## The Princeton Review ${ }^{\text {s }}$

BETTER STRATEGIES. HIGHER SCORES.

## Cracking the New York City


(Specialized High Schools Admissions Test)

## 3rd Edition

## The help you need to get into the

 high school you want.- 2 full-length practice tests with detailed answer explanations
- Review questions and drills to help maximize your score
- Essential math and verbal content review to prepare for the redesigned test


# Cracking the New York City <br>  <br> (Specialized High Schools Admissions Test) 

## 3rd Edition

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- Access additional Reading passages and drills for the new Revising/Editing section and print out extra bubble sheets as needed
- Check to see if there have been any corrections to this edition
- Get our take on any additional late-breaking updates to the SHSAT

Look For These Icons Throughout The Book

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## Part I

Orientation

## Chapter 1 <br> Profiles of the Specialized High Schools

The Specialized High Schools Admissions Test-from now on, we'll refer to it simply as the SHSAT-helps determine entry into a specific field of eight schools. The following is a very basic list of information about these schools. For more information, refer to the schools.nyc.gov/shs website.

## Bronx High School of Science

75 West 205th Street
Bronx, New York 10468
School: (718) 817-7700
Fax: (718) 733-7951
E-mail: golanc@bxscience.edu
Website: www.bxscience.edu
Number of Students: 2,979
2017 Acceptance Rate: 5\% (941 out of 19,198) Languages: French, Greek, Italian, Japanese, Latin, Mandarin, Spanish The Brooklyn Latin School
223 Graham Avenue
Brooklyn, New York 11206
Phone: (718) 366-0154
Fax: (718) 381-3012
E-mail: parents@brooklynlatin.org
Website: www.brooklynlatin.org
Number of Students: 724
2017 Acceptance Rate: 2.6\% (425 out of 16,621) Languages: Greek, Latin, Spanish Brooklyn Technical High School 29 Fort Greene Place
Brooklyn, New York 11217
Phone: (718) 804-6400
Fax: (718) 260-9245
E-mail: info@bths.edu
Website: www.bths.edu
Number of Students: 5,682

2017 Acceptance Rate: $8 \%(1,923$ out of 23,329$)$ Languages: French, Italian, Latin, Mandarin, Spanish High School for Mathematics, Science and Engineering at City College (HSMSE) 240 Convent Avenue
Manhattan, New York 10031
Phone: (212) 281-6490
Fax: (212) 281-6918
E-mail: info@hsmse.org
Website: www.hsmse.org
Number of Students: 463
2017 Acceptance Rate: 1\% (199 out of 18,863) Languages: German, Spanish

High School of American Studies at Lehman College 2925 Goulden Avenue
Bronx, New York 10468
Phone: (718) 329-2144
Fax: (718) 329-0792
E-mail: atrebofiore@schools.nyc.gov
Website: www.hsas-lehman.org
Number of Students: 377
2017 Acceptance Rate: <1\% (146 out of 16,737) Languages: French, Greek, Italian, Japanese, Latin, Russian, Spanish Queens High School for the Sciences at York College 94-50 159th Street
Jamaica, New York 11433
Phone: (718) 657-3181
Fax: (718) 657-2579
E-mail: Adejesus7@schools.nyc.gov
Website: www.ghss.org

Number of Students: 462
2017 Acceptance Rate: 1\% (179 out of 16,402) Languages: French, Mandarin, Spanish Staten Island Technical High School 485 Clawson Street
Staten Island, New York 10306
Phone: (718) 667-3222
Fax: (718) 987-5872
E-mail: BMalefant@schools.nyc.gov
Website: www.siths.org
Number of Students: 1,312
2017 Acceptance Rate: 2.2\% (339 out of 15,278) Languages: Russian

## Stuyvesant High School

345 Chambers Street
Manhattan, New York 10282
Phone: (212) 312-4800
Fax: (212) 587-3874
E-mail: 02M475@schools.nyc.gov
Website: stuy.enschool.org
Number of Students: 3,365
2017 Acceptance Rate: 4.1\% (926 out of 22,393) Languages: French, German, Italian, Japanese, Latin, Mandarin, Spanish The Fiorello H. LaGuardia High School of Music \& Arts and Performing Arts is also a specialized high school, but it handles admission via auditions, not the SHSAT. You can find more information about this school at their website, www.laguardiahs.org.

## Chapter 2

The SHSAT, The Princeton Review, and You

## WHAT IS THE SPECIALIZED HIGH SCHOOLS ADMISSIONS TEST?

The SHSAT, revised in 2017 and updated again in 2018, is a three-hour test that is administered to New York City residents in October/November. It is taken by eighth-grade and first-time ninth-grade students, and the results of this test help to determine admission into the following specialized high schools: The Bronx High School of Science; the Brooklyn Latin School; Brooklyn Technical High School; High School of Mathematics, Science and Engineering at the City College of New York; High School of American Studies at Lehman College; Queens High School for the Sciences at York; Staten Island Technical High School; and Stuyvesant High School.

A Changing Test
This book accounts for all of the question types and content made available as of the 20172018 administration of the test (including a latebreaking update to the Fall 2018 test). Should anything else change, we'll update your free
student tools with walk-throughs of new question types, so be sure to register your book and check online!

For the most part, the test consists of multiple-choice questions that alternate between ABCD and EFGH; there is no penalty for guessing. In the Math section, five questions are grid-ins, for which you will bubble in the numeric answer itself.

The test is divided into two sections, each with various parts. You have three hours total to finish both sections, and can freely move between the two.

1. English Language Arts (ELA)

- 57 multiple-choice questions
- Part A includes 9 to 11 Revising/Editing questions, split between standalone questions and those based on one passage of approximately 350 words.
- Part B includes 46 to 48 Reading Comprehension questions; expect six to seven reading passages, including prose and poetry, each of which can be up to 900 words long.


## 2. Mathematics

- 5 grid-in problems
- 52 multiple-choice questions
- These questions pull from New York City's math curriculum through Grade 7.
- Calculators are not permitted.

There are actually two versions of the SHSAT. This book covers the eighth-grade test. The ninth-grade test is very similar, but it includes math from the Grade 8 curriculum.

## HOW DOES SCORING WORK?

You may hear about two kinds of scores in connection with the SHSAT: raw scores and scaled scores. Here's how they work.

## Raw Scores

There are 114 questions on the SHSAT, but 20 of them (10 in each section) are embedded field test items-that is to say, they're unscored. Of the remaining 94, each one is worth a point.

Gotta Answer 'Em All
Sadly, there's no way to tell in advance if a question is going to count or not, so answer everything. Remember, there's no penalty for guessing.

## Scaled Scores

Because the different versions of the SHSAT are not exactly the same, these raw scores are now translated into a scaled score that accounts for these inconsistencies. It is possible to get a 400 in each of the two sections, making an 800 the highest possible composite score.

## How Important Are SHSAT Scores?

Admission to the specialized high schools is based entirely on your SHSAT score. However, how that SHSAT score is weighted depends on a few other key factors.

## Ranking Your Choices

The first thing you will fill out on your answer sheet is your order of preference. You must fill in at least one school. After the approximately 28,000 students have all been graded, the highest scorers will be granted admission to their firstchoice schools. This will continue down the list; if your first-choice school has been filled by the time your score is reached, you will gain admission to your highest-choice school that still remains.


Tell Them What You Want (What You Really, Really Want)
Perhaps you noticed the acceptance rates in Chapter 1 and think that you're best off putting a school with a higher percentage toward the top of your list. No need! When it's your turn,

## Summer Discovery Programs

Some of the specialized high schools participate in the Discovery Program, which provides a second chance for students who were very close to their school's cut-off score. If you and your school are eligible, and your score qualifies, you'll be able to attend a summer program that, if completed, will grant you admission to that school.

This is an extremely small program that impacts approximately 200 students; speak with your school counselor to see if you and your desired school qualify. More information can be found at schools.nyc.gov/ChoicesEnrollment/High/ specialized.

## HOW DO I REGISTER FOR THE SHSAT?

If you're interested in taking the SHSAT, you need to let your school's guidance counselor know. They will provide you with an SHSAT Test Ticket; this ticket will have the exact date and time of the test, as well as the specific testing site that you'll be going to. If you have a conflict, tell your counselor immediately.

## Before the Test

$\checkmark$ Double-check all of the personal information on the ticket to make sure it is accurate. This is especially important if you require and qualify for any testing accommodations.
$\checkmark$ Sign the ticket. Have your parents/guardians sign it as well.
$\checkmark$ Rank the schools that you wish to attend in order of preference. This is more for your reference, but it's not something you want to figure out at the test itself.
$\checkmark$ Make sure you take your signed ticket to the test site.

## WHAT IS THE PRINCETON REVIEW?

The Princeton Review is the leader in test prep. Our goal is to help students everywhere crack standardized tests, from the SHSAT to the PSAT, SAT, and ACT, as well as graduate-level exams like the GRE and GMAT. Starting from humble beginnings in 1981, The Princeton Review is now the nation’s largest SAT preparation company. We offer courses in more than 500 locations in 20 different countries, as well as online, and we publish best-selling books.

## The Princeton Review Way

This book will show you how to crack the SHSAT by teaching you to:

- extract important information from tricky test questions
- take full advantage of the limited time allowed
- systematically answer questions-even if you don't fully understand them
- apply test-taking strategies to use the test to your advantage


## Practice Makes Perfect

Some of the strategies employed in this book may be different from those that you've used before. As a result, it may take time to integrate them into your testtaking approach. This book is equipped with two full-length SHSAT-like practice tests and a lot of independent practice questions designed to give you that time and practice. It's up to you to use them, however, and to figure out what works best for you.

To that end, we also recommend that you get a copy of the Specialized High Schools Student Handbook, which is available for free through the New York City Board of Education. This booklet contains another two full practice tests, and by working through these with our techniques, you will be able to find the most efficient strategies for test day. (Familiarity with the test should also increase your confidence.)

Copies of this handbook are available online at schools.nyc.gov/ ChoicesEnrollment/High/specialized; you should also be able to request one from your guidance counselor.

Are you ready? Let’s get cracking!

## Chapter 3

Pacing Yourself for the SHSAT

## WHAT DO I DO WITH THIS BOOK?

Not only is time management a big portion of any timed test-it's also a big portion of studying for any test. In both cases, you're trying to figure out the most efficient way to do something-to study or to solve-in a limited amount of time, and the answer to both is the same:

## Plan ahead.

So far as this book goes, begin by figuring out what you need. We recommend beginning with the diagnostic test, timing yourself and sitting in similar conditions to those of the actual test.

## When You Take a Practice Test

- Time yourself strictly. Use a timer, watch, or stopwatch that will ring and take note of where you were when you ran out of time. If you use a phone for this, make sure that you do not use any other functions, as you will not have access to your phone during the test.
- Take the test in one sitting. The real exam is three hours with no breaks, so while you might find it advantageous to pause for a few minutes to catch your breath and collect your thoughts between sections, be aware that time will keep counting down. Weigh the benefits of any mind-clearing break against that ticking clock, and work on building up your endurance for the real test.
- Fill your answers into an actual bubble sheet. We’ve provided one before each of the two practice tests in this book, and you can print additional copies from your online student tools (if you don't like the idea of marking up this book). The point is, you need to be comfortable transferring answers to a separate sheet, especially if you're planning to skip questions or jump between sections.
- Each bubble must be filled in thoroughly, with no other marks in the answer area.
- Periodically check to make sure you're filling out the right bubbles for
the questions you're on. If you fill in the wrong bubble, it won't matter if you've worked out the problem correctly in your test booklet. All that matters to the machine scoring your test is the mark of your No. 2 pencil.

Once three hours have elapsed, take note of how much of the test you completed, and identify the question types or content areas that slowed you down the most. After that, go through and answer any remaining questions and then check to see which ones you answered incorrectly.

## Make a Plan

Once you know where you're stumbling-is it a speed issue? content issue? both?-you can start tackling the content sections of this book.

## A Need For Speed

If you find yourself running out of time, work on applying The Princeton Review strategies to the questions in this book. The SHSAT is testing more than just your content knowledge. It wants to know that you can manage your time effectively. For example, working through all the math for a question isn't always the most time-efficient way to get the right answer. (You'll learn more about this later in Part IV.)

## Discontent with the Content

If there are questions you're misreading or that you fundamentally don't understand how to solve, pay close attention to the Crack It questions in the relevant content sections.

Once you feel more comfortable with the SHSAT, take the second practice test. Continue to study the sections you're struggling with and then move on to the additional tests found in the Specialized High Schools Student Handbook at the Department of Education's website: schools.nyc.gov/ChoicesEnrollment/High/ specialized.

## On the Day of the SHSAT

Your preparation doesn't end once you feel comfortable with the content and pace of the test. The night leading up to the test is critical.

- Wake up refreshed from at least eight hours of sleep the night before. The stress of a last-minute cram isn't worth the sluggishness exhaustion can cause.
- Eat a filling breakfast; you don't want to have distracting hunger pains during the test.
- Food for thought: bring snacks and water. Even if you finish early, you cannot leave the testing room until the 3 hours is up.
- Do not bring electronic devices such as tablets to the test. Cell phones will be collected before the test begins; in short, bring only what you need.
- Do bring sharpened Number 2 pencils and a sturdy eraser. You can bring a highlighter, ergonomic grip for your pencil, or even a magnifying lens.
- If you have not been to the test site before, try to familiarize yourself with the route so you know how long it will take to get there.
- Aim to arrive at the testing center 30 minutes before the time printed on your SHSAT Test Ticket. You can always use any extra time to shake out any last minute nerves, and it's good to have the cushion in case something goes wrong in transit.
- Remember that you do not have to work out every question on the test to get a good score. Don't let yourself become rushed. Pace yourself.


## PACING ON THE SHSAT

Many people believe that to do well on a test, it is important to answer every question. While this may be true of some of the tests you take in school, it's often not the case on standardized tests like the SHSAT. On this test, it is very possible to score well without attempting all of the questions; in fact, you might even improve your scores by answering fewer questions.

If that seems dubious to you, consider this: on the SHSAT, you get points only for answering a question correctly. The more time you spend on one question, the less time you'll have for the others, which means you might have to guess on
them. But all of these questions are worth the same number of points, which means you might end up working hard on a question that you still get wrong, and then guessing on a question that you could have easily answered, if only you'd had the time to read it. Because of this, it's important that you first give your attention to problems you think you can answer. You can always come back to thornier questions later.

Jump. Jump. Jump Around
Questions are not presented in order of difficulty, so skim questions, tackling the ones that look most up your alley first.

## Chapter 4

Strategies for the SHSAT

Here are some general strategies for the SHSAT, followed by some specific ones for the English Language Arts and Math sections.

## EMBRACE YOUR POOD

Embrace your what now? POOD! This stands for "Personal Order of Difficulty." We spoke a bit in the previous chapter about pacing yourself on the test. It's important to re-emphasize that here. The questions on the SHSAT pull from a lot of different content areas, but they're all worth exactly one raw point each.

## One Pass, Two Pass

Within any section, you will find three types of questions:

- those you can answer easily without spending very much time
- those that, if you had all the time in the world, you could solve
- those that you have absolutely no idea how to tackle

To maximize your time, begin the test with the section that you feel most comfortable and confident with—either ELA or Math. Within that section, solve only the first type of question, and once you've solved them all, you'll know how many of the second type you have time to return to. You may want to take all the questions for a Reading passage all in one go, just so that the information you've just read remains fresh. To that end, begin by reading the passages that have topics that seem familiar or interesting to you.

## Bungle-free Bubbling

If you're going to be jumping around from question to question, the most efficient way is to bubble in each question as you answer it, making sure that the number matches. If you're going straight through a section, try first circling all the answers on a
page and then transferring them all at once to the answer sheet. This helps you to fully focus on each task, whether it's answering questions or bubbling them in. Just don't wait until the end. Marks in your booklet aren't worth anything!

## PROCESS OF ELIMINATION

Here's a question you will not see on the SHSAT, but which will show you how powerful Process of Elimination (POE) can be.

What is the capital of Malawi?
A. New York
B. Paris
C. London
D. Lilongwe

There are two ways to get this question right. The first is if you already have the specialized knowledge that the capital of Malawi is Lilongwe. If you don't, however, you can still get the question right-and that's by knowing what the capital of Malawi isn't. The computer that's scoring your test isn't going to know which method you used, and even if it did, it wouldn't care: a right answer is all that matters. Try to get into the habit of looking at a question and asking, "What are the wrong answers?" instead of "What is the right answer?"


By using POE this way, you can leave yourself with fewer answers to choose between. Statistically, this means that you'll pick right answers more often than not. Your baseline odds are $25 \%-1$ right answer out of 4 . Eliminate one choice, and your odds are now $33 \%$; get rid of another, and they shoot up to $50 \%$. That's the great thing about guessing on a standardized test like the SHSAT. Outside of the five Grid-In questions on the Math section, when you have trouble finding
the correct answer, you can use POE to improve your chances. You don't have to come up with the answer from scratch. You just have to identify it.

## The Answers Are Already There

The SHSAT helps even further, in that the majority of questions are contextbased, as opposed to our sample geography question, which requires pre-existing knowledge. The right answer in the Math section has to work for the question, which means that you can sometimes use it to check your work, time permitting. Likewise, the ELA questions are referring to passages or sentences. If something doesn't match, it may be wrong.

In these cases, you may want to place physical marks next to each answer choice as you go.
$\checkmark$ Put a check mark next to an answer that you feel confident about.
$\sim$ Put a squiggle next to an answer that seems correct.
? Put a question mark next to something you think is wrong.
A Cross out the letter of any answer choice you KNOW is wrong.
Once you've reviewed every choice, these icons make it easier to use POE in an educated way. For instance, take the following question:

What is the capital of Qatar?
E. Paris
F. Dukhan
G. Lilongwe
H. Doha

In your initial pass, you strikethrough (E), Paris, and you put a question mark next to (G), Lilongwe, because that doesn’t seem right. You put a squiggle next to (H), Doha, because you think you've heard it before. You're still taking a guess, but it's a more educated one, and over the course of the test, each small, smart choice can have a big impact on your final raw score. By the way, you can use any symbols you want-just be consistent.

## COLUMN OF THE DAY (COTD)

Another benefit of the SHSAT is that there's no penalty for guessing. Say that you can't eliminate any answers, or you're about to run out of time. In this case, pick a Column of the Day and simply bubble down that column (A/E, B/F, C/G, or $\mathrm{D} / \mathrm{H}$ ) at the very end of the test. This is a simple way to make sure that you've answered every remaining question as quickly as possible. Also, while there's no guarantee that the answers are evenly distributed, if they are, this might give you a slight edge.

COTD should absolutely be an afterthought; it's far more important and helpful to your score to use the full three hours of the SHSAT to thoughtfully eliminate answer choices. But for questions you don't know at all and when time is up, COTD is better than having no strategy at all.

## SHSAT MATH STRATEGIES

Math questions have to make sense within the context of the question, which means that there are a variety of strategies you can use on the multiple-choice questions to help find the answer from a different direction.

## Guesstimating

Your answers on the SHSAT need to be exact. However, you don't need to be as precise when eliminating choices!

AGE OF FERRIS WHEEL RIDERS

| Number of Years Old | Percentage of Riders |
| :---: | :---: |
| $\leq 10$ | $3 \%$ |
| $11-30$ | $42 \%$ |
| $31-50$ | $35 \%$ |
| $51-69$ | $14 \%$ |
| $\geq 70$ | $6 \%$ |

The festival organizer is looking over the percent distribution for the day's Ferris wheel riders to see how many purchased full-price tickets. If discounted tickets are sold to those 10 and under and those 70 and over,
and there were 500 riders, how many full-price tickets were sold?
A. 45
B. 245
C. 355
D. 455

Without doing any calculation, think about this question. Nearly half the riders$50 \%$-were 11 to 30 years old, and they bought full-price tickets. This means that both (A) and (B) are likely too small, and you can use POE to rule them out. Estimating the percentage would reveal that nearly $90 \%$ paid full-price, which makes the only possible answer 455, (D).

Guesstimating can also be used to check your work and confirm an answer. If you accidentally found the number of discounted tickets sold, you'd have answered (A). But if you consider how many people bought full-price tickets, you'll realize something is wrong with that answer, and you'll avoid getting trapped.

Here's another good example:
A group of three men buys a one-dollar lottery ticket that wins $\$ 400$. If the one dollar they paid for the ticket is subtracted and the remainder of the prize money is divided equally among the men, how much will each man receive?
E. $\$ 62.50$
F. $\$ 75.00$
G. $\$ 133.00$
H. $\$ 200.00$

To solve this mathematically, you would take $\$ 400$, subtract $\$ 1$, and then divide
the remainder by 3 . But with a little bit of logic, you wouldn’t have to do any of that.

The lottery ticket won $\$ 400$. If there were four men, each one would have won about $\$ 100$. However, there were only three winners, which means each one won more than $\$ 100$, which instantly rules out (E) and (F). Next, consider (H): this is saying that each of the three men won half of the jackpot-but that's not possible. The only answer left is (G), which is correct.

## Plugging In

The SHSAT will often ask you questions about real-life situations in which the numbers have been replaced with variables. For example:

Mark is two inches taller than John, who is four inches shorter than Frank. If $f$ represents Frank's height in inches, then in terms of $f$, what is an expression for Mark's height?
A. $f+4$
B. $f+2$
C. $f$
D. $f-2$


You might stumble on the representation of this problem if you aren't used to thinking of people's heights in terms of variables. To get around that, just pick a number to replace the variable with. As a bonus, since you're the one picking numbers, you can choose ones that you find easiest to work with. They don't even have to be realistic: in this example, let's say that Frank is 10 inches tall, or to put it in terms of the math, $f=10$.

We know that John is four inches shorter than Frank, so $j=6$. We also know that Mark is two inches taller than John, so $m=8$. What you're looking for is the
answer choice which, when you input $f=10$, yields Mark's height, 8 . The only one that fits is (D), $f-2$.

## Percentages and Fractions

There are a few other types of questions for which you might find it convenient to use Plugging In. For instance, questions that deal with percentages and fractions can be tricky; choosing convenient numbers can make them easier to process.

The price of a suit is reduced by half and then the resulting price is reduced by $10 \%$. The final price is what percent of the original price?
E. $10 \%$
F. 25\%
G. $40 \%$
H. $45 \%$

When dealing with percents, the easiest number to work with is generally 100 . In this case, if 100 is reduced by half, it is 50 , and if that is then reduced by $10 \%$, it is 45 . The final price is 45 , which is $45 \%$ of the original price, which means that $(\mathrm{H})$ is correct.

## Plugging In the Answers (PITA)

When the question provides you with an algebraic equation and the answer choices are numbers, you can use a more specific version of this strategy: Plugging In the Answers (PITA). The difference in this case is that instead of choosing your own numbers, you're testing the ones that have been so generously provided by the test writers.

Aaron's bookcase currently holds $b$ books, but is capable of holding up to 80 books. If Aaron loans 3 books to his friend, Sonya, the bookcase is now three-quarters full. What is the value of $b$ ?
A. 54
B. 57
C. 60
D. 63


There's no need to make up your own numbers-in fact, it might be rather timeconsuming to do so. We know that the answer is going to be $60,63,66$, or 69 , so there's no need to test any other numbers. Note that when the answers are in ascending or descending order-and they usually will be on the SHSAT-you can save even more time by starting with one of the two middle choices.

In this case, try starting with (B): if you have 57 books and take 3 away, you'll have 54 books, which is fewer than 60, three-quarters of the bookshelf's capacity. Because you're going to need more books, you can also eliminate (A), and move up to the next choice. In this case, you can also eliminate (C), which is clearly incorrect because it's 60 before subtracting any books. That makes (D) the right answer.

The goal of these math strategies is to help you avoid algebraic mistakes, and to answer a question if you don't remember exactly how to solve it head-on. These can sometimes take a little more time, however, so take time throughout this book to really practice them. Find the methods that work best for you!

## SHSAT ELA STRATEGIES

The trick to eliminating choices for Reading passages is to remember that the answers are always in the text. Watch out for answers that fall into any of the following categories, as they're probably wrong.

## Extreme Answers

An extreme answer is too negative or too positive, or it uses absolute words like "all," "every," "never," or "always." If you have two answers that express a similar idea, there's a good chance that the one that's stated firmly is probably
wrong, while the other, more moderate one, is probably correct.

## Complicated Answers

The Reading passages are not lengthy (though they might sometimes seem that way). As a result, there's little room to cover broad topics, make definitive claims, or have complicated conclusions. An answer should be to the point, and if it's not, it's likely trying to mask the fact that it wasn't actually mentioned in the passage. These choices might use words that do appear in the passage, so make sure there's more to that answer than just a key word. Along this same line, know that if an answer does reiterate something from the passage, it's more likely to be paraphrased than directly repeated.

## Nonsense Answers

Passages are likely to be based on real-world events or claims, which means that if a choice sounds ridiculous, it's probably wrong. If you understand what the passage is saying, this will also help you eliminate contradictory answers-the ones that claim the opposite of what the author is saying.

## Absent Answers

Above all else, remember that all answers are supported by the text. If you can't find evidence in the passage for something, then it's the wrong answer.

## Except/Not/Least Questions

As of the most recent SHSAT, this type of question appears far less frequently, but because it flips things around, you should be prepared in case it makes an appearance. What you need to know about this tricky type of question, which always uses the cue word "except," "least," or "not," the right answer is the one that would normally be wrong.

Instead of changing your entire approach, just cross out the cue word so that the question is normal again, and then read through each choice, using POE to place a "T" (for true) or an "F" (for false) next to each choice. (You can also just cross out any answer that's not true.) In the end, whichever answer choice is crossed
out or has the $F$ next to it is the correct answer.

## Part II

Diagnostic Practice Test

## Chapter 5 Practice Test 1

Click here to download a PDF of Practice Test 1.

# PART 1—ENGLISH LANGUAGE ARTS 

Suggested Time-90 Minutes
57 QUESTIONS

## REVISING/EDITING

QUESTIONS 1-11
IMPORTANT NOTE
The Revising/Editing section (Questions 1-11) is in two parts: Part A and Part B.

## REVISING/EDITING Part A

DIRECTIONS: Read and answer each of the following questions. You will be asked to recognize and correct errors in sentences or short paragraphs. Mark the best answer for each question.

1. Read this paragraph.
(1) Art Spiegelman's graphic novel Maus: A Survivor's Tale is one of the most acclaimed works of literature of the 20th century. (2) Maus is the true story of Spiegelman's parents, Vladek and Anja, Jews who survived the Holocaust in Poland and then emigrated to America after World War II. (3) In Part I of Maus, Spiegelman uses text and illustrations to depict the terror Vladek and Anja felt
during the Nazi occupation of Poland, Part II focuses on the Spiegelmans' lives in America after the war. (4) Part II won the Pulitzer Prize for Literature in 1992.

Which sentence should be revised to correct a run-on?
A. sentence 1
B. sentence 2
C. sentence 3
D. sentence 4
2. Read this sentence.
J.D. Salinger's first novel The Catcher in the Rye, was published in 1951, when he was 32 years old.

Which edit should be made for this sentence?
E. insert comma after novel
F. delete comma after Rye
G. insert comma after Salinger's
H. delete comma after 1951
3. Read these sentences.
(1) Many nature preserves are located far from major cities.
(2) Some big cities have managed to establish their own nature

What is the best way to combine these sentences to clarify the relationship between the ideas?
A. Because some big cities have managed to establish their own nature preserves, many nature preserves are located far from major cities.
B. Many nature preserves are located far from major cities, and some big cities have managed to establish their own nature preserves.
C. Yet some big cities have managed to establish their own nature preserves, many nature preserves are located far from major cities.
D. While many nature preserves are located far from major cities, some big cities have managed to establish their own nature preserves.
4. Read this sentence.

Uninteresting and a little loud, I left the concert early.

How should this sentence be revised?
E. I left the concert early because it was uninteresting and a little loud.
F. Uninteresting and a little loud, the concert was left early.
G. I left the concert, uninterested and a little loud.
H. Early, uninteresting, and a little loud, I left the concert.

## REVISING/EDITING Part B

DIRECTIONS: Read the passage below and answer any questions following it. You will be asked to improve the writing quality of the passage and to correct errors so that each passage follows the conventions of standard written English. You may reread the passage if you need to. Mark the best answer for each question.

## Bluefin Tuna: An Endangered Dinner

(1) Although sushi lovers may never know it, their eating habits may be contributing to the worldwide shortage of Pacific bluefin tuna. (2) A recent study jointly published by several conservation agencies recommended that bluefin tuna be added to the U.S. National Marine Fisheries Service's list of endangered species. (3) Adding the bluefin tuna to this list would ensure the popular fish is given extreme protection from exploitation and other kinds of harm.
(4) Obviously, let's ask: how did we get here? (5) Much of this sharp decline can be blamed on people's insatiable appetite for raw tuna. (6) Scientists estimate that bluefin tuna have been depleted to 3 percent of their pre-exploitation population, or the average number of fish in the oceans before humans began commercially fishing. (7) Originally a favorite of the Japanese diet, tuna sashimi is now prized by eaters all over the world, only heightening the demand for bluefin tuna. (8) Still, the majority of fish caught today is sent to Japan.
(9) Another reason that Pacific bluefin tuna stocks have decreased by 97 percent relates to fishing practices. (10) According to marine biologists, the majority of the bluefin tuna that are caught are very young fish that have not matured to adulthood. (11) Most of the world's captured tuna never had a chance to procreate. (12) The stock of tuna has decreased sharply.
(13) And many experts say that as long as tuna remains popular among consumers, the troubling cycle will continue unabated. (14) On a normal day at Tokyo's busy Tsukiji fish market, a single bluefin tuna can sell for tens of thousands of dollars. (15) In fact, in 2013, Kiyoshi Kimura, a Japanese restaurateur, paid $\$ 1.76$ million for a tuna-a record price. (16) Indeed, it is easy to understand why fishermen continue to eagerly search the Pacific Ocean for
the endangered fish: they can make a fortune selling just one fish.
(17) Some chefs think serving bluefin tuna in restaurants is unconscionable, given their low population numbers. (18) They recognize that action needs to be taken, and have started to pursue other, more sustainable methods. (19) In the United States, many prominent chefs have endorsed the Bluefin Boycott in the name of conservation.
5. Which revision of sentence 4 best maintains the formal style established in the passage?
A. How did this happen? is a question you should ask.
B. First of all, I must back up for a second: how did we get here?
C. It is worth analyzing the causes of the tuna shortage.
D. The causes of the tuna shortage: let's talk about them.
6. Where should sentence 5 be moved to improve the organization of the second paragraph (sentences 4-8)?
E. between sentences 3 and 4
F. between sentences 6 and 7
G. between sentences 7 and 8
H. the end of the paragraph (after sentence 8)
7. Which revision of sentence 8 uses the most precise language?
A. Still, about 80 percent of the bluefin tuna caught today is sent to Japanese consumers.
B. Still, the majority of fish caught today is sent to Japanese consumers.
C. Still, about 80 percent of the bluefin tuna caught today is sent to Japan.
D. Still, the majority of bluefin tuna caught today is sent to Japanese consumers.
8. What is the best way to combine sentences 11 and 12 to clarify the relationship between ideas?
E. Because most of the world's captured tuna never had a chance to procreate, the stock of tuna has decreased sharply.
F. Despite that most of the world's captured tuna never had a chance to procreate, the stock of tuna has decreased sharply.
G. The stock of tuna has decreased sharply, yet most of the world's captured tuna never had a chance to procreate.
H. Because the stock of tuna has decreased sharply, most of the world's captured tuna never had a chance to procreate.
9. Which transition word or phrase should be added to the beginning of sentence 17 ?
A. And
B. In addition
C. Moreover
D. However
10. Read this sentence.

Legendary sushi chef Jiro Ono, for instance, has taken to finding alternative fish to take the place of bluefin tuna in his menu.

Where should this sentence be added to best support the ideas in the fifth paragraph (sentences 17-19)?
E. between sentences 16 and 17
F. between sentences 17 and 18
G. between sentences 18 and 19
H. at the end of the paragraph (after sentence 19)
11. Which concluding sentence should be added after sentence 19 to support the argument presented in the passage?
A. Ultimately, it will take a more global consensus between scientists, chefs, and consumers to truly reverse the devastation facing the bluefin tuna.
B. In the end, not much can be done to save the bluefin tuna as it races toward extinction.
C. Ultimately, scientists need to conduct more studies to identify the best way to protect the bluefin tuna.
D. If Japanese fishing of bluefin tuna continues at present levels, the status of the tuna is precarious.

## READING COMPREHENSION

QUESTIONS 12-57
DIRECTIONS: Read the passage below and answer the questions following it. Base your answers on information contained only in the passage. You may reread a passage if you need to. Mark the best answer for each question.

The following excerpt, from Mark Twain's 1876 The Adventures of Tom Sawyer, details Tom's first encounter with a new neighbor.

Tom skirted the block, and came round into a muddy alley that led by the back of his aunt's cow-stable. He presently got safely beyond the reach of capture 5 and punishment, and hastened toward the public square of the village, where two "military" companies of boys had met for conflict, according to previous appointment. Tom was General of one of these armies, Joe Harper (a bosom friend) General of the other. These two great commanders did not condescend to fight in person-that being better suited to the still smaller fry-but sat
15 together on an eminence and conducted
the field operations by orders delivered through aides-de-camp. Tom's army won a great victory, after a long and hard-fought battle. Then the dead were counted, prisoners exchanged, the terms of the next disagreement agreed upon, and the day for the necessary battle appointed; after which the armies fell into line and marched away, and Tom turned homeward alone.

As he was passing by the house where Jeff Thatcher lived, he saw a new girl in the garden-a lovely little blue-eyed creature with yellow hair plaited into two long-tails, white summer frock and embroidered pantalettes ${ }^{1}$. The freshcrowned hero fell without firing a shot.

A certain Amy Lawrence vanished out of his heart and left not even a memory of herself behind. He had thought he loved her to distraction; he had regarded his passion as adoration; and behold it was only a poor little evanescent partiality. He had been months winning her; she had confessed hardly a week ago; he had been the happiest and the proudest boy in the world only seven short days, and here in one instant of time she had gone out of his heart like a casual stranger whose visit is done.

He worshipped this new angel with furtive eye, till he saw that she had discovered him; then he pretended he did not know she was present, and began to "show off" in all sorts of absurd boyish ways, in order to win her admiration. He kept up this grotesque foolishness for some time; but by-and-by, while he was in the midst of some dangerous gymnastic performances, he glanced aside and saw that the little girl was wending her way toward the house. Tom came up to the fence and leaned on it, grieving, and hoping she would tarry yet awhile longer. She halted a moment on the steps and then moved toward the door. Tom heaved a great sigh as she put her foot on the threshold. But his face lit up, right away, for she tossed a pansy over the fence a moment before she disappeared.

The boy ran around and stopped within a foot or two of the flower, and then shaded his eyes with his hand and 70 began to look down street as if he had discovered something of interest going on in that direction. Presently he picked up a straw and began trying to balance it on his nose, with his head tilted far back; and as he moved from side to side, in his efforts, he edged nearer and nearer toward the pansy; finally his bare foot rested upon it, his pliant toes closed upon it, and he hopped away with the treasure and disappeared round the corner. But only for a minute-only while he could button the flower inside his jacket, next his heart-or next his stomach, possibly, for he was not much posted in anatomy, and not hypercritical, anyway.

He returned, now, and hung about the fence till nightfall, "showing off," as before; but the girl never exhibited
${ }_{90}$ herself again, though Tom comforted himself a little with the hope that she had been near some window, meantime, and been aware of his attentions. Finally he strode home reluctantly, with his poor head full of visions.

[^0]12. How can the perspective of the narrator best be described?
E. from Tom Sawyer's point of view
F. from an all-knowing perspective
G. from the girl's point of view
H. from the perspective of the townspeople who witness Tom's adventures
13. The author most likely includes the phrase "Then the dead were counted, prisoners exchanged, the terms of the next disagreement agreed upon, and the day for the necessary battle appointed" (lines 18-22) in order to A. convey the seriousness of the conflict between the boys.
B. suggest that Tom and his friends strictly adhered to traditional rules and customs of warfare.
C. treat the "warfare" as seriously as Tom and his friends might.
D. lament the endless nature of the boys' fighting.
14. The phrase "The fresh-crowned hero fell without firing a shot" (lines 3132) suggests that $\quad$ E. with one glance, Tom fell helplessly in love with "the new girl."
F. Tom's gun malfunctioned, preventing him from properly fighting in the battle.
G. upon seeing the appealing young girl, Tom fell flat on his face in shock.
H. Tom forgot about his victory crown once he spotted "the new girl."
15. What is the best way to describe Tom's reaction to seeing the new girl on the street?
A. indifference
B. embarrassment
C. enthusiasm
D. disinterest
16. Which of the following best explains Tom's behavior toward the pansy in the fourth paragraph?
E. He is unsure if it would be appropriate to pick up the flower
immediately.
F. He thinks his "showing off" has failed, so he gives up and heads home with the pansy.
G. He is so delighted by the girl's pansy that he cannot hide his glee.
H. He wishes to impress the girl, if she is still watching, by carrying the flower off in an unusual way.
17. Why does the author mention that Tom will "button the flower inside his jacket, next his heart-or next his stomach, possibly" (lines 82-84)?
A. Tom is not savvy enough to differentiate between the location of his heart and stomach.
B. Tom is so distracted by his new crush that he mistakenly buttons his flower in the wrong place.
C. Tom's anatomy is highly unusual in that his stomach is located close to his heart.
D. Tom metaphorically places the flower "next to his heart," but in reality he places it in his trouser pocket.
18. The attitude of the new girl towards Tom is
E. enamored.
F. unclear.
G. appalled.
H. humored.
19. The author most likely mentions Amy Lawrence in order to
A. explain that Tom is faithful to Amy and loves her very much.
B. demonstrate the fickleness of Tom's affections.
C. compare her sense of style with the new girl's.
D. introduce a new character the reader is about to meet.

Many contemporary scientists believe that the universe began in a giant explosion, or "big bang." At least ten billion years ago, they surmise, a state of extremely high temperature and pressure caused the universe to expand rapidly from a compressed state. Soon after, there was significant cooling, and at this point there were probably many types of elementary particles present. As further cooling occurred, the first nuclei were formed. Gradually, over millions of years, the universe began to take on the characteristics that we observe today.

One of the major assumptions upon which the big bang theory is dependent is called the cosmological principle. This principle states that our observations of general properties of the universe do not depend on our location or the direction in which we look. Based on this assumption, scientists believe that the explosion known as the big bang occurred not at any particular point, but instantaneously throughout space.

Evidence to support the big bang theory comes mainly from observations of galaxies out beyond our Milky Way. These galaxies are so far away that, by the time light reaches us from there, millions of years have passed. Distant objects are seen as they appeared when the light we receive first left them, providing us with a picture of the early universe.

In addition, the discovery of weak radiation has given support to the big bang theory of the origin of the universe. This low-level static, observed coming from every direction in space, is regarded as the remains of the initial fireball. At the very beginning, the temperature of the fireball was unimaginably hot. This heat accounts for the radiation we now detect.

Another important piece of evidence is the amount of an element called Helium-4 found in the universe. If the big bang theory is correct, we would expect to find that Helium-4 makes up about $25 \%$ of the matter in the universe. When the universe was only one or two minutes old, it was much too hot to make Helium-4. Then for about 14 minutes, the temperature was perfect for making this element through a process called nuclear fusion. By the time the universe was about 17
${ }_{60}$ minutes old, the universe had cooled down too much to make Helium-4. These 14 minutes of fusion would generate enough Helium- 4 to make up $25 \%$ of the matter. In fact, that is what we see today, supporting the big bang theory.

It is true that nuclear fusion has always taken place in stars, as well. But all of the stars that have ever existed could not possibly create anywhere near the amount of Helium-4 that exists. So, nuclear fusion when the universe was in its infancy is a logical hypothesis to explain all the Helium-4 we observe.
20. Which of the following best tells what the passage is about?
E. a proof of the origin of the universe
F. an explanation of how scientists derived the age of the universe
G. a consideration of one major use of static
H. a discussion of some evidence for one theory of the origin of the universe
21. In order to support the big bang theory, the author mentions which of the following?
A. the results of Galileo's research
B. the discovery of low-level radiation
C. the presence of lead in the initial fireball
D. the exact temperature of the initial fireball
22. The discussion of the change in temperature, in lines $4-10$, implies that E. gradually, the earth moved into a period known today as an ice age. F. nuclei have always been present.
G. elementary particles are never found under extreme temperatures.
H. nuclei did not exist under the extreme heat of the initial explosion.
23. The discussion of the cosmological principle suggests that A. scientists sometimes have to base their theories on other theories.
B. scientists will never be able to concretely prove the big bang theory.
C. we cannot make direct observations of the characteristics of the universe.
D. scientists are able to identify the precise location at which the big bang took place.
24. In what way is measurement of the speed of light useful for scientists researching the big bang theory?
E. Scientists can measure the speed and force of the explosion.
F. The light distorts the images the scientists are observing.
G. Scientists can estimate the time period in which the observations they make now actually occurred.
H. Scientists are able to observe the explosion as it happened.
25. According to the fourth paragraph, what is one characteristic of the static that caused scientists to associate it with the big bang?
A. The radiation was created as the universe cooled.
B. The low-level static does not seem to be coming from one specific place.
C. The intial fireball was comprised mostly of static.
D. The initial explosion was equivalent in magnitude to a nuclear explosion.

The perception of canis lupus, the wild wolf, as humanity's great antagonist is one that pervades Western literature. It is canis lupus who dispossesses
5 the Three Little Pigs, who devours Little Red Riding Hood's granny, and who eventually shows up after Peter cried "wolf!" too many times. This image likely originated in the Middle ${ }^{10}$ Ages. During the Black Plague, human cultivation of herd animalssheep, cattle, and the like-came to a standstill. Ravenous wolves, no longer able to stalk their traditional prey
15 (herd animals), turned their attentions to human beings. Men soon came to see the wolf as "the Devil, red-tongued, sulfur-breathed, and yellow-eyed."

Ironically, canis lupus and humans have lived together, usually peacefully, for two million years-since the wolf evolved to its present form. About 13,000 years ago, Paleolithic hunter-gatherers bred wolves as domestic dogs, canis familiaris. Why wolves? We can't say for sure. There are all kinds of possibilities. One prevalent view today posits that wolves gathered around the Paleolithic humans' campsite looking for food. Some wolves may have been less frightened and more willing to approach closely. Through natural selection these wolves' offspring were the first steps of the wolves' evolution into dogs.

But why did the humans welcome the dogs? Perhaps our ancestors admired the beast's power, intelligence, and social interactions. The wolf pack, say many biologists, is just a more primitive version of the human huntergatherer social structure. They have a community in which it is their social interdependence that aids them all in survival. Sound familiar?

People may have domesticated the wolf also because of wolves' expressiveness.

Through body language and facial expressions, wolves communicate their moods, from anger and aggression to happiness and submission. Early humans surely picked up on the signals, just as the wolves probably picked up on the humans' own methods of communication. Domestication may have been a natural step once the two species had developed a way to understand one another's moods.

To capture their prey, wolves use tactics so similar to those of hunter-gatherer ${ }_{60}$ teams that anthropologists suspect early humans learned to track large animals by watching wolves do the same. Wolves may "appoint" one pack member as a decoy who will lunge at and run around the prey. This allows the rest of the pack to take its victim by surprise. Wolf packs may also form two teams, one to stampede its prey toward the other. In these and other ways, the wolves use their
${ }_{70}$ various strengths to achieve success as a group. Some may be physically powerful, some watchful and clever, but if the pack is successful all will sleep with a full belly by nightfall.

75 These theories ring true to anyone who has a dog today. After all, all modern dogs (no matter how different they seem from the wolf) communicate in the same way as do wolves, and a family's pet dog so becomes a member of the "pack," just as those first domesticated wolves must have done thousands of years ago.
26. Which of the following best tells what this passage is about?
E. how humans domesticated the wild wolf
F. some speculations about the history of relations between wolves and humans G. how wolves live
H. wolves in fairy tales
27. According to the passage, all of the following are true about wolves except that they A. hunt herd animals.
B. are skilled hunters.
C. have always coexisted peacefully with humans.
D. used hunting methods that may have served as a model for early man's hunting methods.
28. According to the passage, domestic dogs E. are descendants of canis lupus.
F. are "red-tongued" and "sulfur-breathed."
G. are classic examples of hunter-gatherers.
H. attack herd animals.
29. The author states that our perception of the wolf as evil probably originated during the Middle Ages because it was then that A. human beings first started to tell bedtime stories.
B. human beings first started to breed dogs.
C. the term canis lupus was first coined.
D. wild wolves preyed on people.
30. The author provides which of the following as an example of herd animals?
E. cattle
F. wolves
G. Paleolithic men
H. large animals
31. The author uses the word "ironically" at the beginning of the second paragraph because it is ironic that $\quad$ A. human beings died by the millions during the Black Plague.
B. humans would so despise an animal that had so infrequently been their
enemy.
C. Paleolithic wolves were more primitive than Paleolithic humans.
D. wolves have more physical power than intelligence.

The term Inka means ruler and is the basis for the name of the Inca Empire; after conquering the Inca, the Spaniards adopted the Spanish spelling Inca. Be5 fore the Spaniards' arrival, however, the Inca called their empire Tawantinsuyu. This means "four parts together," as the empire was divided into four regions whose corners met at Cusco (in Peru), the capital of the empire.

Inca territory covered much of South America, including Peru as well as parts of Ecuador, Bolivia, Argentina, Chile, and Columbia. Originally a small tribe in Cusco, the Inca formed the Kingdom of Cusco and then expanded through the leadership of Pachacuti-Cusi Yupanqui. He sent spies to regions he wanted to incorporate into the empire to determine the strength of those regions. He then offered the leaders the opportunity to join the Inca Empire, promising greater wealth. These rulers quite often accepted, and Pachacuti brought the rulers' children to Cusco to learn about the Incan political system.

As the Inca Empire expanded its reach, Pachacuti created the central Incan government, Tahuantinsuyu, and four
30 local governments headed by loyal leaders. After Pachacuti's death, his son took his place, as did his son's son years later. Under this family, the creation of the empire was swift, taking place
between 1438 and 1533. Thereafter, the empire was even more swiftly dismantled, as the Spanish conquered Incan territory. The last of the Incan territory was taken by 1572 .

The Inca are known today for their architecture, including the worldfamous Machu Pichu. Their temples were constructed without mortar to bind the stones together, but the stones fit together so beautifully that a steel blade cannot be inserted between different stones. The Inca are also known for creating units of measurement, such as the length between the thumb and forefinger and the distance of a wingspan, and for an understanding of astronomy. Indeed, the Inca developed two calendars, one based on the cycle of the sun and the other on the lunar cycle. Time within a day was measured by the travel of the sun.

> Unfortunately, much information about the Inca remains a mystery, because many aspects of Incan culture were destroyed by the Spanish invaders. Among the outstanding questions is whether the Inca developed a written language. For years, archaeologists believed they had not, but more recent theories suggest that the Inca may have used knotted strings, known as quipu, at least as a means of storing numerical data. Whether the quipu conveyed other information remains a subject of inquiry and debate.
32. Which of the following best tells what this passage is about?
E. how Pachacuti expanded the Inca Empire
F. the solar and lunar calendars used by the Inca
G. the destruction of aspects of Incan culture by the Spanish
H. a brief history of the Inca Empire and its accomplishments
33. Which of the following best describes the relationship between Pachacuti and some leaders of nearby regions?
A. Pachacuti feared the regional leaders’ growing power.
B. The leaders were persuaded by Pachacuti's offer to join the Empire.
C. Pachacuti admired the leaders for their skills in battle.
D. The leaders feared Pachacuti's military might and harsh reputation.
34. Which of the following statements about Incan architecture is best supported by the passage?
E. Its stones are known to fit together well.
F. Machu Pichu is the only remaining example.
G. It utilized both stones and steel.
H. The Spanish disliked the architecture.
35. What aspect of the Inca Empire "remains a mystery" (line 58)?
A. what the Spanish hoped to gain by invading
B. whether the Incan people had a written language
C. when the Inca developed their calendars
D. how the Spanish succeeded in destroying the Inca Empire
36. What role did Pachacuti's spies play in the Inca Empire?
E. They warned Pachacuti of plots to overthrow him.
F. They learned about the Spaniards' plan for invasion.
G. They informed Pachacuti about certain territories.
H. They eventually supported the invading Spaniards.
37. Which of the following statements about "quipu" (line 67) would the author be most likely to agree?
A. They prove the existence of a written language.
B. There remains some debate about their significance.
C. The Spaniards destroyed all existing quipu.
D. They can represent numbers but not letters.

Georgia O'Keeffe is known primarily for her large paintings of flowers, images of New York City architecture, and subjects relating to New Mexico. Born 5 in 1887, O'Keeffe knew she wanted to be an artist at the age of ten. After studying art at the School of the Art Institute of Chicago and the Art Students League of New York, O'Keeffe abandoned her dream of being an artist because she did not believe she was talented enough in the techniques she had been taught.

Four years later, however, she discovered new, more modern artistic ideas and returned to school to master-and then teach-the new approach. Early in 1916, the owner of an important art gallery, Alfred Stieglitz, showed some of O'Keeffe's charcoal drawings, which he said were among "the purest, finest, sincerest things" he had seen in his gallery. Later, he organized a collection of her oil paintings and watercolors. O'Keeffe and

During the 1920s, O'Keeffe began painting extreme close-ups of flowers and other natural forms. The exhibitions of her work (at Stieglitz's gallery, among others) established her as one of the most important artists in America. During this period, she also painted scenes of New York City, but by the end of the decade, O'Keeffe wanted to find a new source of inspiration. She found that inspiration in New Mexico, whose desert colors and distinctive natural architecture led her to paint an entirely new subject matter.

O'Keeffe's popularity continued to grow throughout the 1930s and 1940s, and her work was featured in major institutions, such as the Museum of Modern Art. During the 1950s (following her husband's death in 1946), O'Keeffe traveled around the world, painting scenes of what she saw, including a series of paintings based on her views of the clouds she saw from airplanes. She continued to paint using oil paints until 1972, when her eyesight deteriorated. She thereafter limited herself to pencil and charcoal. In 1977, O'Keeffe received the Presidential Medal of
55 Freedom from President Gerald Ford.

Following her death in 1986, many of O'Keeffe's works were left to the Georgia O'Keeffe Foundation and later the Georgia O'Keeffe Museum in Santa Fe. In 2006, a newly discovered fossil of an ancient crocodile-like creature found near O'Keeffe's former New Mexico home was named Effigia okeeffeae ("O'Keeffe's Ghost") to honor her for the ${ }_{65}$ numerous paintings she drew from her home, which was called Ghost Ranch.
38. Which of the following best tells what the passage is about?
E. the flowers in O’Keeffe's work
F. a brief history of modern art
G. O'Keeffe's career as an artist
H. modern artists of the twentieth century
39. According to the author, why was a fossil named for O'Keeffe?
A. because O’Keeffe left her works to the Georgia O’Keeffe Museum
B. because the fossil was found near the source of many O'Keeffe paintings C. because "O'Keeffe's Ghost" was one of her most popular paintings D. because the crocodile-like creature was featured in an O'Keeffe painting
40. Why did O'Keeffe decide not to become an artist after her initial schooling?
E. She was devoted to her marriage to Alfred Stieglitz.
F. Her art was rejected by several galleries.
G. Her flower paintings were not yet popular among the public.
H. She felt unsure about her artistic abilities.
41. How did O’Keeffe react to her declining eyesight (lines 50-53)?
A. She retired to New Mexico.
B. She stopped using oil paints.
C. She spent more time traveling.
D. She showed her work at museums.
42. What prompted O’Keeffe to paint images of New Mexico?
E. She found a source of inspiration there.
F. Alfred Stieglitz asked her to do so.
G. She wanted to establish a museum in Santa Fe.
H. The flowers of New Mexico were particularly beautiful.
43. Which of the following best describes O'Keeffe as an artist?
A. celebrated and significant
B. frustrated and insecure
C. dependent upon her husband
D. consistent in her vision

This excerpt from "Crossing Brooklyn Ferry," written in 1856, describes the poet Walt Whitman's experience of riding the ferry from Manhattan to Brooklyn in the half hour before sunset.

Flow on, river! flow with the flood-tide, and ebb with the ebb-tide!
Frolic on, crested and scallop-edg'd waves!
5 Gorgeous clouds of the sunset! drench with your splendor me, or the men and women generations after me!
Cross from shore to shore, countless crowds of passengers!
10 Stand up, tall masts of Mannahatta! stand up, beautiful hills of Brooklyn!
Throb, baffled and curious brain! throw out questions and answers!
Suspend here and everywhere, eternal float of solution!
Gaze, loving and thirsting eyes, in the house or street or public assembly!

Sound out, voices of young men! loudly and musically call me by my nighest ${ }^{1}$ name!
Live, old life! play the part that looks back on the actor or actress!
Play the old role, the role that is great or small according as one makes it!
25 Consider, you who peruse me, whether I may not in unknown ways be looking upon you;
Be firm, rail over the river, to support those who lean idly, yet haste with the hasting current;
Fly on, sea-birds! fly sideways, or wheel in large circles high in the air;
Receive the summer sky, you water, and faithfully hold it till all downcast eyes have time to take it from you!
Diverge, fine spokes of light, from the shape of my head, or any one's head, in the sunlit water!
Come on, ships from the lower bay! pass up or down, white-sail'd schooners, sloops, lighters!
Flaunt away, flags of all nations! be duly lower'd at sunset!
Burn high your fires, foundry chimneys! cast black shadows at nightfall! cast red and yellow light over the tops of the houses!
${ }^{1}$ Most direct
44. How can this poem best be described?
E. a paean to the ocean as an object with boundless energy F. a reflection on the communal experience of going from one place to another G. a declaration of love for a great city and its inhabitants
H. a memory of a past experience that the poet yearns for
45. What is the effect of mentioning "the men and women generations after me" (lines 6-7)?
A. Whitman imagines how different it will be for his descendants to look at the sun as they cross the river.
B. Whitman fathoms himself as just one small person among millions more who will experience the crossing.
C. Whitman invites the modern-day reader to think of a time he or she rode on a ferry.
D. Whitman offers a thought on the ever-growing population of New York City.
46. What is the author suggesting with the phrase "Suspend here and everywhere, eternal float of solution!" (lines 14-15)?
E. The ferry has provided him with a momentary clarity he wishes could be extended longer.
F. He wishes that the ferry would stop moving forward and, instead, pause to float in the river.
G. The experience of crossing the river has given him all the solutions to his problems.
H. If only the ferry would travel to every shore, he could find what he is looking for.
47. The first lines of the final stanza (lines 18-20) describe A. the poet's realization that the ferry is filled with actors and actresses who are calling out to him.
B. a forceful plea to live one's life, despite one's old age.
C. the frightening moment of seeing a stranger and looking into his or her
eyes.
D. Whitman's desire to seize the moment and to be fully in charge of his destiny.
48. Consider the following lines (33-38) from the poem: Receive the summer sky, you water, and faithfully hold it till all downcast eyes have time to take it from you!
Diverge, fine spokes of light, from the shape of my head, or any one's head, in the sunlit water!

Why might Whitman have included these lines?
E. to demonstrate the science behind the sun's UV rays and their interactions with water F. to reflect on how seeing a bouncing ray of sun can cheer a disheartened person G. to show an unseen connectedness between humans and nature
H. to suggest that a sunbeam hitting a human's head can offer not just physical warmth but spiritual sustenance
49. The poet most likely ended each thought with an exclamation point in order to A. demonstrate the speaker's agitation and troubled state of mind.
B. indicate the high volume with which the poem ought to be read aloud.
C. convey the speaker's joyous, excited attitude about the objects he sees and describes.
D. express an almost shouting quality that will awaken the reader to action.
50. In the line "Stand up, tall masts of Mannahatta!" (line 10), the narrator means E. to draw the reader's attention to an antiquated spelling of "Manhattan."
F. that the buildings of Manhattan strike him like tall ships.
G. to say that he sees ships in Mannahatta.
H. to shock the reader by inexplicably calling out to inanimate objects.

The latter half of the eighteenth century was to chemistry what Galileo's time had been to physics. The outstanding achievement was basing chem-
5 istry on sound, precise, quantitative measurement. The balance, symbol of the modern chemist of the late 1700 s , enabled chemists to substitute precise weights for the crude approximations of 10 the medieval alchemist.

This refined approach helped lead to a number of important discoveries. One of the most significant was the clarification of the distinction between a true chemical reaction and the mere process of mixing. This distinction had been dimly perceived before; mixtures displayed a blend of the properties of their components, depending on the relative proportions making up the mixture. A chemical reaction, however, might produce a substance totally unlike the materials that went into its formation. For example, common water arises from the union of two gases: oxygen and hydrogen. Similarly, the putty-like metal sodium reacts with the green gas chlorine to form ordinary table salt. But the explanation for the difference seemed hazy until the analytic balance revealed the key. By
careful weighing, chemists found that mixtures could be formed in any desired proportions, but chemical reactions were produced by an exact recipe. The constituents had to be present in some exact proportion of weight; if the proportions were not exact, less of the resultant substance was produced.

Finally, the founders of modern chemistry clarified yet another distinction between types of substances. They classified some as elements, which could not be broken down into other substances, and others as compounds, which could.

45 The whole picture was terribly inviting to an atomist. All one needed to do was identify the elements as representing the different kinds of atoms, chemical compounds as substances formed by attaching atoms of different elements together, and mixtures as a free mingling of independent atoms without any ties between them. But atomism, and indeed the whole intellectual style
55 of imaginary model building that lay behind it, was mainly the province of physicists. One such, the Italian Amedeo Avogadro, pushed the atomic
idea in chemistry well before the end of the eighteenth century. But while his arguments satisfied many of his own colleagues, the vast majority of chemists remained skeptical of such wild talk. Throughout its history, chemistry has tended to be a far more conservative science than physics, sticking close to its empirical roots and disdaining abstractions and speculations. The chemists paid little attention to atomism until one of their own, the English chemist Thomas Dalton, brought it forcefully to their attention by showing that the atomic structure of matter could explain the peculiar regularities that kept popping up in the recipes uncovered by the analytic balance.
51. Which of the following best tells what this passage is about?
A. the contrast between chemistry and physics
B. the contributions of Amedeo Avogadro to chemistry
C. the importance of the interdependence of chemistry and physics
D. a summary of some key innovations in chemistry in the late 1700 s
52. The phrase "precise, quantitative measurement" (lines 5-6) refers to the E. mathematical nature of physics.
F. new discoveries in chemical research in the eighteenth century.
G. better methods of weighing and measuring in the eighteenth century.
H. reliance upon the measurements of alchemists.
53. According to the passage, a chemical reaction
A. produces water.
B. results from mixing.
C. can produce a substance different from its original materials.
D. can be made without an exact recipe.
54. The term "exact recipe" in line 34 is used by the author in order to $\mathbf{E}$. show how mixtures and chemical reactions are the same.
F. illustrate the difference between quantitative and proportional measurements.
G. demonstrate that the crude methods of early chemists were more like cooking than science.
H. show the important distinction between a simple mixture and a chemical reaction.
55. The author would agree with which of the following about atomists?
A. Their theories were readily accepted by chemists.
B. They discovered that elements could not be broken down.
C. Their theories were not studied until the nineteenth century.
D. Historically, they were often at odds with chemists.
56. Which of the following is not discussed in the passage?
E. the difference between simple mixtures and chemical reactions
F. the differences between elements and compounds
G. the early developments of modern chemistry
H. methods by which scientists could split the atom
57. Why does the author mention the Italian physicist Amedeo Avogadro?
A. to discredit the wild ideas that Avogadro put forth at the end of the eighteenth century
B. to provide an example of an early advocate of atomism
C. to introduce the scientist who discovered the distinction between a true chemical reaction and the process of mixing D. to offer a dissenting opinion from the prevailing ideas of the chemist Thomas Dalton

# PART 2—MATHEMATICS 

Suggested Time-90 Minutes

## 57 QUESTIONS

IMPORTANT NOTES:
(1) Formulas and definitions of mathematical terms and symbols are not provided.
(2) Diagrams other than graphs are not necessarily drawn to scale. Do not assume any relationship in a diagram unless it is specifically stated or can be figured out from the information given.
(3) Assume that a diagram is in one plane unless the problem specifically states that it is not.
(4) Graphs are drawn to scale. Unless stated otherwise, you can assume relationships according to appearance. For example, (on a graph) lines that appear to be parallel can be assumed to be parallel; likewise for concurrent lines, straight lines, collinear points, right angles, etc.
(5) Reduce all fractions to the lowest terms.

## GRID-IN PROBLEMS

QUESTIONS 58-62
DIRECTIONS: Solve each problem. On the answer sheet, write your answer in the boxes at the top of the grid. Start on the left side of each grid. Print only one number or symbol in each box. DO NOT LEAVE A BOX BLANK IN THE MIDDLE OF AN ANSWER. Under each box, fill in the circle that matches the number or symbol you wrote above. DO NOT FILL IN A CIRCLE UNDER AN UNUSED BOX.
58. In an inventory of 100 sodas, 65 are diet and 30 are caffeine-free. Of the diet sodas, 22 are caffeine-free. If a soda in the inventory is selected at random, what is the probability that it is neither diet nor caffeine-free?
59. $\frac{1}{2} \times \frac{8}{40} \times \frac{15}{25} \times \frac{3}{2}=$
60. What is the sum of all the distinct prime factors of 48 ?
61.


What is the area of the shaded region above?
62. A rectangular box has a base with length 4 and width 5 . If the height of the box is twice its length, what is the surface area of the box?

# MULTIPLE CHOICE PROBLEMS 

## QUESTIONS 63-114

DIRECTIONS: Solve each problem. Select the best answer from the choices given. Mark the letter of your answer on the answer sheet. You can do your figuring in the test booklet or on paper provided by the proctor. DO NOT MAKE ANY MARKS ON YOUR ANSWER SHEET OTHER THAN FILLING IN YOUR ANSWER CHOICES.
63. A pair of shoes was marked down by $20 \%$ on Monday. On Friday the price was reduced by $10 \%$. If the original price of the shoes was $\$ 60.00$, what was the final sale price?
A. $\$ 48.00$
B. $\$ 43.20$
C. $\$ 42.20$
D. $\$ 42.00$
64.


The above figure is composed of two squares with areas as shown. What is the value of $v$ ?
E. 70
F. 90
G. 140
H. 160
65. A recipe for 27 servings of punch calls for 12 gallons of orange juice. How many gallons of orange juice would be needed for 45 servings of the same punch?
A. 16
B. 20
C. 24
D. 33
66. A mint has two machines for making coins. One machine can make a coin every 5 seconds and the other machine can make a coin every 6 seconds. How many coins can be made by the two machines combined in 1 minute?
E. 12
F. 22
G. 24
H. 36
67. If $q+r+s=117$, and $q=s=4 r$, then $r=$
A. 13
B. 18
C. 30
D. 42
68.


Based on the number line above, which of the following statements must be true?
E. $A+B>2$
F. $A+C>2$
G. $C-B>1$
H. $C-A>1$
69. If the sum of two numbers is 29 and their difference is 7 , what is the larger of the two numbers?
A. -11
B. 11
C. 12
D. 18
70. A compact disc is placed in a player that randomly selects and plays songs from the compact disc. The compact disc contains 3 ballads, 4 instrumental pieces, $x$ dance tracks and no other pieces. If the probability that the first song played will be a ballad is $\frac{1}{4}$, what is the value of $x$ ?
E. 4
F. 5
G. 9
H. 12
71.

Division of 150 Tests Given, by Month


Based on the graph above, what was the first month in which, by the end of that month, at least one-third of the 150 tests had been given?
A. December
B. January
C. February
D. March
72. If $5 q=3 r=30$, then $q r=$
E. 32
F. 40
G. 50
H. 60
73. A teacher has a class of 12 students. She must select 4 to which to give a year-end award certificate. How many groups of 4 students can the teacher select?
A. 24
B. 495
C. 6,720
D. 11,880
74. $\frac{5^{2}+5}{5}=$
E. 5
F. 6
G. 25
H. 26
75. Alexis traveled in a car at an average speed of 130 kilometers per hour for 6 hours and 24 minutes. What distance did she travel?
A. 780 km
B. 806 km
C. 811 km
D. 832 km
76. Michele is assigned to do 50 pushups in gym class. She has done 20 in 40 seconds. Assuming the same rate, how many more seconds will it take Michele to complete the assignment?
E. 40
F. 60
G. 80
H. 100
77. Kai rides his bicycle up the hill to school and back every day. He travels twice as fast downhill as uphill, and it takes 30 minutes round-trip. If he travels at a rate of 18 miles per hour on the trip home, how far away is his school?
A. 3
B. 6
C. 9
D. 12
78. During a rainstorm, an empty swimming pool with rectangular dimensions of 9 feet long and 20 feet wide was filled to a depth of 6 inches. What volume of water, in feet, rained into the pool?
E. 60 cu ft
F. 90 cu ft
G. 180 cu ft
H. $1,080 \mathrm{cu} \mathrm{ft}$
79.

| Subject | Grade |
| :--- | :---: |
| Math | A |
| English |  |
| Chemistry | D |
| Physics |  |
| French | B |

Conversions: $A=4, B=3, C=2, D=1$, Fail $=0$.
The chart above is a partially completed report card for Karen. If grade point average is defined as the average of the Conversions (listed above) associated with each grade, what is the minimum grade point average she can receive if she took all the subjects listed above and did not fail any of them?
A. 1.6
B. 2.0
C. 2.4
D. 2.8
80. An eight-sided die with faces numbered 1 through 8 is rolled twice. What is the probability that the first roll will be 3 and the second roll will be 8 if the die can never be rolled as a 6 ?
E. $\frac{1}{8}$
F. $\frac{2}{7}$
G. $\frac{1}{49}$
H. $\frac{2}{49}$
81. A car dealer sells $b$ blue cars and $r$ red cars in one day. Which of the following equations represents the statement, "The number of blue cars sold is three times the number of red cars sold?"
A. $3 r=b$
B. $3 b=r$
C. $b+3=r$
D. $b=r+3$
82.


Line segment $R T$ has a length of 12 . Segment $S T$ is $\frac{1}{6}$ of $R T$. What is the ratio of the length of segment $S T$ to the length of segment $R S$ ?
E. $\frac{1}{6}$
F. $\frac{1}{5}$
G. $\frac{5}{6}$
H. 5
83. Points $A$ and $G$ on a number line are 16 units apart, and point $D$ is the midpoint of $A G$. Point $X$, which is located at -3 on the number line, is the midpoint of $A D$. Which of the following is a possible midpoint of $X G$ ?
A. 6
B. 5
C. 3
D. 1
84. A store orders an awning with 16 inch lettering. The store is willing to accept lettering size within $4 \%$ of the ordered size. What is the maximum size of lettering that the store will accept?
E. 17.28
F. 16.64
G. 15.36
H. 0.64
85.


In the figure above, $x=$
A. $25^{\circ}$
B. $65^{\circ}$
C. $155^{\circ}$
D. $165^{\circ}$
86. The number of minutes in 3.4 hours is which of the following?
E. 204
F. 214
G. 215
H. 224
87. $\left(2 a^{3} b^{2} t^{5}\right)^{3}=$
A. $2 a^{6} b^{8} t^{8}$
B. $6 a^{6} b^{5} t^{8}$
C. $8 a^{9} b^{6} t^{15}$
D. $8 a^{27} b^{8} t^{125}$
88. A fair die with numbers from 1 to 6 is rolled twice. What is the probability that the result of both rolls will be 1 ?
E. $\frac{1}{36}$
F. $\frac{1}{12}$
G. $\frac{1}{6}$
H. $\frac{1}{3}$
89.


In the figure above, $A B=3 \frac{1}{4}$. What is the position of point $B$ ?
A. $4 \frac{3}{8}$
B. $4 \frac{7}{8}$
C. $5 \frac{1}{8}$
D. $5 \frac{3}{8}$
90. In a game, players may exchange 240 garks for 180 praps. At this rate, how many garks are equal to 1 prap?
E. $\frac{3}{4}$
F. $\frac{1}{2}$
G. $\frac{4}{3}$
H. 3
91. Eric has $\$ 3.50$. Of this amount, he owes his mother $14 \%$, and his sister $76 \%$. How much money will Eric have left after he pays his debts?
A. $\$ 0.10$
B. $\$ 0.35$
C. $\$ 0.70$
D. $\$ 0.84$
92.


For the figures above, the perimeter of triangle $A B C$ equals the perimeter of triangle $D E F$. If triangle $A B C$ is equilateral, what is the length of side $A B$, rounded to the nearest whole number?
E. 7
F. 10
G. 14
H. 21
93. The ratio of apples to bananas at a fruit stand is 5 to 4 . The ratio of cherries to bananas is 2 to 5 . What is the ratio of apples to cherries?
A. 5:2
B. 4:5
C. $25: 8$
D. 2:1
94. What is the least integer greater than -3.7 ?
E. -4
F. -3
G. 0
H. 4
95. If $x=-2$ and $y=4$ then $x^{2} y+\frac{y}{x}=$
A. -18
B. -14
C. 14
D. 18
96.

LOOPS AROUND THE TRACK

|  | Most Loops Run | Range |
| :---: | :---: | :---: |
| Team A | 74 | 14 |
| Team B | 77 | 19 |
| Team C | 78 | 12 |

The Redfield Roadrunner Club is divided into three teams, A, B, and C. Each team was challenged to run around a track as many times as possible. The table shows each team's greatest number of loops run and
the range of loops run within that team. What is the overall range of loops run for all three teams?
E. 9
F. 11
G. 18
H. 20
97. A fruit stand sells melons, pears and grapes in the ratio of 2:3:5, respectively. If the stand sold 45 pears, what is the total number of melons and grapes sold?
A. 50
B. 90
C. 105
D. 150
98. In Country X, there are 30,000 doctors. If 18,000 of the doctors treat children and 15,000 of the doctors perform surgery, what is the lowest possible number of doctors who treat children and perform surgery?
E. 12,000
F. 9,000
G. 6,000
H. 3,000
99. In a recent local election, between $60 \%$ and $70 \%$ of 1,100 registered voters cast a ballot. What is the minimum number of registered voters who did not cast a ballot?
A. 330
B. 440
C. 660
D. 770
100. At a baseball stadium, 2 hot dogs are sold for every 5 fans in attendance. If there are 30,000 fans in attendance, how many hot dogs are sold?
E. 6,000
F. 9,000
G. 12,000
H. 15,000
101. Express $27.8913 \times 100$ in scientific notation.
A. $0.278913 \times 10^{4}$
B. $2.78913 \times 10^{2}$
C. $2.78913 \times 10^{3}$
D. $27.8913 \times 10^{2}$
102. If set $Q$ contains all even integers from 4 to 36 inclusive, and set $Z$ contains all multiples of 3 , how many integers are common to both sets?
E. 4
F. 5
G. 6
H. 7
103. A candy bar is advertised as costing "three for $\$ 1.00$." If Jackie bought 21 candy bars at this price, how much money would she have saved if the price had been reduced to "seven for $\$ 2.00$ " ?
A. $\$ 1.00$
B. $\$ 1.50$
C. $\$ 3.00$
D. $\$ 7.00$
104. A carpeting company charges $\$ 2.50$ for every square foot of carpeting. At this rate, what would be the company charge for 21 square feet of carpeting?
E. $\$ 8.40$
F. $\$ 42.00$
G. $\$ 52.50$
H. \$67.50
105. A jogger completes one lap around a circular lake with radius $=r$. Another jogger completes one lap around a lake with a radius $=r+4$. How much farther does the second jogger run?
A. 8
B. $8 \pi$
C. 16
D. $16 \pi$
106. Marcello is $m$ years old, and Celia is 6 years younger than Marcello. In 4 years, Marcello will be twice as old as Celia will be. How old is Marcello today?
E. 4
F. 6
G. 8
H. 10
107. A telephone call costs $\$ 1.50$ for the first three minutes, and 20 cents for each additional minute. What is the cost of a nine minute call?
A. $\$ 2.20$
B. $\$ 2.70$
C. \$3.30
D. $\$ 4.50$
108. Jelena has five pictures to hang on her wall, from left to right. How many different arrangements of the pictures can she make?
E. 10
F. 20
G. 60
H. 120
109. If $n \geq 1$, then $1^{n}+1^{n+1}+1^{n+2}+1^{n+3}+1^{n+4}+1^{n+5}=$
A. 72
B. 15
C. 6
D. 5
110.


In the rectangle shown above, what is the area of the shaded triangle?
E. 24
F. 32
G. 48
H. 64
111. A building is in the shape of a rectangular solid with a square base. A
scale model of the building is created with a base with sides of 5 inches and a height of 20 inches. If the height of the building is 80 feet, what is the area of the base of the building?
A. 20 sq ft
B. 100 sq ft
C. 225 sq ft
D. 400 sq ft
112. If $x+2 y=6$, and $z=7$, what is the value of $2 x+4 y+z$ ?
E. 12
F. 15
G. 19
H. 25
113.

| TEMPERATURE IN CITY X ON DEC. 6 |  |
| :---: | :---: |
| $6 \mathrm{a} . \mathrm{m}$. | $10^{\circ}$ below zero |
| $8 \mathrm{a} . \mathrm{m}$. | $2^{\circ}$ below zero |
| $10 \mathrm{a} . \mathrm{m}$. | $15^{\circ}$ above zero |

What is the average (arithmetic mean) of the temperatures shown on the table above?
A. $3^{\circ}$ above zero
B. $1^{\circ}$ above zero
C. $1^{\circ}$ below zero
D. $2^{\circ}$ below zero
114. If 100 grams is half the mass of one object and twice the mass of another, the mass of the heavier object is how many grams more than the mass of the lighter object?
E. 200
F. 150
G. 100
H. 50

THIS IS THE END OF THE TEST.
IF TIME REMAINS, YOU MAY CHECK YOUR ANSWERS TO PART 1 AND PART 2. BE SURE THAT THERE ARE NO STRAY MARKS, PARTIALLY FILLED ANSWER CIRCLES, OR INCOMPLETE ERASURES ON YOUR ANSWER SHEET.

## Chapter 6

Practice Test 1: Answers and Explanations

## PRACTICE TEST 1 ANSWERS

ELA Part A<br>1. C<br>2. E<br>3. D<br>4. E

ELA Part B
5. C
6. F
7. A 8.E
9. D
10. G
11. A

## Reading Comprehension

12. F
13. C
14. E
15. C
16. H
17. A 18. F
18. B
19. H
20. B
21. H
22. A $24 . \mathrm{G}$
23. B
24. F
25. C
26. E
27. D
28. E
29. B
30. H
31. B
32. E
33. B
34. G
35. B
36. G
37. B
38. H
39. B
40. E
41. A 44. F
42. B
43. E
44. D
45. G
46. C
47. F
48. D
49. G
50. C
51. H
52. D
53. H
54. B

## Math

58. . 27
59. . 09
60.5
60. 6
61. 184
62. B
63. H
64. B
65. F
66. A 68. H
67. D
68. F
69. B
70. H
71. B
72. F
73. D
74. F
75. A 78. F
76. B
77. G
78. A 82. F
79. C
80. F
81. C
82. E
83. C
84. E
85. C
86. G
87. B
88. E
89. C
90. F
91. C
92. H
93. C
94. H
95. A 100. G
96. C
97. G
98. A 104. G
99. B
100. G
101. B
102. H
103. C
104. E
105. D
106. G
107. B
108. F

## ELA Part A

1. C A run-on sentence occurs when a writer tries to combine two or more thoughts without the proper punctuation. Remember the rules about punctuation: STOP, HALF-STOP, and GO. Sentence 1 is a simple complete sentence, so eliminate (A). Sentence 2 correctly sets off Vladek and Anja in commas, since it is nonessential to the main sentence; eliminate (B). Sentence 4 is a simple complete sentence, so eliminate (D). In (C), the writer has combined two complete thoughts with GO punctuation (comma); two complete thoughts can be joined only by STOP or HALF-STOP punctuation. Choice (C) is the correct answer.
2. E The answer choices make it clear that commas are being tested. One rule of commas is to use a comma before and after unnecessary information. Choice (E) puts a comma after novel, which would mean the phrase The Catcher in the Rye is unnecessary or nonessential to the sentence. Put your finger over the phrase to check if the sentence still works grammatically: "J.D. Salinger’s first novel was published in 1951, when he was 32 years old." That works! Choose (E), as the other answer choices will result in a grammatically incorrect sentence. Remember: if you can't cite a reason to use a comma, don't!
3. D Start by finding the relationship between the two sentences. The first statement says that many nature preserves are far away from cities; at the same time, the second statement says that some big cities have created their own nature preserves. Choice (A) makes little sense: it is not true that many nature preserves are far away because big cities have established their own. This answer reverses the order of the statements, which affects the meaning. Eliminate (C), which is also grammatically incorrect, for this reason as well. Down to two! Choice (B) is peculiar because and could work, but it doesn't do the best job describing the relationship between the two ideas. Choice (D) nicely explains the contradiction within the statements: while many nature preserves are located far from cities, some big cities have established their own preserves. Choose (D).
4. E This question is testing misplaced modifiers. The modifying phrase "uninteresting and a little loud" must be followed by what it modifies. In this example, the pronoun "I" follows. Surely the writer does not mean to call himself "uninteresting and a little loud," but, rather, means to call the concert uninteresting and a little loud. Choice (H) maintains this problem, so eliminate it; (G) implies that when the writer left the concert, he was uninterested, so delete it. Finally, (F) awkwardly uses the passive voice to correct the misplaced modifier, but it does not mention who or what leaves early. Only (E) can work, as it successfully corrects the misplaced modifier.

## ELA Part B

5. C This question asks about maintaining formal style. While this is a subjective topic, the SHSAT has some clear standards. The use of the first person allows you to eliminate (B) and (D). You can also cut (A), both because it directly addresses the reader and because it is wordy. Choice (C) nicely aligns with the tone of the rest of the passage, and it is the correct answer.
6. $\mathbf{F} \quad$ The second paragraph discusses the causes that led to the extreme shortage of bluefin tuna. Take note of the pronoun used in sentence 5 (this sharp decline), which suggests that the sentence before sentence 5 should introduce the decline. Sentence 5 also introduces the idea of an "appetite for raw tuna," indicating that the sentence that follows will discuss this idea further. Sentence 7 deals with the Japanese diet of tuna sashimi, while sentence 8 mentions where fish are sent. Only sentence 6 introduces the decline ("bluefin tuna have been depleted to 3 percent of their pre-exploitation population"), so sentence 5 should come between sentences 6 and 7, (F).
7. A On a precision question, remember that the SHSAT values answer choices with specific, relevant, and precise details (like statistics). Look for the differences between choices in order to POE. The phrase "80 percent" is more precise than "the majority," so eliminate (B) and (D). Of the two remaining, (A) is slightly more precise, since it says that the tuna is sent to Japanese consumers, which is more specific
than "Japan" generally. Choose (A).
8. E Start by identifying the relationship between the two sentences-do the sentences agree or disagree with one another? The writer is saying that the stock of tuna has decreased sharply as a result of young tuna being captured before they can procreate. Choice (F) uses "despite," but this would work only if there were an unexpected second clause. However, the decrease of tuna is to be expected, so eliminate ( F ). Similarly, the use of "yet" in (G) suggests a reversal where there is none. And $(\mathrm{H})$ switches the order of the sentences, which does affect the meaning, since it inverts the causal relationship the original sentence describes. Only (E) will work: because most of the world's captured tuna never had a chance to procreate, the stock of tuna has decreased sharply.
9. D When answering questions about transition words, recall that they usually fall into two categories: agreement or departure. Notice that the previous paragraph explains how popular the bluefin tuna is, while the new paragraph discusses how some chefs refuse to serve the fish at all. This is a departure, so POE any transition words that are not departure words: and, in addition, and moreover. Only (D) remains.
10. G To begin, understand what the fifth paragraph is about. A quick skim reveals that the topic is individuals who will not serve bluefin tuna because of its endangered status. The phrase "for instance" implies that the new sentence will support that claim, so it cannot precede the paragraph; eliminate (E). Make sure you check the flow both ways; the sentence can follow 17, but cannot precede 18, so eliminate (F). Choice (G) looks fine, so keep it for now, but check (H) just to be sure. In this case, (H) would not make sense because the example of the Japanese chef does not follow from a sentence about American chefs and a boycott in the United States. Thus, (G) is the best choice.
11. A For a general question like this, POE! Choice (B) says that little can be done to save the bluefin tuna, whereas the passage never made that claim and, in fact, the final paragraph mentions efforts to help the tuna. Eliminate (B). Also, (C) can be crossed out because the passage does identify that stopping the overfishing of bluefin tuna can protect
the species. Choice (D) might be true, but the passage as a whole is not about Japanese fishing practices, so it too can be eliminated. Choice (A) is best because it weaves together many threads that make up the argument of the passage.

## Reading Comprehension

12. $\mathbf{F}$ To answer this question requires a good understanding of the passage as a whole, as it asks about the perspective of the narrator. After reading the passage, jump directly to POE, as there will not be a specific place to find the answer to this question. Eliminate (E) because, while Tom seems to be the central character in this excerpt, the narration comments on the events outside of Tom's personal perspective. (A tell-tale sign of a first-person narrator-i.e., Tom himself-would be many uses of the first-person pronoun I. That pronoun does not appear at all in this passage). Eliminate (G) because the reader hardly gets any information from the new girl's perspective. Finally, eliminate (H), as there is no mention of the townspeople who might have witnessed Tom’s behavior in this excerpt. That leaves ( F ), an "all knowing" narrator who sees everything and everyone in the story at the same time. (This is often referred to as an omniscient narrator.) 13. C Here is a specific question, which gives you a place to go in the passage. Find the part of the story where the narrator says, "Then the dead were counted..." Refer to your headlines in the margin for this first paragraph, which mentions Tom's pretend battling with his friends. Jump to POE with this knowledge in mind. Eliminate (A) because it is clear from the passage that Tom and his friends are not in real danger, but are merely pretending to fight. For this reason, also eliminate (D) because the narrator does not suggest that their fighting is something to "lament." (That would be making an assumption beyond the text!). Down to two. Choice (B) can be eliminated because nowhere does the text say that these rituals are, in fact, the traditional rules and customs of warfare. (This could be true, but don't choose it!) The correct answer is (C): the narrator treats the boys' "warfare" as seriously as Tom and his friends do.
13. $\mathbf{E}$ This question asks you to analyze a single phrase in the context of the
passage. Use the line numbers to guide you to the second paragraph. Here, Tom notices the "new girl" outside the Thatcher house and falls down defeated, without firing his weapon. There's no battle occurring in this paragraph, so eliminate (F). Eliminate (H) because we are given no information that Tom is wearing an actual crown or that he forgot it. The remaining choices, (E) and (G), come down to the literal sense versus the metaphoric sense. Based on the rest of this passage, it is clear that Tom has fallen in love in this moment, not that he has literally fallen on the ground. Choice ( E ) is the correct answer.
14. C Use your headlines from paragraphs 2 and 3 to answer this question. In the passage, Tom falls in love with the new girl at first sight, and begins to worship "this new angel." Move directly to POE. Eliminate (A) and (D) because Tom is actively interested in the new girl, so he would not be indifferent or disinterested. He "pretended he did not know she was present" but only so as "to win her admiration." Eliminate (B) because embarrassment or shyness is not Tom's reaction upon seeing the new girl; the third paragraph describes his actions as "foolishness," but that doesn't stop Tom, if he's even aware of it. The correct answer is (C).
15. H Refer to your fourth paragraph headlines or notes to answer this question. Recall that Tom performs many tricks with the flower that the new girl leaves for him. The question asks how we can explain these tricks, or, in other words, why he performs the tricks. Use POE: eliminate (E) because, while it is true Tom does stop to think about whether he should pick up the flower, it is not because he worries about being inappropriate. Next, eliminate (F) because we get no evidence that Tom thinks his "showing off" has failed. Finally, eliminate (G). Tom is certainly delighted by the girl's pansy, but he does not perform tricks because he is full of glee; rather, he performs the tricks in order to gain the favor of the new girl, who he thinks may be watching him. Thus, the correct answer is (H).
16. A Here is another specific question about the narrator's descriptions. Use the line numbers to jump to the final paragraph, and reread the text around the quotation. Notice that the author says Tom placed the flower "next his heart-or next his stomach, possibly, for he was not
much posted in anatomy, and not hypercritical, anyway." By reading the full sentence, it becomes clear that Tom does not have a very good sense of where his heart is located on his body. Eliminate all other answer choices, and select (A), which matches exactly what the text says. The other choices describe events that the author never stated.
17. F This question asks how the new girl feels about Tom. If you cannot remember how she feels about Tom, that might be all you need to know! Use your headlines to recall that all we know about the girl is that she tosses a flower over her fence and quickly disappears. But what about her attitude towards Tom? POE! Eliminate (E) and (G) because the reader gets no evidence that she loves or hates Tom. And (H) can also be eliminated, as the flower itself is not evidence that she finds Tom funny or charming. The best answer is (F), unclear. At this point in the story, the reader simply does not know how the new girl feels about Tom.
18. B This question asks why the author brings up Amy Lawrence. That's a great lead word-skim the passage to find where this character is mentioned: paragraph 2. Once Tom sees the new girl, "a certain Amy Lawrence vanished out of his heart and left not even a memory of herself behind." Jump to POE. Eliminate (A) because the passage conveys the opposite: Tom is not, in fact, faithful to Amy. The passage does not compare the two girls' sense of style, so eliminate (C). Finally, (D) is not a very strong reason for the author to mention Amy Lawrence; moreover, the reader does not really meet Amyshe's an offstage character. The much better choice is (B), as the author demonstrates the fickleness of Tom's affections by explaining how quickly he fell out of love with his last crush, Amy Lawrence. The correct answer is (B).
19. $\mathbf{H}$ This is a general question, so use your headlines to POE any answers that don't match the themes in the passage. Cross off ( E ) because the passage does not prove how the universe was created; it offers evidence to support the big bang theory. Similarly, eliminate (F) because the passage is not about how old the universe is, but rather how it began. Finally, delete (G) because static appears in the passage, but it is not the main idea. Select (H) because the passage does, in
fact, discuss one theory (the big bang theory) about the origin of the universe.
20. B Use your headlines and general knowledge of this passage to quickly POE this question. Choices (A) and (C) discuss ideas that never appear in the passage, so eliminate them. Down to two! Delete (D) because the "exact temperature" mentioned in the passage is only vaguely given as "unimaginably hot." The correct answer is (B), the discovery of low-level radiation.
21. H This specific question directs you to lines $4-10$, so reread those lines to get the question correct. The author states that after the intense heat of the big bang explosion, a cooling occurred and, subsequently, the first nuclei were formed. Eliminate (E) because the cooling did not usher in an ice age. Choice ( F ) is incorrect because the passage states that nuclei were first created after the cooling period. Finally, eliminate (G) because it is too extreme: the passage does not mention that elementary particles are never found under extreme temperatures. The correct answer is $(\mathrm{H})$; the nuclei did not exist amidst the heat of the big bang explosion.
22. A This question provides the lead words "cosmological principle." Use headlines or skimming to arrive at paragraph 2. Cross off the answer choices that do not align with the claims in this paragraph. Eliminate (B) because it is extreme: the passage does not state that scientists will never be able to prove the big bang theory. Delete (C) because the cosmological principle states that we can make observations about the characteristics of the universe regardless of our location or the direction we look. Finally, eliminate (D) because the passage states that scientists believe that the big bang did not occur at a particular location. Thus, the answer is (A), because scientists have to base the big bang theory on the cosmological principle, another theory.
23. G Lacking line numbers, search for lead words. "The speed of light" leads to the third paragraph, which focuses on observing light that has taken millions of years to travel to our galaxy; this allows scientists to see parts of the universe as it was millions of years ago. Now jump to POE. Eliminate (E) because the passage does not mention calculating
the speed or force of the big bang explosion. Choice (F) does not make a lot of sense when paired with the question. How could distorting the images the scientists are observing possibly be useful to scientists in understanding the big bang? Cross out (F). Finally, remove (H) because scientists are not able to see the actual explosion; they see the objects as they appeared millions of years ago. The correct answer is (G). Recall the original question: in what way is measurement of the speed of light useful? If scientists know the speed of light, they can approximate how long it took the light to arrive in the present, and, therefore, they can know what time period their observations hail from.
24. B The question tells you to look at the fourth paragraph; do so. Your headlines should tell you this paragraph is about low-level static from the universe. POE choices that do not agree with the content of this paragraph. Eliminate (A) because the passage says that the radiation occurs as "the remains of the initial fireball," not as a result of cooling. Moreover, remove (C) because the initial fireball is not known to be mostly static; the static resulted from the fireball. Finally, delete ( D ) because the passage never states that the magnitude of the big bang was akin to a nuclear explosion. The correct answer is (B), as the passage says the static comes from every direction as opposed to "one specific place."
25. F If you don't immediately recognize the answer to a general question from your skim of the passage, refer back to your headlines and POE. Eliminate (G) and (H) immediately because the passage as a whole is focused neither on how wolves live nor on their appearances in fairy tales, although it mentions each of those topics briefly. Down to two! You can eliminate (E) because, although it is mentioned in paragraph 4, it does not best represent the passage as a whole. Moreover, the passage does not specifically explain, as (E) says, "how humans domesticated the wild wolf." Only (F) accounts for the many "speculations" and theories about the complicated relationship between wolves and humans. The correct answer is (F).
26. C Like any question with "except/not/least," follow the basic strategy: use your pencil to cross out the "except," "not," or "least." Then,
mark each answer choice with a T for true or F for false. There will be 3 T's and just 1 F ; you know the odd one out is your answer, so choose it. Put a T next to (A), (B), and (D) because the passage states that wolves are skilled hunters (paragraph 5), and may have inspired the hunting methods of early humans (paragraph 3). Put an F next to (C) because the passage states that "wolves, no longer able to stalk their traditional prey, turned their attentions to human beings." The correct answer is (C).
27. E Use the lead words "domestic dogs" to track down your headlines related to the dog. Notice paragraphs 2 and 3 chronicle the evolutionary journey from wolves, canis lupus, to domestic dogs, canis familiaris. Knowing this, jump to POE. Eliminate (F) because the adjectives "red-tongued" and "sulfur-breathed" refer to the medieval wolves in paragraph 1. Similarly, remove (G) and (H) because, according to the passage, it is wolves that are huntergatherers and a menace to herd animals. The question asks about dogs, so the only answer can be (E): they are descendants of canis lupus.
28. D Using the phrase "Middle Ages" as a guide, find the first paragraph, which discusses popular myths surrounding wolves. According to this paragraph, the terrifying image of the wolf "likely originated in the Middle Ages," when, during the Black Plague, "ravenous wolves... turned their attentions to human beings." Now jump to the answer choices and find one that matches the passage's claims. The correct answer is (D), that wild wolves preyed on people in the Middle Ages.
29. E This question asks about an example of a herd animal. Skim back through the first paragraph to find that herd animals are mentioned in line 11. Directly following herd animals, the author gives these examples: "sheep, cattle, and the like." Choose (E), cattle, and move on.
30. B This specific question tells you exactly where to read: the beginning of the second paragraph. Your job is to find out why the author uses the word "ironically." Irony is something that is the opposite of what would be expected. The sentence in the passage says that, ironically,
humans have coexisted peacefully for two million years. Reading a few sentences before this one reveals that humans have developed a mythologized fear of wolves. The irony lies in this contradiction: wolves are not known to be dangerous to humans, but many humans are afraid of them nonetheless. Use POE to cut (A) because wolves were not the cause of the Black Plague. Choice (C) can be deleted because the passage does not say Paleolithic wolves were more primitive than Paleolithic humans. Similarly, eliminate (D) because the author does not claim that wolves have more physical power than intelligence. Choose (B). It is ironic that "humans would so despise an animal that had so infrequently been their enemy."
31. H A general question like this can best be answered by using your headlines. Jump right to POE. Eliminate (E) because the beginning of the passage mentions Pachacuti, but he is not the central focus. Similarly, delete (F) and (G) because they are included in the text, but are not the main idea. Only (H) summarizes the entire passage: a brief history of the Inca Empire and its accomplishments.
32. B Using your headlines, recall that Pachacuti features heavily in paragraph 2-so start there. The question asks to describe the relationship between Pachacuti and leaders of nearby regions. With paragraph 2 as a reference, POE answer choices that don't match what the text says. Eliminate (A) because the author gives no evidence that fear motivated Pachacuti. Also eliminate (C) because the passage does not mention the other leaders' military skills; finally, cross off (D) because the author never states that the other leaders feared Pachacuti; in fact, most leaders accepted Pachacuti's offer to join the empire. The correct answer is (B).
33. E Notice that the author describes Incan architecture in the fourth paragraph. POE any choices that do not agree with the passage's claims. Eliminate (F) because, while Machu Pichu is a well-known example of Incan architecture, it is not the only one. Choice (G) is a trap because the words "stones" and "steel" appear together in the passage, but the Incan temples were not made with steel. Finally, eliminate (H) because the Spanish invaders destroyed much of Incan culture, but not because they disliked the architecture. The correct
answer is (E).
34. B This question asks what part of the Inca Empire "remains a mystery" and provides a line reference, so use the question to read only what you need: the final paragraph (lines 57-70). The text says that archaeologists do not know whether the Inca developed a written language. According to the answer choices, the only one that matches the text is (B). Choose (B) and move on.
35. G You are asked what role Pachacuti's spies played in the Inca Empire. Refer back to your headlines to find that this topic is discussed in paragraph 2. POE choices that do not reflect the content in paragraph 2. Remove (E) because the author offers no proof that the spies warned their leader of plots against him. Eliminate (F) and (H) because, according to the passage, Pachacuti's spies had nothing to do with the invading Spanish. Choice (G) is the correct answer, because his spies evaluated the strength of neighboring tribes.
36. B This specific question is about "quipu" from line 67, so reread to remember what a quipu is. It is a series of knotted strings that Incans used to "store numerical data." The author then says that there is inquiry and debate about other information the quipu may have stored. The sentence prior to line 67 wonders whether the Inca had a written language, so you can infer that some scholars believe that Incans may have used quipu to write their language, in addition to their figures. Now POE. Eliminate (A) because the quipu do not prove the existence of the written language; and cross out (D) because the author does not say quipu cannot represent letters. Finally, eliminate (C) because there is no evidence in the passage to support the claim that the Spaniards destroyed all of the quipu. The correct answer is (B): there is still debate about the significance of quipu. This works because some scholars believe quipu represented letters, but others did not.
37. G Use your headlines to answer this general question about the passage and to make it easier to POE without having to reread everything. Eliminate ( E ) because the main idea of the passage is not the flowers O’Keeffe painted, although it is one topic; also eliminate (F) because the theme of this passage is not so broad as to include all of modern
art. Similarly, cross out (H) because "modern artists of the twentieth century" is too broad. The passage is just about O'Keeffe's career as an artist. The correct answer is (G).
38. B The lead word "fossil" can lead you to the final paragraph of the passage, which explains that archaeologists in New Mexico found a fossil near O'Keeffe's home, and named the crocodile-like creature Effigia okeeffeae. According to the passage, the name honored her "for the numerous paintings she drew from her home, which was called Ghost Ranch." Use POE at this point. Eliminate (A) because, while it is true that O’Keeffe left many of her works to the Georgia O'Keeffe Museum, this does not explain why the fossil was named for her. Also delete (C) and (D) because the author states neither that O'Keeffe painted "O'Keeffe's Ghost" nor that she painted the crocodile-like creature. The correct answer is (B): because the fossil was found near New Mexico, the source of many O’Keeffe paintings.
39. H This question asks why O'Keeffe decided not to become an artist after her initial schooling. The end of the first paragraph states that she "abandoned her dream of being an artist because she did not believe she was talented enough in the techniques she had been taught." Now look at the answer choices. Based on the quotation from the passage, the only answer that works is $(\mathrm{H})$ : she felt insecure about her artistic abilities.
40. B This specific question asks how O'Keeffe reacted to her declining eyesight. Helpfully, the question provides lines $50-53$, so search for the answer in that part of the text. Notice that the author says after O’Keeffe's eyesight deteriorated, "she thereafter limited herself to pencil and charcoal." Eliminate (A) and (C) because the text does not say she relocated or traveled as a result of her declining vision. Finally, eliminate (D); her vision problems did not lead to her showing her work at museums. The correct answer is (B); if she was using only pencil and charcoal, then she was no longer using oils.
41. E This question asks why O’Keeffe painted images of New Mexico. Headlines can be useful on a question like this. The third paragraph speaks to how the artist exhibits her work in New York, but then finds
inspiration in the New Mexican landscape. Use POE based on the information in paragraph 3. Cross out (F) because Stieglitz does not ask O'Keeffe to focus on New Mexico. Eliminate (G) because the passage does not say her initial goal was to establish a museum. Choice (H) is a bit of a trap, because O'Keeffe is particularly famous for painting large, close-up studies of flowers-but the passage does not mention this as a reason for venturing to New Mexico, so eliminate ( H ). The correct answer is ( E ), which matches with the passage.
42. A This is a very general question about O'Keeffe as an artist, and might be easiest to answer once you've gone through the other questions. (You can also just skim.) Use POE to eliminate (B) because, while O'Keeffe is insecure initially after finishing art school, her career in general is not one of frustration or insecurity. The passage never says that she is dependent on her husband, so cross off (C). Finally, the passage shows a deep variety in her interests over her career, spanning New York City scenes to New Mexican landscapes, so "consistent in her vision" is not the best description of her. Choice (A) is correct.
43. $\mathbf{F} \quad$ This general question requires a sense of the poem as a whole. Use your reading of the poem (as well as the informational blurb that precedes it) to tackle this question. Jump to POE on this and most general questions. Eliminate (E) because, while the first line encourages the river and tide to flow forever, the rest of the poem concerns many other ideas. Also remove (G) for this reason: the poet mentions New York City only briefly here; it is not the central theme of the piece. Finally, remove (H) because the sense of this poem is the present, not the past. (Notice all the present tense verbs: "flow," "gaze," "sound out," etc.) The correct answer is (F).
44. B Use the line numbers to help answer this specific question. Notice that the poet, Whitman, writes, "Gorgeous clouds of the sunset! drench with your splendor me, or the men and women generations after me!" Here, Whitman invites the sun to shine down for generations after him on this spot of the river. Be careful about what this question is asking! It asks what the effect of this line is. POE! Eliminate (A) because the poem never imagines that the river will look different than it does
now. Next, eliminate (C) and (D) because the poet does not explicitly ask the reader to think of a time she rode the ferry; and the billowing population of New York City appears nowhere. The correct answer, then, is (B): the poet is just one person of millions more who will cross the river.
45. E This specific question leads you to the phrase "suspend here and everywhere, eternal float of solution" in the first stanza. Note how the previous line ("Throb, baffled and curious brain! throw out questions and answers!") sets up the idea that the majestic river crossing can inspire new questions and that being on the ferry can also provide "solution." Consider (F). Whitman is not literally suggesting that the ferry stop moving and float aimlessly on the river, so eliminate (F). Next, eliminate (H) because in saying "suspend here and everywhere," Whitman most likely wants to remain aboard the ferry, not continue to the shore. Finally, eliminate (G) because it is too extreme of a choice, given the word "all." The writer does not claim to find all the solutions to his problems, though the river crossing does bring him some solutions. The correct answer is (E): he does not literally wish the ferry would stop moving, but he wishes that the clarifying experience of the river crossing could be extended.
46. D Go directly to the first few lines of the second stanza in order to answer this specific question. Using these lines, move to POE and eliminate any choices that are not consistent with the poem. Eliminate (A) because, although actors and actresses are mentioned, they are not in fact aboard the vessel with the narrator (he is conjuring them in his mind). Next, eliminate (B) because in the line "Live, old life!" the author does not mean to say that someone is at an advanced age; he simply encourages the listener to "live." Finally, eliminate (C) because while these lines describe the feeling of locking eyes with a stranger, the experience is not defined as "frightening." The correct answer is (D), as these lines can be read as a call to action, to live as fully as possible.
47. G This question asks you why these lines would be included in the poem. Review the lines and their place in the poem as a whole. The narrator speaks here about the relationship between the sun and the
water and how they can bounce off a human's head. At this point, use POE. Eliminate (E) because it is clear Whitman is not concerned with the science behind the sun's rays. Then eliminate both (F) and (H) because while they mention the notion of bringing cheer or warmth, this does not appear in the text itself. The correct choice is (G), as Whitman merely muses about the relationship between humans and nature; it would be reading a bit beyond the text to say that seeing this relationship brings joy to people.
48. C This is a broad question about the poem's punctuation. You've probably noticed that almost every line ends with an exclamation point; this question gives you some options about how that changes how you read the poem. Move directly to POE as there is no specific place to read for this question. Eliminate (A) because the poet does not have a "troubled state of mind"; in fact, the opposite may be true. Next, eliminate (B) and (D), as an exclamation point is not merely a measure of volume; it conveys other qualities, such as excitement. Choice (C) captures that quality perfectly: with the exclamation points, we hear the speaker's joyous, excited attitude about the objects he sees and describes. The correct answer is (C).
49. F This specific question asks you to analyze the line "Stand up, tall masts of Mannahatta!" in the context of the poem. The narrator says these lines as he crosses the ferry between Manhattan and Brooklyn. Using this knowledge, move directly to POE. Eliminate (H) because it is the most outlandish answer choice; we are not led to believe that his calling to the island of Manhattan is to either shock the reader or engage with inanimate objects. Similarly, (E) can be eliminated because the old-fashioned spelling of Manhattan is not the main idea of the excerpt. Finally, eliminate (G) because the narrator probably does not literally see ships in Manhattan; he is most likely using "masts" metaphorically, to describe the tall buildings of Manhattan. This idea comes across in (F), the correct answer choice.
50. D This general question can best be answered using headlines. Move directly to POE. Eliminate (A) because the passage does not contrast chemistry and physics; it is a passage about chemistry that mentions physics. Remove (B) because, although Avogadro is mentioned, he is
not the main focus of the passage. Finally, eliminate (C) because the interdependence of chemistry and physics is mentioned, but it is by no means the central claim of the passage. The text is best described as a "summary of some key innovations in chemistry in the late 1700s." Choose (D).
51. G This question refers to a specific quotation in lines 5-6, so begin there. The text says that chemists based their work on precise, quantitative measurements and then explains how the "balance... enabled chemists to substitute precise weights for the crude approximations of the medieval alchemist." Use this information to POE choices that do not match the text. Cross off (E) because this paragraph discuses chemistry, not physics; (F) can be removed because "new discoveries" is too vague to explain what the precise measurements are. Finally, eliminate (H) because the 18th-century chemists sought to move past the methods of the medieval alchemists. The "precise, quantitative measurement" refers to the better method of weighing, or the balance. The correct answer is (G).
52. C Use the lead words "chemical reaction" to find the part of the passage that mentions this topic. Notice that it first appears in paragraph 2, where the author says that chemists discovered the difference between mixing and a true chemical reaction. The main point here is that a simple mixture creates a blend of the two ingredients, while a true chemical reaction can "produce a substance totally unlike the materials that went into its formation." Look at the answer choices to see which one matches. Choice (C) is the correct answer.
53. H This question asks why the author uses the term "exact recipe" in line 34. Find the context for this phrase in the passage. The author writes that "mixtures could be formed in any desired proportions, but chemical reactions were produced by an exact recipe." Now POE. Eliminate (E) because the passage makes clear that mixtures and chemical reactions are not the same. Next, delete (F) because the author does not draw a distinction between quantitative and proportional measurements, even though forms of those words appear in this passage (don't be tricked by that!). Finally, eliminate (G) because the author never compares science to cooking. The correct
answer is ( H ).
54. D Use the lead word "atomist" to guide your process with this question, tracing it back to its introduction in the fourth paragraph. The author mentions that atomism was, at first, mainly the province of physicists and not chemists, as chemists were skeptical of the atomists' abstract theories. Use POE to get rid of choices that do not match the claims of paragraph 4. Eliminate (A) because chemists did not readily accept atomists' ideas. Cross off (B) because chemists, not atomists, discovered that elements could not be broken down. Last, this passage focuses on developments in the late 18th century, so atomists were not ignored until the 19th century; eliminate (C). Choose (D), because atomists were, in fact, often at odds with chemists.
55. H This question is phrased in a tricky way, asking you which of the following is not discussed in the passage. Follow the basic strategy for except/not/least questions: use your pencil to cross out the "except," "not," or "least." Then, mark each answer choice with a T for True or an F for False. There will be 3 Trues and just 1 False; you know the odd one out is your answer, so select it. In the case of this question, this process makes the question a lot simpler: "which of the following is discussed in the passage?" Put a T next to (E) because one of the main themes of the passage is the difference between mixtures and chemical reaction. Similarly, (F) and (G) should have Ts because the passage does discuss elements, compounds, and the early developments of modern chemistry. The only one that does not appear in the passage is $(\mathrm{H})$; the author does not discuss how scientists could split the atom. Choice (H) is the odd one out, so it is the correct answer.
56. B This question asks why the author mentions Amedeo Avogadro. Skim the passage for this scientist's name. Notice in the fourth paragraph, the author writes that the Italian physicist Avogadro "pushed the atomic idea in chemistry well before the end of the eighteenth century." POE! Eliminate (A) because the author does not discredit Avogadro's ideas. Next, eliminate (C) because Avogadro is not the scientist who discovered the distinction between a true chemical reaction and the process of mixing. Finally, cross off (D) because
rather than dissenting from Thomas Dalton, Avogadro's ideas aligned with Dalton's. The correct answer is (B). The author mentions Avogadro to provide an example of an early advocate of atomism.

## Math

58. . 27 The question asks for probability, so use the formula probability $=$ the number of what you want. There are 100 sodas in the the total number inventory, so the total number is 100 . To get the number of what you want, use the group formula: Total = Group $1+$ Group $2+$ Neither Both. The Total is 100. Let Group 1 be the number of diet sodas, which is 65 . Let Group 2 be the number of caffeine-free sodas, which is 30 . Both is the diet sodas that are also caffeine-free, which is 22 . Let Neither be represented by $n$. Therefore, $100=65+30+n-22$. Simplify the right side to get $100=73+n$. Subtract 73 from both sides to get $n=27$. Since the question asks for the probability that the soda is neither diet nor caffeine-free, the number you want is 27 , and the probability is $\frac{27}{100}$, which in decimal form is .27 .
59. .09 Before multiplying the fractions, reduce $\frac{8}{40}$ by 8 to get $\frac{1}{5}$ and reduce $\frac{15}{25}$ by 5 to get $\frac{3}{5}$. Multiply across the numerators and denominators to
get $\frac{(1 \times 1 \times 3 \times 3)}{(2 \times 5 \times 5 \times 2)}$, or $\frac{9}{100}$. For a grid-in problem, this must be converted to decimal form, making .09 your answer.
60. 5 Find the prime factors of 48 using a factor tree. Find two factors of 48: 6 and 8. Find two factors of 6: 2 and 3. Since both 2 and 3 are prime, do not branch off of them. Go to the other branch and find two factors of 8: 2 and 4. 2 is prime, so don't branch off of it. Find two factors 4: 2 and 2. Do not branch off any more. The result is this.


The question asks for the sum of the distinct prime factors. Distinct means different, so don't repeat any numbers. The distinct prime factors are 2 and 3 , so the sum is $2+3=5$.
61. 6 The question asks for the area of the shaded region. The shaded region is a rectangle, so use the area formula for a rectangle, which is $A=l w$. The length is 2 and the width is 3 , so the area is $A=(2)(3)=6$.
62. 184 The question asks for the surface area of a rectangular box, so use the formula $S A=2 l w+2 w h+2 l h$. The length is 4 , and the width is 5 . The height is twice the length, so $h=2 l=2(4)=8$. Therefore, the surface area is $S A=2(4)(5)+2(5)(8)+2(4)(8)=40+80+64=184$.
63. B The question asks for the final sale price. Start with the original, which is $\$ 60.00$. The price is marked down by $20 \%$ on Monday. Take
$20 \%$ of $\$ 60.00$ to get $\frac{20}{100} \times \$ 60.00=\frac{1}{5} \times \$ 60.00=\frac{\$ 60.00}{5}=$ $\$ 12.00$. Since the price is marked down by $\$ 12.00$, the price is now $\$ 60.00-\$ 12.00=\$ 48.00$. The price was then marked down by $10 \%$ on Friday, so take $10 \%$ of $\$ 48.00$ to get $\frac{10}{100} \times \$ 48.00=\frac{1}{10} \times \$ 48.00$ $=\frac{\$ 48.00}{10}=\$ 4.80$. Subtract $\$ 4.80$ from the price to get a final sale price of $\$ 48.00-\$ 4.80=\$ 43.20$, which is (B).
64. H The question asks for the value of $v$. The segment with length $v$ is a combination of one side of each square, so determine the lengths of the sides. The areas are given, so use the formula for the area of a square: $A=s^{2}$. The square on the left has an area of 4,900 , so $4,900=$ $s^{2}$. Take the square root of both sides to get $s=70$. Thus, the square on the left has a side of 70 . The square on the right has an area of 8,100 , so $8,100=s^{2}$. Take the square root of both sides to get $s=90$. The value of $v$ is the sum of the lengths of a side of each square, so $v$ $=70+90=160$, which is $(\mathrm{H})$.
65. B The question asks for the number of gallons of orange juice needed for 45 servings. The question provides the relationship between servings of punch and gallons of orange juice, so set up the proportion $\frac{27 \text { servings of punch }}{12 \text { gallons of orange juice }}=\frac{45 \text { servings of punch }}{g \text { gallons of orange juice }}$. Crossmultiply to get $27 \mathrm{~g}=(12)(45)$. Rather than multiply on the right side, divide both sides by 27 to get $g=\frac{(12)(45)}{27}$. Reduce the fraction.

Reduce 12 and 27 by 3 to get $g=\frac{(4)(45)}{9}$. Reduce 9 and 45 by 9 to get $g=\frac{(4)(5)}{1}$, which is (B).
66. F The question asks how many coins the machines can make in 1 minute. Determine the number of coins each machine can make in one minute and add. When a question asks about rates, use proportions.

The first machine can make 1 coin every 5 seconds. Using like units, set up the proportion $\frac{1 \text { coin }}{5 \text { seconds }}=\frac{x \text { coins }}{60 \text { seconds }}$. Cross-multiply to get $5 x=60$. Divide by 5 to get $x=12$. Do the same for the other machine. That machine produces 1 coin every 6 seconds, so set up the proportion $\frac{1 \text { coin }}{6 \text { seconds }}=\frac{y \text { coins }}{60 \text { seconds }}$. Cross-multiply to get $6 y=60$. Divide by 6 to get $x=10$. Therefore, the first machine makes 12 coins in a minute and the second makes 10 . To determine how much they make combined, add to get $12+10=22$, which is $(\mathrm{F})$.
67. A The question asks for the value of $r$, so get an equation in terms of $r$. The first is $q+r+s=117$. From the second equation, $q=s=4 r$, so substitute $4 r$ for $q$ and $s$ into the first equation to get $4 r+r+4 r=117$. Combine like terms to get $9 r=117$. Divide both sides by 9 to get $r=$ 13 , which is (A).
68. H Estimate values for $A, B$, and $C$ based on their positions on the number line. $A$ is slightly to the right of -1 , so estimate it as -0.9 . $B$ is
slightly to the right of 1 , so estimate it as 1.1 . $C$ is about as far to the right of 1 as 1 is from 0 , so estimate it as 2 . Go through the choices. Choice (E) is $-0.9+1.1>2$. This is false, so eliminate (E). Choice (F) is $-0.9+2>2$. This is false, so eliminate (F). Choice (G) is $2-1.1>$ 1. This is false, so eliminate (G). Select the only remaining choice, which is (H).
69. D The question asks for the larger of two numbers with a sum of 29 and a difference of 11 . One option is to translate the two statements into a system of equations. However, an easier approach is to Plug In the Answers. First, note that one of the answers is negative. If the larger number is negative, the smaller must be negative too. The sum of two negatives must be negative and, therefore, cannot be 29. Eliminate (A). Try the middle of the remaining three choices, which is (C). Let the larger number be 12. If the difference between the two numbers is 7 , then the smaller number is $12-7=5$, so the sum is $12+5=17$. The sum is supposed to be 29, so (C) is incorrect. The answer has to be greater. Try ( D ), which is 18 . If the larger number is 18 , then the smaller number is $18-7=11$, so the sum is $18+11=29$. This is consistent with the information in the question, so the answer is (D).
70. F The question asks for the value of $x$, which is the number of dance tracks. The probability that the first song selected will be a ballad is $\frac{1}{4}$ . Probability is equal to $\frac{\text { the number of what you want }}{\text { the total number }}$. There are 3 ballads, so this is the number of what you want. Set up a proportion to find the total number of tracks on the compact disc. Let $t$ represent the total to get $\frac{1}{4}=\frac{3}{t}$. Cross-multiply to get $t=12$, so the total number of tracks is 12. There are 3 ballads, 4 instrumental pieces, and $x$ dance
tracks, so $3+4+x=12$. Simplify to get $7+x=12$. Subtract 7 from both sides to get $x=5$, which is (F).
71. B The question asks for the first month in which at least one-third of the 150 tests had been given by the end of the month. One-third of 150 is $\frac{1}{3} \times 150=\frac{150}{3}=50$, so find the first month in which at least 50 tests had been given. The wording in this question is tricky. When the question asks for when at least 50 tests had been given, you must include previous months. Go through the months one at a time. In September, 5 tests were given. In October, 8 tests were given, so a total of $5+8=13$ had been given. This is less than 50 , so continue. In November, 11 tests were given, so a total of $13+11=24$ tests had been given. This is still less than 50, so continue. In December, 14 tests were given, so a total of $24+14=38$ tests had been given. This is still less than 50, so continue. In January, 18 tests were given, so a total of $38+18=56$ tests had been given by the end of the month. Since at least 50 tests had been given by the end of the month, stop here. The answer is January, which is (B).
72. $\mathbf{H}$ The question asks for $q r$. The equation $5 q=3 r=30$ can be rewritten as two equations: $5 q=30$ and $3 r=30$. Solve the first for $q$ and the second for $r$. Divide both sides of $5 q=30$ by 5 to get $q=6$. Divide both sides of $3 r=30$ by 3 to get $r=10$. Therefore, $q r=(6)(10)=60$, which is (H).
73. B The question asks how many groups of 4 students the teacher can select. When a question asks for how many ways to form a group, write a dash for each spot. The teacher selects 4 students, so make 4 dashes. Now fill in the number of possible students for each spot. There are 12 students in the class, so there are 12 possible students for the first spot. Once that student is selected, there are 11 students left for the second spot, 10 left for the third spot, and 9 left for the fourth spot. Now determine whether order matters. Since nothing in the question specifies any difference among the certificates or anything else to indicate position or role of any kind, order does not matter.

Because order doesn't matter and there are 4 spots, divide by $4 \times 3 \times$
$2 \times 1$. Therefore, the number of groups the teacher can select is $\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$. Simplify before multiplying. Reduce 12 and 4 by 3 to get $\frac{3 \times 11 \times 10 \times 9}{1 \times 3 \times 2 \times 1}$. Now cancel the 3 's to get $\frac{1 \times 11 \times 10 \times 9}{1 \times 1 \times 2 \times 1}$. Now reduce 10 and 2 by 2 to get $\frac{1 \times 11 \times 5 \times 9}{1 \times 1 \times 1 \times 1}$
$=11 \times 5 \times 9=495$, which is (B).
74. F Work out the expression one step at a time. Begin with the exponent. $5^{2}=25$, so $\frac{25+5}{5}$. Add 25 and 5 to get $\frac{30}{5}$. Divide 30 by 5 to get 6, which is ( F ).
75. D The question asks for the distance traveled. When the question involves rates, use proportions. The car drives at 130 kilometers per hour for 6 hours and 24 minutes, so set up the proportion $\frac{130 \mathrm{~km}}{1 \mathrm{hr}}=$ $\frac{x \mathrm{~km}}{6 \mathrm{hr} 24 \mathrm{~min}}$. Always use like units in a proportion; therefore, use another proportion to convert 24 minutes into hours. There are 60 minutes in 1 hour, so $\frac{60 \mathrm{~min}}{1 \mathrm{hr}}=\frac{24 \mathrm{~min}}{y \mathrm{hr}}$. Cross-multiply to get $60 y=$ 24. Divide both sides by 60 to get $y=\frac{24}{60}=\frac{2}{5}=0.4$. Rewrite the first proportion as $\frac{130 \mathrm{~km}}{1 \mathrm{hr}}=\frac{x \mathrm{~km}}{6.4 \mathrm{hr}}$. Cross-multiply to get $x=(130)(6.4)$ $=832$, which is $(D)$.
76. F The question asks how many more seconds it will take Michele to complete the assignment. She has already done 20 of the 50 required pushups, so she must do $50-20=30$ more pushups. She completes
pushups at a rate of 20 every 40 seconds, so set up the proportion $\frac{20 \text { pushups }}{40 \text { seconds }}=\frac{30 \text { pushups }}{x \text { seconds }}$. Cross-multiply to get $20 x=1,200$. Divide both sides by 20 to get $x=60$, which is ( F ).
77. A The question asks how far away Kai’s school is. He bikes uphill to school and travels twice as fast downhill home. If he travels twice as fast downhill, it takes twice as long uphill. Let $t$ be the time it takes to travel home and $2 t$ be the time it takes to travel to school. Since he
travels a total of 30 minutes for the round trip, $2 t+t=30$. Combine
like terms to get $3 t=30$. Divide both sides by 3 to get $t=10$, so it
takes 10 minutes to ride home. He travels 18 miles per hour on the trip
home, so set up the proportion $\frac{18 \mathrm{mi}}{1 \mathrm{hr}}=\frac{x \mathrm{mi}}{10 \mathrm{~min}}$. Use the like units
in the proportion, so convert 10 minutes into hours. There are 60
minutes in 1 hour, so use the proportion $\frac{60 \mathrm{~min}}{1 \mathrm{hr}}=\frac{10 \mathrm{~min}}{y \mathrm{hr}}$. Crossmultiply to get $60 y=10$. Divide both sides by 60 to get $y=\frac{10}{60}=\frac{1}{6}$. Rewrite the first proportion as $\frac{18 \mathrm{mi}}{1 \mathrm{hr}}=\frac{x \mathrm{mi}}{\frac{1}{6} \mathrm{hr}}$. Cross-multiply to get $x=18\left(\frac{1}{6}\right)=\frac{18}{6}$, which is (A).
78. $\mathbf{F}$ The question asks for the volume of the water in the rectangular pool.

The volume of a rectangular solid is $V=l w h$. The length is 9 and the width is 20 . The height is 6 inches. The answers are in cubic feet, so convert 6 inches into feet. There are 12 inches in a foot, so use the proportion $\frac{12 \mathrm{in}}{1 \mathrm{ft}}=\frac{6 \mathrm{in}}{x \mathrm{ft}}$. Cross-multiply to get $12 x=6$. Divide both sides by 12 to get $x=\frac{6}{12}=\frac{1}{2}$. Therefore, the volume is $V=(9)(20)$ $\left(\frac{1}{2}\right)=180\left(\frac{1}{2}\right)=\frac{180}{2}=90$, which is (F).
79. B The question asks for the minimum grade point average if Karen didn't fail any courses. The two missing grades in her report card are English and Physics. If she didn't fail those courses, the minimum she could have gotten in each is a D . The grade point average is the average of the conversion, so use the average pie. There are 5 grades, so this is the number of things. The total is the sum of the
conversions. Karen had one A, which is a 4 , one B, which is a 3 , and 3 D's, which are 1 's. Therefore, the total is $4+3+1+1+1=10$. Fill these into the pie to get


Divide to get an average of $10 \div 5=2$, which is (B).
80. G The question asks for "and" probability. Find the individual probabilities and multiply. Probability is equal to the number of what you want. An eight-sided die is rolled. the total number However, the die can never be rolled as a 6 , so there are only 7 possible outcomes. The first roll has to be a 3 . There is only one 3 , so the probability is $\frac{1}{7}$. Similarly, there is only one 8 , so the probability that the second roll will be an 8 is also $\frac{1}{7}$. Therefore, the probability that the first roll will be a 3 and the second roll will be an 8 is $\frac{1}{7} \times \frac{1}{7}=$ $\frac{1}{49}$, which is (G).
81. A Translate the statement, "The number of blue cars sold is three times the number of red cars sold." The number of blue cars sold is represented by $b$. The word is translates to $=$. The phrase three times translates to $3 \times$. Finally, the number of red cars sold is represented by
$r$. Therefore, the statement translates to $b=3 \times r$, which simplifies to $b=3 r$, which can be rewritten as $3 r=b$, or (A).
82. F The question asks for the ratio of $S T$ to $R S . R T$ has a length of 12 , so $R S+S T=12$. Segment $S T$ is $\frac{1}{6}$ of $R T$, so $S T=\frac{1}{6}(12)=\frac{12}{6}=2$. Substitute this value for $S T$ into the first equation to get $R S+2=12$. Subtract 2 from both sides to get $R S=10$. Therefore, the ratio of $S T$ to $R S$ is $\frac{2}{10}=\frac{1}{5}$, which is ( F ).
83. C Draw the number as it is described by the question. $A$ and $G$ are 16 units apart. $D$ is the midpoint of $A G$, so $A D=D G=\frac{16}{2}=8$. $X$ is the midpoint of $A D$, so $A X=X D \frac{8}{2}=4$. Since $X D=4, D=-3+4=1$. Since $D G=8, G=1+8=9$.


The question asks for a possible midpoint of $X G$. The midpoint is the average of the values, so the midpoint is $\frac{-3+9}{2}=\frac{6}{2}=3$, which is (C).
84. F The store is willing to accept lettering with $4 \%$ of the ordered size.

The ordered size is 16 inches, so take $4 \%$ of 16 , which is $\frac{4}{100} \times 16=$ $\frac{4}{100} \times \frac{16}{1}=\frac{64}{100}=0.64$. The store is willing to accept lettering within 0.64 inches of 16 . Therefore, the maximum size that the store is willing to accept is 0.64 above 16 . This is $16+0.64=16.64$, which is (F).
85. C The question asks for the value of $x$. Start with the known angles. The triangle on the left has a right angle and a $25^{\circ}$ angle. The sum of the measures of the angles in any triangle is $180^{\circ}$. Let the missing angle be $a^{\circ}$. For that triangle, $25+90+a=180$. Simplify to get $115+a=$ 180. Subtract 115 from both sides to get $a=65$. Remember that adjacent angles are supplementary. In this case, the adjacent angles to $65^{\circ}$ measure $180^{\circ}-65^{\circ}=115^{\circ}$. The angle opposite $65^{\circ}$, known as the vertical angle, is equal, so it also has a measure of $65^{\circ}$. Since you now know two of the three angles of the triangle on the right, the third can be determined. Let the missing angle be $b^{\circ}$. Therefore, $65+90+b=$ 180. Simplify to get $155+b=180$. Subtract 155 from both sides to get $b=25$.


Since $25^{\circ}+x$ forms a straight line, $x=180-25=155$, which is (C).
86. E The question asks for a unit conversion, so use proportions. There are 60 minutes in 1 hour, so set up $\frac{60 \mathrm{~min}}{1 \mathrm{hr}}=\frac{x \mathrm{~min}}{3.4 \mathrm{hr}}$. Cross-multiply to get $x=(60)(3.4)=204$, which is $(\mathrm{E})$.
87. C Use exponent rules to simplify the expression. Apply the exponent to each factor. Apply the exponent to 2 to get $2^{3}=8$. Eliminate (A) and (B), which don't have 8 as the coefficient. Apply the exponent to $a^{3}$ to get $\left(a^{3}\right)^{3}$. Because a number with an exponent is raised to another exponent, simplify by multiplying the exponents to get $a^{3 \times 3}=a^{9}$. Eliminate (D), which doesn't have $a^{9}$. The only choice remaining is (C).
88. E Because the question asks for the probability that both rolls will be 1 , this is an example of "and" probability. Multiply the probability that each individual roll will be 1 , using a fair die numbered 1 through 6 . There are six numbers, with one 1 , so the probability that the first roll will be a 1 is $\frac{1}{6}$. The die is not different the second time it is rolled, so the probability that the second roll will be a 1 is also $\frac{1}{6}$. Therefore, the probability that both rolls will be a 1 is $\frac{1}{6} \times \frac{1}{6}=\frac{1}{36}$, or (E).
89. $C$ If $A B=3 \frac{1}{4}$, then the position of $B$ is $3 \frac{1}{4}$ more than the position of $A$. Since the position of $A$ is $1 \frac{7}{8}$, the position of $B$ is $1 \frac{7}{8}+3 \frac{1}{4}$. Mixed
numbers can be tricky to do arithmetic with, so first convert them to improper fractions. $1 \frac{7}{8}=1+\frac{7}{8}=\frac{8}{8}+\frac{7}{8}=\frac{15}{8}$, and $3 \frac{1}{4}=3+\frac{1}{4}=\frac{12}{4}$ $+\frac{1}{4}=\frac{13}{4}$. Therefore, the position of $B$ is $\frac{15}{8}+\frac{13}{4}=\frac{15}{8}+\frac{26}{8}=\frac{41}{8}$. The answers are in the form of mixed numbers. Convert $\frac{41}{8}$ to a mixed number using long division.


Dividing 41 by 8 gives you 5 with 1 left over, so $\frac{41}{8}=5 \frac{1}{8}$, which is (C).
90. G The question asks about rates, so use proportions. 240 garks are equal to 180 praps, so to find how many garks equal 1 prap, set up $\frac{240 \text { garks }}{180 \text { praps }}=\frac{x \text { garks }}{1 \text { praps }}$. Cross-multiply to get $180 x=240$. Divide both sides by 180 to get $x=\frac{240}{180}=\frac{24}{18}=\frac{12}{9}=\frac{4}{3}$, which is (G).
91. B The question asks how much Eric will have left after paying his debts.

He owes $14 \%$ of what he has to his sister and $76 \%$ to his mother.

Therefore, his debts total $14 \%+76 \%=90 \%$, so he will be left with
$100 \%-90 \%=10 \%$. He started with $\$ 3.50$, so he will be left with $10 \%$ of the $\$ 3.50$, which is $\frac{10}{100} \times \$ 3.50=\frac{1}{10} \times \$ 3.50=\frac{\$ 3.50}{10}=$ $\$ 0.35$, which is (B).
92. E Since the perimeters of the two triangles are equal, find the sum of the sides of triangle $D E F$, which is $4+8+9=21$. Therefore, the perimeter of triangle $A B C=21$. The question asks for the length of $A B$, which is one of the sides of triangle $A B C$. Since triangle $A B C$ is equilateral, the length of each side is $21 \div 3=7$, which is ( E ).
93. C The question asks for the ratio of apples to cherries. On a ratio question, use the ratio box. Because there are two ratios given, use two ratio boxes. The ratio of apples to bananas is 5 to 4 , and the ratio of cherries to bananas is 2 to 5 .


There are no actual numbers given, so plug in numbers that fit the ratios. Bananas are part of both ratios, so make the number of bananas a multiple of 4 and 5. Try 20. In the first ratio box, add across the top row to get $5+4=9$ under Total. Multiply downward. Since $4 \times 5=$ 20, set 5 as the multiplier and repeat 5 across the middle row. Multiply downward to get 25 in the bottom row under Apples and 45 in the bottom row under Total. Now do the same for the second box. Add across the top row to get $2+5=7$. Multiply downward. Since $5 \times 4=$ 20 , set 4 as the multiplier and repeat 4 across the middle row. Multiply
downward to get 8 in the bottom row under Cherries and 28 in the bottom row under Total.

| Apples | Bananas | Total | Ratio <br> Multiplier <br> Actual \#'s |
| :---: | :---: | :---: | :---: |
| 5 | 4 | 9 |  |
| 5 | 5 | 5 |  |
| 25 | 20 | 45 |  |
| Cherries | Bananas | Total |  |
| 2 | 5 | 7 | Ratio |
| 4 | 4 | 4 | Multiplier |
| 8 | 20 | 28 | Actual \#'s |

The question asks for the ratio of apples to cherries. There are 25 apples and 8 cherries, so the ratio is $25: 8$, which is (C).
94. $\mathbf{F}$ The question asks for the least integer greater than -3.7 . Put -3.7 on a number line.


The least integer greater than or equal to -3.7 is the first integer to the right of -3.7 . That integer is -3 , which is ( F ).
95. C Substitute $x=-2$ and $y=4$ to get $(-2)^{2}(4)+\frac{4}{-2}$. Use order of operations. Start with exponents. The square of a negative is positive, so the expression simplifies to (4)(4) $+\frac{4}{-2}$. Now do multiplication and division to get $16+(-2)$. Addition of a negative is the same as subtraction, so the result is $16-2=14$, which is (C).
96. H The question asks for the range, which is the difference between the greatest and the least. Find the greatest and least for each team, and
select the greatest and least overall. The greatest for each team is given: 74 for Team A, 77 for Team B, and 78 for Team C. Therefore, the greatest overall is 78 . Find the least overall. The range is given for each team. Find the least for each team by subtracting the range from the greatest. Therefore, the least for Team A is $74-14=60$, the least for Team B is $77-19=58$, and the least for Team C is $78-12=66$. Thus, the least overall is 58 . Subtract the least overall from the greatest overall to get $78-58=20$, which is $(\mathrm{H})$.
97. C The question is about ratios, so use the ratio box. It is a three part ratio, so add an extra column. Otherwise, the box works the same way. The ratio is 2:3:5 and the actual number of pears is 45 .

| Melons | Pears | Grapes | Total |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 5 |  |
|  |  |  |  |
|  | 45 |  |  |
|  | Ratio |  |  |
| Multiplier |  |  |  |
| Actual \#'s |  |  |  |

Add across the top row to get $2+3+5=10$ under Total. Multiply downward. Since $3 \times 15=45$, the multiplier is 15 . Fill in 15 across the middle row. Multiply downward to get $2 \times 15=30$ in the bottom row under Melons, $5 \times 15=75$ in the bottom row under Grapes, and $10 \times 15=150$ in the bottom row under Total.

| Melons | Pears | Grapes | Total |
| :---: | :---: | :---: | :---: |
| 2 | 3 | 5 | 10 |
|  | Ratio |  |  |
| 15 | 15 | 15 | 15 |
| Multiplier |  |  |  |
| 30 | 45 | 75 | 150 |

The question asks for the total number of melons and grapes sold. Add the two actual numbers to get $30+75=105$, which is (C).
98. H The question asks about overlapping groups, so use the group formula: Total $=$ Group $1+$ Group $2+$ Neither - Both. There is a total of 30,000 . Let Group 1 be the number of doctors who treat children, which is 18,000 , and Group 2 be the number of doctors who perform surgery, which is 15,000 . Fill these into the formula to get $30,000=$ $18,000+15,000+$ Neither - Both. The question asks for the lowest possible number of doctors who treat children and perform surgery,
which is Both. Let Neither be $N$ and Both be B. Isolate B. Simplify to get $30,000=33,000+N-B$. Subtract 33,000 from both sides to get -$3,000=N-B$. Subtract $N$ from both sides to get $-3,000-N=-B$. Divide both sides by -1 to get $3,000+N=B$. To find the least possible value of $B$, make $N$ as small as possible, so let $N=0$. In that case, $B=3,000+0=3,000$, which is $(\mathrm{H})$.
99. A The question asks for the minimum number who did not cast a ballot.

To find the minimum who did not cast a ballot, find the maximum who did cast a ballot. The greatest possible percent of registered voters who did cast a ballot is $70 \%$, so the minimum percent that did not cast a ballot is $100 \%-70 \%=30 \%$. Take $30 \%$ of the 1,100 registered voters to get $\frac{30}{100} \times 1,100=\frac{3}{10} \times \frac{1,100}{1}=\frac{3}{1} \times \frac{110}{1}=330$, which is (A).
100. G The question says 2 hot dogs are sold for every 5 fans, so use proportions. To determine how many hot dogs are sold to the 30,000 fans, set up $\frac{2 \text { hot dogs }}{5 \text { fans }}=\frac{x \text { hot dogs }}{30,000 \text { fans }}$. Cross-multiply to get $5 x=$ 60,000 . Divide both sides by 5 to get $x=12,000$, which is (G).
101. C The question asks for $27.8913 \times 100$ in scientific notation. Scientific notation must have the decimal point after the first non-zero digit. Eliminate (A) and (D), which are not in scientific notation. Multiply 27.8913 by 100 . To multiply by 100 (a 1 with two 0 's), move the decimal two places to the right to get 2789.13. To convert this to
scientific notation, move the decimal point 3 places to the left to get it after the first non-zero digit. Because the decimal point was moved 3 times, multiply by $10^{3}$ to get $2.78913 \times 10^{3}$, which is (C).
102. G The question asks how many integers are common to both sets, so list out the elements of both sets. Set Q contains the even integers from 4 to 36 , inclusive, so $Q=\{4,6,8,10,12,14,16,18,20,22,24,26,28$, 30, 32, 34, 36\}. Set Z contains all the multiples of 3 . There are no restrictions in size, so the set will continue infinitely in both directions but will include $\{\ldots,-3,0,3,6,9,12,15,18,21,24,27,30,33,36$, $39, \ldots\}$. Count the common numbers in both sets: $6,12,18,24,30,36$. There are 6 numbers, so the answer is (G).
103. A The question asks how much Jackie would have saved, so subtract the amount she spent by the amount she would have spent. Jackie bought 21 candy bars at 3 for $\$ 1$. To find the cost of the 21 candy bars, set up the proportion: $\frac{21 \text { candy bars }}{\$ x}=\frac{3 \text { candy bars }}{\$ 1}$. Cross-multiply to get $3 x=21$. Divide by 3 to get $x=7$, so she spent $\$ 7$ on the candy bars. To determine what she would have spent had the cost been 7 for $\$ 2$, set up the proportion $\frac{21 \text { candy bars }}{\$ y}=\frac{7 \text { candy bars }}{\$ 2}$. Cross-multiply to get $7 y=42$. Divide both sides by 7 to get $y=6$, so she would have spent $\$ 6$. Therefore, she would have saved $\$ 7-\$ 6=\$ 1$, which is (A).
104. G The question involves rates, so use proportions. The company charges
$\$ 2.50$ for every square foot. To determine the amount charged for 21
square feet, set up $\frac{\$ 2.50}{1 \mathrm{sq} \mathrm{ft}}=\frac{\$ x}{21 \mathrm{sq} \mathrm{ft}}$. Cross-multiply to get $x=$ $21(\$ 2.50)=\$ 52.50$, which is $(G)$.
105. B The question asks how much farther the second jogger runs, so subtract the distance the second jogger runs from the distance the first jogger runs. The distance around a circular lake is the circumference of the lake. The formula of the circumference is $C=2 \pi r$. The first jogger completes a lap around a lake with radius $r$, so the first jogger runs a distance of $2 \pi r$. The second jogger completes a lap around a lake with a radius of $r+4$, so the second jogger runs a distance of $2 \pi(r+4)=2 \pi r+8 \pi$. Therefore, to find how much farther the second jogger runs subtract the distances to get $(2 \pi r+8 \pi)-2 \pi r=8 \pi$, which is $(\mathrm{B})$.
106. G Translate the information in the question into equations. Celia is 6 years younger than Marcello translates to $C=m-6$, where $C$ is Celia's age now. In 4 years, Marcello will be twice as old as Celia will be. In 4 years, Marcello will be $m+4$, and Celia will be $C+4$. If Marcello will be twice as old as Celia, then $m+4=2(C+4)$. Substitute the expression for $C$ from the first equation into the second to get $m+4=2(m-6+4)$. Simplify the right side to get $m+4=2(\mathrm{~m}$ -2 ). Distribute the 2 to get $m+4=2 m-4$. Add 4 to both sides to get $m+8=2 m$. Subtract $m$ from both sides to get $m=8$, which is (G). (You can also just plug in the answer choices to find the one that works. When Marcello is 8 , Celia is 2, and four years later, he'll be 12 and she'll be 6 , which satisfies the condition that he be twice her age.)
107. B The question asks for the cost of a nine-minute call. A three-minute call costs $\$ 1.50$. It costs 20 cents for each additional minute. Go one minute at a time. A four-minute call costs $\$ 1.50+$ $\$ 0.20=\$ 1.70$. A five-minute call costs $\$ 1.70+\$ 0.20=\$ 1.90$. A sixminute call costs $\$ 1.90+\$ 0.20=\$ 2.10$. A seven-minute call costs $\$ 2.10+\$ 0.20=\$ 2.30$. An eight-minute call costs $\$ 2.30+\$ 0.20=$ $\$ 2.50$. Finally, a nine-minute call costs $\$ 2.50+\$ 0.20=\$ 2.70$, which is $(\mathrm{B})$.
108. H The question asks for the number of possible arrangements, so write a dash for each spot. Jelena has 5 pictures to hang, so make 5 dashes. Now fill in the number of possible pictures for each spot. There are 5 pictures, so there are 5 possible pictures for the first spot. Once that picture is selected, there are 4 pictures left for the second spot, 3 left for the third spot, 2 left for the fourth spot, and 1 left for the fifth spot. Now determine whether order matters. Because the question specifies from left to right, order matters. Because order matters, multiply. Therefore, the number of groups the teacher can select is $5 \times 4 \times 3 \times 2$ $\times 1=120$, which is $(\mathrm{H})$.
109. C The question asks for the value of $1^{n}+1^{n+1}+1^{n+2}+1^{n+3}+1^{n+4}+$ $1^{n+5}$. Without the value of $n$, this may appear to be impossible to answer. However, 1 raised to any power is equal to 1 . Therefore, $1^{n}+$ $1^{n+1}+1^{n+2}+1^{n+3}+1^{n+4}+1^{n+5}=1+1+1+1+1+1=6$, which is (C).
110. $\mathbf{E}$ The question asks for the area of the shaded triangle. The formula for the area of a triangle is $A=\frac{1}{2} b h$. The base of the triangle can be any side. The known side is the one that is also the top of the rectangle, which is 8 . The height has to be perpendicular to the base. Since the outer figure is a rectangle, the side of length 6 is perpendicular to the side of length 8 , so it can be used as the height. Therefore, the area is $A=\frac{1}{2}(8)(6)=24$, which is $(E)$.
111. D The question asks for the area of the base of the building. The base of the building is a square, so use the formula for the area of a square: $A$
$=s^{2}$. Find the side of the base of the building. The question refers to a scale model, so use proportions. A height of 20 inches in the scale model corresponds with a building height of 80 ft . The side of the base of the model is 5 inches, so set up the proportion $\frac{20 \mathrm{in}}{80 \mathrm{ft}}=\frac{5 \mathrm{in}}{x \mathrm{ft}}$. Cross-multiply to get $20 x=400$. Divide both sides by 20 to get $x=$ 20. Therefore, the side of the base in the building is 20 , so the area of the base is $A=20^{2}=400$, which is (D).
112. G The question asks for the value of $2 x+4 y+z$. Substitute $z=7$ to get $2 x+4 y+7$. The question also says $x+2 y=6$. If you multiply both sides by 2 , you'll get that $2 x+4 y=12$, and can just plug that in to get $12+7$. If you don't notice this, you can also just plug in a value for one and solve for the other. Suppose $y=2$. If $y=2$, then $x+2(2)=6$. Simplify to get $x+4=6$. Subtract 4 from both sides to get $x=2$. Substitute $x=2$ and $y=2$ to get 2(2) $+4(2)+7=4+8+7=19$, which is (G).
113. B The question asks for an average, so use the average pie. Find the total and the number of things. Because there are three times shown, the number of things is 3 . To find the total, take the sum of the temperature. Note that below zero refers to negative temperatures, and above zero refers to positive temperatures. Therefore, the total is ( -10 ) $+(-2)+15=3$. Put these into the average pie.


Divide the total by the number of things, to get $3 \div 3=1$. This is positive, so it is $1^{\circ}$ above zero, which is (B).
114. F Translate the information in the question. 100 grams is half the mass of one object and twice the mass of another. This translates into two separate equations. First, 100 grams is half the mass of one object. The word is translates to $=$, so the statement translates to $100=\frac{1}{2} x$, where $x$ is the mass of the first object. 100 is also twice the mass of another. Similarly, this translates to $100=2 y$, where $y$ is the mass of the second object. The question asks how many more grams the mass of the heavier object is than the mass of the lighter object. Find the masses of the two objects and subtract. Multiply both sides of $100=$ $\frac{1}{2}$ by 2 to get $200=x$. Divide both sides for $100=2 y$ by 2 to get $50=$ y. Subtract the two masses to get $200-50=150$, which is (F).

## Part III <br> SHSAT English Language Arts Subject Review

Chapter 7 SHSAT Revising/Editing Part A

## REVISING QUESTIONS

Part A of the new Revising/Editing section on the SHSAT's ELA test will have about five stand-alone questions that refer to a boxed sentence or paragraph and then ask you to fix something. Although these items all begin with an instruction to "read" something, your best bet is to start with the question itself. That's because the question tells you what you're looking for.

There are three types of questions you'll encounter in this section. First, there's the specific question, which tells you exactly what needs to be fixed, like a runon sentence or a misplaced modifier. Because these questions generally refer to a paragraph of text, knowing exactly what you're looking for helps to avoid distractions and save time. In fact, as soon as you find the sentence with that specific error, you can stop reading.

Second, there's the editing question, in which the answer choices offer exact changes. These questions are generally just about a single sentence, so you can read it yourself to see if you spot anything; if you do, try to find that error listed in the choices. If you're stuck, however, you can simply check each choice, looking for the one that improves the sentence. For both this first and second type of question, you'll want to review your basic grammar, which is covered in the next section of this chapter.

The third type of question is a bit trickier, as it offers a series of sentences-not a paragraph—and asks you to combine them. In this case, you're looking for the most concise and accurate sentence. As long as that choice matches all of the information listed in the box and keeps the relationship between them the same, you've got the right answer. More practice with concision, which can sometimes be tough to spot, can be found after the grammar section of this chapter.

This Just In
The College Board has twice revised the ELA section of this test, first removing scrambled
paragraphs and logical reasoning questions in 2017, and again by changing the distribution of

Revising/Editing questions and the types of passages in 2018. As we send this book to print, we have done our best to accurately represent the test, but be sure to $\log$ in to your free online student tools to check for any lastminute updates.

## GRAMMAR

Grammar can sometimes seem overwhelming, with rules in place to govern a wide range of usages. The good news is that the SHSAT tends to test a rather narrow subset of those rules. Moreover, fewer than five questions on the SHSAT revolve entirely around grammar. If you focus on the following common grammatical mistakes, you should feel comfortable spotting and fixing them on the test.


Pace Yourself
If you just took the Diagnostic Practice Test, you might want to give yourself a day or two-
time permitting!-before diving into content
review. It's helpful to digest and analyze the results of your test before diving right back in.

## Verb Usage

It's always a good idea to check how verbs are being used across any given section. That's because verbs tell you what a sentence is doing, and when verbs switch tenses or disagree with nouns, that sentence is doing something wrong.

The main categories of verb usage that you will be tested on are subject-verb agreement, tense, and voice.

## Subject-Verb Agreement

Subjects and verbs must agree with each other. Singular verbs go with singular nouns, and plural verbs go with plural verbs.

Always check to see that a verb agrees with its subject. If there are extra words between the subject and verb, cover them up!

The students write an essay about grammar.
In this case, the subject and verb are next to each other, so there is less work required to see if they agree.

Some sentences, however, are more complex. If there are extra words between the subject and the verb, cover them up.

Examining the various changes in the stalagmite level helps researchers draw conclusions.

Look at the verb "helps." What is the subject of that verb? Don't be fooled: it's not "stalagmite level" or "changes." The subject is an -ing word, "examining." If you are confused, get rid of all the stuff in the way: "Examining....helps researchers." Examining is singular, so the singular verb "helps" works.

## Tense

Verb tense shows the timing of an action. The past tense describes events that have already occurred. The present tense speaks to things that are currently occurring or always occurring. The future tense refers to things that have not yet occurred.

| Simple Past | Simple Present | Simple Future |
| :--- | :--- | :--- |
| I walked through the <br> park yesterday. | I walk to school every <br> day. | I will walk to the store <br> tomorrow. |
| Past Continuous | Present Continuous | Future Continuous |
| I was walking last night. | I am walking two miles <br> per day. | I will be walking down- <br> town later. |
| Past Perfect | Present Perfect | Future Perfect |
| I had walked to the store <br> already, so I didn't want <br> to walk there again. | I have walked a lot this <br> year. | I will have walked 200 <br> miles by the end of the <br> year. |
| Past Perfect <br> Continuous | Present Perfect <br> Continuous | Future Perfect <br> Continuous |
| I had been walking for <br> five minutes when I ran <br> into him. | I have been walking for a <br> while, so I'll take a break <br> now. | I will have been walking <br> for two hours by that <br> point, so I'll break for <br> lunch. |

Read this sentence.

While the number of bicyclists in the city has increased steadily in the last ten years, the number of bike lanes will not grow since 2015.

Which edit should be made to correct the sentence?
A. change increased to increase
B. change will to has
C. change will not grow to has not grown
D. change has increased to will have increased

## Here's How to Crack It

Use POE! Choice (A) will result in "has increase," which is not any verb tense, so it can be deleted. Similarly, "has not grow" is not a proper tense, so (B) is
incorrect. Choice (D) results in the future perfect tense, but the sentence is talking about an increase that has already happened "in the last ten years," so future tense will not work. Thus, (C) is correct: "has not grown" is the past continuous tense, which aligns perfectly with the past continuous "has increased."


Mood
Mood is unlikely to be tested on the SHSAT, but comes in three forms. The indicative ("I read the book") announces something. The imperative ("Read the book, Laura!") demands something. The subjunctive ("It is crucial that he read the book") expresses something that is desired.

## Voice

Can you tell the difference between "I am studying for the SHSAT" and "The SHSAT is being studied by me"? Even though the same thing is happening in both instances, the first example is more direct and active, while the second is indirect and passive. These represent the two ways to "voice" a sentence.

The quick way to spot the difference is to identify the subject of the sentence. Is the subject the "doer" or "agent" of the action? If so, the sentence is active:

The bee stung the horse.
If something else is acting on the subject, the sentence is passive:
The horse was stung by the bee.
The active voice is almost always preferred because it states exactly what happened. The more evasive passive voice is useful only in hiding information, such as identifying who or what did something. The active sentence "Robert yelled at the hotel concierge" is clearer than the passive "The hotel concierge
was yelled at." (By whom?)

Read this paragraph.
(1) Cleaning up the city is a massive task. (2) Every night thousands of piles of garbage are collected. (3) To avoid inconveniencing residents, city leaders work with private companies to schedule pick-ups at times when the fewest vehicles will be on the street. (4) With increased communication between residents and urban planners, everyone wins.

Which sentence should be revised to use the active voice?
E. sentence 1
F. sentence 2
G. sentence 3
H. sentence 4

## Here's How to Crack It

Check to make sure each sentence has a clear subject, and that you know who or what is doing the action. In sentence 2, you learn that piles of garbage are collected, but pause and ask yourself: who is collecting the garbage? Does the passage name that person or persons? If not, the sentence is written in the passive voice and should be revised. Choice ( F ) is the correct answer.

## Pronouns

Verbs aren't the only part of speech that needs to agree. Pronouns are essentially words that stand in for other words. Consider a passage about Ralph Waldo

Emerson; rather than continuing to write out the full name, the author might use "he." This is an acceptable substitution, but only if it's clear who "he" is referring to. If Henry David Thoreau were being discussed alongside Emerson, think of the confusion generated by even a simple sentence like "He greeted him."

On the SHSAT, when encountering a pronoun problem, the first step is always asking yourself: what word is this pronoun trying to replace? Once you identify this word-called the antecedent-make sure the pronoun agrees with the word it replaces in both number and gender. (For example, "she" and not "he" for Mrs. Gonzales, and "they" for "the students.") Here is a list of common pronouns.

| Personal Pronouns | Possessive Pronouns |
| :--- | :--- |
| I / Me | My/Mine |
| We/Us | Our/s |
| You | Your/s |
| She/Her | $\mathrm{Her} / \mathrm{s}$ |
| $\mathrm{He} / \mathrm{Him}$ | His |
| It | Its |
| They | Their/s |

A. change writer to writers
B. change their to they're
C. change their to his or her
D. change each to all

## Here's How to Crack It

Always be careful about identifying what your pronouns are replacing. In this case, the antecedent of "their" is "each writer." If an antecedent is singular, its pronoun must also be singular, and vice versa. "Each writer" refers to each individual writer, so it is singular. Their is a plural pronoun, so it is incorrect. Search for the singular pronouns in the answer choices: the only option is "his or her." Thus, (C) is correct.

## Review Exercise

Fill in each blank with an appropriate pronoun. The answers are in a sidebar on this page.

1. Citizens want reliable access to high-speed Internet, and $\qquad$ grow easily frustrated with service interruptions.
2. My colleague invited me to $\qquad$ wedding.
3. This necktie belongs to you. $\qquad$ is $\qquad$ .
4. The report is well-written. However, $\qquad$ argument is weak.

## Punctuation

There are lots of punctuation marks and lots of uses, but all you have to remember is complete or incomplete. If you can divide a sentence into its complete or incomplete parts, you have done most of the work in figuring out which punctuation mark you need.

Punctuation marks can be divided into three main categories:

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STOP
; :
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.
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,

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\section*{FANBOYS}
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!

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?

FANBOYS refers to the most common coordinating conjunctions For, And, Nor, But, Or, Yet, and So.

When you are linking ideas,

STOP punctuation can link ONLY complete ideas.
HALF-STOP punctuation must be preceded by a complete idea. GO punctuation can link anything EXCEPT two complete ideas.

Let's see an example.
Scholars have long suspected a missing chapter; new evidence supports that claim.

The semicolon is a STOP punctuation mark, meaning it can link only complete ideas. The phrases on either side of the semicolon are, in fact, complete ideas: "Scholars have long suspected a missing chapter" and "new evidence supports that claim" can stand on their own as sentences. Therefore, they are complete phrases. The semicolon-or any STOP or HALF-STOP punctuation-can work in this case. The only option that would be unacceptable is GO punctuation: "Scholars have long suspected a missing chapter, new evidence supports that claim" is an example of a run-on sentence. Linking two complete ideas with just a comma must be avoided at all times.

A run-on sentence is a sentence in which the writer has not properly joined two ideas.

If a sentence has a STOP or HALF-STOP mark, you can use a trick to discover if it is acceptable. Draw a vertical line through that punctuation and read each part individually. Adjust your punctuation based on whether you have two complete phrases or an incomplete and complete phrase.

\section*{Commas}

The two main uses of commas are already known to you: (1) as GO punctuation and (2) as STOP punctuation. A comma plus a coordinating conjunction can link two complete ideas the same way any STOP mark can, such as a period or semicolon. There are three more important uses of commas you may be tested on.

\section*{1. Before and After Unnecessary Information}

Rule: If a part of your sentence can be removed and NOT affect the meaning, it is unnecessary, so set it off in commas.

I met my brother, an investment banker, for lunch.
If you are unsure if something is necessary, put your finger over the phrase and see if the sentence still works. Here, "I met my brother...for lunch" is a perfectly fine sentence, proving that "an investment banker" is unnecessary and should be set off in commas.

\section*{2. Between Two Adjectives}

Rule: If two adjectives modify the same noun, there must be a comma between them. Otherwise, the writing has a distinct lack of clarity.

Incorrect: His professor was a stern uncompromising man.
Correct: His professor was a stern, uncompromising man.


What the adjective modifies is important. In the phrase "heavy science textbook," you are describing a science textbook that is heavy, not a textbook that is "heavy" and "science." As a quick rule, if you could say "and" between the two adjectives, use a comma.

\section*{3. After a Transition Word}

Rule: Use a comma after a transition word that begins a sentence.
Incorrect: The city's power grid went down. Consequently citizens panicked.
Correct: The city's power grid went down. Consequently, citizens panicked.

If you use a transition word in the middle of a sentence, use a pair of commas to
set off the word. This is covered by rule 1, Before and After Unnecessary Information. For example, look at the following sentence: Leaders urged everyone to remain calm. Most people, however, did not heed that order.

The transition word "however" is an unnecessary part of the sentence. You can remove it and the sentence will still work.

Read this paragraph.
(1) Because of their close proximity to water, coastal cities are believed to be at the highest risk for climatechange related flooding. (2) Scientists predict that in the next 50-100 years sea levels will rise dramatically. (3) Seaside cities had better prepare for rising sea levels, they need to develop detailed plans for emergency procedures. (4) In addition, cities will have to evaluate the wisdom of constructing new buildings in areas that are sure to become vulnerable to flooding.

Which sentence should be revised to correct a run-on?
E. sentence 1
F. sentence 2
G. sentence 3
H. sentence 4

\section*{Here's How to Crack It}

A run-on is a sentence in which the writer has not properly joined two ideas. Sentence 1 contains an incomplete idea followed by a complete idea; GO punctuation works. Eliminate (E). Sentence 2 is one complete idea. Eliminate (F). Sentence 3 has two complete ideas linked by a comma. Red flag. Can you ever link two complete ideas with just a comma? No. Choose (G).

\section*{Misplaced Modifiers}

A misplaced or "dangling" modifier is a phrase that is separated from the subject it is modifying. A dangling modifier should be avoided, as it can yield a meaning contrary to what the writer intends-or even a hilarious meaning.

Take a look at this example.
INCORRECT: Old and a bit smelly, I decided to throw away the pasta.
The misplaced modifier is underlined. Who is old and a bit smelly? The speaker, or the pasta? Because the subject of the modifier does not immediately follow the underlined phrase, it is actually implied that the speaker ("I") is old and a bit smelly. Here's one way to rewrite it:
CORRECT: I decided to throw away the pasta because it was old and a bit smelly.

Now, there is no confusion about what is old and a bit smelly.

Read this paragraph.
(1) Sitting at home and staring at the television, an overwhelming number of options are available to watch.
(2) Yet a growing mass of Americans are turning to online video streaming services, in a trend nicknamed "cutting the cord." (3) In fact, according to a recent study, 73\% of Americans age 18-28 said their primary source of television entertainment came from such streaming services. (4) Experts predict that number to increase sharply over the next decade, giving traditional television networks cause for alarm.

Which sentence should be revised to correct a misplaced modifier?
A. sentence 1
B. sentence 2
C. sentence 3
D. sentence 4

\section*{Here's How to Crack It}

Misplaced or dangling modifiers typically (but not always) occur at the beginnings of sentences, so start there. Read the introductory phrase in sentence 1 and ask yourself if the subject being modified is 1 ) named at all or 2) named but separate from the modifying phrase. In this case: who is sitting at home and staring at the television? The writer never says, so sentence 1 needs to be rewritten so that the subject of the modifier is clear to the reader. Choice (A) is correct.

\section*{PART A COMBINING QUESTIONS}

In Part A, you may be asked to combine two or even three sentences into one, using one of the options in the answer choices. The key with these types of questions is to understand the relationship between the sentences and pick a choice that makes sense with that relationship.

When combining sentences, make sure that you do not accidentally change the meaning.

Once you have eliminated answer choices that do not reflect the relationship between the sentences, choose the most concise answer choice-that is, the one that is shortest and most to the point. The idea behind combining sentences is to eliminate extra words.

Read these sentences.
(1) The unification of New York City's private and public subway lines began in 1940.
(2) This unification allowed officials to more efficiently manage all of the city's train tracks.

What is the best way to combine the two sentences in order to clarify the relationship between the ideas?
A. The unification of New York City's private and public subway lines allowed officials to more efficiently manage all of the city's train tracks, and the unification began occurring in 1940.
B. Despite the unification of New York City's private and public subway lines in 1940, officials were able to more efficiently manage all of the city's train tracks.
C. The unification of New York City's private and public subway lines, which allowed officials to more efficiently manage all of the city's train tracks, began in 1940.
D. In 1940 the unification of New York City's private and public subway lines began, and this unification allowed officials to more efficiently manage all of the city's train
tracks.

\section*{Here's How to Crack It}

Remember the basic approach: do not read the sentence(s) just because the test tells you to. First, read the question to understand what's being asked of you. Figure out the relationship between the two ideas, and then jump into POE. The sentences say that the unification began in 1940 and it allowed officials to efficiently run the subway. Choice (B) can be eliminated immediately because the word "despite" contradicts the relationship between the two ideas; "because" would be a more appropriate word. Choices (A) and (D) do not contradict the meaning of the sentences, but notice how wordy they are. Both of these choices use the word "unification" twice. They have reordered a lot of words, but they have not combined the two ideas concisely, so eliminate them. This leaves (C), which is the correct and most concise answer.

\section*{ELA REVISING/EDITING PART A DRILL}
1. Read this sentence.

As a matter of fact dozens of localities around the country are debating changing their time zones, a move that makes many residents suspicious; few people like interruptions to their routine, daily lives.

Which edit should be made to correct this sentence?
A. delete the comma after routine
B. insert a comma after fact
C. delete the comma after zones
D. insert a comma after localities
2. Read this paragraph.
(1) Laura has a very particular procedure when she paints. (2) First, she puts on her painting smock. (3) Next, she filled up a large glass of water for the brushes. (4) Lastly, she brings out the paints themselves, and gets to work on her latest masterpiece.

Which sentence should be revised to correct an inappropriate shift in verb tense?
E. sentence 1
F. sentence 2
G. sentence 3
H. sentence 4
3. Read this sentence.

Each member of the city council was asked whether they agreed with the sentiment behind the not-for-profit organization's petition.

Which edit should be made to correct this sentence?
A. change they to he or she
B. change each to all
C. change member to members
D. change they to it
4. Read these sentences.
(1) The age of the average New York City theatergoer is 44 years old.
(2) Theater companies have tried to attract younger audiences with discounted tickets, free merchandise, and nontraditional productions.

What is the best way to combine these sentences in order to clarify the relationship between the two ideas?
E. The age of the average New York City theatergoer is 44 years old, and theater companies have tried to attract younger audiences with discounted tickets, free merchandise, and nontraditional productions.
F. Because theater companies have tried to attract younger audiences with discounted tickets, free merchandise, and nontraditional productions, the age of the average New York City theatergoer is 44 years old.
G. Theater companies have tried to attract younger audiences with discounted tickets, free merchandise, and nontraditional productions, so the age of the average New York City theatergoer is 44 years old.
H. Because the age of the average New York City theatergoer is 44 years old, theater companies have tried to attract younger audiences with discounted tickets, free merchandise, and nontraditional productions.

\section*{PART A DRILL EXPLANATIONS}
1. B Use POE. Choice (A) would have us remove the comma after routine, but that breaks the rule that you need a comma between two adjectives that precede the same noun. Eliminate (A). There is no reason to add a comma after localities, so eliminate (D). Finally, deleting the comma after zones would create a run-on sentence. Thus, (B) is correct, and it corresponds with the rule: add a comma after a transition word or phrase that begins a sentence.
2. G Look to keep the tense consistent throughout this passage. Sentences 1 and 2 are in the simple present tense, but notice sentence 3 uses the past tense verb "filled." Meanwhile, sentence 4 continues in the present tense. Sentence 3 is the odd one out. In describing an every day task like Laura's, there is no need to slip into the past tense for just one sentence. Choose (G).
3. A Whenever you see pronouns, find the original word they replace. In this instance, the pronoun they refers to "each member." They is a plural pronoun, but be careful: "each member" is a singular noun, referring to one member at a time. Thus, you need a singular pronoun to replace "each member." Cross out (D) because it refers to inanimate objects. The only singular option left is (A), he or she.
4. \(\mathbf{H}\) In a combining sentences question, be sure not to alter the meaning of the ideas. First, identify the relationship: as a result of the average age of theatergoers being high, theater companies have tried to attract younger audiences. Notice that (F) and (G) have reversed the order of the ideas (which is fine), and have also changed the meaning. Choice ( F ) implies that the average age is high because theatre companies are trying to attract younger audiences, which is the opposite of what the passage says, so eliminate (F). Similarly, cross out (G) for this reason: it inverts the causality of the events. Down to two! Choices (E) and (H) are close, but (E) uses "and," which, while grammatically acceptable, doesn't do as good a job as (H) at explaining why theater companies started behaving in this new way. "And" suggests that
these two separate events occurred, but "because" clearly explains the relationship between the two ideas: because the average age was high, theaters tried to attract younger audiences. This makes (H) the correct answer.

\section*{Chapter 8} SHSAT Revising/Editing Part B

\section*{REVISING/EDITING PART B}

Part B of the ELA section of the SHSAT revolves around short, passage-based questions. Some of the conventions of passage reading, which we'll cover in the next chapter, can be applied here, but the key takeaway here is that each question refers to a numbered sentence. If you're pressed for time, consider temporarily skipping those questions that refer to broad ideas about the passage's argument or topic-the main idea. Instead, focus on the questions about precision and organization, as these can often be answered by reading around the indicated sentence.

\section*{SAMPLE PASSAGE}

Your goal in this section is to make sure that a given passage adheres to the conventions of standard written English. While that may seem overwhelming, don't worry: you do not have to memorize every rule of standard written English! All you need to know is how the SHSAT likes its passages: precise, concise, and well-organized.

Read through this passage and see if you can identify poorly structured sections. Then compare what you found to the following sections, which show you how the SHSAT might test your ability to revise them.

\section*{Finding the Green in an Urban Jungle}
(1) New York City's population is growing faster than ever. (2) Its residents need greater access to public parks and green spaces. (3) Social studies have determined that when people, particularly urban dwellers, have access to green spaces, they are healthier, happier, and more productive at work. (4) We, the citizens of New York, have a long way to go before we are there.
(5) For much of the 20th century, green spaces were not a major priority for the city's planners. (6) The latter half of that century prized the automobile above all else. (7) Rather than valuing where people could walk among plant life, leaders instead valued more
paved roads for cars to rumble down. (8) A leading early advocate of improving the power of local neighborhoods against large, dominating real estate developers was Jane Jacobs.
(9) A good measure of adequate green space is calculating how many residents live within a 10 -minute walk of a park. (10) Social scientists reported that in the 1980s, far fewer New Yorkers lived within the 10 -minute zone than they do today.
(11) The new century brought more attention to the lack of green space in the city, and saw new, imaginative efforts at remedying a longstanding problem. (12) Perhaps the most visible sign of progress was the 2009 opening of the High Line in Manhattan's Chelsea neighborhood. (13) Developers took advantage of the nowdefunct New York Central Railroad's abandoned elevated railway tracks and transformed the unused lot into a welcoming walkway abutted by trees and dozens of species of wild plants. (14) The High Line has quickly become one of New York City's most popular tourist attractions. (15) The Parks Department says that millions of people have visited the High Line during the year. (16) Following the High Line's success, the city has planned to unveil more green spaces in the near future.
(17) Critics have pointed out that the High Line's sudden popularity has sharply raised property values in the adjacent neighborhood, leading to some longtime residents being unable to afford their homes. (18) On the other hand, many residents complain of seeing their once-quiet blocks now inundated with herds of tourists, snarling pedestrian and vehicular traffic. (19) In the end, everyone acknowledges that the benefits of building more green space will far outshine the costs for most New Yorkers.

\section*{Precision}

The SHSAT favors highly accurate and specific writing. Remember: no matter how pleasant-sounding or poetic a sentence is, it cannot contain vague or unclear statements.

When asked about precision, choose the answer that has the most specific language and details, including numbers.
1. Which revision of sentence 10 uses the most precise language?
A. Social scientists reported that in 1986 far fewer New Yorkers lived within the 10-minute zone than they did in 2016.
B. Social scientists reported that between 1986 and 2016 the number of New Yorkers who live within a 10 -minute walk of a park increased by 40 percent.
C. Social scientists reported that between 1986 and 2016 the number of New Yorkers who live within a 10 -minute walk of a park increased a great deal.
D. Social scientists reported that over a thirty-year stretch, the number of New Yorkers who live within a 10-minute walk of a park increased by 40 percent.

\section*{Here's How to Crack It}

When searching for the most precise answer, look for exact words and numbers. Choice (A) improves the original sentence by adding specific years, but it still contains the vague phrase "far fewer." Similarly, (C) includes the span of years, but it is imprecise in referencing the number of New Yorkers who lived in that area, saying only that it increased "a great deal." Choice (D) includes the detail of a 40 percent increase, but it neglects to mention any specific years (a thirtyyear stretch could be any stretch of years). Only (B) mentions both the exact years and the exact percent increase, so it is the most precise choice.

\section*{Concision}

In addition to being precise in writing, the SHSAT also values being concise. This type of question may appear in both Part A and Part B.

Concise writing expresses ideas in the fewest words possible, free of any unnecessary phrases or sentences.

Unlike precision questions, concision questions will not have the category name in the question. Instead, you may be asked to combine two sentences into one smaller sentence.

Tip: When combining sentences, make sure that you do not accidentally change the meaning. Concision cannot come at the sake of clarity.
2. What is the best way to combine sentences 1 and 2 to clarify the relationship between the ideas?
E. New York City's population is growing faster than ever; meanwhile, its residents need greater access to public parks and green spaces.
F. Residents need greater access to public parks and green spaces, so New York City's population is growing faster than ever.
G. New York City's population is growing faster than ever, since its residents need greater access to public parks and green spaces.
H. Because New York City's population is growing faster than ever, its residents need greater access to public parks and green spaces.

\section*{Here's How to Crack It}

The question asks you to clarify the relationship between the two ideas, so first identify that relationship. From reading the passage, it is clear that the fastgrowing population of New York City creates a need for more green space to be built. Use POE. Choice (E) uses "meanwhile," which links the two ideas but does not highlight the causal relationship. "Meanwhile" makes it seem as if the two things are happening at the same time, not necessarily that they are related. Cross out ( F ) because it has reversed the causal relationship: the population of the city is not dependent on people needing green space. Eliminate (G) for the same reason: "since" implies that the need for green space results in population growth. Only (H) crystallizes the relationship: because the city's population is
growing, its residents need greater access to public parks and green spaces. Choice (H) is correct.

Additionally, you may be asked which phrase or sentence can be deleted because it is not relevant to the paragraph. This, of course, requires knowing the main idea of the paragraph, so make sure you read the entire paragraph-or, if you're running out of time, at least the first and last sentences-to identify the author's main point.

When determining if a sentence is relevant to the passage, look for a clear match: if the sentence does not match the main idea of the passage, delete it!
3. Which sentence is irrelevant to the ideas in the second paragraph (sentences 5-8) and should be deleted?
A. sentence 5
B. sentence 6
C. sentence 7
D. sentence 8

\section*{Here's How to Crack It}

First, skim the second paragraph to remind yourself of the main idea. This paragraph focuses on historical context: in the second half of the 20th century, city planners prioritized automobiles, not people. Once you identify this information, you can more easily find the sentence that doesn't match it. Sentences 5-7 relate to the central theme, while sentence 8 brings up as evidence an activist who fought against real estate developers. Real estate development is not the main idea of the paragraph, so sentence 8 is the irrelevant sentence, and it can be deleted. This means that (D) is the correct answer.

\section*{ORGANIZATION}

Many of the questions in Revising/Editing Part B deal with organizing the sentences and ideas in a passage. You may be asked to adjust a passage's introduction, transitions, or conclusions; add a sentence to better develop the passage's ideas; or find a more logical place in the passage for a particular sentence.

When tackling these questions, remember that you are not being asked anything subjective. The test is not asking you which transition, phrase, or sentence you prefer. A good sign that you are falling for a trap is when you think to yourself, "This answer choice sounds nice." Answers that sound nice cannot be selected on that basis alone. Correct choices will fit the requirements of the questioneven if they would not be your first choice as a writer.

On organization questions, ask yourself:
1. What, specifically, is the question asking?
2. Which choice best answers that question?

\section*{Zoom In Questions}

Some organization questions are concerned with just one transition word or phrase. A good strategy here is to find a reading window. Analyze the given sentence and the previous sentence. Notice if the ideas expressed in these two sentences are in agreement with each other, or if the second sentence departs from the main idea. Most transition words can be divided into two camps: agreement or departure.

\section*{Agreement Transition Words}

And
Also
In addition

\section*{Departure Transition Words}

But
However
On the other hand

Indeed
First, second, third...
Moreover
Likewise
Similarly
Furthermore

In contrast
Although
Even though
In spite of
Instead
Conversely
Rather
Nevertheless
Regardless

Window Shopping
Finding a reading window is a useful strategy for any passage-based question that points to a specific section. The sentence numbers for

Editing/Revising questions and the line numbers for Reading questions are there for a reason: use them to focus in and around that content so that you avoid other distractions.
4. Which transition word or phrase should be added to the beginning of sentence 11?
E. And
F. Also
G. For instance
H. However

\section*{Here's How to Crack It}

Follow the steps!
1. What is the question asking? The question asks for a transition to be added at the beginning of sentence 11 , so you know the SHSAT is testing the use of transitions. Read both sentence 10 and sentence 11 to understand that
sentence 11 departs from the ideas expressed in sentence 10: in "the new century," there is more of a concern about green spaces than there was in the 1980s (sentence 10).
2. Which choice best answers that question? "And," "also," and "for instance" are agreement words, so eliminate them. Choice (H) is the only departure transition word, so select it.

\section*{Linking Questions}

In other instances, the SHSAT will ask you to identify the best sentence to link two paragraphs. In other words, your job is to find the best topic sentence for the paragraph. This time, your window is the entire paragraph in question; you should skim it to make sure you know what the main idea is. Choose the answer that most clearly expresses that main idea.

The Power of Your Words
It's often better to put a main idea in your own words before you read the choices. After all, answer choices can try to trick you, but you (hopefully!) aren't trying to trick yourself.
5. Which sentence can best follow sentence 16 to help develop the ideas of the fifth paragraph (sentences 17-19) ?
A. Green spaces will invigorate the city with much needed public space.
B. Like any significant change in a city's landscape, the High Line was not immune to criticism from community leaders.
C. While green spaces are planned for the future, it is unknown whether people will enjoy them.
D. The High Line solved many of the problems that the city had sought to
address when it first began the planning stage of the project.

\section*{Here's How to Crack It}

You are asked to find the best sentence that develops the ideas of the fifth paragraph, so skim that paragraph to remind yourself what it is about: various criticisms leveled at the High Line project. Choice (A) doesn't mention any criticisms, so delete it. While (C) entertains the idea that people may not enjoy green spaces in the future, it does not address the main idea of the High Line specifically, so eliminate (C). Cross out (D) because it says the opposite of the main idea of the fifth paragraph; this paragraph talks about concerns with the High Line, not that the High Line solved the city's problems. Choose (B).

\section*{Style Questions}

Finally, you may be asked a question about maintaining "formal" style within the passage. Don't be tricked by these questions: they are not as subjective as you think. The test would never ask you to choose between two well-written, appropriately formal sentences; the correct choice will be in stark contrast to the other, highly informal phrases.

In particular, when maintaining a formal style, try to avoid the more informal first or second person ("I/We" and "you"), as well as lazy turns-ofphrase ("things," "stuff," etc.).
6. Which sentence should replace sentence 4 to maintain a formal style?
E. New York City faces many obstacles in its goal to have more robust access to green spaces.
F. All citizens of New York: you have a long way to go before you achieve more access to green spaces.
G. There are many things hindering New York City’s goal of creating more
access to green spaces.
H. We, the citizens of New York, have a long way to go before we get more robust access to green spaces.

\section*{Here's How to Crack It}

Remember to look out for the first or second person; their usage is considered informal. Choice (F) employs the second person, so eliminate it. Likewise, (H) uses the first person plural, "we." Cross it out. Notice that (G) contains one of those common lazy turns-of-phrase, "things." What things? Choice (E) does the best job, replacing "things" with "obstacles" and maintaining formal tone throughout.

\section*{ELA REVISING/EDITING PART B DRILL}

\section*{From Barber to Surgeon}
(1) Barbers and surgeons have a lot of things in common. (2) Although some members of the medical profession would like to ignore the connection, it is too true to be omitted from the pages of history. (3) Being a barber is an old profession. (4) In the distant past, the art of surgery and the trade of barber were combined. (5) In all parts of the ancient civilized world, the barber acted as a kind of surgeon. (6) He practiced phlebotomy, the art of blood-letting, and he dressed wounds. (7) As in modern society, the barber shops in 420 B.C.E. Greece were known as places to share gossip.
(8) Until the twelfth century, religious people took care of men's bodies as well as their souls. (9) Monks practiced surgery and medicine. (10) Barbers learned a lot from the monks, whom they assisted in surgical operations. (11) The practice of surgery involved the shedding of blood. (12) Monks felt that this practice did not suit the functions of the clergy. (13) After much discussion, in 1163, the clergy were forbidden to act as surgeons, but they were allowed to provide medicine. (14) That same year, the Pope issued an important decree about religious pilgrimages.
(15) The new law satisfied the barbers, and they quickly took advantage of the opportunities now available to them. (16) In London, the barbers advertised their blood-letting by putting blood in their windows to attract the attention of the public. (17) This practice upset people, so a new law was passed eventually, telling the barbers in London to pour the blood somewhere else.
(18) Eventually, the barbers in London grouped together to form a guild. (19) At first, its purpose was for social interaction only. (20) Later, it became like more of a trade union. (21) As time progressed the London Company of Barbers increased in importance. (22) A
change of title occurred in 1540, and the barbers were then called the Company of Barber-Surgeons. (23) Although the surgeons and the barbers were separated by Act of Parliament in 1745, the barber-surgeons continued to practice for a long time.
1. Which sentence should replace sentence 1 to best introduce the main claim of the passage?
A. Today's surgeons can trace their roots to ancient Greece.
B. The surgeon of modern times evolved from the ancient but humble position of the barber.
C. Barbers and surgeons have a lot of things they disagree on.
D. The history of barbers is a complex and fascinating one.
2. Which sentence is irrelevant to the argument presented in the passage and should be deleted?
E. sentence 3
F. sentence 14
G. sentence 18
H. sentence 22
3. What is the best way to combine sentences 11 and 12 to clarify the relationship between ideas?
A. The practice of surgery involved the shedding of blood, since monks felt that this practice did not suit the functions of the clergy.
B. The practice of surgery involved the shedding of blood and, lastly, monks felt that this practice did not suit the functions of the clergy.
C. Because the practice of surgery involved the shedding of blood, monks felt that this practice did not suit the functions of the clergy.
D. Monks felt that the practice of surgery did not suit the functions of the clergy, although it involved the shedding of blood.
4. Which revision of sentence 17 uses the most precise language?
E. This practice upset people, so a new law was passed in 1307 telling the barbers in London to pour the blood somewhere else.
F. This practice upset a great many people, so a new law was passed eventually, telling the barbers in London to pour the blood elsewhere
G. This practice upset people, so a new law was passed a number of years later, telling the barbers in London to pour the blood into the river.
H. This practice upset people, so a new law was passed in 1307 telling the barbers in London to pour the blood into the river.
5. Which sentence could be added to follow and support sentence 3?
A. Another very old and significant profession is the court jester.
B. The earliest references to surgery can be found in the literature of the Renaissance.
C. There are references to it in the Book of the Prophet Ezekiel that describe shaving the head as a sign of mourning.
D. Shaving and hair-cutting has been a documented human behavior for thousands of years, but not as prevalent as other habits such as bathing.

\section*{PART B DRILL EXPLANATIONS}
1. B After reading the whole passage, reflect on the main idea: the position of the modern surgeon emerged from the barber. You might even consider the title, "From Barber to Surgeon," which implies a historical progression. Choice (A) may be true, but is ancient Greece the main focus of the passage? No, so eliminate (A). Similarly, the passage is not concerned with the disagreements between the two professions, so eliminate (C). Finally, (D) deals only with barbers, whereas the passage describes barbers and surgeons. Only (B) matches the main idea of the passage.
2. F This passage concerns the historical progression from barbers to modern-day surgeons, so read each sentence and evaluate whether it is relevant or irrelevant to the passage. Choice (G) mentions the barbers forming a guild, which supports the main idea; cross off (G). Likewise, (H) can be eliminated because it mentions a trade union called the Company of Barber-Surgeons, a direct reference to the passage's main idea. Choice (E) claims that barbers enjoy a long history, which matches the main idea. Therefore, (F) is irrelevant, because it raises the topic of religious pilgrimages, which is not the focus of this passage.
3. C This is a concision question; it asks you to combine two sentences into one. Read carefully to understand the relationship between the two sentences: Monks felt that the practice didn't suit the clergy because surgery involved the shedding of blood. Choice (A) implies that because monks felt the practice unsuitable, the practice involved the shedding of blood. This makes little sense, and transforms the meaning, so eliminate (A). Similarly, the use of "lastly" in (B) implies that there were previous examples given where there are not, so eliminate (B). Finally, (D) reverses the relationship between the clauses with the word "although"; they felt the practice did not suit the function of the clergy not "although" it involved the shedding of blood, but "because." Thus, (C) will work.
4. H On precision questions, remember to look for answer choices that employ the most specific details and (if applicable) numbers. Notice (E) and (H) mention a precise date that the law was passed, whereas ( F ) and ( G ) continue to use vague language ("eventually" or "a number of years later"). Eliminate (F) and (G) for this reason. Down to two! Choice (E) still uses the imprecise "somewhere else," whereas (H) says exactly where barbers should pour the blood: "into the river." Choice (H) is the most precise.
5. C Sentence 3 says "Being a barber is an old profession," which means that you need a choice that would support this claim, or give some evidence. Notice that while (A) and (B) may be true, they do not support the claim that barbers are an old profession. Cross out (A) and (B). Down to two! While (D) is initially about hair-cutting, the second clause contradicts the first clause-it says bathing is an even older habit-thus making (D) a poor choice to support the previous sentence. Choice (C) is the best because it gives specific evidence to support the claim in sentence 3 , while maintaining a consistent style with the rest of the passage.

\section*{Chapter 9 \\ Reading Comprehension}

\section*{WHAT'S READING ALL ABOUT ON THE SHSAT?}

You know how to read for pleasure, sprawled out on a couch, taking your time, and you know how to read for class, focused on highlighting the key phrases that sum up the overall ideas of each paragraph. The SHSAT's Reading Comprehension section requires you to read in a different manner, even for its poetry and prose passages. There's a lot of information to process in these long and often dense passages. If you try to memorize everything, you may forget more than you remember.

This next part may sound a bit crazy, but don't try to learn or remember everything. You don't get any points for reading the passage well, so focus entirely on a different goal: answering questions correctly. Skim the text, not the questions, and use smart test-taking strategies to maximize what you get out of each passage.

\section*{PASSAGE TYPES}

The test-taking strategies for reading also mean choosing wisely the order in which to tackle passages. On the SHSAT, you'll find passages that pull from biography to history and those that discuss scientific discoveries and developments alongside contemporary opinion pieces and human interest stories, as well as the occasional essay on something cultural, like classical musical or a particular style of art. You'll also find at least one literary passage and a poem, both of which can pull from just about any genre, from sci-fi to realism to satire or allegory.

The less fascinated you are by a given subject, the more effort you may have to put in to deciphering what it’s talking about. On the other hand, this also means you'll be less likely to try to answer questions based on pre-existing information -remember that the answers are always directly supported by the text. Because of this, you'll want to refer back to your POOD (Personal Order of Difficulty) and figure out which passages you'd rather read early on in the test and which you'd rather tackle later on. You don't have to do them all in a row!

Fact and Fiction
As of the 2018 revision, you will encounter literary texts on the SHSAT: at least one prose passage and one poem. This book has a variety of content that's at least as complex as what you'll find on the test, but you should still
pick up some outside sources-anthologies, newspapers, etc.-so that you get more comfortable with the differences between poems, stories, and informational texts.

\section*{HOW TO READ A PASSAGE}

You don't have to memorize anything from the passage-you can flip between it and the questions as often as you like. Your first pass should be to quickly identify the author's central ideas; don't get bogged down in the details.

Remember: you get points on the SHSAT only for answering questions, not for reading the passage. As much as possible, skim the passage!

\section*{Skim for the Main Idea}

Skimming does not mean quickly running your eyes up and down a page. It's a form of active reading, in which you are looking for the most important thing in a passage. This is a skill that improves with practice, but it helps to keep asking what the author's point is. These points should connect, so if what you're reading in one paragraph has nothing to do with what you got from the previous one, you might have identified the wrong main idea.

You are allowed to mark up your question booklet, so do so! Don't keep your thoughts to yourself; jot quick reminders to yourself so you can more quickly move between the questions

If you do not have time to skim a full paragraph, read at least the first and last sentences. Many authors place their main ideas in the first sentence (the topic sentence) or the last sentence (conclusion). These are both excellent places to skim for the main idea.

Fiction and Fact
For the most part, the guidelines in this chapter also apply to literary passages. However, because prose and poetry sometimes use metaphors and other elements of subtext, you may want to skim a little less. If you've ever watched The Daily Show or been sarcastic to a friend, you'll appreciate that how a thing is said can change its meaning.

In the following example, an excerpt from Francis M. Walters's Physiology and Hygiene for Secondary Schools, actively engage with the text and search for answers to these questions:
1. What is the topic of the passage?
2. What is the most important thing the author says about the topic?
3. Do the other sentences in the passage either lead up to or support this idea?

Disease, which is some upset of the vital functions, may be due to a variety of causes. Some of these causes, such as hereditary defects, are remote and beyond the control of the individual. Others are the result of negligence in the observance of well-recognized hygienic rules. Others still are of the nature of influences, such as climate, the house in which one lives, or one's method of gaining a livelihood, that produce changes in the body-imperceptible at the time, but, in the long run, laying foundations of disease. And last, and strongest, are the minute living organisms, called germs, that find their way into the body.

The three questions above are all you need to figure out the main idea. The topic of the passage can be found in the very first sentence: disease. The most important thing the author says about the topic also appears in the first sentence: disease may have many different causes. Finally, the other sentences in this paragraph do not present a new main idea or counter-argument; notice that by listing the categories of causes of diseases, these sentences simply provide evidence for the main idea you already found.

\section*{Headline}

The next step is to synthesize the information you found in skimming for the main idea, and use your pencil to jot it down next to the paragraph in your test booklet. Think of this task as writing a headline for the paragraph. In newspapers and magazines, headlines grab the reader's attention and summarize what the article is about. Similarly, your headline should reduce the whole paragraph to a simple statement. Headlines should be 3-10 words in length and should not be formal sentences. Remember: no one else will see these headlines besides you. All that matters is that you know what the headline means!

Headlining ensures that you will not forget what you read later on. A quick look at your headline should tell you everything you need to know about that paragraph.

What might the headline be for the paragraph on diseases? "Disease and its various causes" or "So many causes of disease!" are both good options, since they match exactly the main idea you found from the first sentence of the paragraph. A poor headline would be "Germs are strong" because, while this is a true detail taken from the final sentence, it is not the main idea of this paragraph. The paragraph is about the causes of disease, and germs are just one of those causes.

If you write a headline for every paragraph you read, you will create a strong outline for the entire passage. This will help a

Give headlining a try.

\section*{Headlining Practice}
1. The cinema of attractions is based primarily on showing. It is an exhibitionist style, and this was the original style of film. An early cinema of attractions film is characterized by the actor or actress looking directly at the camera and thereby making direct contact with the audience. For example, one of the Lumiere brothers’ films showed passengers disembarking from a boat trip. As they walked down the gangplank and passed the camera, each passenger looked directly into the lens.

\section*{Headline:}
2. There is one kind of body tissue that has no nerves, no blood vessels, and no lymphatic system (as do many tissues), but performs many crucial functions. That tissue is cartilage, a fundamental biological material that serves two purposes: it helps to shape the body and then helps to support it. In the embryo, cartilage forms a framework on which most of the major bones of the body take form. Then, in the growing body, cartilage forms growth plates at the tips of bones. The bones extend their length from these cartilaginous growth plates. Throughout life, cartilage continues to be important.

\section*{Headline:}
3. When Maria Theresa ascended the throne at the age of twentythree, there was no established Austrian unity. Although the kingdom was mostly Catholic, the only real unifying trend was the allegiance to a common sovereign, and outside the "Core Lands" this sovereign did not exercise effective control. Because of this lack of unity, the only powers that Austria possessed were those of
size, population, and natural resources. In the past these powers might have been sufficient to help Austria fill her role in European affairs, her main duty traditionally being to shift the balance of power away from the Bourbons. But now Austria was being threatened by Frederick of Prussia, and the necessity for strong monarchical control, specifically over a mighty armed force, became obvious.

\section*{Headline:}

\section*{Possible Headlines}

Here are the headlines we came up with; if what you wrote is close to these ideas, then you're in good shape.
1. Possible headline: Cinema of attractions actors look at audience 2. Possible headline: Cartilage is important!
3. Possible headline: Maria Theresa needed to be a strong ruler

\section*{READING COMPREHENSION QUESTIONS}

Use your skimming and headlining skills to go through the following passage and then review the different types of questions that the SHSAT might ask about it. (Actual passages on the SHSAT can be up to twice as long; this is just a sample that is being used to present question types.)

Immediately following the dramatic end of World War II came a realization that the United States now had to turn its attention inward. Years of fighting battles around the globe had drained the country of important resources. Many industries (such as housing) suffered, as both materials and workers were used elsewhere in the war effort. Once the soldiers began returning, it became clear that new jobs and new homes were among their biggest needs. The homes needed to be affordable, since few people had had the time or ability to save much during the war.

It was in this situation that many house developers saw a business opportunity. Amid such a pressing demand for new homes, developer William Levitt realized the need for a new method of building. He sought a way to build homes cheaper and faster than ever before.

He wasn't the only developer to realize this, but he was one of the best in making it happen. He applied the same ideas to homes that Henry Ford had used 50 years earlier in making cars. Levitt did not build a factory with an assembly line of fully formed houses rolling out of some giant machine. Instead, he adapted the assembly line formula into a system in which the workers, rather than the product, moved for a streamlined, efficient building process.

35 Previously, a developer who completed four homes a year had been moving at a good pace. Levitt planned to build that many each week, and succeeded. He created specialized teams that focused
40 on only one job each and moved up and down the streets of new homes. Teams of foundation-builders, carpenters, roofers, and painters worked faster by sticking to just one task as they moved, factorystyle, from house to house. The time and money saved allowed Levitt to build cheap homes of good value.

With this new approach, Levitt oversaw the building of some of the first towns that would eventually be called suburbs-planned communities outside the city. Some critics blame developers like Levitt for turning farmland into monotonous, characterless towns. However, most agree that his contribution to the country following a bitter war was mostly positive. He did vary the style of home from street to street, and his work on simpler home features was influenced by the work of architecture great Frank Lloyd Wright.

In the end, Levitt's success speaks for itself. After his first success-building thousands of homes in Long Island, New York-he went on to found several more "Levittowns" in Pennsylvania, New Jersey, and elsewhere. Levitt gave home buyers what they wanted: nice pieces of land with nice homes on top. In a way, by creating houses that so many families could afford, William Levitt made the American dream a more affordable reality.

\section*{Main Idea}

The first question is usually the broadest, asking for the passage's main idea, or the main purpose. The test writers are expecting a question like this to force you to read the entire passage, but if you've taken good headlining notes, you can often narrow down the choices with POE.
1. Which of the following best tells what this passage is about?
A. the events that occurred during the final days of World War II B. whether suburban housing is unaffordable
C. one person's significant contributions to an industry
D. a complete biography of William Levitt

\section*{Here's How to Crack It}

If you'd only skimmed the first paragraph, (A) would look very appealing. But WWII isn't mentioned anywhere else, so eliminate (A). Choice (D) can also be tempting, since William Levitt's name shows up in almost every paragraph. However, these paragraphs aren't talking about Levitt's history, so much as what Levitt invented, so eliminate (D). Of the remaining choices, (B) is very specific, and none of the paragraphs talk about housing being unaffordable. This leaves (C), which is the correct answer: the passage is all about Levitt's contributions to building homes.

\section*{Specific Questions}

Another type of question will ask how a specific part of a passage, either a set of lines or a full paragraph, helps the author to make his or her point.
2. The author talks about "monotonous, characterless towns" in line 54 in order to E. offer a negative view of assembly-line style houses.
F. suggest a needed return to simplicity after the bitterness of the war.
G. demonstrate the ways in which Levitt improved the design of these towns.
H. offer a definition of the suburbs.

\section*{Here's How to Crack It}

You can zip directly to the fifth paragraph to see what the author's talking about.

The sentence in question refers to "critics" who "blame developers like Levitt," and it seems clear from the contrast to the rest of the paragraph that the author included this to offer a negative view, (E). All of the other choices use words from the paragraph, but they don't fit the author's purpose. There's no call to return to simplicity, so eliminate (F). The "monotonous" quote isn't something that's considered an improvement, so eliminate (G). And while these towns were called suburbs, "characterless towns" are not provided as a definition for the suburbs, so eliminate (H). The correct answer is (E).

\section*{Supporting Evidence}

Some questions ask you to identify the answer choice that best fits the passage. Again, this isn't what best fits your knowledge of the subject, or what your personal opinion might be of the topic. The answer will always be in the text itself.
3. Which of the following statements about William Levitt is best supported by the passage?
A. He invented the word "suburb."
B. He was unconcerned with the style of the homes he built.
C. His ultimate business goal was to make cars as Henry Ford did.
D. His efficient methods helped make homes more affordable.

\section*{Here's How to Crack It}

Use POE: if a choice isn't supported by the passage, you can eliminate it. "Suburb" is mentioned in the fifth paragraph, but not as a word coined by Levitt, so eliminate (A). Levitt "did vary the style of home from street to street" and was "influenced by the work of architecture great Frank Lloyd Wright," neither of which suggests that he was unconcerned with style; eliminate (B). Although Levitt modeled his factory-style building off Ford's process, he wasn’t interested in making cars, so eliminate (C). The correct answer is (D), which is also what was summarized in the last paragraph: "By creating homes that so many families
could afford, William Levitt made the American dream a more affordable reality."

\section*{POETRY AND PROSE PASSAGES}

As of the 2018 revision to the SHSAT, the Reading Comprehension section will include at least one work of fiction and one poem. You'll find sample passages in Practice Tests 1 and 2, and while these were developed from texts found in seventh-grade curriculum, we intentionally left them on the denser side. As a compromise, we have included a few footnoted definitions and introductory blurbs that may not be included on the actual SHSAT.

These passages are designed to err on the side of caution and, if anything, overprepare you for the material you'll face on the day of the test. If continued changes to the test necessitate it, we will post alternative passages online to provide you with as thorough a preparation for the SHSAT as possible.

\section*{Literary Questions}

The English section of the SHSAT has changed twice in quick succession-2017 and again in 2018. If we learn of any new question types after this book goes to print, we will post examples and strategies to your free online student tools, so be sure to check there!

\section*{SHSAT READING COMPREHENSION DRILL}

Read the passage below and answer the questions following it. Base your answers on information contained only in the passage. You may reread a passage if you need to.

Etymology, the study of words and word roots, may sound like the kind of thing done by boring librarians in small, dusty rooms. Yet etymologists actually have a uniquely interesting job. They are in many ways just like archaeologists digging up the physical history of people and events. The special aspect of etymology is that it digs up history, so to speak, through the words and phrases that are left behind.

The English language, in particular, is a great arena in which to explore history through words. As a language, English 15 has an extraordinary number of words. This is in part due to its ability to adapt foreign words so readily. For example, "English" words such as kindergarten (from German), croissant (from French), and cheetah (from Hindi) have become part of the language with little or no change from their original sounds and spellings. So English-language etymologists have a vast world of words to explore.

Another enjoyable element of etymology for most word experts is solving word mysteries. No, etymologists do not go around solving murders, cloaked in intrigue like the great fictional detective Sherlock Holmes. What these word experts solve are mysteries surrounding the origins of some of our most common words.
\({ }_{35}\) One of the biggest questions Englishlanguage experts have pursued is how English came to have the phrase OK. Though it is one of the most commonly used slang expressions, its exact beginning is a puzzle even to this day. Even its spelling is not entirely consistentunless you spell it okay, it's hard to even call it a word.

Etymologists have been able to narrow OK's origin down to a likely, though not certain, source. It became widely used around the time of Martin Van Buren's run for president in 1840. His nickname was Old Kinderhook. What troubles
50 word experts about this explanation is that the phrase appeared in some newspapers before Van Buren became well known. As a result, it's unlikely that Van Buren could be called its primary 55 source. Like bloodhounds following a faint scent, etymologists will doubtless keep searching for the initial source. However, it is clear that OK's popularity and fame have exceeded those of the
\({ }_{60}\) American president to whom it has been most clearly linked.
1. Which of the following best tells what the passage is about?
A. the history of the English language
B. enjoyable aspects of the study of words
C. the origin of the phrase "OK"
D. ways to distinguish English and non-English words
2. The details in the passage suggest that etymologists are best described as E. pursuers of the sources of words.
F. scientists of the five senses.
G. archaeologists of extinct languages.
H. creators of dictionaries.
3. The author includes the details about "OK" in paragraphs 4 and 5 in order
to A. discuss another non-English word.
B. illustrate etymology in action.
C. describe a common American phrase.
D. revisit the legacy of Martin Van Buren
4. Based on the details in the second paragraph, why does the author use the words "kindergarten," "croissant," and "cheetah"?
E. to demonstrate words rarely used in English F. to show areas of dispute among etymologists
G. to present words similarly spelled or spoken in two languages \(\quad \mathbf{H}\). to reveal the mystery of words with unknown origins

\section*{SHSAT READING COMPREHENSION DRILL EXPLANATIONS}
1. B This is a main idea question, so remember to check your headlines in the margins of the text; they will give you a good idea of what the passage is about. After you identify your headlines, jump right to POE. Eliminate (A) because, while the passage mentions history in passing, the overall idea is about the interesting and detective-like work etymologists carry out. You learn that etymologists dig up history, but you don't learn a history of the English language. Choice (C) is about the origin of the word OK; cross out (C) because it is briefly mentioned as evidence, but it is not the main idea of the passage. Finally, (D) can be eliminated because nowhere in the passage does the author raise this topic of distinguishing between English and non-English words. Remember: if it's not in the passage, you should not choose it. Go with (B).
2. E Use your notes for the first two paragraphs to answer this question; in those two paragraphs, the profession of etymology is introduced. You may see that the very first sentence says etymology is "the study of words and word roots," or you may see the end of the third paragraph: what etymologists "solve are mysteries surrounding the origins of some of our most common words." At this point, you know enough to POE. Quickly discard (F) and (H) because they are irrelevant; the five senses and dictionary-making are not mentioned in the passage. Down to two! Careful with (G); etymologists are indeed archeologists of languages, but not extinct languages: the passage mentions English and French, which are very much not-extinct. Thus, (E) is the correct answer.
3. B This is a specific question, giving you an exact place to look for the answer. Recall, from your headlines, that paragraphs 4 and 5 detail the roots of \(O K\) and the controversy surrounding the theory that it dates from Martin Van Buren's presidential campaign. It is also important to understand precisely what this question asks: it is not asking what happens in the passage, but rather why the author included it. POE!

Delete (A) because OK is not a non-English word. You can remove (D) because, while Van Buren is mentioned here, his legacy is not the main focus of this paragraph or the passage as a whole. Finally: (B) and (C) are close, but cross out (C) because it is true that \(O K\) is a common American phrase, but the author is doing more than simply describing it. These paragraphs offer the reader an example of scholars practicing etymology, so choose (B).
4. G Here is another specific question, which helpfully prompts you to the second paragraph. Remember: you're being asked why the author includes these words that English has been able to adapt very easily from their foreign roots. You can eliminate (A) because words like "kindergarten" and "croissant" are not rarely used in English; they are very common. Choice (F) would make sense only if the author introduced these words and then presented a disagreement among etymologists over, say, the word "cheetah." No such disagreement appears, so cross out (F). Finally, (H) can be removed because the author does not use, for instance, "kindergarten" to explain that the word has an unknown origin; the author explains that this word comes directly from German. In this case, there is no mystery to solve for etymologists. So (G) is the correct answer, as it provides the reader clear examples of the English language's ability to assimilate foreign words.

\section*{Part IV \\ SHSAT Math Subject Review}

\section*{Chapter 10 \\ The Number System}

\section*{INTRODUCTION}

Unlike ELA, the Mathematics content of the SHSAT builds on all the content you've learned thus far. Since the test is taken in late October or early November, that means that if you're currently in 8th grade, the SHSAT may cover anything up through the seventh grade. If you're in 9th grade, the SHSAT will have slightly more advanced math, pulling from content through the 8th grade.

The good news is that because the type of material that will be on the Math test is known, you can prepare for it. The bad news is that the SHSAT writers have often gone out of their way to make that material trickier, either by using word problems to present it in unfamiliar ways, or by the fact that you're not provided with formulas or calculators. Fortunately for you, the following chapters will serve to break down all the potential topics and question types, and provide you with strategies you can use when all else fails.


If you just finished reviewing the English Language Arts portion of the book and you have the time to do so, take a break before diving right back into math. These two sections of the test are fairly different, and it may help you to cement what you've learned from ELA before you proceed.

In each of the following lessons, we'll introduce content, walk you through an example or two, and then provide you with a chance to practice on your own. We'll give you a mix of question types and a range of strategies, so don't skip over a section just because you got the first question right. For that matter, because so many topics in mathematics build upon one another, we recommend that you work straight through each lesson, marking anything that's tricky so that you can come back and review it again later.

The Dreaded Common Core
The curriculum currently used for the SHSAT is that of the Common Core. Despite what you may have heard, this refers less to teaching methods and more to a standardized list of
topics. For more, you can refer to their website: http://www.corestandards.org/.

\section*{One Pass, Two Pass}

That said, definitely think about your POOD as you proceed. If you've taken the first Practice Test as a diagnostic and already know which type of questions you're getting wrong, you can jump straight to the material that covers your problem areas. This is, for what it's worth, also how you should take the Math test: skim the questions as you go and start by answering those that you're most confident about.

\section*{Write It Down}

Just because you're scored only on the bubbles you fill in doesn't mean that you can't use your pencil anywhere else. Avoid doing math in your head. Mark up your test booklet. Not only will this help to eliminate careless errors, but it will also give you something to refer to if you have time to check over your work.


Lose Your Calculator
Even now, you might be falling into the old
habit of reaching for a calculator. It's a comfortable tool, but as you work through this book, you're going to want to put it away. You won't be able to use it on the SHSAT, so get used to not using it now.

\section*{FUNDAMENTALS}

Many of the math questions on the SHSAT are word problems. Just as you need to know vocabulary to understand the ELA section of the test, so too do you need to know it for the SHSAT.
\begin{tabular}{|c|c|c|}
\hline Term & Definition & Examples \\
\hline Integer & Any number that does not contain either a fraction or a decimal. & \(14,3,0,-3\) \\
\hline \begin{tabular}{l}
Positive \\
Number
\end{tabular} & Any number greater than zero. & \[
\frac{1}{2}, 1,104
\] \\
\hline \begin{tabular}{l}
Negative \\
Number
\end{tabular} & Any number less than zero. & \[
-\frac{1}{2},-1,-104
\] \\
\hline Even Number & Any number that is evenly divisible by two. Zero is an even number! & \(104,16,2,0,-104\) \\
\hline Odd Number & Any number that is not evenly divisible by two. & \(115,11,1,-1,-11,-115\) \\
\hline Prime Number & Any number that is divisible only by 1 and itself. NOTE: One is not a prime number. & 2, 3, 5, 7, 13, 131 \\
\hline Digit & The numbers from 0 through 9. & 0, 2, 3, 7 \\
\hline Consecutive Number & Any series of numbers listed in the order in which they appear on the number line. & \(3,4,5\) or \(-1,0,1,2\) \\
\hline \begin{tabular}{l}
Distinct \\
Numbers
\end{tabular} & Numbers that are different from one another. & \(2,7,19\) are three distinct numbers. 4 and 4 are not distinct because they are the same number \\
\hline Sum & The result of addition. & The sum of 6 and 2 is 8 because \(6+2=8\). \\
\hline Difference & The result of subtraction. & The difference between 6 and 2 is 4 because
\[
6-2=4
\] \\
\hline Product & The result of multiplication. & The product of 6 and 2 is 12 because \(6 \times 2=12\). \\
\hline Quotient & The result of division. & The quotient when 6 is divided by 2 is 3 , because \(6 \div 2=3\). \\
\hline Square & The result of multiplying a number by itself. & \(6^{2}=6 \times 6=36\) \\
\hline
\end{tabular}

\section*{INTEGERS}

There are plenty of numbers out there-an infinite number, to be exact!-but the SHSAT usually asks for specific types of numbers in the instructions. Integers, whole numbers, real numbers, negative numbers...what's the difference? While there can be some overlap, let's categorize different types of numbers.

\section*{Number Line}

A number line is simply a representation of all numbers, usually marked in equal measurements for scale. These number lines are usually marked by integers, numbers that do not contain decimals or fractions. Numbers such as 5, \(-8,102\) and 0 are examples of integers. They can be positive (greater than zero), negative (less than zero), and zero-they just can't be fractions or decimals. Here is an example of a number line showing integer distances.


The integers from -5 to 5 , inclusive, are shown in the number line above. (Inclusive simply means to include -5 and 5.) The arrows in either direction show that this pattern extends to infinity in either direction. Notice that the circles are colored in on this diagram to show that -5 and 5 are included in the range. However, because the line terminates at -5 and 5 , the range includes only the set of numbers between (and including) them. If the range does not include the outer numbers, the circles will be open at the ends. The number line could start anywhere, positive or negative, to show markers of integers or any other pattern, as well as inequalities. Here is a number line that contains only positive numbers, which counts up by 5 s.


The open circle on this number line means to start at 80 but not to include 80 itself. The bold line pointing in the positive direction represents anything greater than 80 , or \(x>80\).

\section*{Classification of Numbers}

Number lines can count up by a variety of patterns. The SHSAT uses a few other distinctions to classify numbers. Whole numbers are a subset of integers: nonnegative integers. So, is zero a whole number? If you answered yes, you are correct! Zero is not negative, so it qualifies as a whole number, along with all the positive integers.

Numbers can also be classified as positive and negative as we discussed earlier, or as even and odd numbers. Numbers that can be divided by 2 without a remainder are considered even, and those that have a remainder of 1 are considered odd. For example, 4, 8, and 46 are considered even, and 9, 13, and 25 are considered odd.

One last classification you should be ready for on the SHSAT is that of prime numbers. Prime numbers can be divided only by themselves and 1. Important! Since 1 is part of the definition of a prime number, it itself is not considered prime. Therefore, the least prime number is 2 . Also, 2 is the only even prime number, since all other even numbers can be divided by 2 as well as themselves.

\section*{One Is Not Prime!}

Since 1 is part of the definition of a prime number, it itself is not considered prime. Therefore, the least prime number is 2 .

\section*{Prime Numbers}

One application of prime numbers is prime factors. The prime factors of a number can be seen as the building blocks of that number. To find prime factors, use a factor tree. Break the number into two of its factors, and break each of those factors into two of its factors until only prime numbers remain. Look at the prime factors of 24 .


There are three 2's and one 3, so the prime factorization of 24 is \(2 \times 2 \times 2 \times 3\) or \(2^{3} \times 3\). The results will be the same no matter how the factor tree is drawn.


In this case, the result is still \(2^{3} \times 3\). This is because the number is always the product of its prime factors: \(24=2^{3} \times 3\).

What is the difference between the greatest prime factor of 100 and the greatest prime factor of 77 ?
A. 2
B. 3
C. 5
D. 6

\section*{Here's How to Crack It}

The question asks for the difference between the greatest prime factor of 100 and the greatest prime factor of 77, so find the prime factors of each. Start with two factors of 100. Draw a factor tree. Break 100 down into 10 and 10. Then break each 10 down to 2 and 5 .


Therefore, \(100=2^{2} \times 5^{2}\), so the greatest prime factor of 100 is 5 . Now, do the same for 77 . The only factors of 77 (other than 1 and itself) are 11 and 7 .


Therefore, \(77=7 \times 11\), so the greatest prime factor of 77 is 11 . The question asks for the difference, so the answer is \(11-5=6\), which is (D).

\section*{Properties of Zero and One}

While there are infinite integers out there, zero and one are special. Here are some properties of zero and one:
- Anything multiplied by one is itself. i.e., \(7 \cdot 1=7\)
- Anything divided by one is itself. i.e., \(7 \div 1=7\)
- Anything raised to the first power is itself. i.e., \(7^{1}=7\)
- Anything plus zero is itself. i.e., \(7+0=7\)
- Anything minus zero is itself. i.e., \(7-0=7\)
- Anything multiplied by zero is zero. i.e., \(7 \cdot 0=0\)
- Anything divided by zero is undefined. i.e., \(7 \div 0=\) ERROR
- Anything raised to the zero power is one. i.e., \(7^{0}=1\).

When you perform an operation on a number that results in that same number, this is sometimes called an "identity" property. This
occurs for addition and subtraction when 0 is
used, and for multiplication and division when 1
is used.

\section*{Practice Problems}
1. How many prime numbers are less than 20 ?
A. 7
B. 8
C. 9
D. 10
2. How many positive odd factors of 72 are between 13 and 36 ?
E. 0
F. 1
G. 2
H. 3
3. One set of numbers contains consecutive integers from 4 to 16, inclusive. Another set contains even integers between 0 and 20. How many integers appear in both sets?


\section*{Explanations}
1. B Start by listing the prime numbers less than \(20: 2,3,5,7,11,13,17\), and 19. Did you include 1 ? Remember, 1 is not prime because it is part of the definition of a prime number. The correct answer is (B).
2. \(\mathbf{E} \quad 72\) has a couple factors that fit into the range, but read the question carefully. The question asks for how many odd factors are between 13 and 36 . The only factors that lie in that range are 18 and 24, but they are even. (While 36 is an even factor, it is not included because the question specifies "between.") Therefore, the correct answer is (E).
3. 7 First list both sets of numbers. The first set contains \(4,5,6,7,8,9,10\), \(11,12,13,14,15\), and 16 . The second set contains \(2,4,6,8,10,12\), 14,16 , and 18 . Notice that the first set is inclusive (including 4 and 16), while the other set is not since it says between. The numbers contained by both sets are \(4,6,8,10,12,14\), and 16 , so there are seven integers. The correct answer is 7 .

\section*{BASIC OPERATIONS}

Basic mathematic operations are the cornerstone of solving any word problem, equation, or expression, so knowing how to work with each operation is very
important. Let's take a look at the basic operations, how they work, and how they fit together when combined.

Common Root Commute
Commutative contains the same root as commute. When you commute to and from school, you are traveling the same distance, regardless of which way you are headed.

\section*{Addition, Subtraction, Multiplication, and Division}

The words more, less, sum, and difference can be used to describe addition and subtraction on the SHSAT. Think of addition as combining two values to create a total, whereas subtraction is the result of taking a value away from another to find what is left. Addition and multiplication can be done in any order and are thus commutative. This means that \(4+3\) is the same as \(3+4\), and \(7 \times 3\) is the same as \(3 \times 7\). When there are multiple items to add or subtract, work left to right in an equation. When you see words like more than, and, or sum, think of adding the numbers together. The same is true with words such as less or the difference between two numbers: think subtraction. The words product, times, quotient, factor, and divisor crop up quite often in word problems on SHSAT, and all refer to multiplication or division.


You might want to add these, and any other potentially confusing words, to the list of fundamental vocabulary at the start of Chapter

Here are some of the terms you might find on the test that refer to multiplication or division:
\begin{tabular}{|l|l|}
\hline Term & Definition \\
\hline &
\end{tabular}
\begin{tabular}{|l|l|} 
Factor & A number that is multiplied by another to form a product. \\
\hline Product & \begin{tabular}{l} 
The result of multiplying two or more numbers (factors) \\
together.
\end{tabular} \\
\hline Multiple & \begin{tabular}{l} 
If a number can be evenly divided by another, that number is \\
a multiple of the divisor. Likewise, if you multiply one number \\
by any other, both numbers are factors of that product.
\end{tabular} \\
\hline Divisor & \begin{tabular}{l} 
The number that is divided into another number to create a \\
quotient. 6 is the divisor in the equation \(24 \div 6=4\), where 4 is \\
the quotient.
\end{tabular} \\
\hline Quotient & The result of dividing one number by another (the divisor). \\
\hline
\end{tabular}

Factor This Into Your Studying
Is 6 a factor or a multiple of 6 ? Trick question: It's both! Factors 6 and 1 create a product of 6, and \(6 \cdot 1\) is the first multiple of 6 .

\section*{Long Division}

Division on the SHSAT will often call for more complex applications than those that you can easily memorize off the top of your head. In these cases, you can use long division to find your quotient. Go column by column to divide the divisor into the larger number using the following steps.

Try dividing 74 by 3 . Place 74 under the division sign and 3 to the left of the division sign.
\[
3 \longdiv { 7 4 }
\]
1. Divide. First, divide 7 by 3 . Since 3 goes into 7 two times, write a 2 above the division sign, directly above the 7 .
2. Multiply. Next, multiply 2 by 3, the divisor, and place the product, 6 , directly under the 7 . Be sure to keep your columns neat!
\[
\begin{gathered}
2 \\
3 \longdiv { 7 4 } \\
-6
\end{gathered}
\]
3. Subtract. Subtract \(7-6\) to find a difference of 1 .

4. Repeat. Carry the 4 from 74 down to the 1 and repeat the steps. Divide 3 into 14 as in step 1 , multiply the result by 3 , and subtract the product from 14 to find the difference.


Since there is a result other than 0 at the end, this shows that 74 is not divisible by 3 , and thus has a remainder, an amount that is left over. This is very important on many SHSAT questions, which will often ask about the remainder, either directly or through a word problem. To find the remainder, you have to do the long division! Now, look at a question that might be found on the SHSAT:

Stephanie is making Halloween care packages for each of her 8 friends. She buys a package of 100 fun-sized candies and wants to give each friend an equal amount of candies. What is the least amount of candies that Stephanie could have left
over?
A. 1
B. 2
C. 3
D. 4

\section*{Here's How to Crack It}

The leftover candy is a real-life scenario of a remainder. Use long division to find how many times Stephanie can divide 100 candies by 8 friends and find the remainder to be 4. (You can also use your times tables to recognize that \(8 \cdot 12\) is 96 , and that \(100-96\) is 4 .) Because there is no way for you equally divide those remaining 4 pieces, the answer is (D).

\section*{Exponents}

Exponents appear as little superscripts to the right of the number or parenthetical expression they pertain to. The superscript number stands for the number of times the term is multiplied by itself. Therefore, \(6^{2}\) means to multiply 6 by itself twice: \(6 \cdot 6=36\). Similarly, \(6^{5}\) would stand for \(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6\). A few rules to remember regarding exponents:
- A negative exponent is equivalent to the reciprocal: \(6^{-2}=\frac{1}{6^{2}}\)
- Anything raised to the 0 th power is equal to \(1: 6^{0}=1\)
- Anything raised to the 1st power is itself: \(6^{1}=6\)
- Zero to any power is zero.
- One raised to any power is one.

\section*{Roots}

The opposite of exponents are roots, shown with a radical sign \((\sqrt{)}\). The
number underneath a radical sign is the result of squaring, cubing, etc. For example, \(\sqrt{36}=6\) because \(36=6 \cdot 6\). Radicals can also find roots other than square roots through a number on the outside of the radical: \(\sqrt[3]{64}=4\) because 4 - \(4 \cdot 4=4^{3}=64\).

To solve \(x^{2}=25\), take the square root of both sides to get \(\sqrt{x^{2}}=\sqrt{25}\), and \(x=\) 5. However, this is an incomplete solution. Remember that the square of a negative is positive. While it is true that \(5^{2}=25\), it is also true that \((-5)^{2}=25\), so the complete solution is \(\sqrt{x^{2}}= \pm \sqrt{25}\), and \(x= \pm 5\). Roots can similarly be used as the reverse of higher exponents. For example, \(3^{3}=27\). To express this cube in reverse, use a cube root, which is indicated by the symbol, " \(\sqrt[3]{ }\)." For example, since \(3^{3}=27\), it is said that \(\sqrt[3]{27}=3\).Therefore, if \(x^{3}=27\), then, \(\sqrt[3]{x^{3}}=\sqrt[3]{27}\), and \(x=3\). The cube of a negative is a negative, \((-3)^{3}=-27\), so 3 is the only possible solution of \(x^{3}=27\). When solving an equation with an odd exponent, there is no need for any " \(\pm\) " symbol.

Ninth Grade Content
This will likely appear only on the 9th grade SHSAT.

If \(3 x^{2}-3=9\), what could be a value of \(x\) ?
A. -4
B. -2
C. 3
D. 4

\section*{Here's How to Crack It}

Solve for \(x\). Start by isolating \(x^{2}\). Add 3 to both sides to get \(3 x^{2}=12\). Divide both sides by 3 to get \(x^{2}=4\). Now take the square root of both sides to get \(\sqrt{x^{2}}= \pm\) \(\sqrt{4}\), which simplifies to \(x= \pm 2\). Although 2 is not a choice, -2 is, which makes (B) the correct answer.

\section*{Order of Operations}

The correct order of operations is one of the most important concepts to master in math! Just as you wouldn't try to assemble a project without following the steps in the blueprint, you wouldn't want to attempt to solve a problem at random. You might get the right answer, but it's more likely that you'll get something different. In this case, there's a mnemonic device that you can use to remember the order: Please Excuse My Dear Aunt Sally, or PEMDAS.


PEMDAS stands for:
Parentheses, Exponents, Multiplication \& Division, Addition \& Subtraction

The \(P\) stands for Parentheses. If there are parentheses in an equation, always begin by simplifying the expression within them. Only then should you move on.

The E stands for Exponents. If there are any exponents in the equation, solve these next.

M and D are interchangeable. When there are multiple terms to multiply or divide, work from left to right in the equation.

A and S stand for Addition and Subtraction. Like M and D (but only after all multiplication and division have been resolved), addition and subtraction should be done from left to right in the equation.

How would you attack this problem?
\[
16-45 \div(2+1)^{2} \times 4+5=
\]

To solve a problem like this, use PEMDAS. The order of operations is:
Parentheses
Exponents
Done at the same time Multiplication
from left to right Division
Addition Done at the same time
Subtraction from left to right
You can remember the order of operations using this phrase:

\section*{Please Excuse My Dear Aunt Sally.}

Now, let's give it a try:
\[
16-45 \div(2+1)^{2} \times 4+5=
\]

\section*{1. PARENTHESES:}
\[
\begin{aligned}
& 16-45 \div(2+1)^{2} \times 4+5= \\
& 16-45 \div(3)^{2} \times 4+5=
\end{aligned}
\]

\section*{2. EXPONENTS:}
\[
\begin{aligned}
& 16-45 \div(3)^{2} \times 4+5= \\
& 16-45 \div 9 \times 4+5=
\end{aligned}
\]
3. MULTIPLICATION AND DIVISION (from left to right):
\[
\begin{aligned}
& 16-\underline{45 \div 9} \times 4+5= \\
& 16-\underline{5 \times 4}+5= \\
& 16-20+5=
\end{aligned}
\]
4. ADDITION AND SUBTRACTION (from left to right):
\(16-20+5=\)
\(-4+5=1\)
Just take it one step at a time and the math is straightforward!

Simplify the following expression: \(4 \times \frac{(1+4 \times 2)^{2}}{4+5}\).
A. \(\frac{81}{5}\)
B. \(\frac{260}{9}\)
C. 36
D. 45

First, start within the parentheses, where there is both addition and multiplication. Multiplication comes first, so multiply \(4 \cdot 2\). The result is \(4 \times\) \(\frac{(1+8)^{2}}{4+5}\). Now, add: \(4 \times \frac{9^{2}}{4+5} .9\) is the most simplified form of the expression inside the parentheses, so now move on to exponents. \(9^{2}=81\), so the expression will read \(4 \times \frac{81}{4+5}\). Ready to multiply? Not so fast! The denominator should be treated as its own parentheses, so add \(4+5\) before multiplying: \(4 \times \frac{81}{9}\). Now, you are ready to multiply and divide. Simplify \(81 \div 9=9\), and \(4 \times 9=36\). The correct answer is (C).

Now, let's take a look these concepts in the context of a problem close to what will be on the SHSAT:

Two trains are traveling in the same direction on separate tracks. One travels 560 miles in 4 hours, while the other travels 450 miles in 3 hours. What is the difference, per hour, in the distance they travel?
A. 1
B. 10
C. 14
D. 15

\section*{Here's How to Crack It}

Since the question asks for the difference per hour, find how many miles per hour each train is traveling. Per is a word that signifies division, so divide the miles by the hours of each train to find that the first train travels \(\frac{560}{4}=140 \mathrm{mph}\) and the second train travels \(\frac{450}{3}=150 \mathrm{mph}\). To find the difference, use subtraction: \(150 \mathrm{mph}-140 \mathrm{mph}=10 \mathrm{mph}\). The correct answer is (B).

\section*{Practice Problems}
1. \(200(4-0.1)^{2}+50=\)

2. A certain number, \(x\), is added to 40, and the result is doubled. If four less than the final result is 90 , what is \(x\) ?
A. 3
B. 5
C. 7
D. 9
3. On a class field trip to the zoo, a quarter of the students reported that the their favorite exhibit was the penguins. A quarter of the remaining students reported that the lions were their favorite. Three-fifths of the remaining students liked the reptiles the best, and all the rest liked the giraffes the best. If there are 80 students in the class, how many liked the giraffes the most?
E. 18
F. 24
G. 27
H. 45

\section*{Explanations}
1. 3092 Remember your order of operations for this question. Work inside the parentheses first to find that \(4-0.1=3.9\). Next, square 3.9 to find the parenthetical expression in its simplest form, 15.21. Multiply by 200, and then add 50 to find that \(200(15.21)+50=3092\), which is the correct answer.
2. C This is a great opportunity to Plug In the Answers. Try one of the middle answer choices to find if it is too big or too small. Consider (C). If \(x=7\), first add it to 40 to find that \(x+7=47\). Multiply 47 by 2 to find the result to be 94 . Four less than this result is \(94-4=90\), which is what the question describes. The correct answer is (C).
3. E Take this question in bite-sized pieces. If there are 80 students in the class, first find how many students liked the penguins best. A quarter of the 80 students is \(80(1 / 4)=20\). Subtract this from the total and there are \(80-20=60\) students remaining. A quarter of the remaining students who liked the lions is \(60(1 / 4)=15\). Subtract this from 60 to find the new total of \(60-15=45\). Three-fifths of the remaining 45
students (who liked the reptiles best) is \(45(3 / 5)=27\). Subtract this from 45 to find that there were \(45-27=18\) students remaining who liked the giraffes the best. The correct answer is (E).

\section*{ABSOLUTE VALUE}

In math, sometimes the value of a number is less important than the absolute value, which is the distance that the number is from 0 . In an equation or an expression, the absolute value is indicated by the symbol ||. For example, |2| denotes "the absolute value of 2. ."

\section*{Finding Absolute Values}

Finding absolute values is simple. For a positive number, the distance that a number is from zero is equal to the value of the number. Therefore, the absolute value of a positive number is equal to the value of the number itself. Therefore,
\[
\begin{aligned}
|3| & =3 \\
|4.2| & =4.2 \\
\left|\frac{1}{2}\right| & =\frac{1}{2}
\end{aligned}
\]

There's a little more involved in finding the absolute value of a negative number but not much more. Consider the distance between a negative and 0 on the number line. Consider -2 . Since -2 is 2 units to the left of 0 on the number line, the distance between -2 and 0 is 2 , so \(|-2|=2\). Thus, to find the absolute value of a negative number, just drop the negative sign. Therefore,
\[
\begin{aligned}
|-3| & =3 \\
|-4.2| & =4.2 \\
\left|-\frac{1}{2}\right| & =\frac{1}{2}
\end{aligned}
\]

What about \(|0|\) ?
The absolute value is the distance from 0. Since there is no distance between 0 and \(0,|0|\)

\section*{Distance Between Numbers}

Absolute value is, by definition, the distance between a number and 0 . However, it can also be used to determine the distance between any two numbers on a number line. To find the distance, take the absolute value of the distance between the numbers. Look at the number line below.


Notice that 3 is 2 units to the right of 1 , so the distance between them is 2 . This can also be determined by finding the absolute value of the distance between them. Since \(|3-1|=|2|=2\), the distance between 3 and 1 is 2 . The order of the numbers doesn't matter, since \(|1-3|=|-2|\), which is also equal to 2 .

It works the same way for negative numbers. The distance between -4 and -2 is
\[
|-4-(-2)|=|-4+2|=|-2|=2
\]

Furthermore, it also works the same way to find the distance between a positive and a negative number. The distance between 1 and -2 is
\[
|1-(-2)|=|1+2|=|3|=3
\]

\section*{Real-Life Applications}

It may seem that absolute values don't have useful real-life applications. However, negative numbers can show up in real life in surprising ways. In these cases, absolute values are useful toward their understanding.

For example, sometimes account balances can be negative, which means the account holder has a debt. The absolute value of a negative balance is the amount of debt. Therefore, if a bank account has a balance of \(-\$ 100\), the account holder owes the bank \(|-\$ 100|=\$ 100\).

If Death Valley has an elevation of -232 ft and Mount Whitney has an elevation of \(14,505 \mathrm{ft}\), what is the different between the elevations of Death Valley and Mount Whitney? (Negative elevations indicate distances below sea level and positive elevations indicate distances above sea level.)
A. 232 ft
B. \(14,273 \mathrm{ft}\)
C. \(14,505 \mathrm{ft}\)
D. \(14,737 \mathrm{ft}\)

\section*{Here's How to Crack It}

The question asks for the distance between two points, so take the absolute value of the difference to get \(|14,405-(-232)|=|14,737|=14,737\), which is (D).

\section*{Absolute Values Drill}
1. \(|2-3 \times 4|-|8 \div 2-3|=\)
A. -1
B. 5
C. 9
D. 19
2. On a number line, what is the distance between -5 and -2 ?
3. Denise has overdrawn in her bank account and has a balance of \(-\$ 52\). If she then deposits \(\$ 36\) into the account, what is the value of her account?
E. a debt of \$88
F. a debt of \(\$ 16\)
G. a balance of \$16
H. a balance of \$88

\section*{Explanations:}
1. C Use PEMDAS. The absolute value signs serve as parentheses, so first evaluate each of those. In \(|2-3 \times 4|\), first multiply to get \(|2-12|\). Then, subtract to get \(|-10|\). To get the absolute value of -10 , drop the negative to get \(|-10|=10\). Now evaluate \(|8 \div 2-3|\). First divide to get \(|4-3|\). Then, subtract to get |1|. Then take the absolute value of 1 to get \(|1|=1\). Therefore, \(|2-3 \times 4|-|8 \div 2-3|=10-1=9\), which is (C).
2. 3 To find the distance between two points on a number line, take the absolute value of the difference. Therefore, the distance between -5 and -2 is \(|-5-(-2)|=|-5+2|=|-3|=3\). Therefore, the answer is 3 .
3. F Initially, the account has a balance of \(-\$ 52\), which indicates a debt of \$52. Denise deposits \(\$ 36\) into the account. Since a deposit adds money into an account, add \(\$ 36\) to \(-\$ 52\) to get \(-\$ 52+\$ 36=-\$ 16\). There is still a negative balance, so this is a debt of \(\$ 16\), which is ( F ).

\section*{FACTORS}

Factors describe any number that divides evenly (with no remainder) into another number. For example, 2 is a factor of 6 because it goes into 6 a total of 3 times, with no remainder. On the other hand, 5 isn't a factor of 6 because it does not produce an integer quotient. Take a moment to identify the other factors of 6 . From the example, 3 must also be a factor, as it goes into 6 a total of 2 times. Don't stop there, however: remember that because of the multiplicative identity property, 1 is a factor of every number, which also means that the number itself is its own factor. The factors of 6 are \(1,2,3\), and 6 .

\section*{Factor Pairs}

The SHSAT features more advanced factoring problems, which means that you'll want to be comfortable quickly finding all of the factors of a number. Rather than checking every number, a faster and efficient way of finding factors involves factor pairs. Factor pairs are two numbers that multiply to the original number you are finding the factors of. This allows you to find two factors at a time.

Find the factors of 12.
1 and \(12(1 \times 12=12)\)
2 and \(6(2 \times 6=12)\)
3 and \(4(3 \times 4=12)\)

When finding factor pairs, you should always start with 1 and the number you are trying to find the factors of. By finding this factor pair you will know the range of factors (the lowest and highest) that exist for that number.

Then, working from the bottom of the range, figure out the next highest factor. When you find it, you've also found its pair at the top of the range. Repeat this process until you've reached a number you've already used in a pair.

Find the factors of 24.
- Start with 1 and 24.
- What is the next number after 1 that 24 is divisible by?

Answer: 2
- The product of 2 and what number is 24 ?

Answer: 12
- This is your next factor pair (2 and 12).
- Repeat this until your factor pairs meet in the middle.
- 1 and 24
- 2 and 12
- 3 and 8
- 4 and 6
- This is your final list of factors: \(1,2,3,4,6,8,12\), and 24.


Squares
Some numbers, like 36 , have a factor that pairs with itself: \(6 \times 6=36\). If you hit one of these, you've found all the factors.

\section*{Divisibility Rules}

For finding the factors of larger numbers, knowing basic divisibility rules will help:
\begin{tabular}{|l|l|}
\hline \begin{tabular}{l} 
A number is \\
divisible by
\end{tabular} & If... \\
\hline 2 & it is even (ends in \(0,2,4,6\), or 8 ) \\
\hline 3 & the sum of the digits is divisible by 3 \\
\hline 4 & \begin{tabular}{l} 
the last two digits grouped together are divisible by 4 \\
(remember that 0 is divisible by every number)
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|l|l|}
\hline 5 & it ends in 0 or 5. \\
\hline 9 & the sum of the digits is divisible by 9. \\
\hline 10 & it ends in 0. \\
\hline
\end{tabular}

The SHSAT isn't likely to ask you about a number as large as 186,624-imagine trying to find all of those factor pairs!-but you can use these rules to quickly analyze it. This even number (2) has a sum of digits equal to \(27(3,9)\) and ends in 24 (4); because 2 and 3 are factors, 6 is as well.

These rules can be combined to find rules for larger numbers. For instance, if a number is divisible by both 2 and 3, then it is divisible by 6 , as 6 is \(2 \times 3\).

\section*{NUMBER SYSTEM DRILL}
1. How many factors does the number 48 have?

2. What is the smallest number that can fill the blank of 23,47_ so that the resulting number is divisible by 6 ?
A. 2
B. 3
C. 4
D. 5
3. If a certain number has a factor of 20 and is also divisible by 9 , which of the following is the greatest that number can be?
E. 20
F. 80
G. 540
H. 740
4. How many positive odd factors of 66 are between 1 and 33, inclusive?


\section*{NUMBER SYSTEM DRILL EXPLANATIONS}
1. 10 This problem asks you to count the number of factors. To approach this problem, look for factor pairs. 1 and 48 are the smallest and largest factors of 48 respectively. Work your way towards the middle until you have listed all of the factor pairs ( 1 and 48, 2 and 24, 3 and 16,4 and 12,6 and 8 ). After counting all of these factors you get 10 total factors.
2. A Plug In the Answers! The problem asks for the smallest number to go in the blank that would make the resulting number divisible by 6 . Remember that to be divisible by 6 , a number has to be divisible by both 2 and 3 . The rule for 2 stipulates that the number must be even, which means you can eliminate (B) and (D). Of the remaining choices, start with (A) because the problem is asking for the smallest number that would work. If you plug in 2 , the larger number \((23,472)\) will now be even and therefore be divisible by 2 . To test if it is divisible by 3 , add up all of the digits: \(2+3+4+7+2=18\). Since the sum of the digits \((18)\) is divisible by 3 , the whole number \((23,472)\) is divisible by 3 . Since 23,472 is divisible by both 2 and 3 , it is divisible by 6 and therefore (A) is the answer.
3. G This problem is essentially asking for the largest number listed that is divisible by both 9 and 20. Use POE to approach this problem. Remember that any number divisible by 20 is a number that would be divisible by both 10 and 2 (which multiply to 20 ). Since the sum of the digits of \(740(7+4+0=11)\) is not divisible by 9 , then 740 is not divisible by 9 and can be eliminated. Choice ( G ) is the next largest. Since the sum of the digits of \(540(5+4+0=9)\) is divisible by 9 , then 540 is divisible by 9 . Therefore, (G) is the largest number listed that is divisible by both 9 and 20 .
4. 4 To find the positive odd factors of 66 between 1 and 33, inclusive, you should first list all of the factors of 66 . The factor pairs are 1 and 66,2 and 33, 3 and 22, 6 and 11 . Of these factors, 1, 3, 11, and 33 are the only factors that fit the problem's criteria.

\section*{MULTIPLES}

Multiples are the result of multiplying a given integer by any integer (including negatives and zero). For example, to find multiples of 5 , multiply the number 5 by various integers.
\[
\begin{aligned}
& 5 \times 1=5 \\
& 5 \times 2=10 \\
& 5 \times 3=15 \\
& 5 \times 4=20
\end{aligned}
\]

Therefore, \(5,10,15\), and 20 are all multiples of 5 . Similarly,
\[
\begin{gathered}
5 \times 0=0 \\
5 \times-1=-5 \\
5 \times-2=-10 \\
5 \times-3=-15
\end{gathered}
\]
so \(0,-5,-10\), and -15 are all multiples of 5 , as well.
Another way to think about multiples is to consider them the result of "counting by a number." For example, to find the positive multiples of 4, "count by 4" to get \(4,8,12,16,20,24\), etc.

\section*{Least Common Multiple}

Sometime a question will ask about the least common multiple of two numbers. The least common multiple is the smallest positive integer that is a multiple of both of the numbers. For example, consider the least common multiple of 3 and 4. The positive multiples of 3 are \(3,6,9,12,15,18,21,24,27,30,33,36\), etc. The positive multiples of 4 are \(4,8,12,16,20,24,28,32,36,40\), etc. The least common multiple is the least number of both lists, which is 12 . Notice that 24 and 36 are also on both lists. These are also considered common multiples of 3
and 4, but not the least common multiple.

What is the least common multiple of 5 and 10 ?


\section*{Here's How to Crack It}

Though it may be tempting to multiply 5 and 10 to get 50 , the product of the two numbers is not always the least common multiple and is not in this case. The easiest way to find the least common multiple is to list positive multiples of the larger number until finding one that is also a multiple of the smaller one. The first positive multiple of 10 is 10 . Since 10 is a multiple of 5 , this must be the least common multiple. The answer is 10 .

\section*{MULTIPLES DRILL}
1. In the set of consecutive integers from 20 to 50, inclusive, how many integers are multiples of neither 2 nor 5 ?
A. 3
B. 4
C. 12
D. 16
2. What is the least common multiple of 45,15 , and 75 ?
E. 90
F. 150
G. 225
H. 675
3. A sugar company ships out sugar boxes that weigh 2 pounds, 5 pounds, or 6 pounds. All orders are shipped using only full boxes and only one size of box. If an order for \(n\) pounds of sugar can be shipped using any of the three sizes of boxes, what is the least possible value of \(n\) ?


\section*{MULTIPLES DRILL EXPLANATIONS}
1. C The question asks how many integers are multiples of neither 2 nor 5 . Take the integers from 20 to 50, inclusive, and discount any that are multiples of 2 or 5 . The integers that are multiples of 2 are the even integers, so discount the evens. The remaining integers are the odd integers: 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, and 49. Multiples of 5 end in 5 or 0 , so discount the numbers that end in 5 or \(0: 25,35\), and 45 . The remaining numbers are \(21,23,27,29,31,33\), \(37,39,41,43,47\), and 49 . Count the remaining numbers. There are 12 , so the answer is (C).
2. G The question asks for the least common multiple of 45,15 , and 75 , so go to the choices, starting with the least and find one that is a common multiple to all three. Choice (E) is 90 . Since \(90=45 \times 2\), 90 is a multiple of 45 . Since \(90=15 \times 6,90\) is a multiple of 15 . However, since \(90 \div 75\) is not a whole number, 90 is not a multiple of 75 . Eliminate (E). Try (F). Since \(150 \div 45\) is not a whole number, 150 is not a multiple of 45 . Eliminate (F). Try (G). Since \(225=45 \times 5,225\) is a multiple of 45 . Since \(225=15 \times 15,225\) is a multiple of 15 . Since \(225=75 \times 3,225\) is a multiple of 75 . Therefore, 225 is a common multiple of 45,15 , and 75 . Since it is the least choice remaining, it must be the least common multiple, so the answer is (G).
3. 30 If an order can be used with full boxes of a certain kind of box, the weight of the order must be a multiple of the weight of the box. The question asks for the least value of \(n\), for which an order of \(n\) pounds can be shipped using any size box. Therefore, \(n\) must be a multiple of 2,5 , and 6 . Since it asks for the least value of \(n\), it asks for the least common multiple of 2,5 , and 6 . List the multiples of the largest number, 6 , until one is a multiple of 2 and 5 , as well. The first positive multiple of 6 is 6 , which is not a multiple of 5 . The next is 12 , which is not a multiple of 5 . The next is 18 , which is not a multiple of 5 . The next is 24 , which is not a multiple of 5 . The next is 30 , which is a multiple of both 2 and 5 . Therefore, the least common multiple of 2 , 5 , and 6 is 30 , so 30 is the answer.

\section*{FRACTIONS}

A fraction is just another way of representing division. For instance, \(\frac{3}{8}\) is simply saying 3 divided by 8 (which is equal to 0.375 written as a decimal). Fractions are made up of a numerator (the top number) and a denominator (the bottom number). If you have a pizza pie that has 8 slices, and 3 of them are mushroom slices, you can say that \(\frac{3}{8}\) of the pie are mushroom slices. The numerator (3) represents the number of mushroom slices. The denominator (8) represents the number of slices in the whole pizza pie.

\section*{Reducing Fractions}

Two equivalent fractions can be represented in different ways. Picture a pizza pie that has been cut into 6 slices. If you eat 3 of the slices, that means you ate half
of the pie. Now picture that same pizza pie, but now cut into 8 slices. To eat half of this pie, you would need to eat 4 slices. From this example you can see that a half can be written as \(\frac{3}{6}\) or \(\frac{4}{8}\).

\section*{No "Small" Parts}

It's important to remember that when we're comparing fractions
on their own, we're talking about equally sized parts of a whole
-in this example, we're talking about identical pizzas. That's why \(\frac{1}{8}\) (which is 0.125 ) is greater than \(\frac{1}{10}\) (which is 0.1 ). However, \(\frac{1}{8}\) of something that is 16 inches would be 2 inches, whereas \(\frac{1}{10}\) of something that is 30 inches would be 3 inches.

Just keep this in mind.

Pizza 1

\[
\frac{3}{6}=\frac{1}{2}
\]
\[
\frac{4}{8}=\frac{1}{2}
\]

To reduce fractions, you must divide the numerator and denominator by a number they are both divisible by. The number used must be the same for both the numerator and the denominator.

Reduce \(\frac{16}{32}\).
\(\frac{16}{32} \rightarrow \frac{16 \div 2}{32 \div 2}=\frac{8}{16} \rightarrow \frac{8 \div 2}{16 \div 2}=\frac{4}{8} \rightarrow \frac{4 \div 2}{8 \div 2}=\frac{2}{4} \rightarrow \frac{2 \div 2}{4 \div 2}=\frac{1}{2}\)

The above reduction took multiple steps because we kept dividing the numerator and denominator by 2 . Because the SHSAT is a timed test, you'll want to try to find the largest number that both the numerator and denominator are divisible by.

In the fraction above, both 16 and 32 are divisible by 16 . If we had done this from the start, we would've gotten the answer of \(\frac{1}{2}\) in a single step. That said, don't waste time searching for the perfect number. If you get stuck, try numbers
like \(2,3,5\), or 10 and keep reducing until you get the final simplified fraction.

You'll Care More If You're Careless
Every timed test is a compromise between speed and accuracy. If you don't immediately see a large common divisor, pick a smaller one. A quick, extra step that puts you on the correct path is going to be better than a shortcut that steers you wrong.

\section*{Reduction Practice}
1. \(\frac{4}{16}\)
2. \(\frac{20}{45}\)
3. \(\frac{2,048}{256}\)

\section*{Reduction Answers}
1. The greatest common factor here is \(4 ; 4 \div 4=1\) and \(16 \div 4=4\), for a total of \(\frac{1}{4}\).
2. The greatest common factor here is \(5 ; 20 \div 5=4\) and \(45 \div 5=9\), for a total of \(\frac{5}{9}\).
3. The greatest common factor here is \(256 ; 2,048 \div 256=8\), and \(256 \div 256=\) 1 , which makes the answer 8 .

\section*{Adding/Subtracting Fractions}

There are three methods to adding/subtracting fractions, depending on the complexity of the question. You should become comfortable with all of them.

\section*{Method 1: Same Denominators}

If the denominators of the fractions are the same, you can just add or subtract the numerators as usual. It might be tempting at first to add both the numerators and denominators, but remember that the denominator expresses a whole. If you have half a pizza \(\left(\frac{1}{2}\right)\) and add another half \(\left(\frac{1}{2}\right)\), you are taking two parts of a twopart whole, which gives you 1 . If you got \(\frac{2}{4}\), you'd somehow have wound up with \(\frac{1}{2}\).

This works the same way for subtraction. If you have \(\frac{5}{8}\) and \(\frac{3}{8}\), the result is \(\frac{(5-3)}{8}\), or \(\frac{2}{8}\). The denominator clarifies that you're dealing with equal parts, so you're subtracting 3 slices of an 8 -slice pie from the 5 slices that are left.

Things get trickier if the denominators aren't the same: how would you add \(\frac{2}{3}\) of a pizza and \(\frac{3}{4}\) of a pizza?

\section*{Method 2: Lowest Common Denominator}

Earlier, you saw how to reduce fractions to an equivalent form. You could, however, go in the other direction-multiplying both the numerator and denominator by the same number instead of dividing. In this case, you are trying
to get the denominators to be the same, which means you're looking for what is called the "lowest common denominator" (LCD).
\(\frac{2}{3}+\frac{3}{4}\)
Step 1: Write out some of the multiples for 3 and 4. If you compare the lists (3, \(6,9,12,15,18,21,24\) and \(4,8,12,16,20,24)\), you'll see that 12 is the smallest number that both 3 and 4 go into. This lowest common denominator will be the denominator of the answer.
\[
\begin{aligned}
\frac{2}{3}+\frac{3}{4}= & \frac{}{12} \\
& \downarrow \\
& \text { Lowest } \\
& \text { Common } \\
& \text { Denominator }
\end{aligned}
\]

How Low Can You Go?
If you look at those lists of multiples, you may notice that 24 is also on both lists. That's a valid equivalent, but it isn't the least, which means that the math would be more complicated, and if you had to express your answer in lowest terms, you would likely have another step. Why make the SHSAT harder?

Step 2: Now that you've found the denominator, you need to identify how many times each denominator goes into the lowest common denominator.


Step 3: You can't just multiply the denominators by those results. You can, however, multiply both the numerator and denominator by that result, because \(4 / 4\) is equivalent to 1 , and any fraction times 1 is equal to itself. (This is the identity property of multiplication.)


Step 4: Because this is an addition problem, add the two products from step 3. If this were a subtraction problem we would subtract the two products.


It can sometimes be difficult to find the lowest common denominator of the two fractions. In this case, there's another method.

\section*{Method 3: Bowtie}
\(\frac{2}{3}+\frac{3}{4}=\) ?
Step 1: Multiply diagonally from the denominator of the first fraction to the numerator of the second fraction and from the denominator of the second fraction to the numerator of the first fraction. Write the products by the fraction where the respective arrows are pointing.
\[
\begin{gathered}
2 \times 4=8 \\
\frac{2}{3} \\
\square
\end{gathered}+\begin{gathered}
3 \times 3 \\
4
\end{gathered}
\]

Step 2: In addition problems, add the two resulting products from step 1 and write the result in the numerator of the answer. In subtraction problems, you would subtract the two resulting products from step 1.
\[
\begin{gathered}
2 \times 4=8 \\
\frac{2}{3}
\end{gathered} \quad \begin{aligned}
& 3 \times 3=9 \\
& \frac{3}{4}
\end{aligned}
\]

Step 3: Multiply the denominators from left to right and write the product in the denominator of the answer.
\[
\quad 2 \times 4=8 \quad \frac{2}{3}{\underset{x}{x}+3=9}_{+}^{3}=\frac{3+9}{12}=\frac{17}{12}
\]

By the end of this method, you should have three arrows drawn that resemble a bow tie. This is a straightforward way to remember which arrow to draw and in what direction.

Follow the arrows: they tell you where to write the products.

You can use any method you like on the SHSAT, but the bowtie tends to be the most straightforward. It might be a little more time-consuming, as it sometimes requires additional reduction once you obtain your answer, but it always works.

Try all three methods. Don't just look for the fastest one-find the most reliable one.

\section*{Adding/Subtracting Practice}

Find the totals for each expression below.
1. \(\frac{5}{7}+\frac{9}{10}\)
2. \(\frac{6}{9}-\frac{2}{5}\)
3. \(\frac{14}{15}+\frac{3}{4}\)

\section*{Adding/Subtraction Fractions Practice Answers}
1. \(\frac{5}{7} \underset{~}{\text { 50 }}+\frac{9^{63}}{10}=\frac{50+63}{70}=\frac{113}{70}\)
2. \(\frac{6}{9} \longleftrightarrow \frac{2^{18}}{5}=\frac{30-18}{45}=\frac{12}{45}=\frac{4}{15}\)
3. \(\frac{56}{15} \longrightarrow \frac{3^{45}}{4}=\frac{56+45}{60}=\frac{101}{60}\)

\section*{Fraction Comparisons}

Now that you know how to find equivalent fractions, you can also tackle another type of problem that appears on the SHSAT: comparing fractions. Although you don't have to add or subtract, you do have to follow the same steps. If two fractions have the same denominator, the larger fraction is the one with the larger numerator. For instance, \(\frac{6}{7}\) is larger than \(\frac{5}{7}\) because 6 is greater than 5 . If the fractions don't have the same denominator, make them equivalent, and then compare the numerators.

Which fraction is larger, \(\frac{4}{9}\) or \(\frac{3}{8}\) ?
1. Draw one arrow from the denominator of the first fraction to the numerator
of the second fraction and another arrow from the denominator of the second fraction to the numerator of the first fraction.

2. Multiply the two pairs of numbers connected by arrows and write the resulting product on the side that arrow is pointing to.
\[
4 \times 8=32
\]
3. The larger fraction is the one paired with the larger product. Since 32 is larger than \(27, \frac{4}{9}>\frac{3}{8}\).

\section*{Fraction Comparison Practice}

Circle the larger fraction in each of the following examples. The answers are on this page.
1. \(\frac{7}{9}\) vs. \(\frac{2}{4}\)
2. \(\frac{16}{25}\) vs. \(\frac{4}{5}\)
3. \(\frac{9}{12}\) vs. \(\frac{7}{10}\)

\section*{Multiplying Fractions}

Multiplying fractions is the most straightforward of all the mathematical operations to perform with fractions. Simply multiply the numerators across the top and denominators across the bottom. Reduce when necessary.
\[
\frac{5}{6} \times \frac{7}{9}=?
\]
\[
\frac{5}{6} \times \frac{7}{9}=\frac{5 \times 7}{6 \times 9}=\frac{35}{54}
\]

\section*{Reduction Tricks}

You can sometimes save a bit of time when multiplying by reducing as you go.
For instance, take the expression \(\frac{20}{6} \times \frac{15}{7}\). If you just go straight across, you'll get \(\frac{300}{42}\). This will then likely need to be reduced. Here are two quick tips. 42

First, reduce all terms before multiplying. In this case, you can divide the top and bottom of \(\frac{20}{6}\) to get the equivalent term \(\frac{10}{3}\). Next, check to see if you can reduce
diagonally. Note: This is a special trick that you can do only when multiplying.
(You're really just identifying the individual factors that are going to be a part of the final numerator and denominator.)
\[
\frac{10}{\beta_{1}} \times \frac{15^{5}}{7}
\]

Now when you multiply across, you're left with \(\frac{50}{7}\), not \(\frac{300}{42}\).

These steps can be taken in either order. In this case, reducing \(\frac{20}{6}\) to \(\frac{10}{3}\) makes it clearer that 3 is a factor of both 6 and 15 .

\section*{Finding the Reciprocal}

Dividing fractions is very similar to multiplying fractions, but there's one important, additional step, and that's finding the reciprocal of a fraction. To do so, switch the numerator and the denominator. The reciprocal of \(\frac{3}{4}\) is \(\frac{4}{3}\).

\section*{Dividing Fractions}

Use the mnemonic "Keep, Change, Flip." Keep the first fraction, Change the division sign to multiplication, and Flip the second fraction (find the reciprocal).
\[
\begin{gathered}
\frac{4}{9} \div \frac{5}{3}=? \\
\text { Keep: } \frac{4}{9} \div \frac{5}{3}= \\
\text { Change: } \frac{4}{9} \times \frac{5}{3}= \\
\text { Flip: } \frac{4}{9} \times \frac{3}{5}= \\
\text { Answer: } \frac{4 \times 3}{9 \times 5}=\frac{12}{45}=\frac{4}{15}
\end{gathered}
\]

\section*{Mixed Numbers}

Sometimes integers and fractions will be put together like \(1 \frac{1}{2}\) or \(-3 \frac{4}{5}\). These are known as mixed numbers. Mixed numbers have equivalent values that are improper fractions. Improper fractions are fractions in which the numerator is greater than the denominator. Often it will be necessary to convert a mixed number into an improper fraction. To do this, find the sum of the integer and the fraction.
\[
1 \frac{1}{2}=1+\frac{1}{2}=\frac{2}{2}+\frac{1}{2}=\frac{3}{2}
\]

For negative numbers, treat the mixed number like a positive and keep the negative sign.
\[
-3 \frac{4}{5}=-\left(3 \frac{4}{5}\right)=-\left(3+\frac{4}{5}\right)=-\left(\frac{15}{5}+\frac{4}{5}\right)=-\frac{19}{5}
\]

To convert an improper fraction to a mixed number, use the fact that fractions express division. Divide the numerator by the denominator and take the remainder.

Convert \(\frac{11}{5}\) to a mixed number.
\[
5 \begin{array}{r}
2 \\
\frac{10}{1}
\end{array}
\]

The quotient is 2 with a remainder of 1 . Make 2 the integer, 1 the numerator, and keep 5 as the denominator to get
\[
\frac{11}{5}=2 \frac{1}{5}
\]

\section*{FRACTIONS DRILL}
1. Adam bought one-sixth of a collector's baseball cards. If the collector had 42 baseball cards, how many baseball cards did Adam buy?
A. 6
B. 7
C. 8
D. 12
2. Which of the following is the greatest?
E. \(\frac{3}{4}+\frac{7}{5}\)
F. \(\frac{2}{9} \div \frac{4}{27}\)
G. \(\frac{4}{7}-\frac{1}{8}\)
Н. \(\frac{6}{7} \times \frac{4}{5}\)
3. Allan and Suzy are splitting a pizza pie. Allan ate \(\frac{1}{4}\) of the pie and Suzy ate \(\frac{7}{25}\) of the pie. How much of the pie is left uneaten? 25

4. Erick sleeps \(\frac{2}{3}\) of every day. Over 6 days, he sleeps the equivalent of how many full days?
A. \(\frac{1}{4}\)
B. \(\frac{2}{3}\)
C. 2
D. 4
5. \(4 \frac{5}{6}+3 \frac{1}{2}+2 \frac{1}{6}-1 \frac{2}{3}\)

What is the value of the expression above?
E. \(\frac{41}{6}\)
F. \(\frac{53}{6}\)
G. \(\frac{67}{6}\)
H. \(\frac{73}{6}\)

\section*{FRACTIONS DRILL EXPLANATIONS}
1. B One-sixth is the same as \(\frac{1}{6}\). Since there are 42 cards and Adam bought \(\frac{1}{6}\) of them, you can multiply \(\frac{1}{6} \times \frac{42}{1}\) to find the number of cards Adam bought. From this product, you get \(\frac{42}{6}=7\). Therefore, the answer is (B).
2. \(\mathbf{E} \quad\) To find the answer you will have to perform each of the operations to get the result. Choice (E) is the addition of \(\frac{3}{4}+\frac{7}{5}=\frac{15+28}{20}=\frac{43}{20}\). Choice (F) is \(\frac{2}{9} \div \frac{4}{27}=\frac{2}{9}=\frac{27}{4}=\frac{54}{36}=\frac{3}{2}\). Choice (G) is \(\frac{4}{7}-\frac{1}{8}=\) \(\frac{32-7}{56}=\frac{25}{56}\). Choice (H) is \(\frac{6}{7} \times \frac{4}{5}=\frac{24}{35}\). Now that you have the values of the four answer choices, you can see that one answer choice has a fraction greater than 2, one answer choice has a fraction that is greater than 1, and two answer choices have fractions that are less than 1. For this reason, you can eliminate (F), (G), and (H). The answer is (E).
3. 47 To find out how much pizza was eaten in total you must add the two fractions of pizza that Allan and Suzy ate: \(\frac{1}{4}+\frac{7}{25}=\frac{25}{100}+\)
\(\frac{28}{100}=\frac{53}{100}\). A whole is represented by 1 , so in this case, you can say that the whole pizza would be \(\frac{100}{100}\), which is equivalent to 1 . Therefore, the remaining fraction of pizza would be \(\frac{100}{100}-\frac{53}{100}=\frac{47}{100}\), which in decimal form is .47 .
4. D Since Erick sleeps \(\frac{2}{3}\) of every day, to find out the equivalent number of days he sleeps over 6 days, you must multiply \(\frac{2}{3} \times 6\). Convert 6 to an equivalent fraction \(\left(\frac{6}{1}\right)\) to make the multiplication clearer: \(\frac{2}{3} \times \frac{6}{1}=\frac{12}{3}\), so the answer is (D), 4 days.
5. F Mixed numbers are difficult to work with, so convert all of the mixed numbers to improper fractions: \(4 \frac{5}{6}=\frac{24}{6}+\frac{5}{6}=\) \(\frac{29}{6}, 3 \frac{1}{2}=\frac{6}{2}+\frac{1}{2}=\frac{7}{2}, 2 \frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}\), and \(1 \frac{2}{3}=\frac{3}{3}+\frac{2}{3}=\frac{5}{3}\).
Now find a common denominator. A common multiple to 2 , 3 , and 6 is 6 , so use this as the common denominator: \(\frac{7}{2} \times \frac{3}{3}=\frac{21}{6}\) and \(\frac{5}{3} \times \frac{2}{2}=\frac{10}{6}\). Therefore, \(4 \frac{5}{6}+3 \frac{1}{2}+2 \frac{1}{6}-1 \frac{2}{3}=\frac{29}{6}+\frac{21}{6}+\frac{13}{6}-\frac{10}{6}\). Combine the numerators to get \(\frac{29+21+13-10}{6}=\frac{53}{6}\), which is (F).

\section*{DECIMALS}

Decimals are another way of expressing fractions. Some examples of decimals are \(0.5,1.37\), and -51.2 . In many ways, decimals behave similarly to integers. In integers, the different digits have different place values. For example, in the number 62, 6 represents the tens digit and 2 represents the ones (or units) digit. Therefore, \(62=(6 \times 10)+(2 \times 1)\). This is also true of decimals. For decimals, however, the place values continue to get smaller past the ones digit. Consider the number 62.394.


The digits past the decimal point (.) have place values less than 1 . The first digit
after the decimal point is the tenths digit, the second is the hundredths digit, the third is the thousandths digit, etc. Therefore, \(62.394=(6 \times 10)+(2 \times 1)+\) \(\left(3 \times \frac{1}{10}\right)+\left(9 \times \frac{1}{100}\right)+\left(4 \times \frac{1}{1,000}\right)\). Furthermore, the number \(62.3940=(6 \times 10)\) \(+(2 \times 1)+\left(3 \times \frac{1}{10}\right)+\left(9 \times \frac{1}{100}\right)+\left(4 \times \frac{1}{1,000}\right)+\left(0 \times \frac{1}{10,000}\right)\).

Since \(\left(0 \times \frac{1}{10,000}\right)=0\), this is equal to \((6 \times 10)+(2 \times 1)+\left(3 \times \frac{1}{10}\right)+\left(9 \times \frac{1}{100}\right)+\)
\(\left(4 \times \frac{1}{1,000}\right)\), which is 62.394 . As you can see, adding 0 's to the end of a decimal does not have any effect on the value of the decimal.

\section*{Converting Decimals to Fractions}

To convert decimals to fractions, take advantage of the meaning of decimal
places. Remember that \(62.394=(6 \times 10)+(2 \times 1)+\left(3 \times \frac{1}{10}\right)+\left(9 \times \frac{1}{100}\right)+\) \(\left(4 \times \frac{1}{1,000}\right)\). Simplify this to get \(62.394=60+2+\frac{3}{10}+\frac{9}{100}+\frac{4}{1,000}\). Use 1,000 as a common denominator to get \(62.394=\frac{60,000}{1,000}+\frac{2,000}{1,000}+\frac{300}{1,000}+\) \(\frac{90}{1,000}+\frac{4}{1,000}=\frac{62,394}{1,000}\). Notice that the numerator is the original number with the decimal point removed and that the denominator matches the place value of the last digit. This will always be the case. Therefore,
\[
\begin{gathered}
0.2=\frac{2}{10}=\frac{1}{5} \\
1.45=\frac{145}{100}=\frac{29}{20} \\
0.236=\frac{236}{1,000}=\frac{59}{250}
\end{gathered}
\]

\section*{Adding and Subtracting Decimals}

Adding and subtracting decimals is not that different from adding and subtracting integers. To add \(1.2+4.37\), just stack them. Make sure that the decimal points are lined up.
\[
\begin{array}{r}
1.2 \\
+4.37 \\
\hline
\end{array}
\]

Now, add a 0 to the end of 1.2 , so that the number of digits after the decimal point will be the same in both numbers.
\[
\begin{array}{r}
1.20 \\
+4.37 \\
\hline
\end{array}
\]

Bring down the decimal point, and add normally.
\[
\begin{array}{r}
1.20 \\
+4.37 \\
\hline 5.57
\end{array}
\]

Subtraction works the same way. For example, to subtract 23.34 from 135.8, rewrite 135.8 as 135.80 , and line up the decimal points.

\title{
135.80 \\ \(-23.34\)
}

Bring down the decimal and subtract normally, including regrouping to get
\[
\begin{array}{r}
7 \\
135.8^{1} 0 \\
-23.34 \\
\hline 112.46
\end{array}
\]

\section*{Multiplying Decimals}

Multiplying decimals is also like multiplying integers. To multiply 2.43 by 1.2, begin by ignoring the decimals and multiply as if it were 243 times 12 . There is no need to line up the decimal points.


Now determine where the decimal point belongs. The number of digits after the decimal point in the product has to be equal to the total number of digits following the decimal points between the two factors. In this case, 2.43 has 2 digits after the decimal point, and 1.2 has 1 digit after the decimal. Therefore, the answer must have \(1+2=3\) digits after the decimal point, which turns 3916 into 3.916.

\section*{Dividing Decimals}

With a little bit of manipulation, you can divide decimals the same way as you divide integers. That's because you can multiply both numbers by 10 to slowly move the decimal place out of the divisor. For instance, take \(8.64 \div 2.7\).

Set these two numbers up as you would any other division problem.
\[
2 . 7 \longdiv { 8 . 6 4 }
\]

Now move the decimal in both numbers one place to the right-the equivalent of multiplying both by 10 .
\[
2 7 \longdiv { 8 6 . 4 }
\]

Mark the decimal above so you don't forget about its position:
\[
2 7 \longdiv { 8 6 . 4 }
\]

Now just divide normally, ignoring the decimal point on the bottom.
\[
\begin{array}{r}
3 7 \longdiv { 8 6 . 4 } \\
\frac{81}{54} \\
\hline 54 \\
\hline 0
\end{array}
\]

Therefore, \(8.64 \div 2.7=3.2\).
\[
\frac{(3+0.4)^{2}}{0.5}=
\]


\section*{Here's How to Crack It}

Remember to use PEMDAS. Start with what's inside the parentheses: \(3+0.4\). Rewrite 3 as 3.0. Stack 3.0 and 0.4 with the decimals lined up to get
\[
\begin{array}{r}
3.0 \\
+0.4 \\
\hline 3.4
\end{array}
\]
simplifying the expression to \(\frac{(3.4)^{2}}{0.5}\). Next is the exponent. To solve \(3.4^{2}\), multiply 3.4 by 3.4 to get
\[
\begin{array}{r}
3.4 \\
\times 3.4 \\
\hline 136 \\
+1020 \\
\hline 11.56
\end{array}
\]

Remember to put two digits after the decimal point in the product, since there are a total of two digits after the decimal points in the two factors. Thus, the expression simplifies to \(\frac{11.56}{0.5}\). Now divide. Get an integer in the denominator by moving the decimal place in the both the number and the denominator by 1 to the right to get \(\frac{115.6}{5}\). Divide. 5 doesn't go into 1 but goes into 11 twice. \(5 \times 2=\) 10, so subtract 10 from 11 to get 1 left over. Carry down the 5 . The 5 goes into 15 a total of 3 times. \(5 \times 3=15\), so subtract 15 from 15 to get 0 left over. Carry up the decimal point, and bring down the 6 . The 5 goes into 6 one time. \(5 \times 1=\) 5, so subtract 5 from 6 and get 1 left over. Because this doesn't go in evenly, add a 0 to the end of 115.6 and bring down the 0 . The 5 goes into 10 two times. Since this goes in evenly, stop here. The scratch work should look like this


Therefore, the answer is 23.12 .

\section*{Converting Fractions to Decimals}

To convert fractions to decimals, remember that the fraction bar is the same as division. Therefore, the conversion can be done using long division. For example, take the fraction \(\frac{3}{20}\). Divide 20 into 3 .
\[
2 0 \longdiv { 3 }
\]

20 doesn't go into 3 , so add a decimal point and a 0 . Carry up the decimal point to find
\[
\begin{array}{r}
0.1 \\
20 \lcm{3.0} \\
\frac{20}{10}
\end{array}
\]

In other words, 20 goes into 30 one time with 10 left over. Since it doesn't go in evenly, add another 0 . Bring down the 0 . The 20 goes into 100 five times. Since it goes in evenly, stop here.
\[
\begin{array}{r}
0.15 \\
20 \lcm{3.00} \\
\frac{20}{100} \\
\frac{100}{0}
\end{array}
\]

Therefore, \(\frac{3}{20}=0.15\).

\section*{Scientific Notation}

Decimals come in handy as a way of shortening larger numbers. For example, the number 5,400,000 can be cumbersome to work with. As an alternative, use scientific notation. Move the decimal to after the first digit. To avoid changing the value, multiply by \(10^{n}\), where \(n\) is the number of places, the decimal had to be moved. In this case, move the decimal point in \(5,400,000\) six times to get 5.4 \(\times 10^{6}\). Similarly,
\[
\begin{gathered}
100=1 \times 10^{2} \\
75=7.5 \times 10^{1} \\
25,374,100=2.53741 \times 10^{7}
\end{gathered}
\]

\section*{DECIMALS DRILL}
1. \([(-2.4)-(-1.3)+3]-[4+1.7]=\)

2. Joan goes to the store and buys five pounds of peanuts and three pounds of cashews for a total of \(\$ 17.75\). If cashews cost \(\$ 1.75\) per pound, what is the price per pound for the peanuts?
A. \(\$ 1.80\)
B. \(\$ 2.50\)
C. \(\$ 3.00\)
D. \(\$ 4.17\)
3. \(\frac{4}{7}=0 . \overline{571428}\)

In the decimal above, the repeating pattern begins with 5 . What is the 105th digit after the decimal?
E. 1
F. 4
G. 5
H. 8
4. If \(\frac{17}{n}\), what is the value of \(n\) ?
A. 10
B. 20
C. 50
D. 100
5. Which of the following is equivalent to \(3.2 \times 10^{5}\) ?
E. 3,200
F. 32,000
G. 320,000
H. 3,200,000

\section*{DECIMALS DRILL EXPLANATIONS}
1. -3.8 Use PEMDAS and start with the brackets and parentheses. Within the first set of brackets, do addition and subtraction in order from left to right: \((-2.4)-(-1.3)=-2.4+1.3=-1.1\). Now add 3 to get \(-1.1+3=\) 1.9. Go to the second set of brackets: \(4+1.7=5.7\). Finally, subtract 5.7 from 1.9 to find that \(1.9-5.7=-3.8\).
2. B The question asks for the price per pound for the peanuts. She buys three pounds of cashews for \(\$ 1.75\) per pound. To get the total cost of the cashews, multiply \(\$ 1.75\) by 3.
\[
\begin{array}{r}
1.75 \\
\times 3 \\
\hline 5.25
\end{array}
\]

Since she spent \(\$ 5.25\) on cashews and a total of \(\$ 17.75\) on cashews and peanuts, subtract \(\$ 5.25\) from \(\$ 17.75\) to get the total cost of the peanuts.
17.75
\(-5.25\)
12.50

Since she spent \(\$ 12.50\) on five pounds of peanuts, divide by 5 to get the price per pound.
\[
\begin{gathered}
2.50 \\
5 \begin{array}{c}
12.50 \\
\frac{10}{25} \\
\frac{25}{0}
\end{array}
\end{gathered}
\]

Therefore, the price per pound for the peanuts is \(\$ 2.50\), which is (B).
3. E The question asks for the 105th digit after the decimal. Because the digits are in a repeating pattern, count the number of digits in the pattern to get that there are six. Divide 105 by 6, and take the remainder. Since the remainder is 3 , the digit is equal to the third digit after the decimal, which is 1 . The answer is ( E ).
4. B The question asks for the value of \(n\). Get the two sides of the equation into the same form, by turning 0.85 into a fraction. Because the last digit of 0.85 is in the hundredths place, \(0.85=\frac{85}{100}\). Reduce this fraction by 5 to get \(\frac{17}{20}\). Therefore, \(\frac{17}{n}=\frac{17}{20}\). Since the numerators are equal, the denominators are equal, so \(n=20\), which is (B).
5. G To change a number in scientific notation into standard form, move the decimal point to the right by a number of times equal to the exponent on 10 . For \(3.2 \times 10^{5}\), move the decimal point in 3.2 to the right five times, adding in zeros as needed. Therefore, \(3.2 \times 10^{5}=\) 320,000 , which is (G).

\section*{Chapter 11}

Ratios and Proportional Relationships

\section*{PERCENTS}

Some SHSAT question will ask about different types of part-whole relationships. One of those types of relationships is percents. Percent means per hundred. For example, if there are 100 marbles, \(20 \%\) of which are blue, then there are 20 blue marbles. This relationship holds at this same rate even if the whole isn't 100. Imagine there are 200 marbles and \(20 \%\) are blue. Since there are 20 blue marbles per hundred marbles, there are 40 blue marbles. Similarly, if there are 50 marbles and \(20 \%\) are blue, then there are 10 blue marbles.

\section*{Translating Percent Questions}

Very often, the key to solving a percent question is to take the English in the question and translate it into math. Once you rephrase the question as an equation, you can solve it.
\begin{tabular}{|l|l|}
\hline English & Math Equivalent \\
\hline percent (\%) & \(\overline{100}\) \\
\hline of & times \((\times)\) \\
\hline what & variable \((x, y, z\), etc.) \\
\hline is, are, were, did, etc. & equals ( \(=\) ) \\
\hline
\end{tabular}
\(60 \%\) of what is 75 ?
A. 30
B. 45
C. 105
D. 125

Translate the question from English into Math. \(60 \%\) translates to \(\frac{60}{100}\). The word of translates to " \(\times\)." The word what translates to a variable. (We'll use \(x\).) The word is translates to " \(=\)." Therefore, \(60 \%\) of what is 75 translates to \(\frac{60}{100} x=75\). Solve the equation. First, reduce the fraction \(\frac{60}{100}\) to \(\frac{3}{5}\). Then, multiply both sides of \(\frac{3}{5} x=75\) by 5 to get \(3 x=375\). Finally, divide both sides by 3 to get \(x=\) 5

125 , which is (D).

\section*{Percent Change}

Some percent questions will be about percent change. Whether the question asks for percent increase, percent decrease, percent greater than, percent less than, percent discount, or any other kind of percent change or difference, use the formula percent change \(=\frac{\text { difference }}{\text { original }} \times 100\)

The difference simply refers to the positive difference between the two quantities. If the question asks for a percent increase or percent greater, then the smaller quantity is the original. If the question asks for a percent decrease or percent less, then the bigger quantity is the original.

If an item that regularly costs \(\$ 25\) is on sale for \(\$ 20\), what is the percent discount?
E. 5\%
F. \(20 \%\)
G. 25\%
H. 30\%

Here's How to Crack It
The question asks for a percent discount, so use the formula \(\frac{\text { difference }}{\text { original }} \times 100\). The difference is the positive difference between the quantities, which is \(\$ 25-\) \(\$ 20=\$ 5\). Because the question asks for the percent discount, there is a decrease, so the greater number, \(\$ 25\), is the original. Therefore, the percent discount is \(\frac{\$ 5}{\$ 25} \times 100=\frac{500}{25}=20\), which is \((\mathrm{F})\).

\section*{PERCENTS DRILL}
1. Juliet has sold 24 of her building block sets. This is \(40 \%\) of the original number of building block sets in her original collection. How many building block sets were in her original collection?

2. Kevin bought 3 bottles of soda for \(\$ 1.50\) each. If there is a \(6 \%\) sales tax, how much did Kevin pay in total for the soda?
A. \(\$ 1.59\)
B. \(\$ 4.23\)
C. \(\$ 4.77\)
D. \(\$ 5.04\)
3. John has 24 carrots and eats 8 of them. The number of carrots he had originally is what percent greater than the number he had after eating the 8 carrots?
E. 8\%
F. 16\%
G. \(33 \%\)
H. \(50 \%\)
4. A tennis league requires that all tennis balls used must have a diameter of 2.6 inches with a maximum allowable error of \(2 \%\). What is the least possible diameter of a tennis ball that can be used in this league?
A. 0.052
B. 2.548
C. 2.62
D. 2.652

\section*{PERCENTS DRILL EXPLANATIONS}
1. 60 Translate the information in the question. The second sentence says, This is \(40 \%\) of the original number of building block sets in her original collection. The word this refers to what comes before it, which is the number of building block sets she sold, which is 24 . The word is translates to " \(=\)." \(40 \%\) translates to \(\frac{40}{100}\). The word of translates to "×." Finally, the original number of building block sets in her collection is unknown, so use a variable, such as \(C\). In that case, the question translates to \(24=\frac{40}{100} C\). Reduce \(\frac{40}{100}\) to \(\frac{2}{5}\). Multiply both sides of \(24=\frac{2}{5} C\) by 5 to get \(120=2 C\). Divide both sides by 2 to get \(60=C\). Therefore, the original number of building block sets in her collection is 60 .
2. C The question asks for the total amount that Kevin pays for the three sodas. The sodas cost \(\$ 1.50\) each, so 3 sodas cost \(3 \times \$ 1.50=\$ 4.50\). There is also a \(6 \%\) sales tax. Take \(6 \%\) of the cost of the sodas to get \(\frac{6}{100}(\$ 4.50)=\$ 0.27\). Add the tax to the cost to get \(\$ 4.50+\$ 0.27=\)
\(\$ 4.77\), which is (C).
3. H The question asks for percent greater, so use the formula \(\frac{\text { difference }}{\text { original }} \times\) 100. The difference is the number of carrots John ate, which is 8 . Because the question asks for percent greater, the smaller is the original. In the beginning, John had 24 carrots. After eating 8, he had \(24-8=16\) carrots. Since 16 is less, this is the original. Therefore, the percent difference is \(\frac{8}{16}(100)=50\), which is \((\mathrm{H})\).
4. B There is a maximum allowable error of \(2 \%\), so take \(2 \%\) of the diameter to get \(\frac{2}{100} \times 2.6=\frac{5.2}{100}=0.052\). Therefore, the ball must have a diameter within 0.052 inches of the 2.6 inch requirement. Therefore, the minimum diameter a ball can have is \(2.6-0.052=\) 2.548, which is (B).

\section*{RATIOS AND PROPORTIONS}

\section*{Ratios}

A ratio is like a recipe because it says how much of different ingredients go into a mixture. For example:

To make punch, mix two parts grape juice with three parts orange juice.

This ratio says that for every two units of grape juice, add three units of orange juice. It doesn't matter what the units are; if working with ounces, mix two ounces of grape juice with three ounces of orange juice to get five ounces of punch. If working with gallons, mix two gallons of grape juice with three gallons of orange juice. How much punch would there be? Five gallons.

What if the recipe called for 20 gallons of punch? To work through this kind of question, organize the information given using a ratio box.
\begin{tabular}{|l|l|l|}
\hline Grape Juice & Orange Juice & Total \\
\cline { 1 - 3 } 2 & 3 & \\
\hline & Ratio \\
\hline & & \\
\hline & & 20 \\
Multiplier \\
Actual Number
\end{tabular}

To begin the ratio box, fill in the ratio in the top row and the actual numbers in the bottom row. The ratio of grape juice to orange juice is \(2: 3\), and the actual amount of total punch is 20 gallons. To fill in the rest of the box, start by adding across the top row to get 5 under the total. Now, multiply downward. 5 times what number is 20 ? 4 . Fill in 4 in the multiplier row. All numbers in the multiplier row are equal, so fill 4 into each column in that row. Multiply downward again to get that the actual number of gallons of grape juice is \(2 \times 4=\) 8 and that the actual number of gallons of orange juice is \(3 \times 4=12\).
\begin{tabular}{|l|l|l|}
\hline Grape Juice & Orange Juice & Total \\
\cline { 1 - 3 } 2 & 3 & 5 \\
\hline 4 & 4 & 4 \\
\hline 8 & 12 & 20 \\
\hline Ratio \\
\hline Multiplier \\
Actual Number
\end{tabular}

A mixture of paint contains blue paint, red paint, and yellow paint in a ratio of 5:2:1. How many gallons of red paint are needed to create two gallons of this mixture?
A. 0.25
B. 0.5
C. 1.25
D. 16

\section*{Here's How to Crack It}

The question involves a ratio, so set up a ratio box. This is a three-part ratio, so add an extra column. Otherwise, the box works the same way
\begin{tabular}{|l|l|l|l|}
\hline Blue & Red & Yellow & Total \\
\hline 5 & 2 & 1 & \\
\hline & & & \\
\hline & & & 2 \\
\hline
\end{tabular}

Add across the top row to get 8 in the total column. To get the multiplier, determine what you'd need to multiply 8 by to get 2 . This is \(\frac{2}{8}=\frac{1}{4}=0.25\). Fill in 0.25 across the middle row. Multiply downward in each of the other columns:
\(5 \times 0.25=1.25,2 \times 0.25=0.5\), and \(1 \times 0.25=0.25\).
\begin{tabular}{|l|l|l|l|}
\hline Blue & Red & Yellow & Total \\
\hline 5 & 2 & 1 & 8 \\
\hline 0.25 & 0.25 & 0.25 & 0.25 \\
\hline 1.25 & 0.5 & 0.25 & 2 \\
\hline
\end{tabular}

The question asks for how much red paint is needed, so look in the bottom row under red. It says 0.5 , which is (B).

\section*{Proportions}

Some questions will involve two ratios set equal to each other. This equation is called a proportion. For example, if 20 gallons of punch are needed to serve 160 people, how many gallons are needed to serve 240 people? Set the known relationship equal to the unknown.
\[
\frac{20 \text { gallons }}{160 \text { people }}=\frac{x \text { gallons }}{240 \text { people }}
\]

Make sure to write the units. Put like units in the numerators and like units in the denominators. To solve, cross-multiply: multiply each numerator equal to the denominator on the other side of the equation, and set the products equal. In this case, the result is \((20)(240)=160 x\)

Multiply on the left side to get
\[
4,800=160 x
\]

Divide both sides by 160 to get \(x=30\). Therefore, 30 gallons are needed to serve 240 people.

A certain school has 4 girls for every 3 boys. If there are a total of 360 boys in the school, how many girls are there?
E. 90
F. 120
G. 270
H. 480

\section*{Here's How to Crack It}

Set the known equal to the unknown: \(\frac{4 \text { girls }}{3 \text { boys }}=\frac{x \text { girls }}{360 \text { boys }}\). Cross-multiply to get \(3 x=1,440\). Divide both sides by 3 to get \(x=480\), which is \((H)\).

\section*{Conversions}

One application of proportions is unit conversions. How many yards are in 12 feet? There are 3 feet in one yard, so set up a proportion.
\[
\frac{3 \mathrm{feet}}{1 \text { yard }}=\frac{12 \mathrm{feet}}{x \text { yards }}
\]

Cross-multiply to get \(3 x=12\). Divide both sides by 3 to get \(x=4\). Therefore, there are 4 yards in 12 feet.
\[
\begin{gathered}
1 \text { mile }=0.87 \text { nautical miles } \\
1 \text { mile }=1.61 \text { kilometers }
\end{gathered}
\]

Using the conversion above, approximately how many kilometers is equal to 2 nautical miles?
A. 0.54
B. 1.1
C. 1.9
D. 3.7

\section*{Here's How to Crack It}

The question asks about the relationship between nautical miles and kilometers.
Set the known equal to the unknown. Since both 0.87 nautical miles and 1.61
kilometers are equal to 1 mile, they are equal to each other. This is the known relationship. Set it equal to the unknown to get \(\frac{0.87 \text { nautical miles }}{1.61 \text { kilometers }}=\frac{2 \text { nautical miles }}{x \text { kilometers }}\). Cross-multiply to get \(0.87 x=3.22\).

Divide both sides by 0.87 to get 3.7 , which is (D).

\section*{Rate Problems}

Proportions are also useful for problems involving rates. Rates are amounts of distances divided by time. This rate can be put into ratio form, meaning that equal rates can be put into proportion form. For example, if a car travels at a constant speed of 30 miles per hour, how far does it travel in 4 hours? To find this, set up a proportion. Any time a question uses the word per in a rate question, take this to mean 1.
\[
\frac{30 \text { miles }}{1 \text { hour }}=\frac{x \text { miles }}{4 \text { hours }}
\]

Cross-multiply to get \(x=120\), so the car travels 120 miles per hour.

Randall can type 70 words in 2 minutes. How many words can he type in 6 minutes?
E. 35
F. 140
G. 210
H. 420

\section*{Here's How to Crack It}

Set up the proportion \(\frac{70 \text { words }}{2 \text { minutes }}=\frac{x \text { words }}{6 \text { minutes }}\). Cross-multiply to get \(2 x=420\).
Divide by 2 to get \(x=210\), which is (G).

\section*{Group Questions}

Some questions will ask about overlapping groups within a whole. Very often, this kind of question will actually use the words both and neither. If this is the
case, use the group formula: Total \(=\) Group \(1+\) Group \(2+\) Neither - Both
For example, a class has 10 students who are wearing hats, 7 students who are wearing gloves, 3 students who are wearing both hats and gloves, and 2 students who are wearing neither hats not gloves. How many students are in the class? Let Group 1 be the students who are wearing hats and Group 2 be the students who are wearing gloves. Use the formula to get Total \(=10+7+2-3=16\).

Of the 1,000 fans at a baseball game, 525 are eating hot dogs and 434 are drinking sodas. What is the greatest possible number of fans who are neither eating hot dogs nor drinking sodas?
A. 21
B. 382
C. 455
D. 546

\section*{Here's How to Crack It}

The question uses the word neither, so use the group formula: Total = Group \(1+\) Group \(2+\) Neither - Both. Let Group 1 be the fans eating hot dogs and Group 2 be the fans drinking sodas to get \(1,000=525+434+N-B\). The question asks for Neither, so isolate \(N\). Simplify the right side to get \(1,000=979+N-B\). Subtract 979 from both sides to get \(21=N-B\). Add \(B\) to both sides to get \(21+B\) \(=N\). To get the greatest possible value of \(N\), find the greatest possible value of \(B\). \(B\) represents the number of fans who are both eating hot dogs and drinking sodas. To find the greatest possible number, assume everyone drinking soda is also eating a hot dog. (Note that since more people are eating hot dogs, it cannot be the case that everyone eating a hot dog is also drinking a soda.) If everyone drinking a soda is also eating a hot dog, \(B=434\). Thus \(N=21+434=455\), which is (C).

\section*{RATIO AND PROPORTIONS DRILL}
1. A jar of 36 coins contains only pennies, nickels, and dimes. If the ratio of pennies to nickels is \(4: 3\) and the ratio of nickels to dimes is \(3: 5\), how many pennies are in the jar?
A. 9
B. 12
C. 15
D. 18
2. In a scale drawing of a building, the building is 5 inches wide and 12.5 inches tall. If the building is 43.75 feet tall, how wide is the building, in feet?

3.
\[
\begin{aligned}
& 1 \text { inch }=4.5 \text { auchels } \\
& 1 \text { inch }=0.9 \text { hunecks }
\end{aligned}
\]

Based on the conversion above, how many auchels are in one huneck?
E. 5
F. 6
G. 7
H. 8
4. Jen removes 3 posters from her wall each day. If she begins with 25 posters, how many posters will she have remaining on her wall after 4 days?
A. 9
B. 12
C. 13
D. 16

\section*{RATIO AND PROPORTIONS DRILL EXPLANATIONS}
1. B The question provides ratios, so use a ratio box. The ratio of pennies to nickels is \(4: 3\) and the ratio of nickels to dimes is \(3: 5\). Since the number corresponding to nickels is the same in both ratios, the ratios can be combined into one ratio. The ratio of pennies to nickels to dimes is \(4: 3: 5\). Fill these into the top row of the ratio box. The actual number of coins in the box is 36 , so fill this into the bottom row under Total. Add across the top row to get 12 under Total. Since \(12 \times 3=36\), 3 is the multiplier. Fill this across the middle row. Multiply downward in each column to get \(4 \times 3=12\) pennies, \(3 \times 3=9\) nickels, and \(5 \times 3\) \(=15\) dimes.
\begin{tabular}{|l|l|l|l|}
\hline Pennies & Nickels & Dimes & Total \\
\hline 4 & 3 & 5 & 12 \\
\hline 3 & 3 & 3 & 3 \\
\hline 12 & 9 & 15 & 36 \\
\hline
\end{tabular}

The question asks for the number of pennies. Since there are 12 pennies, the answer is (B).
2. 17.5 When a question provides a scale, use proportions. A drawing height of 12.5 inches corresponds with a building height of 43.75 feet. To determine what building width corresponds with a drawing width of 5 inches, set up the proportion \(\frac{12.5 \text { inches }}{43.75 \text { feet }}=\frac{5 \text { inches }}{x \text { feet }}\). Crossmultiply to get \(12.5 x=218.75\). Divide both sides by 12.5 to get \(x=\) 17.5 , so this is the answer.
3. \(\mathbf{E}\) The question asks for how many auchels are in one huneck. Since 4.5 auchels and 0.9 hunecks are both equal to 1 inch, they are equal to each other. To determine the number of hunecks in one auchel, set up the proportion \(\frac{4.5 \text { auchels }}{0.9 \text { hunceks }}=\frac{x \text { auchels }}{1 \text { huneck }}\). Cross-multiply to get 4.5 \(=0.9 x\). Divide both sides by 0.9 to get \(x=5\), which is \((\mathrm{E})\).
4. C The question says she removes 3 posters each day for 4 days. Determine the total number of posters removed in 4 days using the proportion \(\frac{3 \text { posters }}{1 \text { day }}=\frac{x \text { posters }}{4 \text { days }}\). Cross-multiply to get \(x=12\). However, this is not the final answer. The question asks for how many posters she will have remaining on her wall. She begins with 25 and removes 12 , so she will have \(25-12=13\) remaining. The answer is (C).

\section*{Chapter 12 \\ Expressions and Equations}

\section*{ALGEBRAIC EQUATIONS AND INEQUALITIES}

Some equations will have unknown values called variables. Variables will take the forms of letters such as \(x\) or \(y\). An equation with variables and no exponents is called a linear equation.

\section*{Solving Linear Equations}

Solving linear equations requires algebra, which is basically backward arithmetic. Take the equation
\[
3 x+5=17
\]

To find the value of \(x\), isolate \(x\) by applying an operation that is the opposite of the one shown. Note that because you're working in reverse, you'll need to follow the order of operations in the opposite order, so start with addition and subtraction. The equation adds 5 to \(3 x\), so do the opposite and subtract 5 . Whatever is done to one side of the equation must be done to the other side of the equation, so subtract 5 from both sides.
\[
\begin{array}{r}
3 x+5=17 \\
-5-5 \\
\hline 3 x \quad=12
\end{array}
\]

Now go to multiplication and division. Since \(x\) is multiplied by 3, divide both sides by 3 .
\[
\begin{gathered}
\frac{3 x}{3}=\frac{12}{3} \\
x=4
\end{gathered}
\]

You can check that \(x=4\) by plugging 4 into \(x\) in the original equation:
\[
\begin{gathered}
3(4)+5=17 \\
12+5=17
\end{gathered}
\]

That last statement is true, which means that \(x=4\) is a valid solution to this equation.

For what value of \(y\) is \(6 y=2 y+20\) ?


\section*{Here's How to Crack It}

The question asks for the value of \(y\), so isolate \(y\). First, however, make sure only one term in the equation has \(y\). Since two terms have \(y\), combine like terms. Subtract \(2 y\) from both sides to get \(4 y=20\). Divide both sides by 4 to get \(y=5\), which is the correct answer.

\section*{Distributive Law}

In the order of operations of arithmetic, operations inside parentheses are performed first. Therefore, in algebra, operations inside the parentheses are performed last. However, there is an alternative method to handling addition or subtraction inside parentheses called the Distributive Law.
\[
\begin{aligned}
& a(b+c)=a b+a c \\
& a(b-c)=a b-a c
\end{aligned}
\]

For example, look at the equation
\[
3(x-2)=18 .
\]

One solution is to divide both sides by 3 to get
\[
x-2=6 .
\]

Then, add 2 to both sides get \(x=8\).
An alternative method is to use the Distributive Law. Start with
\[
3(x-2)=18 .
\]

Distribute three on the left side. Multiply 3 by \(x\) and by 2 to get
\[
3 x-6=18
\]

Add 6 to both sides to get
\[
3 x=24 .
\]

Divide both sides by 3 to get \(x=8\).

What is the value of \(x\) if \(5(x+3)-(3 x-4)=9\) ?
E. -5
F. -1
G. 3
H. 6

\section*{Here's How to Crack It}

Because there are two different sets of parentheses, each will have to be handled individually using the distributive law. First, distribute the 5 in the first term to get \(5 x+15-(3 x-4)=9\). The negative sign in front of \((3 x-4)\) is the same as multiplying it by -1 , so distribute -1 to find \(5 x+15-3 x+4=9\). Combine like terms \(5 x-3 x=2 x\) and \(15+4=19\), and you get \(2 x+19=9\). Subtract 19 from both sides to get \(2 x=-10\). Divide both sides by 2 to get \(x=-5\), which is ( E ).

\section*{Inequalities}

Let's begin with a simple number sentence. If the variable \(x\) equals 10, this can be written as
\[
x=10
\]

However, if there is another variable, \(y\), that is less than 10 , this requires a symbol that means less than. For less than, use the symbol <. So, write the inequality this way:
\[
y<10
\]

Now, suppose that you need to show that yet another variable, \(n\), is greater than 10. You will need to use a symbol that means greater than. For greater than, we use the symbol >. So, we write the inequality this way:
\[
n>10
\]

Notice that the open side of the less than and greater than signs faces the greater number or term.

Sometimes a variable might be less than or equal to a number. Or, a variable might be greater than or equal to a number. In these cases, a horizontal line is included just underneath the \(<\) or \(>\). Think of the horizontal line as part of the
equal sign. This is how to write it:
\[
\begin{gathered}
p \leq 10 \\
q \geq 10
\end{gathered}
\]

Each of these can be expressed graphically using a number line.
\begin{tabular}{|l|c|}
\hline\(y<10\) & \(\longleftrightarrow\) \\
\hline\(n>10\) & \(\longleftrightarrow\) \\
\hline\(p \leq 10\) & \(\longleftrightarrow\) \\
\hline\(q \geq 10\) & \(\longleftrightarrow\) \\
\hline
\end{tabular}

We're Not Number Lyin'
We told you that number lines would be important back in Chapter 10. Sketching or visualizing complicated inequalities can help you to avoid basic mistakes.

Graph less than by shading the number line to the left and using an open circle. Graph greater than by shading the number line to the right and using an open circle. Graph less than or equal to by shading the number line to the left and using a closed circle. Graph greater than or equal to by shading the number line to the right and using a closed circle.

Solving inequalities is just like solving equations but with an extra rule. Start with the inequality
\[
5 x-2<7 x+4
\]

Just like in an equation, isolate the variable. Subtract \(7 x\) from both sides to get
\[
-2 x-2<4
\]

Add 2 to both sides to get
\[
-2 x<6
\]

Divide both sides by -2 . The extra rule with inequalities is:

When both sides of an equation are multiplied or divided by a negative number, the inequality sign must be flipped.

Thus, diving both sides by -2 results in
\[
x>-3
\]

Which number line below provides the solution to the inequality \(-6 \leq 3 x \leq\) 9 ?
A.

B.

C.

D.


\section*{Here's How to Crack It}

Begin by isolating \(x\). Even though it is a three-part inequality, the method is the same. Since \(x\) is multiplied by 3, divide all three parts by 3. Because 3 is a positive number, do not flip the sign. The result is \(-2 \leq x \leq 3\). Therefore, the graph has to be shaded to the right of -2 but to the left of 3 . The graph that does this is (A).

\section*{Translating Word Problems}

Sometimes the key to word problems is to translate the information in the question into a linear equation.
\begin{tabular}{|l|l|}
\hline \multicolumn{2}{|c|}{ English to Math Translation Table } \\
\hline English & Math \\
\hline what, how much, how many & Variable \\
\hline \begin{tabular}{l} 
is, are, was, were, (any form of the \\
verb to be), equals, makes
\end{tabular} & \(=\) \\
\hline of, product, times & Multiplication \\
\hline quotient, divided by, per & Division \\
\hline sum, total, more than & Addition \\
\hline difference, less than & Subtraction \\
\hline Percent & \(\geq\) \\
\hline At least & \(\leq\) \\
\hline No more than & \\
\hline
\end{tabular}

You might recall some of these terms from the table on this page. That's all that translations are: identifying math vocabulary and using the corresponding symbols for each term.

What is \(40 \%\) of half the difference between 284 and 178 ?


\section*{Here's How to Crack It}

Translate each part from English to math. First, what translates to a variable. Use \(x\). Then, is translates to \(=\). Then, \(40 \%\) translates to \(\frac{40}{100}\). Then, of translates to multiplication. Then, half translates to \(\frac{1}{2}\) ( ). Finally, the difference between 284 and 178 translates to 284-178. Therefore, the whole question translates to \(x=\) \(\frac{40}{100} \times \frac{1}{2}(284-178)\). Simplify the right side. Subtract inside the parentheses to
get \(x=\frac{40}{100} \times \frac{1}{2}(106)\). Rewrite 106 as \(\frac{106}{1}\) to get \(x=\frac{40}{100} \times \frac{1}{2} \times \frac{106}{1}\). Multiply the fractions to get \(x=\frac{4,240}{200}=\frac{106}{5}=21.2\).

Jamie is renting a scooter for a 45 -mile drive around the island. Gas costs \(\$ 2.80\) per gallon. If Jamie's scooter gets 30 miles per gallon, how much does Jamie spend on gas for his drive?
A. \(\$ 1.50\)
B. \(\$ 2.40\)
C. \(\$ 3.60\)
D. \(\$ 4.20\)

\section*{Here's How to Crack It}

The question asks how much Jamie spends on gas for his drive. The question says that gas costs \(\$ 2.80\) per gallon, so \(2.80=\frac{\text { dollars }}{\text { gallons }}\). To get the number of dollars, determine the number of gallons. Jamie's scooter gets 30 miles per gallon, so \(30=\frac{\text { miles }}{\text { gallons }}\). The drive is 45 miles, so \(30=\frac{45}{g}\), where \(g\) is the number of gallons. Multiply both sides by \(g\) to get \(30 g=45\). Divide both sides by 30 to get \(g=\frac{45}{30}=1.5\). Therefore, using the first equation, \(2.80=\frac{d}{1.5}\), where \(d\) is the number of dollars. Multiply both sides by 1.5 to get \(d=4.20\), which is (D).

\section*{ALGEBRAIC EQUATIONS AND INEQUALITIES DRILL}
1. If \(5 a+3 b=15\), what is \(a\), in terms of \(b\) ?
A. \(a=\frac{3}{5} b+3\)
B. \(a=-\frac{3}{5} b+3\)
C. \(a=\frac{3}{5} b+15\)
D. \(a=-\frac{3}{5} b+15\)
2. Which number line below shows the solution set for the inequality \(6 x-20\) \(<2<3 x+1\) ?
E.

F.

G.

H.

3. Jennifer has at least 20 more dollars than Andrew. Which of the following describes the relationship between the number of dollars Jennifer has ( \(j\) ) and the number of dollars Andrew has (a) ?
A. \(20-j \leq a\)
B. \(20-a \leq j\)
C. \(j-a \leq 20\)
D. \(j-a \geq 20\)
4. A jar with a maximum capacity of \(m\) marbles, currently holds 24 . If 9 marbles are removed, the jar will be \(\frac{1}{3}\) full. What is the value of \(m\) ?
E. 5
F. 11
G. 45
H. 99
5. To run a lemonade stand, Jennifer has fixed costs of \(\$ 20\) per hour. Each cup of lemonade costs her \(\$ 0.20\) to make, and she charges \(\$ 1\) per cup of lemonade. If she sells 50 cups of lemonade in an hour, what is her profit for that hour?
A. \(\$ 10\)
B. \(\$ 20\)
C. \$30
D. \(\$ 40\)

\section*{ALGEBRAIC EQUATIONS AND INEQUALITIES DRILL EXPLANATIONS}
1. B The question asks for \(a\), in terms of \(b\), so isolate \(a\). Subtract \(3 b\) from both sides to get \(5 a=-3 b+15\). Divide both sides by 5 to get \(a=-\frac{3}{5}\) \(b+3\), which is (B).
2. \(\mathbf{F}\) Because there are terms including \(x\) in two parts of the three-part inequality, it has to be solved by breaking it into to two separate inequalities: \(6 x-20<2\) and \(2<3 x+1\). Start with \(6 x-20<2\). Add 20 to both sides to get \(6 x<22\). Divide both sides by 6 to get \(x<\frac{22}{6}=\) \(\frac{11}{3}=3 \frac{2}{3}\). Now, go to \(2<3 x+1\). Subtract 1 from both sides to get \(1<\) \(3 x\). Divide both sides by 3 to get \(\frac{1}{3}<x\). Put the two together to get \(\frac{1}{3}<\) \(x<3 \frac{2}{3}\). Shade the number line to the left of \(\frac{1}{3}\) and to the right of \(3 \frac{2}{3}\), which is (F).
3. D Translate the statement into an inequality. First, Jennifer translates to \(j\). Then, has at least translates to \(\geq\). Then 20 more dollars than Andrew translates to \(20+a\). Therefore, the statement translates to \(j \geq 20+a\). However, this is not a choice. To make this equivalent to a choice, subtract \(a\) from both sides to get \(j-a \geq 20\), which is (D).
4. G Translate the information into an equation. The question says, If 9
marbles are removed, the jar will be \(\frac{1}{3}\) full. The jar currently holds 24 marbles, so if 9 are removed, there will be \(24-9\) remaining in the jar. This would make the jar \(\frac{1}{3}\) full. Since the jar has a maximum capacity of \(m\) marbles, \(\frac{1}{3}\) full translates to \(\frac{1}{3} m\). Therefore, \(24-9=\frac{1}{3} m\). Simplify the left side to get \(15=\frac{1}{3} m\). Divide both sides by \(\frac{1}{3}\), which is the same as multiplying by 3 , to get \(m=45\), which is (G).
5. B To find the profit, do Revenue - Cost. She sells 50 cups of lemonade for \(\$ 1\) each, so her revenue is \(50 \times \$ 1=\$ 50\). To get the cost, add the fixed cost to the cost of the cups of lemonade. The cost for each cup is \(\$ 0.20\), so the cost for 50 cups is \(50 \times \$ 0.20=\$ 10\). Add the \(\$ 20\) fixed cost to get a total cost of \(\$ 10+\$ 20=\$ 30\). Therefore, the profit is \(\$ 50\) \(-\$ 30=\$ 20\), which is (B).

\section*{FUNKY FUNCTIONS}

Though this type of question is much less common on the redesigned SHSAT, you may still want to familiarize yourself with these steps in the event that it appears. That's because this is a weird type of question, one that uses unfamiliar symbols to test your ability to follow instructions.

\section*{Simon Says}

The operations you're already familiar with have straightforward rules. Any new function you encounter on the test just combines a bunch of them. Think of it as a game of Simon Says, in which you do exactly what the instructions say. For instance, if \(\& d=d+5\) and the question asks for \(\& 13\), treat the 13 as \(d\), and place it in the equation anywhere you see \(d\). Some include multiple variables, but the process remains the same. Follow the ordering of the variables and plug in exactly as shown in the example. For example, if \(a^{*} b=b-a\), place the number
in the \(a\) spot where the \(a\) is in the equation and likewise for \(b\). Let's give it a try.

For all real numbers, \(a^{b}=a(b-a)\).
\(7^{4}=\) ?
A. 21
B. 12
C. -12
D. -21

\section*{Here's How to Crack It}

The definition contains two variables, so it is imperative that you place the numbers in exactly the right order. For \(7^{4}, 7\) replaces \(a\) and 4 replaces \(b\) in the equation: \(7^{4}=7(4-7)\). Now, follow the order of operations to simplify the answer. Work inside the parentheses first to find \(7^{4}=7(-3)\), and then multiply 7 and -3 , which yields -21 . The correct answer is (D).

Sometimes, the SHSAT will give the result of a definition, which is a great opportunity to use PITA. Simply follow the definition exactly as you see it, paying careful attention to variable placement, and you will know how to do the rest!

For all real numbers, \(c \# d=(d-c)(c+d)\). Which of the following is equal to 24 ?
E. 6\#5
F. \(1 \# 5\)
G. 5\#1
H. 5\#6

\section*{Here's How to Crack It}

Use PITA to test each answer choice, being careful to follow the order of
operations. Because this is an unknown function, you can begin with any choice -you won't necessarily know which of these will yield larger or smaller answers. Try (E): \(6 \# 5=(5-6)(6+5)=(-1)(11)=-11\), so eliminate (E). Try (F) next: \(1 \# 5=(5-1)(1+5)=(4)(6)=24\). This choice works, so the correct answer is \((\mathrm{F})\).

FUNKY FUNCTIONS DRILL
1. For all real numbers \(n, \$ n=10 n-10\). If \(\$ n=120\), what does \(n\) equal?
A. 11
B. 12
C. 13
D. 120
2. For all real numbers \(d\) and \(y, d_{\dot{c}} y=(d \times y)-(d+y)\). [Example: \(3 \dot{¿} 2=(3\) \(\times 2)-(3+2)=6-5=1\) ]
\((2 \dot{¿} 4) \times(3 \dot{¿} 6)=\)
E. \(\left(9 \dot{¿}^{3}\right)+3\)
F. \((6 \dot{c} 4)+1\)
G. \((5 \dot{¿} 3)+4\)
H. \((8\) ¿ 4\()+2\)

\section*{FUNKY FUNCTIONS DRILL EXPLANATIONS}
1. C Since the question asks for a specific outcome, 120, this is an excellent opportunity to use PITA to solve. Start with one of the middle numbers to determine if the answer is too large, too small, or just right. Consider (C): \(\$ 13=10(13)-10=130-10=120\). Because this is the correct result, you do not need to test the other answers. The correct answer is (C).
2. E This is a multistep, time-consuming question! It might be good to do this one on a second pass and find quicker points first. First, follow the exact instructions (remember, this is a game of Simon Says) to find the result of the expression \((2 \dot{~} 4) \times(3 ; 6) .(2 \dot{¿} 4) \times(3 \dot{i})=[(2\) \(\times 4)-(2+4)] \times[(3 \times 6)-(3+6)]\). Simplify the equation by working inside the parentheses first: [(8) - (6)] \(\times[(18)-(9)]\), and further to [2] \(\times[9]=18\). Next, use PITA to test the answer choices to see which of the choices is equal to 18. It may be difficult to know the trends in the answer choices, so start with (E): \((9\) ¿ 3\()+3=(9 \times 3)-(9+3)+3\). Work inside the parentheses first to find that (27) - (12) +3 , and then work left to right with addition and subtraction: \(27-12=15\), and 15 \(+3=18\). This is the correct outcome, so there is no need to test the other choices. The correct answer is (E).

Chapter 13
Geometry

Geometry questions make up about 15-20\% of the SHSAT, and cover a broad range of topics. You'll often find different geometric facts combined into a single question, so if you're getting questions of this sort wrong, use this section to pinpoint exactly where the errors are occurring.

\section*{ANGLES}

There are four types of angles categorized by their sizes and are measured in degrees. The four types of angles are acute, right, straight, and obtuse.
\begin{tabular}{|l|l|l|l|l|}
\hline Type of Angles & Acute Angles & Right Angles & \begin{tabular}{l} 
Obtuse \\
Angles
\end{tabular} & \begin{tabular}{l} 
Straight \\
Angles
\end{tabular} \\
\hline \begin{tabular}{l} 
Degree \\
Measurement
\end{tabular} & Less than \(90^{\circ}\) & \(90^{\circ}\) & \begin{tabular}{l}
\(90^{\circ}-180^{\circ}\) \\
(exclusive)
\end{tabular} & \(180^{\circ}\) \\
\hline
\end{tabular}

To visualize the types of angles, it is good to start with right angles. Typically, a right angle is marked with a small square where it measures \(90^{\circ}\). The angle below pictures this.


Acute angles are smaller than right angles by definition (see below).


On the other hand, obtuse angles are larger than right angles but smaller than straight angles (see below).


Bisecting Angles
When the SHSAT refers to a bisected angle, it refers to an angle that has been split in half. By bisecting an angle, you are splitting the original angle into two equal, smaller angles.

\section*{Categorizing Angles Practice}

Name the type of angle. The answers are in a sidebar on this page.
1. \(40^{\circ}\)
2. \(180^{\circ}\)
3. \(120^{\circ}\)
4. \(75^{\circ}\)
5. \(90^{\circ}\)

\section*{Naming Angles}

Angles are categorized not only by their degree measures, but also by the lines that create them. For example, let's look at the angle pictured below.


The point at which the two lines intersect is called the vertex. In a figure that pictures an angle formed by two lines, the vertex is all you need to specify which angle you're referring to. The angle shown above would be called "angle \(B\)," which could also be written with an angle sign as \(\angle B\).

However, vertices made up of more than two lines create multiple angles. See the example below.


This vertex is formed by lines \(X W, Y W\), and \(Z W\). Since there are multiple angles
formed in this case, it is unacceptable to simply call the above angle \(\angle W\). This would make it impossible to distinguish between the two smaller acute angles ( \(25^{\circ}\) and \(37^{\circ}\) ) and the larger acute angle \(\left(25^{\circ}+37^{\circ}=62^{\circ}\right)\). To clarify, you would include three points in the name (usually written in alphabetic order) with the vertex point in the middle. The \(25^{\circ}\) angle is \(\angle X W Y\) or \(\angle Y W X\), the \(37^{\circ}\) angle is \(\angle Y W Z\) or \(\angle Z W Y\), and the \(62^{\circ}\) angle is \(\angle X W Z\) or \(\angle Z W X\).

\section*{Naming Angles Practice}

Identify each angle by its three-point name.

3.


Naming Angles Practice Answers
1. \(\angle D E F\) or \(\angle F E D\)
2. \(\angle M N O\) or \(\angle O N M\)
3. \(\angle G H I\) or \(\angle I H G\)

\section*{Complementary and Supplementary Angles}

As we saw with \(\angle X W Z\) above, larger angles can be made up of smaller angles. For example, in the figure below, \(\angle A B C=40^{\circ}\) and \(\angle C B D=30^{\circ}\). This means that the larger angle can be found by adding together the sum of its parts: \(\angle A B D\) \(=70^{\circ}\).


A special case occurs when the two smaller angles add up to either \(90^{\circ}\) or \(180^{\circ}\). In the diagram below, \(\angle F G I\) is a right angle. This means that \(x+y=90^{\circ}\).


Because they add up to \(90^{\circ}, \angle F G H\) and \(\angle H G I\) are called complementary angles. The SHSAT may have you incorporate this knowledge by telling you that \(y\) is equal to \(20^{\circ}\) and then asking you to solve for \(x\). Since the two angles are complementary, you would simply subtract \(y\) from \(90^{\circ}\) to get \(x=70^{\circ}\).

If two smaller angles add up to \(180^{\circ}\) to make a straight line, as seen with \(\angle J K M\) and \(\angle L K M\) below, they are called supplementary angles.


If you are told that \(y=60\), then you are able to figure out that \(x=120\) the same way we did above.

\section*{ANGLES DRILL}
1. Which of the following pairs of angles could equate to supplementary angles?
A. Two acute angles
B. An obtuse angle and an acute angle
C. Two obtuse angles
D. A right angle and an acute angle
2.


In the figure above, points \(B, F\), and \(E\) are collinear. What is the sum of the bisected angle and the smallest acute angle?
E. \(80^{\circ}\)
F. \(100^{\circ}\)
G. \(140^{\circ}\)
H. \(180^{\circ}\)
3.


In the figure above, \(\angle R U T\) and \(\angle T U V\) are supplementary. If \(\angle S U V\) is twice \(\angle R U S\), what is the measure of \(\angle S U V\) ?
A. \(20^{\circ}\)
B. \(40^{\circ}\)
C. \(100^{\circ}\)
D. \(120^{\circ}\)

\section*{ANGLES DRILL EXPLANATIONS}
1. B Supplementary angles are angles that add up to \(180^{\circ}\). Two acute angles, which are each less than \(90^{\circ}\), could never add up to \(180^{\circ}\) and thus (A) is eliminated. Two obtuse angles, which are each greater than \(90^{\circ}\), would add up to more than \(180^{\circ}\) and thus (D) is eliminated. Similarly, a right angle measuring \(90^{\circ}\) and an obtuse angle measuring more than \(90^{\circ}\) would add up to more than \(180^{\circ}\) and thus (C) is eliminated. However, an acute angle and an obtuse angle could add up to \(180^{\circ}\) and therefore be supplementary, which makes the answer (B).
2. \(\mathbf{H} \quad\) The bisected angle in this figure is \(\angle C F E\), which measures \(140^{\circ}\left(70^{\circ}\right.\) \(+70^{\circ}\) ). The smallest acute angle in the figure is angle \(\angle B F C\), which measures \(40^{\circ}\). Therefore, the sum of these two angles is \(180^{\circ}\). Additionally, you could realize that all three angles make up the straight line \(B E\) and therefore sum to \(180^{\circ}\), (H).
3. D Since \(\angle R U T\) and \(\angle T U V\) are supplementary, they add up to \(180^{\circ}\), forming line \(R V\). Since line \(R V\) can also be broken up into \(\angle R U S\) and \(\angle S U V\), you know that these two angles also add up to \(180^{\circ}\). If you let \(x=\angle R U S\) and \(2 x=\angle S U V\), you can set up the following: \(x+2 x=180^{\circ}\)
\[
\begin{aligned}
3 x & =180^{\circ} \\
x & =60^{\circ}
\end{aligned}
\]

Therefore, \(\angle S U V=2 x=2\left(60^{\circ}\right)=120^{\circ}\), which is (D).

\section*{SHAPES}

In addition to knowing angles, you'll need to know various shapes, which are defined by a mixture of the number of sides they have and the type of angles they contain.

\section*{Triangles}

A triangle is any three-sided figure. The sum of the angles in all triangles is
\(180^{\circ}\). Triangles can come in many shapes and forms.
Any triangle with two equal sides is an isosceles triangle. If two sides of a triangle are equal, then the angles opposite those sides are always equal. Similarly, if two angles of a triangle are equal, then the sides opposite those angles are always equal, so the triangle is isosceles.


This particular isosceles
triangle has two equal sides (of length 6) and therefore two equal angles ( \(40^{\circ}\) in this case).

\[
\begin{gathered}
n=65^{\circ} \\
y=9
\end{gathered}
\]

An equilateral triangle is a triangle that has three equal sides. If all the sides are equal, then all the angles must be equal. Each angle in an equilateral triangle equals \(60^{\circ}\).

A right triangle is a triangle with one \(90^{\circ}\) angle. The side opposite the \(90^{\circ}\) angle is known as the hypotenuse, while the other two sides are known as the legs.


This is a right triangle.
It is also an isosceles triangle.
What does that tell you?
For all right triangles, if two sides are known, the third side can be determined using the Pythagorean Theorem:

\[
a^{2}+b^{2}=c^{2}
\]

When using the Pythagorean Theorem, remember that \(a\) and \(b\) represent the two legs of the right triangle, while \(c\) represents the hypotenuse.


If \(b=45\), then \(v^{2}=\)
A. 32
B. 25
C. 16
D. 5

\section*{Here's How to Crack It}

Fill in \(b=45\). Two angles of the triangle are known, so the third can be determined. Call the missing angle \(x^{\circ}\). Because the angles in a triangle have a sum of 180, \(90+45+x=180\). Simplify the left side to get \(135+x=180\). Subtract 135 from both sides to get \(x=45\). Since there are two \(45^{\circ}\) angles, this triangle is isosceles. Since the side opposite one of the \(45^{\circ}\) angles is equal to 4 , so is the side opposite the other \(45^{\circ}\). Since two sides of a right triangle are known, the Pythagorean Theorem can be used to find the third. According to the Pythagorean Theorem, \(a^{2}+b^{2}=c^{2}\), where \(a\) and \(b\) are the legs and \(c\) is the hypotenuse. The two legs are both 4 and the hypotenuse is \(v\), so \(4^{2}+4^{2}=v^{2}\). Simplify the left side to get \(16+16=v^{2}\) and \(32=v^{2}\). The question asks for the value of \(v^{2}\), so there is no need to continue further. The value of \(v^{2}\) is 32 , which is ( A ).

\section*{Quadrilaterals}

A quadrilateral is any four-sided figure. The angles in any quadrilateral have a sum of \(360^{\circ}\). Although quadrilaterals also come in shapes and forms, only a few of the more common types should be expected on the SHSAT.

A parallelogram is a quadrilateral in which opposite sides are parallel. There are other key properties to parallelograms, as well. Opposite sides are equal, opposite angles are equal, and adjacent angles are supplementary.

To give some perspective: A rectangle is a parallelogram in which all angles are right angles. A square is a rectangle in which all four sides are equal.


In the parallelogram above, what is the value of \(y\) ?


\section*{Here's How to Crack It}

The question asks for the value of \(y\) and states that the figure is a parallelogram, so use the angle properties of the parallelogram: opposite angles are equal and adjacent angles are supplementary. The two given angles, \(y^{\circ}\) and \(64^{\circ}\), are adjacent, and so they are supplementary, or equal to \(180^{\circ} .180^{\circ}-64^{\circ}=y^{\circ}\), which means \(y=116\).

\section*{Circles}

Not all shapes are made up of straight lines and angles. One example of such a shape is called a circle. A circle is a collection of points that are a fixed distance from a center point.


A radius is a segment from the center to any point on the outside of the circle. Each radius of a circle has the same length. A diameter is any segment through the center whose end points are on the outside of the circle. Any two diameters of a circle have the same length, which is twice the length of any radius of the circle.


In the circle above with center \(O\), what is the value of \(x\) ?
A. 4
B. 5
C. 6
D. 7

\section*{Here's How to Crack It}

The question asks for the value of \(x\). The lengths of the radius and diameter are given, in terms of \(x\). Since the length of the diameter is always twice the length of the radius, set up the equation \(3 x-3=2(x+1)\). Distribute on the right side to get \(3 x-3=2 x+2\). Add 3 to both sides to get \(3 x=2 x+5\). Subtract \(2 x\) from both sides to get \(x=5\), which is (B).

\section*{SHAPES DRILL}
1.


12
In the right triangle above, what is the value of \(a\) ?
A. 3
B. 4
C. 9
D. 15
2.


In the figure above, \(V W X Y\) is a parallelogram. The measure of \(\angle Z X W\) is \(40^{\circ}\), and the measure of \(\angle X Z W\) is \(75^{\circ}\). What is the measure of \(\angle Y V Z\) ?
E. 55
F. 65
G. 115
H. 125
3. A scale drawing of a circular painting has a radius of 10 centimeters. If the actual painting has a diameter of 50 inches, one centimeter in the drawing
represents how many inches in the actual painting?


\section*{SHAPES DRILL EXPLANATIONS}
1. A Use the Pythagorean Theorem, \(a^{2}+b^{2}=c^{2}\), to get \((3 a)^{2}+12^{2}=(5 a)^{2}\). Simplify to get \(9 a^{2}+144=25 a^{2}\). Subtract \(9 a^{2}\) from both sides to get \(144=16 a^{2}\). Divide both sides by 16 to get \(9=a^{2}\). Take the square root of both sides to get \(a=3\), which is (A).
2. \(\mathbf{G}\) The question asks for the measure of \(\angle Y V Z\), which is an angle of a parallelogram, so use the angle properties of the parallelogram: opposite angles are equal and adjacent angles are supplementary. There are two angles known in triangle \(X W Z\), so find the third. Let \(\angle V W X\) have a measure of \(a^{\circ}\). Since the sum of the angles in a triangle is \(180^{\circ}, 75+40+a=180\). Simplify the left side to get \(115+a=180\). Subtract 115 from both sides to get \(a=65\). Since \(\angle V W X\) and \(\angle Y V W\) are adjacent angles in a parallelogram, they are supplementary. Let \(\angle Y V W\) have a measure of \(b^{\circ} .65+b=180\). Subtract 65 from both sides to get \(b=115\), which is (G).
3. 2.5 The question refers to a scale, so use proportions. The radius of the drawing is 10 centimeters, and the diameter of the painting is 50
inches. Make sure to compare like things. Since the diameter is twice
the radius, the diameter of the drawing is 20 centimeters. The question
asks how many inches are represented by 1 centimeter on the drawing, so set up the proportion \(\frac{20 \text { centimeters }}{50 \text { inches }}=\frac{1 \text { centimeter }}{x \text { inches }}\)
.Cross-multiply to get \(20 x=50\). Divide both sides by 20 to get \(x=\)
\[
\frac{50}{20}=\frac{5}{2}=2.5 .
\]

\section*{FORMULAS}

Many geometry questions will involve the use of formulas to determine things like perimeter, area, surface area, and volume. These formulas are not provided on the test, so they have to be memorized.

\section*{Perimeter}

The perimeter of a figure is the distance around the outside of a figure. To find the perimeter, find the sum of the individual sides. For example, look at the pentagon below.


10
The perimeter of the pentagon is \(6+6+8+8+10=38\).
Most of the time, formulas are not needed for perimeter. It's as simple as the sum of the sides. Sometimes, however, a formula can come in handy.

Perimeter of an equilateral triangle: \(P=3 s\), where \(s\) is a side of the triangle.

Perimeter of a Rectangle: \(P=2 l+2 w\), where \(l\) is the length
and \(w\) is the width.

Perimeter of a Square: \(P=4 s\), where \(s\) is the side of the square.

Circles do not have sides, so the term perimeter is not used there. Instead, the term used for circles is circumference. Both words describe the same concept: the distance around a figure. To find the circumference of a circle, one of two formulas is needed.

Circumference of a Circle: \(C=2 \pi r\), where \(r\) is the radius of the circle.

Circumference of a Circle: \(C=\pi d\), where \(d\) is the diameter of the circle.

The ratio of the length of a rectangle to its width is \(3: 5\). If the perimeter of the rectangle is 640 , what is the length of the rectangle?
A. 120
B. 200
C. 240
D. 400

\section*{Here's How to Crack It}

The question refers to a ratio, so use the ratio box. Create a column for the length, width, and the total. The total is the sum of the length and the width. The question provides the perimeter. The formula for the perimeter of a rectangle is \(P\) \(=2 l+2 w\). Substitute \(P=640\) to get \(640=2 l+2 w\). Divide both sides by 2 to get \(320=l+w\). Put 320 in the Actual \#'s row under Total.
\begin{tabular}{|l|l|l|l}
\multicolumn{1}{l}{ Length } & \multicolumn{1}{l}{ Width } & \multicolumn{1}{l}{ Total } \\
\hline 3 & 5 & & \begin{tabular}{l} 
Ratio \\
Multiplier
\end{tabular} \\
\hline & & & \begin{tabular}{l} 
Actual \#'s
\end{tabular} \\
\hline & & 320 &
\end{tabular}

Add across the top row to get 8 under Total. Multiply downward, and since \(8 \times\) \(40=320\), fill in 40 across the multiplier row. Multiply downward to get \(3 \times 40=\) 120 under Length and \(5 \times 40=200\) under Width.
\begin{tabular}{|l|l|l|l}
\multicolumn{1}{l}{ Length } & Width & \multicolumn{1}{l}{ Total } \\
\hline 3 & 5 & 8 & Ratio \\
\hline 40 & 40 & 40 & \begin{tabular}{l} 
Multiplier \\
Actual \#'s
\end{tabular} \\
\hline 120 & 200 & 320 &
\end{tabular}

The question asks for the length, which is 120 , so the answer is (A).
\(\qquad\)

\section*{Area}

Whereas perimeter looks at the distance around a figure's border, area looks at the amount of space take up within it. The formulas that determine area vary based on the figure.

Area of a Triangle: \(A=\frac{1}{2} b h\), where \(b\) is the base and \(h\) is the
height of the triangle.

For a triangle, any side can be selected as the base. Usually, the bottom side is selected but doesn't have to be. The height is the length of the segment that is perpendicular to the base and connects to the opposite angle. The height doesn't have to be one of the sides. In fact, for triangles that aren't right triangles, the
height will never be one of the sides.


Notice in the example on the right that the height could fall outside the triangle.

Area of a Parallelogram: \(A=b h\), where \(b\) is the base and \(h\) is the height.

The height of the parallelogram works the same way as the height of the triangle. It is the length of the segment that is perpendicular to the base and the side opposite the base. Unless the parallelogram is a rectangle, the height will never be one of the sides.


If the parallelogram is a rectangle, the formula still applies. However, the height of the rectangle is one of the sides. In that case, a simplified version of the formula can be used.

Area of a Rectangle: \(A=l w\), where \(l\) is the length and \(w\) is the width of the rectangle.

Similarly, since a square is a rectangle, which is a parallelogram, both of these
formulas also apply to a square. However, because both the length and width are equal, an even more simplified version of the formula can be used.

Area of a Square: \(A=s^{2}\), where \(s\) is a side of the square.

Even though a circle doesn't have any sides, its area can still be determined using a formula.

Area of a Circle: \(A=\pi r^{2}\), where \(r\) is the radius of the circle.

A circle has a circumference of \(10 \pi\). What is the circle's area?
A. \(5 \pi\)
B. \(10 \pi\)
C. \(20 \pi\)
D. \(25 \pi\)

\section*{Here's How to Crack It}

The question asks for the circle's area, so use the formula \(A=\pi r^{2}\). To find the area, the radius is needed. The question says that the circumference of the circle is \(10 \pi\), so use the circumference formula that involves radius: \(C=2 \pi r\). Substitute \(C=10 \pi\) to get \(10 \pi=2 \pi r\). Divide both sides by \(\pi\) to get \(10=2 r\). Now, divide both sides by 2 to get \(5=r\). Substitute this value of \(r\) into the original area formula to get \(A=\pi(5)^{2}=25 \pi\), which is (D).

\section*{Coordinate Geometry}

Some geometry questions on the SHSAT will involve the coordinate plane. The coordinate plane has a horizontal \(x\)-axis intersecting a vertical \(y\)-axis. That point of intersection is called the origin. Points on the coordinate plane have coordinates in the form \((x, y)\). The \(x\)-coordinate is based on the point's horizontal position. Points to the left of the \(y\)-axis have negative \(x\)-coordinates, and points to the right of the \(y\)-axis have positive \(x\)-coordinates. The absolute value of the \(x\) coordinate is the horizontal distance to the \(y\)-axis. Similarly, the \(y\)-coordinate is based on the vertical position. Points above the \(x\)-axis have positive \(y\) coordinates, and points below the \(y\)-axis have negative coordinates with absolute values equal to the point's vertical distance to the \(x\)-axis. Look at the coordinate plane below.


In the example above, point \(A\) has a positive \(x\) and \(y\) value (3,2), and point \(C\) has a negative \(x\) and \(y\) value ( \(-3,-2\) ). Can you identify the value of points \(B\) and \(D\) ? One is \((2,-3)\), and the other is \((-2,3)\) : always be really careful that you're putting the \(x\) value first.

\section*{Surface Area}

The calculations for figures thus far have been for two-dimensional figures, but the SHSAT might sometimes add a third dimension. One of the ways of measuring three-dimensional figures is using surface area. The surface area of a three-dimensional figure is the sum of the areas of all the two-dimensional faces. Like perimeter, this often does not require a specific formula. However, some formulas may be useful.


What is the surface area of the right triangular prism shown above?
A. 36
B. 48
C. 60
D. 72

\section*{Here's How to Crack It}

To find the surface area of the prism, find the area of each face. In a prism, opposite sides are congruent, so the two triangles have the same area. They are
right triangles, so use the legs as the base and height. The area of a triangle can be found using the formula \(A=\frac{1}{2} b h=\frac{1}{2}(3)(4)=6\). The other faces are rectangles, so use the formula \(A=l w\). The rectangle on the bottom is 4 by 4 , so \(A\) \(=l w=(4)(4)=16\). The upright rectangle is 3 by 4 , so \(A=l w=(3)(4)\). The slanted rectangle is 5 by 4 , so \(v=l w=(5)(4)=20\). Surface area is the sum of the areas of the two triangles and three rectangles, which is \(6+6+16+12+20=\) 60 , or (C).

\section*{Volume}

Another way of measuring three-dimensional figures is using volume. Volume can be thought of as being for three-dimensional figures what area is for twodimensional figures: the amount of space the figure takes up. There are three main volume formulas that will be needed on the SHSAT.

Volume of a Prism: \(V=B h\), where \(B\) is the area of the base and \(h\) is the height of the prism.

Volume of a Rectangular Prism: \(V=l w h\), where \(l\) is the length, \(w\) is the width, and \(h\) is the height of the prism.

Volume of a Cube: \(V=s^{3}\), where \(s\) is the length of the edge or
"side" of the cube.

A fish tank in the shape of a hexagonal prism has a height of 7 ft . When full, the fish tank holds 210 cubic feet of water. What is the area of the hexagonal base?
A. 3 sq ft
B. 10 sq ft
C. 18 sq ft
D. 30 sq ft

\section*{Here's How to Crack It}

Although the question asks for the area of a hexagonal base, no knowledge of hexagons is needed. The question gives the volume of the prism, so use the formula \(V=B h\). To find the area of the base, find the value of \(B\). The volume is 210 and the height is 7 . Plug these into the formula to get \(210=7 B\). Divide both sides by 7 to get \(B=30\), which is (D).

\section*{FORMULAS DRILL}
1.


What is the perimeter of the figure above?
A. 20
B. 24
C. 26
D. 28
2.


What is the area of the right triangle above?
E. 30
F. 60
G. 78
H. 156
3. A cube has a surface area of 150 . What is the volume of the cube?
A. 5
B. 25
C. 100
D. 125
4.


What is the perimeter of the shaded rectangle shown in the figure above?
E. \(a+b+5\)
F. \(5+b-a\)
G. \(5(b-a)\)
H. 2(5) \(+2(b-a)\)
5. Kumail rides a bicycle that has a wheel with a diameter of 63 centimeters. If he makes 2 revolutions per second, what is his speed in centimeters per second?
(Use the approximation \(\frac{22}{7}\) for л.) A. 126
B. 198
C. 396
D. 792

\section*{FORMULAS DRILL EXPLANATIONS}
1. D To find the perimeter, add the sum of the sides on the outside of the figure. Since the two segments of length 4 are not on the outside, do not count these. The known sides are of length 3,6 , 3 , and 6 . Find the two missing sides, which are the slanted sides on the left and right of the figure. In each case, the missing side forms the hypotenuse of a right triangle, so use the Pythagorean Theorem: \(a^{2}+b^{2}=c^{2}\). Set \(a=3\) and \(b=4\) to get \(3^{2}+4^{2}=c^{2}\). Square 3 and 4 to get \(9+16=c^{2}\). Add 9 and 16 to get \(25=c^{2}\). Take the square root of both sides to get \(c=5\). The two missing sides are 5 and 5 , so the perimeter is \(5+5+3+3+\) \(6+6=28\), which is (D).
2. \(\mathbf{E} \quad\) The formula for the area of a triangle is \(A=\frac{1}{2} b h\). The base is given as 12. It may be tempting to use 13 as the height, but be careful. The height has to be perpendicular to the base. To find the perpendicular side, use the Pythagorean Theorem: \(a^{2}+b^{2}=c^{2}\). The hypotenuse, which is 13 , is \(c\), so \(a^{2}+12^{2}=13^{2}\). Square 12 and 13 to get \(a^{2}+144\)
\(=169\). Subtract 144 from both sides to get \(a^{2}=25\). Take the square root of both sides to get \(a=5\). Therefore, the height is 5 , which makes \(A=\frac{1}{2}(5)(12)=30\), or \((\mathrm{E})\).
3. D The question asks for the volume of the cube, so use the formula \(V=\) \(s^{3}\). Find the length of the side. The question says that the surface area is 150 . The formula for surface area of a cube is \(S A=6 s^{2}\), so \(150=\) \(6 s^{2}\). Divide both sides by 6 to get \(25=s^{2}\). Take the square root of both sides to get \(s=5\). Substitute this into the volume formula to get \(V=5^{3}\)
\(=125\), which is (D).
4. \(\mathbf{H} \quad\) To find the perimeter of a rectangle, use the formula, \(P=2 l+2 w\). The length of the figure is its distance from the \(y\)-axis to the edge of the figure, which is 5 . The height of the figure is the distance from the top of the figure to the bottom, which is the difference between the \(y\) coordinates of the top and bottom, which is \(b-a\). Therefore, the perimeter is \(P=2(5)+2(b-a)\), or \((\mathrm{H})\).
5. C When a question mentions revolution, use the fact that the distance a wheel travels in one revolution is equal to the circumference. The formula for circumference is \(C=\pi d\). The diameter is 63 , and the question says to use \(\frac{22}{7}\) for \(\pi\), so \(C=\frac{22}{7} \times 63=\frac{22}{7} \times \frac{63}{1}=\frac{22}{1} \times \frac{9}{1}=\)
198. The question asks for the speed in centimeters per second. Kumail makes 2 revolutions per second, so he travels \(2 \times 198=396\) centimeters per second, which is (C).

\section*{OVERLAPPING FIGURES}

SHSAT geometry questions love to include different types of information, whether that's using angles to find sides or information about a shape to calculate something about another shape.

Don't be intimidated by the language that's being used, or by the number of shapes. Break these down to their simpler parts and you will realize that the questions are often rather straightforward. To begin with, be solid on the following terms: Inscribed: drawn inside

\section*{Circumscribed: drawn outside}

Tangent: touching at only one point
Second, remember this very important rule:

If there is no figure, be sure to draw one for yourself.

Finally, determine what the figures have in common. Try writing down what you know versus what you need. Use the information given about one figure and determine how to get information about the other figure.


A circle is inscribed in the square above. If the perimeter of the square is 16 , what is the circumference of the circle?
A. \(2 \pi\)
B. \(4 \pi\)
C. \(8 \pi\)
D. \(16 \pi\)

\section*{Here's How to Crack It}

The question involves overlapping figures, so determine what is common to both figures. When a circle is inscribed in a square, the diameter of the circle has the same length as the side of the square. Use information about the square to determine the diameter of the circle. The square has a perimeter of 16 . The formula for the perimeter of a square is \(P=4 s\), so \(16=4 \mathrm{~s}\). Divide both sides by 4 to get \(s=4\). Therefore, the diameter of the circle is also 4 . The question asks for the circumference. Use the formula \(C=\pi d\) to get \(C=4 \pi\), which is (B).

Here's another rule that can help for questions of this nature:

Connect the points that are not already connected.

That is, just because a figure is given, that doesn't mean you can't continue to add to it. If two intersecting points of the figure are not connected by a straight line, go ahead and draw one! This may help you to literally make connections with how the measurements relate to multiple shapes at once.

Consider the following diagram:


Now, try drawing a line to connect the intersecting points together:


Hey, look! By drawing this line, you not only found the height of the triangle, but also the diameter of the circle. This measurement will help you to solve many questions of this nature.

\section*{The Power of Guesstimating}

Questions with diagrams can sometimes be solved by estimating amounts and values instead of the time-consuming "math" way of solving problems. Sometimes, it is necessary to do the work to arrive at the correct answer, but if
the answer choices are spread out enough, it can be possible to use the power of estimation to eliminate choices. Unless the figures are marked as not drawn to scale, you can trust that the figures are to scale.

Here are some really helpful numbers to keep in mind: \(\pi \cong 3, \sqrt{2} \cong 1.4\), and \(\sqrt{3} \cong 1.7\). Both the square roots are about 1.5, give or take, which is a useful estimation. To keep them separate, think of 2/14, Valentine's Day, and 3/17, St. Patrick's Day!


A circle is inscribed in square \(P Q R S\). What is the area of the shaded region?
A. \(16-6 \pi\)
B. \(16-4 \pi\)
C. \(16-3 \pi\)
D. \(16-2 \pi\)

\section*{Here's How to Crack It}

This is a great opportunity to guesstimate how much of the area is shaded, and decide which answer choice most closely matches. First find the area of the entire square. Recall that the area of a square is \(s^{2}\), so \(4^{2}=16\). Now, the shaded area looks like perhaps a quarter of the entire area, or at least something close to
that. Therefore, the answer must be something close to 4 . Now, evaluate the answer choices to see which one is closest, remembering that \(\pi\) is roughly 3 .

A: \(16-6(3)=-2\). Yikes! How can something have a negative area?
B: \(16-4(3)=4\). Keep it.
C: \(16-3(3)=7\). Seems a bit on the large side...almost half the entire area?
D: \(16-2(3)=10\). Definitely too big.
Okay, you say. What if I need to solve it the math way? The overlapping area is the circle inscribed in the square, and the key to any overlapping figure question is to find the points of the figure in which the shapes touch one another.


By connecting points that are not already connected, you can see that the side of the square is equal to the diameter of the circle. If the diameter of the circle is 4 , the radius must be 2 . The area of a circle is \(A=\pi r^{2}\), so the area of the circle is \(A\) \(=\pi(2)^{2}\). The shaded region is the difference of the square and the circle, so \(16-\) \(4 \pi\).

The correct answer is (B).

FIGURES DRILL
1.


In the figure above, a quarter circle with center \(O\) contains a triangle \(O Q R\). Line segment \(O Q\) is 4 . What is the length of \(Q R\) ?
A. 4
B. \(4 \sqrt{3}\)
C. 6
D. \(8 \sqrt{3}\)
2.


In the figure above, rectangle \(A B C D\) is inscribed in circle \(O\). If \(A O C\) is a straight line, what is the area of the circle?
E. 10
F. 25
G. \(25 \pi\)
H. \(100 \pi\)
3. A square is inscribed in a circle with a radius of 5 . What is the area of the square?

4.


In the figure above, triangle \(A B C\) is equilateral and \(A B\) has a length of 6 . Point \(D\) bisects \(A C\). What is the length of \(B D\) (not shown)?
A. 3
B. \(3 \sqrt{3}\)
C. 6
D. \(6 \sqrt{3}\)
5. A square garden is surrounded on all sides by a 2 -foot-wide walking path. If the garden has an area of 25 , what is the area of the walking path?


\section*{FIGURES DRILL EXPLANATIONS}
1. A The question gives some important information to take into account: this is a quarter circle, so \(\angle P O R\) is a right angle. This means that \(\angle Q O R\) must be \(60^{\circ}\) since \(30^{\circ}+60^{\circ}=90^{\circ}\). The question also states that line segment \(O Q\) is 4 . This is a special line segment, as it is a radius. This means that \(O P\) and \(O R\) are also 4 since they are radii. Since \(\triangle O Q R\) has two sides that are the same, \(O Q\) and \(O R\), the angles opposite them must also be the same. All angles in a triangle total \(180^{\circ}\) and \(\angle Q O R=60^{\circ}\), so the other two equal angles must equal \(120^{\circ}\). Divide this in two to find that angles \(\angle Q R O\) and \(\angle O Q R\) are also \(60^{\circ}\). This means that \(\triangle O Q R\) is an equilateral triangle, so all the sides and angles are equal. \(Q R\) is equal to 4 . The correct answer is (A).
2. G Right off the bat, you can eliminate (E) and (F), as the area of the rectangle alone is 48 . You can also assume the figure is drawn to scale, which means that (H), approximately 314 (over six times greater than the rectangle), is way too big. To check the math, you need to realize that because points \(A\) and \(C\) are on the circle and the straight line between them (a chord) passes through the the center of the circle, \(O\), line \(A O C\) is the diameter of the circle. It is also the hypotenuse of both triangles it creates, \(A B C\) and \(A C D\). Using the Pythagorean Theorem (or recognizing that this is part of a 3:4:5 special triangle), that diameter, \(c\), is equal to \(\sqrt{\left(6^{2}+8^{2}\right)}\), or \(\sqrt{100}\), which is 10 . This means that the radius of the circle is 5 , and plugging that into the area of the circle \(\left(\mathrm{A}=\pi r^{2}\right)\) yields a total of \(25 \pi\), or \((\mathrm{G})\).
3. 50 Remember to draw a figure when it is not given to you. It is always a great idea to connect the points that are not connected, so draw the diagonals of the square. This not only creates the diagonal of the square, but also the diameter of the circle. If the radius is 5 , the two diagonals create isosceles right triangles, each with a length of 5 from
the center of the circle.


Use the Pythagorean Theorem to find the hypotenuse of the triangle, which is the side of the square. \(5^{2}+5^{2}=c^{2}\), which yields \(5 \sqrt{2}\). Plug this into the formula for the area of a square to find that \((5 \sqrt{2})^{2}=5^{2} \times\) \(\sqrt{2^{2}}=25 \times 2=50\). The correct answer is 50 .
4. B Be sure to mark the figure carefully with the instructions given in the question. The triangle is equilateral, which means that all sides and angles are equal. Since \(A B\) is 6 , all sides of the triangle are also 6 , and all the angles are \(60^{\circ}\). Point \(D\) bisects segment \(A C\), which means it divides the segment. Draw segment \(B D\). By creating this line segment, it creates two 30-60-90 right triangles. If you know the proportions of this special right triangle, \(x, x \sqrt{3}\) and \(2 x\), respectively, you can solve \(B D\) right away to get \(3 \sqrt{3}\). If you forgot or did not know this proportion, you can still solve \(B D\) using the Pythagorean Theorem: \(3^{2}\) \(+b^{2}=6^{2}\). The correct answer is (B).
5. 56 No figure is given, so draw one! To find the area of the walking path, subtract the area of the garden from the area of the garden and walking path together. The area of the garden is 25 , so find the area of the walking path and garden together. The walking path and garden together form a larger square, so use the formula, \(A=s^{2}\). Find the
length of the side. First, find the side of the garden. The area is 25 , so \(25=s^{2}\). Take the square root of both sides to get \(s=5\). The 2 foot wide walking path surrounds the garden on all sides like below.


Therefore, the length of the side of the larger square is \(2+5+2=9\), so the total area is \(A=s^{2}=9^{2}=81\). Remember that you're looking for just the area of the walking path, so you'll now have to subtract the area of the garden: \(81-25=56\).

Chapter 14 Statistics and Probability

\section*{PROBABILITY}

Probability describes how likely it is that something is going to happen. You might not have quantified it before, but it's something you do every day, whether you're flipping a coin or wondering if you'll need to take an umbrella. To make probability less vague, many people express the odds of something happening in terms of percents. For instance, a standard coin has two equal results: heads ( \(50 \%\) ) and tails ( \(50 \%\) ). If the weather report says that there's a \(90 \%\) chance of rain, you probably want to take that umbrella. You might also see probability expressed as a number between 0 and 1-the chance of seeing a flying elephant being 0 and the chance of the sun rising tomorrow being 1 .

Don't be distracted by the objects in the question or the method with which they're being assessed. Whether it's \(20 \%\) or 0.2 , whether dealing with marbles, cards, coins, food, or anything else, a probability question essentially boils down to finding a specific number of possibilities out of the total number of possibilities.

Percents Make Per-sense
Remember that the word percent translates to

> Therefore, a probability of \(0 \%\) is the same
> 100
> as a probability \(\frac{0}{100}=0\), and a probability of \(100 \%\) is the same as a probability of \(\frac{100}{100}=1\).

> Any probability between 0\% and \(100 \%\) is
> equivalent to a probability between 0 and 1 .

\section*{Independent Probability}

The basic formula for probability is
\[
\text { probability }=\frac{\text { the number of what you want }}{\text { the total number }}
\]

For example, if a bag with 11 total gumballs has 6 red gumballs, the probability of randomly selecting a red gumball is \(\frac{6}{11}\), or 0.54 , or \(54 \%\).

A gumball is randomly selected from a jar that contains 5 blue gumballs, 15 red gumballs, and 30 yellow gumballs. What is the probability that the randomly selected gumball is red?
A. \(\frac{1}{10}\)
B. \(\frac{3}{10}\)
C. \(\frac{3}{7}\)
D. \(\frac{3}{5}\)

\section*{Here's How to Crack It}

The question asks for probability, so use the formula probability \(=\frac{\text { the number of what you want }}{\text { the total number }}\). The question asks for the probability that a randomly selected gumball is red, so the number you want is the number of red gumballs, which is 15 , and the total number is the number of gumballs, which is \(5+15+30=50\). Therefore, the probability is \(\frac{15}{50}\). This is not
a choice, so reduce the fraction. Both the numerator and denominator are multiples of 5 , so divide both by 5 to get \(\frac{3}{10}\), which is (B).

\section*{"Not" Probability}

So-called "not" probability is just the probability that something will not happen.

To find this, determine the probability that the event will happen and subtract that number from 1. It's often easier than calculating the probability of all other possible outcomes and adding all of those together to get the answer. Remember, the bag with 11 total gumballs has 6 red gumballs. The probability of randomly selecting a red gumball is \(\frac{6}{11}\), so the probability of not selecting a red gumball is \(1-\frac{6}{11}=\frac{5}{11}\).

\section*{"Or" Probability}

Sometimes the question will ask for the probability of one event or another event taking place. To find this, add the individual probabilities. For example, take that same bag with 11 gumballs, of which are 6 red. Of the remaining 5,3 are blue gumballs and 2 are yellow gumballs. The probability of getting a red gumball or a blue gumball is \(\frac{6}{11}+\frac{3}{11}=\frac{9}{11}\). The probability of getting a blue gumball of a
yellow gumball is \(\frac{3}{11}+\frac{2}{11}=\frac{5}{11}\). Notice that this is the same as the probability of not getting a red gumball. The probability of getting a red gumball, a blue gumball, or a yellow gumball is \(\frac{6}{11}+\frac{3}{11}+\frac{2}{11}=\frac{11}{11}=1\). The probability is 1 , because red, blue, and yellow are the only colors of gumballs in the bag, so one these colors must be drawn.
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|c|}{ COLORS OF THE RAINBOW SURVEY } \\
\hline Color & Number of Students \\
\hline Red & 13 \\
\hline Orange & 15 \\
\hline Yellow & 17 \\
\hline Green & 31 \\
\hline Blue & 24 \\
\hline Indigo & 6 \\
\hline Violet & 29 \\
\hline
\end{tabular}

The students at a school were surveyed and asked for their favorite color of the rainbow. What is the probability that a randomly selected student chose either red or yellow as her favorite color?
A. \(\frac{13}{135}\)
B. \(\frac{17}{135}\)
C. \(\frac{2}{9}\)
D. \(\frac{4}{9}\)

\section*{Here's How to Crack It}

The question asks for the probability that a randomly selected student choses either red or yellow, so add the probability that the student selects red to the probability that the student selects yellow. To find each of these, determine the total number of students in the poll. The total number is \(13+15+17+31+24\) \(+6+29=135\). There are 13 students who select red, so the probability of selecting red is \(\frac{13}{135}\). There are 17 students who select yellow, so the probability of selecting yellow is \(\frac{17}{135}\). Therefore, the probability of selecting red or yellow is \(\frac{13}{135}+\frac{17}{135}=\frac{30}{135}=\frac{10}{45}=\frac{2}{9}\), which is (C).

\section*{"And" Probability}

On SHSAT probability questions, it is important to decide if there are any events that happen independently from each other, or if one event affects another. For instance, if a question asks about flipping a coin four times, does one coin flip influence the next? The answer is no. A coin flip will come out to either heads or tails every time, so the probability of getting tails on each trial is \(\frac{1}{2}\). However, in 2
some cases, one event does affect the next, which is where you'll want to know about "and" probability. This concept still uses basic probability. Take the probability that each individual event happens and multiply them.

Once again, consider the bag with 11 gumballs, including 6 red gumballs. If two gumballs are selected at random, what is the probability that they will both be red? The probability that the first gumball will be red is \(\frac{6}{11}\). However, the probability that the second gumball will be red is different. Once one red gumball is removed, only 5 red gumballs and 10 total gumballs remain, so the probability is \(\frac{5}{10}\). To find the probability that both are red, multiply the two individual probabilities to get \(\frac{6}{11} \times \frac{5}{10}=\frac{6}{11} \times \frac{1}{2}=\frac{3}{11} \times \frac{1}{1}=\frac{3}{11}\).

Josh has 3 T-shirts, 4 long-sleeved shirts, and 7 tank tops. If he selects one shirt at random, what is the probability that both shirts will be tank tops?
A. \(\frac{3}{13}\)
B. \(\frac{1}{4}\)
C. \(\frac{6}{13}\)
D. \(\frac{1}{2}\)

\section*{Here's How to Crack It}

To find the probability that both shirts will be tank tops, find the probability that each individual shirt will be a tank top. There are 7 tank tops and \(3+4+7=14\) total shirts, so the probability that the first shirt will be a tank top is \(\frac{7}{14}\). Once one tank top is removed, there are 6 tank tops and 13 total shirts remaining, so the probability that the second shirt will be a tank top is \(\frac{6}{13}\). Therefore, the probability that both will be tank tops is \(\frac{7}{14} \times \frac{6}{13}=\frac{1}{2} \times \frac{6}{13}=\frac{1}{1} \times \frac{3}{13}=\frac{3}{13}\), which is \((\mathrm{A})\).

\section*{Predictions Using Probability}

Sometimes the SHSAT will ask for predictions using probability. If a coin is flipped 100 times, it is likely that there will be close to 50 heads and 50 tails. If a six-sided die is rolled 60 times, it is likely that there will be about 10 rolls of each side. While the result will not likely match the prediction exactly, it is still fair to make a prediction using the formula
\# of successful attempts = (probability)(total \# of attempts)

Rosa is playing a certain card game that uses a deck of 20 cards with two red cards, three black cards, four blue cards, four green cards, and seven pink cards. Each time a card is drawn, it is shuffled back randomly into the deck. If a card is drawn 100 times, what is the number of times that Rosa can expect that a pink card will be drawn?
A. 7
B. 15
C. 20
D. 35

\section*{Here's How to Crack It}

The question asks for a prediction, so use the formula \# of successful attempts = (probability)(total \# of attempts). There are 20 cards in the deck, 7 of which are pink, so the probability of selecting a pink card is \(\frac{7}{20}\). A card is drawn 100 times, so the total \# of attempts is 100. Therefore, the \# of successful attempts is \(\frac{7}{20} \times 100=\frac{700}{20}=35\), which is \((D)\).

\section*{PROBABILITY DRILL}
1. A basket of marbles contains 15 blue marbles. If the probability of not selecting a blue marble is \(\frac{4}{9}\), how many marbles are in the basket?
A. 12
B. 18
C. 24
D. 27
2. A bowl of fruit contains 4 apples, 6 kiwis, and 5 oranges. If one fruit is selected from the bowl at random, what is the probability that it will be an apple or a kiwi?
E. \(\frac{4}{35}\)
F. \(\frac{1}{3}\)
G. \(\frac{2}{3}\)
H. \(\frac{31}{35}\)
3. A box of cookies has 2 chocolate chip, 4 pecan, 7 oatmeal raisin, and 3 peanut butter cookies. If two cookies are selected at random, what is the probability that both cookies will be pecan?
A. \(\frac{1}{20}\)
B. \(\frac{1}{9}\)
C. \(\frac{1}{4}\)
D. \(\frac{1}{5}\)
4. Neha rolled a loaded die 25 times and found that the probability of rolling a 6 is \(48 \%\). If she rolls the same die 125 more times, what is the total number of times she can expect to roll a 6 ?

5. A jar of cookies contains 5 chocolate chip, 4 oatmeal raisin, 4 snicker doodles, and 2 red velvet. Sandy chooses two cookies from the jar without replacement. What is the probability that she will choose a chocolate chip cookie first and a red velvet cookie second?
E. \(\frac{1}{21}\)
F. \(\frac{2}{15}\)
G. \(\frac{10}{21}\)
Н. \(\frac{7}{15}\)

\section*{PROBABILITY DRILL EXPLANATIONS}
1. D The question asks for how many marbles are in the basket. The probability of not selecting a blue marble is \(\frac{4}{9}\), so the probability of selecting a blue marble is \(1-\frac{4}{9}=\frac{9}{9}-\frac{4}{9}=\frac{5}{9}\). Use the probability formula: \(\quad\) probability \(=\frac{\text { the number of what you want }}{\text { the total number }}\). The probability is \(\frac{5}{9}\), and the number of what you want is the number of blue marbles, which is 15 . The question asks for the total number, so set this equal to \(x\) to get \(\frac{5}{9}=\frac{15}{x}\). Cross-multiply to get \(5 x=135\). Divide both sides by 5 to get \(x=27\), which is (D).
2. G The question asks for the probability that the fruit selected will be an apple or a kiwi, so get the sum of the probabilities that the fruit will be an apple and that the fruit will be a kiwi. There is a total of \(4+6+5=\) 15 fruit. There are 4 apples, so the probability that the fruit is an apple is \(\frac{4}{15}\). There are 6 kiwis, so the probability that the fruit is a kiwi is \(\frac{6}{15}\) . Therefore, the probability that the fruit is an apple or a kiwi is \(\frac{4}{15}+\) \(\frac{6}{15}=\frac{10}{15}=\frac{2}{3}\), which is (G).
3. A The question asks for the probability that both cookies are pecan so multiply the probabilities that each individual cookie will be pecan. There are 4 pecan cookies and \(2+4+7+3=16\) total cookies, so the probability that the first cookie will be pecan is \(\frac{4}{16}\). Once one pecan cookie is removed, there are 3 remaining pecan cookies and 15 total cookies remaining, so the probability that the second cookie will be pecan is \(\frac{3}{15}\). Multiply the two to get \(\frac{4}{16} \times \frac{3}{15}=\frac{1}{4} \times \frac{1}{5}=\frac{1}{20}\), which is (A).
4. 72 The question asks for how many times Neha can expect to roll a 6, so use the formula \# of successful attempts = (probability)(total \# of attempts). The probability is \(48 \%\), which can be rewritten as \(\frac{48}{100}\). She rolls the die 125 more times after the initial 25 , so the total \# of attempts is \(125+25=150\). Therefore, the number of times she can expect to roll a 6 is \(\frac{48}{100} \times 150=\frac{48}{100} \times \frac{150}{1}=\frac{48}{2} \times \frac{3}{1}=\frac{24}{1} \times \frac{3}{1}=\) 72.
5. E On the first trial, Sandy chooses a chocolate chip cookie, which has a probability of \(\frac{5}{15}\). On the second trial, she chooses a red velvet
cookie. There are 2 red velvet cookies, but now there are only 14 cookies remaining in the jar to choose from. The probability for the second trial is \(\frac{2}{14}\). Multiply these together since these are independent of each other: \(\frac{5}{15} \times \frac{2}{14}=\frac{1}{3} \times \frac{1}{7}=\frac{1}{21}\). The correct answer is (E).

\section*{DATA ANALYSIS}

Whenever data is given, there is a need to simplify the data to express useful information. Different pieces of data are helpful to express different types of information.

Deliciously Useful
Pi may be an irrational number, but creating an average pie is a quite rational strategy.

\section*{Average (Mean)}

One common way of measuring data is the average, also known as the mean. There are three parts of every average problem: total, number, and average. Most SHSAT problems on averages will provide two of the three pieces and ask for the third. To help organize the given information, use the average pie.


The average pie organizes all of your information visually. It clearly and cleanly shows the relationships between the pieces of the pie:
- TOTAL \(=(\#\) of items \() \times\) (Average)
- \(\#\) of items \(=\frac{\text { Total }}{\text { Average }}\)
- Average \(=\frac{\text { Total }}{\# \text { of items }}\)

For example, if a student had three biology tests and scored 82, 94, and 97, what is the average for all three tests? First, find the total by adding the three scores to get \(82+94+97=273\). There are three tests, so the number of items is 3 . Put these two numbers into the average pie.


Divide to find the average: \(273 \div 3=91\).

In her bowling league, Charlotte bowls 4 games with an average score of 80. What would she need to score in her fifth game to increase her average to 85 ?
A. 90
B. 95
C. 100
D. 105

\section*{Here's How to Crack It}

Because there are two averages mentioned in this question, draw an average pie for each average. The first is the average score of 80 for Charlotte’s first four games. Draw an average pie for this.


Multiply to get a total of 320 . The second average mentioned is the goal average of 85 after 5 games. Make a new average pie for this.


The question asks what she would need to score on her fifth game to get the average to 85 . To do this Charlotte needs to bring her total from 320 to 425 , so to find the score she needs in the fifth game, subtract the two totals to get 425 - 320 \(=105\), which is \((\mathrm{D})\).

\section*{Median}

The median is the middle number in an ordered list of numbers. Find the median in a set of numbers.

List \(S:\{2,4,7,12,16\}\)
To find the median, start narrowing down to our middle number. Cross out the smallest number in the list and the biggest number in the list. That gets rid of 2 and 16. Next cross out the next smallest and the next largest. That gets rid of 4 and 12 . What's left over must be the median, so the median is 7 .

Now, look at a trickier example.
List \(C:\{91,42,36,24,64,78\}\)
There are two things that make this example trickier. The first is that the elements have to form an ordered list, so begin by reordering the elements of the list.

List \(C:\{24,36,42,64,78,91\}\)

Next, start crossing off sets of numbers. Remember, only cross out a small number if there is large number to cross out as well. You can cross off 24 and 91, and then you can cross off 36 and 78 . Be careful, however-42 cannot be crossed out without crossing out 64, and then there'd be no number left for the median! In this case, take the two remaining middle numbers and find the average. The average is considered the median. Therefore, the median of List \(C\) is \(\frac{42+64}{2}=\frac{106}{2}=53\).

\section*{Mode}

The mode is the number that shows up most often in a list. "Mode" sounds a lot like "most," which make make it easier to remember. The mode is also the most direct one to figure out, because it doesn't require any actual math. Look at the following list of numbers.

\section*{List \(P:\{3,18,42,3,14,3,18\}\)}

Which number shows up most frequently in List \(P\) ? 3. Thus, the mode of List \(P\) is 3 . Even though there are two 18 's, 18 is not the mode because there are more 3 's than 18 's. The mode describes only the number that shows up most often.

\section*{Range}

The range of a list is the difference between the greatest number and the least number of that list. Look at a sample list.

List A: \{101, 47, 36, 74, 98, 74\}
Find the least and greatest numbers on this list. The least number is 36 , and the greatest is 101. Therefore, the range is \(101-36=65\).

On Monday, the temperature had a high of \(80^{\circ} \mathrm{F}\) and a range of \(34^{\circ} \mathrm{F}\). On Tuesday, the high temperature had a high of \(72^{\circ} \mathrm{F}\) and a range of \(28^{\circ} \mathrm{F}\). What is the overall range of temperatures for Monday and Tuesday?
A. \(28^{\circ} \mathrm{F}\)
B. \(31^{\circ} \mathrm{F}\)
C. \(34^{\circ} \mathrm{F}\)
D. \(36^{\circ} \mathrm{F}\)

\section*{Here's How to Crack It}

The question asks for the range, so determine the highest and lowest temperatures for the two days. The high temperature of Monday was \(80^{\circ}\), and the high temperature of Tuesday was \(72^{\circ}\), so the high for the two days was \(80^{\circ}\).

To find the low temperature for the two days, start by finding the low temperature for each day. The range for Monday was \(34^{\circ}\), so the low was \(80^{\circ}\) \(34^{\circ}=46^{\circ}\). The range for Tuesday was \(28^{\circ}\), so the low was \(72^{\circ}-28^{\circ}=44^{\circ}\). Therefore, the low for the two days was \(44^{\circ}\), so the range for the two days was \(80^{\circ}-44^{\circ}=36^{\circ}\), which is (D).

\section*{DATA ANALYSIS DRILL}
1. If Ricardo averages 18 points per game in his first five basketball games, what is the least number of points he could average after 6 games?

2. Jenny had an average score of 84 on her five history tests for the term. Her first two tests had an average of 81 . What was her average score on the last three tests?
A. 82.5
B. 86
C. 87
D. 88.5
3. If List \(C\) has a median of 42 and a range of 12 , which of the following cannot be a member of List \(C\) ?
E. 29
F. 30
G. 41
H. 52

\section*{DATA ANALYSIS DRILL EXPLANATIONS}
1. 15 There are two averages mentioned in the question, so use two average pies. In his first five games, Ricardo averaged 18 points. Use the average pie to get a total of \(5 \times 18=90\).


The question asks for the least number of points he could average after 6 games. Because the total after five games is 90 , the least the total can be after six games is also 90 : it's what his total would be if he scores 0 points in the sixth game. This would give him a total of 90 in six games to get an average of 15 , which is the correct answer.

2. B There are three averages mentioned, so use three average pies. For her five tests, her average score is 84 . Draw the pie to get a total of \(5 \times 84\) \(=420\).


The average of the first two tests is 81 , so draw another pie to get that her first two tests have a total of \(2 \times 81=162\).


The question asks for the average of the last three tests. Draw a new average pie.


The question asks for the average, so find the total. To get the total of the last three tests, subtract the total of all five tests by the total of the first two tests to get \(420-162=258\). Fill this into the average pie to get an average of \(258 \div 3=86\), which is (B).

3. \(\mathbf{E} \quad\) The range of the set is 12 , so the difference between the greatest and least member of the list is 12 . Because of this, there cannot be two numbers in the list with a difference greater than 12 . Therefore, no member of the set can be more than 12 away from the median (or any other member of the set). Since the median is 12 , no member of the set can be less than \(42-12=30\) or greater than \(42+12=54\). Since 29 is less than 30 , it cannot be a member of the set, so the answer is (E).

\section*{CHARTS}

You need to be careful that you read chart questions properly. Follow these three steps and you'll be well on your way to mastering any chart question.
1. Read any text that accompanies the chart. It is important to know what the chart is showing and what scale the numbers are on.
2. Read the question.
3. Refer to the chart and find the specific information you need.

Here is a sample chart of Club membership by state, 2016 and 2017:
\begin{tabular}{|l|c|c|}
\hline \multicolumn{1}{|c|}{ State } & \(\mathbf{2 0 1 7}\) & \(\mathbf{2 0 1 7}\) \\
\hline California & 300 & 500 \\
\hline Florida & 225 & 250 \\
\hline Illinois & 200 & 180 \\
\hline Massachusetts & 150 & 300 \\
\hline Michigan & 150 & 200 \\
\hline New Jersey & 200 & 250 \\
\hline New York & 400 & 600 \\
\hline Texas & 50 & 100 \\
\hline
\end{tabular}

There are lots of different questions that you can answer based on the information in this chart. For instance:

What is the difference between the number of members who came from New York in 2016 and the number of members who came from Illinois in 2017?

\section*{Here's How to Crack It}

This question asks you to look up two simple pieces of information and then do
a tiny bit of math. First, the number of members who came from New York in 2016 was 400 . Second, the number of members who came from Illinois in 2017 was 180. Finally, look back at the question. It asks you to find the difference between these numbers: \(400-180=220\). Done.

Does the New SHSAT Even Share?
As of 2017, SHSAT math questions tend to be independent of one another and don't share data. However, we've provided you with both kinds of questions, just so that you're ready for anything.

What was the percent increase in members from New Jersey from 2016 to 2017?

\section*{Here's How to Crack It}

In 2016, there were 200 club members from New Jersey. In 2017 there were 250 members from New Jersey. That represents an increase of 50 members. So to determine the percent increase, you will need to ask yourself "50 (the increase) is what percent of 200 (the original amount)?"

Translated, this becomes:
\[
50=\frac{g}{100} \times 200
\]

With a little bit of manipulation, this equation becomes \(50=2 g\), or \(25=g\).
So from 2016 to 2017, there was a 25 percent increase in the number of members from New Jersey. Good work!

Which state had as many club members in 2017 as a combination of Illinois, Massachusetts, and Michigan had in 2016?
A. California
B. Florida
C. New York
D. Texas

\section*{Here's How to Crack It}

First, take a second to look up the number of members who came from Illinois, Massachusetts, and Michigan in 2016 and add them together.
\[
200+150+150=500
\]

Which state had 500 members in 2017? California, (A). That's all there is to it!
\(\qquad\)

\section*{CHARTS DRILL}

Questions 1-4 refer to the following information.
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{ SPENDING ON ENERGY IN THOUSANDS OF DOLLARS } \\
\hline District & 2016 & 2017 \\
\hline A & 400 & 600 \\
\hline B & 500 & 700 \\
\hline C & 200 & 350 \\
\hline D & 100 & 150 \\
\hline E & 600 & 800 \\
\hline
\end{tabular}
1. In 2016, which district spent twice as much on energy as district A spent in 2017 ?
A. A
B. C
C. D
D. E
2. Which district spent the most on energy in 2016 and 2017 combined?
E. A
F. B
G. D
H. E
3. The total increase in energy expenditure in these districts, from 2016 to 2017, is how many dollars?
A. \(\$ 600\)
B. \(\$ 800\)
C. \(\$ 600,000\)
D. \(\$ 800,000\)
4. The mean spending on energy per district in 2016 is how much less than the mean spending on energy per district in 2017 ?
E. \(\$ 160,000\)
F. \$360,000
G. \(\$ 520,000\)
H. \(\$ 800,000\)

\section*{CHARTS DRILL EXPLANATIONS}
1. D The question asks for which district spent twice as much on energy in 2017 as District A spent in 2016. Because all numbers in the table represent thousands of dollars, just use the numbers given. Look in the District A row and the 2016 column to find that District A spent 400 in 2016. Find the district that spent \(2 \times 400=800\) in 2017. Look at the 2017 column and find 800. It is found in the row for District \(E\), so the answer is (D).
2. \(\mathbf{H}\) The question asks for the district that spent the most on energy in 2016 and 2017 combined. Add the two years individually for each district. Because all numbers in the table represent thousands of dollars, just use the numbers given. District A spent a total of \(400+\) \(600=1,000\). District B spent a total of \(500+700=1,200\). District C spent a total of \(200+350=550\). District D spent a total of \(100+150\) \(=250\). District E spent a total of \(600+800=1,400\). The district with the greatest total is District E , so the answer is ( H ).
3. D To get the total increase, get the increase for each district and take the sum. For District A, the increase is \(600-400=200\). For District B, the increase is \(700-500=200\). For District C, the increase is \(350-\) \(200=150\). For District D, the increase is \(150-100=50\). For District E, the increase is \(800-600=200\). Therefore, the total increase is 200 \(+200+150+50+200=800\). However, remember that this is in thousands of dollars. Therefore, the actual increase is \(\$ 800,000\), which is (D).
4. \(\mathbf{E} \quad\) Get the mean spending for each year and find the difference. To get the mean spending for 2016, find the total and divide by the number of districts. The total is \(400+500+200+100+600=1,800\). There are five districts, so use the average pie to get an average of 360 .


Do the same for 2017 . The total is \(600+700+350+150+800=\) 2,600 . Again, there are five districts so use the average pie to get an average of 520 .


Therefore, the average in 2016 was \(520-360=160\) less than in 2017. Remember that the numbers given are in thousands of dollars, so the actual difference is \(\$ 160,000\), which is ( E ).

\section*{GRAPHS}

Some questions will ask you to interpret a graph. You should be familiar with both pie and bar graphs. These graphs are generally drawn to scale (meaning that the graphs give an accurate visual impression of the information), so you can always guess based on the figure if you need to.

The way to approach a graph question is exactly the same as the way to approach a chart question. Follow the same three steps:
1. Read any text that accompanies the graph. It is important to know what the graph is showing and what scale the numbers are on.
2. Read the question.
3. Refer back to the graph and find the specific information you need.

This is how it works:


Figure 1
The graph in Figure 1 shows Emily's clothing expenditures for the month of October. On which type of clothing did she spend the most money?
A. shoes
B. shirts
C. socks
D. hats

\section*{Here's How to Crack It}

This one is straightforward. You can look at the pieces of the pie and identify the largest, or you can look at the amounts shown in the graph and choose the largest one. Either way, the answer is (A), because Emily spent more money on shoes than on any other clothing items in October.


Emily spent half of her clothing money on which two items?
A. shoes and pants
B. shoes and shirts
C. hats and socks
D. socks and shirts

\section*{Here's How to Crack It}

Again, you can find the answer to this question two different ways. You can look for which two items together make up half the chart, or you can add up the total amount of money Emily spent ( \(\$ 240\) ) and then figure out which two items made up half (or \$120) of that amount. Either way is just fine, and either way, the right answer is (B), shoes and shirts.

\section*{GRAPHS DRILL}

Questions 1-3 refer to the following graph.
Number of Streaming Service
Subscriptions by Students

1. Carl has as many streaming service subscriptions as which two other students combined?
A. Abe and Ben
B. Ben and Dave
C. Abe and Dave
D. Ben and Ed
2. Which one student accounts for one-fourth of subscriptions accounted for in the graph?
E. Ben
F. Carl
G. Dave
H. Ed
3. If a subscription of one of the students is selected at random, what is the probability the subscription belongs to Ed?
A. 1

16
B. \(\frac{1}{8}\)
C. \(\frac{1}{7}\)
D. \(\frac{1}{5}\)

\section*{GRAPHS DRILL EXPLANATIONS}
1. C Begin by finding how many subscriptions Carl has. Go to the top of Carl's bar and trace to the vertical axis to get 5 . Determine which combination in the choices has a sum of 5. Choice (A) is Abe and Ben. According to the graph, Abe has 2 subscriptions and Ben has 4 to get a total of \(2+4=6\), so eliminate (A). Choice (B) is Ben and Dave. Ben has 4 subscriptions and Dave has 3 to get a total of \(4+3=\) 7, so eliminate (B). Choice (C) is Abe and Dave. Abe has 2 subscriptions and Dave has 3 to get a total of \(2+3=5\). This is the same as Carl, so the answer is (C).
2. \(\mathbf{E}\) First, determine the total number of subscriptions accounted for in the graph. Abe has 2, Ben has 4, Carl has 5, Dave has 3, and Ed has 2, so the total is \(2+4+5+3+2=16\). The question asks for who has onefourth the total, so find the student with \(16\left(\frac{1}{4}\right)=4\). This is the number Ben has, so the answer is (E).
3. B The question asks for probability, so use the formula probability \(=\frac{\text { the number of what you want }}{\text { the total number of items }}\). First, determine the total number of subscriptions. Abe has 2, Ben has 4, Carl has 5, Dave has 3, and Ed has 2, so the total is \(2+4+5+3+2=16\). The question asks for the probability that the subscription chosen belongs to Ed. Since Ed has 2 subscriptions, the number you want is 2 . Therefore, the
probability is \(\frac{2}{16}=\frac{1}{8}\), which is (B).

\section*{Chapter 15 \\ Grid-In Problems}

\section*{GRID-INS}


No, this isn't some sort of weird-looking multiple-choice question. It's a grid-in, which is basically a standardized open-ended question. You might at first be alarmed that there aren't any answer choices, and that does mean that there are five questions you can't guess on. But for the most part, you can solve these with the same strategies you've been using all along.

Plugging In is still often a good idea, although you'll have to supply the numbers, and you can sometimes even ballpark angles and work backward to check that they fit. Moreover, there are only four columns for digits (there’s a fifth column that indicates if a number is positive or negative), so there's a limit to how complex a number can be. This, in turn, often limits the difficulty of the question.

One recommendation: carefully write your answers from left to right. A right answer won't count if you leave a blank in the middle or bubble two numbers in the same column.

\author{
Fraction Infraction
}

Look closely at the grid and you'll see there's no way to bubble in a fraction. That means you'll have to convert fractions to decimal form.

Rather than reducing to the lowest form, you
may want to try to get a denominator of 100,
because then you simply have to put a decimal before the numerator as you grid it in.

\section*{Break It Down: Bite-Sized Pieces}

As you know, the SHSAT loves to ask long, wordy questions that have multiple steps that lead to a specific value. With a multiple-choice question, it might be a good idea to use PITA in such a case. While this is not possible with grid-ins, all the other steps that you would take in those types of questions apply here: break the question down into small, digestible pieces until you arrive at the answer. Let's give it a try.

The product of two negative, consecutive numbers is 2 . If 3 is subtracted from the smaller number and 4 is added to the larger number, what is the sum of the two subsequent numbers?

\section*{Here's How to Crack It}

Take it piece by piece. The first clue is that the product of two negative, consecutive numbers is 2 . Only -1 and -2 will yield a product of 2 . The next piece of information is that 3 is subtracted from the smaller number, so subtract 3 from -2 , which results in -5 . Similarly, 4 is added to the larger number, so -1 \(+4=3\). The sum of the two subsequent numbers means to add the two new numbers together. \(-5+3=-2\). The correct answer is -2 . (Don't forget to bubble in the negative sign!)

The Grid-Ins test a wide variety of topics, from geometry to algebra. Here's another sample problem you might see on the test.


What is the value of the missing angle in the image above?

\section*{Here's How to Crack It}

Just like with your regular geometry questions, use what you know about vertical angles and supplementary angles to find the missing angle. Remember that all angles in a triangle add up to \(180^{\circ}\), so find the other two angles and subtract them from \(180^{\circ}\) to find the missing angle. The top of the triangle is opposite the \(85^{\circ}\) angle, so it must also be \(85^{\circ}\). The leftmost angle of the triangle is supplementary to the \(120^{\circ}\) angle, meaning that the two angles add up to \(180^{\circ}\). Therefore, the supplementary angle to \(120^{\circ}\) is \(60^{\circ}\). Since the two known angles of the triangle are \(60^{\circ}\) and \(85^{\circ}\), subtract them from \(180^{\circ}\) : \(180^{\circ}-85^{\circ}-60^{\circ}=35^{\circ}\).
\(\qquad\)
There might even be a problem you can use Plugging In on. Check this one out.

A certain dress goes on sale for \(20 \%\) off. The dress then goes on clearance for \(25 \%\) off the already reduced price. What fraction of the original price is the resulting price?

\section*{Here's How to Crack It}

Since there are no starting or ending values, use Plugging In. Start with \(\$ 100\) for the original value, a very convenient number to use for percentages. The dress goes on sale for \(20 \%\) off, so find \(20 \%\) of \(\$ 100\) and then subtract it from the total. \(20 \%\) of \(\$ 100\) is \(\$ 20\), so subtract this from \(\$ 100\) to find \(\$ 80\) as the sale price. This goes on clearance for an extra \(25 \%\) off, and \(25 \%\) of \(\$ 80\) is \(\$ 20\). Subtract this from \(\$ 80\) to find the final price is \(\$ 60\). To find the answer to the question, take
the clearance price over the original price to find the fraction 60/100. You can reduce this further, but note that you cannot grid in fractions-so it's more important that you be able to reduce this to a decimal. In this case, 60/100 is fairly recognizable as 60 .

Take a Congratulatory Breather!
Once you finish the following Grid-In Drill, you'll have reached the end of the content review in this book. Whether you're now planning to go back to the ELA section or to take Practice Test 2, if you have the time, take a day or at least a few hours off so that your mind can settle all the subjects you've been studying. Especially when you take a test, you want to be as focused as possible.

\section*{GRID-IN DRILL}
1. What is the value of \(h\) if \(4 h=6-4 h\) ?

2.


In the figure above, what is the value of \(n\) ?

3. What is the sum of all the distinct prime factors of 48 ?

4. A farmer owns sheep and goats in a 5:4 ratio. How many total animals does the farmer have if she owns 45 sheep?

5. Sheri's math test is comprised of multiple-choice and non-multiple-choice questions. A fifth of the problems on Sheri's math test are multiple-choice. If there are 60 non-multiple-choice questions, how many total questions are on the test?


\section*{GRID-IN DRILL EXPLANATIONS}
1. . 75 In this case, it is not possible to Plug In, so solve it the old-fashioned way. Isolate \(h\) on one side by adding \(4 h\) to both sides: \(8 h=6\). Divide by 8 on both sides to find that \(h\) is \(\frac{6}{8}\), which reduces to \(\frac{3}{4}\). You can't grid in a fraction, however, so you'll either have to recognize that \(\frac{3}{4}\) is equivalent to .75 , use division to calculate this, or multiply the numerator and denominator by 25 so that you have a fraction in the form of \(\frac{x}{100}\), as \(x\) will then be your answer. If you have time, you can now plug this answer into the original equation to check it.
2. 55 There are \(180^{\circ}\) in a triangle, so use this information to fill in the missing angles to find \(n\). Since you know \(35^{\circ}\) and \(100^{\circ}\) of the larger triangle, find the missing angle to that triangle: \(180^{\circ}-100^{\circ}-35^{\circ}=\) \(45^{\circ}\). The \(45^{\circ}\) angle is also part of the small triangle created by the overlapping area of the two larger triangles. \(80^{\circ}\) is opposite the top angle of that triangle, so use these two angles to find \(n\) : \(180^{\circ}-80^{\circ}-\) \(45^{\circ}=55^{\circ}\). The correct answer is 55 .
3. 5 Use a prime factorization tree to find the prime factors of 48:


Remember that distinct means different, so count 2 only once. The only distinct prime factors here are 2 and 3 . The question asks for the sum of these, so \(2+3=5\). The correct answer is 5 .
4. 81 Remember that ratios are groups of things. If the ratio is \(5: 4\), that means there are multiples of each of these numbers. The question states that there are 45 sheep, and the 5 correlates to the sheep from the ratio. \(5 \times 9=45\), so the multiple here is 9 . Therefore, multiply \(4 \times\) \(9=36\) to find the number of goats on the farm. Add 45 and 36 to find the total number of animals: 81 . The correct answer is 81 .
5. 7560 equals \(\frac{4}{5}\) of the total number of questions on the exam. Translate this into math to find that \(60=\left(\frac{4}{5}\right) x\), where \(x\) is the total number of questions on the test. Multiply by the reciprocal on both sides to find that \(60\left(\frac{5}{4}\right)=x=75\). The correct answer is 75 .

Part V
Final Practice Test

\section*{Chapter 16 Practice Test 2}

Click here to download a PDF of Practice Test 2.

\title{
PART 1—ENGLISH LANGUAGE ARTS
}

Suggested Time-90 Minutes
57 QUESTIONS

\section*{REVISING/EDITING}

QUESTIONS 1-11
IMPORTANT NOTE
The Revising/Editing section (Questions 1-11) is in two parts: Part A and Part B.

\section*{REVISING/EDITING Part A}

DIRECTIONS: Read and answer each of the following questions. You will be asked to recognize and correct errors in sentences or short paragraphs. Mark the best answer for each question.
1. Read this sentence.

Indeed, why do most people throughout the world, people of every religion and culture play and enjoy music?

Which edit should be made to this sentence?
A. insert a comma after culture
B. delete the comma after indeed
C. insert a comma after people
D. delete the comma after world
2. Read this paragraph.
(1) Insects do not have spoken communications, but they have other forms of communication that are equally effective. (2) Ants and bees, for example, are able to share large amounts of complex information without talking in two completely different, silent ways. (3) Bees did this by dancing out complex patterns that tell other bees where they can find food or water. (4) Ants, on the other hand, leave behind specific scent trails to mark paths to food and danger.

Which sentence should be revised to correct a verb tense error?
E. sentence 1
F. sentence 2
G. sentence 3
H. sentence 4
3. Read these sentences.
(1) As you see, going to the amusement park is no trivial or simple fun-filled event.
(2) Going to the amusement park is a harrowing and serious series of important decisions.

What is the best way to join these sentences in order to highlight the relationship between the ideas?
A. Going to the amusement park is no trivial or simple fun-filled event, but, on the one hand, a harrowing and serious series of important decisions.
B. Going to the amusement park is no trivial or simple fun-filled event, but rather a harrowing and serious series of important decisions.
C. Going to the amusement park is a harrowing and serious series of important decisions, and it is no trivial or simple fun-filled event, also.
D. Going to the amusement park is no trivial or simple fun-filled event, or a harrowing and serious series of important decisions.
4. Read this sentence.

Born in the late nineteenth century, the public official Robert Moses was responsible for many improvements to New York City.

What is the most precise revision of this sentence?
E. Born in 1888, New York City Parks Commissioner Robert Moses was responsible for many of the highways, bridges, tunnels, and parks that New Yorkers use on a daily basis.
F. Born in the late nineteenth century, New York City Parks Commissioner Robert Moses was responsible for many of the highways, bridges, tunnels, and parks that New Yorkers use on a daily basis.
G. Born in 1888, the public official Robert Moses was responsible for many improvements to New York City.
H. Born in the late nineteenth century, the public official Robert Moses was
responsible for many of the highways, bridges, tunnels, and parks that New Yorkers use on a daily basis.

\section*{REVISING/EDITING Part B}

DIRECTIONS: Read the passage below and answer any questions following it. You will be asked to improve the writing quality of the passage and to correct errors so that each passage follows the conventions of standard written English. You may reread the passage if you need to. Mark the best answer for each question.

\section*{Bitcoin: Past, Present, and Future}
(1) One of the biggest financial stories in the last few years has been the rise of Bitcoin. (2) Some speculators are trying to buy as much Bitcoin as they can; some economists see Bitcoin as a trend that will fade; and some engineers are racing to build software to make Bitcoin more accessible to the public. (3) Undoubtedly, Bitcoin is a hot commodity that investors are chasing after, but how wise is it to purchase Bitcoin?
(4) Bitcoin is a digital currency launched in the 2000s by a person whose real identity is not known. (5) It is fitting that Nakamoto has shrouded his identity, since so much of Bitcoin revolves around privacy. (6) Before the advent of Bitcoin, every transaction you made with your credit card online could be monitored by a bank or government. (7) Most banks charge customers high fees to send money internationally over the Internet. (8) The promise of Bitcoin is a low- or no-fee, untraceable, encrypted currency (or cryptocurrency) that is completely decentralized from any bank. (9) Bitcoin allows individuals to quickly send payments to one another without dealing with a third party. (10) Whereas this process done the traditional way can take several days to complete, with Bitcoin it occurs within minutes.
(11) But what is Bitcoin? (12) What's unique about Bitcoin is not the currency itself, but rather the network it lives on. (13) You can think of Bitcoin like a digital token that can be bought or sold. (14) That in itself is not revolutionary. (15) Most currencies are controlled by a central bank or authority. (16) Bitcoin exists on a decentralized, anonymous web of computers. (17) Additionally, participants can see and validate Bitcoin transactions that occur in an online ledger, called the blockchain.
(18) For all its advantages, Bitcoin has drawbacks. (19) For one, its price is highly volatile, jumping up and down by thousands of dollars in mere days. (20) Moreover, many observers see the current rush to buy Bitcoin reflective of a bubble that is destined to pop. (21) Any investor looking to invest in Bitcoin should know that it comes with many risks.
5. Which sentence should replace sentence 3 to best introduce the main idea of the passage?
A. Amid the global Bitcoin mania, it is best to step back and ask: where did Bitcoin come from?
B. Many people have heard of Bitcoin, but there remains widespread confusion across the world about what precisely Bitcoin is, how it began, and where it is headed in the global economy.
C. Bitcoin is a digital cryptocurrency, meaning it can be used securely and anonymously, but critics wonder about the significance of cryptocurrencies for democracies.
D. There are two ways one can take advantage of Bitcoin: by using it as a means of exchange, or by buying Bitcoin and hoping to sell it later at a higher price.
6. What is the most precise revision of sentence 4 ?
E. Bitcoin-BTC is a frequently used abbreviation-is a digital currency launched in 2009 by a person whose real identity is not known.
F. Bitcoin is a digital currency launched in 2009 by Satoshi Nakamoto, whose real identity is not known.
G. Bitcoin is a digital currency launched in the 2000s by Satoshi Nakamoto,
whose real identity is not known.
H. Bitcoin is a completely and totally digital currency launched in 2009 by Satoshi Nakamoto, whose real identity is not known.
7. What transition word or phrase should be added to the beginning of sentence 7 ?
A. However
B. Although
C. Whereas
D. Additionally
8. Where should sentence 12 be moved to improve the organization of the third paragraph (sentences 11-17)?
E. between sentences 13 and 14
F. between sentences 14 and 15
G. between sentences 16 and 17
H. the end of the paragraph (after sentence 17)
9. Read this sentence.

When one person pays another, all the computers in the Bitcoin network are sent a new "block" detailing the transaction, thus eliminating the need for a third party-like a credit card company -to process the payment.

Where should this sentence be added to best support the ideas in the third paragraph?
A. between sentences 13 and 14
B. between sentences 14 and 15
C. between sentences 16 and 17
D. the end of the paragraph (after sentence 17)
10. What is the best way to combine sentences 15 and 16 in order to establish the relationship between the ideas?
E. While most currencies are controlled by a central bank or authority, Bitcoin exists on a decentralized, anonymous web of computers.
F. Bitcoin exists on a decentralized, anonymous web of computers because most currencies are controlled by a central bank or authority.
G. Despite most currencies being controlled by a central bank or authority, yet Bitcoin exists on a decentralized, anonymous web of computers.
H. Bitcoin exists on a decentralized, anonymous web of computers, so most currencies are controlled by a central bank or authority.
11. Which conclusion sentence should be added after sentence 21 to support the ideas raised in the passage as a whole?
A. Ultimately, until the true identity of Satoshi Nakamoto is revealed, the uncertainty in Bitcoin's future is likely to continue.
B. In the end, the future is Bitcoin's; the digital currency will most likely surpass traditional currencies like dollars, euros, and yen in a matter of decades.
C. Nonetheless, if properly understood and managed, Bitcoin may be an innovative way to send, receive, and even make money in the digital century.
D. The risks associated with Bitcoin are currently too high, and, in conclusion, investors should avoid the digital currency at all costs.

\section*{READING COMPREHENSION}

QUESTIONS 12-57
DIRECTIONS: Read the passage below and answer the questions following it. Base your answers on information contained only in the passage. You may reread a passage if you need to. Mark the best answer for each question.

Crete was settled around the year 5000 B.C. by travelers from Greece who journeyed across sixty miles of open water to reach this island in the Mediterranean 5 Sea. With a jagged spine of mountains cutting across the land, the settlers realized that the potential for farming was very limited. As a means of survival, the people of Crete focused their attention on their geographical assets: the waters surrounding the island, their strategic position in the Mediterranean, and the natural harbors of the gently sloping northern coasts. Thus, the Cretans were the first people to develop a modern commercial economy based mainly on seafaring and trade.

Using evidence from several sources, we can understand some of the details of Cretan life. Archaeological findings support the legend that the people of Crete were skillful builders. The island's forests provided timber for building ships. Their ships were sea-worthy and
must have required careful design and expert construction. Their ships had high, sharply curved bows and sterns, and keels covered in bronze that stood out beyond the prow, a design which shows that the boats were intended for ramming into enemy ships. This naval tactic, developed in Crete, dominated sea warfare during this time.

By the time of the Minoan civilization (2700 to 1450 B.C., the Bronze Age), Crete had become an important center of civilization. The buildings were tall for their time (several stories high), and their architecture was known for massive columns, courtyards, and grand staircases. Sometime around 1700 B.C., many of the palaces were destroyed, perhaps as a result of an earthquake or war, but the Cretans rebuilt them. The ruins of many of these grand palaces have been discovered and restored.

The success of the Cretans in trading gave people leisure time which could be spent doing amusing things; bullfights are one example of a popular pastime. Art works show people dressed in colorful clothes, large decorated hats, and fancy jewelry. As the culture developed, Crete became known for its sculpture, pottery and fresco painting. The remains of villages suggest that the society was middle-class. The palace of the priest-kings, the Minos, was as much a commercial center as it was a 60 palace, demonstrating the importance of trade in the Cretan culture. Because it is relatively poor in resources, however, Crete's history of reliance upon trade has continued, even to the modern day.

65 Today, Crete-the largest Greek island and fifth largest island in the Mediterranean Sea-is a popular tourist destination, partly because so many remains from ancient Crete have been discovered. Knossos, the largest architectural site on Crete and likely the political and cultural center of Crete during the Bronze Age, is among the most visited sites on the island.
12. Which of the following best tells what this passage is about?
E. a description of an early culture
F. a demonstration of how ships are built
G. a proof of the origin of the settlers of Crete
H. a study of the advantages of a trade-oriented economy
13. According to the passage, which of the following statements about Crete is not true?
A. The land was not suitable for the development of an agricultural economy.
B. The location of the island helped settlers to make a good living by trade.
C. The island is surrounded by a border of jagged cliffs.
D. The island provided a large supply of timber.
14. According to the passage, the people of Crete were known as skillful builders because E. they dressed well.
F. they designed a keel which was used in sea warfare.
G. the population had no leisure time.
H. the palace of Minos was a big building.
15. According to the passage, which of the following describes the most important characteristic of the civilization of Crete?
A. It was the first to develop bullfighting as a sport.
B. It was an upper-middle class society.
C. It was the first to start an economy based on trade and seafaring.
D. It was preoccupied with war.
16. According to the passage, the function of "keels covered in bronze" was E. to show religious devotions.
F. to replace wood in building ships.
G. to improve the land for farming.
H. to ram enemy ships.
17. The final paragraph of the passage suggests that
A. modern Cretans'
reliance on trade is due as much to necessity as to ingenuity.
B. modern Cretans do not rely on trade to the extent that ancient Cretans did.
C. modern Cretans rely on trade only to sell manufactured goods.
D. modern Cretans deeply respect the tradition and history of maritime trade.

The following excerpt, from Jack London's 1904 novel The Sea-Wolf, depicts the narrator's harrowing experience aboard a ferry near the San Francisco Bay.

Then everything happened, and with inconceivable rapidity. The fog seemed to break away as though split by a wedge, and the bow of a steamboat emerged, trailing fog-wreaths on either side like seaweed on the snout of Leviathan. I could see the pilot-house and a white-bearded man leaning partly out of it, on his elbows. He was clad in a 10 blue uniform, and I remember noting how trim and quiet he was. His quietness, under the circumstances, was terrible. He accepted Destiny, marched hand in hand with it, and coolly measured the stroke. As he leaned there, he ran a calm and speculative eye over us, as though to determine the precise point of the collision, and took no notice whatever when our pilot, white with rage, shouted, "Now you've done it!"

On looking back, I realize that the remark was too obvious to make rejoinder necessary.
"Grab hold of something and hang on," the red-faced man said to me. All his bluster had gone, and he seemed to have caught the contagion of preternatural calm. "And listen to the women scream," he said grimly-almost bit30 terly, I thought, as though he had been through the experience before.

The vessels came together before I could follow his advice. We must have been struck squarely amidships, for I
35 saw nothing, the strange steamboat having passed beyond my line of vision. The Martinez heeled over, sharply, and there was a crashing and rending of timber. I was thrown flat on the 40 wet deck, and before I could scramble to my feet I heard the scream of the women. This it was, I am certain,-the most indescribable of blood-curdling sounds,--that threw me into a panic. I 45 remembered the life-preservers stored in the cabin, but was met at the door and swept backward by a wild rush of men and women.

What happened in the next few minutes I do not recollect, though I have a clear remembrance of pulling down life-preservers from the overhead racks, while the red-faced man fastened them about the bodies of an hysterical group 55 of women. This memory is as distinct and sharp as that of any picture I have seen. It is a picture, and I can see it now,-the jagged edges of the hole in the side of the cabin, through which the grey fog swirled and eddied; the empty upholstered seats, littered with all the evidences of sudden flight, such as packages, hand satchels, umbrellas, and wraps; the red-faced man, stumping gallantly around on his artificial legs and buckling life-preservers on all comers; and finally, the screaming bedlam of women. This it was, the screaming of the women, that most tried my nerves.

70 I remember the scene impelled me to sudden laughter, and in the next instant I realized I was becoming hysterical myself; for these were women of my own kind, like my mother and sisters, 75 with the fear of death upon them and unwilling to die. And I remember that the sounds they made reminded me of the squealing of pigs under the knife of the butcher, and I was struck with 80 horror at the vividness of the analogy. They wanted to live, they were helpless, like rats in a trap, and they screamed.

I descended to the lower deck. The Martinez was sinking fast, for the water was very near. Numbers of the passengers were leaping overboard. Others, in the water, were clamouring to be taken aboard again. No one heeded them. A cry arose that we were
sinking. I was seized by the consequent panic, and went over the side in a surge of bodies. How I went over I do not know, though I did know, and instantly, why those in the water were so desirous of getting back on the steamer. The water was cold-so cold that it was painful. The pang, as I plunged into it, was as quick and sharp as that of fire. It bit to the marrow. It was like the grip of death. I gasped with the anguish and shock of it, filling my lungs before the life-preserver popped me to the surface. The taste of the salt was strong in my mouth, and I was strangling with the acrid stuff in my throat and lungs.

But it was the cold that was most distressing. I felt that I could survive but a few minutes. People were struggling and floundering in the water about me. I could hear them crying out to one another. And I heard, also, the sound of oars. Evidently the strange steamboat had lowered its boats. As the time went by I marveled that I was still alive. I had no sensation whatever in my lower limbs, while a chilling numbness was wrapping about my heart and creeping into it.

120 The noises grew indistinct, though I heard a final and despairing chorus of screams in the distance, and knew that the Martinez had gone down. Later,how much later I have no knowledge, -
\({ }_{125}\) I came to myself with a start of fear. I was alone. I could hear no calls or cries-only the sound of the waves, made weirdly hollow and reverberant by the fog.
18. This passage can best be summarized as E. a ferry passenger's recollections of a tragedy.
F. a play-by-play account of how a ship sinks when struck by another.
G. an unreliable report on a mythic disappearance.
H. one passenger's journal entry that replays a horrifying collision.
19. According to the passage, what was most likely the cause of the shipwreck?
A. ill intent on the part of the steamboat
B. poor weather conditions
C. the inexperience of the Martinez's crew
D. the hand of fate
20. What does the red-faced man's response to the collision suggest?
E. He is not a stranger to maritime disasters.
F. He is extremely uneasy about the situation.
G. He looks forward to hearing the women's screams.
H. He believes the Martinez can be saved and acts to do so.
21. The narrator is not alarmed by the collision until A. the red-faced man
informs him that the ship will certainly sink.
B. he struggles to locate the life vests.
C. he hears the women shrieking.
D. the ship begins taking on some water.
22. Why does the narrator start laughing?
E. He finds the panic of others genuinely amusing.
F. He sees the red-faced man laughing, and follows his lead.
G. He imagines the shipwreck as a cruel joke.
H. It is a brief, involuntary reaction to the chaos on deck.
23. The author describes the water as so cold that "the pang, as I plunged into it, was as quick and sharp as that of fire" (lines 97-98) in order to A. give the reader a sense of how painfully warm the water was.
B. highlight the irony in cold water feeling like hot fire.
C. educate the reader on typical water temperatures in the Pacific Ocean.
D. demonstrate how dangerous the ocean is for humans.
24. In the final two paragraphs, how can the attitude of the narrator best be described?
E. resigned
F. enraged
G. resentful
H. entrapped
25. The passage suggests that in times of crisis A. people become their true selves, despite all efforts to behave otherwise.
B. all people shriek in terror.
C. time seems to slow down or speed up at will, and one can feel outside of one's body.
D. humans’ survival instinct automatically kicks in and leads them to make wise choices.

Kangaroos are fascinating creatures because they are so different from our usual idea of a mammal. Unlike most mammals, kangaroos rear their young 5 in a pouch, and hop to get around. Their long, powerful hind legs are used for jumping, and their thick tail gives them balance. Their forelimbs are used almost like human hands. Despite these well-
10 known characteristics, prevalant among all species of kangaroos, there are many lesser-known variations; species vary tremendously in such distinguishing characteristics as habitat, color, social patterns and size (they range in length from nine inches to more than eight feet).

Kangaroos are marsupials, a type of mammal characterized by the pouch in which females carry their young. It is believed that marsupials evolved separately from so-called placental mammals, such as humans. The earliest known marsupial lived in China about 125 million years ago. While some marsupials of that time remained in Asia, many traveled across great distances.

In most continents, marsupials were not able to compete successfully with placental mammals, so many marsupials became extinct. The opossum, found in the Americas, is a notable exception. In Australia, on the other hand, marsupials thrived, none more so than the kangaroo. Indeed, there are more than 60 species of kangaroo, ranging in size from many small species to the red kangaroo, the largest of the species. One reason kangaroos were able to evolve and survive is that they had few predators in Australia. The kangaroo is such a unique evolutionary success that it is the national emblem of Australia.
Some people consider marsupials, including the kangaroo, to be a primitive form of mammal, and they point to the opossum in support of their claim. Their view is reflected in the statements they make about the Virginia opossum, the only marsupial native to North America. There is some evidence for such statements: fossils of marsupials that existed at the same time as dinosaurs, some 100 million years ago, have been classified as belonging to the same group as the Virginia opossum.

But even if the opossum can in some ways be considered a "living fossil," the
same cannot be said about the kangaroo.
These Australian marsupials of the Macropodidae ("big feet") family have changed recently in order to adapt to a changing environment. This is a very \({ }^{65}\) common occurrence in the evolutionary patterns of all animals. In fact, almost every family of animals, from fish to insects to mammals, has changed in some way since its ancient ancestors
roamed the earth. Even plants and trees, seemingly the same as they were eons ago, exhibit characteristics that are relatively new. In the case of Macropodidae, they evolved from small was probably related to the spread of grassland areas in Australia between ten and fifteen thousand years ago.
26. Which of the following best tells what this passage is about?
E. a comparison of the opossum and the kangaroo
F. the competition between marsupials and dinosaurs
G. the evolutionary background of kangaroos
H. how kangaroos are not really mammals
27. The author of the passage most likely agrees with which of the following statements?
A. Opossums are smarter than kangaroos.
B. Dinosaurs ate kangaroo meat.
C. Kangaroos were once extinct.
D. Kangaroos are interesting animals.
28. The author of the passage is most likely E. a scientist.
F. a businessman.
G. a cowboy.
H. an Australian.
29. The author most likely quotes the exact size ranges in lines 15-16 in order to A. shock the reader.
B. demonstrate one way in which different species of kangaroos vary greatly.
C. contradict earlier research.
D. prove that not all kangaroos are important.
30. The phrase "living fossil" in line 59 refers to which of the following?
E. a dinosaur
F. a kangaroo
G. an opossum
H. an Australian marsupial
31. What does the passage imply about the kangaroo's development?
A. The kangaroo has adapted to its surroundings more than some other marsupials.
B. The kangaroo had a development almost identical to that of the opossum.
C. The kangaroo is similar to a dinosaur.
D. The native habitat of the kangaroo is North America.

In 1989, the Civil Rights memorial was dedicated in Montgomery, Alabama. The memorial, the first dedicated to the civil rights movement, consists of two distinct
5 pieces of black Canadian granite. One piece carries this quote from Martin Luther King, Jr.: "until justice rolls down like waters and righteousness like a mighty stream." The other piece 10 is a circular slab, placed low enough to be touched by visiting children. On this slab is a time line chronicling the major events of the movement, from the Supreme Court's Brown vs. the Board of Education ruling in 1954 to assassination of Martin Luther King, Jr. in 1968. Other events noted on the memorial include the Montgomery Alabama Bus Boycott, the March on
20 Washington (the "I Have a Dream" speech), and the signing of the Civil Rights Act.

The names of 40 men, women, and children killed during the civil rights movement also appear on the circular slab. Three types of martyrs are commemorated: those who were killed during acts of non-violent protest, those whose deaths were embraced by civil rights leaders to help create momentum for further change, and those who were murdered by anti-movement agitators in order to intimidate other civil rights activists. The intention of those who built the memorial was to represent how each martyr added to the momentum of the struggle, and therefore did not die in vain.

The memorial was designed by Maya
Lin, a Chinese-American woman from Ohio. At the age of 21, while a student at Yale, Lin won a competition to design the Vietnam Veterans Memorial in Washington, D.C. Eight years later, Lin was asked to design the Civil Rights Memorial. In creating her design for the memorial, Lin was inspired by King's "mighty stream" quote. The form of the monument and the time-line concept took shape: water flows from a fountain over the names of the 40 martyrs.

The monument was commissioned by the Southern Poverty Law Center. The center was founded in the sixties by two
lawyers and provided legal defense for the men and women of the civil rights movement. Today, the center is a not-forprofit organization. The main function of the center is to remind the world of
60 what has passed and educate our future leaders. The memorial is a symbol of these goals.

The Memorial is located on an open plaza near two historically important buildings: the capitol steps where the 1965 voting rights march ended, and the church at which King was pastor during the Montgomery Bus Boycott of 1955-1956. According to the Southern
Poverty Law Center, "The plaza is a contemplative area-a place to remember the Movement, to honor those killed during the struggle, to appreciate how far the country has come in its quest for equality and consider how far it has to go."

> When the monument was dedicated, family members of the 40 martyrs touched the names. Many of the family members, public officials, and onlookers, including Maya Lin, were deeply moved on the occasion-and still today, many visitors to the Civil Rights Memorial speak of it as an emotional and powerful experience.
32. Which of the following best tells what this passage is about?
E. the civil rights movement and its winners and losers
F. a unique institution called the Southern Poverty Law Center
G. a description of one important memorial of the civil rights movement
H. an analysis of the impact of Brown vs. the Board of Education
33. Which of the following events is most probably commemorated on the time line of the Civil Rights Memorial?
A. the end of the Civil War in 1865
B. the passage of the 14th Amendment to the Constitution in 1868
C. the passage of the 19th Amendment to the Constitution in 1920
D. the March on Washington in 1963
34. According to the passage, the 40 people whose names appear on the monument all share in common the fact that \(\quad \mathbf{E}\). they were killed during acts of non-violent protest.
F. they were all personal friends of Martin Luther King, Jr.
G. their deaths were related to the civil rights movement in some way.
H. all of them had children who frequently visit the memorial.
35. According to the passage, anti-civil rights agitators murdered some civil
rights activists in order to A. provide civil rights leaders with martyrs around which to rally their cause.
B. influence the Supreme Court to overturn their decision in Brown vs. the Board of Education.
C. add to the momentum of the civil rights struggle.
D. scare others who were involved in the civil rights movement.
36. The author would most likely agree with which statement?
E. There is more prejudice today than in the sixties.
F. It is important to continue to educate people about the civil rights movement.
G. All participants in the civil rights movement were guilty.
H. There are no other important memorials to the civil rights movement.
37. The Southern Poverty Law Center is
B. an educational institution.
C. a museum.
D. an art center.
A. a construction company.

Paul Broca was a major figure in the development of both medicine and anthropology in the mid-nineteenth century. He performed distinguished out, as Broca inferred on only fragmentary evidence, is to an important extent localized in and controlled by Broca's area. It was one of the first discoveries of a separation of function between the left and right hemispheres of the brain. But most importantly, it was one of the first indications that specific brain functions exist in particular locales in the brain, and that there is a connection between the anatomy of the brain and what the brain does.

Paul Broca died in 1880, perhaps of the very sort of aneurysm that he had studied so brilliantly. At the moment of his death he was working on a comprehensive study of brain anatomy.

It is difficult to hold Broca's brain without wondering whether in some sense Broca was still in there-his wit, his skeptical mind, his abrupt gesticulations when he talked, his quiet and sentimental moments. Might there be preserved in the configuration of neurons a recollection of the triumphant moment when he argued before the combined medical
faculties (and his father, overflowing with pride) on the origins of aphasia?
50 Where do we go when we die? Is Paul Broca still there in his formalin-filled bottle? Perhaps the memory traces have decayed, although there is good evidence from modern brain investigations
55 that a given memory is redundantly stored in many different places in the brain. Might it be possible at some future time, when neurophysiology has advanced substantially, to reconstruct the memories or insights of someone long dead? And would that be a good thing? It would be the ultimate breach of privacy. But it would also be a kind of practical immortality, because, especially for a man like Broca, our minds are clearly a major aspect of who we are.
38. Which of the following best tells what this passage is about?
E. the author's reflections on Paul Broca
F. the importance of specific brain functions being attributed to specific parts of the brain G. the overlap of medicine and anthropology
H. the location of memory within the brain and some implications of his work
39. From lines 15-20, it can be determined that Broca is most famous today for A. his work as an anthropologist.
B. his discovery of an area of the brain that controls speech.
C. his investigations of the limbic area.
D. his invention of the tiny region to the left of the cerebral cortex.
40. From the discussion of Broca's research on articulate speech (lines 20-24), it can be inferred that E. Broca would have arrived at a different conclusion had he not relied on fragmentary evidence.
F. Broca's inference was incomplete and insignificant.
G. Broca was successful in locating the part of the brain responsible for articulate speech.
H. because of only fragmentary evidence, Broca was unable to find the part of the brain that controlled speech.
41. Which of the following is not discussed in the passage?
A. Broca's significant contribution to the study of the brain
B. the growing indications that specific brain functions exist in particular sections of the brain \(\quad\) C. memories being stored in various places in the brain
D. Broca's discoveries, which solved all of the mysteries of the brain
42. In what way was Paul Broca's death ironic?
E. Although he studied infant mortality for many years, he died an old man.
F. He was killed by the very people he was trying to save.
G. After devoting most of his life to medicine, he made an unexpected switch to anthropology.
H. He seems to have died from the same disorder that he was studying.
43. The questions posed in lines \(44-52\) demonstrate the author's interest in A. the physical preservation of memory and identity within the brain.
B. Broca's victory over the combined medical faculties in the debate over aphasia.
C. the appreciation of Broca's father in neurophysiology.
D. the arrangements of neurons that cause aphasia.

John Philip Sousa was no Beethoven. Nevertheless, he made a contribution to music. When you say "a Sousa march," almost everyone will know what you are talking about. All of his marches bear the stamp of a vigorous, clear-cut, original musical personality. Above all they are intensely practical. Sousa began studying traditional music theory when he was six. As a teenager, it looked as if he would follow in the footsteps of his father, a trombonist. In 1880, as a member of the U.S. Marine Corps, he became the bandmaster of the Marine Band, where he gradually became known as a bandmaster of incredible precision. The marches he wrote, first for the Marine Band and later for his own band, were intended to set the pace for marching men.

A march is a piece of music with a strong rhythm, usually written for the express purpose of playing while people (often military personnel) march. A march's tempo is usually set at the rate established during Napoleon's reign: 120 beats per minute, although funeral marches are half as fast. Notable composers of marches include Beethoven, Mozart, Mahler, and Wagner, but the person whose name is most commonly associated with marches is Sousa.

Sousa marches are simple. Their tunes are so uncomplicated, so easy to catch, so melodic, that you can easily underrate them. Simple as they may be, they are unmistakably Sousa's. It took inspiration to write them, even if it was only a small amount of inspiration. Perhaps this simplicity accounts for Sousa's marches' popularity. After all, of the many great but complicated marches Wagner wrote for his massive operas, most people are familiar with only one: the wedding march from Lohengrin is the source of the familiar "here comes the bride" music played at many weddings. Not coincidentally, that march may be Wagner's most simple.

On the other hand, many of Sousa's marches are instantly recognizable, even if we don't know their names or even that Sousa wrote them. Among Sousa's most famous marches are "Stars and Stripes Forever," "The Washington Post," and the Marine Corp's official march, "Semper Fideles."
\[
\begin{aligned}
& \text { It can be added that a Sousa march } \\
& \text { does stir one important thing--na- } \\
& 60 \text { tional pride. Today, John Philip Sousa's } \\
& \text { marches can be heard at a variety of } \\
& \text { patriotic events. When you attend a } \\
& \text { Memorial Day parade, or a spectacular } \\
& \text { Fourth of July fireworks display, you } \\
& 65 \text { notice that the tunes of Sousa are the } \\
& \text { overwhelming favorites of marching } \\
& \text { bands. When played, even the most un- } \\
& \text { patriotic of people suddenly rise to the } \\
& \text { occasion, sometimes marching along } \\
& \text { 70 } \\
& \text { with the beat and whistling the simple } \\
& \text { yet moving melodies. If one's music } \\
& \text { were the measure of one's pride in a } \\
& \text { nation, then John Philip Sousa can be } \\
& \text { recognized as being intensely proud of } \\
& 75 \\
& \text { America. Likewise, if listening to music } \\
& \text { can inspire patriotism, then when we } \\
& \text { listen to Sousa's marches we almost } \\
& \text { automatically become enveloped with a } \\
& \text { patriotism unheard of, except perhaps } \\
& 80 \\
& \text { in times of national emergencies. }
\end{aligned}
\]
44. Which of the following best tells what this passage is about?
E. It sets forth a new definition of artistic greatness.
F. It describes the worth of the music of John Philip Sousa.
G. It emphasizes the importance of band music.
H. It proves that Sousa was one of the greatest musicians of all time.
45. The author would be most likely to use which of the following terms in describing the music of John Philip Sousa?
A. memorable
B. peaceful
C. jazzy
D. stupid
46. Sousa's work is "practical" (line 8) in the sense that it is E. often imitated by classical musicians.
F. easy to forget.
G. easily learned by children.
H. used for actual military marching.
47. The discussion of the simplicity of Sousa's marches in lines 33-39 suggests that A. Sousa's marches are childish.
B. the simplicity of Sousa's marches is unnecessary.
C. simple music is superior to complex music.
D. the simplicity of Sousa's marches does not make them any less admirable.
48. What does the passage imply about the quality of Sousa's work?
E. It is inferior to composers such as Beethoven and Wagner.
F. Despite their simplicity, his marches merit consideration as true national classics.
G. It is a highly complex combination of rhythm, melody, and beat.
\(\mathbf{H}\). The works are good to listen to, but have little practical application.
49. Which of the following is not mentioned as an aspect of Sousa's compositions?
A. Marching bands use them very frequently.
B. They are not equal to the complexity of works by Beethoven.
C. The music is original and clearly the work of Sousa.
D. Wagner's marches were an early influence on Sousa's works.
50. The final paragraph of the passage suggests that E. Sousa's music can unleash a patriotism in the listener.
F. most people would prefer to listen to Mozart.
G. Sousa's music encourages citizens to join the military.
H. everyone recognizes and enjoys "Semper Fideles."

The poem "My Lost Youth," written in 1855 by Henry Wadsworth Longfellow, lyrically describes the poet's childhood.

Often I think of the beautiful town
That is seated by the sea;
Often in thought go up and down
The pleasant streets of that dear old
5 town,
And my youth comes back to me. And a verse of a Lapland song Is haunting my memory still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

I can see the shadowy lines of its trees, And catch, in sudden gleams, The sheen of the far-surrounding seas, And islands that were the Hesperides Of all my boyish dreams.

And the burden of that old song, It murmurs and whispers still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

I remember the black wharves and the slips,

And the sea-tides tossing free;

Is singing and saying still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

I remember the bulwarks by the shore, And the fort upon the hill; The sunrise gun, with its hollow roar, The drum-beat repeated o'er and o'er, And the bugle wild and shrill.

And the music of that old song
Throbs in my memory still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

I remember the sea-fight far away, How it thundered o'er the tide! And the dead captains, as they lay In their graves, o'erlooking the tranquil bay, Where they in battle died. And the sound of that mournful song

Goes through me with a thrill:
"A boy's will is the wind's will,
And the thoughts of youth are long, long thoughts."

I can see the breezy dome of groves, The shadows of Deering's Woods; And the friendships old and the early loves
Come back with a Sabbath sound, as of doves

In quiet neighborhoods.
And the verse of that sweet old song,

It flutters and murmurs still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

I remember the gleams and glooms that dart

Across the school-boy's brain;
The song and the silence in the heart, That in part are prophecies, and in part Are longings wild and vain. And the voice of that fitful song Sings on, and is never still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."
so There are things of which I may not speak;

There are dreams that cannot die; There are thoughts that make the strong heart weak,
85 And bring a pallor into the cheek,
And a mist before the eye.
And the words of that fatal song
Come over me like a chill:
"A boy's will is the wind's will,
\({ }_{90}\) And the thoughts of youth are long, long thoughts."

Strange to me now are the forms I meet
When I visit the dear old town; But the native air is pure and sweet, And the trees that o'ershadow each wellknown street,

As they balance up and down, Are singing the beautiful song,
Are sighing and whispering still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."

And Deering's Woods are fresh and fair, And with joy that is almost pain 105 My heart goes back to wander there, And among the dreams of the days that were,

I find my lost youth again.
And the strange and beautiful 110 song,

The groves are repeating it still:
"A boy's will is the wind's will, And the thoughts of youth are long, long thoughts."
51. What is the best way to describe the tone of this poem?
A. romantic
B. nostalgic
C. tragic
D. comedic
52. A "Lapland song" (line 7) most likely refers to E. a song from the narrator's childhood.
F. a song that is native to the narrator's family.
G. a mournful, bleak song.
H. a song meant for traditional dancing.
53. Consider the following excerpt (lines 56-61).

I can see the breezy dome of groves,
The shadows of Deering's Woods;
And the friendships old and the early loves
Come back with a Sabbath sound, as of doves
In these lines, the author describes
A. his memories coming back as birds flying in the wind.
B. the comforting religious practices of his youth.
C. the menacing presence of a local forest.
D. the graves of his lost loved ones.
54. What is suggested by the second to last stanza?
E. The air quality of the speaker's hometown has gained a new sweetness.
F. The shadows of the trees hide the true beauty of the speaker's hometown.
G. Returning to his hometown makes the speaker cry.
H. Many things have changed about the speaker’s hometown, but its natural environment has not.
55. What is most likely the reason for the repetition of the refrain ("A boy's will is the wind's will")?
A. ensure the reader not forget the text of the Lapland song
B. echo the structure of a song, which is fittingly the topic of the poem itself
C. adhere to the required poetic structure of Longfellow's era
D. let the reader know when each stanza is coming to an end
56. The phrase "And with joy that is almost pain / My heart goes back to wander there," (lines 104-105) expresses E. how joyous it always is to see one's hometown after a long time away.
F. the bittersweet feeling of living in the past.
G. the rejuvenating quality of a long walk in the woods.
H. the narrator's desire to move on from his past and live fully in the present, despite the costs.
57. The narrator can best be described as A. someone thinking about his hometown and remembering his youth.
B. a person who is very fond of old songs.
C. an older person who has a deeply personal and emotional connection to nature.
D. someone who is realizing he has been very lucky in his life.

\title{
PART 2—MATHEMATICS
}

Suggested Time-90 Minutes

\section*{57 QUESTIONS}

IMPORTANT NOTES:
(1) Formulas and definitions of mathematical terms and symbols are not provided.
(2) Diagrams other than graphs are not necessarily drawn to scale. Do not assume any relationship in a diagram unless it is specifically stated or can be figured out from the information given.
(3) Assume that a diagram is in one plane unless the problem specifically states that it is not.
(4) Graphs are drawn to scale. Unless stated otherwise, you can assume relationships according to appearance. For example, (on a graph) lines that appear to be parallel can be assumed to be parallel; likewise for concurrent lines, straight lines, collinear points, right angles, etc.
(5) Reduce all fractions to the lowest terms.

\section*{GRID-IN PROBLEMS}

QUESTIONS 58-62
DIRECTIONS: Solve each problem. On the answer sheet, write your answer in the boxes at the top of the grid. Start on the left side of each grid. Print only one number or symbol in each box. DO NOT LEAVE A BOX BLANK IN THE MIDDLE OF AN ANSWER. Under each box, fill in the circle that matches the number or symbol you wrote above. DO NOT FILL IN A CIRCLE UNDER AN UNUSED BOX.
58. A car drives at a rate of 40 miles per hour. The car must drive a total of 350 miles. How many hours will it take the car to finish the trip?
59. If \(36 a=45 \div b\), then \(a b=\)
60. Malcolm Little High School has sophomores, juniors, and seniors, but no freshmen. The ratio of sophomores to juniors is 5 to 4 , and the ratio of juniors to seniors is 3 to 2 . If there are 300 sophomores, how many seniors are there?
61. A class must select a president, a vice-president, and a treasurer from among 10 students. No student can hold more than one position. How many arrangements of the three positions can the class make?
62. If \(\frac{a}{b}<\frac{b}{a}, a^{2}=-2 a\), and \(a b=18\), what is the value of \(b\) ?

\title{
MULTIPLE CHOICE PROBLEMS
}

\section*{QUESTIONS 63-114}

DIRECTIONS: Solve each problem. Select the best answer from the choices given. Mark the letter of your answer on the answer sheet. You can do your figuring in the test booklet or on paper provided by the proctor. DO NOT MAKE ANY MARKS ON YOUR ANSWER SHEET OTHER THAN FILLING IN YOUR ANSWER CHOICES.
63. What is the decimal representation of \(1,000+1+\frac{1}{1,000}\) ?
A. \(1,001.01\)
B. 1,001.001
C. 1,000.101
D. \(1,001.0001\)
64. A track runner records his times on the 400 meter dash. If the range of his times is 9 seconds and one of his times is 58 seconds, then which of the following cannot be one of his times?
E. 48 seconds
F. 50 seconds
G. 66 seconds
H. 67 seconds
65.
\[
\begin{aligned}
& 1 \text { dollar }=0.96 \text { klurbs } \\
& 1 \text { dollar }=0.64 \text { flourns }
\end{aligned}
\]

Using the conversion above, how many klurbs is equal to 1 flourn?
A. 0.66
B. 0.75
C. 1.33
D. 1.50
66. A square has an area of 4 square yards. What is the area of the square in square feet? (1 yard \(=3\) feet) \(\quad\) E. 6
F. 12
G. 16
H. 36
67.
\begin{tabular}{|l|c|c|}
\hline \multicolumn{1}{|c|}{ Plant Type } & \begin{tabular}{c} 
\% of Plant Species \\
in Region \(\boldsymbol{A}\)
\end{tabular} & \begin{tabular}{c} 
\% of Plant Species \\
in Region B
\end{tabular} \\
\hline symbiotic & 0.65 & 34 \\
\hline parasitic & 0.337 & 10 \\
\hline coniferous & 72 & 0.65 \\
\hline deciduous & 27 & 36 \\
\hline epiphytic & 0.013 & 19.35 \\
\hline TOTAL & 100 & 100 \\
\hline
\end{tabular}

What fraction of the plant species in Region \(A\) are parasitic?
A. \(\frac{337}{100}\)
B. \(\frac{337}{1,000}\)
C. \(\frac{337}{10,000}\)
D. \(\frac{337}{100,000}\)
68.


What is the area of the trapezoid shown?
E. 32
F. 40
G. 56
H. 80
69. Four fences are connected to partially enclose a yard. Each of the four fences is 15 feet long, and two larger fences of equal length are connected to complete the enclosure. If the six fences that make the enclosure measure a total of 118 feet, what is the length of one of the larger fences?
A. 16 ft
B. 29 ft
C. 58 ft
D. 60 ft
70.
\[
\frac{2 a-6}{a+3}
\]

How many integer values of \(a\) will allow the expression above to be an integer if \(1 \leq a \leq 7\) ?
E. 1
F. 2
G. 3
H. 4
71. If \(x=-5\), what is the value of \(3 x^{2}-4 x+2\) ?
A. -53
B. -19
C. 57
D. 97
72. A student attempts to measure the length of a table using a ruler and gets a measure of 4.4 inches. However, the student later learns that the ruler is imprecise and that all actual lengths are within \(10 \%\) of the measured length. What is the maximum actual length of the table, in inches?
E. 4.44
F. 4.50
G. 4.80
H. 4.84
73. A painter creates a blend of green paint using blue paint and yellow paint in a ratio of \(3: 4\). The painter creates 35 gallons of the blend. How many gallons of blue paint did the painter use?
A. 5
B. 7
C. 15
D. 20
74. Nancy is selecting clothes to pack for her weekend vacation. She has 12 shirts and must pack 3 . How many different sets of shirts can she pack?
E. 6
F. 220
G. 960
H. 1,320
75. Eric is now 22 years old and in 5 years will be three times as old as Alan will be. How old is Alan now?
A. 4
B. 9
C. 11
D. 27
76. All the students in a class either walk or take the bus to school on all days they attend. On a particular day, of all the students who walk to school, \(30 \%\) are absent and \(75 \%\) of the students who took the bus were present. If \(40 \%\) of the students take the bus to school, what percent of the students were present that day?
E. 28
F. 42
G. 58
H. 72
77. Four coins are flipped. What is the probability that all four coins will be heads?
A. \(\frac{1}{16}\)
B. \(\frac{1}{8}\)
C. \(\frac{1}{4}\)
D. \(\frac{1}{2}\)
78.


The number line above shows the solution to which of the following inequalities?
E. \(-2 \leq \frac{y}{2} \leq 4\)
F. \(-2 \leq 2 y \leq 4\)
G. \(-4 \leq \frac{y}{2} \leq 2\)
H. \(-4 \leq 2 y \leq 2\)
79. Stephen can type 120 words per minute. At this rate, how many words can he type in 1 hour?
A. 360
B. 720
C. 3,600
D. 7,200
80. A fruit stands sells 3 bananas for every 5 apples it sells. If the fruit stand sells a total of 24 bananas in a day, how many apples did it sell?
E. 20
F. 40
G. 80
H. 120
81. A rectangular solid has a volume of 350 . The height of the solid is 7 and the width is twice the length. What is the width of the solid?
A. 2.5
B. 5
C. 7.5
D. 10
82. A jar of gumballs contains 7 red gumballs, 4 blue gumballs, and 5 green gumballs. Two gumballs are selected at random. What is probability that both gumballs are not green?
E. \(\frac{25}{256}\)
F. \(\frac{1}{12}\)
G. \(\frac{11}{24}\)
H. \(\frac{121}{256}\)
83. On Tuesday, \(\frac{2}{3}\) of a large block of ice melted. On Wednesday, \(\frac{1}{2}\) of the remaining ice melted. If the block of ice then weighed 60 pounds, how much did it weigh, in pounds, at the beginning of the day on Tuesday?
A. 480
B. 450
C. 360
D. 180
84. Simplify the following expression:
\[
\frac{a}{b}\left[\left(\frac{2 a-2 b}{2 a+2 b}\right)\left(\frac{2 a+2 b}{2 a-2 b}\right)\right]
\]
(Note: \(a \neq b\) and \(a \neq-b\).)
E. \(a\)
F. \(\frac{a^{2}}{b}\)
G. \(\frac{a(2 a-2 b)}{b}\)
H. \(\frac{a^{2}(2 a+2 b)}{b}\)
85. A coin purse has 3 pennies, 5 nickels, 4 dimes, and 1 quarter. If a coin is selected at random, what is the probability that the coin selected will be a nickel?
A. \(\frac{5}{8}\)
B. \(\frac{5}{13}\)
C. \(\frac{1}{8}\)
D. \(\frac{1}{13}\)
86.

\section*{PETS PER STUDENT IN SCHOOL}
\begin{tabular}{|c|c|}
\hline Number of Pets per Student & Number of Students \\
\hline 0 & 35 \\
\hline 1 & 49 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline 2 & 24 \\
\hline 3 & 16 \\
\hline 4 or more & 11 \\
\hline
\end{tabular}

The table above shows the result of a poll a student conducted about the number of pets each of the students in her school has. What percent of the students polled have at least 3 pets?
E. 27\%
F. 20\%
G. 11\%
H. 8\%
87. A class has 25 students, 5 of whom are wearing black sneakers. If two students are selected at random, what is the probability that both students will be wearing black sneakers?
A. \(\frac{1}{30}\)
B. \(\frac{1}{25}\)
C. \(\frac{1}{6}\)
D. \(\frac{1}{5}\)
88. If \(a=2 b+1\), then, in terms of \(b\), what is the value of \(4 a-5\) ?
E. \(4 b-4\)
F. \(4 b+3\)
G. \(8 b-4\)
H. \(8 b-1\)
89.


On the number line above, \(A B=\frac{1}{6}\). Which of the following is a possible value of \(X\) ?
A. 3.382
B. 3.438
C. 3.584
D. 3.614
90. If \(20 \%\) of the science students at Central High study only physics, \(25 \%\) study only chemistry, and the rest study only biology, what is the smallest number of science students who could be studying biology?
E. 4
F. 9
G. 11
H. 20
91.


If the lines in the figure above intersect as shown, what is the value of \(a\) \(+b\) ?
A. 40
B. 80
C. 140
D. 180
92.


On the number line above, \(A B: B C: C D\) is \(3: 8: 1\). What is the value of \(\overline{B D}\) ?
E. 2
F. 6
G. 16
H. 18
93. If \(p=5\) and \(q=-4\), then \(3 p-2 p q=\)
A. -35
B. -25
C. 35
D. 55
94. The value of a home increases by \(10 \%\) per year. What is the percent increase in the value of the home after 3 years?
E. \(27.1 \%\)
F. \(30.0 \%\)
G. 31.4 \%
H. 33.1\%
95. What is the greatest prime factor of 770 ?
A. 5
B. 10
C. 11
D. 77
96. If \(\frac{3 n-6}{7-4}=11\), then \(n=\)
E. 9
F. 11
G. 13
H. 15
97. A certain car salesperson receives a bonus of \(2 \%\) for cars sold for \(\$ 40,000\) or less and \(3 \%\) for cars sold for more than \(\$ 40,000\). What is the salesperson's bonus if she sells a car for \(\$ 39,000\) and another car for \$41,000 ?
A. \(\$ 2,010\)
B. \(\$ 2,460\)
C. \(\$ 4,000\)
D. \(\$ 4,100\)
98. Sade watches \(2 \frac{1}{2}\) hours of television each day. How many hours of television does she watch in one week?
E. \(2 \frac{4}{5}\)
F. \(10 \frac{2}{3}\)
G. \(14 \frac{1}{2}\)
Н. \(17 \frac{1}{2}\)
99. Inez ran \(y\) miles. Inez and Darcy ran a total of \(3 y+7\) miles. How many miles, in terms of \(y\), did Darcy run?
A. \(\frac{3 y+7}{y}\)
B. \(\frac{3 y-7}{y}\)
C. \(2 y+7\)
D. \(2 y-7\)
100. \(-3 a(5 b-4 c)=\)
E. \(-2 a b c\)
F. \(60 a b c\)
G. \(-15 a b+12 c\)
H. \(-15 a b+12 a c\)
101.


In the figure above, two circles have radii of \(a\) and \(b\). If \(a=\frac{1}{3} b\), and the sum of the circumferences of the two circles is \(64 \pi\), then what is the value of \(b\) ?
A. 8
B. 12
C. 24
D. 32
102.


On the number line above, the hash marks are evenly spaced. What is the value of \(C\) ?
E. \(-\frac{2}{9}\)
F. \(-\frac{1}{7}\)
G. \(\frac{5}{63}\)
H. \(\frac{2}{21}\)
103. Jamie can read an article in 10 minutes. Rich can read an article in 15 minutes. At this rate, how many articles can Jamie and Rich read in 2 hours?
A. 20
B. 22
C. 24
D. 26
104. Based on the approximation 1 meter \(=3.281\) feet, how many feet are in 4.4 kilometers?
E. 1.3411
F. 14.4364
G. 1,341.1
H. \(14,436.4\)
105.


The prism above has bases that are equilateral triangles with perimeter 18 , and \(A B=9\). What is the surface area of the prism, excluding the
A. 54
B. 108
C. 162
D. 216
106. A museum is being built to display 3,100 paintings. If each room in the museum can hold 60 paintings, what is the minimum number of rooms that the museum must contain?
E. 50
F. 51
G. 52
H. 60
107. At 4 p.m., a 6 -foot tall man cast a shadow 10 feet long. At the same hour, how long would the shadow of a 15 -foot tall lamp post have been?
A. 9
B. 21
C. 25
D. 27
108.
\[
a=10 b=7 c=5 d
\]

What is the least possible value for \(a\) if \(b, c\), and \(d\) are positive integers?
E. 50
F. 70
G. 170
H. 350
109. \(15^{5} \div 15^{3}=\)
A. 15
B. \(15^{2}\)
C. \(15^{3}\)
D. \(15^{8}\)
110. What is the value of \(n\) if the mean of \(5,7,9\), and \(n\) is 7 ?
E. 6
F. 7
G. 8
H. 9
111.


In the figure above, the measure of angle \(C=\)
A. 90
B. 60
C. 50
D. 45
112.


Which of the following points can be found in the shaded area above?
E. \(\left(\frac{1}{2}, \frac{3}{4}\right)\)
F. \(\left(\frac{1}{2},-1\right)\)
G. \(\left(-\frac{1}{2},-\frac{3}{4}\right)\)
H. \(\left(-1 \frac{1}{2}, \frac{3}{4}\right)\)
113. Andre has 5 sessions of math class for every 3 sessions of history class. If he has 24 sessions of history class, how many total sessions of math and history class does he have?
A. 8
B. 16
C. 40
D. 64
114. Juliet is working in a call center and receives 3 calls per minute. At this rate, how many calls does she receive per hour?
E. 20
F. 90
G. 180
H. 240

THIS IS THE END OF THE TEST.
IF TIME REMAINS, YOU MAY CHECK YOUR ANSWERS TO PART 1 AND PART 2. BE SURE THAT THERE ARE NO STRAY MARKS, PARTIALLY FILLED ANSWER CIRCLES, OR INCOMPLETE ERASURES ON YOUR ANSWER SHEET.

\section*{Chapter 17}

\section*{Practice Test 2: Answers and Explanations}

\section*{PRACTICE TEST 2 ANSWERS}

ELA Part A
1. A 2. G
3. B
4. E

ELA Part B
5. B
6. F
7. D
8. F
9. D
10. E
11. C

\section*{Reading Comprehension}
12. E
13. C
14. F
15. C
16. H
17. B
18. E
19. B
20. E
21. C
22. H
23. B
24. E
25. C
26. G
27. D
28. E
29. B
30. G
31. A 32. G
33. D
34. G
35. D
36. F
37. B
38. E
39. B
40. G
41. D
42. H
43. A 44. F
45. A 46. H
47. D
48. F
49. D
50. E
51. B
52. E
53. A 54. H
55. B
56. F
57. A

\section*{Math}
58. 8.75
59. 1.25
60. 160
61. 720
62. -9
63. B
64. E
65. D
66. H
67. D
68. G
69. B
70. F
71. D
72. H
73. C
74. F
75. A 76. H
77. A 78. H
79. D
80. F
81. D
82. G
83. C
84. F
85. B
86. F
87. A 88. H
89. B
90. G
91. B
92. H
93. D
94. H
95. C
96. G
97. A 98. H
99. C
100. H
101. C
102. H
103. A 104. H
105. C
106. G
107. C
108. F
109. B
110. F
111. D
112. E
113. D
114. G

\section*{ELA Part A}
1. A The answer choices make it pretty clear that the question is testing comma usage rules. Look at the existing two commas; the first one is necessary, as it sets apart a transition word, so eliminate (B). The second one is needed to separate a phrase in apposition, so eliminate (D). The remaining choices make it clear that the comma after world is setting apart unnecessary or nonessential information, so put your finger over people and see if you can still read the sentence. You can't, so eliminate (C). Choice (A) is correct; if you cover up "people of every religion and culture," the sentence still makes sense.
2. G The question tells you that there's a verb tense error, so skim the paragraph to see which tense is used throughout. This text is an explanatory passage about insects, so it logically employs the present tense throughout. Look for the odd one out! Sentence 3 uses the past tense "did." Don’t waste time with POE here; choose (G) and move on.
3. B This question asks you to combine two sentences while maintaining and clarifying their relationship. Begin, then, by noting that the two sentences are in sharp contrast. You want an answer choice that reconciles this contradiction into one sentence. Use POE to narrow things down: Eliminate (A) because the idiom "on the one hand" does not fit this context; it would need an accompanying "on the other hand." Choice (D) can be crossed off because it alters the relationship: the writer believes going to the park is harrowing. Between (B) and (C), eliminate (C) because its phrasing is not concise, although it does express the relationship. Choice (B) combines the sentences much more succinctly and is the correct answer.
4. \(\mathbf{E}\) The question tells you that it's testing precision directly, so look specifically for the use of appropriate details, such as numbers and dates, in the answer choices. POE those that retain vague phrases. Eliminate ( F ) and ( H ) because they retain the vague "born in the late nineteenth century"; (E) and (G) are more precise with "1888." Down to two! Eliminate (G) because it mentions "many improvements." Choice (E) provides the most precise details.

\section*{ELA Part B}
5. B To answer this question, you need to use your headlines to quickly identify the main idea of the passage is. This brief passage explores the roots of Bitcoin, how it is used today, and its future-so your answer choice should align with that summary. Eliminate (A) and (D) because they address only one of the topics: Bitcoin's roots and how to make money with Bitcoin today. Finally, (C) may be true, but the passage doesn't mention the relationship between Bitcoin and
democracies-so cross it off. The correct answer is (B).
6. \(\mathbf{F} \quad\) The question announces it is looking for precision, so start looking for any vague details in sentence 4 . The most precise choice has as many appropriate details as possible, including numbers. Eliminate (E) because, even though it adds a new detail, it retains the vague "a person whose real identity is not known." Similarly, eliminate (G) because it retains the vague "launched in the 2000s," and the date 2009 is preferred. There is only a slight difference between (F) and (H), the phrase "completely and totally," which does not add any significant new detail to the sentence. Simply calling it "a digital currency" is fine. Choice ( F ) is the most precise revision.
7. D When choosing an appropriate transition word, look to see if the second sentence agrees or departs from the previous one. Look at the passage and notice that sentence 6 continues to explain the preBitcoin method of sending money digitally. The passage says banks can monitor transactions and charge fees, so look for an agreement transition word. Cross off the departure transition words in (A), (B), and (C). The correct answer is (D).
8. \(\mathbf{F}\) On a question like this, refer to the paragraph and sentence in question, and look for clues that can lead you to another place in the paragraph. Sentence 12 says "What's unique about Bitcoin is not the currency itself, but rather the network it lives on." You notice that the first part of the paragraph describes what Bitcoin is and then describes the Bitcoin network. The key sentence to notice is sentence 14, which says that Bitcoin itself is "not revolutionary." "Revolutionary" is a relative of "unique"; it’s a good match. It would logically make sense to call Bitcoin itself not revolutionary, and then explain what might be revolutionary or unique. Sentence 12 flows nicely after sentence 14 , so choose (F).
9. D Answering this question correctly requires knowing what the main idea of the third paragraph is. Either by using your headlines or skimming, notice that this paragraph describes how Bitcoin works today-its use as a token and its collective network. The sentence that you are given explains how the blockchain works, so find the sentence
in which the blockchain is introduced: sentence 17. This sentence can logically follow sentence 17 to offer supporting evidence about the blockchain. The correct answer is (D).
10. \(\mathbf{E}\) On a combining sentences question like this, be sure to create one sentence without altering the relationship between the two ideas in any way. First, understand that relationship: most currencies are run by a central bank, but Bitcoin is totally decentralized and run by an anonymous network. Eliminate (F) and (H) as they’ve suggested causality with "because" and "so." Bitcoin is not decentralized because other currencies are controlled by a central bank. And most currencies are not controlled by a central bank as a result of Bitcoin being decentralized; that's not what the passage says. Down to two! Eliminate (G) because the "yet" in the sentence does not work with the "despite" also being there (additionally it creates a comma splice). Therefore, choose (E).
11. C Use your headlines and knowledge of the passage to jump right to POE on a general question like this. It helps to have done the other questions before a general one like this, because it gives you a good deal of information about its argument. Cross out (A) because, while this may be true, the main argument of the passage is not about Nakamoto's identity; it is about Bitcoin more broadly. Eliminate (B) and (D) because the passage says neither that Bitcoin will surpass other currencies nor that investors should stay away from the currency. The correct answer is (C), which aligns nicely with the tone of the passage as a whole: curious and open-minded about the possibilities of Bitcoin, but not its most fervent defender.

\section*{Reading Comprehension}
12. E This is a general question, so once you've skimmed the passage, use your headlines to move directly into POE. First, eliminate (F) because, while shipbuilding is mentioned in this text, it is not the author's main purpose. Next, eliminate (G) because the passage does speak about the origin of Crete, but its origin story takes up a mere
sentence, and the passage is not trying to "prove" or defend itself against counter-claims. Finally, eliminate (H) because this passage does not analyze trade-oriented economies generally; it discusses one trade-oriented economy, Crete. The correct answer is (E).
13. C Like any question with except/not/least, follow the basic strategy: use your pencil to cross out the "except," "not," or "least." Then, mark each answer choice with a T for true or F for false. There will be 3 Trues and just 1 False; you know the odd one out is your answer, so select it. Put a T next to (A), (B), and (D) because the passage states that the land was not suitable for an agricultural economy (lines 6-8); that the location of the island helped settlers make a good living by trade (lines \(14-17\) ); and that the island provided a large supply of timber (lines 22-24). Choice (C) uses deceptive language: the author states that the island is surrounded by jagged mountains, not jagged cliffs. This makes (C) the false statement and therefore the correct answer.
14. F This question asks why the people of Crete were known as skillful builders. Using your headlines from the passage is the best strategy for this question. You can also skim for the term "skillful builders," which appears in paragraph 2; start there. This paragraph mentions shipbuilding as evidence of Cretan building skills. POE based on this paragraph. Eliminate (E) because being dressed well has nothing to do with building well; eliminate ( G ) because leisure time is unrelated to building and, moreover, in paragraph 4, the author reports that Cretans did have leisure time. Finally, remove (H) because the author does not claim that the size of the palace of Minos is reflective of Cretan building skills. The correct answer is (F); the ship's sophisticated keel serves as evidence of the high quality of the building.
15. C Here is a general question best answered using POE with reference to your headlining notes on the passage. Remember to delete answer choices that are not the most important characteristic of Crete, as the question asks you to do. Eliminate (A) because bullfighting is mentioned in passing and is not a central claim. Similarly, cross off (B) because the passage suggests Crete was a middle-class society, not an upper-middle class society. And although the reader hears about

Crete having enemies, the culture is not "preoccupied" with war-so eliminate (D). The correct answer is (C).
16. \(\mathbf{H}\) This specific question asks what the function was of "keels covered in bronze." These are helpful lead words that guide you to paragraph 2 and the description of Cretan ships. The author says that the expertly constructed bows and sterns with keels covered in bronze "show that the boats were intended for ramming into enemy ships." The function of the bronze-covered keels, therefore, was to defeat enemy ships, which matches exactly with (H).
17. B This question asks the purpose of the final paragraph. Refresh your memory of this paragraph with a glance at the headline, or, time permitting, a brief skim. Then move into POE based on what the passage says. Eliminate (A) because modern Cretans, according to the passage, do not rely on trade. Eliminate (C) for this same reason. Finally, even if modern Cretans deeply respect the traditions of maritime trading, the last paragraph does not discuss it-so cross out (D). The correct answer is (B). Because the final paragraph says that modern Cretans rely on tourism, you can infer that, as (B) says, the modern economy of Crete does not rely on trade like its ancient culture did.
18. E This is a general question asking you to summarize the passage as a whole. Having skimmed or read in detail the passage, you can move directly to POE. Eliminate (F) because the excerpt's main focus is not on the mechanics of how any ship sinks, but, rather, the story of one particular shipwreck. Also eliminate (G) because the word "unreliable" disqualifies this answer; additionally, you do not know that this disappearance is "mythic." Then eliminate (H) because you do not have evidence that this excerpt comes from a journal entry. The best choice is (E).
19. B This question asks about the cause of the shipwreck, so check your headlines in the first part of the passage, where the shipwreck occurs. Note that the steamboat appears out of nowhere from behind the thick fog that surrounds the ferry. Eliminate (A) because this does not suggest that the steamboat meant to collide with the Martinez.

Similarly, eliminate (C) because the author does not say that the inexperience of the crew results in the deadly accident. Finally, eliminate (D) because the weather conditions at sea are a much better explanation for the situation than "the hand of fate." Moreover, the poor weather conditions are actually referenced in the text. The correct answer is (B).
20. E The red-faced man's reaction to the collision should bring you to the third paragraph, where he tells the narrator to "grab hold of something and hang on." Using this part of the text as reference, move to POE. You can eliminate (F) because, if anything, the red-faced man appears assured and confident in this time of crisis-not nervous. Also eliminate ( G ) because the man does indeed mention the women's screams, but he does not say he is looking forward to them; he anticipates them. Eliminate (H) because the red-faced man seems to know the ship is doomed and makes no act to save the ship, only the lives of those on it. His calm reaction to collision, as the narrator reports, suggests that the red-faced man has experienced shipwrecks before and knows what to expect. The correct answer, then, is (E).
21. C Rely on headlines or your skimming to answer this question. In the fifth paragraph, the narrator describes the scene of panic aboard the ship ("...the empty upholstered seats, littered with all the evidences of sudden flight, such as packages, hand satchels, umbrellas, and wraps..."), but only becomes alarmed when he hears the screaming of the women. In fact, he says, "This it was, the screaming of the women, that most tried my nerves." Only (C) works; the shrieking of the women terrifies him.
22. H This question asks why the narrator starts laughing, which is a helpful lead word because the narrator's laughter is a surprising detail for a man aboard a sinking vessel. Recall that the loud shrieks of the women "impelled me to sudden laughter, in the next instant I realized I was becoming hysterical myself." Based on your reading of this moment, jump to POE. Eliminate (F) immediately because the redfaced man does not appear in this scene. Next, eliminate (E) and (G), as you have no proof that the narrator considers this circumstance literally funny. Careful: all you know is that the narrator laughed; you
do not know for a fact that he is seeing something funny. Use that detail to arrive at (H), the correct answer. This is, of course, supported by the text: the hysteria of the women has provoked a surprising reaction in the narrator.
23. B This question asks you to analyze a specific sentence from the passage (lines 97-98). This question is unique in that it asks you to think about the meaning of the words apart from the passage as a whole. All you need is to read the given text and use POE to cross off choices that are not supported by the language the author uses. Quickly eliminate (A) because the water is certainly not warm; it is cold. Next, eliminate (C) because the author does not mention that this water temperature is typical of the Pacific Ocean. Finally, eliminate (D), because it doesn't speak to the overall danger of the ocean, only a specific situation encountered by the narrator. The correct answer is (B).
24. E This question directs you to the final two paragraphs of the passage. Check your headlines or skim these paragraphs (if you have the time), and POE any choices that are inconsistent with the excerpt. Eliminate (F) because the narrator does not appear mad or upset there. Also eliminate (G) because it would be unclear whom the narrator resents, if he were resentful. Last, eliminate (H) because nothing particular is entrapping the narrator; he is adrift at sea, which is in some ways the opposite of being entrapped. The correct answer is (E), resigned, which matches well with the text. Consider when he says, "I felt that I could survive but a few minutes," for instance.
25. C Here is a general question, so you should use your knowledge of the passage as a whole to move directly to POE. The question asks about what the passage says people generally do in a time of crisis. You can eliminate (B) immediately because it is the most extreme; the passage does not claim all people shriek in terror. Next, eliminate (A), as neither the narrator (nor another character) emerge from the crisis as a more "authentic" version of themselves; this idea, even if it sounds true to you, is not explored in this excerpt, so you cannot choose it. Finally, eliminate (D) because, while it may be true that the ferry passenger's survival instincts kicked in, this certainly did not lead to universally wise decisions-in fact, in this story many passengers
became hysterical and panicked to the point of inhibiting their wisdom. The correct answer is (C). Time seems to slow down or speed up at will, and one can feel outside of one's body. This choice is expressed through the narrator's frequent statements like "Then everything happened, and with inconceivable rapidity" or "What happened in the next few minutes I do not recollect" or "Later,-how much later I have no knowledge,-I came to myself..."
26. G This is a general question that can best be answered using headlines of the passage as a whole. Jump right to POE on a question like this. Eliminate (E) because the opossum appears only in paragraph 4; comparing the opossum to the kangaroo is not the main idea. Cross off ( F ), as the author briefly mentions marsupials and dinosaurs coexisting (paragraph 4), but does not say they were in competition. Finally, eliminate (H) because the author explains that kangaroos are "so different from our usual idea of a mammal," which is different than saying "kangaroos are not really mammals." The correct answer is \((\mathrm{G})\).
27. D This question asks which of the following statements the author would most likely agree with. For a question like this, use POE and evaluate whether each statement is supported by the passage or not. Eliminate (A) because the author never says that opossums are smarter than kangaroos. Nowhere does the reader learn that dinosaurs ate kangaroo meat, so delete (B). Choice (C) can be removed because if kangaroos were once extinct, how could they still be populating Australia? (Besides, the author never makes this claim.) The correct answer is (D).
28. E This is a very broad question about the identity of the author. Having skimmed the passage and written headlines, move directly to POE. Cross off (F) and (G) because they are outlandish answers (the author is neither a businessman nor a cowboy). Then eliminate (H) because, while the author mentions Australia, there is no evidence that he or she is an Australian. Based on the biological and evolutionary evidence presented in the passage, it is reasonable to assume that the author is a scientist. Choose (E).
29. B This specific question asks why the author would mention the exact size ranges of a kangaroo, and it draws your attention to lines 15-16 -so read five lines above and five lines below from there. Notice that the author offers several pieces of evidence for the claim that kangaroos vary a lot within the species (line 11). This is precisely what (B) expresses, so choose it and move on. The other answer choices are unrelated to what the text says.
30. G When asked about the meaning of "living fossil" in line 59, read that sentence in the context of the passage. It reads, "But even if the opossum can in some ways be considered a 'living fossil,'...." This links "living fossil" to the opossum. The correct answer is (G).
31. A This general question asks about the kangaroo's development. Use your headlines to find the best place in the passage to locate this answer. Recall that paragraph 5 focuses on the kangaroo's evolutionary trends, and the adjustments it has made. According to the author, "if the opossum can in some ways be considered a 'living fossil,' the same cannot be said about the kangaroo"; kangaroos "have changed recently in order to adapt to a changing environment." POE! Eliminate (C) and (D) immediately because a kangaroo is clearly not similar to a dinosaur, and kangaroos are not native to North America. (You may have already known this, but it's better to confirm with lines from the text, like 61: "These Australian marsupials....") Down to two! Notice that (B) says the kangaroo had an identical development to the opossum, but the text explicitly states that opossums are "living fossils," while kangaroos have adapted. Eliminate (B), and choose (A).
32. G This is a general question. If you're not sure what the main idea of the passage is after skimming it and noting the headlines, start using POE. Eliminate ( E ) because the passage is about the monument, not the civil rights movement itself. Remove (F) because the Southern Poverty Law Center commissioned the monument, but it is not the main focus of the passage. Finally, remove (H) because Brown vs. the Board of Education is briefly mentioned as one of the events commemorated on the memorial; the passage, however, is about the memorial. The correct answer is (G).
33. D This question asks about the content on the memorial itself. Let your headlines guide you to the first paragraph, which describes the monument itself. The author writes that on the slab "is a time line chronicling the major events of the movement, from the Supreme Court's Brown vs. the Board of Education ruling in 1954 to the assassination of Martin Luther King, Jr. in 1968." This range of dates - 1954 to 1968—and the following sentence (lines 17 to 22), which details some of the specific events, should help to answer the question. The only choice left is (D), the March on Washington in 1963.
34. G Your headlines and skimming tell you that paragraph 2 discusses why there are 40 names inscribed on the monument. Search that paragraph for information on those names, and use it to POE. Careful: the question is asking what all 40 names have in common. Eliminate (E) because not all 40 individuals were killed during non-violent protests; remove ( F ) because the passage does not say that these people were friends of Martin Luther King, Jr. Don’t be fooled by the imagery of children visiting the memorial-paragraph 1 mentions that the slab is so low that children could touch it; eliminate (H). The correct answer is \((\mathrm{G})\).
35. D This question asks why anti-civil rights agitators murdered some civil rights activists. Use your headlines to notice that paragraph 2 discusses the 40 names that are commemorated on the monument's circular slab. The writer states that "those who were murdered by antimovement agitators" were killed "in order to intimidate other civil rights activists." Choices (A) and (C) do not agree with that quotation, so eliminate them; the murderers did not seek to leave the civil rights leaders with martyrs, and they hardly wished to add momentum to the civil rights movement. Additionally, the passage does not say the killings were tied to the Brown vs. Board decision, so remove (B). Choose (D).
36. F For this question that asks which of the following statements the author would most likely agree with, jump directly to POE, as you have probably read enough of the passage at this point to have a
decent understanding of the author's perspective. Cross out any choices that do not have support in the passage. The author never says that there is more prejudice today, so eliminate (E). Similarly, he or she does not claim that everyone involved in the movement was guilty, or that this memorial is the only important memorial-so eliminate (G) and (H). The correct answer is (F).
37. B Use the lead words "Southern Poverty Law Center" to skim the passage for information on this organization. Paragraph 4 says that today, "The main function of the center is to remind the world of what has passed and educate our future leaders." Time for POE! The center is clearly not a construction company or an art center, so immediately eliminate (A) and (D). Down to 2! "Museum" may be tempting, but remember that the passage does not say the center is a museum, so you cannot choose it. The correct answer is (B), an educational institution.
38. E This is a broad, general question. If you're not sure what the main idea of the passage is after skimming it and noting the headlines, start using POE. Eliminate (G) because anthropology is mentioned only in passing here. Next, eliminate (H) because it does not account for Paul Broca, who is the central figure in the passage; his is ambiguous in this case, as it could be the author or Broca. This is also a good reason for eliminating ( F ); even though some of the passage is about brain functions and their location in the brain, the main ideas are about Broca and his contributions to the study of the brain. The correct answer is ( E ).
39. B This specific question leads you to lines 15-20 and asks why Broca is famous today. Read those lines in paragraph 2, which say that Broca discovered "a small region in the third convolution of the left frontal lobe of the cerebral cortex." In addition, "articulate speech...is to an important extent localized in and controlled by Broca's area." Now, POE based on this information. Cross off (A) because this part of the passage is discussing his discovery as a brain anatomist. Eliminate (D) because Broca did not invent the tiny region to the left of the cerebral cortex. Careful not to choose (C)! Although the passage does say that Broca investigated the limbic region, he is not most famous today for
that accomplishment. The correct answer is (B).
40. G This specific question asks about Broca's research on articulate speech. Find the place in the passage the question mentions (lines 2024). The author says, "Articulate speech, it turns out, as Broca inferred on only fragmentary evidence, is to an important extent localized in and controlled by Broca's area." This question asks what inference you can make from this excerpt. Delete (E) because you have no proof that Broca would have made a different conclusion. Eliminate ( F ) because this is the opposite of what is true; this paragraph is about how significant Broca's inference was. Finally, cross off (H) because the text clearly states that Broca was able to find the part of the brain that controls speech. The correct answer is (G).
41. D For a question with except/not/least, follow the basic strategy: use your pencil to cross out the "except," "not," or "least." Then, mark each answer choice with a T for true or F for false. There will be 3 Trues and just 1 False; you know the odd one out is your answer, so select it. In the case of this question, this process makes the question a lot simpler: "Which of the following is discussed in the passage?" Write a T next to (A), (B), and (C) because the text does discuss Broca's contributions (paragraphs 1-2), the discovery that specific brain functions exist in particular sections of the brain (paragraph 2), and that memory may be stored in various places (paragraph 4). Write an F next to (D), as the author does not claim that Broca solved all the mysteries of the brain. The False choice is (D), and therefore the correct answer.
42. H This question asks why Broca's death was ironic. Irony is something that is the opposite of what would be expected. Search your headlines for mention of Broca's death (paragraph 3); he died "of the very sort of aneurysm that he had studied so brilliantly." If the irony of that statement isn’t apparent, use POE. Eliminate (E) and (F) because Broca did not study infant mortality, and he was not murdered. Then delete (G) because the author does not mention a late-in-life switch to anthropology. The correct answer is (H): it is ironic that Broca died from the same disorder he was studying.
43. A This specific question asks about lines 44-52 in the passage, so look there. These lines contain a wish on the author's part to look into Broca's brain and perhaps locate a memory. That memory is when Broca successfully argued in front of the medical faculties, but that is not important to the author's larger ideas-so eliminate (B). Eliminate (C) because Broca's father's role in neurophysiology is not mentioned. Finally, eliminate (D) because the passage does not discuss an arrangement of neurons causing aphasia. Be careful not to fall for the deceptive language: the author does imagine a configuration of neurons, but it is not the cause of aphasia. The correct answer is (A). The author is interested more generally in the physical preservation of memory and identity, using Broca as an example.
44. F This general question can best be answered by relying on your headlines. If you're not sure what the main idea of the passage is after skimming it and noting the headlines, start using POE. Eliminate (E) because the author does not create a new definition of artistic greatness. Similarly, the passage does not prove that Sousa is one of the greatest musicians of all time, so eliminate (H). Down to two! Choice (G) can be deleted because, although band music is a major topic in the text, the central figure in this passage is John Philip Sousa. The correct answer is ( F ).
45. A This is a broad question that can best be tackled by POE if you've skimmed a good deal of the passage. The author does not think Sousa's music is stupid, so eliminate (D). (You might also remember that the SHSAT will not use derogatory terms to describe subject matter.) Next, there is no reference to jazz music or to quiet, thoughtful music-this music is meant to be played while soldiers go to war, after all—so eliminate (C) and (B). The correct answer is (A), memorable, which is precisely what the author says in lines 33-35: "Their tunes are so uncomplicated, so easy to catch, so melodic."
46. H This specific question asks what the author means by saying Sousa's music is practical (line 8). Practical means that there is real-life purpose to Sousa's music, so POE with that in mind. Eliminate (E) because the passage does not say Sousa's music is imitated by
classical musicians. Eliminate (F) because it says precisely the opposite of the passage; his marches were, in fact, "uncomplicated, easy to catch, melodic" (lines 34-35). Finally, eliminate (G) because Sousa's marches may be easily learned by children, but the passage does not explicitly say that and it is not the reason Sousa's work is considered practical. The correct answer is (H): the marches are practical because they are used for actual military marching.
47. D This question asks what lines 33-39 suggest about the simplicity of Sousa's marches. Since this is a specific question, jump right to those line numbers. The author states that his tunes are very catchy and simple, adding "simple as they may be,...it took inspiration to write them." POE using this information. Eliminate (A) because the author says the marches are simple, but never says they are childish. Eliminate (B) since the passage does not claim that such simplicity in music is unnecessary. Similarly, eliminate (C) because the author does not claim that simplicity is superior to complexity. The only choice that agrees with what the author does say is (D): the simplicity of Sousa's marches does not make them any less admirable.
48. F This broad question asks about the quality of Sousa's work. There is not a specific place to look in the passage, so POE is a good place to start. Get rid of things you know are not true. Eliminate (E) because the author never makes a judgment like this comparing Sousa to Wagner (although he does say "Sousa is no Beethoven."). Eliminate ( G ) because the main adjective the author uses to describe Sousa's marches is "simple," not "complex." Finally, eliminate (H) because, according to the passage, "above all, [Sousa's marches] are intensely practical." The correct answer is ( F ).
49. D For a question with except/not/least, follow the basic strategy: use your pencil to cross out the "except," "not," or "least." Then, mark each answer choice with a T for true or F for false. There will be 3 Trues and just 1 False; you know the odd one out is your answer, so select it. In the case of this question, this process makes the question a lot simpler: "which of the following is mentioned as an aspect of Sousa's compositions?" Place a T next to (A), (B), and (C) because the passage does say that marching bands use his works (lines 65-67),
that "Sousa was no Beethoven" (line 1), and that his compositions are original and "unmistakably Sousa's" (line 37). Place an F next to (D) because the author compares the simplicity of Wagner's and Sousa's marches, but does not claim that one influenced the other. Choice (D) is the False choice, and so the correct answer.
50. E This question asks what the final paragraph suggests, so check your headlines from that paragraph or briefly skim those lines if you have not already done so. Now move to POE. Eliminate (F) immediately because nowhere does this paragraph state that people prefer Mozart. Next, eliminate (H) because the author does not state that everyone enjoys "Semper Fideles": this is both extreme and in the previous paragraph. Down to two! There's a slight difference in (E) and (G). Does Sousa's music really encourage people to enlist in the military? Or does the music inspire patriotic feelings? Check what the passage says; it doesn't say anything about joining the military, so eliminate (G). The correct answer is (E), which is supported by this paragraph's first sentence: "a Sousa march does stir one important thing-national pride."
51. B This general question about tone can best be answered by POE, based on your reading of the poem. (There is no direct place in the poem to reference to get this question, just an overall "mood.") Eliminate (D) because even if you've read just a little portion of this poem, you know that it is most certainly not comedic. Similarly, the poem has a melancholy in it, but not necessarily a tragedy (we can't point to a singular horrible event that ruins the narrator's life, or kills him). Thus, remove (C). Finally, remove (A) because, while there are traces of youthful romances in the poem, the overall tone of the piece cannot be said to be romantic. The only remaining choice is the best choice: nostalgic, (B).
52. E This specific question asks what a Lapland song most likely means. Use the line numbers to return to the poem and investigate. The narrator writes, "... and my youth comes back to me. / And a verse of a Lapland song / Is haunting my memory still..." Don't worry! Outside knowledge of the location of Lapland is not required to answer this question. (The SHSAT will never ask you to rely on
outside knowledge.) Use POE to determine the most likely meaning of a Lapland song. Eliminate (G) and (H) because you have no evidence that the Lapland song is meant for dancing or that something about Lapland songs is inherently mournful (although the poem's song happens to be mournful). Finally, eliminate (F) because, although it could be true that this song is native to the narrator's family, the text does not say this. The correct answer is (E), as holds true for the rest of the poem: this Lapland song is a song the narrator remembers from his childhood.
53. A This specific question asks you to analyze a small excerpt from the poem. Reread the lines carefully, and POE any answer choices that are not supported in the passage. Eliminate (B) because that answer is deceptive; you might connect Sabbath to a religious practice, but that would be a mistake because the excerpt is not specifically about religion. Next, eliminate (C) because, although the narrator does bring up the groves and shadows of Deering's Woods, he does not explicitly call the forest "menacing." Finally, you can eliminate (D) because this choice wants you to confuse "grove" (actually in the text) and "grave" (not in the excerpt). The correct answer is (A), which corresponds well with the lines "friendships old and the early loves / Come back with a Sabbath sound, as of doves."
54. H This question asks you to analyze the second to last stanza, so start by rereading that part. Notice that this excerpt is about the narrator not recognizing the "forms" when he revisits his home town, but appreciating the "native air." Based on this information, move to POE. Eliminate (E) because the narrator does admire the sweet air, but he does not say that the sweetness is a new quality. Next, eliminate (F), as the trees are mentioned but not that they hide the beauty of the town. Finally, eliminate (G), because this stanza does not mention the speaker crying. The correct answer is (H): many things have changed about the speaker's hometown, but its natural environment has not.
55. B Here you are asked a broad question about the repetition of the poem's refrain. Jump right to POE, based on your reading of this poem and the effect of hearing the repeated text over and over. Eliminate (C) because this choice would require outside knowledge of
poetry customs in Longfellow's era (and remember that passages on the SHSAT do not require outside knowledge). Next, eliminate (A) and (D) because both of these reasons are unlikely to be the most important reason for repeating a piece of text, as they are both a bit too simplistic-especially when considering (B), which aptly captures a metaphor at play in the poem: the repeats of the Lapland song echo the structure of a song, which is fittingly the topic of the poem itself.
56. F This specific question asks you to explain the meaning of the phrase "And with joy that is almost pain / My heart goes back to wander there." Refer to the stanza in the poem that contains this passage. Note that the speaker says he wanders "among the dreams of the days that were." Move to POE. Eliminate (G) because this excerpt does not discuss walking in the woods. Next, eliminate (H) because the narrator does not say he wishes to move on fully from the past, and the poem as a whole explores a longing for the past. Down to two! Eliminate ( E ) because joy alone is not the chief emotion expressed in the line; the line also includes pain, which matches well with (F): the bittersweet feeling of living in the past. The correct answer is (F).
57. A This is a general question about the identity of the narrator that can best be answered by moving directly to POE, based on your reading of the passage. Eliminate (D) because the narrator never says he is lucky. Also eliminate (B) because it is not that the speaker likes old songs in general, but that he likes a particular song from his youth. Lastly, eliminate (C) because, although the poem expresses a fondness for nature, the central fondness is for the narrator's hometown. Hence, the best answer choice is (A).

\section*{Math}
58. 8.75

The question mentions rates, so use proportions. The car drives at 40 miles per hour and must drive 350 miles, so set up \(\frac{40 \mathrm{mi}}{1 \mathrm{hr}}=\frac{350 \mathrm{mi}}{x \mathrm{hr}}\).

Cross-multiply to get \(40 x=350\). Divide both sides by 40 to get \(x=\) \(\frac{350}{40}=\frac{35}{4}\). Convert this to its decimal equivalent, 8.75 , so that you can grid it in.
59. 1.25 The question asks for the value of \(a b\). There is only one equation, so there is no way to determine the value of each variable individually. Instead, isolate the expression \(a b\). Multiply both sides of the equation by \(b\) to get \(36 a b=45\). Divide both sides by 36 to get \(a b=\frac{45}{36}=\frac{5}{4}=\) 1.25 , which is the correct answer.
60. 160 The question involves ratios, so use the ratio box. There are two ratios in the question, so use two boxes. The ratio of sophomores to juniors is 5 to 4 and there are 300 sophomores, so set up the box as below.
\begin{tabular}{|c|c|l|}
\multicolumn{1}{c}{ Sophomore } & \multicolumn{1}{c}{ Junior } & \multicolumn{1}{c}{ Total } \\
\hline 5 & 4 & \\
\hline & & \\
\hline 300 & & \\
\hline Ratio \\
Multiplier \\
Actual \#'s
\end{tabular}

Add across the top row to get 5 under total. Multiply downward. Because \(3 \times 80=240\), the multiplier is 80 . Fill in 80 across the middle row. Multiply downward again. Fill in \(2 \times 80=160\) in the bottom row under Junior and \(5 \times 80=400\) under Total.
\begin{tabular}{|c|c|c|}
\multicolumn{1}{c}{ Sophomore } & \multicolumn{1}{c}{ Junior } & \multicolumn{1}{c}{ Total } \\
\hline 5 & 4 & 9 \\
\hline 60 & 60 & 60 \\
\hline Ratio \\
\hline Multiplier \\
\hline 300 & 240 & 540 \\
\hline
\end{tabular}

The ratio of juniors to seniors is 3 to 2 . Set up another box. Keep 240 as the actual number of juniors.
\begin{tabular}{|c|c|l|}
\multicolumn{1}{c}{ Junior } & Senior & \multicolumn{1}{c}{ Total } \\
\hline 3 & 2 & \\
\hline & & \\
\hline 240 & & \\
\hline Ratio \\
Multiplier \\
Actual \#'s
\end{tabular}

Add across the top row to get 5 under total. Multiply downward. Because \(3 \times 80=240\), the multiplier is 80 . Fill in 80 across the middle row. Multiply downward again. Fill in \(2 \times 80=160\) in the bottom row under Junior and \(5 \times 80=400\) under Total.
\begin{tabular}{|c|c|c|c}
\multicolumn{1}{c}{ Junior } & Senior & \multicolumn{1}{c}{ Total } \\
\hline 3 & 2 & 5 & Ratio \\
\hline 80 & 80 & 80 & Multiplier \\
\hline 240 & 160 & 400 & Actual \#'s \\
\hline
\end{tabular}

The question asks for the number of seniors, which is 160 .
61. 720 The question asks for the number of possible arrangements, so write a dash for each position. The class has 3 positions to select, so make 3 dashes. Now fill in the number of possible students for each position. There are 10 students, so there are 10 possible selections for president. No student can hold more than one position, so once that student is selected, there are 9 students left for vice-president, and 8 students left for treasurer. Now determine whether order matters. Because the question asks about arrangements, order matters. Therefore, the number of arrangements the class can select is \(10 \times 9 \times 8=720\).
62. -9 The question asks for the value of \(b\). Start by solving the first equation
for \(a\). Divide both sides of \(a^{2}=-2 a\) by \(a\) to get \(a=-2\). Substitute this value of \(a\) into \(a b=18\) to get \(-2 b=18\). Divide both sides by -2 to get \(b=-9\), which is the correct answer.
63. B The question asks for the decimal representation of \(1,000+1+\frac{1}{1,000}\). Start by adding the whole numbers to get \(1,001+\frac{1}{1,000}\). Now deal with the \(\frac{1}{1,000}\). Since the thousandths position is three spaces to the right of the decimal point, you know that \(\frac{1}{1,000}=0.001\). Therefore, \(1,001+\frac{1}{1,000}=1,001+0.001=1,001.001\), which is \((B)\).
64. E The question provides the range, which is defined as the difference between the greatest and least numbers in a list. The question says that one of his times is 58 seconds. However, the question does not specify where 58 falls (least, greatest, or anywhere in between). The question asks for what cannot be one of his times, so determine the greatest and least possible numbers. To find the greatest possible time, assume 58 is the least time. If it's the least time and the range is 9 seconds, the greatest time would be \(58+9=67\) seconds. To find the least possible time, assume 58 is the greatest possible time. If that's the case, then the least time is \(58-9=49\) seconds. Therefore, the times all must be from 49 to 67 , inclusive. Since 48 is outside this, it is not a possible time, so the answer is (E).
65. D The question involves conversion, so use proportions. Because 0.96 klurbs and 0.64 flourns are both equal to 1 dollar, 0.96 klurbs \(=0.64\)
flourns. To determine the number of klurbs equal to 1 flourn, set up \(\frac{0.96 \text { klurbs }}{0.64 \text { flourns }}=\frac{x \text { klurbs }}{1 \text { flourn }}\). Cross-multiply to get \(0.64 x=0.96\). Divide
both sides by 0.64 to get \(\frac{0.96}{0.64}=\frac{96}{64}=\frac{48}{32}=\frac{24}{16}=\frac{12}{8}=\frac{6}{4}=1.5\), which is (D).
66. H Always do unit conversions in one dimension. Even though 1 yard is 3 feet, 1 square yard is not 3 square feet. Determine the length of one side of the square. The area is 4 square yards. The formula for area of a square is \(A=s^{2}\). Substitute \(A=4\) to get \(4=s^{2}\). Take the square root of both sides to get \(s=2\). Therefore, the side of the square is 2 yards. To find the area in square feet, find the side of the square in feet. To do conversions, use a proportion: \(\frac{1 \text { yard }}{3 \text { feet }}=\frac{2 \text { yards }}{x \text { feet }}\). Cross-multiply to get \(x=6\). Therefore, the side of the square is 6 feet, and the area is \(A=s^{2}=6^{2}=36\) square feet, which is (H).
67. D The question asks for what fraction of the plant species in Region A is parasitic. Go to the table under Region A. There is 0.337 in the parasitic row and 100 under the total row, so the fraction is \(\frac{0.337}{100}\). This is not a choice. The choices all have 337 in the numerator, so move the decimal three places to the right in the numerator and denominator to get \(\frac{337}{100,000}\), which is (D).
68. G The question asks for the area of a trapezoid. When a question asks for the area of an unusual shape, break the shape into more normal shapes. In this case, break the figure into a rectangle and a right triangle as below.


Because opposite sides of a rectangle are equal, fill in the other vertical side of the rectangle as 8 and the other horizontal side of the rectangle as 4 . Because the bottom side of the trapezoid is 10 , the portion of the bottom side that's not in the rectangle is \(10-4=6\). Determine the area of the rectangle and of the right triangle. The area of the rectangle can be determined using the formula \(A=l w\). The length is 4 and the width is 8 , so the area is \(A=(4)(8)=32\). The area of the right triangle can be determined using the formula \(A=\frac{1}{2} b h\). The base is 6 and the height is 8 , so the area is \(A=\frac{1}{2}(6)(8)=24\). The area of the trapezoid is the sum of the areas of the rectangle and the right triangle, which is \(32+24=56\), which is (G).
69. B The question asks for the length of one of the larger fences. Translate the information in the question. Two of the larger fences and four of the smaller fences complete an enclosure of 118 ft , so \(2 l+4 s=118\). The length of the smaller fences is 15 ft , so \(2 l+4(15)=118\). Solve for l. Simplify the left side to get \(2 l+60=118\). Subtract 60 from both sides to get \(2 l=58\). Divide both sides by 2 to get \(l=29\), which is (B).
70. \(\mathbf{F}\) The question asks how many values of \(a\) will make the expression an integer if \(1 \leq a \leq 7\). Plug in the values of \(a\) from 1 to 7 and see which values are integers. Plug in \(a=1\) to get \(\frac{2(1)-6}{1+3}=-\frac{4}{4}=-1\), which is an integer. Plug in \(a=2\) to get \(\frac{2(2)-6}{2+3}=-\frac{2}{5}\), which is not an integer. Plug in \(a=3\) to get \(\frac{2(3)-6}{3+3}=\frac{0}{6}=0\), which is an integer. Plug in \(a=4\) to get \(\frac{2(4)-6}{4+3}=\frac{2}{7}\), which is not an integer. Plug in \(a=\) 5 to get \(\frac{2(5)-6}{5+3}=\frac{4}{8}\), which is not an integer. Plug in \(a=6\) to get \(\frac{2(6)-6}{6+3}=\frac{6}{9}\), which is not an integer. Plug in \(a=7\) to get \(\frac{2(7)-6}{7+3}=\) \(\frac{8}{10}\), which is not an integer. Count the ones that are integers. There are 2, which is ( F ).
71. D The question asks for the value of \(3 x^{2}-4 x+2\) if \(x=-5\). Substitute \(x=\) -5 to get \(3(-5)^{2}-4(-5)+2=3(25)+20+2=75+20+2=95+2=\) 97 , which is (D).
72. H The actual length is within \(10 \%\) of the measured length. The measured length is 4.4 , so \(10 \%\) is \(\frac{10}{100} \times 4.4=\frac{44}{100}=0.44\).

Therefore, the actual length is within 0.44 of 4.4. To get the maximum actual length, add 0.44 to 4.4 to get 4.84 , which is (H).
73. C The question is about ratios, so use the ratio box. The ratio of blue paint to yellow paint is 3:4 and the actual number of total gallons is 35.
\begin{tabular}{|c|c|c|c}
\multicolumn{1}{c}{ Blue } & \multicolumn{1}{c}{ Yellow } & \multicolumn{1}{c}{ Total } \\
\hline 3 & 4 & \\
\hline & & & \begin{tabular}{l} 
Ratio \\
Multiplier \\
\hline
\end{tabular} \\
\hline & & 35 & Actual \#'s
\end{tabular}

Add across the top row to get \(3+4=7\) under Total. Multiply downward. Since \(7 \times 5=35\), the multiplier is 5 . Fill in 5 across the middle row. Multiply downward to get \(3 \times 5=15\) in the bottom row under Blue, and \(4 \times 5=20\) in the bottom row under Yellow.
\begin{tabular}{|c|c|c|}
\multicolumn{1}{c}{ Blue } & \multicolumn{1}{c}{ Yellow } & \multicolumn{1}{c}{ Total } \\
\hline 3 & 4 & 7 \\
\hline 5 & 5 & 5 \\
\hline 15 & 20 & Ratio \\
\hline Multiplier \\
\hline & Actual \#'s
\end{tabular}

The question asks for the total number of gallons of blue paint used. Look in the Actual \#'s row under Blue to find 15, which is (C).
74. F The question asks how many sets of 3 shirts Nancy can select. When a question asks for how many ways to form a group, write a dash for each spot. Nancy selects 3 shirts, so make 3 dashes. Now fill in the number of possible shirts for each spot. She has 12 shirts, so there are

12 possible shirts for the first spot. Once that shirt is selected, there are 11 shirts left for the second spot, and 10 left for the third spot.

Now determine whether order matters. Since nothing in the question specifies any order or position, order does not matter. Because order doesn't matter and there are 3 spots, divide by \(3 \times 2 \times 1\). Therefore, the number of groups Nancy can select is \(\frac{12 \times 11 \times 10}{3 \times 2 \times 1}\). Simplify before multiplying. Reduce 12 and 3 by 3 to get \(\frac{4 \times 11 \times 10}{1 \times 2 \times 1}\). Now reduce 4 and 2 by 2 to get \(\frac{2 \times 11 \times 10}{1 \times 1 \times 1}=2 \times 11 \times 10=220\), which is (F).
75. A The question is asking for Alan's age. Translate the information in the question: in 5 years, Eric will be three times as old as Alan will be. Let \(E\) be Eric's age now and \(A\) be Alan's age now. In 5 years, Eric will be \(E+5\) and Alan will be \(A+5\). If at that time, Eric will be three times as old as Alan, then \(E+5=3(A+5)\). The question says that Eric is now 22, so substitute \(E=22\) to get \(22+5=3(A+5)\). To find Alan's age, solve for \(A\). Start by simplifying the left side to get \(27+\) \(3(A+5)\). Divide both sides by 3 to get \(9=A+5\). Subtract 5 from both sides to get \(A=4\), which is (A).
76. H The students can be grouped by whether they walk to school or take the bus. Separately, the students can be grouped as whether they are present or absent. Because the students can be grouped two different ways, use a ratio box.


Because the question deals only with percents, put \(100 \%\) in the bottom right. \(40 \%\) of the students take the bus to school, so fill in 40 in the bottom row under Bus. The rest walk to school, so put \(100 \%\) \(40 \%=60 \%\) in the bottom row under Walk. The question says that \(30 \%\) of the students who walk to school were absent. Take \(30 \%\) of the \(60 \%\) who walk to get \(\frac{30}{100} \times 60 \%=\frac{3}{10} \times \frac{60}{1} \%=\frac{3}{1} \times \frac{6}{1} \%=18 \%\). Fill this into the Walk column and Absent Row. Similarly, 75\% of the students who take the bus to school were present, so take \(75 \%\) of the \(40 \%\) who take the bus to get \(\frac{75}{100} \times 40 \%=\frac{3}{4} \times \frac{40}{1} \%=\frac{3}{1} \times \frac{10}{1} \%=\) 30\%. Fill this into the Bus column and Present Row.
\begin{tabular}{|c|c|c|c}
\multicolumn{1}{c}{ Walk } & \multicolumn{1}{c}{ Bus } & \multicolumn{1}{c}{ Total } \\
\hline & \(30 \%\) & & \\
\hline Present \\
\hline \(18 \%\) & & & Absent \\
\hline \(60 \%\) & \(40 \%\) & \(100 \%\) & Total \\
\hline
\end{tabular}

Fill in the rest of the boxes. Subtract \(60 \%\) by \(18 \%\) to get \(42 \%\) in the top row under Walk. Subtract \(40 \%\) by \(30 \%\) to get \(10 \%\) in the middle row under Bus. Add \(42 \%\) to \(30 \%\) to get \(72 \%\) in the top right. Add \(18 \%\) to \(10 \%\) to get \(28 \%\) in the middle right.
\begin{tabular}{|c|c|c|}
\multicolumn{1}{c}{ Walk } & \multicolumn{1}{c}{ Bus } & \multicolumn{2}{c}{ Total } \\
\hline \(42 \%\) & \(30 \%\) & \(72 \%\) \\
\hline Present \\
\hline \(18 \%\) & \(10 \%\) & \(28 \%\) \\
\hline Absent \\
\hline \(60 \%\) & \(40 \%\) & \(100 \%\) \\
\hline
\end{tabular}

The question asks what percent of the students were present. Go to the Present row in the Total Column to get \(72 \%\), which is (H).
77. A The question asks for the probability that all four coins selected will be heads. Get the probability that each individual coin will be heads and multiply. The probability of a coin being flipped to heads is always \(\frac{1}{2}\). Therefore, the probability that all four will be heads is \(\frac{1}{2} \times\) \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times=\frac{1}{16}\), which is (A).
78. \(\mathbf{H}\) The question asks which inequality the number line is the solution to. The number line is the closed interval from -2 to 1 , which can be written as \(-2 \leq y \leq 1\). Determine which of the choices has this inequality. Go one choice at a time. Multiply all three parts of (E) to get \(-4 \leq y \leq 2\). Eliminate (E). Divide all three parts of (F) by 2 to get \(1 \leq y \leq 2\). Eliminate (F). Multiply all three parts of (G) by 2 to get -8 \(\leq y \leq 4\). Eliminate (G). Divide all three parts of (H) to get \(-2 \leq y \leq 1\). This matches the number line, so the answer is ( H ).
79. D The question involves rates, so use proportions. Stephen can type 120 words per minute, so to determine the number of words he can type in 1 hour, set up \(\frac{120 \text { words }}{1 \mathrm{~min}}=\frac{x \text { words }}{1 \mathrm{hr}}\). Like units must be used in a
proportion, so replace 1 hour with 60 minutes to get \(\frac{120 \text { words }}{1 \mathrm{~min}}=\frac{x \text { words }}{60 \mathrm{~min}}\). Cross-multiply to get \(x=7,200\), which is (D).
80. F Set up a proportion. Since the fruit stand sold 24 bananas and sold 3 bananas for every 5 apples, set up \(\frac{3 \text { bananas }}{5 \text { apples }}=\frac{24 \text { bananas }}{x \text { apples }}\). Crossmultiply to get \(3 x=120\). Divide both sides by 3 to get \(x=40\), which is \((\mathrm{F})\).
81. D The question provides the volume of a rectangular solid, so use the formula \(V=l w h\). The volume is 350 , the height is 7 , and the width is twice the length. Fill these into the formula to get \(350=(l)(2 l)(7)\). Simplify the right side to get \(350=14 l^{2}\). Divide both sides by 14 to get \(25=l^{2}\). Take the square root of both sides to get \(5=l\). The question asks for the width. Since the width is twice the length, the width is \(2 \times 5=10\), which is (D).
82. G To determine the probability that both gumballs are not green, determine the probability that each individual gumball is not green and multiply. There are a total of \(7+4+5=16\) gumballs. Of the 16 , \(16-5=11\) are not green, so the probability that the first gumball will not be green is \(\frac{11}{16}\). Once that gumball is selected, there is one fewer gumball that is not green and one fewer gumball overall. Therefore,
the probability that the second gumball will not be green is \(\frac{10}{15}\). Therefore, the probability that both gumballs are not green is \(\frac{11}{16} \times \frac{10}{15}\) \(=\frac{11}{16} \times \frac{2}{3}=\frac{11}{8} \times \frac{1}{3}=\frac{11}{24}\), which is (G).
83. C The question asks how much the ice weighed on Tuesday, which is at the beginning. Because the question asks for the starting point, plug in the answers. Begin with one of the middle choices. Try (B), which is 450. On Tuesday, \(\frac{2}{3}\) of the ice melted. Take \(\frac{2}{3}\) of 450 to get \(\frac{2}{3} \times 450=\) \(\frac{900}{3}=300\). Since 300 pounds melted, there are \(450-300=150\) pounds remaining. On Wednesday, \(\frac{1}{2}\) of the remaining ice melted, so \(\frac{1}{2} \times 150=\frac{150}{2}=75\) pounds melted, leaving \(150-75=75\) pounds. According to the question, 60 pounds should be remaining, so eliminate (B). The answer must be less, so eliminate (A), as well. Try (C), which is 360 . If there are 360 pounds of ice on Tuesday, then \(\frac{2}{3} \times\) \(360=\frac{720}{3}=240\) pounds melted on Tuesday, leaving 360-240= 120. On Wednesday, \(\frac{1}{2} \times 120=\frac{120}{2}=60\) pounds melted, leaving 120 \(-60=60\). This is consistent with the information in the question, so the answer is (C).
84. F The question looks to simplify the expression \(\frac{a}{b}\left[a\left(\frac{2 a-2 b}{2 a+2 b}\right)\left(\frac{2 a+2 b}{2 a-2 b}\right)\right]\). Don't be deterred by how complicated this looks; it can be simplified quickly. Look at \(\left(\frac{2 a-2 b}{2 a+2 b}\right)\left(\frac{2 a+2 b}{2 a-2 b}\right)\). In multiplication, common factors of one numerator and one denominator can be cancelled even if they are not part of the same fraction. In each fraction, the numerator cancels with the denominator of the other fraction, making the product 1. Therefore, \(\frac{a}{b}\left[a\left(\frac{2 a-2 b}{2 a+2 b}\right)\left(\frac{2 a+2 b}{2 a-2 b}\right)\right]=\frac{a}{b}[a(1)]=\frac{a}{b}\left[\frac{a}{1}\right]=\frac{a^{2}}{b}\), which is (F).
85. B The question asks for probability, which is equal to the number of what you want. The question asks for the the total number
probability of selecting a nickel, so the number of what you want is equal to the number of nickels, which is 5 . The total number is equal to the total number of coins, which is \(3+5+4+1=13\). Therefore, the probability of selecting a nickel is \(\frac{5}{13}\), which is (B).
86. \(\mathbf{F}\) The question asks what percent of the students have at least three pets, so you'll have to translate the data into a suitable equation. The word what translates to a variable. Use \(x\). The word percent translates to

100 . Of translates to \(\times\). For the students, determine the total number of students by adding all the numbers in the poll to get \(35+49+24+\) \(16+11=135\). The word have translates to \(=\). For at least 3 pets, get the sum of the number of students with three pets and the number of students with four or more pets to get \(16+11=27\). The question translates to \(\frac{x}{100} \times 135=27\). Solve for \(x\). Multiply both sides by 100 to get \(135 x=2,700\). Divide both sides by 135 to get \(\frac{2,700}{135}=\frac{900}{45}=\frac{100}{5}=20\), which is \((\mathrm{F})\).
87. A To find the probability that both students will be wearing black sneakers, find the probability that each individual student will be wearing black sneakers and multiply. There are 25 students, 5 of whom are wearing black sneakers, so the probability that the first student is wearing black sneakers is \(\frac{5}{25}\). Once that student is selected, there will be one fewer student with black sneakers and one fewer student overall, so the probability that the second student will be wearing black sneakers is \(\frac{4}{24}\). Therefore, the probability that both students will be wearing black sneakers is \(\frac{5}{25} \times \frac{4}{24}=\frac{1}{5} \times \frac{1}{6}=\frac{1}{30}\),
which is (A).
88. H The question asks for the value of \(4 a-5\) in terms of \(b\). The question states that \(a=2 b+1\), so substitute this expression to get \(4 a-5=4(2 b\) \(+1)-5\). Distribute 4 to get \(8 b+4-5\). Combine like terms to get \(8 b-\) 1 , which is (H).
89. B If \(A B=\frac{1}{6}\) and \(B=3 \frac{7}{12}\), then \(A=3 \frac{7}{12}-\frac{1}{6}=3 \frac{7}{12}-\frac{2}{12}=3 \frac{5}{12}\). Therefore, \(3 \frac{5}{12}<X<3 \frac{7}{12}\). The answer choices are in decimal form, so convert the endpoints to decimals using long division. Divide 5 by

12 to get
\[
\begin{gathered}
1 2 \longdiv { 5 . 4 1 \overline { 6 6 } } \\
\frac{48}{20} \\
\frac{12}{80} \\
\frac{72}{80}
\end{gathered}
\]

Divide 7 by 12 to get
\(1 2 \longdiv { 0 . 5 8 \overline { 3 3 } }\)
\(\frac{60}{100}\)
\(\frac{96}{40}\)
\(\frac{36}{40}\)

Therefore, \(3.41 \overline{6} .<X<3.58 \overline{3}\) The only choice between these two is (B).
90. G The question asks for the smallest number, so start with the smallest answer. Suppose there are 4 students taking biology. Since \(20 \%\) of the student study only physics and \(25 \%\) study only chemistry, then \(100 \%\)
\(-20 \%-25 \%=55 \%\) of the students study only biology. To determine the number of students, set up the equation \(\frac{55}{100} s=4\). Solve for \(s\). Reduce the fraction to get \(\frac{11}{20} s=4\). Multiply both sides by 20 to get \(11 s=80\). Divide both sides by 11 to get \(s=\frac{80}{11}=7 \frac{3}{11}\). Since the number of students cannot be a fraction, this cannot be correct. Eliminate (E). Try (F), which is 9 . Once again, \(55 \%\) of students study
only biology, so set up \(\frac{55}{100} s=9\), which simplifies to \(\frac{11}{20} s=9\).
Multiply both sides by 20 to get \(11 s=180\). Divide both sides by 11 to get \(s=\frac{180}{11}=16 \frac{4}{11}\). Once again, this is not an integer, so eliminate (F). Try (G), which is 11 . Set up the equation \(\frac{11}{20} s=11\). Multiply both sides by 20 to get \(11 s=220\). Divide both sides by 11 to get \(s=20\). This is an integer. Make sure the number of students who study physics and the number of students who student chemistry are also integers. \(20 \%\) of the students study only physics, so \(\frac{20}{100} \times 20=\frac{400}{100}\) \(=4\) students study only physics. \(25 \%\) of the students study only chemistry, so \(\frac{25}{100} \times 20=\frac{500}{100}=5\) students study only chemistry. Since they are all integers, 11 is a possible number of students who study only biology, so the answer is (G).
91. B The question asks for the value of \(a+b\). Both the angle with measure \(a^{\circ}\) and the angle with measure \(b^{\circ}\) form straight lines when combined with the angle of measure \(140^{\circ}\). Straight lines have \(180^{\circ}\), so \(a+140=\) 180 and \(b+140=180\). Subtract 140 from both sides of both equations to get \(a=40\) and \(b=40\). Therefore, \(a+b=40+40=80\), which is (B).
92. H The question is about ratios, so use the ratio box. It is a three-part ratio, so add an extra column. Otherwise, the box works the same way. The ratio is \(3: 8: 1\). The total is the sum of the parts. The sum of \(A B, B C\), and \(C D\) is the length of segment \(A D\). This is the difference
between the values of \(D\) and \(A\), which is \(17-(-7)=17+7=24\). Fill this in the Actual \#'s row under Total.
\begin{tabular}{|c|c|c|c|}
\multicolumn{1}{c}{\(\boldsymbol{A B}\)} & \(\boldsymbol{B C}\) & \(\boldsymbol{C D}\) & \multicolumn{1}{c}{ Total (AD) } \\
\hline 3 & 8 & 1 & \\
\hline & & & \\
\hline & & & \\
\hline & & & 24 \\
\hline
\end{tabular}

Add across the top row to get \(3+8+1=12\) under Total. Multiply downward. Since \(12 \times 2=24\), the multiplier is 2 . Fill in 2 across the middle row. Multiply downward to get \(3 \times 2=6\) in the bottom row under \(A B, 8 \times 2=16\) in the bottom row under \(B C\), and \(1 \times 2=2\) in the bottom row under \(C D\).
\begin{tabular}{|c|c|c|c|c|}
\multicolumn{1}{c}{\(\boldsymbol{A B}\)} & \(\boldsymbol{B C}\) & \(\boldsymbol{C D}\) & \multicolumn{1}{c}{ Total (AD) } \\
\hline 3 & 8 & 1 & 12 & Ratio \\
\hline 2 & 2 & 2 & 2 & Multiplier \\
\hline 6 & 16 & 2 & 24 & Actual \#'s \\
\hline
\end{tabular}

The question asks for the value of \(\overline{B D}\), which is \(B C+D C=16+2=\) 18 , which is (H).
93. D The question asks for the value of \(3 p-2 p q\) and provides the values \(p\) \(=5\) and \(q=-4\). Substitute these values to get 3(5) -2(5)(-4) = \(15+\) \(40=55\), which is (D).
94. \(\mathbf{H}\) To find the percent increase in the price of the home, assume the value of the home began as \(\$ 100\) and increase the value of \(10 \%\) each year. \(10 \%\) of 100 is 10 , so after 1 year, the value was \(\$ 100+\$ 10=\$ 110\). Take \(10 \%\) of the new value. \(10 \%\) of 110 is \(\frac{10}{100} \times 110=\frac{1}{10} \times 110=\) 11 , so the value of the home after two years is \(\$ 110+\$ 11=\$ 121\). Take \(10 \%\) of the new value one more time. \(10 \%\) of 121 is \(\frac{10}{100} \times 121\)
\(=\frac{1}{10} \times 121=\frac{121}{10}=12.1\), so the value of the home after three years is
\(\$ 121+\$ 12.1=\$ 133.1\). To find percent increase, use the formula \(\frac{\text { difference }}{\text { original }} \times 100\). In this case the difference is \(\$ 133.1-\$ 100=\$ 33.1\).
The original is \(\$ 100\), so the percent increase is \(\frac{33.1}{100} \times 100=33.1\), which is \((\mathrm{H})\).
95. C The question asks for the greatest prime factor of 770. Immediately eliminate (B) and (D) because they are not prime numbers. To find the prime factors of a number, use a factor tree. Start with two factors of 770. Use 77 and 10. Find two factors of 77: 7 and 11. These are both prime, so don't branch off these. Find two factors of 10: 2 and 5. These are also prime, so don't branch off these.


Therefore, the prime factors of 770 are \(2,5,7\), and 11 . The greatest prime factor is 11 , which is (C).
96. G Solve the equation \(\frac{3 n-6}{7-4}=11\) for \(n\). Simplify the denominator to get \(\frac{3 n-6}{3}=11\). Multiply both sides by 3 to get \(3 n-6=33\). Add 6 to both sides to get \(3 n=39\). Divide both sides by 3 to get \(n=13\), which is \((\mathrm{G})\).
97. A The question asks for the bonus if the salesperson sells a car for \$39,000 and another for \(\$ 41,000\). Cars sold for \(\$ 40,000\) or less yield a bonus of \(2 \%\), so she receives a bonus of \(\frac{2}{100} \times \$ 39,000=\frac{\$ 78,000}{100}\) \(=\$ 780\) for the car she sold for \(\$ 39,000\). Cars sold for more than \(\$ 40,000\) yield a bonus of \(3 \%\), so she receives a bonus of \(\frac{3}{100} \times\) \(\$ 41,000=\frac{\$ 123,000}{100}=\$ 1,230\) for the car she sold for \(\$ 41,000\). Therefore, the total bonus she receives is \(\$ 780+\$ 1,230=\$ 2,010\), which is (A).
98. \(\mathbf{H}\) This question provides the rate at which Sade watches television in a day, so use proportions. To determine the number of hours Sade watches in one week, set up the proportion \(\frac{2 \frac{1}{2} \text { hours }}{1 \text { day }}=\frac{x \text { hours }}{1 \text { week }}\). Always use like units in a proportion, so replace 1 week with 7 days to get \(\frac{2 \frac{1}{2} \text { hours }}{1 \text { day }}=\frac{x \text { hours. }}{7 \text { days }}\) Cross-multiply to get \(x=\left(2 \frac{1}{2}\right)(7)\). Convert the mixed number into an improper fraction to get \(x=\) \(\left(2+\frac{1}{2}\right)(7)=\left(\frac{4}{2}+\frac{1}{2}\right)(7)=\left(\frac{5}{2}\right)\left(\frac{7}{1}\right)=\frac{35}{2}\). There are mixed numbers in the choices, so convert using long division.
\[
\begin{gathered}
17 \\
2 \longdiv { 3 5 } \\
\underline{2} \\
15 \\
\underline{14}
\end{gathered}
\]

The result of \(35 \div 2\) is 17 with a remainder of 1 , or \(17 \frac{1}{2}\), which is (H).
99. C Translate the information in the question. Let \(D\) be the number miles Darcy ran. The question says that Inez ran \(y\) miles and Inez and Darcy together ran \(3 y+7\), so \(y+D=3 y+7\). The question asks for the number of miles Darcy ran, so isolate \(D\). Subtract \(y\) from both sides to get \(D=2 y+7\), which is (C).
100. H Simplify the expression \(-3 a(5 b-4 c)\). Distribute \(-3 a\). Don’t forget to apply the negative to both terms inside the parentheses, which will cancel the negative in the second term. The result is \(-15 a b+12 a c\), which is (H).
101. C The question asks for the value of \(b\), which is the radius of one of the circles. The question says that the sum of the circumferences is \(64 \pi\). The formula for circumference of a circle is \(C=2 \pi r\). The two circles have radii of \(a\) and \(b\), respectively. The circle with radius \(a\) has circumferences \(C=2 \pi a\), and the circle with radius \(b\) has circumference \(C=2 \pi b\). Therefore, the sum of the circumferences is
\(2 \pi a+2 \pi b=64 \pi\). The question says that \(a=\frac{1}{3} b\), so substitute to get \(2 \pi\) \(\left(\frac{1}{3} b\right)+2 \pi b=64 \pi\). Simplify the right side to get \(\frac{2 \pi b}{3}+2 \pi b=64 \pi\). To get rid of the denominator, multiply both sides by 3 to get \(2 \pi b+\) \(6 \pi b=192 \pi\). Combine like terms on the left side to get \(8 \pi b=192 \pi\).

Divide both sides by \(\pi\) to get \(8 b=192\). Divide both sides by 8 to get \(b\) \(=24\), which is (C).
102. H The question says that the hash marks are evenly spaced, so determine the length between each hash mark. The distance from \(X\) to \(Y\) is \(\frac{1}{3}-\) \(\left(-\frac{2}{9}\right)=\frac{1}{3}+\frac{2}{9}=\frac{3}{9}+\frac{2}{9}=\frac{5}{9}\). Count the hash marks from \(X\) to \(Y\). Do not count \(X\). There are 7 hash marks. Therefore, the distance between each hash mark is \(\frac{5}{9} \div 7=\frac{5}{9} \times \frac{1}{7}=\frac{5}{63}\). To determine the value of \(C\), start by counting the hash marks from \(X\) to \(C\). Again, don't count \(X\). There are 4 hash marks, so the distance from \(X\) to \(C\) is \(4 \times \frac{5}{63}=\frac{20}{63}\). Add this distance to the value of \(X\) to get \(-\frac{2}{9}+\frac{20}{63}=-\frac{14}{63}+\frac{20}{63}=\frac{6}{63}\) \(=\frac{2}{21}\), which is (H).
103. A When a question asks about rates, use proportions. The question asks
how many articles the two can read in 2 hours. Determine how many articles each one can read individually in 2 hours and then add. Jaime can read 1 article every 10 minutes, so in 2 hours, he could read \(x\) articles: \(\frac{1 \text { article }}{10 \text { minutes }}=\frac{x \text { articles }}{2 \text { hours }}\). Make sure to use like units. There are 60 minutes in 1 hour, so to determine the number of minutes in 2 hours, set up another proportion to convert: \(\frac{60 \text { minutes }}{1 \text { hour }}=\frac{m \text { minutes }}{2 \text { hours }}\). Cross-multiply to get \(m=120\). Therefore, 2 hours is 120 minutes; change the original proportion to \(\frac{1 \text { article }}{10 \text { minutes }}=\frac{x \text { articles }}{120 \text { minutes }}\). Cross-multiply to get \(10 x=120\). Divide by 10 to get \(x=12\), so Jamie can read 12 articles in 2 hours. Do the same for Rich. Rich reads 1 articles every 15 minutes, so set up the proportion \(\frac{1 \text { article }}{15 \text { minutes }}=\frac{y \text { articles }}{120 \text { minutes }}\). Cross-multiply to get \(15 y=120\). Divide by 15 to get \(y=8\), so Rich reads 8 articles in 2 hours. To get how much they read combined, add to get \(12+8=20\), which is (A).
104. \(\mathbf{H}\) Because this is a conversion question, use proportions. 1 meter \(=\) 3.281 feet, so to get the number of feet in 4.4 kilometers, set up the
proportion \(\frac{1 \text { meter }}{3.281 \text { feet }}=\frac{4.4 \text { kilometers }}{x \text { feet }}\). Always use like units, so
convert 4.4 kilometers to meters, using the proportion
\(\frac{1 \text { kilometer }}{1,000 \text { meters }}=\frac{4.4 \text { kilometers }}{y \text { feet }}\). Cross-multiply to get \(y=4,400\).
Rewrite the first proportion as \(\frac{1 \text { meter }}{3.281 \text { feet }}=\frac{4,400 \text { meters }}{x \text { feet }}\). Cross-
multiply to get \(x=14,436.4\), which is \((\mathrm{H})\).
105. C The question asks for the surface area excluding the triangular bases. The remaining faces are rectangles, so use the formula for the area of a rectangle: \(A=l w\). The length of each of the rectangles is a side of an equilateral triangle with perimeter 18. Since the lengths of all sides of an equilateral triangle are equal, and perimeter is the sum of the lengths of sides, the length of the side can be found by dividing the perimeter by 3 to get \(18 \div 3=6\). The width of each of the rectangles is equal to \(A B\), which is 9 . Therefore, the area of each of the rectangles is \(A=6 \times 9=54\). The surface area is the area of the three sides together, which is \(54 \times 3=162\), which is (C).
106. G The question asks for the minimum number of rooms that the museum must contain. The museum is being built to display 3,100 paintings and each room can hold 60 paintings. To find the minimum number of rooms, fill each individual room with the maximum number of paintings. To find the number of rooms of 60 paintings needed to display 3,100 paintings, divide 3,100 by 60 . First, write \(\frac{3,100}{60}\) and reduce by 10 to get \(\frac{310}{6}\). Long division can help you here:


The result is 51 with a remainder of 4 . Because there are paintings left over, you'll have 51 rooms of 60 paintings with the remaining 4 paintings going into a 52nd room. The total number of rooms needed is 52 , which is ( G ).
107. C When a question involves comparing shadows, use proportions. A 6foot tall man casts a 10 -foot shadow. To determine the length of the shadow of a 15 -foot lamp post, set up \(\frac{6 \mathrm{ft}}{10 \mathrm{ft}}=\frac{15 \mathrm{ft}}{x \mathrm{ft}}\). Cross-multiply to get \(6 x=150\). Divide both sides by 6 to get \(x=25\), which is (C).
108. F The question asks for the least possible value of \(a\), so plug in the answers, starting with the least choice. Start with (E), which is 50 .

Plug \(a=50\) into \(a=10 b=7 c=5 d\), and determine whether \(b, c\), and \(d\) are integers. Divide both sides of \(50=10 b\) by 10 to get \(5=b\). This is an integer, so continue. Divide both sides of \(50=7 c\) by 7 to get \(\frac{50}{7}=\) \(c\). Since the value of \(c\) is not an integer, eliminate (E). Try (F), which is 70 . Divide both sides of \(70=10 b\) by 10 to get \(10=b\). This is an
integer, so continue. Divide both sides of \(70=7 c\) by 7 to get \(10=c\).

This is an integer, so continue. Divide both sides of \(70=5 d\) by 5 to get \(14=d\). This is also an integer. Since \(b, c\), and \(d\) are all integers, the answer is (F).
109. B The question asks for which choice is equivalent to \(15^{5} \div 15^{3}\). When dividing numbers with exponents and the same base, subtract the exponents. In this case, \(15^{5} \div 15^{3}=15^{5-3}=15^{2}\), which is (B).
110. F The question involves averages, so use the average pie. The average of \(5,7,9\), and \(n\) is 7 . There are four numbers with an average of 7 , so put these into the average pie.


Multiply to get a total of \(4 \times 7=28\). The total is equal to the sum of the numbers, so \(5+7+9+n=28\). Simplify the left side to get \(21+n\) \(=28\). Subtract 21 from both sides to get \(n=7\), which is \((F)\).
111. D The question asks for the measure of angle \(C\), which is one of the angles in the triangle. The sum of the measures of all the angles of any triangle is 180 , so \(A+B+C=180\). Angle \(B\) is marked as a right angle, so its measure is \(90^{\circ}\). Substitute \(B=90\) to get \(A+90+C=\) 180. Subtract 90 from both sides to get \(A+C=90\). The lengths of sides \(A B\) and \(B C\) are both 11 . In a triangle, equal sides are opposite equal angles. Therefore, since angle \(C\) and angle \(A\) are opposite side
\(A B\) and side \(B C\), respectively, \(A=C\). Substitute \(A=C\) into the equation to get \(C+C=90\). Combine like terms to get \(2 C=90\). Divide both sides by 2 to get \(C=45\), which is (D).
112. E The question asks which point is found in the shaded region. Eliminate the choices that are not found within the shaded region. Start with the \(x\)-coordinates. The \(x\)-coordinates of all the points in the shaded region go from -1 to 1 , inclusive. Eliminate (H), which has an \(x\)-coordinate less than -1 . The \(y\)-coordinates of all the points in the shaded region go from 0 to 1 . Eliminate (F) and (G), which have a \(y\) coordinate less than 0 . The only choice remaining is ( E ).
113. D The question describes a ratio, so use the ratio box. The ratio of math sessions to history sessions is 5:3 and the actual number of history sessions is 24.
\begin{tabular}{|c|c|l|}
\multicolumn{1}{c}{ Math } & \multicolumn{1}{c}{ History } & \multicolumn{1}{c}{ Total } \\
\hline 5 & 3 & \\
\hline & & \\
\hline & 24 & \\
\hline & Ratio \\
Multiplier \\
& Actual \#'s
\end{tabular}

Add across the top row to get \(5+3=8\) under Total. Multiply downward. Since \(3 \times 8=24\), the multiplier is 8 . Fill in 8 across the middle row. Multiply downward to get \(5 \times 8=40\) in the bottom row under Math, and \(8 \times 8=64\) in the bottom row under Total.
\begin{tabular}{|c|c|c|}
\multicolumn{1}{c}{ Math } & History & \multicolumn{1}{c}{ Total } \\
\hline 5 & 3 & 8 \\
\hline 8 & 8 & 8 \\
\hline 40 & Ratio \\
\hline Multiplier \\
\hline & 24 & 64 \\
\hline
\end{tabular}

The question asks for the total number of math and history sessions. Look in the Actual \#'s row under Total to find 64, which is (D).
114. G The question involves rates, so use proportions. Juliet receives 3 calls
per minute, so to determine the number of calls she receives per hour, set up the proportion \(\frac{3 \text { calls }}{1 \text { minute }}=\frac{x \text { calls }}{1 \text { hour }}\). Like units must be used in a proportion, so replace 1 hour with 60 minutes to get \(\frac{3 \text { calls }}{1 \text { minute }}=\frac{x \text { calls }}{60 \text { minutes }}\). Cross-multiply to get \(x=180\), which is (G).

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[^0]:    ${ }^{1}$ undergarments that cover the legs

