	0.05	0.121	919	2.00	0.2
10.0	10.5	10.4	10.5	10.4	0.0289	0.102	0.0289	0.05	0.121	919	2.00	0.2
24.0	24.3	24.2	24.3	24.2	0.0289	0.102	0.0289	0.05	0.121	919	2.00	0.2
30.0	30.3	30.4	30.3	30.4	0.0289	0.102	0.0289	0.05	0.121	919	2.00	0.2
40.0	40.5	40.4	40.5	40.4	0.0289	0.102	0.0289	0.05	0.121	919	2.00	0.2

(d)



(e)

Obj	Std	$f(x)$	$[y - f(x)^2]$
6.0	6.45	6.414243	0.001279
10.0	10.45	10.40818	0.001749
24.0	24.25	24.38697	0.018762
30.0	30.35	30.37788	0.000777
40.0	40.45	40.36273	0.007616
		Σ	0.030182
		u_{fit}	0.100303 bar

(f)

7.8.6
(a) (b)

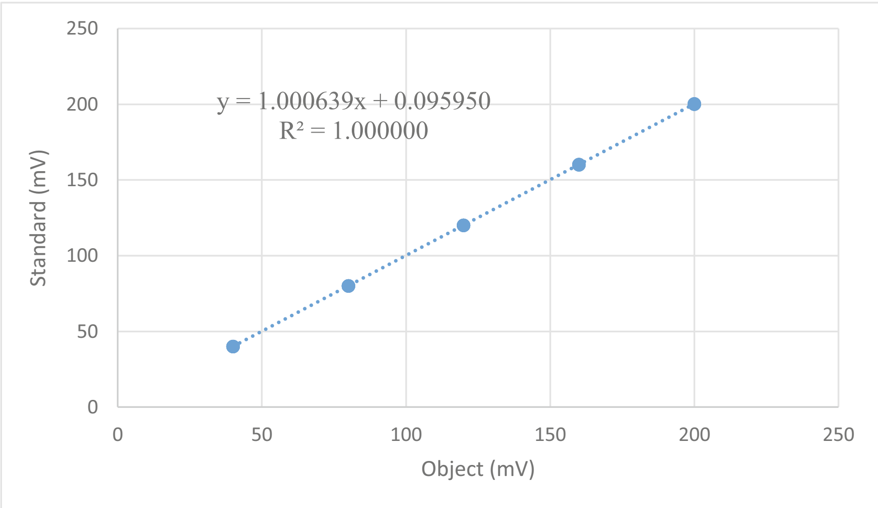
Object	Standard[a] (mV)				Error (mV)	Error (%)
(mV)	V1	V2	V3	V4		
40.00	40.110	40.150	40.160	40.120	−0.16	0.08
80.00	80.117	80.157	80.137	80.127	−0.16	0.08
120.00	120.146	120.166	120.186	120.186	−0.19	0.09
160.00	160.225	160.175	160.165	160.175	−0.22	0.11
200.00	200.205	200.225	200.255	200.265	−0.27	0.13

[a]Values are corrected by the bias found in the calibration certificate

(c)

Object	Standard (mV)				u	u	u	u	u	υ eff	k	U
(mV)	V1	V2	V3	V4	Type A	Obj resol	std	parasite	comb			mV
40.00	40.110	40.150	40.160	40.120	0.0119	0.00289	0.001	0.00115	0.01234	3.5	3.31	0.04
80.00	80.117	80.157	80.137	80.127	0.0085	0.00289	0.001	0.00115	0.00914	3.9	3.31	0.03
120.00	120.146	120.166	120.186	120.186	0.0096	0.00289	0.001	0.00115	0.01012	3.7	3.31	0.03
160.00	160.225	160.175	160.165	160.175	0.0135	0.00289	0.001	0.00115	0.01393	3.4	3.31	0.05
200.00	200.205	200.225	200.255	200.265	0.0138	0.00289	0.001	0.00115	0.01415	3.3	3.31	0.05

(d)



(e)

Object	Standard	$f(x)$	$[y-f(x)]^2$
40.00	40.135	40.11990	0.00023
80.00	80.135	80.14390	8.8E−05
120.00	120.171	120.16790	9.6E−06
160.00	160.185	160.19190	4.8E−05
200.00	200.238	200.21590	0.00047
		Σ	0.00084
		u fitting	0.01673 mV

(f)

Object	Std	u	u	u	u	u	u			U
bar	bar	Type A	Obj. resol	Std. resol	std	fitting	comb	v_{eff}	k	bar
6.0	6.45	0.0289	0.102	0.0289	0.05	0.1003	0.157	18	2.16	0.3
10.0	10.45	0.0289	0.102	0.0289	0.05	0.1003	0.157	18	2.16	0.3
24.0	24.25	0.0289	0.102	0.0289	0.05	0.1003	0.157	18	2.16	0.3
30.0	30.35	0.0289	0.102	0.0289	0.05	0.1003	0.157	18	2.16	0.3
40.0	40.45	0.0289	0.102	0.0289	0.05	0.1003	0.157	18	2.16	0.3

Object	Standard (mean)	*u*	*u*	*u*	*u*	*u*	*u*	*υ* eff	*k*	*U*
mV	mV	Type A	Obj resol	std	parasite	fitting	comb			mV
40.00	40.135	0.0119	0.00289	0.001	0.00115	0.01673	0.020794	5.7	2.65	0.06
80.00	80.135	0.0085	0.00289	0.001	0.00115	0.01673	0.019069	4.7	2.87	0.05
120.00	120.171	0.0096	0.00289	0.001	0.00115	0.01673	0.019555	5.1	2.65	0.05
160.00	160.185	0.0135	0.00289	0.001	0.00115	0.01673	0.021773	6.0	2.52	0.05
200.00	200.238	0.0138	0.00289	0.001	0.00115	0.01673	0.021916	6.1	2.52	0.06

Chapter 8

8.9.1. (b)
8.9.2. (a)
8.9.3. $y = 910$ bar

$$T_L = 900 \text{ bar} \quad U = 10 \text{ bar} \rightarrow u = 5 \text{ bar}$$

$$P_c = \varphi\left(\frac{y - T_L}{u}\right) = \varphi\left(\frac{910 - 900}{5}\right) = \varphi(2) = 0.9772 \rightarrow 97.72\%$$

8.9.4. $P_c = \varphi\left(, \frac{T_U - y}{u}\right) - \varphi\left(, \frac{T_L - y}{u}\right)$

$$T_U = 34.0 \ {}^\circ\text{C} \quad T_L = 30 \ {}^\circ\text{C} \quad u(y) = 0.5 \ {}^\circ\text{C}$$

$$y = 34 \ {}^\circ\text{C} \quad P_c = \varphi\ \left(\frac{34 - 34}{0.5}\right) - \varphi\ \left(\frac{30 - 34}{0.5}\right) = \varphi(0) - \varphi(-16) = 0.5 - 0$$
$$= 0.5\ (50\%)$$

$$y = 32 \ {}^\circ\text{C} \quad P_c = \varphi\left(\frac{34 - 32}{0.5}\right) - \varphi\left(\frac{30 - 32}{0.5}\right) = \varphi(4) - \varphi(-4) = 1 - 0$$
$$= 1\ (100\%)$$

$$y = 33 \ {}^\circ\text{C} \quad P_c = \varphi\left(\frac{34 - 33}{0.5}\right) - \varphi\left(\frac{30 - 33}{0.5}\right) = \varphi(2) - \varphi(-6) = 0.9772 - 0$$
$$= 0.9772\ \ (97.72\%)$$

8.9.5. (c)
8.9.6. $T_U = 500$ mg/kg $\quad U = 4$ mg $\rightarrow u = 2$ mg $\quad P = 0.99$

$$P = \varphi\left(\frac{y - T_U}{u}\right) = \varphi\left(\frac{y - 500}{2} = z\right) = 0.99 \rightarrow z = 2.33$$

$$2.33 = \left(\frac{y - 500}{2}\right) \rightarrow y = 504{,}66 \cong 505 \text{ mg/kg}$$

8.9.7. (d)

8.9.8.

$$P = \varphi\left(\frac{T_L - y}{u}\right) = \varphi\left(\frac{715 - y}{2.5} = z\right) = 0.999 \rightarrow z = 3.08$$

$$3.08 = \left(\frac{715 - y}{2.5}\right) \rightarrow y = 707\ \mathrm{kg/m^3}$$

8.9.9.

(a) $(83 \pm 3)\ ^\circ\mathrm{C}$
(b) TUR = $TL/U \rightarrow U = TL/\mathrm{TUR} = 3\ ^\circ\mathrm{C}/3 = 1\ ^\circ\mathrm{C}$

8.9.10.

(a) $T_L = 44.0\ ^\circ\mathrm{C}$ and $T_U = 50.0\ ^\circ\mathrm{C}$
(b) TUR = $TL/U = 5 \rightarrow U = TL/\mathrm{TUR} = 3\ ^\circ\mathrm{C}/5 = 0.6\ ^\circ\mathrm{C}$

$$w = 1.5\ \ U = 0.9\ ^\circ\mathrm{C} \rightarrow A_L = T_L + w = 44.9\ ^\circ\mathrm{C}\ \text{and}\ A_U = T_U - w = 49.1\ ^\circ\mathrm{C}$$

Chapter 9

9.5.1

(R)	The calibration certificate is a technical record that enables the user to measure instrument conformity assessment
(W)	The laboratory that performs the measuring instrument calibration will never make any recommendation on the periodicity of calibration
(R)	The calibration laboratory does not usually adjust the measurement instrument; however, results should be reported before and after adjustment if this is done
(R)	The certificate must present calibration results in the units of measure used and declare the measurement uncertainty, scope factor, and confidence level used
(W)	The environmental conditions of the calibration site only need to be declared, if they affect the calibration result

9.5.2

Object (bar)	Standard (bar)				Error (bar)	Error (%)
	Charge 1	Discharge 1	Charge 2	Discharge 2		
5.0	5.00	5.20	5.25	5.25	0.25	0.42
15.0	15.25	15.55	15.00	15.50	0.55	0.92
25.0	25.00	25.55	25.50	25.55	0.55	0.92
35.0	35.25	35.00	35.50	35.25	0.50	0.83
45.0	44.55	45.05	45.00	45.50	0.55	0.92
55.0	56.00	56.00	55.55	55.50	1.00	1.67
60.0	60.00	60.00	60.00	60.00	0.00	0

Class 1.0 → MPE = 1.0 %. The 55.0 bar error was 1.67 %, higher than the MPE. Therefore, the gauge does not meet class 1.0.

9.5.3

- Unique identification that all components are recognized as part of a complete report and an identification to the end of the document.
- Presentation of the method used in calibration.
- Date of calibration.
- Declaration that the results apply only to the calibrated instrument.
- Declaration that the certificate should only be reproduced completely.

9.5.4
Table 9.9 Burettes (ASTM E287)

Capacity (mL)	MPE (mL) ±	
	Class A	Class B
10	0.02	0.04
25	0.03	0.06
50	0.05	0.10
100	0.10	0.20

$$V = 25.05 \text{ mL} \;\rightarrow\; \text{Error} = (25.05–25)\ \text{mL} = 0.05\ \text{mL} < \text{Class B error} = 0.10\ \text{(OK)}$$

$$U = 0.01\ \text{mL}$$

$$U_{\text{maximum}} = E + U = (0.05 + 0.01)\ \text{mL} = 0.06\ \text{mL} < AC = 0.1\ \text{mL}\ \text{(OK)}$$

9.5.5 We know that for a 95 % decision rule (ISO 14253-1:2017), we must use a guard band $w = 0.83\ U_{\text{maximum}}$

Width: $A_L = T_L + U_{\text{maximum}} = (24.988 + 0.00166)\ \text{mm} = 24.990\ \text{mm}$

$$A_U = T_U - U_{\text{maximum}} = (25.012 - 0.00166)\ \text{mm} = 25.010\ \text{mm}$$

Diameter: $AL = T_L + U_{\text{maximum}} = (958.5 + 0.415)\ \text{mm} = 958.9\ \text{mm}$

$$A_U = T_U - U_{\text{maximum}} = (961.5 - 0.415)\ \text{mm} = 961.1\ \text{mm}$$

9.5.6 According to Table 8.11, for a false positive of less than 2.5%, we must adopt a w guard band equal to AC.

$$A_L = T_L + U_{\text{maximum}} = (14.95 + 0.017)\ \text{mm} = 14.97\ \text{mm}$$

$$A_U = T_U - U_{\text{maximum}} = (15.05 - 0.017)\ \text{mm} = 15.03\ \text{mm}$$

9.5.7 (b)
9.5.8 (a)
9.5.9 (b)
9.5.10 (c)
9.5.11 (a)
9.5.12 (b)
9.5.13 (c)
9.5.14 (b)
9.5.15 (b)

References

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