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Fifth Edition

Fixed Income Analysis

WILEY

FIXED INCOME ANALYSIS

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FIXED INCOME ANALYSIS

Fifth Edition

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WILEY

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PREFACE

We are pleased to bring you *Fixed Income Analysis*, which provides authoritative and up-to-date coverage of how investment professionals analyze and manage fixed-income portfolios. As with many of the other titles in the CFA Institute Investment Series, the content for this book is drawn from the official CFA Program curriculum. As such, readers can rely on the content of this book to be current, globally relevant, and practical.

The content was developed in partnership by a team of distinguished academics and practitioners, chosen for their acknowledged expertise in the field, and guided by CFA Institute. It is written specifically with the investment practitioner in mind and provides numerous examples and practice problems that reinforce the learning outcomes and demonstrate real-world applicability.

The CFA Program curriculum, from which the content of this book was drawn, is subjected to a rigorous review process to assure that it is:

- Faithful to the findings of our ongoing industry practice analysis
- Valuable to members, employers, and investors
- Globally relevant
- Generalist (as opposed to specialist) in nature
- Replete with sufficient examples and practice opportunities
- Pedagogically sound

The accompanying workbook is a useful reference that provides Learning Outcome Statements, which describe exactly what readers will learn and be able to demonstrate after mastering the accompanying material. Additionally, the workbook has summary overviews and practice problems for each chapter.

We hope you will find this and other books in the CFA Institute Investment Series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran striving to keep up to date in the ever-changing market environment. CFA Institute, as a long-term committed participant in the investment profession and a not-for-profit global membership association, is pleased to provide you with this opportunity.

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Anchored by a practice-based curriculum, the CFA Program Body of Knowledge reflects the knowledge, skills, and abilities identified by professionals as essential to the investment decision-making process. This body of knowledge maintains its relevance through a regular, extensive survey of practicing CFA charterholders across the globe. The curriculum covers 10 general topic areas, ranging from equity and fixed-income analysis to portfolio management to corporate finance—all with a heavy emphasis on the application of ethics in professional practice. Known for its rigor and breadth, the CFA Program curriculum highlights principles common to every market so that professionals who earn the CFA designation have a thoroughly global investment perspective and a profound understanding of the global marketplace.

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ABOUT THE CFA INSTITUTE SERIES

CFA Institute is pleased to provide you with the CFA Institute Investment Series, which covers major areas in the field of investments. We provide this series for the same reason we have been chartering investment professionals for more than 50 years: to lead the investment profession globally by promoting the highest standards of ethics, education, and professional excellence for the ultimate benefit of society.

The books in the CFA Institute Investment Series contain practical, globally relevant material. They are intended both for those contemplating entry into the extremely competitive field of investment management as well as for those seeking a means of keeping their knowledge fresh and up to date. This series was designed to be user friendly and highly relevant.

We hope you find this series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran ethically bound to keep up to date in the ever-changing market environment. As a long-term, committed participant in the investment profession and a not-for-profit global membership association, CFA Institute is pleased to provide you with this opportunity.

THE TEXTS

Alternative Investments is the definitive guide to help students and professionals understand non-traditional asset classes, including real estate, commodities, infrastructure, private equity, private credit, and hedge funds. This book provides readers the foundational knowledge to recognize the many distinguishing characteristics of alternative investments—higher fees, less regulation than traditional investments, concentrated portfolios, unique legal and tax considerations, and more. Through a series of strategically designed learning objectives and high-level chapter summaries, learners will develop an understanding of the value, risks, and processes associated with non-traditional investments.

Derivatives is a key resource for anyone interested in the role of derivatives within comprehensive portfolio management. Via accessible prose and real-world examples, a general discussion of the types of derivatives leads to a detailed examination of each market and its contracts—including forwards, futures, options and swaps, and a look at credit derivative markets and their instruments. This vital text offers a conceptual framework for understanding the fundamentals of derivatives. By the end of the book, readers will recognize the different types of derivatives, their characteristics, and how and why derivatives are essential to risk management.

The *Portfolio Management in Practice* three-volume set meets the needs of a wide range of individuals, from graduate-level students focused on finance to practicing investment professionals. This set within the CFA Institute Investment Series delivers complete coverage of the foundational issues surrounding modern portfolio management, examining everything from asset allocation strategies to risk management frameworks. Readers are guided through the full portfolio management process with accessible chapters that distill the knowledge, skills, and abilities needed to succeed in today's fast-paced financial world. Key topics outlined in this set include forming capital market expectations, principles and processes of asset allocation, considerations specific to high-net-worth individuals and institutions, ESG integration, and more. Discover the most comprehensive overview of portfolio management on the market with *Portfolio Management in Practice, Volume 1: Investment Management, Portfolio Management in Practice, Volume 2: Asset Allocation,* and *Portfolio Management in Practice, Volume 3: Equity Portfolio Management.*

Quantitative Investment Analysis, 4th Edition focuses on the primary quantitative tools needed by today's professional investor. Readers begin with classic time value of money, discounted cash flow applications, and probability models applied to investment management problems. The text then develops a critical skill that challenges many professionals—the ability to distinguish useful information from the overwhelming quantity of available data. Chapters dealing with correlation and hypothesis testing lead into regression modeling and machine learning that ultimately demonstrates how to extract insights from Big Data found in investment management. The final chapter of *Quantitative Investment Analysis* covers portfolio concepts and takes the reader beyond the traditional capital asset pricing model (CAPM) type of tools and into the more practical world of multifactor models and arbitrage pricing theory.

International Financial Statement Analysis, 4th Edition is designed to address the ever-increasing need for investment professionals and students to think about financial statement analysis from a global perspective. The text is a practically oriented introduction to financial statement analysis that is distinguished by its combination of a true international orientation, a structured presentation style, and abundant illustrations and tools covering concepts as they are introduced. The authors cover this discipline comprehensively and with an eye to ensuring the reader's success at all levels in the complex world of financial statement analysis.

Equity Asset Valuation, 4th Edition integrates accounting and finance concepts to deliver a comprehensive collection of valuation models and guide novice investors and experienced practitioners alike in determining which models are appropriate for specific situations. This updated edition of the *Equity Asset Valuation* text blends theory and practice, details contemporary techniques used to determine the intrinsic value of an equity security and demonstrates how these techniques are applied in both foreign and domestic markets. Readers will find clear, example-driven coverage of technical analysis, fundamental analysis, and an array of valuation techniques.

All titles in the Investment Series are also available on https://www.wiley.com/ for purchase directly from the publisher in print and eBook formats.

Qualified instructors may request evaluation copies for university classroom use at https://secure.wiley.com/digitalevaluationcopyrequest/. *Restrictions apply.*

FIXED INCOME ANALYSIS

PART I

FIXED INCOME ESSENTIALS

CHAPTER 1

FIXED-INCOME SECURITIES: DEFINING ELEMENTS

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LEARNING OUTCOMES

The candidate should be able to:

- describe basic features of a fixed-income security;
- describe content of a bond indenture;
- compare affirmative and negative covenants and identify examples of each;
- describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities;
- describe how cash flows of fixed-income securities are structured;
- describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender.

1. INTRODUCTION AND OVERVIEW OF A FIXED-INCOME SECURITY

- **describe basic features of a fixed-income security**

Fixed-income securities constitute the most prevalent means of raising capital globally based on total market value. These instruments allow governments, companies, and other issuers to borrow from investors, promising future interest payments and the return of principal, which are contractual (legal) obligations of the issuer. Fixed-income securities are the largest source of capital for government, not-for-profit, and other entities that do not issue equity. For private companies, fixed-income investors differ from shareholders in not having ownership rights. Payments of interest and repayment of principal (amount borrowed) are a higher priority claim on the company's earnings and assets compared with the claim of common shareholders. Since fixed-income claims rank above shareholder claims in the capital structure, a company's fixed-income securities have, in theory, lower risk than their common shares.

Financial analysts who master these and other fixed-income concepts have a distinct edge over their peers for several reasons. First, given the nature of fixed-income cash flows and their preponderance across issuers and regions, these instruments form the basis for risk versus return comparisons both across and within specific jurisdictions. For example, as bonds issued by the US Treasury and other developed market central governments are viewed as having little to no default risk, they serve as building blocks in determining the time value of money for less certain cash flows. Fixed-income securities also fulfill an important role in portfolio management as a prime means by which individual and institutional investors can fund known future obligations, such as tuition payments or retirement obligations. Finally, while the correlation of fixed-income returns with common share returns varies, adding fixed-income securities to portfolios that include common shares can be an effective way of obtaining diversification benefits.

Among the questions to be addressed are the following:

- Which features define a fixed-income security, and how do they determine the scheduled cash flows?
- What are the legal, regulatory, and tax considerations associated with a fixed-income security, and why are they important for investors?
- What are the common interest and principal payment structures?
- What types of provisions may affect the disposal or redemption of fixed-income securities?

Note that the terms "fixed-income securities," "debt securities," and "bonds" are often used interchangeably by experts and non-experts alike. We will also follow this convention, and where any nuance of meaning is intended, it will be made clear. Moreover, the term "fixed income" is not to be understood literally: Some fixed-income securities have interest payments that change over time.

1.1. Overview of a Fixed-Income Security

A **bond** is a contractual agreement between the issuer and the bondholders. Three important elements that an investor needs to know about when considering a fixed-income security are:

- The bond's features, including the issuer, maturity, par value, coupon rate and frequency, and currency denomination. These features determine the bond's scheduled cash flows and, therefore, are key determinants of the investor's expected and actual return.
- The legal, regulatory, and tax considerations that apply to the contractual agreement between the issuer and the bondholders.
- The contingency provisions that may affect the bond's scheduled cash flows. These contingency provisions are options providing either issuers or bondholders certain rights affecting the bond's disposal or redemption.

This section describes a bond's basic features and introduces yield measures. The legal, regulatory, and tax considerations and contingency provisions are discussed in subsequent sections.

1.1.1. Basic Features of a Bond

All bonds, regardless of issuer, are characterized by the same basic features, which include maturity, par or principal amount, coupon size, frequency, and currency.

1.1.1.1. Issuer Many entities issue bonds: private individuals, such as the musician David Bowie; national governments, such as Singapore or Italy; and companies, such as BP, General Electric, or Tata Group.

Bond issuers are classified into categories based on the similarities of these issuers and their characteristics. Major types of issuers include the following:

- Supranational organizations, such as the World Bank or the European Investment Bank;
- Sovereign (national) governments, such as the United States or Japan. As sovereign bonds are backed by the full faith and credit of each respective government, they usually represent the lowest risk and most secure bonds in each market;
- Non-sovereign (local) governments, such as the State of Minnesota in the United States, the Catalonia region in Spain, or the City of Edmonton in Alberta, Canada;
- Quasi-government entities (i.e., agencies that are owned or sponsored by governments), such as postal services in many countries—for example, Correios in Brazil, La Poste in France, or Pos in Indonesia;
- Companies (i.e., corporate issuers). A distinction is often made between financial issuers (e.g., banks and insurance companies) and non-financial issuers; and
- Special legal entities (i.e., special purpose entities) that use specific assets, such as auto loans and credit card debt obligations, to guarantee (or secure) a bond issue known as an asset-backed security that is then sold to investors.

Market participants often classify fixed-income markets by the type of issuer, which leads to the identification of three bond market sectors: the government and government-related sector (i.e., the first four types of issuers just listed), the corporate sector (the fifth type listed), and the structured finance sector (the last type listed).

While several major local fixed-income markets, such as China and Japan, are dominated by government issuers, a significant portion of the US bond market consists of corporate issuance and asset-backed securities in addition to US treasury securities issued by the federal government.

Asset-backed securities (ABS) are created from a process called securitization, which involves moving assets from the owner of the assets into a special legal entity. This special legal entity then uses the securitized assets as guarantees to back (secure) a bond issue, leading to the creation of ABS. Assets that are typically used to create ABS include residential and commercial mortgage loans (mortgages), automobile (auto) loans, student loans, bank loans, and credit card debt. Many elements discussed in this chapter apply to both traditional bonds and ABS. Considerations specific to ABS are discussed in a separate chapter on asset-backed securities.

Bondholders are exposed to credit risk—that is, the risk of loss resulting from the issuer failing to make full and timely payments of interest and/or repayments of principal. Credit risk is inherent to all debt investments. Bond markets are sometimes classified into sectors based on the issuer's creditworthiness as judged by credit rating agencies. The three largest credit rating agencies are Moody's Investors Service, Standard & Poor's, and Fitch Ratings. One major distinction is between investment-grade and non-investment-grade bonds, also called high-yield or speculative bonds. For example, bonds rated Baa3 or higher by Moody's and BBB– or higher by Standard & Poor's and Fitch are considered investment grade. Although a variety of considerations enter into distinguishing the two sectors, the promised payments of investment-grade bonds are perceived as less risky than those of non-investment-grade bonds because of profitability and liquidity considerations. Some regulated financial intermediaries, such as banks and life insurance companies, may face explicit or implicit limitations of holdings of non-investment-grade bonds. The investment policy statements of some investors may also include constraints or limits on such holdings. From the issuer's perspective, an investment-grade credit rating generally allows easier access to bond markets and at lower interest rates than does a non-investment-grade credit rating.

1.1.1.2. Maturity The maturity date of a bond refers to the date when the issuer is obligated to redeem the bond by paying the outstanding principal amount. The **tenor** is the time remaining until the bond's maturity date. Tenor is an important consideration in analyzing a bond's risk and return, as it indicates the period over which the bondholder can expect to receive interest payments and the length of time until the principal is repaid in full.

Maturities typically range from overnight to 30 years or longer. Fixed-income securities with maturities at issuance (original maturity) of one year or less are known as **money market securities**. Issuers of money market securities include governments and companies. Commercial paper and certificates of deposit are examples of money market securities. Fixed-income securities with original maturities that are longer than one year are called **capital market securities**. Although very rare, **perpetual bonds**, such as the consols issued by the sovereign government in the United Kingdom, have no stated maturity date.

1.1.1.3. Par Value The **principal amount**, **principal value**, or simply **principal** of a bond is the amount that the issuer agrees to repay the bondholders on the maturity date. This amount is also referred to as the par value, or simply par, face value, nominal value, redemption value, or maturity value. Bonds can have any par value.

In practice, bond prices are quoted as a percentage of their par value. For example, assume that a bond's par value is $1,000. A quote of 95 means that the bond price is $950 (95% × $1,000). When the bond is priced at 100% of par, the bond is said to be trading at par. If the bond's price is below 100% of par, such as in the previous example, the bond is trading at a discount. Alternatively, if the bond's price is above 100% of par, the bond is trading at a premium.

1.1.1.4. Coupon Rate and Frequency The coupon rate or nominal rate of a bond is the interest rate that the issuer agrees to pay each year until the maturity date. The annual amount of interest payments made is called the coupon. A bond's coupon is determined by multiplying its coupon rate by its par value. For example, a bond with a coupon rate of 6% and a par value of $1,000 will pay annual interest of $60 (6% × $1,000).

Coupon payments may be made annually, such as those for German government bonds or Bunds. Many bonds, such as government and corporate bonds issued in the United States or government gilts issued in the United Kingdom, pay interest semi-annually. Some bonds make quarterly or monthly interest payments. The acronyms QUIBS (quarterly interest bonds) and QUIDS (quarterly income debt securities) are used by Morgan Stanley and Goldman Sachs, respectively, for bonds that make quarterly interest payments. Many **mortgage-backed securities** (MBS), which are ABS backed by residential or commercial mortgages, pay interest monthly to match the cash flows of the underlying assets. If a bond has a coupon rate of 6% and a par value of $1,000, the periodic interest payments will be $60 if coupon payments are made annually, $30 if they are made semi-annually, $15 if they are made quarterly, and $5 if they are made monthly.

A **plain vanilla bond** or **conventional bond** pays a fixed rate of interest. In this case, the coupon payment does not change during the bond's life. However, there are bonds that pay a floating rate of interest; such bonds are called **floating-rate notes** (FRNs) or **floaters**. The coupon rate of an FRN includes two components: a **market reference rate (MRR)** plus a spread. The spread, also called margin, is typically constant and is expressed in basis points (bps). A **basis point** is equal to 0.01% (i.e., 100 basis points equals 1%). The spread is set when the bond is issued based on the issuer's creditworthiness at issuance: The higher the issuer's credit quality, the lower the spread. The MRR, however, resets periodically. Thus, as the MRR changes, the coupon rate and coupon payment change accordingly.

The market reference rate is a collective name for a set of rates covering different currencies for different maturities, ranging from overnight to one year. These rates have historically included the London Interbank Offered Rate (Libor), the Euro Interbank Offered Rate (Euribor), the Hong Kong Interbank Offered Rate (Hibor), or the Singapore Interbank Offered Rate (Sibor) for issues denominated in US dollars, euros, Hong Kong dollars, and Singapore dollars, respectively. The process of phasing out Libor and moving to new market reference rates is discussed in a subsequent CFA Level I chapter.

For example, assume that the coupon rate of an FRN that makes semi-annual interest payments in June and December is expressed as the six-month MRR + 150 bps. Suppose that in December 20X0, the six-month MRR is 3.25%. The interest rate that will apply to the payment due in June 20X1 will be 4.75% (3.25% + 1.50%). Now suppose that in June 20X1, the six-month MRR has decreased to 3.15%. The interest rate that will apply to the payment due in December 20X1 will decrease to 4.65% (3.15% + 1.50%).

All bonds, whether they pay a fixed or floating rate of interest, make periodic coupon payments except for **zero-coupon bonds**. Zero-coupon, or **pure discount bonds**, are issued at a discount to par value and are redeemed at par. The interest earned on a zero-coupon bond is implied and equal to the difference between the par value and the purchase price. For example, if the par value is $1,000 and the purchase price is $950, the implied interest is $50.

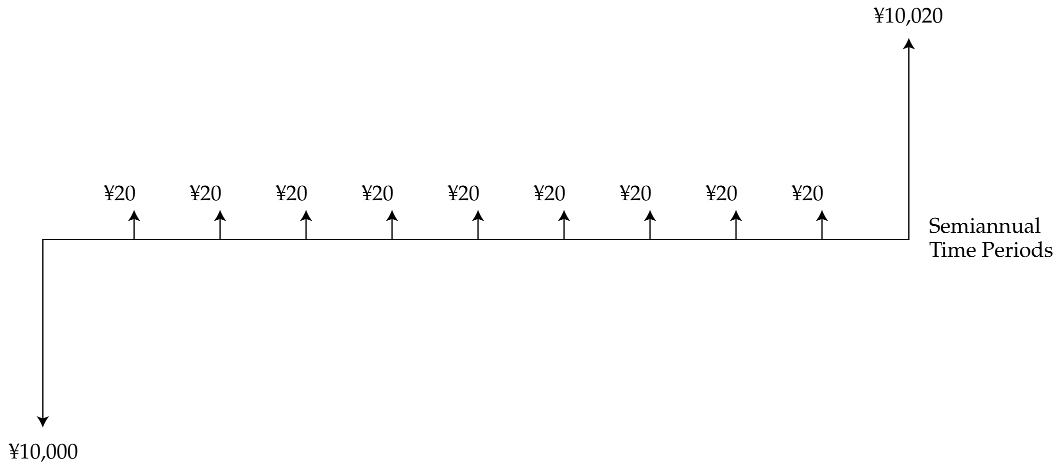
1.1.1.5. Currency Denomination Bonds can be issued in any currency, although a large number of bond issues are made in either euros or US dollars. The currency of issue may affect a bond's attractiveness. Borrowers in emerging markets often elect to issue bonds in euros or US dollars because doing so makes the bonds more attractive to international investors than bonds in a domestic currency that may be illiquid or not freely traded. Issuers may also choose to borrow in a foreign currency if they expect cash flows in that currency to offset interest payments and principal repayments. If a bond is aimed solely at a country's domestic investors, it is more likely that the borrower will issue in the local currency.

Dual-currency bonds make coupon payments in one currency and pay the par value at maturity in another currency. For example, assume that a Japanese company needs to finance a long-term project in the United States that will take several years to become profitable. The Japanese company could issue a yen/US dollar dual-currency bond. The coupon payments in yen can be made from the cash flows generated in Japan, and the principal can be repaid in dollars once the project becomes profitable.

Currency option bonds can be viewed as a combination of a single-currency bond plus a foreign currency option. They give bondholders the right to choose the currency in which they want to receive interest payments and principal repayments. Bondholders can select one of two currencies for each payment.

Exhibit 1 brings all the basic features of a bond together and illustrates how these features determine the cash flow pattern for a plain vanilla bond. The bond is a five-year Japanese government bond (JGB) with a coupon rate of 0.4% and a par value of ¥10,000. Interest payments are made semi-annually. The bond is priced at par when it is issued and is redeemed at par.

EXHIBIT 1 Cash Flows for a Plain Vanilla Bond



The downward-pointing arrow in Exhibit 1 represents the cash flow paid by the bond investor (received by the issuer) on the day of the bond issue—that is, ¥10,000. The upward-pointing arrows are the cash flows received by the bondholder (paid by the issuer) during the bond's life. As interest is paid semi-annually, the coupon payment is ¥20 [(0.004 × ¥10,000) ÷ 2] every six months for five years—that is, 10 coupon payments of ¥20. The last payment is equal to ¥10,020 because it includes both the last coupon payment and the payment of the par value.

EXAMPLE 1

1. An example of a sovereign bond is a bond issued by:
   A. the World Bank.
   B. the city of New York.
   C. the federal German government.
2. The risk of loss resulting from the issuer failing to make full and timely payment of interest is called:
   A. credit risk.
   B. systemic risk.
   C. interest rate risk.
3. A money market security *most likely* matures in:
   A. one year or less.
   B. between 1 and 10 years.
   C. over 10 years.
4. If the bond's price is higher than its par value, the bond is trading at:
   A. par.
   B. a discount.
   C. a premium.
5. A bond has a par value of £100 and a coupon rate of 5%. Coupon payments are made semi-annually. The periodic interest payment is:
   A. £2.50, paid twice a year.
   B. £5.00, paid once a year.
   C. £5.00, paid twice a year.

6. The coupon rate of a floating-rate note that makes payments in June and December is expressed as six-month MRR + 25 bps. Assuming that the six-month MRR is 3.00% at the end of June 20XX and 3.50% at the end of December 20XX, the interest rate that applies to the payment due in December 20XX is:
   A. 3.25%.
   B. 3.50%.
   C. 3.75%.
7. The type of bond that allows bondholders to choose the currency in which they receive each interest payment and principal repayment is a:
   A. pure discount bond.
   B. dual-currency bond.
   C. currency option bond.

Solution to 1: C is correct. A sovereign bond is a bond issued by a national government, such as the federal German government. A is incorrect because a bond issued by the World Bank is a supranational bond. B is incorrect because a bond issued by a local government, such as the city of New York, is a non-sovereign bond.

Solution to 2: A is correct. Credit risk is the risk of loss resulting from the issuer failing to make full and timely payments of interest and/or repayments of principal. B is incorrect because systemic risk is the risk of failure of the financial system. C is incorrect because interest rate risk is the risk that a change in market interest rate affects a bond's value.

Solution to 3: A is correct. The primary difference between a money market security and a capital market security is the maturity at issuance. Money market securities mature in one year or less, whereas capital market securities mature in more than one year.

Solution to 4: C is correct. If a bond's price is higher than its par value, the bond is trading at a premium. A is incorrect because a bond is trading at par if its price is equal to its par value. B is incorrect because a bond is trading at a discount if its price is lower than its par value.

Solution to 5: A is correct. The annual coupon payment is 5% × £100 = £5.00. The coupon payments are made semi-annually, so £2.50 paid twice a year.

Solution to 6: A is correct. The interest rate that applies to the payment due in December 20XX is the six-month MRR at the end of June 20XX plus 25 bps. Thus, it is 3.25% (3.00% + 0.25%).

Solution to 7: C is correct. A currency option bond gives bondholders the right to choose the currency in which they want to receive each interest payment and principal repayment. A is incorrect because a pure discount bond is issued at a discount to par value and redeemed at par. B is incorrect because a dual-currency bond makes coupon payments in one currency and pays the par value at maturity in another currency.

1.1.2. Yield Measures

Several yield measures are commonly used by market participants. The **current yield** or **running yield** is equal to the bond's annual coupon divided by the bond's price, expressed as a percentage. For example, if a bond has a coupon rate of 6%, a par value of $1,000, and a price of $1,010, the current yield is 5.94% ($60 ÷ $1,010). The current yield is a measure of income that is analogous to the dividend yield for a common share.

The most common yield measure is known as the **yield-to-maturity**, also called the **yield-to-redemption** or **redemption yield**. The yield-to-maturity is the internal rate of return on a bond's expected cash flows—that is, the discount rate that equates the present value of the bond's expected cash flows until maturity with the bond's price. The yield-to-maturity can be considered an estimate of the bond's expected return; it reflects the annual return earned by an investor who purchases the bond today and holds it until maturity, provided they receive all promised cash flows and are able to reinvest all coupons at this same yield. There is an inverse relationship between the bond's price and its yield-to-maturity, all else being equal. That is, the higher the bond's yield-to-maturity, the lower its price. Alternatively, the higher the bond's price, the lower its yield-to-maturity. Thus, investors expecting interest rates to fall (and investors to demand a lower yield-to-maturity) anticipate a positive return from price appreciation. The topic of risk and return of fixed-income securities is covered in more detail in a subsequent chapter.

2. BOND INDENTURE

- **describe content of a bond indenture**
- **compare affirmative and negative covenants and identify examples of each**

As contractual agreements between the issuer and the bondholders, bonds are subject to legal, regulatory, and tax considerations that are important to fixed-income investors.

2.1. Bond Indenture

The **trust deed** is the legal contract that describes the form of the bond, the obligations of the issuer, and the rights of the bondholders. This legal contract is often referred to as the bond **indenture**, particularly in the United States and Canada. The indenture references both the issuer and the features of the bond issue, such as the principal value, the interest or coupon rate, the interest payment dates, the maturity date, and any contingency provisions. The indenture also identifies the funding sources for interest and principal payments and any collateral, credit enhancements, or covenants. **Collateral** is a specific asset or assets or financial guarantees securing the debt obligation above and beyond the issuer's promise to pay. **Credit enhancements** are provisions used to reduce the credit risk of the bond issue. **Covenants** are clauses that specify the rights of the bondholders and any actions that the issuer is obligated to perform or prohibited from performing.

Because it would be impractical for the issuer to enter into a direct agreement with each bondholder, the indenture is usually held by a trustee. The trustee is typically a financial institution acting as a **fiduciary** (or representative protecting investor interests) to ensure the issuer complies with the obligations specified in the indenture and to take action on behalf of the bondholders, when necessary. The trustee's duties are administrative and include the maintenance of required documentation and records; holding beneficial title to, safeguarding, and appraising collateral (if any); invoicing the issuer for interest payments and principal repayments; and holding funds until they are paid, although the cash flow movements from issuers to the trustee are typically handled by the principal paying agent. In the event of default, the discretionary powers of the trustee increase considerably. For example, the trustee is responsible for calling bondholder meetings to discuss actions as well as bringing legal action against an issuer on behalf of bondholders, if necessary.

For a plain vanilla bond, the indenture is often a standard template that is updated for a bond's specific terms and conditions. For exotic bonds, the document is tailored and can often be several hundred pages.

The indenture identifies basic bond features (e.g., investor rights in the event of default) that impact the bond's risk–reward profile, such as:

- the legal identity of the bond issuer and its legal form;
- the source of repayment proceeds;
- the asset or collateral backing (if any);
- the credit enhancements (if any); and
- the covenants (if any).

We consider each of these areas in the following sections.

2.1.1. Legal Identity of the Bond Issuer and Its Legal Form

The bond issuer, identified in the indenture by its legal name, must make all contractual payments assigned to the bond issuer. For a sovereign bond, the legal issuer is usually the office responsible for managing the national budget, such as HM Treasury (Her Majesty's Treasury) in the United Kingdom. The legal issuer may be different from the body that administers the bond issue process. Using the UK example, the legal obligation to repay gilts lies with HM Treasury, but the bonds are issued by the UK Debt Management Office, an executive agency of HM Treasury.

For corporate bonds, the issuer is usually the corporate legal entity—for example, Wal-Mart Stores Inc., Samsung Electronics Co. Ltd., or Volkswagen AG. However, bonds are sometimes issued by a subsidiary of a parent legal entity. In this case, investors should consider the credit quality of the subsidiary, unless the indenture specifies that the bond liabilities are guaranteed by the parent. When rated, subsidiaries often receive a credit rating below that of the parent.

Bonds are sometimes issued by a holding company, or parent legal entity for a group of companies, rather than by one of the operating companies in the group. A holding company may be rated differently from its operating companies, and investors may lack recourse to assets held by those companies. If the bonds are issued by a holding company with few assets to call on in the event of default, investors face more credit risk than if the bonds were issued by an operating company in the group.

For ABS, the legal obligation to repay the bondholders often lies with the special legal entity created by the sponsor or originator, a financial institution in charge of the securitization process. The special legal entity is most frequently referred to as a special purpose entity (SPE) in the United States and a special purpose vehicle (SPV) in Europe, also called a special purpose company (SPC). The legal form for the special legal entity may be a limited partnership, a limited liability company, or a trust. Typically, special legal entities are thinly capitalized, have no independent management or employees, and have no purpose other than the transactions for which they were created.

The sponsor transfers the assets to the special legal entity to carry out a securitization or series of securitizations. A key reason for forming a special legal entity is *bankruptcy remoteness*. The transfer of assets by the sponsor is considered a legal sale; once the assets have been securitized, the sponsor no longer has ownership rights. Any party making claims following the bankruptcy of the sponsor would be unable to recover the assets or their proceeds. As a result, the special legal entity's ability to pay interest and repay the principal should remain intact even if the sponsor were to fail—hence the reason why the special legal entity is also called a bankruptcy-remote vehicle.

2.1.2. Source of Repayment Proceeds

The indenture usually describes how the issuer intends to service the debt (make interest payments) and repay the principal. Generally, the source of repayment for bonds issued by supranational organizations is either the repayment of previous loans made by the organization or the paid-in capital from its members. National governments may also act as guarantors for certain bond issues. If additional sources of repayment are needed, the supranational organization may call on its members to provide funds.

Sovereign bonds are backed by the "full faith and credit" of the national government. As national governments have unique powers to ensure the ability to repay debt—such as the authority to tax economic activity, print money, and control foreign currency reserves—sovereign bonds denominated in local currency are generally considered the safest of all investments. It is therefore highly probable that interest and principal will be paid fully and on time, resulting in lower yields on sovereign bonds than for similar bonds from other local issuers. Government yields are often used as a "risk-free rate" for time value of money calculations as well as a benchmark reference rate for pricing fixed-income securities from other issuers of the same tenor.

There are three major sources for repayment of non-sovereign government debt issues, and bonds are usually classified according to these sources. The first source is through the general taxing authority of the issuer. The second source is from the cash flows of the project the bond issue is financing. The third source is from special taxes or fees established specifically for the purpose of funding interest payments and principal repayments.

The source of payment for corporate bonds is the issuer's ability to generate cash flows, primarily through its operations. These cash flows depend on the issuer's financial strength and integrity. Because corporate bonds carry a higher level of credit risk than otherwise similar sovereign and non-sovereign government bonds, they typically offer a higher yield.

In contrast to corporate bonds, the source of payment for ABS depends on the cash flows generated by one or more of the underlying financial assets, such as mortgages or auto loans, rather than an operating entity. Thus, investors in ABS must pay special attention to the quality of these assets.

2.1.3. Asset or Collateral Backing

Collateral backing is a way to alleviate credit risk. Investors should review where they rank compared with other creditors in the event of default and analyze the quality of the collateral backing the bond issue.

2.1.3.1. Seniority Ranking **Secured bonds** are backed by assets or financial guarantees pledged to ensure debt repayment in the case of default. In contrast, unsecured bonds have no collateral; bondholders have only a general claim on the issuer's assets and cash flows. Thus, unsecured bonds are paid after secured bonds in the event of default. By lowering credit risk, collateral backing increases the bond issue's credit quality and decreases its yield.

A bond's collateral backing might not specify an identifiable asset but instead may be described as the "general plant and infrastructure" of the issuer. In such cases, investors rely on seniority ranking—that is, the systematic way in which lenders are repaid in case of bankruptcy or liquidation. What matters to investors is where they rank compared with other creditors rather than whether an asset of sufficient quality and value is in place to cover their claims. Senior debt has a priority claim over subordinated debt or junior debt. Financial institutions issue a large volume of both senior unsecured and subordinated bonds globally; it is not uncommon to see large and small banks issue such bonds. For example, banks as diverse as

Royal Bank of Scotland in the United Kingdom and Prime Bank in Bangladesh issue senior unsecured bonds to institutional investors.

Debentures are a type of bond that can be secured or unsecured. In many jurisdictions, debentures are unsecured bonds, with no collateral backing assigned to the bondholders. In contrast, in the United Kingdom and in other Commonwealth countries, such as India, debentures are usually backed by an asset or pool of assets assigned as collateral support for the bond obligations and segregated from other creditor claims. Thus, it is important for investors to review the indenture to determine whether a debenture is secured or unsecured. If the debenture is secured, debenture holders rank above unsecured creditors of the company; they have a specific asset or pool of assets that the trustee can call on to realize the debt in the event of default.

2.1.3.2. Types of Collateral Backing Many types of bonds are secured by a form of collateral. Some companies issue collateral trust bonds and equipment trust certificates. **Collateral trust bonds** are backed by securities, such as common shares, other bonds, or other financial assets. These securities are pledged by the issuer and typically held by the trustee. **Equipment trust certificates** are bonds secured by specific types of equipment or physical assets, such as aircraft, railroad cars, shipping containers, or oil rigs. They are often issued to take advantage of the tax benefits of leasing. For example, suppose an airline finances the purchase of new aircraft with equipment trust certificates. The legal title to the aircraft is held by the trustee, which issues equipment trust certificates to investors in the amount of the aircraft purchase price. The trustee leases the aircraft to the airline and collects lease payments from the airline to pay the interest on the certificates. When the certificates mature, the trustee sells the aircraft to the airline, uses the proceeds to retire the principal, and cancels the lease.

Mortgaged property is one of the most common forms of ABS collateral. MBS are debt obligations that represent claims to the cash flows from pools of mortgage loans, most commonly on residential property. Mortgage loans are purchased from banks, mortgage companies, and other originators and then assembled into pools by a governmental, quasi-governmental, or private entity.

Financial institutions, particularly in Europe, issue covered bonds. **Covered bonds** are debt obligations backed by a segregated pool of assets. They are similar to ABS but offer bondholders recourse against both the financial institution and the underlying asset pool. Covered bonds will be addressed in detail in a later chapter.

2.1.4. Credit Enhancements

Credit enhancements refer to a variety of provisions that can be used to reduce the credit risk of a bond issue. Thus, they increase the issue's credit quality and decrease the bond's yield. Credit enhancements are very often used when creating ABS.

The two primary types of credit enhancements are internal and external. Internal credit enhancement relies on structural features regarding the bond issue. External credit enhancement refers to financial guarantees received from a third party, often called a financial guarantor. We describe each type in the following sections.

2.1.4.1. Internal Credit Enhancement The most common forms of internal credit enhancement are subordination, overcollateralization, and reserve accounts.

Subordination, also known as **credit tranching**, is the most popular internal credit enhancement technique. It relies on creating more than one bond class or tranche and ordering the claim priorities for ownership or interest in an asset between the tranches. The cash flows generated by the assets are allocated with different priority to tranches of different seniority. The ordering of the claim priorities is called a senior/subordinated structure, where the tranches of highest seniority are senior followed by subordinated or junior tranches. The subordinated tranches function as credit protection for the more senior tranches, in the sense that the most senior tranche has the first claim on available cash flows. This type of protection is also commonly referred to as a waterfall structure because in the event of default, the proceeds from liquidating assets will first be used to repay the most senior creditors. Thus, if the issuer defaults, losses are allocated from the bottom up—that is, from the most junior to the most senior tranche. The most senior tranche is typically unaffected unless losses exceed the amount of the subordinated tranches, which is why the most senior tranche is usually rated Aaa/AAA.

Overcollateralization refers to the process of posting more collateral than is needed to obtain or secure financing. It represents a form of internal credit enhancement because the additional collateral can be used to absorb losses. For example, a bond issue of $100 million with collateral value of $110 million has excess collateral of $10 million. Over time, the amount of overcollateralization changes. This can happen because of amortization, prepayments, or defaults as well as changes in collateral values. For example, one of the most significant contributors to the 2007–2009 global financial crisis was a valuation problem with the residential housing assets backing MBS. While many properties were originally valued above the value of securities outstanding, as property prices fell and the number of defaults rose, the credit quality of these MBS declined sharply. This resulted in a rapid rise in yields and losses among investors in these securities.

Reserve accounts or **reserve funds** are another internal credit enhancement in the form of either a cash reserve fund or an excess spread account. A cash reserve fund is a cash deposit used to absorb losses. An excess spread account is an allocation of any asset cash flows remaining after paying interest to bondholders. The excess spread (or excess interest cash flow) can be retained and deposited into a reserve account as a first line of protection against losses. In a process called "turboing," the excess spread can be used to retire the principal, with the most senior tranche having the first claim on these funds.

2.1.4.2. External Credit Enhancement The most common forms of external credit enhancement are bank guarantees and surety bonds, letters of credit, and cash collateral accounts.

Bank guarantees and **surety bonds** are very similar in nature because they both reimburse bondholders for any losses incurred if the issuer defaults. However, there is usually a maximum amount that is guaranteed, called the penal sum. The major difference between a bank guarantee and a surety bond is that the former is issued by a bank, whereas the latter is issued by a rated and regulated insurance company. Insurance companies that specialize in providing financial guarantees are typically called monoline insurance companies or monoline insurers. Monoline insurers played an important role in securitization until the 2007–2009 global financial crisis, but they are now a less common form of credit enhancement due to the credit downgrades of these insurers during the crisis.

A **letter of credit** from a financial institution is another form of external credit enhancement for a bond issue. The financial institution provides the issuer with a credit line to reimburse any cash flow shortfalls from the assets backing the issue. Letters of credit have also become a less common form of credit enhancement since the credit downgrades of financial institutions during the financial crisis.

Bank guarantees, surety bonds, and letters of credit expose the investor to third-party (or counterparty) risk—that is, the possibility that a guarantor cannot meet its obligations. A **cash collateral account** mitigates this concern because the issuer immediately borrows the credit enhancement amount and usually invests it in highly rated short-term commercial paper. Because a cash collateral account is an actual deposit rather than a pledge of cash, a cash collateral account provider downgrade will not necessarily result in a downgrade of the bond issue backed by that provider.

2.1.5. Covenants

Bond covenants are legally enforceable rules that borrowers and lenders agree on at the time of a new bond issue. An indenture will frequently include affirmative (or positive) and negative covenants. Affirmative covenants enumerate what issuers are required to do, whereas negative covenants specify what issuers are prohibited from doing.

Affirmative covenants are typically administrative in nature. For example, frequently used affirmative covenants include what the issuer will do with the proceeds from the bond issue and the promise of making the contractual payments. The issuer may also promise to comply with all laws and regulations, maintain its current lines of business, insure and maintain its assets, and pay taxes as they come due. Other examples include a **pari passu** (or "equal footing") clause, which ensures that a debt obligation is treated the same as the borrower's other senior debt instruments, or a **cross-default** clause, which specifies that a borrower is considered in default if they default on another debt obligation. These types of covenants typically do not impose additional costs to the issuer and do not materially constrain the issuer's discretion regarding how to operate its business.

In contrast, negative covenants are frequently costly and materially constrain the issuer's potential business decisions. They protect bondholders from the dilution of their claims, asset withdrawals or substitutions, and suboptimal investments by the issuer. Examples of negative covenants include the following:

- *Restrictions on debt* regulate the issue of additional debt. Maximum acceptable debt usage ratios (sometimes called leverage ratios or gearing ratios) and minimum acceptable interest coverage ratios are frequently specified, permitting new debt to be issued only when justified by the issuer's financial condition.
- *Negative pledges* prevent the issuance of debt that would be senior to or rank in priority ahead of the existing bondholders' debt.
- *Restrictions on prior claims* protect unsecured bondholders by preventing the issuer from using assets that are not collateralized (called unencumbered assets) to become collateralized.
- *Restrictions on distributions to shareholders* restrict dividends and other payments to shareholders, such as share buybacks (repurchases). The restriction typically operates by reference to the borrower's profitability; that is, the covenant sets a base date, usually at or near the time of the issue, limiting dividends and share buybacks to a percentage of earnings or cumulative earnings after that date.
- *Restrictions on asset disposals* limit the amount of assets that can be disposed by the issuer during the bond's life. This limit on cumulative disposals as a percentage of a company's gross assets is intended to protect bondholder claims by preventing a break-up of the company.
- *Restrictions on investments* constrain risky investments by blocking speculative investments and ensuring an issuer devotes its capital to its going-concern business. A companion covenant may require the issuer to stay in its present line of business.
- *Restrictions on mergers and acquisitions* prevent these actions unless the company is the surviving company or the acquirer delivers a supplemental indenture to the trustee expressly assuming the old bonds and terms of the old indenture. These requirements effectively prevent a company from avoiding its obligations to bondholders by selling to another company.

These are only a few examples of negative covenants. The common characteristic of all negative covenants is ensuring that the issuer will not take any actions that would significantly reduce its ability to make interest payments and repay the principal. Bondholders, however, rarely wish to be too specific about how an issuer should run its business because doing so would imply a degree of control that bondholders legally want to avoid. In addition, very restrictive covenants may not be in the bondholders' best interest if they force the issuer to default when default is avoidable. For example, strict restrictions on debt may prevent the issuer from raising new funds that are necessary to meet its contractual obligations; strict restrictions on asset disposals may prohibit the issuer from selling assets or business units and obtaining the necessary liquidity to make interest payments or principal repayments; and strict restrictions on mergers and acquisitions may prevent the issuer from being taken over by a stronger company that would be able to honor the issuer's contractual obligations.

EXAMPLE 2

1. The term *most likely* used to refer to the legal contract under which a bond is issued is:
   A. indenture.
   B. debenture.
   C. letter of credit.
2. The individual or entity that *most likely* assumes the role of trustee for a bond issue is:
   A. a financial institution appointed by the issuer.
   B. the treasurer or chief financial officer of the issuer.
   C. a financial institution appointed by a regulatory authority.
3. The individual or entity *most likely* responsible for the timely payment of interest and repayment of principal to bondholders is the:
   A. trustee.
   B. primary or lead bank of the issuer.
   C. treasurer or chief financial officer of the issuer.
4. The major advantage of issuing bonds through a special legal entity is:
   A. bankruptcy remoteness.
   B. beneficial tax treatments.
   C. greater liquidity and lower issuing costs.
5. The category of bond *most likely* repaid from the repayment of previous loans made by the issuer is:
   A. sovereign bonds.
   B. supranational bonds.
   C. non-sovereign bonds.
6. The type of collateral used to secure collateral trust bonds is *most likely*:
   A. securities.
   B. mortgages.
   C. physical assets.
7. The external credit enhancement that has the *least* amount of third-party risk is a:
   A. surety bond.
   B. letter of credit.
   C. cash collateral account.

8. An example of an affirmative covenant is the requirement:
   A. that dividends will not exceed 60% of earnings.
   B. to insure and perform periodic maintenance on financed assets.
   C. that the debt-to-equity ratio will not exceed 0.4 and times interest earned will not fall below 8.0.
9. An example of a covenant that protects bondholders against the dilution of their claims is a restriction on:
   A. debt.
   B. investments.
   C. mergers and acquisitions.

Solution to 1: A is correct. The contract between a bond issuer and the bondholders is very often called an indenture or deed trust. The indenture documents the terms of the issue, including the principal amount, the coupon rate, and the payments schedule. It also provides information about the funding sources for the contractual payments and specifies whether there are any collateral, credit enhancement, or covenants. B is incorrect because a debenture is a type of bond. C is incorrect because a letter of credit is an external credit enhancement.

Solution to 2: A is correct. The issuer chooses a financial institution with trust powers, such as the trust department of a bank or a trust company, to act as a trustee for the bond issue.

Solution to 3: A is correct. Although the issuer is ultimately the source of the contractual payments, it is the trustee that ensures timely payments. Doing so is accomplished by invoicing the issuer for interest payments and principal repayments and holding the funds until they are paid.

Solution to 4: A is correct. A special legal entity is a bankruptcy-remote vehicle. Bankruptcy remoteness is achieved by transferring the assets from the sponsor to the special legal entity. Once this transfer is completed, the sponsor no longer has ownership rights. If the sponsor defaults, no claims can be made to recover the assets that were transferred or the proceeds from the transfer to the special legal entity.

Solution to 5: B is correct. The source of payment for bonds issued by supranational organizations is either the repayment of previous loans made by the organization or the paid-in capital of its member states. A is incorrect because national governments rely on their taxing authority and money creation to repay their debt. C is incorrect because non-sovereign bonds are typically repaid from the issuer's taxing authority or the cash flows of the project being financed.

Solution to 6: A is correct. Collateral trust bonds are secured by securities, such as common shares, other bonds, or other financial assets. B is incorrect because MBS are secured by mortgages. C is incorrect because equipment trust certificates are backed by physical assets, such as aircraft, railroad cars, shipping containers, or oil rigs.

Solution to 7: C is correct. The third-party (or counterparty) risk for a surety bond and a letter of credit arises from both being future promises to pay. In contrast, a cash collateral account allows the issuer to immediately borrow the credit-enhancement amount and then invest it.

Solution to 8: B is correct. Affirmative covenants indicate what the issuer "must do" and are administrative in nature. A covenant requiring the issuer to insure and perform periodic maintenance on financed assets is an example of an affirmative covenant. A and C are incorrect because they are negative covenants; they indicate what the issuer cannot do.

Solution to 9: A is correct. A restriction on debt typically takes the form of a maximum acceptable debt usage ratio or a minimum acceptable interest coverage ratio. Thus, it limits the issuer's ability to issue new debt that would dilute the bondholders' claims. B and C are incorrect because they are covenants that restrict the issuer's business activities by preventing the company from making investments or being taken over, respectively.

3. LEGAL, REGULATORY, AND TAX CONSIDERATIONS

- **describe how legal, regulatory, and tax considerations affect the issuance and trading of fixed-income securities**

Fixed-income securities are subject to different legal and regulatory requirements across jurisdictions, depending on where they are issued and traded as well as who holds them.

An important consideration for investors is where the bonds are issued and traded because it affects the laws and regulations that apply. The *global bond* markets consist of national bond markets and the Eurobond market. A *national bond* market includes all the bonds that are issued and traded in a specific country and are denominated in the currency of that country. Bonds issued by entities that are incorporated in that country are called *domestic bonds*, whereas bonds issued by entities that are incorporated in another country are called *foreign bonds*. If Ford Motor Company issues bonds denominated in US dollars in the United States, for example, these bonds will be classified as domestic. If Volkswagen Group and Toyota Motor Corporation (or their German or Japanese subsidiaries) issue bonds denominated in US dollars in the United States, these bonds will be classified as foreign. Foreign bonds very often receive nicknames. For example, foreign bonds are called "kangaroo bonds" in Australia, "maple bonds" in Canada, "panda bonds" in China, "samurai bonds" in Japan, "kimchi bonds" in South Korea, "matryoshka bonds" in Russia, "matador bonds" in Spain, "bulldog bonds" in the United Kingdom, and "Yankee bonds" in the United States. National regulators may make distinctions both between and among resident and non-resident issuers, and they may have different requirements regarding the issuance process, the level of disclosures, or the restrictions imposed on the bond issuer and/or the investors as to who can purchase the bonds.

Governments and companies have issued foreign bonds in London since the 19th century, and foreign bond issues expanded in such countries as the United States, Japan, and Switzerland during the 1980s. But the 1960s saw the emergence of another bond market: the Eurobond market. The Eurobond market was created primarily to bypass the legal, regulatory, and tax constraints imposed on bond issuers and investors, particularly in the United States. Bonds issued and traded on the Eurobond market are called **Eurobonds**, and they are named after the currency in which they are denominated. For example, Eurodollar and Euroyen bonds are denominated in US dollars and Japanese yens, respectively. Bonds that are denominated in euros are called euro-denominated Eurobonds.

Eurobonds are issued outside the jurisdiction of any single country, are usually unsecured, and may be denominated in any currency, including the issuer's domestic currency. They are underwritten by an international syndicate—that is, a group of financial institutions from different jurisdictions—and mostly sold to investors in Europe, the Middle East, and Asia. Eurobonds denominated in US dollars cannot be sold to US investors at the time of issue because they are not registered with the US Securities and Exchange Commission (SEC). In the past, Eurobonds typically were **bearer bonds**, meaning that the trustee did not keep records of who owned the bonds; only the clearing system knew who the bond owners were. Eurobonds, domestic, and foreign bonds are now **registered bonds** for which ownership is recorded by either name or serial number.

A global bond is one issued simultaneously in the Eurobond market and in at least one domestic bond market, ensuring sufficient demand for large bond issues and access to all fixed-income investors regardless of location. For example, the World Bank is a regular issuer of global bonds. Many market participants refer to foreign bonds, Eurobonds, and global bonds as international bonds as opposed to domestic bonds.

Domestic bonds, foreign bonds, Eurobonds, and global bonds are subject to different legal, regulatory, and tax requirements. They also have different interest payment frequencies and interest payment calculation methods that affect bond cash flows and prices. Note, however, that the currency in which a bond is denominated has a stronger effect on its price than where the bond is issued or traded. This is because market interest rates have a strong influence on a bond's price, and the market interest rates that affect a bond are those associated with the currency in which the bond is denominated.

As the emergence and growth of the Eurobond market illustrates, legal and regulatory considerations affect the dynamics of the global fixed-income markets. Exhibit 2 compares the amounts of total and international debt outstanding for the 15 countries that were the largest debt issuers at the end of December 2019. The reported amounts are based on the residence of the issuer.

EXHIBIT 2 Total and International Debt Securities by Residence of Issuer at the End of December 2019

Issuers	Total Debt Securities (US$ billions)	International Debt Securities (US$ billions)
United States	41,232	2,356
China	14,726	217
Japan	12,825	480
United Kingdom	6,288	3,288
France	4,670	1,433
Germany	3,548	1,290
Canada	3,361	971
Italy	3,194	840
Netherlands	2,193	2,109
Spain	1,972	532
Australia	1,959	595
Luxembourg	1,084	936
Ireland	824	893
Denmark	793	124
Belgium	729	167

Source: Based on data from the Bank for International Settlements, Table C1, available at www.bis.org/statistics/secstats.htm (updated 3 June 2020).

EXAMPLE 3

1. An example of a domestic bond is a bond issued by:
   A. LG Group from South Korea, denominated in British pounds, and sold in the United Kingdom.
   B. the UK Debt Management Office, denominated in British pounds, and sold in the United Kingdom.
   C. Wal-Mart from the United States, denominated in US dollars, and sold in various countries in North America, Europe, the Middle East, and Asia Pacific.
2. A bond issued by Sony in Japan, denominated in US dollars but not registered with the SEC, and sold to an institutional investor in the Middle East, is *most likely* an example of a:
   A. Eurobond.
   B. global bond.
   C. foreign bond.

Solution to 1: B is correct. A domestic bond is issued by a local issuer, denominated in local currency, and sold in the domestic market. Gilts are British pound-denominated bonds issued by the UK Debt Management Office in the United Kingdom. Thus, they are UK domestic bonds. A is incorrect because a bond issued by LG Group from South Korea, denominated in British pounds, and sold in the United Kingdom is an example of a foreign bond. C is incorrect because a bond issued by Wal-Mart from the United States, denominated in US dollars, and sold in various countries in North America, Europe, the Middle East, and Asia Pacific is most likely an example of a global bond, particularly if it is also sold in the Eurobond market.

Solution to 2: A is correct. A Eurobond is a bond that is issued internationally, outside the jurisdiction of any single country. Thus, a bond issued by Sony from Japan, denominated in US dollars but not registered with the SEC, is an example of a Eurobond. B is incorrect because global bonds are bonds that are issued simultaneously in the Eurobond market and in at least one domestic bond market. C is incorrect because if Sony's bond issue were a foreign bond, it would be registered with the SEC.

3.1. Tax Considerations

Bond interest is usually taxed at the ordinary income tax rate, which is typically the same tax rate that an individual would pay on wage or salary income. Tax-exempt securities are the exception to this rule. For example, interest income received by holders of local government bonds, called municipal bonds in the United States, is often exempt from federal income tax and from the income tax of the state in which the bonds are issued. The tax status of bond income may also depend on where the bond is issued and traded. For example, some domestic bonds pay their interest net of income tax. Other bonds, including some Eurobonds, make gross interest payments.

In addition to earnings from interest, a bond investment will generate a capital gain or loss if sold prior to maturity at a price different from the purchase price. This change will generate a capital gain if the bond price has increased or a capital loss if the bond price has decreased. Capital gains or losses usually face different tax treatment from taxable income, which often varies for long-term and short-term capital gains. For example, capital gains recognized over a year after the original bond purchase may be taxed at a lower long-term capital gains tax rate, whereas capital gains recognized within a year of bond purchase may be taxed as a short-term capital gain that equals the ordinary income tax rate. Exceptions exist, and not all countries have a separate capital gains tax. Differences in national and local tax legislation can add further complexity to aggregate country capital gains tax treatment.

For bonds issued at a discount, the tax status of the original issue discount is an additional tax consideration. The original issue discount is the difference between the par value and the original issue price. For example, the United States includes a prorated portion of the discount in interest income every tax year, while Japan does not. Original issue discount tax treatment varies by country, and Exhibit 3 illustrates the potential importance of this tax consideration.

EXHIBIT 3 Original Issue Discount Tax Provision

Assume a hypothetical country, Zinland, where the local currency is the zini (Z). The market interest rate in Zinland is 10%, and both interest income and capital gains are taxed. Companies A and B issue 20-year bonds with a par value of Z1,000. Company A issues a coupon bond with an annual coupon rate of 10%. Investors buy Company A's bonds for Z1,000. Every year, they receive and pay tax on their Z100 annual interest payments. When Company A's bonds mature, bondholders receive the par value of Z1,000. Company B issues a zero-coupon bond at a discount. Investors buy Company B's bonds for Z148.64. They do not receive any cash flows until Company B pays the par value of Z1,000 when the bonds mature.

Company A's bonds and Company B's bonds are economically identical in the sense that they have the same maturity (20 years) and the same yield to maturity (10%). Company A's bonds make periodic payments, however, whereas Company B's bonds defer payment until maturity. Investors in Company A's bonds must include the annual interest payments in taxable income. When they receive their original Z1,000 investment back at maturity, they face no capital gain or loss. Without an original issue discount tax provision, investors in Company B's bonds do not have any taxable income until the bonds mature. When they receive the par value at maturity, they face a capital gain on the original issue discount—that is, on Z851.36 (Z1,000 – Z148.64). The purpose of an original issue discount tax provision is to tax investors in Company B's bonds the same way as investors in Company A's bonds. Thus, a prorated portion of the Z851.36 original issue discount is included in taxable income every tax year until maturity. This allows investors in Company B's bonds to increase their cost basis in the bonds so that at maturity, they face no capital gain or loss.

Some jurisdictions also have tax provisions for bonds bought at a premium. They may allow investors to deduct a prorated portion of the amount paid in excess of the bond's par value from their taxable income every tax year until maturity. For example, if an investor pays $1,005 for a bond that has a par value of $1,000 and matures five years later, she can deduct $1 from her taxable income every tax year for five years. This deduction may be a choice rather than a requirement, with an investor able to decide whether to deduct a prorated portion of the premium each year or to deduct nothing and declare a capital loss when the bond is redeemed at maturity.

EXAMPLE 4

1. The coupon payment is *most likely* to be taxed as:
   A. ordinary income.
   B. short-term capital gain.
   C. long-term capital gain.
2. Assume that a company issues bonds in the hypothetical country of Zinland, where the local currency is the zini (Z). There is an original issue discount tax provision in Zinland's tax code. The company issues a 10-year zero-coupon bond with a par value of Z1,000 and sells it for Z800. An investor who buys the zero-coupon bond at issuance and holds it until maturity *most likely*:
   A. has to include Z20 in his taxable income every tax year for 10 years and has to declare a capital gain of Z200 at maturity.
   B. has to include Z20 in his taxable income every tax year for 10 years and does not have to declare a capital gain at maturity.
   C. does not have to include anything in his taxable income every tax year for 10 years but has to declare a capital gain of Z200 at maturity.

Solution to 1: A is correct. Interest income is typically taxed at the ordinary income tax rate, which may be the same tax rate that individuals pay on wage and salary income.

Solution to 2: B is correct. The original issue discount tax provision requires the investor to include a prorated portion of the original issue discount in his taxable income every tax year until maturity. The original issue discount is the difference between the par value and the original issue price—that is, Z1,000 − Z800 = Z200. The bond's maturity is 10 years. Thus, the prorated portion that must be included each year is Z200 ÷ 10 = Z20. The original issue discount tax provision allows the investor to increase his cost basis in the bond so that when the bond matures, the investor faces no capital gain or loss.

4. PRINCIPAL REPAYMENT STRUCTURES

- **describe how cash flows of fixed-income securities are structured**

The most common payment structure by far is that of a plain vanilla bond, as depicted in Exhibit 1. These bonds make periodic, fixed coupon payments and a lump-sum payment of principal at maturity. But there are other structures regarding both the principal repayment and the interest payments. This section discusses the major schedules observed in the global fixed-income markets. Schedules for principal repayments and interest payments are typically similar for a particular type of bond, such as 30-year US Treasury bonds. However, payment schedules vary considerably between types of bonds, such as government bonds versus corporate bonds.

4.1. Principal Repayment Structures

How the amount borrowed is repaid is an important consideration for investors as it affects credit risk. Any provision that periodically retires some of the principal amount outstanding is a way to reduce credit risk.

4.1.1. Bullet, Fully Amortized, and Partially Amortized Bonds

The payment structure of a plain vanilla bond has been used for nearly every government bond ever issued as well as for most corporate bonds. Such a bond is also known as a **bullet bond** because the entire payment of principal occurs at maturity.

In contrast, an **amortizing bond** has a payment schedule that calls for periodic payments of interest and repayments of principal. A fully amortized bond is characterized by a fixed periodic payment schedule that reduces the bond's outstanding principal amount to zero by the maturity date. A partially amortized bond also makes fixed periodic payments until maturity, but only a portion of the principal is repaid by the maturity date. Thus, a **balloon payment** is required at maturity to retire the bond's outstanding principal amount.

Exhibit 4 illustrates the differences in the payment schedules for a bullet bond, a fully amortized bond, and a partially amortized bond. For the three bonds, the principal amount is $1,000, the maturity is five years, the coupon rate is 6%, and interest payments are made annually. The market interest rate used to discount the bonds' expected cash flows until maturity is assumed to be constant at 6%. The bonds are issued and redeemed at par. For the partially amortized bond, the balloon payment is $200 at maturity.

EXHIBIT 4 Example of Payment Schedules for Bullet, Fully Amortized, and Partially Amortized Bonds

Bullet Bond

Year	Investor Cash Flows	Interest Payment	Principal Repayment	Outstanding Principal at the End of the Year
0	–$1,000.00			$1,000.00
1	60.00	$60.00	$0.00	1,000.00
2	60.00	60.00	0.00	1,000.00
3	60.00	60.00	0.00	1,000.00
4	60.00	60.00	0.00	1,000.00
5	1,060.00	60.00	1,000.00	0.00

Fully Amortized Bond

Year	Investor Cash Flows	Interest Payment	Principal Repayment	Outstanding Principal at the End of the Year
0	–$1,000.00			
1	237.40	$60.00	$177.40	$822.60
2	237.40	49.36	188.04	634.56
3	237.40	38.07	199.32	435.24
4	237.40	26.11	211.28	223.96
5	237.40	13.44	223.96	0.00

Partially Amortized Bond

Year	Investor Cash Flows	Interest Payment	Principal Repayment	Outstanding Principal at the End of the Year
0	–$1,000.00			
1	201.92	$60.00	$141.92	$858.08
2	201.92	51.48	150.43	707.65
3	201.92	42.46	159.46	548.19
4	201.92	32.89	169.03	379.17
5	401.92	22.75	379.17	0.00

Investors pay \$1,000 now to purchase any of the three bonds. For the bullet bond, they receive the coupon payment of \$60 (6% × \$1,000) every year for five years. The last payment is \$1,060 because it includes both the last coupon payment and the principal amount.

For the fully amortized bond, the annual payment, which includes both the coupon payment and the principal repayment, is constant. Thus, this annual payment can be viewed as an annuity. This annuity lasts for five years; its present value, discounted at the market interest rate of 6%, is equal to the bond price of \$1,000. Therefore, the annual payment is \$237.40. The first year, the interest part of the payment is \$60 (6% × \$1,000), which implies that the principal repayment part is \$177.40 (\$237.40 – \$60). This repayment leaves an outstanding principal amount, which becomes the basis for the calculation of the interest the following year, of \$822.60 (\$1,000 – \$177.40). The second year, the interest part of the payment is \$49.36 (6% × \$822.60), the principal repayment part is \$188.04 (\$237.40 – \$49.36), and the outstanding principal amount is \$634.56 (\$822.60 – \$188.04). The fifth year, the outstanding principal amount is fully repaid. Note that the annual payment is constant, but over time the interest payment decreases while the principal repayment increases.

The partially amortized bond can be viewed as the combination of two elements: a five-year annuity plus the balloon payment at maturity. The sum of the present values of these two elements is equal to the bond price of \$1,000. As for the fully amortized bond, the discount rate is the market interest rate of 6%, making the constant amount for the annuity \$201.92. This amount represents the annual payment for the first four years. For Years 1 through 4, the split between interest and principal is done the same way as for the fully amortized bond. The interest part of the payment is equal to 6% multiplied by the outstanding principal at the end of the previous year; the principal repayment part is equal to \$201.92 minus the interest part of the payment for the year; and the outstanding principal amount at the end of the year is equal to the outstanding principal amount at the end of the previous year minus the principal repayment for the year. In Year 5, investors receive \$401.92; this amount is calculated either as the sum of the interest payment (\$22.75) and the outstanding principal amount (\$379.17) or as the constant amount of the annuity (\$201.92) plus the balloon payment (\$200). As for the fully amortized bond, the interest payment decreases and the principal repayment increases over time. Because the principal amount is not fully amortized, interest payments are higher for the partially amortized bond than for the fully amortized bond, except the first year when they are equal.

Exhibit 4 does not address the complexity of the repayment structure for some bonds, such as many ABS. For example, MBS face prepayment risk, which is the possible early repayment of mortgage principal. Borrowers usually have the right to prepay mortgages, which typically occurs when a current homeowner purchases a new home or when homeowners refinance their mortgages because market interest rates have fallen.

EXAMPLE 5

1. The structure that requires the largest repayment of principal at maturity is that of a:
   A. bullet bond.
   B. fully amortized bond.
   C. partially amortized bond.

2. A plain vanilla bond has a maturity of 10 years, a par value of £100, and a coupon rate of 9%. Interest payments are made annually. The market interest rate is assumed to be constant at 9%. The bond is issued and redeemed at par. The principal repayment the first year is *closest* to:
   A. £0.00.
   B. £6.58.
   C. £10.00.
3. Relative to a fully amortized bond, the coupon payments of an otherwise similar partially amortized bond are:
   A. lower or equal.
   B. equal.
   C. higher or equal.

Solution to 1: A is correct. The entire repayment of principal occurs at maturity for a bullet (or plain vanilla) bond, whereas it occurs over time for fully and partially amortized bonds. Thus, the largest repayment of principal at maturity is that of a bullet bond.

Solution to 2: A is correct. A plain vanilla (or bullet) bond does not make any principal repayment until the maturity date. B is incorrect because £6.58 would be the principal repayment for a fully amortized bond.

Solution to 3: C is correct. Except at maturity, the principal repayments are lower for a partially amortized bond than for an otherwise similar fully amortized bond. Consequently, the principal amounts outstanding—and, therefore, the amounts of interest payments—are higher for a partially amortized bond than for a fully amortized bond, all else equal. The only exception is the first interest payment, which is the same for both repayment structures. This is because no principal repayment has been made by the time the first coupon is paid.

4.1.2. Sinking Fund Arrangements

A **sinking fund arrangement** is another approach that can be used to achieve the same goal of periodically retiring the bond's principal outstanding. The term sinking fund refers to an issuer's plans to set aside funds over time to retire the bond. Originally, a sinking fund was a specified cash reserve that was segregated from the rest of the issuer's business for the purpose of repaying the principal. More generally today, a sinking fund arrangement specifies the portion of the bond's principal outstanding, perhaps 5%, that must be repaid each year throughout the bond's life or after a specified date. This repayment occurs whether or not an actual segregated cash reserve has been created.

Typically, the issuer will forward repayment proceeds to the bond's trustee. The trustee will then either redeem bonds to this value or select by lottery the serial numbers of bonds to be paid off. The bonds for repayment may be listed in business newspapers, such as the *Wall Street Journal* or the *Financial Times*.

Another type of sinking fund arrangement operates by redeeming a steadily increasing amount of the bond's notional principal (total amount) each year. Any remaining principal is then redeemed at maturity. It is common to find utility and energy companies in the

United States, the United Kingdom, and the Commonwealth countries that issue bonds with sinking fund arrangements that incorporate such a provision.

Another common variation is for the bond issue to include a call provision, which gives the issuer the option to repurchase the bonds before maturity. The issuer can usually repurchase the bonds at the market price, at par, or at a specified sinking fund price, whichever is the lowest. To allocate the burden of the call provision fairly among bondholders, the bonds to be retired are selected at random based on serial number. Usually, the issuer can repurchase only a small portion of the bond issue. Some indentures, however, allow issuers to use a doubling option to repurchase double the required number of bonds.

Sinking fund arrangements have several benefits and drawbacks. The benefits include a structure that ensures a formal plan exists for debt retirement and also reduces credit risk due to a reduction of default risk at maturity. Sinking fund drawbacks include investor reinvestment risk, which is the risk associated with having to reinvest cash flows at an interest rate below the current yield-to-maturity. If the serial number of an investor's bonds is selected, the bonds will be repaid and the investor will have to reinvest the proceeds. If market interest rates have fallen since the investor purchased the bonds, the investor probably will not be able to purchase a bond offering the same return. Another potential disadvantage for investors occurs if the issuer has the option to repurchase bonds at below market prices. For example, an issuer could exercise a call option to buy back bonds at par on bonds priced above par. In this case, investors would suffer a loss.

Exhibit 5 illustrates an example of a sinking fund arrangement.

EXHIBIT 5 Example of a Sinking Fund Arrangement

The notional principal of the bond issue is £200 million. The sinking fund arrangement calls for 5% of the outstanding principal amount to be retired in Years 10 through 19, with the outstanding balance paid off at maturity in 20 years.

Year	Outstanding Principal at the Beginning of the Year (£ millions)	Sinking Fund Payment (£ millions)	Outstanding Principal at the End of the Year (£ millions)	Final Principal Repayment (£ millions)
0			200.00	
1 to 9	200.00	0.00	200.00	
10	200.00	10.00	190.00	
11	190.00	9.50	180.50	
12	180.50	9.03	171.48	
13	171.48	8.57	162.90	
14	162.90	8.15	154.76	
15	154.76	7.74	147.02	
16	147.02	7.35	139.67	
17	139.67	6.98	132.68	
18	132.68	6.63	126.05	
19	126.05	6.30	119.75	
20	119.75			119.75

EXHIBIT 5 (Continued)

There is no repayment of the principal during the first nine years. Starting the 10th year, the sinking fund arrangement calls for 5% of the outstanding principal amount to be retired each year. In Year 10, £10 million (5% × £200 million) are paid off, which leaves an outstanding principal balance of £190 million. In Year 11, the principal amount repaid is £9.50 million (5% × £190 million). The final repayment of the remaining balance (£119.75 million) is a balloon payment at maturity.

5. COUPON PAYMENT STRUCTURES

- **describe how cash flows of fixed-income securities are structured**

A coupon is the interest payment that the bond issuer makes to the bondholder. A conventional bond pays a fixed periodic coupon over a specified time to maturity. Most frequently, the coupon is paid semi-annually for sovereign and corporate bonds; this is the case in the United States, the United Kingdom, and such Commonwealth countries as Bangladesh, India, and New Zealand. Eurobonds usually pay an annual coupon, although some Eurobonds make quarterly coupon payments. Most bonds issued in the eurozone have an annual coupon.

Fixed-rate coupons are not the only coupon payment structure, however. A wide range of coupon types is offered in the global fixed-income markets to meet the differing needs of both issuers and investors.

5.1. Floating-Rate Notes

Floating-rate notes do not have a fixed coupon; instead, their coupon rate is linked to an external reference rate, such as Euribor. Thus, an FRN's interest rate will fluctuate periodically during the bond's life, following the changes in the reference rate. Therefore, the FRN's cash flows are not known with certainty. Large issuers of FRNs include government-sponsored enterprises (GSEs), such as the Federal Home Loan Banks (FHLB), the Federal National Mortgage Association ("Fannie Mae"), and the Federal Home Loan Mortgage Corporation ("Freddie Mac") in the United States, as well as banks and financial institutions in Europe and Asia Pacific. It is rare for national governments to issue FRNs because investors in sovereign bonds generally prefer fixed coupon bonds.

Almost all FRNs have quarterly coupons, although counterexamples do exist. FRNs usually pay a fixed spread over the specified reference rate. A typical coupon rate may be the three-month US dollar MRR + 20 bps (i.e., MRR + 0.20%) for a US dollar-denominated bond or the three-month Euribor + 20 bps for a euro-denominated FRN.

Contrary to plain vanilla, which are fixed-rate securities that decline in value in a rising interest rate environment, FRNs are less affected when interest rates increase because their coupon rates vary with market interest rates and are reset at regular, short-term intervals. Thus, FRNs have little interest rate risk—that is, the risk that a change in market interest rate affects a bond's value. FRNs are frequently favored by investors who expect that interest rates will rise. That said, investors still face credit risk when investing in FRNs. If an issuer's credit risk does not change from one coupon reset date to the next, the FRN's price generally will stay close to the par value. However, if there is a change in the issuer's credit quality that affects the perceived credit risk associated with the bond, the price of the FRN will deviate from its par value. A higher level of credit risk will lead to a lower price.

Additional features observed in FRNs may include a floor or a cap. A floor (floored FRN) prevents the coupon from falling below a specified minimum rate. This feature benefits the bondholders, who are guaranteed that the interest rate will not fall below the specified rate during a time of falling interest rates. In contrast, a cap (capped FRN) prevents the coupon from rising above a specified maximum rate. This feature benefits the issuer because it sets a limit to the interest rate paid on the debt during a time of rising interest rates. It is also possible to have a collared FRN, which includes both a cap and a floor.

An inverse or reverse FRN, or simply an inverse floater, is a bond whose coupon rate has an inverse relationship to the reference rate. The basic structure is the same as an ordinary FRN except for the direction in which the coupon rate is adjusted. When interest rates fall, the coupon rate on an ordinary FRN decreases; in contrast, the coupon rate on a reverse FRN increases. Thus, inverse FRNs are typically favored by investors who expect interest rates to decline.

5.2. Step-Up Coupon Bonds

The coupon of a **step-up coupon bond**, which may be fixed or floating, increases by specified margins at specified dates. An example of a bond with a step-up coupon is a ten-year callable bond issued by the Federal Home Loan Bank on 3 August 2016. The initial coupon rate was 1.25% and steps up to 1.50% on 3 August 2018, to 2.00% on 3 August 2020, to 2.50% on 3 August 2022, to 3.00% on 3 August 2023, to 4.00% on 3 August 2024, and finally to 6.00% on 3 August 2025 for the final year. The bond was first callable at par on 3 August 2018, at the time of the first step up.

Bonds with step-up coupons offer bondholders some protection against rising interest rates, and they may be an important feature for callable bonds. When interest rates increase, there is a higher likelihood that the issuer will not call the bonds, particularly if the bonds have a fixed rate of interest. The step-up coupon allows bondholders to receive a higher coupon, in line with the higher market interest rates. Alternatively, when interest rates decrease or remain stable, the step-up feature acts as an incentive for the issuer to call the bond before the coupon rate increases and the interest expense rises.

Redeeming the bond when the coupon increases is not automatic, however; the issuer may choose to keep the bond despite its increasing cost. This may happen if refinancing the bond is necessary and alternatives are less advantageous for this issuer. For example, a financial crisis may make it difficult for the issuer to refinance. Alternatively, the issuer's credit quality may have deteriorated, which would lead to a higher yield, potentially making the coupon rate on the new bond more expensive than that on the existing bond despite the stepped-up coupon. Although the issuer does not have to call the bond, there is an implicit expectation from investors that it will do so if the market price for the bond is above the call price. Failure to do so may be viewed negatively by market participants and reduce investors' appetite for that issuer's bonds in the future.

5.3. Credit-Linked Coupon Bonds

A **credit-linked coupon bond** has a coupon that changes when the bond's credit rating changes. An example of a bond with a credit-linked coupon is one of British Telecom's bonds maturing in 2030. It was issued with a coupon rate of 8.625%, with a 25 bp increase in coupon for each credit rating downgrade below the bond's credit rating at the time of issuance and a 25 bp decrease for every credit rating upgrade above the bond's credit rating at the time of issuance.

Bonds with credit-linked coupons are attractive to investors who are concerned about the future creditworthiness of the issuer. They may also provide some general protection against a poor economy because credit ratings tend to decline the most during recessions. A potential problem associated with these bonds is that increases in the coupon payments resulting from a downgrade may ultimately result in further deteriorations of the credit rating or even contribute to the issuer's default.

5.4. Payment-in-Kind Coupon Bonds

A payment-in-kind (PIK) coupon bond typically allows the issuer to pay interest in the form of additional amounts of the bond issue rather than as a cash payment. Such bonds are favored by issuers who are concerned that the issuer may face potential cash flow problems in the future. They are used, for example, in financing companies that have a high debt burden, such as companies going through a leveraged buyout (a form of acquisition in which the financing consists primarily of debt). Investors usually demand a higher yield for holding bonds with PIK coupons in exchange for assuming their additional credit risk.

Other forms of PIK arrangements can also be found, such as paying the bondholders with common shares worth the amount of coupon due. With a PIK "toggle" note, the borrower has the option, for each interest period, to pay interest in cash, to make the interest payment in kind, or some mix of the two. Cash payments or payments in kind are frequently at the discretion of the borrower, but whether the payment is made in cash or in kind can be determined by an earnings or cash flow trigger identified in the indenture.

5.5. Deferred Coupon Bonds

A **deferred coupon bond**, sometimes called a **split coupon bond**, pays no coupons for its first few years but then pays a higher coupon than it otherwise normally would for the remainder of its life. Issuers of deferred coupon bonds are usually seeking ways to conserve cash in the years immediately following the bond issue, which may indicate poorer credit quality. Deferred coupon bonds are also common in project financing when the assets being developed do not generate any income during the development phase. A deferred coupon bond allows the issuer to delay interest payments until the project is completed, and the cash flows generated by the assets being financed can be used to service the debt.

One of the main advantages of investing in a deferred coupon bond is that these bonds are typically priced at significant discounts to par. Investors may also find the deferred coupon structure to be very helpful in managing taxes. If taxes due on the interest income can be delayed, investors may be able to minimize taxes. This tax advantage, however, depends on the jurisdiction concerned and how its tax rules apply to deferred coupon payments.

A zero-coupon bond can be thought of as an extreme form of deferred coupon bond. These securities pay no interest to the investor and thus are issued at a deep discount to par value. At maturity, the bondholder receives the par value of the bond as payment. Effectively, a zero-coupon bond defers all interest payments until maturity.

5.6. Index-Linked Bonds

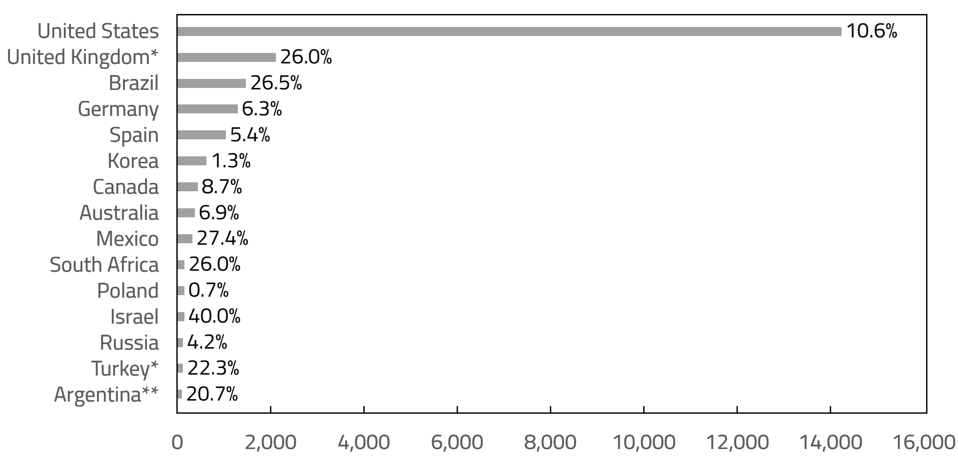
An **index-linked bond** has its coupon payments and/or principal repayment linked to a specified index. In theory, a bond can be indexed to any published variable, including an index reflecting prices, earnings, economic output, commodities, or foreign currencies. **Inflation-linked bonds** are an example of index-linked bonds. They offer investors protection against inflation by linking a bond's coupon payments and/or the principal repayment to an index of consumer prices, such as the UK Retail Price Index (RPI) or the US Consumer Price Index (CPI). The advantage of using the RPI or CPI is that these indexes are well-known, transparent, and published regularly.

Governments are large issuers of inflation-linked bonds, also called **linkers**. The United Kingdom was one of the first developed countries to issue inflation-linked bonds in 1981, offering gilts linked to the UK RPI, its main measure of the rate of inflation. In 1997, the US Treasury introduced Treasury Inflation-Protected Securities (TIPS) linked to the US Consumer Price Index (CPI). Inflation-linked bonds are now more frequently being offered by corporate issuers, including both financial and non-financial companies.

A bond's stated coupon rate represents the nominal interest rate received by the bondholders. But inflation reduces the actual value of the interest received. The interest rate that bondholders actually receive, net of inflation, is the real interest rate; it is approximately equal to the nominal interest rate minus the rate of inflation. By increasing the coupon payments and/or the principal repayment in line with increases in the price index, inflation-linked bonds reduce inflation risk. An example of an inflation-linked bond is the 1% US TIPS that matures on 15 February 2048. The coupon rate remains fixed at 1%, but the principal is adjusted every six months for changes in the CPI.

Exhibit 6 shows some of the national governments that have issued large amounts of inflation-linked bonds. It displays the amount of the linkers and the percentage of the total debt for each country. Sovereign issuers can be grouped into three categories. The first category includes such countries as Brazil and Argentina, who have issued inflation-linked bonds because they were experiencing extremely high rates of inflation when borrowing. Thus, offering inflation-linked bonds was their only available alternative to raise funds. The second category includes the United Kingdom and Australia. These countries have issued inflation-linked bonds to not only add credibility to the government's commitment to disinflationary policies but also to capitalize on investor demand. The third category, which includes the United States, Canada, and Germany, consists of national governments that are most concerned about the social welfare benefits associated with inflation-linked securities. Theoretically, inflation-linked bonds provide investors the benefit of a long-term asset with a fixed real return that is free from inflation risk.

EXHIBIT 6 Government Inflation-Linked Debt in Billions of USD as of Q4 2019 and the Percentage of Total Debt



* Denotes latest data from 2017.

** Denotes latest data from 2018.

Source: Based on data from the Bank for International Settlements, Table C2, available at www.bis.org/statistics/secstats.htm (updated 3 June 2020).

Different methods have been used for linking the cash flows of an index-linked bond to a specified index; the link can be made via the interest payments, the principal repayment, or both. The following examples describe how the link between the cash flows and the index is established, using inflation-linked bonds as an illustration.

- *Zero-coupon-indexed bonds* pay no coupon, so the inflation adjustment is made via the principal repayment only: The principal amount to be repaid at maturity increases in line with increases in the price index during the bond's life. This type of bond has been issued in Sweden.
- *Interest-indexed bonds* pay a fixed nominal principal amount at maturity but an index-linked coupon during the bond's life. Thus, the inflation adjustment applies to the interest payments only. This is essentially a floating-rate note in which the reference rate is the inflation rate instead of an MRR, such as Euribor. These have been issued by insurance companies and major commercial banks but not typically by governments.
- *Capital-indexed bonds* pay a fixed coupon rate, but it is applied to a principal amount that increases in line with increases in the index during the bond's life. Thus, both the interest payments and the principal repayment are adjusted for inflation. Such bonds have been issued by governments in Australia, Canada, New Zealand, the United Kingdom, and the United States.
- *Indexed-annuity bonds* are fully amortized bonds, in contrast to interest-indexed and capital-indexed bonds that are non-amortizing coupon bonds. The annuity payment, which includes both payment of interest and repayment of the principal, increases in line with inflation during the bond's life. Indexed-annuity bonds linked to a price index have been issued by local governments in Australia but not by the national government.

Exhibit 7 illustrates the different methods used for inflation-linked bonds.

EXHIBIT 7 Examples of Inflation-Linked Bonds

Assume a hypothetical country, Lemuria, where the currency is the lemming (L). The country issued 20-year bonds linked to the domestic Consumer Price Index (CPI). The bonds have a par value of L1,000. Lemuria's economy has been free of inflation until the most recent six months, when the CPI increased by 5%.

Suppose that the bonds are zero-coupon-indexed bonds. There will never be any coupon payments. Following the 5% increase in the CPI, the principal amount to be repaid increases to L1,050 [L1,000 × (1 + 0.05)] and will continue increasing in line with inflation until maturity.

Now, suppose that the bonds are coupon bonds that make semi-annual interest payments based on an annual coupon rate of 4%. If the bonds are interest-indexed bonds, the principal amount at maturity will remain L1,000 regardless of the CPI level during the bond's life and at maturity. The coupon payments, however, will be adjusted for inflation. Prior to the increase in inflation, the semi-annual coupon payment was L20 [(0.04 × L1,000) ÷ 2]. Following the 5% increase in the CPI, the semi-annual coupon payment increases to L21 [L20 × (1 + 0.05)]. Future coupon payments will also be adjusted for inflation.

If the bonds are capital-indexed bonds, the annual coupon rate remains 4%, but the principal amount is adjusted for inflation and the coupon payment is based on the inflation-adjusted principal amount. Following the 5% increase in the CPI, the inflation-adjusted principal amount increases to L1,050 [L1,000 × (1 + 0.05)], and the new semi-annual coupon payment is L21 [(0.04 × L1,050) ÷ 2]. The principal amount will continue increasing in line with increases in the CPI until maturity, and so will the coupon payments.

(*continued*)

EXHIBIT 7 (Continued)

If the bonds are indexed-annuity bonds, they are fully amortized. Prior to the increase in inflation, the semi-annual payment was L36.56—the annuity payment based on a principal amount of L1,000 paid back in 40 semi-annual payments with an annual discount rate of 4%. Following the 5% increase in the CPI, the annuity payment increases to L38.38 [L36.56 × (1 + 0.05)]. Future annuity payments will also be adjusted for inflation in a similar manner.

EXAMPLE 6

1. Floating-rate notes *most likely* pay:
   A. annual coupons.
   B. quarterly coupons.
   C. semi-annual coupons.
2. A zero-coupon bond can *best* be considered a:
   A. step-up bond.
   B. credit-linked bond.
   C. deferred coupon bond.
3. The bonds that do **not** offer protection to the investor against increases in market interest rates are:
   A. step-up bonds.
   B. floating-rate notes.
   C. inverse floating-rate notes.
4. The US Treasury offers Treasury Inflation-Protected Securities (TIPS). The principal of TIPS increases with inflation and decreases with deflation based on changes in the US Consumer Price Index. When TIPS mature, an investor is paid the original principal or inflation-adjusted principal, whichever is greater. TIPS pay interest twice a year based on a fixed real coupon rate that is applied to the inflation-adjusted principal. TIPS are *most likely*:
   A. capital-indexed bonds.
   B. interest-indexed bonds.
   C. indexed-annuity bonds.
5. Assume a hypothetical country, Lemuria, where the national government has issued 20-year capital-indexed bonds linked to the domestic Consumer Price Index (CPI). Lemuria's economy has been free of inflation until the most recent six months, when the CPI increased. Following the increase in inflation:
   A. the principal amount remains unchanged but the coupon rate increases.
   B. the coupon rate remains unchanged, but the principal amount increases.
   C. the coupon payment remains unchanged, but the principal amount increases.

Solution to 1: B is correct. Most FRNs pay interest quarterly and are tied to a three-month MRR.

Solution to 2: C is correct. Because interest is effectively deferred until maturity, a zero-coupon bond can be thought of as a deferred coupon bond. A and B are incorrect because both step-up bonds and credit-linked bonds pay regular coupons. For a step-up bond, the coupon increases by specified margins at specified dates. For a credit-linked bond, the coupon changes when the bond's credit rating changes.

Solution to 3: C is correct. The coupon rate on an inverse FRN has an inverse relationship to the market reference rate. Thus, an inverse FRN does not offer protection to the investor when market interest rates increase but rather when they decrease. A and B are incorrect because step-up bonds and FRNs both offer protection against increases in market interest rates.

Solution to 4: A is correct. TIPS have a fixed coupon rate, and the principal is adjusted based on changes in the CPI. Thus, TIPS are an example of capital-indexed bonds. B is incorrect because with an interest-indexed bond, it is the principal repayment at maturity that is fixed and the coupon that is linked to an index. C is incorrect because indexed-annuity bonds are fully amortized bonds, not bullet bonds. The annuity payment (interest payment and principal repayment) is adjusted based on changes in an index.

Solution to 5: B is correct. Following an increase in inflation, the coupon rate of a capital-indexed bond remains unchanged, but the principal amount is adjusted upward for inflation. Thus, the coupon payment, which is equal to the fixed coupon rate multiplied by the inflation-adjusted principal amount, increases.

6. CALLABLE AND PUTABLE BONDS

- **describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender**

A contingency refers to some future event or circumstance that is possible but not certain. A **contingency provision** is a clause in a legal document that allows for some action if the event or circumstance does occur. For bonds, the term **embedded option** refers to various contingency provisions found in the indenture. These contingency provisions provide the issuer or the bondholders the right, but not the obligation, to take some action. These rights are called "options." These options are not independent of the bond and cannot be traded separately—hence the term "embedded." Some common types of bonds with embedded options include callable bonds, putable bonds, and convertible bonds. The options embedded in these bonds grant either the issuer or the bondholders certain rights affecting the disposal or redemption of the bond.

6.1. Callable Bonds

The most widely used embedded option is the call provision. A **callable bond** gives the issuer the right to redeem all or part of the bond before the specified maturity date. The primary reason why issuers choose to issue callable bonds rather than non-callable bonds is to protect themselves against a decline in interest rates. This decline can come either from market interest rates falling or from the issuer's credit quality improving. If market interest rates fall or credit quality improves, the issuer of a callable bond has the right to replace an old, expensive bond issue with a new, cheaper bond issue. In other words, the issuer can benefit from a decline in interest rates by being able to refinance its debt at a lower interest rate. For example, assume that the market interest rate was 6% at the time of issuance and that a company issued a bond with a coupon rate of 7%—the market interest rate plus a spread of 100 bps. Now assume that the market interest rate has fallen to 4% and that the company's creditworthiness has not changed; it can still issue at the market interest rate plus 100 bps. If the original bond is callable, the company can redeem it and replace it with a new bond paying 5% annually. If the original bond is non-callable, the company must carry on paying 7% annually and cannot benefit from the decline in market interest rates.

As illustrated in this example, callable bonds are advantageous to the issuer of the security. Put another way, the call option has value to the *issuer*. Callable bonds present investors with a higher level of reinvestment risk than non-callable bonds; that is, if the bonds are called, bondholders must reinvest funds in a lower interest rate environment. For this reason, callable bonds have to offer a higher yield and sell at a lower price than otherwise similar non-callable bonds. The higher yield and lower price compensate the bondholders for the value of the call option to the issuer.

Callable bonds have a long tradition and are commonly issued by corporate issuers. Although first issued in the US market, they are now frequently issued in every major bond market and in a variety of forms.

The details about the call provision are specified in the indenture. These details include the call price, which represents the price paid to bondholders when the bond is called. The call premium is the amount over par paid by the issuer if the bond is called. There may be restrictions on when the bond can be called, or the bond may have different call prices depending on when it is called. The call schedule specifies the dates and prices at which a bond may be called. Some callable bonds are issued with a *call protection period*, also called lockout period, cushion, or deferment period. The call protection period prohibits the issuer from calling a bond early in its life and is often added as an incentive for investors to buy the bond. The earliest time that a bond might be called is known as the call date.

Make-whole calls first appeared in the US corporate bond market in the mid-1990s and have become more commonplace ever since. A typical make-whole call requires the issuer to make a lump-sum payment to the bondholders based on the present value of the future coupon payments and outstanding principal due to early bond redemption. The discount rate used is usually a pre-determined spread over the yield-to-maturity of an appropriate sovereign bond. The typical result is a redemption value that is significantly greater than the bond's current market price. A make-whole call provision is less detrimental to bondholders than a regular call provision because it allows them to be compensated if the issuer calls the bond. Issuers, however, rarely invoke this provision because redeeming a bond that includes a make-whole provision before the maturity date is costly. Issuers tend to include a make-whole provision as a "sweetener" to make the bond issue more attractive to potential buyers and allow them to pay a lower coupon rate.

Available exercise styles on callable bonds include the following:

- *American-style* call, sometimes referred to as continuously callable, for which the issuer has the right to call a bond at any time starting on the first call date.
- *European-style* call, for which the issuer has the right to call a bond only once on the call date.
- *Bermuda-style* call, for which the issuer has the right to call bonds on specified dates following the call protection period. These dates frequently correspond to coupon payment dates.

EXAMPLE 7

Assume a hypothetical 30-year bond is issued on 15 August 2019 at a price of 98.195 (as a percentage of par). Each bond has a par value of $1,000. The bond is callable in whole or in part every 15 August from 2029 at the option of the issuer. The call prices are shown below.

Year	Call Price	Year	Call Price
2029	103.870	2035	101.548
2030	103.485	2036	101.161
2031	103.000	2037	100.774
2032	102.709	2038	100.387
2033	102.322	2039 and thereafter	100.000
2034	101.955		

1. The call protection period is:
   A. 10 years.
   B. 11 years.
   C. 20 years.
2. The call premium (per $1,000 in par value) in 2033 is *closest* to:
   A. $2.32.
   B. $23.22.
   C. $45.14.
3. The call provision is *most likely*:
   A. a Bermuda call.
   B. a European call.
   C. an American call.

Solution to 1: A is correct. The bonds were issued in 2019 and are first callable in 2029. The call protection period is 2029 – 2019 = 10 years.

Solution to 2: B is correct. The call prices are stated as a percentage of par. The call price in 2033 is $1,023.22 (102.322% × $1,000). The call premium is the amount paid above par by the issuer. The call premium in 2033 is $23.22 ($1,023.22 – $1,000).

Solution to 3: A is correct. The bond is callable every 15 August from 2029—that is, on specified dates following the call protection period. Thus, the embedded option is a Bermuda call.

6.2. Putable Bonds

A put provision gives the bondholders the right to sell the bond back to the issuer at a predetermined price on specified dates. **Putable bonds** are beneficial for the bondholder by guaranteeing a pre-specified selling price at the redemption dates. If interest rates rise after issuance and bond prices fall, the bondholders can put the bond back to the issuer and reinvest the proceeds in bonds that offer higher yields, in line with higher market interest rates.

Because a put provision has value to the *bondholders*, the price of a putable bond will be higher than the price of an otherwise similar bond issued without the put provision. Similarly, the yield on a bond with a put provision will be lower than the yield on an otherwise similar non-putable bond. The lower yield compensates the issuer for the value of the put option to the investor.

The indenture lists the redemption dates and the prices applicable to the sale of the bond back to the issuer. The selling price is usually the par value of the bond. Depending on the terms set out in the indenture, putable bonds may allow buyers to force a sellback only once or multiple times during the bond's life. Putable bonds that incorporate a single sellback opportunity include a European-style put and are often referred to as one-time put bonds. Putable bonds that allow multiple sellback opportunities include a Bermuda-style put and are known as multiple put bonds. Multiple put bonds offer more flexibility for investors, so they are generally more expensive than one-time put bonds.

Typically, putable bonds incorporate one- to five-year put provisions. Their increasing popularity has often been motivated by investors wanting to protect themselves against major declines in bond prices. One benefit of this rising popularity has been an improvement in liquidity in some markets, because the put protection attracts more conservative classes of investors. The global financial crisis that started in 2008 showed that these securities can often exacerbate liquidity problems, however, because they provide a first claim on the issuer's assets. The put provision gives bondholders the opportunity to convert their claim into cash before other creditors.

7. CONVERTIBLE BONDS

- **describe contingency provisions affecting the timing and/or nature of cash flows of fixed-income securities and whether such provisions benefit the borrower or the lender**

A **convertible bond** is a hybrid security with both debt and equity features. It gives the bondholder the right to exchange the bond for a specified number of common shares in the issuing company. Thus, a convertible bond can be viewed as the combination of a straight bond (option-free bond) plus an embedded equity call option. Convertible bonds can also include additional provisions, the most common being a call provision.

From the investor's perspective, a convertible bond offers several advantages relative to a non-convertible bond. First, it gives the bondholder the ability to convert into equity in case of share price appreciation, and thus participate in the equity upside. At the same time, the bondholder receives downside protection; if the share price does not appreciate, the convertible bond offers the comfort of regular coupon payments and the promise of principal repayment at maturity. Even if the share price and thus the value of the equity call option decline, the price of a convertible bond cannot fall below the price of the straight bond. Consequently, the value of the straight bond acts as a floor for the price of the convertible bond.

Because the conversion provision is valuable to *bondholders*, the price of a convertible bond is higher than the price of an otherwise similar bond without the conversion provision. Similarly, the yield on a convertible bond is lower than the yield on an otherwise similar non-convertible bond. However, most convertible bonds offer investors a yield advantage; the coupon rate on the convertible bond is typically higher than the dividend yield on the underlying common share.

From the issuer's perspective, convertible bonds offer two main advantages. The first is reduced interest expense. Issuers are usually able to offer below-market coupon rates due to the conversion feature's value. The second advantage is the elimination of debt if the conversion option is exercised, but this is dilutive to existing shareholders.

Key terms regarding the conversion provision include the following:

- The **conversion price** is the price per share at which the convertible bond can be converted into shares.
- The **conversion ratio** is the number of common shares that each bond can be converted into. The indenture sometimes does not stipulate the conversion ratio but only mentions the conversion price. The conversion ratio is equal to the par value divided by the conversion price. For example, if the par value is €1,000 and the conversion price is €20, the conversion ratio is €1,000 ÷ €20 = 50:1, or 50 common shares per bond.
- The **conversion value**, sometimes called the parity value, is the current share price multiplied by the conversion ratio. For example, if the current share price is €33 and the conversion ratio is 30:1, the conversion value is €33 × 30 = €990.
- The **conversion premium** is the difference between the convertible bond's price and its conversion value. For example, if the convertible bond's price is €1,020 and the conversion value is €990, the conversion premium is €1,020 – €990 = €30.
- Conversion parity occurs if the conversion value is equal to the convertible bond's price. Using the previous two examples, if the current share price is €34 instead of €33, then both the convertible bond's price and the conversion value are equal to €1,020 (i.e., a conversion premium equal to 0). This condition is referred to as parity. If the common share is selling for less than €34, the condition is below parity. In contrast, if the common share is selling for more than €34, the condition is above parity.

Generally, convertible bonds have maturities of five to ten years. First-time or newer issuers are usually able to issue convertible bonds of up to three years in maturity only. Although it is common for convertible bonds to reach conversion parity before maturity, bondholders rarely exercise the conversion option before that time. Early conversion would eliminate the yield advantage of continuing to hold the convertible bond; investors would typically receive in dividends less than they would receive in coupon payments. For this reason, it is common to find convertible bonds that are also callable by the issuer on a set of specified dates. If the convertible bond includes a call provision and the conversion value is above the convertible bond price, the issuer may force the bondholders to convert their bonds into common shares before maturity. For this reason, callable convertible bonds have to offer a higher yield and sell at a lower price than otherwise similar non-callable convertible bonds. Some indentures specify that the bonds can be called only if the share price exceeds a specified price, giving investors more predictability about the share price at which the issuer may force conversion.

A **warrant** is an "attached" rather than embedded option entitling the holder to buy the underlying stock of the issuing company at a fixed exercise price until the expiration date. Warrants are considered yield enhancements; they are frequently attached to bond issues as a "sweetener." Warrants are actively traded in some financial markets, such as the Deutsche Börse and the Hong Kong Stock Exchange.

Several European banks have issued a type of convertible bond called contingent convertible bonds. **Contingent convertible bonds**, nicknamed "CoCos," are bonds with contingent write-down provisions. Two main features distinguish bonds with contingent write-down provisions from the traditional convertible bonds just described. A traditional convertible bond is convertible at the option of the bondholder, and conversion occurs on the upside—that is, if the issuer's share price increases. In contrast, bonds with contingent write-down provisions are convertible on the downside. In the case of CoCos, conversion is automatic if a specified event occurs—for example, if the bank's core Tier 1 capital ratio (a measure of the bank's proportion of core equity capital available to absorb losses) falls below the minimum requirement set by the regulators. Thus, if the bank experiences losses that reduce its equity capital below the minimum requirement, CoCos are a way to reduce the bank's likelihood of default and, therefore, systemic risk—that is, the risk of failure of the financial system. When the bank's core Tier 1 capital falls below the minimum requirement, the CoCos immediately convert into equity, automatically recapitalizing the bank, lightening the debt burden, and reducing the risk of default. Because the conversion is not at the option of the bondholders but automatic, CoCos force bondholders to take losses. For this reason, CoCos must offer a higher yield than otherwise similar bonds.

EXAMPLE 8

1. Which of the following is **not** an example of an embedded option?
   A. Warrant
   B. Call provision
   C. Conversion provision
2. The type of bond with an embedded option that would *most likely* sell at a lower price than an otherwise similar bond without the embedded option is a:
   A. putable bond.
   B. callable bond.
   C. convertible bond.
3. The additional risk inherent to a callable bond is *best* described as:
   A. credit risk.
   B. interest rate risk.
   C. reinvestment risk.
4. The put provision of a putable bond:
   A. limits the risk to the issuer.
   B. limits the risk to the bondholder.
   C. does not materially affect the risk of either the issuer or the bondholder.
5. Assume that a convertible bond issued in South Korea has a par value of ₩1,000,000 and is currently priced at ₩1,100,000. The underlying share price is ₩40,000, and the conversion ratio is 25:1. The conversion condition for this bond is:
   A. parity.
   B. above parity.
   C. below parity.

Solution to 1: A is correct. A warrant is a separate, tradable security that entitles the holder to buy the underlying common share of the issuing company. B and C are incorrect because the call provision and the conversion provision are embedded options.

Solution to 2: B is correct. The call provision is an option that benefits the issuer. Because of this, callable bonds sell at lower prices and higher yields relative to otherwise similar non-callable bonds. A and C are incorrect because the put provision and the conversion provision are options that benefit the investor. Thus, putable bonds and convertible bonds sell at higher prices and lower yields relative to otherwise similar bonds that lack those provisions.

Solution to 3: C is correct. Reinvestment risk refers to the effect that lower interest rates have on available rates of return when reinvesting the cash flows received from an earlier investment. Because bonds are typically called following a decline in market interest rates, reinvestment risk is particularly relevant for the holder of a callable bond. A is incorrect because credit risk refers to the risk of loss resulting from the issuer failing to make full and timely payments of interest and/or repayments of principal. B is incorrect because interest rate risk is the risk that a change in market interest rate affects a bond's value.

Solution to 4: B is correct. A putable bond limits the risk to the bondholder by guaranteeing a pre-specified selling price at the redemption dates.

Solution to 5: C is correct. The conversion value of the bond is ₩40,000 × 25 = ₩1,000,000. The price of the convertible bond is ₩1,100,000. Thus, the conversion value of the bond is less than the bond's price, and this condition is referred to as below parity.

SUMMARY

This chapter introduces the salient features of fixed-income securities while noting how these features vary among different types of securities. Important points include the following:

- The three important elements that an investor needs to know when investing in a fixed-income security are: (1) the bond's features, which determine its scheduled cash flows and thus the bondholder's expected and actual return; (2) the legal, regulatory, and tax considerations that apply to the contractual agreement between the issuer and the bondholders; and (3) the contingency provisions that may affect the bond's scheduled cash flows.
- The basic features of a bond include the issuer, maturity, par value (or principal), coupon rate and frequency, and currency denomination.
- Issuers of bonds include supranational organizations, sovereign governments, non-sovereign governments, quasi-government entities, and corporate issuers.
- Bondholders are exposed to credit risk and may use bond credit ratings to assess the credit quality of a bond.
- A bond's principal is the amount the issuer agrees to pay the bondholder when the bond matures.
- The coupon rate is the interest rate that the issuer agrees to pay to the bondholder each year. The coupon rate can be a fixed rate or a floating rate. Bonds may offer annual, semi-annual, quarterly, or monthly coupon payments depending on the type of bond and where the bond is issued.
- Bonds can be issued in any currency. Such bonds as dual-currency bonds and currency option bonds are connected to two currencies.

- The yield-to-maturity is the discount rate that equates the present value of the bond's future cash flows until maturity to its price. Yield-to-maturity can be considered an estimate of the market's expectation for the bond's return.
- A plain vanilla bond has a known cash flow pattern. It has a fixed maturity date and pays a fixed rate of interest over the bond's life.
- The bond indenture or trust deed is the legal contract that describes the form of the bond, the issuer's obligations, and the investor's rights. The indenture is usually held by a financial institution called a trustee, which performs various duties specified in the indenture.
- The issuer is identified in the indenture by its legal name and is obligated to make timely payments of interest and repayment of principal.
- For asset-backed securities, the legal obligation to repay bondholders often lies with a separate legal entity—that is, a bankruptcy-remote vehicle that uses the assets as guarantees to back a bond issue.
- How the issuer intends to service the debt and repay the principal should be described in the indenture. The source of repayment proceeds varies depending on the type of bond.
- Collateral backing is a way to alleviate credit risk. Secured bonds are backed by assets or financial guarantees pledged to ensure debt payment. Examples of collateral-backed bonds include collateral trust bonds, equipment trust certificates, mortgage-backed securities, and covered bonds.
- Credit enhancement can be internal or external. Examples of internal credit enhancement include subordination, overcollateralization, and reserve accounts. A bank guarantee, a surety bond, a letter of credit, and a cash collateral account are examples of external credit enhancement.
- Bond covenants are legally enforceable rules that borrowers and lenders agree on at the time of a new bond issue. Affirmative covenants enumerate what issuers are required to do, whereas negative covenants enumerate what issuers are prohibited from doing.
- An important consideration for investors is where the bonds are issued and traded, because it affects the laws, regulation, and tax status that apply. Bonds issued in a country in local currency are domestic bonds if they are issued by entities incorporated in the country and foreign bonds if they are issued by entities incorporated in another country. Eurobonds are issued internationally, outside the jurisdiction of any single country and are subject to a lower level of listing, disclosure, and regulatory requirements than domestic or foreign bonds. Global bonds are issued in the Eurobond market and at least one domestic market at the same time.
- Although some bonds may offer special tax advantages, as a general rule, interest is taxed at the ordinary income tax rate. Some countries also implement a capital gains tax. There may be specific tax provisions for bonds issued at a discount or bought at a premium.
- An amortizing bond is a bond whose payment schedule requires periodic payment of interest and repayment of principal. This differs from a bullet bond, whose entire payment of principal occurs at maturity. The amortizing bond's outstanding principal amount is reduced to zero by the maturity date for a fully amortized bond, but a balloon payment is required at maturity to retire the bond's outstanding principal amount for a partially amortized bond.
- Sinking fund agreements provide another approach to the periodic retirement of principal, in which an amount of the bond's principal outstanding amount is usually repaid each year throughout the bond's life or after a specified date.
- A floating-rate note, or floater, is a bond whose coupon is set based on a market reference rate (MRR) plus a spread. FRNs can be floored, capped, or collared. An inverse FRN is a bond whose coupon has an inverse relationship to the reference rate.

- Other coupon payment structures include bonds with step-up coupons, which pay coupons that increase by specified amounts on specified dates; bonds with credit-linked coupons, which change when the issuer's credit rating changes; bonds with payment-in-kind coupons, which allow the issuer to pay coupons with additional amounts of the bond issue rather than in cash; and bonds with deferred coupons, which pay no coupons in the early years following the issue but higher coupons thereafter.
- The payment structures for index-linked bonds vary considerably among countries. A common index-linked bond is an inflation-linked bond, or linker, whose coupon payments and/or principal repayments are linked to a price index. Index-linked payment structures include zero-coupon-indexed bonds, interest-indexed bonds, capital-indexed bonds, and indexed-annuity bonds.
- Common types of bonds with embedded options include callable bonds, putable bonds, and convertible bonds. These options are "embedded" in the sense that there are provisions provided in the indenture that grant either the issuer or the bondholder certain rights affecting the disposal or redemption of the bond. They are not separately traded securities.
- Callable bonds give the issuer the right to buy bonds back prior to maturity, thereby raising the reinvestment risk for the bondholder. For this reason, callable bonds have to offer a higher yield and sell at a lower price than otherwise similar non-callable bonds to compensate the bondholders for the value of the call option to the issuer.
- Putable bonds give the bondholder the right to sell bonds back to the issuer prior to maturity. Putable bonds offer a lower yield and sell at a higher price than otherwise similar non-putable bonds to compensate the issuer for the value of the put option to the bondholders.
- A convertible bond gives the bondholder the right to convert the bond into common shares of the issuing company. Because this option favors the bondholder, convertible bonds offer a lower yield and sell at a higher price than otherwise similar non-convertible bonds.

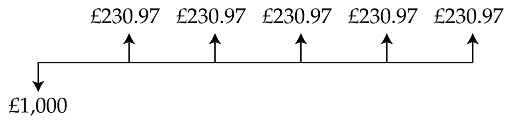
PRACTICE PROBLEMS

1. A 10-year bond was issued four years ago. The bond is denominated in US dollars, offers a coupon rate of 10% with interest paid semi-annually, and is currently priced at 102% of par. The bond's:
   A. tenor is six years.
   B. nominal rate is 5%.
   C. redemption value is 102% of the par value.
2. A sovereign bond has a maturity of 15 years. The bond is *best* described as a:
   A. perpetual bond.
   B. pure discount bond.
   C. capital market security.
3. A company has issued a floating-rate note with a coupon rate equal to the three-month MRR + 65 bps. Interest payments are made quarterly on 31 March, 30 June, 30 September, and 31 December. On 31 March and 30 June, the three-month MRR is 1.55% and 1.35%, respectively. The coupon rate for the interest payment made on 30 June is:
   A. 2.00%.
   B. 2.10%.
   C. 2.20%.

4. The legal contract that describes the form of the bond, the obligations of the issuer, and the rights of the bondholders can be *best* described as a bond's:
   A. covenant.
   B. indenture.
   C. debenture.
5. Which of the following is a type of external credit enhancement?
   A. Covenants
   B. A surety bond
   C. Overcollateralization
6. An affirmative covenant is *most likely* to stipulate:
   A. limits on the issuer's leverage ratio.
   B. how the proceeds of the bond issue will be used.
   C. the maximum percentage of the issuer's gross assets that can be sold.
7. Which of the following *best* describes a negative bond covenant? The issuer is:
   A. required to pay taxes as they come due.
   B. prohibited from investing in risky projects.
   C. required to maintain its current lines of business.
8. A South African company issues bonds denominated in pound sterling that are sold to investors in the United Kingdom. These bonds can be *best* described as:
   A. Eurobonds.
   B. global bonds.
   C. foreign bonds.
9. Relative to domestic and foreign bonds, Eurobonds are *most likely* to be:
   A. bearer bonds.
   B. registered bonds.
   C. subject to greater regulation.
10. An investor in a country with an original issue discount tax provision purchases a 20-year zero-coupon bond at a deep discount to par value. The investor plans to hold the bond until the maturity date. The investor will *most likely* report:
    A. a capital gain at maturity.
    B. a tax deduction in the year the bond is purchased.
    C. taxable income from the bond every year until maturity.
11. A bond that is characterized by a fixed periodic payment schedule that reduces the bond's outstanding principal amount to zero by the maturity date is *best* described as a:
    A. bullet bond.
    B. plain vanilla bond.
    C. fully amortized bond.
12. If interest rates are expected to increase, the coupon payment structure *most likely* to benefit the issuer is a:
    A. step-up coupon.
    B. inflation-linked coupon.
    C. cap in a floating-rate note.
13. Investors who believe that interest rates will rise *most likely* prefer to invest in:
    A. inverse floaters.
    B. fixed-rate bonds.
    C. floating-rate notes.

14. A 10-year, capital-indexed bond linked to the Consumer Price Index (CPI) is issued with a coupon rate of 6% and a par value of 1,000. The bond pays interest semi-annually. During the first six months after the bond's issuance, the CPI increases by 2%. On the first coupon payment date, the bond's:
    A. coupon rate increases to 8%.
    B. coupon payment is equal to 40.
    C. principal amount increases to 1,020.
15. The provision that provides bondholders the right to sell the bond back to the issuer at a predetermined price prior to the bond's maturity date is referred to as:
    A. a put provision.
    B. a make-whole call provision.
    C. an original issue discount provision.
16. Which of the following provisions is a benefit to the issuer?
    A. Put provision
    B. Call provision
    C. Conversion provision
17. Relative to an otherwise similar option-free bond, a:
    A. putable bond will trade at a higher price.
    B. callable bond will trade at a higher price.
    C. convertible bond will trade at a lower price.
18. Which type of bond *most likely* earns interest on an implied basis?
    A. Floater
    B. Conventional bond
    C. Pure discount bond
19. Clauses that specify the rights of the bondholders and any actions that the issuer is obligated to perform or is prohibited from performing are:
    A. covenants.
    B. collaterals.
    C. credit enhancements.
20. Which of the following type of debt obligation *most likely* protects bondholders when the assets serving as collateral are non-performing?
    A. Covered bonds
    B. Collateral trust bonds
    C. Mortgage-backed securities
21. Which of the following *best* describes a negative bond covenant? The requirement to:
    A. insure and maintain assets.
    B. comply with all laws and regulations.
    C. maintain a minimum interest coverage ratio.
22. Contrary to positive bond covenants, negative covenants are *most likely*:
    A. costlier.
    B. legally enforceable.
    C. enacted at time of issue.

23. A five-year bond has the following cash flows:



The bond can *best* be described as a:
   A. bullet bond.
   B. fully amortized bond.
   C. partially amortized bond.
24. Investors seeking some general protection against a poor economy are *most likely* to select a:
   A. deferred coupon bond.
   B. credit-linked coupon bond.
   C. payment-in-kind coupon bond.
25. The benefit to the issuer of a deferred coupon bond is *most likely* related to:
   A. tax management.
   B. cash flow management.
   C. original issue discount price.
26. Which of the following bond types provides the *most* benefit to a bondholder when bond prices are declining?
   A. Callable
   B. Plain vanilla
   C. Multiple put
27. Which type of call bond option offers the *greatest* flexibility as to when the issuer can exercise the option?
   A. Bermuda call
   B. European call
   C. American call
28. Which of the following *best* describes a convertible bond's conversion premium?
   A. Bond price minus conversion value
   B. Par value divided by conversion price
   C. Current share price multiplied by conversion ratio

CHAPTER 2

FIXED-INCOME MARKETS: ISSUANCE, TRADING, AND FUNDING

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LEARNING OUTCOMES

The candidate should be able to:

- describe classifications of global fixed-income markets;
- describe the use of interbank offered rates as reference rates in floating-rate debt;
- describe mechanisms available for issuing bonds in primary markets;
- describe secondary markets for bonds;
- describe securities issued by sovereign governments;
- describe securities issued by non-sovereign governments, quasi-government entities, and supranational agencies;
- describe types of debt issued by corporations;
- describe structured financial instruments;
- describe short-term funding alternatives available to banks;
- describe repurchase agreements (repos) and the risks associated with them.

1. INTRODUCTION

Global fixed-income markets represent the largest subset of financial markets in terms of number of issuances and market capitalization; they bring borrowers and lenders together to allocate capital globally to its most efficient uses. Fixed-income markets include not only publicly traded securities, such as commercial paper, notes, and bonds, but also non-publicly traded loans. Although they usually attract less attention than equity markets, fixed-income markets are more than three times the size of global equity markets. The Institute of International Finance reports that the size of the global debt market surpassed USD 253 trillion in the third quarter of 2019, representing a 322% global debt-to-GDP ratio (1 Institute of International Finance Global Debt Monitor, January 13, 2020).

Debt investors and issuers need to understand how fixed-income markets are structured and how they operate. Debt issuers have financing needs that must be met. For example, a government may need to finance an infrastructure project, a new hospital, or a new school. A start-up technology company may require funds beyond its initial seed funding from investors to expand its business and achieve scale. Financial institutions also have funding needs, and they are among the largest issuers of fixed-income securities.

Among the questions this chapter addresses are the following:

- What are the key bond market sectors?
- How are bonds sold in primary markets and traded in secondary markets?
- What types of bonds are issued by governments, government-related entities, financial companies, and non-financial companies?
- What additional sources of funds are available to banks?

We first present an overview of global fixed-income markets and how these markets are classified. We identify the major issuers of and investors in fixed-income securities, present fixed-income indexes, discuss how fixed-income securities are issued in primary markets, and explore how these securities are then traded in secondary markets. We also examine different bond market sectors and discuss additional short-term funding alternatives available to banks.

2. CLASSIFICATION OF FIXED-INCOME MARKETS

- **describe classifications of global fixed-income markets**
- **describe the use of interbank offered rates as reference rates in floating-rate debt**

Although there is no standard classification of fixed-income markets, many investors and market participants use criteria to structure fixed-income markets and identify bond market sectors. This section starts by describing the most widely used ways of classifying fixed-income markets.

2.1. Classification of Fixed-Income Markets

Common criteria used to classify fixed-income markets include the type of issuer; the bonds' credit quality, maturity, currency denomination, and type of coupon; and where the bonds are issued and traded.

2.1.1. Classification by Type of Issuer

One way of classifying fixed-income markets is by type of issuer, which leads to the identification of four bond market sectors: households, non-financial corporates, government, and financial institutions. Exhibit 1 presents data on global debt markets at the end of the third quarter of 2019 by these sectors. Each sector is broken down into mature markets (United States, Euro area, Japan, and United Kingdom) and emerging markets. Although the sectors are of roughly comparable sizes overall, in emerging markets the debt of non-financial corporates is proportionately higher and the financial sector lower than in the mature markets.

EXHIBIT 1 Global Debt by Sector at End of Q3 2019 in USD

USD Trillion (% of total)	Households	Non-Financial Corporates	Government	Financial Sector	Total
Mature markets	34.3	43.1	52.5	50.2	180.1
	(19.0%)	(23.9%)	(29.2%)	(27.9%)	(100.0%)
Emerging markets	13.2	31.3	16.7	11.3	72.5
	(18.2%)	(43.2%)	(23.0%)	(15.6%)	(100.0%)
Global debt	47.5	74.4	69.2	61.5	252.6
	(18.8%)	(29.5%)	(27.4%)	(24.3%)	(100.0%)

Source: Institute of International Finance Global Debt Monitor, January 2020.

Exhibit 2 displays global debt by sector as a percentage of GDP for various mature and emerging market countries at the end of the third quarter of 2019. In general, debt in proportion to GDP is higher in developed countries. However, debt takes different forms in different countries.

EXHIBIT 2 Global Debt by Sector as a Percentage of GDP, at End of Q3 2019

	Households	Non-Financial Corporates	Government	Financial Sector	Total
United States	74.2%	74.2%	101.8%	77.1%	327.3%
Japan	55.3	101.9	226.3	157.0	540.5
United Kingdom	83.8	81.5	110.3	178.0	453.6
China	55.4	156.7	53.6	42.8	308.5
South Korea	95.1	101.6	40.2	88.8	325.7
Brazil	28.7	42.9	87.9	40.4	199.9
Mexico	16.7	26.4	35.3	16.6	95.0
Israel	41.6	69.6	60.4	10.6	182.2
Nigeria	15.6	8.3	29.2	4.3	57.4

Source: Institute of International Finance Global Debt Monitor, 13 January 2020.

2.1.2. Classification by Credit Quality

Investors who hold bonds are exposed to credit risk, which is the risk of loss resulting from the issuer failing to make full and timely payments of interest and/or principal. Bond markets can be classified based on the issuer's creditworthiness as judged by credit rating agencies. Ratings of Baa3 or above by Moody's Investors Service or BBB– or above by Standard & Poor's (S&P) and Fitch Ratings are considered investment grade. In contrast, ratings below these levels are referred to as non-investment grade, high yield, speculative, or "junk." An important point to understand is that credit ratings assess the issuer's creditworthiness at a certain point in time; they are not a recommendation to buy or sell the issuer's securities. Credit ratings are not static, however; they will change if a credit rating agency perceives that the probability of default for an issuer has changed.

One of the reasons why the distinction between investment-grade and high-yield bond markets matters is because institutional investors may be prohibited from investing in, or restricted in their exposure to, lower-quality or lower-rated securities. Prohibition or restriction in high-yield bond holdings generally arise because of a more restrictive risk–reward profile that forms part of the investor's investment objectives and constraints. For example, regulated banks, life insurance companies, and pension schemes are usually limited to investing in very highly rated securities. In contrast, hedge funds or some sovereign wealth funds (Qatar and Kuwait) have no formal restrictions on what type of assets they can hold or on the percentage split between bond market sectors. Globally, investment-grade bond markets tend to be more liquid than high-yield bond markets.

2.1.3. Classification by Maturity

Fixed-income securities can also be classified by the original maturity of the bonds when they are issued. Securities may be issued with maturities at issuance (original maturity) ranging from overnight to 30, 40, 50, and up to 100 years. For example, large companies including The Walt Disney Company and The Coca-Cola Company have issued 100-year bonds in the past. Some governments have done the same. These bonds are attractive to the issuers because they secure long-term funding. They are also attractive for many investors, such as insurance companies, that have long-term liabilities and need to buy long-duration assets.

2.1.4. Classification by Currency Denomination

One of the critical ways to distinguish among fixed-income securities is by currency denomination. The currency denomination of the bond's cash flows influences the yield that must be offered to investors to compensate for the potential impact of currency movements on bond investment returns. For example, if a bond is denominated in Brazilian real, its price will be primarily driven by the credit quality of the issuer, the general level of interest rates in Brazil, and the economic fundamentals of Brazil to the extent that they will affect the exchange rate, and thus *ex post* bond returns in the investor's domestic currency.

Exhibit 3 presents data on debt as a percentage of GDP for several emerging market countries for local currency (LC) and foreign currency (FC) issuances at the end of the third quarter of 2019. Government debt is almost entirely in the domestic currency, whereas corporate and bank debt differs considerably by country.

EXHIBIT 3 Debt as a Percentage of GDP by Local and Foreign Currency for Several Emerging Markets

	Non-Financial Corporates		Government		Financial Sector	
	LC	FC	LC	FC	LC	FC
Brazil	28.1%	14.9%	87.2%	0.6%	31.9%	8.5%
India	37.9	6.3	66.3	1.9	0.9	3.1
Mexico	9.0	17.3	28.3	7.0	13.9	2.7
Turkey	30.8	36.5	17.9	17.1	5.3	22.2

Note: LC stands for "local currency"; FC stands for "foreign currency."
Source: Institute of International Finance Global Debt Monitor, 13 January 2020.

2.1.5. Classification by Type of Coupon

Another way of classifying fixed-income markets is by type of coupon. Some bonds pay a fixed rate of interest; others, called floating-rate bonds, floating-rate notes (FRNs), or floaters, pay a rate of interest that adjusts to market interest rates at regular, short-term intervals (e.g., quarterly).

2.1.5.1. Demand and Supply of Fixed-Rate vs. Floating-Rate Debt Balance sheet risk management considerations explain much of the demand and supply of floating-rate debt. For instance, the funding of banks—that is, the money banks raise to make loans to companies and individuals—is often short term and issued at rates that change or reset frequently. When a mismatch arises between the interest paid on the liabilities (money the bank borrowed) and the interest received on the assets (money the bank lent or invested), banks are exposed to interest rate risk—that is, the risk associated with a change in interest rate. In an effort to limit the volatility of their net worth resulting from interest rate risk, banks that issue floating-rate debt often prefer to make floating-rate loans and invest in floating-rate bonds or in other adjustable-rate assets. In addition to institutions with short-term funding needs, demand for floating-rate bonds comes from investors who believe that interest rates will rise. In that case, investors will benefit from holding floating-rate investments compared with fixed-rate ones.

On the supply side, issuance of floating-rate debt comes from institutions needing to finance short-term loans, such as consumer finance companies. Corporate borrowers also view floating-rate bonds as an alternative to using bank liquidity facilities (e.g., lines of credit), particularly when they are a lower-cost option, and as an alternative to borrowing long term at fixed rates when they expect interest rates will fall.

2.1.5.2. Reference Rates The coupon rate of a floating-rate bond is typically expressed as a reference rate plus a constant spread or margin. It is primarily a function of the issuer's credit risk at issuance: The lower the issuer's credit quality (the higher its credit risk), the higher the spread. The reference rate, however, resets periodically. Therefore, the coupon rate adjusts to the level of market interest rates each time the reference rate is reset.

Different reference rates are used depending on where the bonds are issued and their currency denomination. In the past, the **London interbank offered rate (Libor)** has long been the reference rate for many floating-rate bonds. Libor reflected the rate at which a panel of banks believed they could borrow unsecured funds from other banks. It was gathered by way of survey, and subjectivity is believed to have made its way into the process. The sidebar describes how Libor is being phased out and set to be replaced by new money market reference rates.

THE PHASEOUT OF LIBOR

Starting in 1986, daily Libor quotations for various currencies and maturities were set by the British Bankers' Association (BBA). Every business day, a select group of banks would submit to the BBA the rates at which they believed they could borrow from other banks in the London interbank market. Over time, the coverage grew to include 10 currencies and 15 borrowing periods. The submitted rates would be ranked from highest to lowest, and the upper and lower four submissions would be discarded. The arithmetic mean of the remaining rates became the Libor setting for a particular combination of currency and maturity. The 150 Libor quotations would then be communicated to market participants for use as reference rates in many different types of debt, including floating-rate bonds.

Problems with Libor first arose during the 2007–2009 global financial crisis, when the perceived default and liquidity risks of major international banks rose significantly. Some banks were allegedly submitting lowered rates to influence the market perception of their credit quality. In addition, some banks were allegedly altering their submitted rates to improve the valuation of their derivatives positions in contracts tied to Libor. In 2014, UK regulatory authorities transferred the administration of Libor from BBA to the International Currency Exchange (ICE). Under ICE, the size for Libor quotations was reduced to five currencies and seven maturities.

By 2017, it became apparent that the lack of activity in interbank borrowing and lending meant that Libor submissions were based more on judgment of market conditions than actual trades. Therefore, the decision was made that the panel of banks would no longer be required to submit quotations after 2021. In anticipation of the eventual demise of Libor, market participants and regulators have been working to develop new alternative money market reference rates. In the United States, it appears that the new rate will be the secured overnight financing rate (SOFR). SOFR, which has begun to be reported daily by the US Federal Reserve, is based on actual transactions in the sale and repurchase agreement ("repo") market. (Repos are described later.) In July 2018, Fannie Mae, a major institution in the US secondary mortgage market, issued USD6 billion in floating-rate notes at SOFR plus a small spread. (Note: based on "Libor: Its Astonishing Ride and How to Plan for Its End," by Liang Wu, a Numerix white paper, February 2018.)

The use of these money market reference rates extends beyond setting coupon rates for floating-rate debt. These rates are also used as reference rates for other debt instruments, including mortgages, derivatives such as interest rate and cross-currency swaps, and many other financial contracts and products. In 2018, nearly USD350 trillion in financial instruments were estimated to be tied to Libor. A major task for the future will be the transition of the terms on these contracts to the new Market Reference Rate (MRR) as Libor is phased out.

2.1.6. Classification by Geography

A distinction is very often made among the domestic bond, foreign bond, and Eurobond markets. Bonds issued in a specific country, denominated in the currency of that country, and sold in that country are classified as domestic bonds if they are issued by an issuer domiciled in that country and foreign bonds if they are issued by an issuer domiciled in another country. Domestic and foreign bonds are subject to the legal, regulatory, and tax requirements that apply in that particular country. In contrast, a Eurobond is issued internationally, outside the jurisdiction of the country in whose currency the bond is denominated. The Eurobond market has traditionally been characterized by fewer reporting, regulatory, and tax constraints than domestic and foreign bond markets. This less constrained environment explains why approximately 80% of entities that issue bonds outside their country of origin choose to do so in the Eurobond market rather than in a foreign bond market. In addition, Eurobonds are attractive for issuers because it enables them to reach out to more investors globally. Access to a wider pool of investors often allows issuers to raise more capital, usually at a lower cost. Along these lines, Eurobond markets are attractive for investors because they offer investors the ability to take on sovereign credit risk without being exposed to risks that are germane to the individual jurisdiction of the issuer countries, such as potential changes to capital control rules, which would restrict the return of capital, and other country risks.

Investors make a distinction between countries with established capital markets (developed markets) and countries where the capital markets are in earlier stages of development (emerging markets). For emerging bond markets, a further distinction is made between bonds issued in local currency and bonds issued in a foreign currency, such as the euro or the US dollar. All else equal, the yield offered by an individual emerging market country for a bond issued denominated in its domestic currency will be higher than the yield offered if the bond were denominated in euros or US dollars. Emerging market currencies are perceived to be riskier, and thus higher yields need to be offered to investors as compensation.

Emerging bond markets are much smaller than developed bond markets. But as demand from local and international investors has increased, issuance and trading of emerging market bonds have risen in volume and value. International investors' interest in emerging market bonds has been triggered by a desire to diversify risk across several jurisdictions in the belief that investment returns across markets are not closely correlated. In addition, emerging market bonds usually offer higher yields (return) than developed market bonds because of the higher perceived risk. Emerging countries typically lag developed countries in the areas of political stability, property rights, and contract enforcement, which often leads to a higher credit risk and higher yields. Many emerging countries, however, are less indebted than their developed counterparts and benefit from higher growth prospects, which appeals to many investors.

Composition of the bond markets varies widely by geography, with many developed and particularly emerging fixed-income markets dominated by sovereign fixed-income securities, with few bonds issued by the private sector and a predominance of bank lending. Potential bond investors may have access to private sector debt securities only indirectly by investing in a bank that holds such loans on its balance sheet. This scenario stands in contrast to markets such as the United States.

2.1.7. Other Classifications of Fixed-Income Markets

There are various other ways of classifying fixed-income markets. Specific market sectors that are of interest to some investors are inflation-linked bonds and, in some jurisdictions, tax-exempt bonds. Issuance of either type of bond tends to be limited to certain types of issuers. Inflation-linked bonds or linkers are typically issued by governments, government-related entities, and corporate issuers that have an investment-grade rating. They offer investors protection against inflation by linking the coupon payment and/or the principal repayment to an index of consumer prices.

Tax-exempt bonds can be issued only in those jurisdictions that recognize such tax exemption. In the United States for example, there is an income tax exemption for some of the bonds issued by governments or by some non-profit organizations. In particular, local governments can issue **municipal bonds** (or **munis**) that are tax exempt (they can also issue taxable municipal bonds, although tax-exempt munis are more frequently issued than taxable munis). Tax-exempt bonds also exist in other jurisdictions. For example, the National Highways Authority of India issues tax-exempt bonds. In countries that implement a capital gains tax, there may be tax exemptions for some types of bonds. In the United Kingdom, for example, government gilts are not subject to capital gains tax.

EXAMPLE 1 Classification of Fixed-Income Markets

1. Which of the following is *most likely* an issuer of bonds?
   A. Hedge fund
   B. Pension fund
   C. Local government
2. A bond issued by a city would *most likely* be classified as a:
   A. supranational bond.
   B. quasi-government bond.
   C. non-sovereign government bond.
3. A fixed-income security issued with a maturity at issuance of nine months is *most likely* classified as a:
   A. capital market security.
   B. money market security.
   C. securitized debt instrument.
4. The price of a bond issued in the United States by a British company and denominated in US dollars is *most likely* to:
   A. change as US interest rates change.
   B. change as British interest rates change.
   C. be unaffected by changes in US and British interest rates.
5. Interbank offered rates are *best* described as the rates at which a panel of banks can:
   A. issue short-term debt.
   B. borrow unsecured funds from other major banks.
   C. borrow from other major banks against some form of collateral.
6. A company issues floating-rate bonds. The coupon rate is expressed as the three-month Libor plus a spread. The coupon payments are *most likely* to increase as:
   A. Libor increases.
   B. the spread increases.
   C. the company's credit quality decreases.

Solution to 1: C is correct. Major issuers of bonds include sovereign (national) governments, non-sovereign (local) governments, quasi-government agencies, supranational organizations, and financial and non-financial companies. A and B are incorrect because hedge funds and pension funds are typically investors in, not issuers of, bonds.

Solution to 2: C is correct. Non-sovereign (local) government bond issuers include provinces, regions, states, and cities. A is incorrect because supranational bonds are issued by international organizations. B is incorrect because quasi-government bonds are issued by agencies that are either owned or sponsored by governments.

Solution to 3: B is correct. Money market securities are issued with a maturity at issuance (original maturity) ranging from overnight to one year. A is incorrect because capital market securities are issued with an original maturity longer than one year. C is incorrect because securitization, which leads to the creation of securitized debt instruments, does not relate to a bond's maturity but to the process that transforms private transactions between borrowers and lenders into securities traded in public markets.

Solution to 4: A is correct. The currency denomination of a bond's cash flows influences which country's interest rates affect a bond's price. The price of a bond issued by a British company and denominated in US dollars will be affected by US interest rates.

Solution to 5: B is correct. Interbank offered rates represent a set of interest rates at which major banks believe they could borrow unsecured funds from other major banks in the interbank money market for different currencies and different borrowing periods ranging from overnight to one year.

Solution to 6: A is correct. The coupon payments of a floating-rate bond that is tied to three-month Libor will reset every three months, based on changes in Libor. Thus, as Libor increases, so will the coupon payments. B is incorrect because the spread on a floating-rate bond is typically constant; it is set when the bond is issued and does not change afterward. C is incorrect because the issuer's credit quality affects the spread and thus the coupon rate that serves as the basis for the calculation of the coupon payments, but only when the spread is set—that is, at issuance.

2.2. Fixed-Income Indexes

A fixed-income index is a multi-purpose tool used by investors and investment managers to describe a given bond market or sector, as well as to track and quantify the performance of investments and investment managers. Most fixed-income indexes are constructed as portfolios of securities that reflect a particular bond market or sector. The index construction—that is, the security selection and the index weighting—varies among indexes. Index weighting may be based on price, value (market capitalization), or total return.

There are dozens of fixed-income indexes globally, capturing different aspects of the fixed-income markets discussed earlier. One of the most popular set of indexes is the Bloomberg Barclays Global Aggregate Bond Index, which represents a broad-based measure of the global investment-grade fixed-rate bond market. It has an index history beginning on 1 January 1990 and contains three important components: the US Aggregate Bond Index (formerly Lehman Aggregate Bond Index), the Pan-European Aggregate Bond Index, and the Asian-Pacific Aggregate Bond Index. These indexes reflect the investment-grade sectors of the US, European, and Asian-Pacific bond markets, respectively.

With respect to emerging markets, one of the most widely followed indexes is the J.P. Morgan Emerging Market Bond Index (EMBI) Global, which includes US dollar-denominated Brady bonds (bonds issued primarily by Latin American countries in the late 1980s under a debt restructuring plan aimed at converting bank loans into tradable securities), Eurobonds, and loans issued by sovereign and quasi-sovereign entities in several emerging markets.

Another popular set of indexes is the FTSE Global Bond Index Series, set up to provide coverage of different classes of securities related to the government and corporate bond markets. It includes indexes of global government bonds, euro-denominated government bonds from emerging markets, sterling- and euro-denominated investment-grade corporate bonds, and covered bonds from Germany and other European Union issuers. Covered bonds are debt obligations issued by banks and backed (secured) by a segregated pool of assets.

Many other fixed-income indexes are available to investors and investment managers to measure and report performance.

It is important to note that constructing a fixed-income index is more challenging than constructing an equity index, because fixed-income markets feature far more securities of different types and more frequent changes in the index as new bonds are issued while others reach maturity.

2.3. Investors in Fixed-Income Securities

The overview of fixed-income markets has so far focused on the supply side. Before discussing bond issuers in greater detail, it is important to consider the demand side because demand for a particular type of bond or issuer may affect supply. After all, market prices are the result of the interaction between demand and supply; neither one can be considered in isolation. For example, an increase in demand for inflation-linked bonds as a result of investors' desire to protect the value of their portfolios against inflation risk may lead governments to issue a greater quantity of this type of bond. By issuing relatively more inflation-linked bonds for which there is demand, a government not only manages to sell its bond issue and obtain the funds required, but it may also benefit from a lower cost of financing.

There are different types of investors in fixed-income securities. Major categories of bond investors include central banks, institutional investors, and retail investors. The first two typically invest directly in fixed-income securities. In contrast, retail investors often invest indirectly, traditionally through fixed-income mutual funds and, during the last two decades, through exchange-traded funds (ETFs), described later.

Central banks use open market operations to implement monetary policy. **Open market operations** refer to the purchase or sale of bonds, usually but not always sovereign bonds issued by the national government. By purchasing (selling) domestic bonds, central banks increase (decrease) the monetary base in the economy. Central banks may also purchase and sell bonds denominated in foreign currencies as part of their efforts to manage the relative value of the domestic currency and their country's foreign reserves.

Institutional investors, including pension funds, hedge funds, charitable foundations and endowments, insurance companies, and banks, represent the largest groups of investors in fixed-income securities. Another major group of investors is sovereign wealth funds, which are state-owned investment funds that tend to have very long investment horizons and aim to preserve or create wealth for future generations.

Finally, retail investors often invest heavily in fixed-income securities because of the attractiveness of relatively stable prices and steady income production.

Fixed-income markets are dominated by institutional investors, in part because of the high informational barriers to entry and high minimum transaction sizes. Fixed-income securities are far more diverse than equity securities because of the variety of types of issuers and securities. In addition, unlike common shares that are primarily issued and traded in organized markets, the issuance and trading of bonds very often occurs in **over-the-counter (OTC) markets**. Thus, fixed-income securities are more difficult to access than equity securities. For these reasons, institutional investors tend to invest directly in bonds, whereas most retail investors prefer to use investment vehicles, such as mutual funds and ETFs.

EXAMPLE 2 Investors in Fixed-Income Securities

1. Open market operations describe the process used by central banks to buy and sell bonds to:
   A. implement fiscal policy.
   B. control the monetary base.
   C. issue and repay government debt.
2. Retail investors *most often*:
   A. do not invest in fixed-income securities.
   B. invest directly in fixed-income securities.
   C. invest indirectly in fixed-income securities through mutual funds or exchange-traded funds.

Solution to 1: B is correct. Open market operations refer to the purchase or sale of bonds, usually sovereign bonds issued by the national government, as a means of implementing monetary policy. By purchasing (selling) bonds, central banks increase (decrease) the monetary base in the economy, thus controlling the money supply. A is incorrect because open market operations help facilitate monetary policy, not fiscal policy (which is the taxing and spending by the national government). C is incorrect because although Treasury departments and some central banks may facilitate the issuance and repayment of government debt, open market operations specifically refer to the implementation of monetary policy.

Solution to 2: C is correct. Retail investors often invest in fixed-income securities because of the attractiveness of relatively stable prices and steady income production. Because most retail investors lack the expertise to value fixed-income securities and are not large enough investors to buy and sell them directly, however, they usually invest in fixed income indirectly through mutual funds and exchange-traded funds.

3. PRIMARY BOND MARKETS

- **describe mechanisms available for issuing bonds in primary markets**

Primary bond markets are markets in which issuers initially sell bonds to investors to raise capital. In contrast, **secondary bond markets** are markets in which existing bonds are subsequently traded among investors. As with all financial markets, primary and secondary bond markets are regulated within the framework of the overall financial system. An established independent regulatory authority is usually responsible for overseeing both the structure of the markets and the credentials of market participants.

3.1. Primary Bond Markets

Issuances in primary bond markets are frequent. Different bond issuing mechanisms are used depending on the type of issuer and the type of bond issued. A bond issue can be sold via a **public offering** (or **public offer**), in which any member of the public may buy the bonds, or via a **private placement**, in which only a selected investor, or group of investors, may buy the bonds.

3.1.1. Public Offerings

Investment banks play a critical role in bond issuance by assisting the issuer in accessing the primary market and by providing an array of financial services. The most common bond issuing mechanisms are underwritten offerings, best-efforts offerings, and auctions. In an **underwritten offering**, also called a **firm commitment offering**, the investment bank guarantees the sale of the bond issue at an offering price that is negotiated with the issuer. Thus, the investment bank, called the **underwriter**, takes the risk associated with selling the bonds. In contrast, in a **best-efforts offering**, the investment bank serves only as a broker. It tries to sell the bond issue at the negotiated offering price only if it is able to for a commission. Thus, the investment bank has less risk and correspondingly less incentive to sell the bonds in a best-efforts offering than in an underwritten offering. An **auction** is a bond issuing mechanism that involves bidding.

3.1.1.1. Underwritten Offerings Underwritten offerings are typical bond-issuing mechanisms for corporate bonds, some local government bonds (such as municipal bonds in the United States), and some asset-backed securities (such as mortgage-backed securities). The underwriting process typically includes six phases.

The underwriting process starts with the determination of the funding needs. Often with the help of an adviser or advisers, the issuer must determine how much money must be raised, the type of bond offering, and whether the bond issue should be underwritten.

Once the issuer has decided that the bond issue should be underwritten, it must select the underwriter, which is typically an investment bank. The underwriter of a bond issue takes the risk of buying the newly issued bonds from the issuer, and it then resells them to investors or to dealers that then sell them to investors. The difference between the purchase price of the new bond issue and the reselling price to investors is the underwriter's revenue. A relatively small-size bond issue may be underwritten by a single investment bank. It is more common for larger bond issues, however, to be underwritten by a group, or syndicate, of investment banks. In this case, the bond issue is referred to as a **syndicated offering**. A lead underwriter invites other investment banks to join the syndicate and coordinates the effort. The syndicate is collectively responsible for determining the pricing of the bond issue and for placing (selling) the bonds with investors.

The third phase of an underwritten offering is to structure the transaction. Before the bond issue is announced, the issuer and the lead underwriter discuss the terms of the bond issue, such as the bond's notional principal (total amount), the coupon rate, and the expected offering price. The underwriter or the syndicate typically organizes the necessary regulatory filings and prepares the offering circular or prospectus that provides information about the terms of the bond issue. The issuer must also choose a trustee, which is typically a trust company or the trust department of a bank, to oversee the master bond agreement. The bond offering is formally launched the day the transaction is announced, usually in the form of a press release. The announcement specifies the new bond issue's terms and conditions, including the bond's features, such as the maturity date, the currency denomination, and the expected coupon range, as well as the expected offering price. The issuer also releases the offering circular or prospectus. The final terms may differ from these terms because of changes in market conditions between the announcement day and the pricing day.

The success of the bond issue depends on the underwriter or syndicate's discernment in assessing market conditions and in pricing the bond issue accordingly. The pricing of the bond issue is, therefore, an important phase of an underwritten offering. Ideally, the bond issue should be priced so that the amount of bonds available equals investors' demand for the bonds. If the offering price is set too high, the offering will be undersubscribed—that is, there will be insufficient demand for the bond issue. As a consequence, the underwriter or syndicate will fail to sell the entire bond issue. Alternatively, if the offering price is set too low, the offering will be oversubscribed. Underwriters may aim at a small oversubscription because it reduces the risk of being unable to sell the entire bond issue. But a large oversubscription indicates that the offering terms were probably unfavorable to the issuer, in that the issuer might have raised the desired amount of capital at a lower coupon rate.

Between the announcement of a bond issue and the end of the subscription period, the underwriter or syndicate must gauge the demand for the bond issue and the price at which the bond should be offered to ensure that the entire bond issue is placed without running the risk of a large oversubscription. There are different ways for underwriters to do so. The bond issue is usually marketed to potential investors, either through an indirect approach—such as an advertisement in a newspaper, a commonly used approach for bond issued by household names—or through direct marketing and road shows, aimed at institutional investors such as pension funds and insurance companies. The underwriter or syndicate may also approach large institutional investors and discuss with them the kind of bond issues they are willing to buy. These buyers are known as the "anchor." For some, but not all, bond issues, the grey market is another way for underwriters to gauge investors' interest. The **grey market**, also called the "when issued" market, is a forward market for bonds about to be issued. Trading in the grey market helps underwriters determine what the final offering price should be.

The pricing day is the last day when investors can commit to buy the bond issue, and it is also the day when the final terms of the bond issue are agreed on. The following day, called the "offering day," the underwriting agreement that includes the bond issue's final terms is signed. The underwriting process then enters the issuing phase. The underwriter or the syndicate purchases the entire bond issue from the issuer, delivers the proceeds, and starts reselling the bonds through its sales network.

The underwriting process comes to an end about 14 days later, on the closing day, when the bonds are delivered to investors. Investors no longer receive a paper settlement; instead, the bond itself is represented by a global note that is typically held by the paying agent.

3.1.1.2. Shelf Registrations A **shelf registration** allows certain authorized issuers to offer additional bonds to the general public without having to prepare a new and separate offering circular for each bond issue. Rather, the issuer prepares a single, all-encompassing offering circular that describes a range of future bond issuances, all under the same document. This master prospectus may be in place for years before it is replaced or updated, and it may be used to cover multiple bond issuances.

Under a shelf registration, each individual offering is prefaced with a short issue announcement document. This document must confirm that there has been no change to material elements of the issuer's business, or otherwise describe any changes to the issuer's financial condition since the master prospectus was filed. Because shelf issuances are subject to a lower level of scrutiny compared with standard public offerings, they are an option only for well-established issuers that have convinced the regulatory authorities of their financial strength. Additionally, certain jurisdictions may allow shelf registrations to be purchased only by "qualified" institutional investors—that is, institutional investors that meet a set of criteria set forth by the regulators.

3.1.1.3. Auctions An auction is a method that involves bidding. It is helpful in providing price discovery (i.e., it facilitates supply and demand in determining prices) and in allocating securities. In many countries, most sovereign bonds are sold to the public via a public auction. For example, in 2017, the United States conducted 277 public auctions and issued approximately $8.5 trillion of new securities such as Treasury bills, notes, bonds, and Treasury Inflation-Protected Securities (TIPS). The public auction process used in the United States is a single-price auction through which all the winning bidders pay the same price and receive the same coupon rate for the bonds. In contrast, the public auction process used in Canada and Germany is a multiple-price auction process, which generates multiple prices and yields for the same bond issue.

The US sovereign bond market is one of the largest and most liquid bond markets globally, so we will illustrate the US single-price auction process. This process includes three phases: announcement, bidding, and issuance. First, the US Treasury announces the auction and provides information about the bond issue, such as the amount of securities being offered, the auction date, the issue date, the maturity date, bidding close times, and other pertinent information.

After the auction announcement is made, dealers, institutional investors, and individual investors may enter competitive or non-competitive bids. With competitive bids, a bidder specifies the rate (yield) that is considered acceptable; if the rate determined at auction is lower than the rate specified in the competitive bid, the investor will not be offered any securities. In contrast, with non-competitive bids, a bidder agrees to accept the rate determined at auction; non-competitive bidders always receive their securities. At the close of the auction, the US Treasury accepts all non-competitive bids and competitive bids in ascending order of their rates (lowest to highest) until the amount of bids is equal to the amount the issuer requires. All bidders receive the same rate, based on the highest accepted bid. This single-price auction process encourages aggressive bidding and potentially results in a lower cost of funds (i.e., lower coupon rate) for the US Treasury because all the winning bidders pay the same price.

On the issue day, the US Treasury delivers the securities to the winning bidders and collects the proceeds from investors. After the auction process is complete, the securities are traded in secondary markets like other securities.

Exhibit 4 shows the results of a US Treasury public auction.

EXHIBIT 4 Results of a US Treasury Public Auction on 23 July 2018

TREASURY NEWS

Department of the Treasury • Bureau of the Fiscal Service

For Immediate Release
July 23, 2018

CONTACT: Treasury Auctions
202-504-3550

TREASURY AUCTION RESULTS

Term and Type of Security	182-Day Bill
CUSIP Number	912796QU6
High Rate [1]	2.140%
Allotted at High	29.02%
Price	98.918111
Investment Rate [2]	2.193%
Median Rate [3]	2.110%
Low Rate [4]	2.085%
Issue Date	July 26, 2018
Maturity Date	January 24, 2019

	Tendered	Accepted
Competitive	$128,928,869,200	$43,206,629,200
Noncompetitive	$793,540,100	$793,540,100
FIMA (Noncompetitive)	$1,000,000,000	$1,000,000,000
Subtotal [5]	**$130,722,409,300**	**$45,000,169,300** [6]
SOMA	$0	$0
Total	**$130,722,409,300**	**$45,000,169,300**

	Tendered	Accepted
Primary Dealer [7]	$98,935,000,000	$20,122,560,000
Direct Bidder [8]	$6,105,000,000	$5,205,000,000
Indirect Bidder [9]	$23,888,869,200	$17,879,069,200
Total Competitive	**$128,928,869,200**	**$43,206,629,200**

[1] All tenders at lower rates were accepted in full.
[2] Equivalent coupon-issue yield.
[3] 50% of the amount of accepted competitive tenders was tendered at or below that rate.
[4] 5% of the amount of accepted competitive tenders was tendered at or below that rate.
[5] Bid-to-Cover Ratio: $130,722,409,300/$45,000,169,300 = 2.90
[6] Awards to TreasuryDirect = $384,924,100.
[7] Primary dealers as submitters bidding for their own house accounts.
[8] Non-Primary dealer submitters bidding for their own house accounts.
[9] Customers placing competitive bids through a direct submitter, including Foreign and International Monetary Authorities placing bids through the Federal Reserve Bank of New York.

Source: Based on information from www.treasurydirect.gov.

The rate determined at auction was 2.140%. T-bills are pure discount bonds; they are issued at a discount to par value and redeemed at par. Investors paid 98.918111% of par—that is, $9,891.81 per $10,000 in par value. The US Treasury received bids for $130.7 billion but raised only $45.0 billion. All the non-competitive bids ($793.5 million) were accepted, but only a third ($43.2 of the $128.9 billion) of competitive bids were accepted. Note that half the competitive bids were submitted with a rate lower than 2.110% (the median rate). All successful bidders, however, received the rate of 2.140%, which is the essential feature of a single-price auction.

Exhibit 4 also identifies the types of bidders. Most US Treasury securities are bought at auction by primary dealers. **Primary dealers** are financial institutions that are authorized to deal in new issues of US Treasury securities. They have established business relationships with the Federal Reserve Bank of New York (New York Fed), which implements US monetary policy. Primary dealers serve primarily as trading counterparties of the New York Fed and are required to participate meaningfully in open market operations and in all auctions of US Treasury securities. They also provide the New York Fed with market information. Institutional investors and central banks are the largest investors in US Treasury securities; only a very small amount of these bonds is purchased directly by individual investors.

3.1.2. Private Placements

A private placement is typically a non-underwritten, unregistered offering of bonds that are sold only to an investor or a small group of investors. Typical investors in privately placed bonds are large institutional investors. A private placement can be accomplished directly between the issuer and the investor(s) or through an investment bank. Because privately placed bonds are unregistered and may be restricted securities that can be purchased by only some types of investors, there is usually no active secondary market to trade them. However, trading may be possible under certain conditions. For example, restricted securities issued under Rule 144A in the United States cannot be sold to the public, but they can be traded among qualified institutional investors. Even if trading is possible, privately placed bonds typically exhibit lower liquidity than publicly issued bonds. Insurance companies and pension funds are major buyers of privately placed bonds because they do not need every security in their portfolios to be liquid and they often value the additional yield offered by these bonds.

Private placements sometimes represent a step in the company's financing evolution between **syndicated loans** (loans from a group of lenders to a single borrower further discussed later) and public offerings. Privately placed bonds are often issued in small aggregate amounts, at times by unknown issuers. Many investors may be unwilling to undertake the credit analysis that is required for a new name, in particular if the offering amount is small. Unlike in a public offering in which the bonds are often sold to investors on a take-it-or-leave-it basis, investors in a private placement can influence the structure of the bond issue, including such considerations as asset and collateral backing, credit enhancements, and covenants. It is common for privately placed bonds to have more customized and restrictive covenants than publicly issued ones. In addition to being able to negotiate the terms of the bonds and align those terms with their needs, investors in private placements are rewarded by receiving the bonds, which is not always the case in public offerings in which investors cannot know for sure when the issue will become available and how many securities they will be allocated.

Private placements are also offered by regular bond issuers, in particular for smaller amounts of capital raised in major currencies, such as US dollars, euros, or sterling. Private placements are usually more flexible than public offerings and allow regular issuers to tailor the bond issue to their own needs.

4. SECONDARY BOND MARKETS

- **describe secondary markets for bonds**

Secondary markets, also called the "aftermarket," are where existing securities are traded among investors. Securities can be traded directly from investor to investor, through a broker or dealer, or directly from issuer to investor by way of bond tender offer. The major participants in secondary bond markets globally are large institutional investors and central banks. The direct presence of retail investors in secondary bonds markets is limited, unlike in secondary equity markets. Retail investors do participate in secondary bond markets, albeit indirectly, through mutual and exchange-traded funds (ETFs) offered by fund management companies.

There are three main ways for secondary markets to be structured: as an organized exchange, as an over-the-counter market, or as a bond tender offer. An **organized exchange** provides a place where buyers and sellers can meet to arrange their trades. Although buy or sell orders may come from anywhere, the transaction must take place at the exchange according to the rules imposed by the exchange. In contrast, with **over-the-counter (OTC) markets**, buy and sell orders initiated from various locations are matched through a communications network. Thus, OTC markets need electronic trading platforms over which users submit buy and sell orders. Bloomberg Fixed Income Electronic Trading platform is an example of such a platform through which dealers stand ready to trade in multiple bond markets globally. Although there is some trading of government bonds and very active corporate bonds on many stock exchanges around the world, the vast majority of bonds are traded in OTC markets. A bond tender offer involves an issuer making an offer to existing bondholders of the company to repurchase a specified number of bonds at a particular price and a specified time. Companies use such offers to restructure or refinance their current capital structure. Investors generally receive a price that is above the market value of the bond, thereby enticing them to participate; yet this price is likely below par, and hence also a win for the issuer. Note, however, that the price at which the bonds are repurchased is not determined in advance at the time of the original issuance of the bond. It merely reflects the market conditions at the time of the repurchase.

The liquidity demands of fixed-income investors have evolved since the early 1990s. The type of investors that would buy and hold a bond to maturity, which once dominated the fixed-income markets, has been supplanted by institutional investors who trade actively. The dynamics of global fixed-income markets reflect this change in the relative demand for liquidity.

We will illustrate how secondary markets work by using the example of Eurobonds. The most important Eurobond trading center by volume is in London, although a large number of market participants are also based in Brussels, Frankfurt, Zurich, and Singapore. Liquidity is supplied by Eurobond market makers, of which approximately 35 are registered with the International Capital Market Association (ICMA). ICMA is an association of banks and other financial institutions that provides a regulatory framework for international bond markets and that is behind much of the established uniform practices observed by all market participants in the Eurobond market.

The key to understanding how secondary bond markets are structured and function is to understand liquidity. Liquidity refers to the ability to trade (buy or sell) securities quickly and easily at prices close to their fair market value. Liquidity involves much more than "how quickly one can turn a bond into cash." This statement implicitly assumes a long position, but some market participants need to buy quickly when covering a short position. The other aspect of liquidity that is often ignored is that speed of trading alone does not constitute a liquid market. One can always buy something quickly by offering a very high price or sell something quickly by accepting a very low price. In a liquid market, trading takes place quickly at prices close to the security's fair market value.

A key indicator and measurement of liquidity is the **bid–offer spread** or **bid–ask spread**, which reflects the prices at which dealers will buy from a customer (bid) and sell to a customer (offer or ask). It can be as low as 5 bps for very liquid bond issues, such as issues of the World Bank, to no price quoted for illiquid issues. A reasonable spread is of the order of 10 bps to

12 bps, whereas an illiquid spread may be in excess of 50 bps. When there is no bid or offer price, the issue is completely illiquid for trading purposes.

Settlement is the process that occurs after the trade is made. The bonds are passed to the buyer, and payment is received by the seller. Secondary market settlement for government and quasi-government bonds typically takes place either on a cash basis or on a $T + 1$ basis. With cash settlement, trading and settlement occur on the same day. With $T + 1$ settlement, settlement takes place the day after the trade date. In contrast, corporate bonds usually settle on a $T + 2$ or $T + 3$ basis, although settlement can extend to $T + 7$ in some jurisdictions. Trades clear within either or both of the two main clearing systems, Euroclear and Clearstream. Settlement occurs by means of a simultaneous exchange of bonds for cash on the books of the clearing system. An electronic bridge connecting Euroclear and Clearstream allows transfer of bonds from one system to the other, so it is not necessary to have accounts at both systems. Both systems operate on a paperless, computerized book-entry basis, although a bond issue is still represented by a physical document, the global note mentioned earlier. All participants in either system will have their own internal account set up, and they may also act as agent for buyers or sellers who do not possess an account.

EXAMPLE 3 Bond Markets

1. Which of the following *best* describes a primary market for bonds? A market:
   A. in which bonds are issued for the first time to raise capital.
   B. that has a specific location where the trading of bonds takes place.
   C. in which existing bonds are traded among individuals and institutions.
2. US Treasury bonds are typically sold to the public via a(n):
   A. auction.
   B. primary dealer.
   C. secondary bond market.
3. In a single-price bond auction, an investor who places a competitive bid and specifies a rate that is above the rate determined at auction will *most likely*:
   A. not receive any bonds.
   B. receive the bonds at the rate determined at auction.
   C. receive the bonds at the rate specified in the investor's competitive bid.
4. A bond purchased in a secondary market is *most likely* purchased from:
   A. the bond's issuer.
   B. the bond's lead underwriter.
   C. another investor in the bond.
5. Corporate bonds will *most likely* settle:
   A. on the trade date.
   B. on the trade date plus one day.
   C. by the trade date plus three days.

Solution to 1: A is correct. Primary bond markets are markets in which bonds are issued for the first time to raise capital. B is incorrect because having a specific location where the trading of bonds takes place is not a requirement for a primary bond market. C is incorrect because a market in which existing bonds are traded among individuals and institutions is the definition of a secondary, not primary, market.

Solution to 2: A is correct. US Treasury bonds are typically sold to the public via an auction. B is incorrect because primary dealers are often bidders in the auction; they are financial institutions that are active in trading US Treasury bonds. C is incorrect because any bond issue coming directly to the market is considered to be in the primary, not the secondary, market.

Solution to 3: A is correct. In a single-price bond auction, a bidder that enters a competitive bid specifies the rate (yield) that is considered acceptable. If the rate specified in the competitive bid is above the rate (yield) determined at auction, the investor will not be offered any securities.

Solution to 4: C is correct. Secondary bond markets are where bonds are traded among investors. A and B are incorrect because a bond purchased from the bond's issuer or from the bond's lead underwriter would happen in the primary, not secondary, market.

Solution to 5: C is correct. Corporate bonds typically settle on a $T + 2$ or $T + 3$ basis—that is, two or three days after the trade date—although settlement can extend to $T + 7$ in some jurisdictions. A and B are incorrect because it is government and quasi-government bonds, not corporate bonds, that typically settle either on a cash basis or on a $T + 1$ basis.

5. SOVEREIGN BONDS

- **describe securities issued by sovereign governments**

National governments issue bonds primarily for fiscal reasons—to fund spending when tax revenues are insufficient to cover expenditures. To meet their spending goals, national governments issue bonds in various types and amounts. This section discusses bonds issued by national governments, often referred to as **sovereign bonds** or sovereigns.

5.1. Characteristics of Sovereign Bonds

Sovereign bonds denominated in local currency have different names in different countries. For example, they are named US Treasuries in the United States, Japanese government bonds (JGBs) in Japan, gilts in the United Kingdom, Bunds in Germany, and obligations assimilables du Trésor (OATs) in France. Some investors or market participants may refer to sovereign bonds as Treasury securities or Treasuries for short, on the principle that the national Treasury department is often in charge of managing a national government's funding needs.

Names may also vary depending on the original maturity of the sovereign bond. For example, US government bonds are named Treasury bills (T-bills) when the original maturity is one year or shorter, Treasury notes (T-notes) when the original maturity is longer than one year and up to 10 years, and Treasury bonds (T-bonds) when the original maturity is longer than 10 years. In Spain, the sovereigns issued by Tesoro Público are named letras del Tesoro, bonos del Estado, and obligaciones del Estado, depending on the sovereign's original maturity: one year or shorter, longer than one year and up to five years, or longer than five years, respectively. Although very rare, some bonds, such as the consols in the United Kingdom, have no stated maturity date.

The majority of the trading in secondary markets is of sovereign securities that were most recently issued, called **on-the-run** securities. The latest sovereign bond issue for a given maturity is also referred to as a **benchmark issue** because it serves as a benchmark against which to compare bonds that have the same features (i.e., maturity, coupon type and frequency, and currency denomination) but that are issued by another type of issuer (e.g., non-sovereign, corporate). As a general rule, as sovereign securities age, they trade less frequently.

One salient difference between money market securities, such as T-bills, and capital market securities, such as T-notes and T-bonds, is the interest provision. T-bills are pure discount bonds; they are issued at a discount to par value and redeemed at par. The difference between the par value and the issue price is the interest paid on the borrowing. In contrast, capital market securities are typically coupon (or coupon-bearing) bonds; these securities make regular coupon payments and repay the par value at maturity. Bunds pay coupons annually, whereas US Treasuries, JGBs, gilts, and OATs make semi-annual coupon payments.

5.2. Credit Quality of Sovereign Bonds

Sovereign bonds are usually unsecured obligations of the sovereign issuer—that is, they are backed not by collateral but by the taxing authority of the national government. When a national government runs a budget surplus, excess tax revenues over expenditures are the primary source of funds for making interest payments and repaying the principal. In contrast, when a country runs a budget deficit, the source of the funds used for the payment of interest and repayment of principal comes either from tax revenues and/or by "rolling over" (refinancing) existing debt into new debt.

Highly rated sovereign bonds denominated in local currency are virtually free of credit risk. Credit rating agencies assign ratings to sovereign bonds, and these ratings are called sovereign ratings. The highest rating (i.e., highest credit quality and lowest credit risk) is AAA by S&P and Fitch and Aaa by Moody's. In 2018, only a handful of sovereign issuers were rated at this (theoretically) risk-free level by these three credit rating agencies, including Germany, Singapore, Canada, Sweden, Norway, Denmark, Luxembourg, Australia, Switzerland, and the Netherlands. Notably, S&P downgraded the United States from AAA to AA+ in 2011 and downgraded the United Kingdom from AAA to AA, two notches, in 2016 following the referendum that approved the United Kingdom's exit from the European Union.

Credit rating agencies distinguish between bonds issued in the sovereign's local currency and bonds issued in a foreign currency. In theory, a government can make interest payments and repay the principal by generating cash flows from its unlimited power (in the short run at least) to tax its citizens. A national government also has the ability to print its own currency, whereas it is restricted in being able to pay in a foreign currency only what it earns in exports or can exchange in financial markets. Thus, it is common to observe a higher credit rating for sovereign bonds issued in local currency than for those issued in a foreign currency. But there are limits to a government's ability to reduce the debt burden. As the sovereign debt crisis that followed the global financial crisis has shown, taxing citizens can only go so far in paying down debt before the taxation becomes an economic burden. Additionally, printing money only serves to weaken a country's currency relative to other currencies over time.

The national government of a country that has a strong domestic savings base has the luxury of being able to issue bonds in its local currency and sell them to domestic investors. If the local currency is liquid and freely traded, the sovereign issuer may also attract international investors who may want to hold that sovereign issuer's bonds and have exposure to that country's local currency. A national government may also issue debt in a foreign currency when there is demand for the sovereign issuer's bonds but not necessarily in the sovereign's local currency. For example, demand from overseas investors has caused national governments such as Switzerland and Sweden to issue sovereign bonds in US dollars and euros. Emerging market countries may also have to issue in major currencies because international investors may be willing to accept the credit risk but not the foreign exchange (currency) risk associated with emerging market bonds. The yield offered for bonds issued in major currencies will be lower because of the lower risk profile. When a sovereign issuer raises debt in a foreign currency, it usually swaps the proceeds into its local currency for use and deployment.

5.3. Types of Sovereign Bonds

National governments issue different types of bonds, some of them paying a fixed rate of interest and others paying a floating rate, including inflation-linked bonds.

5.3.1. Fixed-Rate Bonds

Fixed-rate bonds (i.e., bonds that pay a fixed rate of interest) are by far the most common type of sovereign bond. National governments routinely issue two types of fixed-rate bonds: zero-coupon bonds (or pure discount bonds) and coupon bonds. A zero-coupon bond does not pay interest. Instead, it is issued at a discount to par value and redeemed at par at maturity. Coupon bonds are issued with a stated rate of interest and make interest payments periodically, such as semi-annually or annually. They have a terminal cash flow equal to the final interest payment plus the par value. As mentioned earlier, most sovereign bonds with an original maturity of one year or less are zero-coupon bonds, whereas bonds with an original maturity longer than one year are typically issued as coupon bonds.

5.3.2. Floating-Rate Bonds

The price of a bond changes in the opposite direction from the change in interest rates, a relationship that is fully explained later when discussing the risk and return of fixed-income securities. Thus, investors who hold fixed-rate bonds are exposed to interest rate risk: As interest rates increase, bond prices decrease, which lowers the value of their portfolio. In response to public demand for less interest rate risk, some national governments around the world issue bonds with a floating rate of interest that resets periodically based on changes in the level of a reference rate such as Libor. Although interest rate risk still exists on floating-rate bonds, it is far less pronounced than that on fixed-rate bonds.

Examples of countries where the national government issues floating-rate bonds include Germany, Spain, and Belgium in developed markets and Brazil, Turkey, Mexico, Indonesia, and Poland in emerging markets. The largest sovereign issuer, the United States, began issuing floating-rate bonds in January 2014. Two other large sovereign issuers, Japan and the United Kingdom, have never issued bonds whose coupon rate is tied to a reference rate.

5.3.3. Inflation-Linked Bonds

Fixed-income investors are exposed to inflation risk. The cash flows of fixed-rate bonds are fixed by contract. If a particular country experiences an inflationary episode, the purchasing power of the fixed cash flows erodes over time. Thus, to respond to the demand for less inflation risk, many national governments issue inflation-linked bonds, or linkers, whose cash flows are adjusted for inflation. First issuers of inflation-linked bonds were the governments of Argentina, Brazil, and Israel. The United States introduced inflation-linked securities in January 1997, calling them Treasury Inflation-Protected Securities (TIPS). Other countries where the national government has issued inflation-linked bonds include the United Kingdom, Sweden, Australia, and Canada in developed markets and Brazil, South Africa, and Chile in emerging markets.

As explained in the earlier coverage of the defining elements of fixed-income securities, the index to which the coupon payments and/or principal repayments are linked is typically an index of consumer prices. Inflation-linked bonds can be structured a variety of ways: The inflation adjustment can be made via the coupon payments, the principal repayment, or both. In the United States, the index used is the Consumer Price Index for All Urban Consumers (CPI-U). In the United Kingdom, it is the Retail Price Index (RPI) (All Items). In France, there are two inflation-linked bonds with two different indexes: the French consumer price index (CPI) (excluding tobacco) and the Eurozone's Harmonized Index of Consumer Prices (HICP) (excluding tobacco). Although linking the cash flow payments to a consumer price index reduces inflation risk, it does not necessarily eliminate the effect of inflation completely because the consumer price index may be an imperfect proxy for inflation.

EXAMPLE 4 Sovereign Bonds

1. Sovereign debt with a maturity at issuance shorter than one year *most likely* consists of:
   A. floating-rate instruments.
   B. zero-coupon instruments.
   C. coupon-bearing instruments.
2. Floating-rate bonds are issued by national governments as the *best* way to reduce:
   A. credit risk.
   B. inflation risk.
   C. interest rate risk.
3. Sovereign bonds whose coupon payments and/or principal repayments are adjusted by a consumer price index are *most likely* known as:
   A. linkers.
   B. floaters.
   C. consols.

Solution to 1: B is correct. Most debt issued by national governments with a maturity at issuance (original maturity) shorter than one year takes the form of zero-coupon instruments. A and C are incorrect because floating-rate and coupon-bearing instruments are typically types of sovereign debt with maturities longer than one year.

Solution to 2: C is correct. The coupon rates of floating-rate bonds are reset periodically based on changes in the level of a reference rate such as Libor, which reduces interest rate risk. A is incorrect because credit risk, although low for sovereign bonds, cannot be reduced by linking the coupon rate to a reference rate. B is incorrect because although inflation risk is lower for floating-rate bonds than for fixed-rate bonds, floating-rate bonds are not as good as inflation-linked bonds to reduce inflation risk.

Solution to 3: A is correct because sovereign bonds whose coupon payments and/or principal repayment are adjusted by a consumer price index are known as inflation-linked bonds or linkers. B is incorrect because floaters describe floating-rate bonds that have a coupon rate tied to a reference rate such as Libor. C is incorrect because consols are sovereign bonds with no stated maturity date issued by the UK government.

6. NON-SOVEREIGN, QUASI-GOVERNMENT, AND SUPRANATIONAL BONDS

- **describe securities issued by non-sovereign governments, quasi-government entities, and supranational agencies**

This section covers the bonds issued by local governments and by government-related entities.

6.1. Non-Sovereign Bonds

Levels of government below the national level, such as provinces, regions, states, and cities, issue bonds called non-sovereign government bonds or **non-sovereign bonds**. These bonds are typically issued to finance public projects, such as schools, motorways, hospitals, bridges, and airports. The sources for paying interest and repaying the principal include the taxing authority of the local government, the cash flows of the project the bond issue is financing, or special taxes and fees established specifically for making interest payments and principal repayments. Non-sovereign bonds are typically not guaranteed by the national government.

As mentioned earlier, bonds issued by state and local governments in the United States are known as municipal bonds, and they often offer income tax exemptions. In the United Kingdom, non-sovereign bonds are known as local authority bonds. Other non-sovereign bonds include those issued by state authorities such as the 16 Länder in Germany.

Credit ratings for non-sovereign bonds vary widely because of the differences in credit and collateral quality. Because default rates of non-sovereign bonds are historically low, they very often receive high credit ratings. However, non-sovereign bonds usually trade at a higher yield and lower price than sovereign bonds with similar characteristics. The additional yield depends on the credit quality, the liquidity of the bond issue, and the implicit or explicit level of guarantee or funding commitment from the national government. The additional yield is the lowest for non-sovereign bonds that have high credit quality, are liquid, and are guaranteed by the national government.

6.2. Quasi-Government Bonds

National governments establish organizations that perform various functions for them. These organizations often have both public and private sector characteristics, but they are not actual governmental entities. They are referred to as quasi-government entities, although they take different names in different countries. These quasi-government entities often issue bonds to fund specific financing needs. These bonds are known as **quasi-government bonds** or **agency bonds**.

Examples of quasi-government entities include government-sponsored enterprises (GSEs) in the United States, such as the Federal National Mortgage Association ("Fannie Mae"), the Federal Home Loan Mortgage Corporation ("Freddie Mac"), and the Federal Home Loan Bank (FHLB). Other examples of quasi-government entities that issue bonds include Hydro Quebec in Canada or the Japan Bank for International Cooperation (JBIC). In the case of JBIC's bonds, timely payments of interest and repayment of principal are guaranteed by the Japanese government. Most quasi-government bonds, however, do not offer an explicit guarantee by the national government, although investors often perceive an implicit guarantee.

Because a quasi-government entity typically does not have direct taxing authority, bonds are repaid from the cash flows generated by the entity or from the project the bond issue is financing. Quasi-government bonds may be backed by collateral, but this is not always the case. Quasi-government bonds are usually rated very high by the credit rating agencies because historical default rates are extremely low. Bonds that are guaranteed by the national government receive the highest ratings and trade at a lower yield and higher price than otherwise similar bonds that are not backed by the sovereign government's guarantee.

6.3. Supranational Bonds

A form of often highly rated bonds is issued by supranational agencies, also referred to as multilateral agencies. The most well-known supranational agencies are the International Bank for Reconstruction and Development (the World Bank), the International Monetary Fund (IMF), the European Investment Bank (EIB), the Asian Development Bank (ADB), and the African Development Bank (AFDB). Bonds issued by supranational agencies are called **supranational bonds**.

Supranational bonds are typically plain-vanilla bonds, although floating-rate bonds and callable bonds are sometimes issued. Highly rated supranational agencies, such as the World Bank, frequently issue large-size bond issues that are often used as benchmarks issues when there is no liquid sovereign bond available.

EXAMPLE 5 Non-Sovereign Government, Quasi-Government, and Supranational Bonds

1. Relative to sovereign bonds, non-sovereign bonds with similar characteristics *most likely* trade at a yield that is:
   A. lower.
   B. the same.
   C. higher.
2. Bonds issued by a governmental agency are *most likely*:
   A. repaid from the cash flows generated by the agency.
   B. guaranteed by the national government that sponsored the agency.
   C. backed by the taxing power of the national government that sponsored the agency.

Solution to 1: C is correct. Non-sovereign bonds usually trade at a higher yield and lower price than sovereign bonds with similar characteristics. The higher yield is because of the higher credit risk associated with non-sovereign issuers relative to sovereign issuers, although default rates of local governments are historically low and their credit quality is usually high. The higher yield may also be a consequence of non-sovereign bonds being less liquid than sovereign bonds with similar characteristics.

Solution to 2: A is correct. Most bonds issued by a governmental agency are repaid from the cash flows generated by the agency or from the project the bond issue is financing. B and C are incorrect because although some bonds issued by governmental agencies are guaranteed by the national government or are backed by the taxing power of the national government that sponsored the agency, bonds are most likely repaid first from the cash flows generated by the agency.

7. CORPORATE DEBT: BANK LOANS, SYNDICATED LOANS, AND COMMERCIAL PAPER

- **describe types of debt issued by corporations**

Companies differ from governments and government-related entities in that their primary goal is profit; profitability is an important consideration when companies make decisions, including financing decisions. Companies routinely raise debt as part of their overall capital structure, both to fund short-term spending needs (e.g., working capital) as well as long-term capital investments. We have so far focused on publicly issued debt, but loans from banks and other financial institutions are a significant part of the debt raised by companies. For example, it is estimated that European companies meet 75% of their borrowing needs from banks and only 25% from financial markets. In Japan, these percentages are 80% and 20%, respectively. In the United States, however, debt capital is much more significant: 80% is from financial markets and just 20% from bank lending (SIFMA 2018 Outlook: Through the Looking Glass).

7.1. Bank Loans and Syndicated Loans

A **bilateral loan** is a loan from a single lender to a single borrower. Companies routinely use bilateral loans from their banks, and these bank loans are governed by the bank loan documents. Bank loans are the primary source of debt financing for small and medium-size companies as well as for large companies in countries where bond markets are either underdeveloped or where most bond issuances are from government, government-related entities, and financial institutions. Access to bank loans depends not only on the characteristics and financial health of the company but also on market conditions and bank capital availability.

A syndicated loan is a loan from a group of lenders, called the "syndicate," to a single borrower. A syndicated loan is a hybrid between relational lending and publicly traded debt. Syndicated loans are primarily originated by banks, and the loans are extended to companies but also to governments and government-related entities. The coordinator, or lead bank, originates the loan, forms the syndicate, and processes the payments. In addition to banks, a variety of lenders participate in the syndicate, such as pension funds, insurance companies, and hedge funds. Syndicated loans are a way for these institutional investors to participate in corporate lending while diversifying the credit risk among a group of lenders.

There is also a secondary market in syndicated loans. These loans are often packaged and securitized, and the securities created are then sold in secondary markets to investors.

Most bilateral and syndicated loans are floating-rate loans, and the interest rate is based on a reference rate plus a spread. The reference rate may be MRR, formerly Libor, a sovereign rate (e.g., the T-bill rate), or the prime lending rate, also called the "prime rate." The prime rate reflects the rate at which banks lend to their most creditworthy customers, which tends to vary based on the overnight rate at which banks lend to each other plus a spread. Bank loans can be customized to the borrower's needs. They can have different maturities, as well as different interest payment and principal repayment structures. The frequency of interest payments varies among bank loans. Some loans are bullet loans, in which the entire payment of principal occurs at maturity, and others are amortizing loans, in which the principal is repaid over time.

For highly rated companies, both bilateral and syndicated loans can be more expensive than bonds issued in financial markets. Thus, companies often turn to money and capital markets to raise funds, which allows them to diversify their sources of financing.

7.2. Commercial Paper

Commercial paper is a short-term, unsecured promissory note issued in the public market or via a private placement that represents a debt obligation of the issuer. Commercial paper was first issued in the United States more than a century ago. It later appeared in the United Kingdom, in other European countries, and then in the rest of the world.

7.2.1. Characteristics of Commercial Paper

Commercial paper is a valuable source of flexible, readily available, and relatively low-cost short-term financing. It is a source of funding for working capital and seasonal demands for cash. It is also a source of **bridge financing**—that is, interim financing that provides funds until permanent financing can be arranged. Suppose a company wants to build a new distribution center in southeast China and wants to finance this investment with an issuance of long-term bonds. The market conditions for issuing long-term bonds may currently be volatile, which would translate into a higher cost of borrowing. Rather than issuing long-term bonds immediately, the company may opt to raise funds with commercial paper and wait for a more favorable environment in which to sell long-term bonds.

The largest issuers of commercial paper are financial institutions, but some non-financial companies are also regular issuers of commercial paper. Although the focus of this section is on corporate borrowers, sovereign governments and supranational agencies routinely issue commercial paper as well.

The maturity of commercial paper can range from overnight to one year, but a typical issue matures in less than three months.

7.2.2. Credit Quality of Commercial Paper

Traditionally, only the largest, most stable companies issued commercial paper. Although only the strongest, highest-rated companies issue low-cost commercial paper, issuers from across the risk spectrum can issue commercial paper with higher yields than higher-rated companies. Thus, investors in commercial paper are exposed to various levels of credit risk depending on the issuer's creditworthiness. Many investors perform their own credit analysis, but most investors also assess a commercial paper's credit quality by using the ratings provided by credit rating agencies. Exhibit 5 presents the range of commercial paper ratings from the main credit rating agencies. Commercial paper rated adequate or above (the shaded area of Exhibit 5) is called "prime paper," and it is typically considered investment grade by investors.

EXHIBIT 5 Commercial Paper Ratings

Credit Quality	Moody's	S&P	Fitch
Superior	P1	A1+/A1	F1+/F1
Satisfactory	P2	A2	F2
Adequate	P3	A3	F3
Speculative	NP	B/C	F4
Defaulted	NP	D	F5

In most cases, maturing commercial paper is paid with the proceeds of new issuances of commercial paper, a practice referred to as "rolling over the paper." This practice creates a risk that the issuer will be unable to issue new paper at maturity, referred to as rollover risk.

As a safeguard against rollover risk, credit rating agencies often require that commercial paper issuers secure a **backup line of credit** from banks. The purpose of the backup lines of credit is to ensure that the issuer will have access to sufficient liquidity to repay maturing commercial paper if rolling over the paper is not a viable option. Therefore, backup lines of credit are sometimes called "liquidity enhancement" or "backup liquidity lines." Issuers of commercial paper may be unable to roll over the paper because of either market-wide or company-specific events. For example, financial markets could be in the midst of a financial crisis that would make it difficult to roll over the paper. A company could also experience some sort of financial distress such that it could only issue new commercial paper at significantly higher rates. In this case, the company could draw on its credit lines instead of rolling over its paper. Most commercial paper issuers maintain 100% backing, although some large, high-credit-quality issues carry less than 100% backing. Backup lines of credit typically contain a "material adverse change" provision that allows the bank to cancel the backup line of credit if the financial condition of the issuer deteriorates substantially.

Historically, defaults on commercial paper have been relatively rare, primarily because commercial paper has a short maturity. Each time existing paper matures, investors have another opportunity to assess the issuer's financial position, and they can refuse to buy the new paper if they estimate that the issuer's credit risk is too high. Thus, the commercial paper markets adapt more quickly to a change in an issuer's credit quality than do the markets for longer-term securities. This flexibility reduces the exposure of the commercial paper market to defaults. In addition, corporate managers realize that defaulting on commercial paper would likely prevent any future issuance of this valuable financing alternative.

The combination of short-dated maturity, relatively low credit risk, and a large number of issuers makes commercial paper attractive to a diverse range of investors, including money market mutual funds, bank liquidity desks, corporate treasury departments, and other institutional investors that have liquidity constraints. Most commercial paper investors hold their position to maturity. The result is little secondary market trading except for the largest issues. Investors who wish to sell commercial paper prior to maturity can sell it either back to the dealer, to another investor, or in some cases, directly back to the issuer.

The yield on commercial paper is typically higher than that on short-term sovereign bonds of the same maturity for two main reasons. First, commercial paper is exposed to credit risk unlike most highly rated sovereign bonds. Second, commercial paper markets are generally less liquid than short-term sovereign bond markets. Thus, investors require higher yields to compensate for the lower liquidity. In the United States, the yield on commercial paper also tends to be higher than that on short-term municipal bonds for tax reasons. Income generated by investments in commercial paper is usually subject to income taxes, whereas income from many municipal bonds is tax exempt. Thus, to attract taxable investors, bonds that are subject to income taxes must offer higher yields than those that are tax exempt.

7.2.3. US Commercial Paper vs. Eurocommercial Paper

The US commercial paper (USCP) market is the largest commercial paper market in the world, although there are other active commercial paper markets in other countries. Commercial paper issued in the international market is known as Eurocommercial paper (ECP). Although ECP is a similar instrument to USCP, there are some differences between the two. These differences are shown in Exhibit 6.

EXHIBIT 6 USCP vs. ECP

Feature	US Commercial Paper	Eurocommercial Paper
Currency	US dollar	Any currency
Maturity	Overnight to 270 days[a]	Overnight to 364 days
Interest	Discount basis (instrument is issued at a discount to par value)	Interest-bearing (instrument issued at par and pays interest) or discount basis
Settlement	$T + 0$ (trade date)	$T + 2$ (trade date plus two days)
Negotiable	Can be sold to another party	Can be sold to another party

[a]In the United States, securities with an original maturity greater than 270 days must be registered with the Securities and Exchange Commission (SEC). To avoid the time and expense associated with a SEC registration, issuers of US commercial paper rarely offer maturities longer than 270 days.

A difference between USCP and ECP is related to the interest provision. USCP is typically issued on a discount basis—that is, USCP is issued at a discount to par value and pays full par value at maturity. The difference between the par value and the issue price is the interest paid on the borrowing. In contrast, ECP may be issued at, and trade on, an interest-bearing or yield basis or a discount basis. The distinction between the discount and the interest-bearing basis is explored later in the discussion on fixed-income valuation.

Typical transaction sizes in ECP are also much smaller than in USCP, and it is difficult to place longer-term ECP with investors. The ECP market also exhibits less liquidity than the USCP market.

8. CORPORATE DEBT: NOTES AND BONDS

Companies are active participants in global capital markets and regularly issue corporate notes and bonds. These securities can be placed directly with specific investors via private placements or sold in public securities markets. This section discusses various characteristics of corporate notes and bonds.

8.1. Maturities

There is no universally accepted taxonomy as to what constitutes short-, medium-, and long-term maturities. For our purposes, short term refers to original maturities of 5 years or less; intermediate term to original maturities longer than 5 years and up to 12 years; and long term to original maturities longer than 12 years. Those securities with maturities between 1 and 12 years are often considered notes, whereas securities with maturities greater than 12 years are considered bonds. It is not uncommon, however, to refer to bonds for all securities, irrespective of their original maturity.

In practice, most corporate bonds range in term to maturity between 1 and 30 years. In Europe, however, some bond issues have maturities of 40 or 50 years. In addition, companies and sovereigns have issued 100-year bonds; these are called "century bonds." For example, in 2017 Austria issued EUR 3.5 billion in bonds that will mature in 2117.

The first century bond was issued by the Walt Disney Company in 1993 as part of its **medium-term note** program. Medium-term note (MTN) is a misnomer because, as the Disney example illustrates, MTNs can have very long maturities. From the issuer's perspective, the initial purpose of MTNs was to fill the funding gap between commercial paper and long-term bonds. It is for this reason that they are referred to as "medium term." The MTN market can be broken into three segments: short-term securities that carry floating or fixed rates, medium- to long-term securities that primarily bear a fixed rate of interest, and structured notes.

MTNs have the unique characteristic of being securities that are offered continuously to investors by an agent of the issuer. This feature gives the borrower maximum flexibility for issuing securities on a continuous basis. Financial institutions are the primary issuers of MTNs, in particular short-term ones. Life insurance companies, pension funds, and banks are among the largest buyers of MTNs because they can customize the bond issue to their needs and stipulate the amount and characteristics of the securities they want to purchase. These investors are often willing to accept less liquidity than they would receive with a comparable publicly issued bond because the yield is slightly higher. The cost savings in registration and underwriting often makes MTNs a lower-cost option for the issuer.

8.2. Coupon Payment Structures

Corporate notes and bonds have a range of coupon payment structures. Financial and non-financial companies issue conventional coupon bonds that pay a fixed periodic coupon during the bond's life. They also issue bonds for which the periodic coupon payments adjust to changes in market conditions and/or changes to the issuer's credit quality. Such bonds typically offer investors the opportunity to reduce their exposure to a particular type of risk. For example, FRNs, whose coupon payments adjust to changes in the level of market interest rates, are a way to limit interest rate risk; some of the inflation-linked bonds whose coupon payments adjust to changes in the level of a consumer price index offer a protection against inflation risk; credit-linked coupon bonds, whose coupon payments adjust to changes in the issuer's credit quality, are a way to reduce credit risk. Whether the periodic coupon is fixed or not, coupon payments can be made quarterly, semi-annually, or annually depending on the type of bond and where the bonds are issued and traded.

Other coupon payment structures exist. Zero-coupon bonds pay no coupon. Deferred coupon bonds pay no coupon initially but then offer a higher coupon. Payment-in-kind (PIK) coupon bonds make periodic coupon payments, but not necessarily in cash; the issuer may pay interest in the form of securities, such as bonds or common shares. These types of coupon payment structures increase issuers' flexibility regarding the servicing of their debt.

8.3. Principal Repayment Structures

Corporate note or bond issues have either a serial or a term maturity structure. With a **serial maturity structure**, the maturity dates are spread out during the bond's life; a stated number of bonds mature and are paid off each year before final maturity. With a **term maturity structure**, the bond's notional principal is paid off in a lump sum at maturity. Because there is no regular repayment of the principal outstanding throughout the bond's life, a term maturity structure carries more credit risk than a serial maturity structure.

A sinking fund arrangement is a way to reduce credit risk by making the issuer set aside funds over time to retire the bond issue. For example, a corporate bond issue may require a specified percentage of the bond's outstanding principal amount to be retired each year. The issuer may satisfy this requirement in one of two ways. The most common approach is for the issuer to make a random call for the specified percentage of bonds that must be retired and to pay the bondholders whose bonds are called the sinking fund price, which is typically par. Alternatively, the issuer can deliver bonds to the trustee with a total amount equal to the amount that must be retired. To do so, the issuer may purchase the bonds in the open market. The sinking fund arrangement on a term maturity structure accomplishes the same goal as the serial maturity structure—that is, both result in a portion of the bond issue being paid off each year. With a serial maturity structure, however, the bondholders know which bonds will mature and will thus be paid off each year. In contrast, the bonds retired annually with a sinking fund arrangement are designated by a random drawing.

8.4. Asset or Collateral Backing

Unlike most highly rated sovereign bonds, all corporate debt is exposed to varying degrees of credit risk. Thus, corporate debt is structured with this risk in mind. An important consideration for investors is seniority ranking—that is, the systematic way in which lenders are repaid if the issuer defaults. In the case of secured debt, there is some form of collateral pledged to ensure payment of the debt. In contrast, in the case of unsecured debt, claims are settled by the general assets of the company in accordance with the priority of payments that applies either legally or contractually and as described in the bond indenture. Within each category of debt (secured and unsecured), there are finer gradations of rankings, which are discussed later in the curriculum coverage of credit analysis.

There is a wide range of bonds that are secured by some form of collateral. Companies that need to finance equipment or physical assets may issue equipment trust certificates. Corporate issuers also sell collateral trust bonds that are secured by securities, such as common shares, bonds, or other financial assets. Banks, particularly in Europe, may issue covered bonds, which are a type of debt obligation that is secured by a segregated pool of assets. Asset-backed securities are also secured forms of debt.

Companies can and do default on their debt. Debt secured by collateral may still experience losses, but during a company's bankruptcy proceedings, investors in secured debt usually fare better than investors in unsecured debt. Investors who face a higher level of credit risk typically require a higher yield than investors exposed to very little credit risk.

8.5. Contingency Provisions

Contingency provisions are clauses in the indenture that provide the issuer or the bondholders rights that affect the disposal or redemption of the bond. The three commonly used contingency provisions are call, put, and conversion provisions.

Callable bonds give issuers the ability to retire debt prior to maturity. The most compelling reason for them to do so is to take advantage of lower borrowing rates. By calling the bonds before their maturity date, the issuer can substitute a new, lower-cost bond issue for an older, higher-cost one. In addition, companies may also retire debt to eliminate restrictive covenants or to alter their capital structure to improve flexibility. Because the call provision is a valuable option for the issuer, investors demand compensation *ex ante* (before investing in the bond). Thus, other things equal, investors require a higher yield (and thus pay a lower price) for a callable bond than for an otherwise similar non-callable bond.

Companies also issue putable bonds, which give the bondholders the right to sell the bond back to the issuer at a pre-determined price on specified dates before maturity. Most putable bonds pay a fixed rate of interest, although some bonds may have step-up coupons that increase by specified margins at specified dates. Because the put provision is a valuable option for the bondholders, putable bonds offer a lower yield (and thus have a higher price) than otherwise similar non-putable bonds. The main corporate issuers of putable bonds are investment-grade companies. Putable bonds may offer them a cheaper way of raising capital, especially if the company estimates that the benefit of a lower coupon outweighs the risk associated with the put provision.

A convertible bond is a hybrid security that lies on a continuum between debt and equity. It consists of a long position in an option-free bond and a conversion option that gives the bondholder the right to convert the bond into a specified number of shares of the issuer's common shares. From the issuer's point of view, convertible bonds make it possible to raise funds that may not be possible without the incentive associated with the conversion option. The more common issuers of convertibles bonds are newer companies that have not established a presence in debt capital markets but who are able to present a more attractive package to institutional investors by including an equity upside potential. Established issuers of bonds may also prefer to issue convertible bonds because they are usually sold at a lower coupon rate than otherwise similar non-convertible bonds as a result of investors' attraction to the conversion provision. There is a potential equity dilution effect, however, if the bonds are converted. From the investor's point of view, convertible bonds represent a means of accessing the equity upside potential of the issuer but at a lower risk–reward profile, because there is the floor of the coupon payments in the meantime.

8.6. Issuance, Trading, and Settlement

In the era before electronic settlement, there were some differences in the processes of issuing and settling corporate bonds depending on where the securities were registered. This is no longer the case; the processes of issuing and settling bonds are now essentially the same globally. New corporate bond issues are usually sold to investors by investment banks acting as underwriters in the case of underwritten offerings or brokers in the case of best-efforts offerings. They are then settled via the local settlement system. These local systems typically possess a "bridge" to the two Eurobond systems, Euroclear and Clearstream. As for Eurobonds from the corporate sector, they are all issued, traded, and settled in the same way, irrespective of the issuer and its local jurisdiction.

Most bond prices are quoted in basis points. The vast majority of corporate bonds are traded in OTC markets through dealers that "make a market" in bonds and sell from their inventory. Dealers do not typically charge a commission or a transaction fee. Instead, they earn a profit from the bid–offer spread.

For corporate bonds, settlement differences exist primarily between new bond issues and the secondary trading of bonds. The issuing phase for an underwritten offering usually takes several days. Thus, settlement takes longer for new bond issued than for the secondary trading of bonds, for which settlement is typically on a $T + 2$ or $T + 3$ basis.

EXAMPLE 6 Corporate Debt

1. A loan made by a group of banks to a private company is *most likely*:
   A. a bilateral loan.
   B. a syndicated loan.
   C. a securitized loan.

2. Which of the following statements relating to commercial paper is *most accurate*? Companies issue commercial paper:
   A. only for funding working capital.
   B. only as an interim source of financing.
   C. both for funding working capital and as an interim source of funding.
3. Maturities of Eurocommercial paper range from:
   A. overnight to three months.
   B. overnight to one year.
   C. three months to one year.
4. A bond issue that has a stated number of bonds that mature and are paid off each year before final maturity *most likely* has a:
   A. term maturity.
   B. serial maturity.
   C. sinking fund arrangement.

Solution to 1: B is correct. A loan from a group of lenders to a single borrower is a syndicated loan. A is incorrect because a bilateral loan is a loan from a single lender to a single borrower. C is incorrect because securitization involves moving assets, such as loans, from the owner of the assets into a special legal entity.

Solution to 2: C is correct. Companies use commercial paper as a source of funding working capital and seasonal demand for cash, as well as an interim source of financing until permanent financing can be arranged.

Solution to 3: B is correct. Eurocommercial paper ranges in maturity from overnight to 364 days.

Solution to 4: B is correct. With a serial maturity structure, a stated number of bonds mature and are paid off each year before final maturity. A is incorrect because a bond issue with a term maturity structure is paid off in one lump sum at maturity. C is incorrect because a sinking fund arrangement, such as a serial maturity structure, results in a portion of the bond issue being paid off every year. With a serial maturity structure, however, the bonds are paid off because the maturity dates are spread out during the life of the bond and the bonds that are retired are maturing; the bondholders know in advance which bonds will be retired. In contrast, the bonds retired annually with a sinking fund arrangement are designated by a random drawing.

9. STRUCTURED FINANCIAL INSTRUMENTS

- **describe structured financial instruments**

Structured financial instruments represent a broad sector of financial instruments. This sector includes asset-backed securities (ABS) and collateralized debt obligations (CDOs). CDOs are securities backed by a diversified pool of one or more debt obligations, and like ABS, they are discussed later in curriculum coverage on asset-backed securities. A common attribute of all these financial instruments is that they repackage and redistribute risks.

Our focus in this section is on structured financial instruments apart from ABS and CDOs. These instruments typically have customized structures that often combine a bond and at least one derivative. Some of these instruments are called structured products. The use of derivatives gives the holder of the structured financial instrument exposure to one or more underlying assets, such as equities, bonds, and commodities. The redemption value and often the coupons of structured financial instruments are linked via a formula to the performance of the underlying asset(s). Thus, the bond's payment features are replaced with non-traditional payoffs that are derived not from the issuer's cash flows but from the performance of the underlying asset(s). Although no universally accepted taxonomy exists to categorize structured financial instruments, we will present four broad categories of instruments: capital protected, yield enhancement, participation, and leveraged instruments.

9.1. Capital Protected Instruments

Suppose an investor has $100,000 to invest. The investor buys zero-coupon bonds issued by a sovereign issuer that will pay off $100,000 one year from now. Also suppose the cost of buying the zero-coupon bonds is $99,000. The investor can use the $1,000 left over from the purchase of the zero-coupon bond to buy a call or put option on some underlying asset that expires one year from now. In this context, buying a call (put) option gives the investor the right to buy (sell) the underlying asset in one year at a pre-determined price. The investor will receive $100,000 when the zero-coupon bond matures and may also gain from the upside potential of the option, if any. This combination of the zero-coupon bond and the call option can be pre-packaged as a structured financial instrument called a **guarantee certificate**. The zero-coupon bond provides the investor capital protection; at maturity, the investor will receive 100% of the capital invested even if the option expires worthless. The call (put) option provides upside potential if the price of the underlying asset rises (falls) and a limited downside if the price of the underlying asset falls (rises) or remains unchanged. The downside is limited to the price, often called the premium, paid for the call or put option. In our example, the maximum loss the investor faces is $1,000, which is the price paid for the option.

Capital protected instruments offer different levels of capital protection. A guarantee certificate offers full capital protection. Other structured financial instruments may offer only partial capital protection. Note that the capital protection is only as good as the issuer of the instrument. Should the issuer of guarantee certificates go bankrupt, investors may lose their entire capital. These instruments are naturally more prevalent when the general level of interest rates is high and/or implied volatilities (the key driver of option prices, as we will explain later) are low. The cost of the options in the structure must be absorbed by the forfeited interest.

9.2. Yield Enhancement Instruments

Yield enhancement refers to increasing risk exposure in the hope of realizing a higher expected return. A **credit-linked note (CLN)** is an example of a yield enhancement instrument. Specifically, it is a type of bond that pays regular coupons but whose redemption value depends on the occurrence of a well-defined credit event, such as a rating downgrade or the default of an underlying asset, called the reference asset. If the specified credit event does not occur, the investor receives the par value of the CLN at maturity. But if the specified credit event occurs, the investor receives the par value of the CLN minus the nominal value of the reference asset to which the CLN is linked.

A CLN allows the issuer to transfer the effect of a credit event to investors. Thus, the issuer is the protection buyer and the investor is the protection seller. Investors are willing to buy CLNs because these securities offer higher coupons than otherwise similar bonds. In addition, CLNs are usually issued at a discount. Thus, if the specified credit event does not occur, investors will realize a significant capital gain on the purchase of the CLN.

9.3. Participation Instruments

As the name suggests, a participation instrument is one that allows investors to participate in the return of an underlying asset. Floating-rate bonds can be viewed as a type of participation instrument. As discussed earlier, floaters differ from fixed-rate bonds in that their coupon rate adjusts periodically according to a pre-specified formula. The coupon formula is usually expressed as a reference rate adjusted for a spread. A floater has almost zero interest rate risk because changes in the cash flows limit the effect of changes in interest rates on the floater's price. Thus, floaters give investors the opportunity to participate in movements of interest rates. For example, the Italian government issued in June 2005 floaters set to mature in June 2020. The coupon payments are delivered annually and determined by the formula of 85% of the 10-year constant maturity swap rate, a widely used type of interest rate. Thus, investors who hold these floaters participate partially in movements of the 10-year constant maturity swap rate.

Most participation instruments are designed to give investors indirect exposure to a specific index or asset price. For example, investors who are precluded from investing in equity directly may gain indirect equity exposure by investing in participation instruments that are linked via a formula to the performance of equity indexes. Many structured products sold to individuals are participation instruments linked to an equity index. In contrast to capital protected instruments that offer equity exposure, these participation instruments usually do not offer capital protection.

9.4. Leveraged Instruments

Leveraged instruments are structured financial instruments created to magnify returns and offer the possibility of high payoffs from small investments. An **inverse floater** is an example of a leveraged instrument. As the name suggests, an inverse floater is the opposite of a traditional floater. The cash flows are adjusted periodically and move in the opposite direction of changes in the reference rate. So, when the reference rate decreases, the coupon payment of an inverse floater increases.

A general formula for an inverse floater's coupon rate is as follows:

$$\text{Inverse floater coupon rate} = C - (L \times R)$$

where C is the maximum coupon rate reached if the reference rate is equal to zero, L is the coupon leverage, and R is the reference rate on the reset date. Note that the coupon leverage indicates the multiple that the coupon rate will change in response to a 100-bp change in the reference rate. For example, if the coupon leverage is three, the inverse floater's coupon rate will decrease by 300 bps when the reference rate increases by 100 bps.

Inverse floaters with a coupon leverage greater than zero but lower than one are called deleveraged inverse floaters. Inverse floaters with a coupon leverage greater than one are called leveraged inverse floaters. For example, the Barclays Bank PLC issued a 15-year bond in January 2010 having quarterly payments. The coupon rate was fixed at 7.50% for the first three years and then in January 2013 transformed into a leveraged inverse floater paying 7.50% *minus* the euro three-month Libor. In this case, the coupon leverage is one. Thus, for a 100 bps increase in the euro three-month Libor, the coupon rate of the leveraged inverse floater will decrease by 100 bps. Inverse floaters often have a floor that specifies a minimum coupon rate; for example, a floor may be set at zero to avoid the possibility of a negative interest rate. This inverse floater does not have a maximum coupon rate. At the July 2018 reset, euro three-month Libor was –0.32%, so the coupon rate for the following quarter was 7.82%. Note that the example involves a bond issued in the past, and thus Libor was the reference rate. Future bonds issued will reference a different rate, including prime or MRR.

EXAMPLE 7 Structured Financial Instruments

1. If an investor holds a credit-linked note and the credit event does not occur, the investor receives:
   A. all promised cash flows as scheduled.
   B. all coupon payments as scheduled but not the par value at maturity.
   C. all coupon payments as scheduled and the par value minus the nominal value of the reference asset to which the credit-linked note is linked at maturity.
2. A structured financial instrument whose coupon rate is determined by the formula 5% – (0.5 × Libor) is *most likely*:
   A. a leveraged inverse floater.
   B. a participation instrument.
   C. a deleveraged inverse floater.

Solution to 1: A is correct. If the credit event does not occur, the issuer must make all promised cash flows as scheduled—that is, the regular coupon payments and the par value at maturity.

Solution to 2: C is correct. A structured financial instrument whose coupon rate moves in the opposite direction of the reference rate is called an inverse floater. Because the coupon leverage (0.5) is greater than zero but lower than one, the structured financial instrument is a deleveraged inverse floater. In this example, if the reference rate increases by 100 bps, the coupon rate decreases by 50 bps. A is incorrect because the coupon leverage would have to be higher than one for the structured financial instrument to be a leveraged inverse floater. B is incorrect because a participation instrument is designed to give investors indirect exposure to a particular underlying asset.

10. SHORT-TERM BANK FUNDING ALTERNATIVES

- **describe short-term funding alternatives available to banks**

Funding refers to the amount of money or resources necessary to finance some specific project or enterprise. Accordingly, funding markets are markets in which debt issuers borrow to meet their financial needs. Companies have a range of funding alternatives, including bank loans, commercial paper, notes, and bonds. Financial institutions such as banks have larger financing needs than non-financial companies because of the nature of their operations. This section discusses the additional funding alternatives available to them. The majority of these funding alternatives have short maturities.

Banks, such as deposit-taking (or depository) institutions, typically have access to funds obtained from the retail market—that is, deposit accounts from their customers. It is quite common, however, for banks to originate more loans than they have retail deposits. Thus, whenever the amount of retail deposits is insufficient to meet their financial needs, banks also need to raise funds from the wholesale market. Wholesale funds include central bank funds, interbank deposits, and certificates of deposit. In addition to filling the gaps between loans and deposits, banks raise wholesale funds to minimize their funding cost. At the margin, wholesale funds may be less expensive (in terms of interest expense) than deposit funding. Finally, financial institutions may raise wholesale funds as a balance sheet risk management tool to reduce interest rate risk, as discussed earlier.

10.1. Retail Deposits

One of the primary sources of funding for deposit-taking banks is their retail deposit base, which includes funds from both individual and commercial depositors. There are several types of retail deposit accounts. Demand deposits, also known as checking accounts, are available to customers "on demand." Depositors have immediate access to the funds in their deposit accounts and use the funds as a form of payment for transactions. Because the funds are available immediately, deposit accounts typically pay no interest. In contrast, savings accounts pay interest and allow depositors to accumulate wealth in a very liquid form, but they do not offer the same transactional convenience as demand deposits. Money market accounts were originally designed to compete with money market mutual funds. They offer money market rates of return and depositors can access funds at short or no notice. Thus, money market accounts are, for depositors, an intermediate between demand deposit and savings accounts.

10.2. Short-Term Wholesale Funds

Wholesale funds available for banks include reserve funds, interbank funds, and certificates of deposit.

10.2.1. Reserve Funds

Many countries require deposit-taking banks to place a reserve balance with the national central bank. The reserve funds help to ensure sufficient liquidity should depositors require withdrawal of funds. When a bank cannot obtain short-term funding, most countries allow that bank to borrow from the central bank. In aggregate, the reserve funds act as a liquidity buffer, providing comfort to depositors and investors that the central bank can act as lender of last resort.

Treatment of interest on reserve funds varies among countries, from a low interest payment, to no interest payment, to charges for keeping reserve funds. Additionally, there is an opportunity cost to the banks for holding reserves with the central bank, in that these funds cannot be invested with higher interest or loaned out to consumers or commercial enterprises. Some banks have an excess over the minimum required funds to be held in reserve. At the same time, other banks run short of required reserves. This imbalance is solved through the **central bank funds market**, which allows banks that have a surplus of funds to lend money to banks that need funds for maturities of up to one year. These funds are known as central bank funds and are called "overnight funds" when the maturity is one day and "term funds" when the maturity ranges from two days to one year. The interest rates at which central bank funds are bought (i.e., borrowed) and sold (i.e., lent) are short-term interest rates determined by the markets but influenced by the central bank's open market operations. These rates are termed the **central bank funds rates**.

In the United States, the central bank is the Federal Reserve (Fed). The central bank funds and funds rate are called Fed funds and the Fed funds rate, respectively. Other short-term interest rates, such as the yields on Treasury bills, are highly correlated with the Fed funds rate. The most widely followed rate is known as the Fed funds effective rate, which is the volume-weighted average of rates for Fed fund trades arranged throughout the day by the major New York City brokers. Fed funds are traded between banks and other financial institutions globally and may be transacted directly or through money market brokers.

10.2.2. Interbank Funds

The **interbank market** is the market of loans and deposits between banks. The term to maturity of an interbank loan or deposit ranges from overnight to one year. The rate on an interbank loan or deposit can be quoted relative to a reference rate, such as an interbank offered rate, or as a fixed interest rate. An interbank deposit is unsecured, so banks placing deposits with another bank need to have an interbank line of credit in place for that institution. Usually, a large bank will make a two-way price, indicating the rate at which it will lend funds and the rate at which it will borrow funds for a specific maturity, on demand. Interest on the deposit is payable at maturity. Much interbank dealing takes place on the Reuters electronic dealing system, so that the transaction is done without either party speaking to the other.

Because the market is unsecured, it is essentially based on confidence in the banking system. At times of stress, such as in the aftermath of the Lehman Brothers' bankruptcy in 2008, the market is prone to "dry up" as banks withdraw from funding other banks.

10.2.3. Large-Denomination Negotiable Certificates of Deposit

A **certificate of deposit** (CD) is an instrument that represents a specified amount of funds on deposit for a specified maturity and interest rate. CDs are an important source of funds for financial institutions. A CD may take one of two forms: non-negotiable or negotiable. If the CD is non-negotiable, the deposit plus the interest are paid to the initial depositor at maturity. A withdrawal penalty is imposed if the depositor withdraws funds prior to the maturity date.

Alternatively, a negotiable CD allows any depositor (initial or subsequent) to sell the CD in the open market prior to the maturity date. Negotiable CDs were introduced in the United States in the early 1960s when various types of deposits were constrained by interest rate ceilings. At the time, bank deposits were not an attractive investment because investors earned a below-market interest rate unless they were prepared to commit their capital for an extended period. The introduction of negotiable CDs enabled bank customers to buy a three-month or longer negotiable instrument yielding a market interest rate and to recover their investment by selling it in the market. This innovation helped banks increase the amount of funds raised in the money markets. It also fostered competition among deposit-taking institutions.

There are two types of negotiable CDs: large-denomination CDs and small-denomination CDs. Thresholds between small- and large-denomination CDs vary among countries. For example, in the United States, large-denomination CDs are usually issued in denominations of $1 million or more. Small-denomination CDs are a retail-oriented product, and they are of secondary importance as a funding alternative. Large-denomination CDs, in contrast, are an important source of wholesale funds and are typically traded among institutional investors.

Like other money market securities, CDs are available in domestic bond markets as well as in the Eurobond market. Most CDs have maturities shorter than one year and pay interest at maturity. CDs with longer maturities are called "term CDs."

Yields on CDs are driven primarily by the credit risk of the issuing bank and to a lesser extent by the term to maturity. The spread attributable to credit risk will vary with economic conditions and confidence in the banking system in general and in the issuing bank in particular. As with all debt instruments, spreads widen during times of financial turmoil in response to increased risk aversion.

11. REPURCHASE AND REVERSE REPURCHASE AGREEMENTS

- **describe repurchase agreements (repos) and the risks associated with them**

Repurchase agreements are another important source of funding not only for banks but also for other market participants. A **repurchase agreement** or **repo** is the sale of a security with a simultaneous agreement by the seller to buy the same security back from the purchaser at an agreed-on price and future date. In practical terms, a repurchase agreement can be viewed as a collateralized loan in which the security sold and subsequently repurchased represents the collateral posted. One party is borrowing money and providing collateral for the loan at an interest rate that is typically lower than on an otherwise similar bank loan. The other party is lending money while accepting a security as collateral for the loan.

Repurchase agreements are a common source of money market funding for dealer firms in many countries. An active market in repurchase agreements underpins every liquid bond market. Financial and non-financial companies participate actively in the market as both sellers and buyers of collateral depending on their circumstances. Central banks are also active users of repurchase agreements in their daily open market operations; they either lend to the market to increase the supply of funds or withdraw surplus funds from the market.

11.1. Structure of Repurchase and Reverse Repurchase Agreements

Suppose a government securities dealer purchases a 2.25% UK gilt that matures in three years. The dealer wants to fund the position overnight through the end of the next business day. The dealer could finance the transaction with its own funds, which is what other market participants, such as insurance companies or pension funds, may do in similar circumstances. But a securities dealer typically uses leverage (debt) to fund the position. Rather than borrowing from a bank, the dealer uses a repurchase agreement to obtain financing by using the gilt as collateral for the loan.

A repurchase agreement may be constructed as follows: The dealer sells the 2.25% UK gilt that matures in three years to a counterparty for cash today. At the same time, the dealer makes a promise to buy the same gilt the next business day for an agreed-on price. The price at which the dealer repurchases the gilt is known as the **repurchase price**. The date when the gilt is repurchased, the next business day in this example, is called the **repurchase date**. When the term of a repurchase agreement is one day, it is called an "overnight repo." When the agreement is for more than one day, it is called a "term repo." An agreement lasting until the final maturity date is known as a "repo to maturity."

As in any borrowing or lending transaction, the interest rate of the loan must be negotiated in the agreement. The interest rate on a repurchase agreement is called the **repo rate**. Several factors affect the repo rate:

- The *risk* associated with the collateral. Repo rates are typically lower for highly rated collaterals, such as highly rated sovereign bonds. They increase with the level of credit risk associated with the collateral underlying the transaction.
- The *term* of the repurchase agreement. Repo rates generally increase with maturity because long-term rates are typically higher than short-term rates in normal circumstances.
- The *delivery requirement* for the collateral. Repo rates are usually lower when delivery to the lender is required.
- The *supply and demand conditions* of the collateral. The more in demand a specific piece of collateral is, the lower the repo rate against it because the borrower has a security that lenders of cash want for specific reasons, perhaps because the underlying issue is in great demand. The demand for such collateral means that it is considered to be "on special." Collateral that is not special is known as "general collateral." The party that has a need for collateral that is on special is typically required to lend funds at a below-market repo rate to obtain the collateral.
- The *interest rates of alternative financing* in the money market.

The interest on a repurchase agreement is paid on the repurchase date—that is, at the termination of the agreement. Note that any coupon paid by the security during the repurchase agreement belongs to the seller of the security (i.e., the borrower of cash).

When a repurchase agreement is viewed through the lens of the cash lending counterparty, the transaction is referred to as a **reverse repurchase agreement** or **reverse repo**. In the foregoing example, the counterparty agrees to buy the 2.25% UK gilt that matures in three years and promises to sell it back the next business day at the agreed-on price. The counterparty is making a collateralized loan to the dealer. Reverse repurchase agreements are very often used to borrow securities to cover short positions.

The question of whether a particular transaction is labeled a repurchase agreement or a reverse repurchase agreement depends on one's point of view. Standard practice is to view the transaction from the dealer's perspective. If the dealer is borrowing cash from a counterparty and providing securities as collateral, the transaction is termed a repurchase agreement. If the dealer is borrowing securities and lending cash to the counterparty, the transaction is termed a reverse repurchase agreement.

11.2. Credit Risk Associated with Repurchase Agreements

Each market participant in a repurchase agreement is exposed to the risk that the counterparty defaults, regardless of the collateral exchanged. Credit risk is present even if the collateral is a highly rated sovereign bond. Suppose that a dealer (i.e., the borrower of cash) defaults and is not in a position to repurchase the collateral on the specified repurchase date. The lender of funds takes possession of the collateral and retains any income owed to the borrower. The risk is that the price of the collateral has fallen following the inception of the repurchase agreement, causing the market value of the collateral to be lower than the unpaid repurchase price. Conversely, suppose the investor (i.e., the lender of cash) defaults and is unable to deliver the collateral on the repurchase date. The risk is that the price of the collateral has risen since the inception of the repurchase agreement, resulting in the dealer now holding an amount of cash lower than the market value of the collateral. In this case, the investor is liable for any excess of the price paid by the dealer for replacement of the securities over the repurchase price.

Although both parties to a repurchase agreement are subject to credit risk, the agreement is structured as if the lender of funds is the most vulnerable party. Specifically, the amount lent is lower than the collateral's market value. The difference between the market value of the security used as collateral and the value of the loan is known as the **repo margin**, although the term **haircut** is more commonly used, particularly in the United States. The repo margin allows for some worsening in market value and thus provides the cash lender a margin of safety if the collateral's market value declines. Repo margins vary by transaction and are negotiated bilaterally between the counterparties. The level of margin is a function of the following factors:

- The *length* of the repurchase agreement. The longer the repurchase agreement, the higher the repo margin.
- The *quality* of the collateral. The higher the quality of the collateral, the lower the repo margin.
- The *credit quality* of the counterparty. The higher the creditworthiness of the counterparty, the lower the repo margin.
- The *supply and demand conditions* of the collateral. Repo margins are lower if the collateral is in short supply or if there is a high demand for it.

EXAMPLE 8 Short-Term Funding Alternatives Available to Banks

1. Which of the following are **not** considered wholesale funds?
   A. Interbank funds
   B. Central bank funds
   C. Repurchase agreements
2. A large-denomination negotiable certificate of deposit *most likely*:
   A. is traded in the open market.
   B. is purchased by retail investors.
   C. has a penalty for early withdrawal of funds.
3. From the dealer's viewpoint, a repurchase agreement is *best* described as a type of:
   A. collateralized short-term lending.
   B. collateralized short-term borrowing.
   C. uncollateralized short-term borrowing.
4. The interest on a repurchase agreement is known as the:
   A. repo rate.
   B. repo yield.
   C. repo margin.
5. The level of repo margin is higher:
   A. the higher the quality of the collateral.
   B. the higher the credit quality of the counterparty.
   C. the longer the length of the repurchase agreement.

Solution to 1: C is correct. Wholesale funds refer to the funds that financial institutions lend to and borrow from each other. They include central bank funds, interbank funds, and certificates of deposit. Although repurchase agreements are an important source of funding for banks, they are not considered wholesale funds.

Solution to 2: A is correct. Large-denomination negotiable CDs can be traded in the open market. B is incorrect because it is small-denomination, not large-denomination, negotiable CDs that are primarily purchased by retail investors. C is incorrect because it is non-negotiable, not negotiable, CDs that have a penalty for early withdrawal of funds.

Solution to 3: B is correct. In a repurchase agreement, a security is sold with a simultaneous agreement by the seller to buy the same security back from the purchaser later at a higher price. Thus, a repurchase agreement is similar to a collateralized short-term borrowing in which the security sold and subsequently repurchased represents the collateral posted. A is incorrect because collateralized short-term lending is a description of a reverse repurchase agreement. C is incorrect because a repurchase agreement involves collateral. Thus, it is a collateralized, not uncollateralized, short-term borrowing.

Solution to 4: A is correct. The repo rate is the interest rate on a repurchase agreement. B is incorrect because the interest on a repurchase agreement is known as the repo rate, not repo yield. C is incorrect because the repo margin refers to the difference between the market value of the security used as collateral and the value of the loan.

Solution to 5: C is correct. The longer the length of the repurchase agreement, the higher the repo margin (haircut). A is incorrect because the higher the quality of the collateral, the lower the repo margin. B is incorrect because the higher the credit quality of the counterparty, the lower the repo margin.

SUMMARY

Debt financing is an important source of funds for households, governments, government-related entities, financial institutions, and non-financial companies. Well-functioning fixed-income markets help ensure that capital is allocated efficiently to its highest and best use globally. Important points include the following:

- The most widely used ways of classifying fixed-income markets include the type of issuer; the bonds' credit quality, maturity, currency denomination, and type of coupon; and where the bonds are issued and traded.
- Based on the type of issuer, the four major bond market sectors are the household, non-financial corporate, government, and financial institution sectors.
- Investors distinguish between investment-grade and high-yield bond markets based on the issuer's credit quality.
- Money markets are where securities with original maturities ranging from overnight to one year are issued and traded, whereas capital markets are where securities with original maturities longer than one year are issued and traded.
- The majority of bonds are denominated in either euros or US dollars.
- Investors distinguish between bonds that pay a fixed rate versus a floating rate of interest. The coupon rate of floating-rate bonds is often expressed as a reference rate plus a spread. Interbank offered rates, such as Libor, historically have been the most commonly used

reference rates for floating-rate debt and other financial instruments but are being phased out to be replaced by alternative reference rates.

- Based on where the bonds are issued and traded, investors distinguish between domestic and international bond markets. The latter includes the Eurobond market, which falls outside the jurisdiction of any single country and is characterized by less reporting, regulatory, and tax constraints. Investors also distinguish between developed and emerging bond markets.
- Investors and investment managers use fixed-income indexes to describe bond markets or sectors and to evaluate performance of investments and investment managers.
- The largest investors in bonds include central banks; institutional investors, such as pension funds, hedge funds, charitable foundations and endowments, insurance companies, mutual funds and ETFs, and banks; and retail investors, typically by means of indirect investments.
- Primary markets are markets in which issuers first sell bonds to investors to raise capital. Secondary markets are markets in which existing bonds are subsequently traded among investors.
- There are two mechanisms for issuing a bond in primary markets: a public offering, in which any member of the public may buy the bonds, or a private placement, in which only an investor or small group of investors may buy the bonds either directly from the issuer or through an investment bank.
- Public bond issuing mechanisms include underwritten offerings, best-efforts offerings, shelf registrations, and auctions.
- When an investment bank underwrites a bond issue, it buys the entire issue and takes the risk of reselling it to investors or dealers. In contrast, in a best-efforts offering, the investment bank serves only as a broker and sells the bond issue only if it is able to do so. Underwritten and best-efforts offerings are frequently used in the issuance of corporate bonds.
- The underwriting process typically includes six phases: the determination of the funding needs, the selection of the underwriter, the structuring and announcement of the bond offering, pricing, issuance, and closing.
- A shelf registration is a method for issuing securities in which the issuer files a single document with regulators that describes and allows for a range of future issuances.
- An auction is a public offering method that involves bidding and is helpful both in providing price discovery and in allocating securities. Auctions are frequently used in the issuance of sovereign bonds.
- Most bonds are traded in OTC markets, and institutional investors are the major buyers and sellers of bonds in secondary markets.
- Sovereign bonds are issued by national governments primarily for fiscal reasons. These bonds take different names and forms depending on where they are issued, their maturities, and their coupon types. Most sovereign bonds are fixed-rate bonds, although some national governments also issue floating-rate bonds and inflation-linked bonds.
- Local governments, quasi-government entities, and supranational agencies issue bonds, which are named non-sovereign, quasi-government, and supranational bonds, respectively.
- Companies raise debt in the form of bilateral loans, syndicated loans, commercial paper, notes, and bonds.
- Commercial paper is a short-term unsecured security that companies use as a source of short-term and bridge financing. Investors in commercial paper are exposed to credit risk, although defaults are rare. Many issuers roll over their commercial paper on a regular basis.
- Corporate bonds and notes take different forms depending on the maturities, coupon payment, and principal repayment structures. Important considerations also include collateral backing and contingency provisions.

- Medium-term notes are securities that are offered continuously to investors by an agent of the issuer. They can have short-term or long-term maturities.
- The structured finance sector includes asset-backed securities, collateralized debt obligations, and other structured financial instruments. All of these seemingly disparate financial instruments share the common attribute of repackaging risks.
- Many structured financial instruments are customized instruments that often combine a bond and at least one derivative. The redemption and often the coupons of these structured financial instruments are linked via a formula to the performance of the underlying asset(s). Thus, the bond's payment features are replaced with non-traditional payoffs derived not from the issuer's cash flows but from the performance of the underlying asset(s). Capital protected, yield enhancement, participation and leveraged instruments are typical examples of structured financial instruments.
- Financial institutions have access to additional sources of funds, such as retail deposits, central bank funds, interbank funds, large-denomination negotiable certificates of deposit, and repurchase agreements.
- A repurchase agreement is similar to a collateralized loan. It involves the sale of a security (the collateral) with a simultaneous agreement by the seller (the borrower) to buy back the same security from the purchaser (the lender) at an agreed-on price in the future. Repurchase agreements are a common source of funding for dealer firms and are also used to borrow securities to implement short positions.

PRACTICE PROBLEMS

1. The distinction between investment-grade debt and non-investment-grade debt is *best* described by differences in:
   A. tax status.
   B. credit quality.
   C. maturity dates.
2. A bond issued internationally, outside the jurisdiction of the country in whose currency the bond is denominated, is *best* described as a:
   A. Eurobond.
   B. foreign bond.
   C. municipal bond.
3. When classified by type of issuer, asset-backed securities are part of the:
   A. corporate sector.
   B. structured finance sector.
   C. government and government-related sector.
4. Compared with developed market bonds, emerging market bonds *most likely*:
   A. offer lower yields.
   B. exhibit higher risk.
   C. benefit from lower growth prospects.
5. With respect to floating-rate bonds, a reference rate (such as MRR) is *most likely* used to determine the bond's:
   A. spread.
   B. coupon rate.
   C. frequency of coupon payments.

6. The variability of the coupon rate on a Libor-based floating-rate bond is *most likely* caused by:
   A. periodic resets of the reference rate.
   B. market-based reassessments of the issuer's creditworthiness.
   C. changing estimates by the Libor administrator of borrowing capacity.
7. Which of the following statements is *most accurate*? An interbank offered rate:
   A. is a single reference rate.
   B. applies to borrowing periods of up to 10 years.
   C. is used as a reference rate for interest rate swaps.
8. An investment bank that underwrites a bond issue *most likely*:
   A. buys and resells the newly issued bonds to investors or dealers.
   B. acts as a broker and receives a commission for selling the bonds to investors.
   C. incurs less risk associated with selling the bonds than in a best-efforts offering.
9. In major developed bond markets, newly issued sovereign bonds are *most* often sold to the public via a(n):
   A. auction.
   B. private placement.
   C. best-efforts offering.
10. Which of the following describes privately placed bonds?
   A. They are non-underwritten and unregistered.
   B. They usually have active secondary markets.
   C. They are less customized than publicly offered bonds.
11. A mechanism by which an issuer may be able to offer additional bonds to the general public without preparing a new and separate offering circular *best* describes:
   A. the grey market.
   B. a shelf registration.
   C. a private placement.
12. Which of the following statements related to secondary bond markets is *most accurate*?
   A. Newly issued corporate bonds are issued in secondary bond markets.
   B. Secondary bond markets are where bonds are traded between investors.
   C. The major participants in secondary bond markets globally are retail investors.
13. A bond market in which a communications network matches buy and sell orders initiated from various locations is *best* described as an:
   A. organized exchange.
   B. open market operation.
   C. over-the-counter market.
14. A liquid secondary bond market allows an investor to sell a bond at:
   A. the desired price.
   B. a price at least equal to the purchase price.
   C. a price close to the bond's fair market value.
15. Corporate bond secondary market trading *most often* occurs:
   A. on a book-entry basis.
   B. on organized exchanges.
   C. prior to settlement at $T + 1$.
16. Sovereign bonds are *best* described as:
   A. bonds issued by local governments.
   B. secured obligations of a national government.
   C. bonds backed by the taxing authority of a national government.

17. Which factor is associated with a more favorable quality sovereign bond credit rating?
    A. Issued in local currency, only
    B. Strong domestic savings base, only
    C. Issued in local currency of country with strong domestic savings base
18. Which type of sovereign bond has the lowest interest rate risk for an investor?
    A. Floaters
    B. Coupon bonds
    C. Discount bonds
19. Agency bonds are issued by:
    A. local governments.
    B. national governments.
    C. quasi-government entities.
20. The type of bond issued by a multilateral agency such as the International Monetary Fund (IMF) is *best* described as a:
    A. sovereign bond.
    B. supranational bond.
    C. quasi-government bond.
21. A bond issued by a local government authority, typically without an explicit funding commitment from the national government, is *most likely* classified as a:
    A. sovereign bond.
    B. quasi-government bond
    C. non-sovereign government bond.
22. Which of the following statements relating to commercial paper is *most accurate*?
    A. There is no secondary market for trading commercial paper.
    B. Only the strongest, highly rated companies issue commercial paper.
    C. Commercial paper is a source of interim financing for long-term projects.
23. Eurocommercial paper is *most likely*:
    A. negotiable.
    B. denominated in euros.
    C. issued on a discount basis.
24. For the issuer, a sinking fund arrangement is *most similar* to a:
    A. term maturity structure.
    B. serial maturity structure.
    C. bondholder put provision.
25. When issuing debt, a company may use a sinking fund arrangement as a means of reducing:
    A. credit risk.
    B. inflation risk.
    C. interest rate risk.
26. Which of the following is a source of wholesale funds for banks?
    A. Demand deposits
    B. Money market accounts
    C. Negotiable certificates of deposit
27. A characteristic of negotiable certificates of deposit is:
    A. they are mostly available in small denominations.
    B. they can be sold in the open market prior to maturity.
    C. a penalty is imposed if the depositor withdraws funds prior to maturity.

28. A repurchase agreement is *most* comparable to a(n):
    A. interbank deposit.
    B. collateralized loan.
    C. negotiable certificate of deposit.
29. The repo margin is:
    A. negotiated between counterparties.
    B. established independently of market-related conditions.
    C. structured on an agreement assuming equal credit risks to all counterparties.
30. The repo margin on a repurchase agreement is *most likely* to be lower when:
    A. the underlying collateral is in short supply.
    B. the maturity of the repurchase agreement is long.
    C. the credit risk associated with the underlying collateral is high.

CHAPTER 3

INTRODUCTION TO FIXED-INCOME VALUATION

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LEARNING OUTCOMES

The candidate should be able to:

- calculate a bond's price given a market discount rate;
- identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity);
- define spot rates and calculate the price of a bond using spot rates;
- describe and calculate the flat price, accrued interest, and the full price of a bond;
- describe matrix pricing;
- calculate annual yield on a bond for varying compounding periods in a year;
- calculate and interpret yield measures for fixed-rate bonds and floating-rate notes;
- calculate and interpret yield measures for money market instruments;
- define and compare the spot curve, yield curve on coupon bonds, par curve, and forward curve;
- define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates;
- compare, calculate, and interpret yield spread measures.

1. INTRODUCTION

Globally, the fixed-income market is a key source of financing for businesses and governments. In fact, the total market value outstanding of corporate and government bonds is significantly larger than that of equity securities. Similarly, the fixed-income market, which is also called the debt market or bond market, represents a significant investing opportunity for institutions as well as individuals. Pension funds, mutual funds, insurance companies, and sovereign wealth funds, among others, are major fixed-income investors. Retirees who desire a relatively stable income stream often hold fixed-income securities. Clearly, understanding how to value fixed-income securities is important to investors, issuers, and financial analysts. We focus on the valuation of traditional (option-free) fixed-rate bonds, although other debt securities, such as floating-rate notes and money market instruments, are also covered.

We first describe and illustrate basic bond valuation, which includes pricing a bond using a market discount rate for each of the future cash flows and pricing a bond using a series of spot rates. Valuation using spot rates allows for each future cash flow to be discounted at a rate associated with its timing. This valuation methodology for future cash flows has applications well beyond the fixed-income market. Relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity) are also described and illustrated.

We then turn our attention to how bond prices and yields are quoted and calculated in practice. When bonds are actively traded, investors can observe the price and calculate various yield measures. However, these yield measures differ by the type of bond. In practice, different measures are used for fixed-rate bonds, floating-rate notes, and money market instruments.

We then discuss the maturity or term structure of interest rates, involving an analysis of yield curves, which illustrates the relationship between yields-to-maturity and times-to-maturity on bonds with otherwise similar characteristics. Lastly, we describe yield spreads, measures of how much additional yield over the benchmark security (usually a government bond) investors expect for bearing additional risk.

2. BOND PRICES AND THE TIME VALUE OF MONEY

- **calculate a bond's price given a market discount rate**

Bond pricing is an application of discounted cash flow analysis. The complexity of the pricing depends on the particular bond's features and the rate (or rates) used to do the discounting. This section starts with using a single discount factor for all future cash flows and concludes with the most general approach to bond valuation. The general approach to bond valuation is to use a series of spot rates that correspond to the timing of the future cash flows.

2.1. Bond Pricing with a Market Discount Rate

On a traditional (option-free) fixed-rate bond, the promised future cash flows are a series of coupon interest payments and repayment of the full principal at maturity. The coupon payments occur on regularly scheduled dates; for example, an annual payment bond might pay interest on 15 June of each year for five years. The final coupon typically is paid together with the full principal on the maturity date. The price of the bond at issuance is the present value of the promised cash flows. The **market discount rate** is used in the time-value-of-money calculation to obtain the present value. The market discount rate is the rate of return required by investors given the risk of the investment in the bond. It is also called the **required yield**, or the **required rate of return**.

For example, suppose the coupon rate on a bond is 4% and the payment is made once a year. If the time-to-maturity is five years and the market discount rate is 6%, the price of the bond is 91.575 per 100 of **par value**. The par value is the amount of principal on the bond.

$$\frac{4}{(1.06)^1} + \frac{4}{(1.06)^2} + \frac{4}{(1.06)^3} + \frac{4}{(1.06)^4} + \frac{104}{(1.06)^5} =$$
$$3.774 + 3.560 + 3.358 + 3.168 + 77.715 = 91.575$$

The final cash flow of 104 is the redemption of principal (100) plus the coupon payment for that date (4). The price of the bond is the sum of the present values of the five cash flows. The price per 100 of par value may be interpreted as the percentage of par value. If the par value is USD100,000, the coupon payments are USD4,000 each year and the price of the bond is USD91,575. Its price is 91.575% of par value. This bond is described as trading at a **discount** because the price is below par value.

Suppose that another five-year bond has a coupon rate of 8% paid annually. If the market discount rate is again 6%, the price of the bond is 108.425.

$$\frac{8}{(1.06)^1} + \frac{8}{(1.06)^2} + \frac{8}{(1.06)^3} + \frac{8}{(1.06)^4} + \frac{108}{(1.06)^5} =$$
$$7.547 + 7.120 + 6.717 + 6.337 + 80.704 = 108.425$$

This bond is trading at a **premium** because its price is above par value.

If another five-year bond pays a 6% annual coupon and the market discount rate still is 6%, the bond would trade at par value.

$$\frac{6}{(1.06)^1} + \frac{6}{(1.06)^2} + \frac{6}{(1.06)^3} + \frac{6}{(1.06)^4} + \frac{106}{(1.06)^5} =$$
$$5.660 + 5.340 + 5.038 + 4.753 + 79.209 = 100.000$$

The coupon rate indicates the amount the issuer promises to pay the bondholders each year in interest. The market discount rate reflects the amount investors need to receive in interest each year in order to pay full par value for the bond. Therefore, assuming that these three bonds have the same risk, which is consistent with them having the same market discount rate, the 4% bond offers a "deficient" coupon rate. The amount of the discount below par value is the present value of the deficiency, which is 2% of par value each year. The present value of the deficiency, discounted using the market discount rate, is −8.425.

$$\frac{-2}{(1.06)^1} + \frac{-2}{(1.06)^2} + \frac{-2}{(1.06)^3} + \frac{-2}{(1.06)^4} + \frac{-2}{(1.06)^5} = -8.425$$

The price of the 4% coupon bond is 91.575 (= 100 − 8.425). In the same manner, the 8% bond offers an "excessive" coupon rate given the risk because investors require only 6%. The amount of the premium is the present value of the excess cash flows, which is +8.425. The price of the 8% bond is 108.425 (= 100 + 8.425).

These examples demonstrate that the price of a fixed-rate bond, relative to par value, depends on the relationship of the coupon rate to the market discount rate. Here is a summary of the relationships:

- When the coupon rate is less than the market discount rate, the bond is priced at a discount below par value.
- When the coupon rate is greater than the market discount rate, the bond is priced at a premium above par value.
- When the coupon rate is equal to the market discount rate, the bond is priced at par value.

At this point, it is assumed that the bond is priced on a coupon payment date. If the bond is between coupon payment dates, the price paid will include accrued interest, which is interest that has been earned but not yet paid. Accrued interest is discussed in detail later.

Equation 1 is a general formula for calculating a bond price given the market discount rate:

$$PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N} \tag{1}$$

where

PV = present value, or the price of the bond
PMT = coupon payment per period
FV = future value paid at maturity, or the par value of the bond
r = market discount rate, or required rate of return per period
N = number of evenly spaced periods to maturity

The examples so far have been for an annual payment bond, which is the convention for most European bonds. Asian and North American bonds generally make semiannual payments, and the stated rate is the annual coupon rate. Suppose the coupon rate on a bond is stated to be 8% and the payments are made twice a year (semiannually) on 15 June and 15 December. For each 100 in par value ($FV = 100$), the coupon payment per period is 4 ($PMT = 4$). If there are three years to maturity, there are six evenly spaced semiannual periods ($N = 6$). If the market discount rate is 3% per semiannual period ($r = 0.03$), the price of the bond is 105.417 per 100 of par value.

$$\frac{4}{(1.03)^1} + \frac{4}{(1.03)^2} + \frac{4}{(1.03)^3} + \frac{4}{(1.03)^4} + \frac{4}{(1.03)^5} + \frac{104}{(1.03)^6} = 105.417$$

This bond is trading at a premium above par value because the coupon rate of 4% *per period* is greater than the market discount rate of 3% *per period*. Usually, those interest rates are annualized by multiplying the rate per period by the number of periods in a year. Therefore, an equivalent statement is that the bond is priced at a premium because its stated *annual* coupon rate of 8% is greater than the stated *annual* market discount rate of 6%. Interest rates, unless stated otherwise, are typically quoted as annual rates.

EXAMPLE 1 Bonds Trading at a Discount, at a Premium, and at Par

Identify whether each of the following bonds is trading at a discount, at par value, or at a premium. Calculate the prices of the bonds per 100 in par value using Equation 1. If the coupon rate is deficient or excessive compared with the market discount rate, calculate the amount of the deficiency or excess per 100 of par value.

Bond	Coupon Payment per Period	Number of Periods to Maturity	Market Discount Rate per Period
A	2	6	3%
B	6	4	4%
C	5	5	5%
D	0	10	2%

Solution:

Bond A

$$\frac{2}{(1.03)^1}+\frac{2}{(1.03)^2}+\frac{2}{(1.03)^3}+\frac{2}{(1.03)^4}+\frac{2}{(1.03)^5}+\frac{102}{(1.03)^6}=94.583$$

Bond A is trading at a discount. Its price is below par value because the coupon rate per period (2%) is less than the required yield per period (3%). The deficiency per period is the coupon rate minus the market discount rate, times the par value: (0.02 – 0.03) × 100 = –1. The present value of deficiency is –5.417, discounted using the required yield (market discount rate) per period.

$$\frac{-1}{(1.03)^1}+\frac{-1}{(1.03)^2}+\frac{-1}{(1.03)^3}+\frac{-1}{(1.03)^4}+\frac{-1}{(1.03)^5}+\frac{-1}{(1.03)^6}=-5.417$$

The amount of the deficiency can be used to calculate the price of the bond; the price is 94.583 (= 100 – 5.417).

Bond B

$$\frac{6}{(1.04)^1}+\frac{6}{(1.04)^2}+\frac{6}{(1.04)^3}+\frac{106}{(1.04)^4}=107.260$$

Bond B is trading at a premium because the coupon rate per period (6%) is greater than the market discount rate per period (4%). The excess per period is the coupon rate minus the market discount rate, times the par value: (0.06 – 0.04) × 100 = +2. The present value of excess is +7.260, discounted using the required yield per period.

$$\frac{2}{(1.04)^1}+\frac{2}{(1.04)^2}+\frac{2}{(1.04)^3}+\frac{2}{(1.04)^4}=7.260$$

The price of the bond is 107.260 (= 100 + 7.260).

Bond C

$$\frac{5}{(1.05)^1}+\frac{5}{(1.05)^2}+\frac{5}{(1.05)^3}+\frac{5}{(1.05)^4}+\frac{105}{(1.05)^5}=100.000$$

Bond C is trading at par value because the coupon rate is equal to the market discount rate. The coupon payments are neither excessive nor deficient given the risk of the bond.

Bond D

$$\frac{100}{(1.02)^{10}}=82.035$$

Bond D is a zero-coupon bond, which always will trade at a discount below par value (as long as the required yield is greater than zero). The deficiency in the coupon payments is –2 per period: (0 – 0.02) × 100 = –2.

$$\frac{-2}{(1.02)^1}+\frac{-2}{(1.02)^2}+\frac{-2}{(1.02)^3}+\frac{-2}{(1.02)^4}+\frac{-2}{(1.02)^5}+$$
$$\frac{-2}{(1.02)^6}+\frac{-2}{(1.02)^7}+\frac{-2}{(1.02)^8}+\frac{-2}{(1.02)^9}+\frac{-2}{(1.02)^{10}}=-17.965$$

The price of the bond is 82.035 (= 100 – 17.965).

2.2. Yield-to-Maturity

If the market price of a bond is known, Equation 1 can be used to calculate its **yield-to-maturity** (YTM). The yield-to-maturity is the internal rate of return on the cash flows—the uniform interest rate such that when the future cash flows are discounted at that rate, the sum of the present values equals the price of the bond. It is the *implied* market discount rate.

The yield-to-maturity is the rate of return on the bond to an investor given three critical assumptions:

1. The investor holds the bond to maturity.
2. The issuer makes all the coupon and principal payments in the full amount on the scheduled dates. Therefore, the yield-to-maturity is the *promised* yield—the yield assuming the issuer does not default on any of the payments.
3. The investor is able to reinvest coupon payments at that same yield. This is a characteristic of an internal rate of return.

For example, suppose that a four-year, 5% annual coupon payment bond is priced at 105 per 100 of par value. The yield-to-maturity is the solution for the rate, r, in this equation:

$$105 = \frac{5}{(1+r)^1} + \frac{5}{(1+r)^2} + \frac{5}{(1+r)^3} + \frac{105}{(1+r)^4}$$

Solving by trial-and-error search or using the time-value-of-money keys on a financial calculator obtains the result that $r = 0.03634$. The bond trades at a premium because its coupon rate (5%) is greater than the yield that is required by investors (3.634%). The yield is implied by the market price.

The yield-to-maturity on a bond may be positive or negative. In fact, following the launch of unconventional monetary policies in many regions after the global financial crisis of 2008–2009, yields on many government bonds fell into the negative territory. The market value of negative-yielding bonds reached $17 trillion in August 2019, amounting to roughly 1 in 3 of all investment-grade bonds, according to the Bloomberg Barclays Global-Aggregate Index. Bonds with a negative yield-to-maturity include those issued at higher yields in the past that have experienced significant price appreciation, as well as newly issued zero-coupon sovereign government bonds priced at a premium to par value. Suppose the four-year bond in the previous example were trading at a price of 122.50 per 100 of par value, instead of 105. Using Equation 1, the yield-to-maturity, r, for this bond would be −0.549%.

Yield-to-maturity does not depend on the actual amount of par value in a fixed-income portfolio. For example, suppose a Japanese institutional investor owns a three-year, 2.5% semiannual payment bond having a par value of JPY100 million. The bond currently is priced at JPY98,175,677. The yield per semiannual period can be obtained by solving this equation for r:

$$98.175677 = \frac{1.25}{(1+r)^1} + \frac{1.25}{(1+r)^2} + \frac{1.25}{(1+r)^3} + \frac{1.25}{(1+r)^4} + \frac{1.25}{(1+r)^5} + \frac{101.25}{(1+r)^6}$$

The yield per semiannual period turns out to be 1.571% ($r = 0.01571$), which can be annualized to be 3.142% ($0.01571 \times 2 = 0.03142$). In general, a three-year, 2.5% semiannual bond for *any* amount of par value has an annualized yield-to-maturity of 3.142% if it is priced at 98.175677% of par value.

EXAMPLE 2 Yields-to-Maturity for a Premium, Discount, and Zero-Coupon Bond

Calculate the yields-to-maturity for the following bonds. The prices are stated per 100 of par value.

Bond	Coupon Payment per Period	Number of Periods to Maturity	Price
A	3.5	4	103.75
B	2.25	6	96.50
C	0	60	22.375

Solution:

Bond A

$$103.75 = \frac{3.5}{(1+r)^1} + \frac{3.5}{(1+r)^2} + \frac{3.5}{(1+r)^3} + \frac{103.5}{(1+r)^4};\ r = 0.02503$$

Bond A is trading at a premium, so its yield-to-maturity per period (2.503%) must be lower than its coupon rate per period (3.5%).

Bond B

$$96.50 = \frac{2.25}{(1+r)^1} + \frac{2.25}{(1+r)^2} + \frac{2.25}{(1+r)^3} + \frac{2.25}{(1+r)^4} + \frac{2.25}{(1+r)^5} + \frac{102.25}{(1+r)^6};\ r = 0.02894$$

Bond B is trading at a discount, so the yield-to-maturity per period (2.894%) must be higher than the coupon rate per period (2.25%).

Bond C

$$22.375 = \frac{100}{(1+r)^{60}};\ r = 0.02527$$

Bond C is a zero-coupon bond trading at a significant discount below par value. Its yield-to-maturity is 2.527% per period.

2.3. Relationships between the Bond Price and Bond Characteristics

- **identify the relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity)**

The price of a fixed-rate bond will change whenever the market discount rate changes. The following relationships pertain to the change in the bond price given the market discount rate:

1. The bond price is inversely related to the market discount rate. When the market discount rate increases, the bond price decreases (the inverse effect).
2. For the same coupon rate and time-to-maturity, the percentage price change is greater (in absolute value, meaning without regard to the sign of the change) when the market discount rate goes down than when it goes up (the convexity effect).

3. For the same time-to-maturity, a lower-coupon bond has a greater percentage price change than a higher-coupon bond when their market discount rates change by the same amount (the coupon effect).
4. Generally, for the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their market discount rates change by the same amount (the maturity effect).

Exhibit 1 illustrates these relationships using nine annual coupon payment bonds. The bonds have different coupon rates and times-to-maturity but otherwise are the same in terms of risk. The coupon rates are 10%, 20%, and 30% for bonds having 10, 20, and 30 years to maturity. At first, the bonds are all priced at a market discount rate of 20%. Equation 1 is used to determine the prices. Going across columns, the market discount rate is decreased by 1 percentage point, from 20% to 19%, and next, it is increased from 20% to 21%.

EXHIBIT 1 Relationships between Bond Prices and Bond Characteristics

				Discount Rates Go Down		Discount Rates Go Up	
Bond	Coupon Rate	Maturity	Price at 20%	Price at 19%	% Change	Price at 21%	% Change
A	10.00%	10	58.075	60.950	4.95%	55.405	–4.60%
B	20.00%	10	100.000	104.339	4.34%	95.946	–4.05%
C	30.00%	10	141.925	147.728	4.09%	136.487	–3.83%
D	10.00%	20	51.304	54.092	5.43%	48.776	–4.93%
E	20.00%	20	100.000	105.101	5.10%	95.343	–4.66%
F	30.00%	20	148.696	156.109	4.99%	141.910	–4.56%
G	10.00%	30	50.211	52.888	5.33%	47.791	–4.82%
H	20.00%	30	100.000	105.235	5.23%	95.254	–4.75%
I	30.00%	30	149.789	157.581	5.20%	142.716	–4.72%

The first relationship is that the bond price and the market discount rate move inversely. All bond prices in Exhibit 1 go up when the rates go down from 20% to 19%, and all prices go down when the rates go up from 20% to 21%. This happens because of the fixed cash flows on a fixed-rate bond. The numerators in Equation 1 do not change when the market discount rate in the denominators rises or falls. Therefore, the price (PV) moves inversely with the market discount rate (r).

The second relationship reflects the convexity effect. In Exhibit 1, the percentage price changes are calculated using this equation:

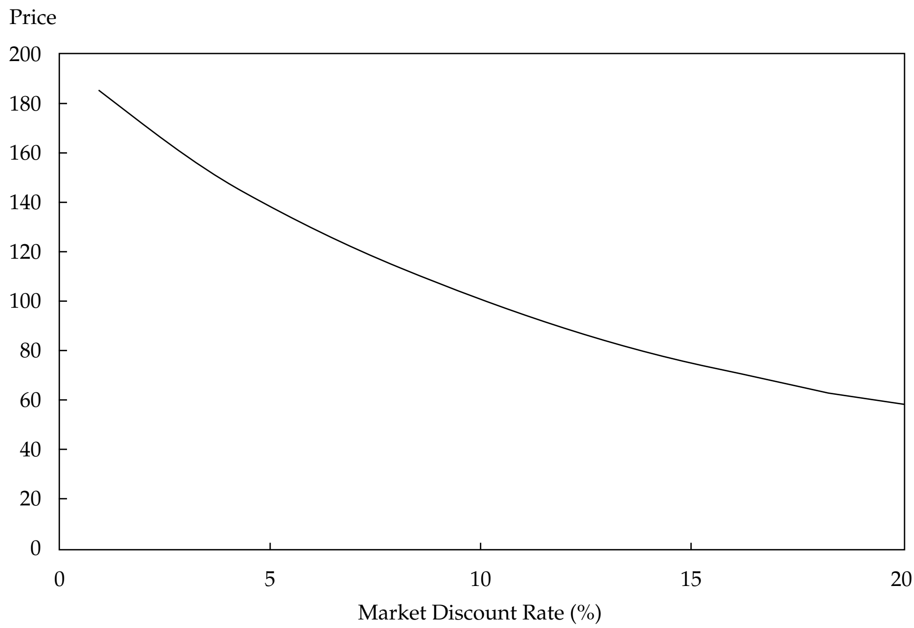
$$\% \text{ Change} = \frac{\text{New price} - \text{Old price}}{\text{Old price}}$$

For example, when the market discount rate on Bond A falls, the price rises from 58.075 to 60.950. The percentage price increase is 4.95%.

$$\% \text{ Change} = \frac{60.950 - 58.075}{58.075} = 0.0495$$

For each bond, the percentage price increases are greater in *absolute value* than the percentage price decreases. This implies that the relationship between bond prices and the market discount rate is not linear; instead, it is curved. It is described as being "convex." The convexity effect is shown in Exhibit 2 for a 10%, 10-year bond.

EXHIBIT 2 The Convex Relationship between the Market Discount Rate and the Price of a 10-Year, 10% Annual Coupon Payment Bond



The third relationship is the coupon effect. Consider Bonds A, B, and C, which have 10 years to maturity. For both the decrease and increase in the yield-to-maturity, Bond A has a larger percentage price change than Bond B and Bond B has a larger change than C. The same pattern holds for the 20-year and 30-year bonds. Therefore, lower-coupon bonds have more price volatility than higher-coupon bonds, other things being equal.

The fourth relationship is the maturity effect. Compare the results for Bonds A and D, for Bonds B and E, and for Bonds C and F. The 20-year bonds have greater percentage price changes than the 10-year bonds for either an increase or a decrease in the market discount rate. In general, longer-term bonds have more price volatility than shorter-term bonds, other things being equal.

There are exceptions to the maturity effect. That is why the word "generally" appears in the statement of the relationship at the beginning of this section. Compare the results in Exhibit 1 for Bonds D and G, for Bonds E and H, and for Bonds F and I. For the higher-coupon bonds trading at a premium, Bonds F and I, the usual property holds: The 30-year bonds have greater percentage price changes than the 20-year bonds. The same pattern holds for Bonds E and H, which are priced initially at par value. The exception is illustrated in the results for Bonds D and G, which are priced at a discount because the coupon rate is lower than the market discount rate. The 20-year, 10% bond has a greater percentage price change than the 30-year, 10% bond. Exceptions to the maturity effect are rare in practice. They occur only for low-coupon (but not zero-coupon), long-term bonds trading at a discount. The maturity effect always holds on zero-coupon bonds, as it does for bonds priced at par value or at a premium above par value.

One final point to note in Exhibit 1 is that Bonds B, E, and H, which have coupon rates of 20%, all trade at par value when the market discount rate is 20%. A bond having a coupon rate equal to the market discount rate is priced at par value on a coupon payment date, regardless of the number of years to maturity.

EXAMPLE 3 Bond Percentage Price Changes Based on Coupon and Time-to-Maturity

An investor is considering the following six annual coupon payment government bonds:

Bond	Coupon Rate	Time-to-Maturity	Yield-to-Maturity
A	0%	2 years	5.00%
B	5%	2 years	5.00%
C	8%	2 years	5.00%
D	0%	4 years	5.00%
E	5%	4 years	5.00%
F	8%	4 years	5.00%

1. Based on the relationships between bond prices and bond characteristics, which bond will go up in price the *most* on a percentage basis if all yields go down from 5.00% to 4.90%?
2. Based on the relationships between the bond prices and bond characteristics, which bond will go down in price the *least* on a percentage basis if all yields go up from 5.00% to 5.10%?

Solution to 1: Bond D will go up in price the most on a percentage basis because it has the lowest coupon rate (the coupon effect) and the longer time-to-maturity (the maturity effect). There is no exception to the maturity effect in these bonds because there are no low-coupon bonds trading at a discount.

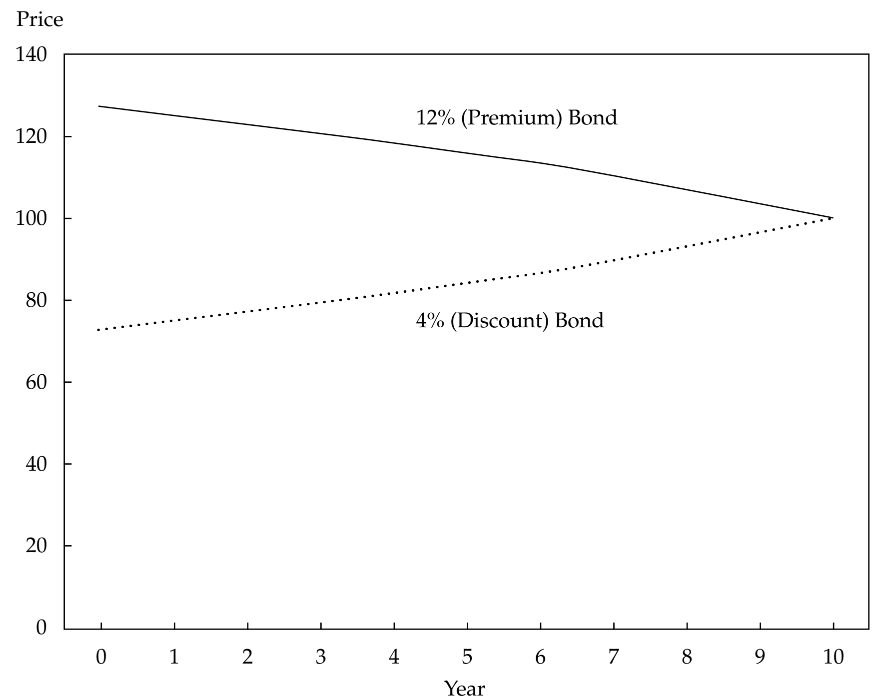
Solution to 2: Bond C will go down in price the least on a percentage basis because it has the highest coupon rate (the coupon effect) and the shorter time-to-maturity (the maturity effect). There is no exception to the maturity effect because Bonds C and F are priced at a premium above par value.

Exhibit 2 demonstrates the impact on a bond price assuming the time-to-maturity does not change. It shows an *instantaneous* change in the market discount rate from one moment to the next.

In practice, bond prices change even if the market discount rate remains the same. As time passes, the bondholder comes closer to receiving the par value at maturity. The **constant-yield price trajectory** illustrates the change in the price of a fixed-income bond over time. This trajectory shows the "pull to par" effect on the price of a bond trading at a premium or a discount to par value. If the issuer does not default, the price of a bond approaches par value as its time-to-maturity approaches zero.

Exhibit 3 shows the constant-yield price trajectories for 4% and 12% annual coupon payment 10-year bonds. Both bonds have a market discount rate of 8%. The 4% bond's initial price is 73.160 per 100 of par value. The price increases each year and approaches par value as the maturity date nears. The 12% bond's initial price is 126.840, and it decreases each year, approaching par value as the maturity date nears. Both prices are "pulled to par."

EXHIBIT 3 Constant-Yield Price Trajectories for 4% and 12% Annual Coupon Payment 10-Year Bonds at a Market Discount Rate of 8%



	0	1	2	3	4	5	6	7	8	9	10
Discount Bond	73.160	75.012	77.013	79.175	81.508	84.029	86.751	89.692	92.867	96.296	100.00
Premium Bond	126.84	124.98	122.98	120.82	118.49	115.97	113.24	110.30	107.13	103.70	100.00

2.4. Pricing Bonds Using Spot Rates

- **define spot rates and calculate the price of a bond using spot rates**

When a fixed-rate bond is priced using the market discount rate, the same discount rate is used for each cash flow. A more fundamental approach to calculate the price of a bond is to use a sequence of market discount rates that correspond to the cash flow dates. These market discount rates are called **spot rates**. Spot rates are yields-to-maturity on zero-coupon bonds maturing at the date of each cash flow. Sometimes these are called "zero rates." Bond price (or value) determined using the spot rates is sometimes referred to as the bond's "no-arbitrage value." If a bond's price differs from its no-arbitrage value, an arbitrage opportunity exists in the absence of transaction costs.

Suppose that the one-year spot rate is 2%, the two-year spot rate is 3%, and the three-year spot rate is 4%. Then, the price of a three-year bond that makes a 5% annual coupon payment is 102.960.

$$\frac{5}{(1.02)^1} + \frac{5}{(1.03)^2} + \frac{105}{(1.04)^3} =$$
$$4.902 + 4.713 + 93.345 = 102.960$$

This three-year bond is priced at a premium above par value, so its yield-to-maturity must be less than 5%. Using Equation 1, the yield-to-maturity is 3.935%.

$$102.960 = \frac{5}{(1+r)^1} + \frac{5}{(1+r)^2} + \frac{105}{(1+r)^3};\ r = 0.03935$$

When the coupon and principal cash flows are discounted using the yield-to-maturity, the same price is obtained.

$$\frac{5}{(1.03935)^1} + \frac{5}{(1.03935)^2} + \frac{105}{(1.03935)^3} =$$
$$4.811 + 4.629 + 93.520 = 102.960$$

Notice that the present values of the individual cash flows discounted using spot rates differ from those using the yield-to-maturity. The present value of the first coupon payment is 4.902 when discounted at 2%, but it is 4.811 when discounted at 3.935%. The present value of the final cash flow, which includes the redemption of principal, is 93.345 at 4% and 93.520 at 3.935%. Nevertheless, the sum of the present values using either approach is 102.960.

Equation 2 is a general formula for calculating a bond price given the sequence of spot rates:

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT + FV}{(1+Z_N)^N} \tag{2}$$

where

Z_1 = spot rate, or zero-coupon yield, or zero rate, for Period 1
Z_2 = spot rate, or zero-coupon yield, or zero rate, for Period 2
Z_N = spot rate, or zero-coupon yield, or zero rate, for Period N

EXAMPLE 4 Bond Prices and Yields-to-Maturity Based on Spot Rates

Calculate the price (per 100 of par value) and the yield-to-maturity for a four-year, 3% annual coupon payment bond given the following two sequences of spot rates.

Time-to-Maturity	Spot Rates A	Spot Rates B
1 year	0.39%	4.08%
2 years	1.40%	4.01%
3 years	2.50%	3.70%
4 years	3.60%	3.50%

Solution:

Spot Rates A

$$\frac{3}{(1.0039)^1} + \frac{3}{(1.0140)^2} + \frac{3}{(1.0250)^3} + \frac{103}{(1.0360)^4} =$$
$$2.988 + 2.918 + 2.786 + 89.412 = 98.104$$

Given Spot Rates A, the four-year, 3% bond is priced at 98.104.

$$98.104 = \frac{3}{(1+r)^1} + \frac{3}{(1+r)^2} + \frac{3}{(1+r)^3} + \frac{103}{(1+r)^4}; r = 0.03516$$

The yield-to-maturity is 3.516%.

Spot Rates B

$$\frac{3}{(1.0408)^1} + \frac{3}{(1.0401)^2} + \frac{3}{(1.0370)^3} + \frac{103}{(1.0350)^4} =$$
$$2.882 + 2.773 + 2.690 + 89.759 = 98.104$$

$$98.104 = \frac{3}{(1+r)^1} + \frac{3}{(1+r)^2} + \frac{3}{(1+r)^3} + \frac{103}{(1+r)^4}; r = 0.03516$$

Given Spot Rates B, the four-year, 3% bond is again priced at 98.104 to yield 3.516%.

This example demonstrates that two very different sequences of spot rates can result in the same bond price and yield-to-maturity. Spot Rates A are increasing for longer maturities, whereas Spot Rates B are decreasing.

3. PRICES AND YIELDS: CONVENTIONS FOR QUOTES AND CALCULATIONS

- **describe and calculate the flat price, accrued interest, and the full price of a bond**

When investors purchase shares, they pay the quoted price. For bonds, however, there can be a difference between the quoted price and the price paid. This section explains why this difference occurs and how to calculate the quoted price and the price that will be paid. It also describes how prices are estimated for bonds that are not actively traded and demonstrates how yield measures are calculated for fixed-rate bonds, floating-rate notes, and money market instruments.

3.1. Flat Price, Accrued Interest, and the Full Price

When a bond is between coupon payment dates, its price has two parts: the **flat price** (PV^{Flat}) and the **accrued interest** (AI). The sum of the parts is the **full price** (PV^{Full}), which also is called the invoice or "dirty" price. The flat price, which is the full price minus the accrued interest, is also called the quoted or "clean" price.

$$PV^{Full} = PV^{Flat} + AI \tag{3}$$

The flat price usually is quoted by bond dealers. If a trade takes place, the accrued interest is added to the flat price to obtain the full price paid by the buyer and received by the seller on the **settlement date**. The settlement date is when the bond buyer makes cash payment and the seller delivers the security.

The reason for using the flat price for quotation is to avoid misleading investors about the market price trend for the bond. If the full price were to be quoted by dealers, investors would see the price rise day after day even if the yield-to-maturity did not change. That is because the amount of accrued interest increases each day. Then, after the coupon payment is made, the quoted price would drop dramatically. Using the flat price for quotation avoids that misrepresentation. It is the flat price that is "pulled to par" along the constant-yield price trajectory shown in Exhibit 3.

Accrued interest is the proportional share of the next coupon payment. Assume that the coupon period has "T" days between payment dates and that "t" days have gone by since the last payment. The accrued interest is calculated using Equation 4:

$$AI = \frac{t}{T} \times PMT \tag{4}$$

where

t = number of days from the last coupon payment to the settlement date
T = number of days in the coupon period
t/T = fraction of the coupon period that has gone by since the last payment
PMT = coupon payment per period

Notice that the accrued interest part of the full price does not depend on the yield-to-maturity. Therefore, it is the flat price that is affected by a market discount rate change.

There are different conventions used in bond markets to count days. The two most common day-count conventions are actual/actual and 30/360. For the actual/actual method, the actual number of days is used, including weekends, holidays, and leap days. For example, a semiannual payment bond pays interest on 15 May and 15 November of each year. The accrued interest for settlement on 27 June would be the actual number of days between 15 May and 27 June (t = 43 days) divided by the actual number of days between 15 May and 15 November (T = 184 days), times the coupon payment. If the stated coupon rate is 4.375%, the accrued interest is 0.511209 per 100 of par value.

$$AI = \frac{43}{184} \times \frac{4.375}{2} = 0.511209$$

Day-count conventions vary from market to market. However, actual/actual is most common for government bonds.

The 30/360 day-count convention often is used for corporate bonds. It *assumes* that each month has 30 days and that a full year has 360 days. Therefore, for this method, there are assumed to be 42 days between 15 May and 27 June: 15 days between 15 May and 30 May and 27 days between 1 June and 27 June. There are assumed to be 180 days in the six-month period between 15 May and 15 November. The accrued interest on a 4.375% semiannual payment corporate bond is 0.510417 per 100 of par value.

$$AI = \frac{42}{180} \times \frac{4.375}{2} = 0.510417$$

The full price of a fixed-rate bond between coupon payments given the market discount rate per period (r) can be calculated with Equation 5:

$$PV^{Full} = \frac{PMT}{(1+r)^{1-t/T}} + \frac{PMT}{(1+r)^{2-t/T}} + \cdots + \frac{PMT + FV}{(1+r)^{N-t/T}} \qquad (5)$$

This is very similar to Equation 1. The difference is that the next coupon payment (PMT) is discounted for the remainder of the coupon period, which is $1 - t/T$. The second coupon payment is discounted for that fraction plus another full period, $2 - t/T$.

Equation 5 is simplified by multiplying the numerator and denominator by the expression $(1 + r)^{t/T}$. The result is Equation 6:

$$\begin{aligned} PV^{Full} &= \left[\frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}\right] \times (1+r)^{t/T} \\ &= PV \times (1+r)^{t/T} \end{aligned} \qquad (6)$$

An advantage to Equation 6 is that PV, the expression in the brackets, is easily obtained using the time-value-of-money keys on a financial calculator because there are N evenly spaced periods. PV here is identical to Equation 1 and is not the same as PV^{Flat}.

For example, consider a 5% semiannual coupon payment government bond that matures on 15 February 2028. Accrued interest on this bond uses the actual/actual day-count convention. The coupon payments are made on 15 February and 15 August of each year. The bond is to be priced for settlement on 14 May 2019. That date is 88 days into the 181-day period. There are actually 88 days from the last coupon on 15 February to 14 May and 181 days between 15 February and the next coupon on 15 August. The annual yield-to-maturity is stated to be 4.80%. That corresponds to a market discount rate of 2.40% per semiannual period. As of the beginning of the coupon period on 15 February 2019, there would be 18 evenly spaced semiannual periods until maturity. The first step is to solve for PV using Equation 1, whereby $PMT = 2.5$, $N = 18$, $FV = 100$, and $r = 0.0240$.

$$PV = \frac{2.5}{(1.0240)^1} + \frac{2.5}{(1.0240)^2} + \cdots + \frac{102.5}{(1.0240)^{18}} = 101.447790$$

The price of the bond would be 101.447790 per 100 of par value if its yield-to-maturity is 2.40% per period on the last coupon payment date. This is not the actual price for the bond on that date. It is a "what-if" price using the required yield that corresponds to the settlement date of 14 May 2019.

Equation 6 can be used to get the full price for the bond.

$$PV^{Full} = 101.447790 \times (1.0240)^{88/181} = 102.624323$$

The full price is 102.624323 per 100 of par value. The accrued interest is 1.215470 per 100 of par value.

$$AI = \frac{88}{181} \times 2.5 = 1.215470$$

The flat price is 101.408853 per 100 of par value.

$$PV^{Flat} = PV^{Full} - AI = 102.624323 - 1.215470 = 101.408853$$

Microsoft Excel users can obtain the flat price using the PRICE financial function: PRICE(DATE(2019,5,14),DATE(2028,2,15),0.05,0.048,100,2,1). The inputs are the settlement date, maturity date, annual coupon rate as a decimal, annual yield-to-maturity as a decimal, par value, number of periods in the year, and code for the day count (0 for 30/360, 1 for actual/actual).

EXAMPLE 5 Calculating the Full Price, Accrued Interest, and Flat Price for a Bond

A 6% German corporate bond is priced for settlement on 18 June 2019. The bond makes semiannual coupon payments on 19 March and 19 September of each year and matures on 19 September 2030. The corporate bond uses the 30/360 day-count convention for accrued interest. Calculate the full price, the accrued interest, and the flat price per EUR100 of par value for three stated annual yields-to-maturity: (A) 5.80%, (B) 6.00%, and (C) 6.20%.

Solution: Given the 30/360 day-count convention assumption, there are 89 days between the last coupon on 19 March 2015 and the settlement date on 18 June 2019 (11 days between 19 March and 30 March, plus 60 days for the full months of April and May, plus 18 days in June). Therefore, the fraction of the coupon period that has gone by is assumed to be 89/180. At the beginning of the period, there are 11.5 years (and 23 semiannual periods) to maturity.

(A) Stated annual yield-to-maturity of 5.80%, or 2.90% per semiannual period:

The price at the beginning of the period is 101.661589 per 100 of par value.

$$PV = \frac{3}{(1.0290)^1} + \frac{3}{(1.0290)^2} + \cdots + \frac{103}{(1.0290)^{23}} = 101.661589$$

The full price on 18 June is EUR103.108770.

$$PV^{Full} = 101.661589 \times (1.0290)^{89/180} = 103.108770$$

The accrued interest is EUR1.483333, and the flat price is EUR101.625437.

$$AI = \frac{89}{180} \times 3 = 1.4833333$$

$$PV^{Flat} = 103.108770 - 1.483333 = 101.625437$$

(B) Stated annual yield-to-maturity of 6.00%, or 3.00% per semiannual period:

The price at the beginning of the period is par value, as expected, because the coupon rate and the market discount rate are equal.

$$PV = \frac{3}{(1.0300)^1} + \frac{3}{(1.0300)^2} + \cdots + \frac{103}{(1.0300)^{23}} = 100.000000$$

The full price on 18 June is EUR101.472251.

$$PV^{Flat} = 100.000000 \times (1.0300)^{89/180} = 101.472251$$

The accrued interest is EUR1.483333, and the flat price is EUR99.988918.

$$AI = \frac{89}{180} \times 3 = 1.4833333$$

$$PV^{Flat} = 101.472251 - 1.483333 = 99.988918$$

The flat price of the bond is a little below par value, even though the coupon rate and the yield-to-maturity are equal, because the accrued interest does not take into account the time value of money. The accrued interest is the interest earned by the owner of the bond for the time between the last coupon payment and the settlement date, 1.483333 per 100 of par value. However, that interest income is not received until the next coupon date. In theory, the accrued interest should be the *present value* of 1.483333. In practice, however, accounting and financial reporting need to consider issues of practicality and materiality. For those reasons, the calculation of accrued interest in practice neglects the time value of money. Therefore, compared with theory, the reported accrued interest is a little "too high" and the flat price is a little "too low." The full price, however, is correct because it is the sum of the present values of the future cash flows, discounted using the market discount rate.

(C) Stated annual yield-to-maturity of 6.20%, or 3.10% per semiannual period:

The price at the beginning of the period is 98.372607 per 100 of par value.

$$PV = \frac{3}{(1.0310)^1} + \frac{3}{(1.0310)^2} + \cdots + \frac{103}{(1.0310)^{23}} = 98.372607$$

The full price on 18 June is EUR99.868805.

$$PV^{Full} = 98.372607 \times (1.0310)^{89/180} = 99.868805$$

The accrued interest is EUR1.483333, and the flat price is EUR98.385472.

$$AI = \frac{89}{180} \times 3 = 1.4833333$$

$$PV^{Flat} = 99.868805 - 1.483333 = 98.385472$$

The accrued interest is the same in each case because it does not depend on the yield-to-maturity. The differences in the flat prices indicate the differences in the rate of return that is required by investors.

3.2. Matrix Pricing

- **describe matrix pricing**

Some fixed-rate bonds are not actively traded. Therefore, there is no market price available to calculate the rate of return required by investors. The same problem occurs for bonds that are not yet issued. In these situations, it is common to estimate the market discount rate and price based on the quoted or flat prices of more frequently traded comparable bonds. These comparable bonds have similar times-to-maturity, coupon rates, and credit quality. This estimation process is called **matrix pricing**.

For example, suppose that an analyst needs to value a three-year, 4% semiannual coupon payment corporate bond, Bond X. Assume that Bond X is not actively traded and that there are no recent transactions reported for this particular security. However, there are quoted prices for four corporate bonds that have very similar credit quality:

- Bond A: Two-year, 3% semiannual coupon payment bond trading at a price of 98.500
- Bond B: Two-year, 5% semiannual coupon payment bond trading at a price of 102.250
- Bond C: Five-year, 2% semiannual coupon payment bond trading at a price of 90.250
- Bond D: Five-year, 4% semiannual coupon payment bond trading at a price of 99.125

The bonds are displayed in a matrix according to the coupon rate and the time-to-maturity. This matrix is shown in Exhibit 4.

EXHIBIT 4 Matrix Pricing Example

	2% Coupon	3% Coupon	4% Coupon	5% Coupon
Two years		98.500		102.250
		3.786%		3.821%
Three years			Bond X	
Four years				
Five years	90.250		99.125	
	4.181%		4.196%	

In Exhibit 4, below each bond price is the yield-to-maturity. It is stated as the yield per semiannual period times two. For example, the yield-to-maturity on the two-year, 3% semiannual coupon payment corporate bond is 3.786%.

$$98.500 = \frac{1.5}{(1+r)^1} + \frac{1.5}{(1+r)^2} + \frac{1.5}{(1+r)^3} + \frac{101.5}{(1+r)^4};\ r = 0.01893; \times 2 = 0.03786$$

Next, the analyst calculates the average yield for each year: 3.8035% for the two-year bonds and 4.1885% for the five-year bonds.

$$\frac{0.03786 + 0.03821}{2} = 0.038035$$

$$\frac{0.04181 + 0.04196}{2} = 0.041885$$

The estimated three-year market discount rate can be obtained with linear interpolation. The interpolated yield is 3.9318%.

$$0.038035 + \left(\frac{3-2}{5-2}\right) \times (0.041885 - 0.038035) = 0.039318$$

Using 3.9318% as the estimated three-year annual market discount rate, the three-year, 4% semiannual coupon payment corporate bond has an estimated price of 100.191 per 100 of par value.

$$\frac{2}{(1.019659)^1} + \frac{2}{(1.019659)^2} + \frac{2}{(1.019659)^3} + \frac{2}{(1.019659)^4} + \frac{2}{(1.019659)^5} + \frac{102}{(1.019659)^6} = 100.191$$

Notice that 3.9318% is the stated annual rate. It is divided by two to get the yield per semiannual period: (0.039318/2 = 0.019659).

Matrix pricing also is used in underwriting new bonds to get an estimate of the **required yield spread** over the **benchmark rate**. The benchmark rate typically is the yield-to-maturity on a government bond having the same or close to the same time-to-maturity. The spread is the difference between the yield-to-maturity on the new bond and the benchmark rate. The yield spread is the additional compensation required by investors for the difference in the credit risk, liquidity risk, tax status, and other risk premiums of the bond relative to the government bond. This spread is sometimes called the **spread over the benchmark**. Yield spreads are often stated in terms of basis points (bps), where one **basis point** equals one-hundredth of a percentage point. For example, if a yield-to-maturity is 2.25% and the benchmark rate is 1.50%, the yield spread is 0.75%, or 75 bps. Yield spreads are covered in more detail later.

Suppose that a corporation is about to issue a five-year bond. The corporate issuer currently has a four-year, 3% annual coupon payment debt liability on its books. The price of that bond is 102.400 per 100 of par value. This is the full price, which is the same as the flat price because the accrued interest is zero. This implies that the coupon payment has just been made and there are four full years to maturity. The four-year rate of return required by investors for this bond is 2.36%.

$$102.400 = \frac{3}{(1+r)^1} + \frac{3}{(1+r)^2} + \frac{3}{(1+r)^3} + \frac{103}{(1+r)^4};\ r = 0.0236$$

Suppose that there are no four-year government bonds to calculate the yield spread on this security. However, there are three-year and five-year government bonds that have yields-to-maturity of 0.75% and 1.45%, respectively. The average of the two yields-to-maturity is 1.10%, which is the estimated yield for the four-year government bond. Therefore, the estimated yield spread is 126 bps over the implied benchmark rate (0.0236 – 0.0110 = 0.0126).

There usually is a different yield spread for each maturity and for each credit rating. The term structure of "risk-free" rates, which is further discussed later, is the relationship between yields-to-maturity on "risk-free" bonds and times-to-maturity. The quotation marks around "risk-free" indicate that no bond is truly without risk. The primary component of the yield spread for many bonds is compensation for credit risk, not for time-to-maturity, and as a result, the yield spreads reflect the **term structure of credit spreads**. The term structure of credit spreads is the relationship between the spreads over the "risk-free" (or benchmark) rates and times-to-maturity. These term structures are covered in more detail later.

The issuer now has an estimate of the four-year yield spread: 126 bps. This spread is a reference point for estimating the five-year spread for the newly issued bond. Suppose that the term structure of credit spreads for bonds of the corporate issuer's quality indicates that five-year spreads are about 25 bps higher than four-year spreads. Therefore, the estimated five-year required yield spread is 151 bps (0.0126 + 0.0025 = 0.0151). Given the yield-to-maturity of 1.45% on the five-year government bond, the expected market discount rate for the newly issued bond is 2.96% (0.0145 + 0.0151 = 0.0296). The corporation might set the coupon rate to be 3% and expect that the bond can be sold for a small premium above par value.

EXAMPLE 6 Using Matrix Pricing to Estimate Bond Price

An analyst needs to assign a value to an illiquid four-year, 4.5% annual coupon payment corporate bond. The analyst identifies two corporate bonds that have similar credit quality: One is a three-year, 5.50% annual coupon payment bond priced at 107.500 per 100 of par value, and the other is a five-year, 4.50% annual coupon payment bond priced at 104.750 per 100 of par value. Using matrix pricing, the estimated price of the illiquid bond per 100 of par value is *closest* to:

A. 103.895.
B. 104.991.
C. 106.125.

Solution: B is correct. The first step is to determine the yields-to-maturity on the observed bonds. The required yield on the three-year, 5.50% bond priced at 107.500 is 2.856%.

$$107.500 = \frac{5.50}{(1+r)^1} + \frac{5.50}{(1+r)^2} + \frac{105.50}{(1+r)^3};\ r = 0.02856$$

The required yield on the five-year, 4.50% bond priced at 104.750 is 3.449%.

$$104.750 = \frac{4.50}{(1+r)^1} + \frac{4.50}{(1+r)^2} + \frac{4.50}{(1+r)^3} + \frac{4.50}{(1+r)^4} + \frac{104.50}{(1+r)^5};\ r = 0.03449$$

The estimated market discount rate for a four-year bond having the same credit quality is the average of two required yields:

$$\frac{0.02856 + 0.03449}{2} = 0.031525$$

Given an estimated yield-to-maturity of 3.1525%, the estimated price of the illiquid four-year, 4.50% annual coupon payment corporate bond is 104.991 per 100 of par value.

$$\frac{4.50}{(1.031525)^1} + \frac{4.50}{(1.031525)^2} + \frac{4.50}{(1.031525)^3} + \frac{104.50}{(1.031525)^4} = 104.991$$

3.3. Annual Yields for Varying Compounding Periods in the Year

- **calculate annual yield on a bond for varying compounding periods in a year**

There are many ways to measure the rate of return on a fixed-rate bond investment. Consider a five-year, zero-coupon government bond. The purchase price today is 80. The investor receives 100 at redemption in five years. One possible yield measure is 25%—the gain of 20 divided by the amount invested, 80. However, investors want a yield measure that is *standardized* to allow for comparison between bonds that have different times-to-maturity. Therefore, yield measures typically are *annualized*. A possible annual rate for this zero-coupon bond is 5% per year—25% divided by five years. But for bonds maturing in more than one year, investors want an *annualized and compounded* yield-to-maturity. Money market rates on instruments maturing in one year or less typically are *annualized but not compounded*. They are stated on a simple interest basis. This concept is covered later.

In general, an annualized and compounded yield on a fixed-rate bond depends on the assumed number of periods in the year, which is called the **periodicity** of the annual rate. Typically, the periodicity matches the frequency of coupon payments. A bond that pays semiannual coupons has a stated annual yield-to-maturity for a periodicity of two—the rate per semiannual period times two. A bond that pays quarterly coupons has a stated annual yield for a periodicity of four—the rate per quarter times four. It is always important to know the periodicity of a stated annual rate.

The periodicity of the annual market discount rate for a zero-coupon bond is arbitrary because there are no coupon payments. For semiannual compounding, the annual yield-to-maturity on the five-year, zero-coupon bond priced at 80 per 100 of par value is stated to be 4.5130%. This annual rate has a periodicity of two.

$$80 = \frac{100}{(1 + r)^{10}};\ r = 0.022565; \times 2 = 0.045130$$

For quarterly compounding, the annual yield-to-maturity is stated to be 4.4880%. This annual rate has a periodicity of four.

$$80 = \frac{100}{(1 + r)^{20}};\ r = 0.011220; \times 4 = 0.044880$$

For monthly compounding, the annual yield-to-maturity is stated to be 4.4712%. This annual rate has a periodicity of 12.

$$80 = \frac{100}{(1 + r)^{60}};\ r = 0.003726; \times 12 = 0.044712$$

For annual compounding, the yield-to-maturity is stated to be 4.5640%. This annual rate has a periodicity of one.

$$80 = \frac{100}{(1 + r)^{5}};\ r = 0.045640; \times 1 = 0.045640$$

This is known as an **effective annual rate**. An effective annual rate has a periodicity of one because there is just one compounding period in the year.

In this zero-coupon bond example, 2.2565% compounded two times a year, 1.1220% compounded four times a year, and 0.3726% compounded 12 times a year are all equivalent to an effective annual rate of 4.5640%. The compounded total return is the same for each expression for the annual rate. They differ in terms of the number of compounding periods per year—that is, in terms of the *periodicity* of the annual rate. For a given pair of cash flows, the stated annual rate and the periodicity are inversely related.

The most common periodicity for US dollar-denominated bond yields is two because most bonds in the US dollar market make semiannual coupon payments. An annual rate having a periodicity of two is known as a **semiannual bond basis yield**, or **semiannual bond equivalent yield**. Therefore, a semiannual bond basis yield is the yield per semiannual period times two. It is important to remember that "semiannual bond basis yield" and "yield per semiannual period" have different meanings. For example, if a bond yield is 2% per semiannual period, its annual yield is 4% when stated on a semiannual bond basis.

An important tool used in fixed-income analysis is to convert an annual yield from one periodicity to another. These are called periodicity, or compounding, conversions. A general formula to convert an annual percentage rate for *m* periods per year, denoted APR_m, to an annual percentage rate for *n* periods per year, APR_n, is Equation 7:

$$\left(1+\frac{APR_m}{m}\right)^m=\left(1+\frac{APR_n}{n}\right)^n \tag{7}$$

For example, suppose that a three-year, 5% semiannual coupon payment corporate bond is priced at 104 per 100 of par value. Its yield-to-maturity is 3.582%, quoted on a semiannual bond basis for a periodicity of two: $0.01791 \times 2 = 0.03582$.

$$104=\frac{2.5}{(1+r)^1}+\frac{2.5}{(1+r)^2}+\frac{2.5}{(1+r)^3}+\frac{2.5}{(1+r)^4}+\frac{2.5}{(1+r)^5}+\frac{102.5}{(1+r)^6};\ r=0.01791$$

To compare this bond with others, an analyst converts this annualized yield-to-maturity to quarterly and monthly compounding. Doing so entails using Equation 7 to convert from a periodicity of $m = 2$ to periodicities of $n = 4$ and $n = 12$.

$$\left(1+\frac{0.03582}{2}\right)^2=\left(1+\frac{APR_4}{4}\right)^4;\ APR_4=0.03566$$

$$\left(1+\frac{0.03582}{2}\right)^2=\left(1+\frac{APR_{12}}{12}\right)^{12};\ APR_{12}=0.03556$$

An annual yield-to-maturity of 3.582% for semiannual compounding provides the same rate of return as annual yields of 3.566% and 3.556% for quarterly and monthly compounding, respectively. A general rule for these periodicity conversions is *compounding more frequently at a lower annual rate corresponds to compounding less frequently at a higher annual rate*. This rule can be used to check periodicity conversion calculations.

Equation 7 also applies to *negative* bond yields. Government bond yields have been negative in several countries, including Switzerland, Germany, Sweden, and Japan. For a simple example, consider a five-year, zero-coupon bond priced at 105 (% of par value). Its yield-to-maturity is –0.971% stated as an effective annual rate for a periodicity of 1.

$$105=\frac{100}{(1+r)^5};\ r=-0.00971$$

Converting that to semiannual and monthly compounding for periodicities of 2 and 12 results in annual yields of –0.973% and –0.975%, respectively.

$$\left(1+\frac{-0.00971}{1}\right)^1=\left(1+\frac{APR_2}{2}\right)^2;\ APR_2=-0.00973$$

$$\left(1+\frac{-0.00971}{1}\right)^1=\left(1+\frac{APR_{12}}{12}\right)^{12};\ APR_{12}=-0.00975$$

Compounding more frequently within the year results in a lower (more negative) yield-to-maturity.

EXAMPLE 7 Yield Conversion Based on Periodicity

A five-year, 4.50% semiannual coupon payment government bond is priced at 98 per 100 of par value. Calculate the annual yield-to-maturity stated on a semiannual bond basis, rounded to the nearest basis point. Convert that annual yield to

A. an annual rate that can be used for direct comparison with otherwise comparable bonds that make *quarterly* coupon payments and
B. an annual rate that can be used for direct comparison with otherwise comparable bonds that make *annual* coupon payments.

Solution: The stated annual yield-to-maturity on a semiannual bond basis is 4.96% ($0.0248 \times 2 = 0.0496$).

$$98 = \frac{2.25}{(1+r)^1} + \frac{2.25}{(1+r)^2} + \frac{2.25}{(1+r)^3} + \frac{2.25}{(1+r)^4} + \frac{2.25}{(1+r)^5} + \frac{2.25}{(1+r)^6} + \frac{2.25}{(1+r)^7} + \frac{2.25}{(1+r)^8} + \frac{2.25}{(1+r)^9} + \frac{102.25}{(1+r)^{10}};\ r = 0.0248$$

A. Convert 4.96% from a periodicity of two to a periodicity of four:

$$\left(1 + \frac{0.0496}{2}\right)^2 = \left(1 + \frac{APR_4}{4}\right)^4;\ APR_4 = 0.0493$$

The annual percentage rate of 4.96% for compounding semiannually compares with 4.93% for compounding quarterly. That makes sense because increasing the frequency of compounding lowers the annual rate.

B. Convert 4.96% from a periodicity of two to a periodicity of one:

$$\left(1 + \frac{0.0496}{2}\right)^2 = \left(1 + \frac{APR_1}{1}\right)^1;\ APR_1 = 0.0502$$

The annual rate of 4.96% for compounding semiannually compares with an effective annual rate of 5.02%. Converting from more frequent to less frequent compounding entails raising the annual percentage rate.

3.4. Yield Measures for Fixed-Rate Bonds

- **calculate and interpret yield measures for fixed-rate bonds and floating-rate notes**

An important concern for quoting and calculating bond yields-to-maturity is the actual timing of the cash flows. Consider a 6% semiannual payment corporate bond that matures on 15 March 2028. Suppose that for settlement on 23 January 2020, the bond is priced at 98.5 per

100 of par value to yield 6.236% quoted on a semiannual bond basis. Its coupon payments are scheduled for 15 March and 15 September of each year. The yield calculation implicitly assumes that the payments are made on those dates. It neglects the reality that 15 March 2020 is a Sunday and 15 March 2025 is a Saturday. In fact, the coupon payments will be made to investors on the following Monday.

Yield measures that neglect weekends and holidays are quoted on what is called **street convention**. The street convention yield-to-maturity is the internal rate of return on the cash flows assuming the payments are made on the scheduled dates. This assumption simplifies bond price and yield calculations and commonly is used in practice. Sometimes the **true yield** is also quoted. The true yield-to-maturity is the internal rate of return on the cash flows using the actual calendar of weekends and bank holidays. The true yield is never higher than the street convention yield because weekends and holidays delay the time to payment. The difference is typically small, no more than a basis point or two. Therefore, the true yield is not commonly used in practice. Sometimes, a **government equivalent yield** is quoted for a corporate bond. A government equivalent yield restates a yield-to-maturity based on a 30/360 day count to one based on actual/actual. The government equivalent yield on a corporate bond can be used to obtain the spread over the government yield. Doing so keeps the yields stated on the same day-count convention basis.

Another yield measure that is commonly quoted for fixed-income bonds is the **current yield**, also called the income or running yield. The current yield is the sum of the coupon payments received over the year divided by the flat price. For example, a 10-year, 2% semiannual coupon payment bond is priced at 95 per 100 of par value. Its current yield is 2.105%.

$$\frac{2}{95} = 0.02105$$

The current yield is a crude measure of the rate of return to an investor because it neglects the frequency of coupon payments in the numerator and any accrued interest in the denominator. It focuses only on interest income. In addition to collecting and reinvesting coupon payments, the investor has a gain if the bond is purchased at a discount and is redeemed at par value. The investor has a loss if the bond is purchased at a premium and is redeemed at par value. Sometimes the **simple yield** on a bond is quoted. It is the sum of the coupon payments plus the straight-line amortized share of the gain or loss, divided by the flat price. Simple yields are used mostly to quote Japanese government bonds, known as "JGBs."

EXAMPLE 8 Comparing Yields for Different Periodicities

An analyst observes these reported statistics for two bonds.

	Bond A	Bond B
Annual coupon rate	8.00%	12.00%
Coupon payment frequency	Semiannually	Quarterly
Years to maturity	5 Years	5 Years
Price (per 100 of par value)	90	105
Current yield	8.889%	11.429%
Yield-to-maturity	10.630%	10.696%

1. Confirm the calculation of the two yield measures for the two bonds.
2. The analyst believes that Bond B has a little more risk than Bond A. How much additional compensation, in terms of a higher yield-to-maturity, does a buyer of Bond B receive for bearing this risk compared with Bond A?

Solution to 1: Current yield for Bond A:

$$\frac{8}{90} = 0.08889$$

Yield-to-maturity for Bond A:

$$90 = \frac{4}{(1+r)^1} + \frac{4}{(1+r)^2} + \cdots + \frac{104}{(1+r)^{10}};\ r = 0.05315;\ \times 2 = 0.10630$$

Current yield for Bond B:

$$\frac{12}{105} = 0.11429$$

Yield-to-maturity for Bond B:

$$105 = \frac{3}{(1+r)^1} + \frac{3}{(1+r)^2} + \cdots + \frac{103}{(1+r)^{20}};\ r = 0.02674;\ \times 4 = 0.10696$$

Solution to 2: The yield-to-maturity on Bond A of 10.630% is an annual rate for compounding semiannually. The yield-to-maturity on Bond B of 10.696% is an annual rate for compounding quarterly. The difference in the yields is *not* 6.6 bps (0.10696 – 0.10630 = 0.00066). It is essential to compare the yields for the same periodicity to make a statement about relative value.

A yield-to-maturity of 10.630% for a periodicity of two converts to 10.492% for a periodicity of four:

$$\left(1 + \frac{0.10630}{2}\right)^2 = \left(1 + \frac{APR_4}{4}\right)^4;\ APR_4 = 0.10492$$

A yield-to-maturity of 10.696% for a periodicity of four converts to 10.839% for a periodicity of two:

$$\left(1 + \frac{0.10696}{4}\right)^4 = \left(1 + \frac{APR_2}{2}\right)^2;\ APR_2 = 0.10839$$

The additional compensation for the greater risk in Bond B is 20.9 bps (0.10839 – 0.10630 = 0.00209) when the yields are stated on a semiannual bond basis. The additional compensation is 20.4 bps (0.10696 – 0.10492 = 0.00204) when both are annualized for quarterly compounding.

If a fixed-rate bond contains an **embedded option**, other yield measures are used. An embedded option is part of the security and cannot be removed and sold separately. For example, a **callable bond** contains an embedded call option that gives the issuer the right to buy the bond back from the investor at specified prices on pre-determined dates. The preset dates usually coincide with coupon payment dates after a **call protection** period. A call protection period is the time during which the issuer of the bond is not allowed to exercise the call option.

Suppose that a seven-year, 8% annual coupon payment bond is first callable in four years. That gives the investor four years of protection against the bond being called. After the call protection period, the issuer might exercise the call option if interest rates decrease or the issuer's credit quality improves. Those circumstances allow the issuer to refinance the debt at a lower cost of funds. The preset prices that the issuer pays if the bond is called often are at a premium above par. For example, the "call schedule" for this bond might be that it is callable first at 102 (per 100 of par value) on the coupon payment date in four years, at 101 in five years, and at par value on coupon payment dates thereafter.

The yield-to-maturity on this seven-year, 8% callable bond is just one of several traditional yield measures for the investment. Others are yield-to-first-call, yield-to-second-call, and so on. If the current price for the bond is 105 per 100 of par value, the yield-to-first-call in four years is 6.975%.

$$105 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8+102}{(1+r)^4}, \quad r = 0.06975$$

The yield-to-second-call in five years is 6.956%.

$$105 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8}{(1+r)^4} + \frac{8+101}{(1+r)^5}; \quad r = 0.06956$$

The yield-to-third-call is 6.953%.

$$105 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8}{(1+r)^4} + \frac{8}{(1+r)^5} + \frac{8+100}{(1+r)^6}; \quad r = 0.06953$$

Finally, the yield-to-maturity is 7.070%.

$$105 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8}{(1+r)^4} + \frac{8}{(1+r)^5} + \frac{8}{(1+r)^6} + \frac{8+100}{(1+r)^7}; \; r = 0.07070$$

Each calculation is based on Equation 1, whereby the call price (or par value) is used for *FV*. The lowest of the sequence of yields-to-call and the yield-to-maturity is known as the **yield-to-worst**. In this case, it is the yield-to-third-call of 6.953%. The intent of this yield measure is to provide to the investor the most conservative assumption for the rate of return.

The yield-to-worst is a commonly cited yield measure for fixed-rate callable bonds used by bond dealers and investors. However, a more precise approach is to use an option pricing model and an assumption about future interest rate volatility to value the embedded call option. The value of the embedded call option is added to the flat price of the bond to get the **option-adjusted price**. The investor bears the call risk (the bond issuer has the option to call), so the embedded call option reduces the value of the bond from the investor's perspective. The investor pays a lower price for the callable bond than if it were option-free. If the bond were non-callable, its price would be higher. The option-adjusted price is used to calculate the **option-adjusted yield**. The option-adjusted yield is the required market discount rate whereby the price is adjusted for the value of the embedded option. The value of the call option is the price of the option-free bond minus the price of the callable bond.

3.5. Yield Measures for Floating-Rate Notes

Floating-rate notes are very different from fixed-rate bonds. The interest payments on a floating-rate note, which often is called a floater or an FRN, are not fixed. Instead, they vary from period to period depending on the current level of a reference interest rate. The interest payments could go up or down; that is why they are called "floaters." The intent of an FRN is to offer the investor a security that has less market price risk than a fixed-rate bond when market interest rates fluctuate. In principle, a floater has a stable price even in a period of volatile interest rates. With a traditional fixed-income security, interest rate volatility affects the price because the future cash flows are constant. With a floating-rate note, interest rate volatility affects future interest payments.

The reference rate on a floating-rate note usually is a short-term money market rate. The three-month Libor (London Interbank Offered Rate) had long been the reference rate for many floating-rate notes. As discussed earlier, Libor is being phased out. We refer to the new rate as the Market Reference Rate (MRR). The principal on the floater typically is non-amortizing and is redeemed in full at maturity. As of June 2020, the reference rate is determined at the beginning of the period, and the interest payment is made at the end of the period. This payment structure is called "in arrears." The most common day-count conventions for calculating accrued interest on floaters are actual/360 and actual/365.

Although there are many varieties of FRNs, only the most common and traditional floaters are covered here. On these floaters, a specified yield spread is added to or subtracted from the reference rate. For example, the floater might reset its interest rate quarterly at MRR plus 0.50%. This specified yield spread over the reference rate is called the **quoted margin** on the FRN. The role of the quoted margin is to compensate the investor for the difference in the credit risk of the issuer and that implied by the reference rate. For example, a company with a stronger credit rating than that of the banks or financial institutions from which MRR is computed may be able to obtain a "sub-MRR" cost of borrowed funds, which results in a negative quoted margin. An AAA rated company might be able to issue an FRN that pays MRR minus 0.25%.

The **required margin** is the yield spread over or under the reference rate such that the FRN is priced at par value on a rate reset date. Suppose that a traditional floater is issued at par value and pays MRR plus 0.50%. The quoted margin is 50 bps. If there is no change in the credit risk of the issuer, the required margin remains at 50 bps. On each quarterly reset date, the floater will be priced at par value. Between coupon dates, its flat price will be at a premium or discount to par value if MRR goes, respectively, down or up. However, if the required margin continues to be the same as the quoted margin, the flat price is "pulled to par" as the next reset date nears. At the reset date, any change in MRR is included in the interest payment for the next period.

Changes in the required margin usually come from changes in the issuer's credit risk. Changes in liquidity or tax status also could affect the required margin. Suppose that on a reset date, the required margin goes up to 75 bps because of a downgrade in the issuer's credit rating. A floater having a quoted margin of 50 bps now pays its investors a "deficient" interest payment. This FRN will be priced at a discount below par value. The amount of the discount is the present value of the deficient future cash flows. That annuity is 25 bps per period for the remaining life of the bond. It is the difference between the required and quoted margins. If the required margin goes down from 50 bps to 40 bps, the FRN will be priced at a premium. The amount of the premium is the present value of the 10 bp annuity for the "excess" interest payment each period.

Fixed-rate and floating-rate bonds are essentially the same with respect to changes in credit risk. With fixed-rate bonds, the premium or discount arises from a difference in the fixed coupon rate and the required yield-to-maturity. With floating-rate bonds, the premium or discount arises from a difference in the fixed quoted margin and the required margin. However, fixed-rate and floating-rate bonds are very different with respect to changes in benchmark interest rates.

The valuation of a floating-rate note needs a pricing model. Equation 8 is a simplified FRN pricing model. Following market practice, the required margin is called the **discount margin**.

$$PV = \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^1} + \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^2} + \cdots + \frac{\frac{(\text{Index} + QM) \times FV}{m} + FV}{\left(1 + \frac{\text{Index} + DM}{m}\right)^N} \quad (8)$$

where

PV = present value, or the price of the floating-rate note
Index = reference rate, stated as an annual percentage rate
QM = quoted margin, stated as an annual percentage rate
FV = future value paid at maturity, or the par value of the bond
m = periodicity of the floating-rate note, the number of payment periods per year
DM = discount margin, the required margin stated as an annual percentage rate
N = number of evenly spaced periods to maturity

This equation is similar to Equation 1, which is the basic pricing formula for a fixed-rate bond given the market discount rate. In Equation 1, *PMT* is the coupon payment *per period*. Here, *annual* rates are used. The first interest payment is the annual rate for the period (Index + QM) times the par value (FV) and divided by the number of periods in the year (m). In Equation 1, the market discount rate per period (r) is used to discount the cash flows. Here, the discount rate per period is the reference rate plus the discount margin (Index + DM) divided by the periodicity (m).

This is a simplified FRN pricing model, for several reasons. First, PV is for a rate reset date when there are N evenly spaced periods to maturity. There is no accrued interest so that the flat price is the full price. Second, the model assumes a 30/360 day-count convention so that the periodicity is an integer. Third, and most important, the same reference rate (Index) is used for all payment periods in both the numerators and denominators. More complex FRN pricing models use projected future rates for Index in the numerators and spot rates in the denominators. Therefore, the calculation for DM depends on the simplifying assumptions in the pricing model.

Suppose that a two-year FRN pays MRR plus 0.50%. Currently, MRR is 1.25%. In Equation 8, Index = 0.0125, QM = 0.0050, and m = 2. The numerators in Equation 8, ignoring the repayment of principal, are 0.875.

$$\frac{(\text{Index} + QM) \times FV}{m} = \frac{(0.0125 + 0.0050) \times 100}{2} = 0.875$$

Suppose that the yield spread required by investors is 40 bps over the reference rate; DM = 0.0040. The assumed discount rate per period is 0.825%.

$$\frac{\text{Index} + DM}{m} = \frac{0.0125 + 0.0040}{2} = 0.00825$$

Using Equation 8 for N = 4, the FRN is priced at 100.196 per 100 of par value.

$$\frac{0.875}{(1 + 0.00825)^1} + \frac{0.875}{(1 + 0.00825)^2} + \frac{0.875}{(1 + 0.00825)^3} + \frac{0.875 + 100}{(1 + 0.00825)^4} = 100.196$$

This floater is priced at a premium above par value because the quoted margin is greater than the discount margin.

A similar calculation is to estimate the discount margin given the market price of the floating-rate note. Suppose that a five-year FRN pays MRR plus 0.75% on a quarterly basis. Currently, MRR is 1.10%. The price of the floater is 95.50 per 100 of par value, a discount below par value because of a downgrade in the issuer's credit rating.

$$\frac{(\text{Index} + QM) \times FV}{m} = \frac{(0.0110 + 0.0075) \times 100}{4} = 0.4625$$

In Equation 8, use $PV = 95.50$ and $N = 20$.

$$95.50 = \frac{0.4625}{\left(1 + \frac{0.0110 + DM}{4}\right)^1} + \frac{0.4625}{\left(1 + \frac{0.0110 + DM}{4}\right)^2} + \cdots + \frac{0.4625 + 100}{\left(1 + \frac{0.0110 + DM}{4}\right)^{20}}$$

This has the same format as Equation 1, which can be used to solve for the market discount rate per period, $r = 0.7045\%$.

$$95.50 = \frac{0.4625}{(1 + r)^1} + \frac{0.4625}{(1 + r)^2} + \cdots + \frac{0.4625 + 100}{(1 + r)^{20}};\ r = 0.007045$$

This can be used to solve for $DM = 1.718\%$.

$$0.007045 = \frac{0.0110 + DM}{4};\ DM = 0.01718$$

If this FRN was issued at par value, investors required at that time a spread of only 75 bps over MRR. Now, after the credit downgrade, investors require an *estimated* discount margin of 171.8 bps. The floater trades at a discount because the quoted margin remains fixed at 75 bps. The calculated discount margin is an estimate because it is based on a simplified FRN pricing model.

The FRN pricing model in Equation 8 similarly applies to adjustable-rate loans made by banks and other non-securitized fixed-income instruments. Because a large portion of a bank's funding comes from short-duration investor deposits, banks prefer to make floating-rate unsecured loans to businesses and individuals, as opposed to fixed-rate loans, in order to help maintain a match between assets and liabilities on the balance sheet. This model is used to determine the appropriate interest rate to pay on demand or certificates of deposit (CDs), as well as to develop scorecards for risk-based floating-rate loan pricing.

EXAMPLE 9 Calculating the Discount Margin for a Floating-Rate Note

A four-year French floating-rate note pays three-month Euribor (Euro Interbank Offered Rate, an index produced by the European Banking Federation) plus 1.25%. The floater is priced at 98 per 100 of par value. Calculate the discount margin for the floater assuming that three-month Euribor is constant at 2%. Assume the 30/360 day-count convention and evenly spaced periods.

Solution: By assumption, the interest payment each period is 0.8125 per 100 of par value.

$$\frac{(\text{Index} + QM) \times FV}{m} = \frac{(0.0200 + 0.0125) \times 100}{4} = 0.8125$$

The discount margin can be estimated by solving for *DM* in this equation.

$$98 = \frac{0.8125}{\left(1 + \frac{0.0200 + DM}{4}\right)^1} + \frac{0.8125}{\left(1 + \frac{0.0200 + DM}{4}\right)^2} + \cdots + \frac{0.8125 + 100}{\left(1 + \frac{0.0200 + DM}{4}\right)^{16}}$$

The solution for the discount rate per period is 0.9478%.

$$98 = \frac{0.8125}{(1 + r)^1} + \frac{0.8125}{(1 + r)^2} + \cdots + \frac{0.8125 + 100}{(1 + r)^{16}};\ r = 0.009478$$

Therefore, $DM = 1.791\%$.

$$0.009478 = \frac{0.0200 + DM}{4};\ DM = 0.01791$$

The quoted margin is 125 bps over the Euribor reference rate. Using the simplified FRN pricing model, it is estimated that investors require a 179.1 bp spread for the floater to be priced at par value.

3.6. Yield Measures for Money Market Instruments

- **calculate and interpret yield measures for money market instruments**

Money market instruments are short-term debt securities. They range in time-to-maturity from overnight sale and repurchase agreements (repos) to one-year bank certificates of deposit. Money market instruments also include commercial paper, government issues of less than one year, bankers' acceptances, and time deposits based on such indexes as Libor and Euribor. Money market mutual funds are a major investor in such securities. These mutual funds can invest only in certain eligible money market securities.

There are several important differences in yield measures between the money market and the bond market:

1. Bond yields-to-maturity are annualized and compounded. Yield measures in the money market are annualized but not compounded. Instead, the rate of return on a money market instrument is stated on a simple interest basis.
2. Bond yields-to-maturity can be calculated using standard time-value-of-money analysis and with formulas programmed into a financial calculator. Money market instruments often are quoted using non-standard interest rates and require different pricing equations than those used for bonds.
3. Bond yields-to-maturity usually are stated for a common periodicity for all times-to-maturity. Money market instruments having different times-to-maturity have different periodicities for the annual rate.

In general, quoted money market rates are either **discount rates** or **add-on rates**. Although market conventions vary around the world, commercial paper, Treasury bills (US government securities issued with a maturity of one year or less), and bankers' acceptances often are quoted on a discount rate basis. Bank certificates of deposit, repos, and such indexes as Libor and

Euribor are quoted on an add-on rate basis. It is important to understand that "discount rate" has a unique meaning in the money market. In general, discount rate means "interest rate used to calculate a present value"—for instance, "market discount rate" as used in our discussion. In the money market, however, discount rate is a specific type of quoted rate. Some examples will clarify this point.

Equation 9 is the pricing formula for money market instruments quoted on a discount rate basis.

$$PV = FV \times \left(1 - \frac{\text{Days}}{\text{Year}} \times DR\right) \tag{9}$$

where

PV = present value, or the price of the money market instrument
FV = future value paid at maturity, or the face value of the money market instrument
Days = number of days between settlement and maturity
Year = number of days in the year
DR = discount rate, stated as an annual percentage rate

Suppose that a 91-day US Treasury bill (T-bill) with a face value of USD10 million is quoted at a discount rate of 2.25% for an assumed 360-day year. Enter FV = 10,000,000, Days = 91, Year = 360, and DR = 0.0225. The price of the T-bill is USD9,943,125.

$$PV = 10{,}000{,}000 \times \left(1 - \frac{91}{360} \times 0.0225\right) = 9{,}943{,}125$$

The unique characteristics of a money market discount rate can be examined with Equation 10, which transforms Equation 9 algebraically to isolate the DR term.

$$DR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{FV}\right) \tag{10}$$

The first term, Year/Days, is the periodicity of the annual rate. The second term reveals the odd character of a money market discount rate. The numerator, $FV - PV$, is the interest earned on the T-bill, USD56,875 (= 10,000,000 − 9,943,125), over the 91 days to maturity. However, the denominator is FV, not PV. In theory, an interest rate is the amount earned divided by the investment amount (PV)—not divided by the total return at maturity, which includes the earnings (FV). Therefore, by design, a money market discount rate *understates* the rate of return to the investor, and it *understates* the cost of borrowed funds to the issuer. That is because PV is less than FV (as long as DR is greater than zero).

Equation 11 is the pricing formula for money market instruments quoted on an add-on rate basis.

$$PV = \frac{FV}{\left(1 + \frac{\text{Days}}{\text{Year}} \times AOR\right)} \tag{11}$$

where

PV = present value, principal amount, or the price of the money market instrument
FV = future value, or the redemption amount paid at maturity including interest
Days = number of days between settlement and maturity
Year = number of days in the year
AOR = add-on rate, stated as an annual percentage rate

Suppose that a Canadian pension fund buys a 180-day banker's acceptance (BA) with a quoted add-on rate of 4.38% for a 365-day year. If the initial principal amount is CAD10 million, the redemption amount due at maturity is found by re-arranging Equation 11 and entering PV = 10,000,000, Days = 180, Year = 365, and AOR = 0.0438.

$$FV = 10,000,000 + \left(10,000,000 \times \frac{180}{365} \times 0.0438\right) = 10,216,000$$

At maturity, the pension fund receives CAD10,216,000, the principal of CAD10 million plus interest of CAD216,000. The interest is calculated as the principal times the fraction of the year times the annual add-on rate. It is added to the principal to determine the redemption amount.

Suppose that after 45 days, the pension fund sells the BA to a dealer. At that time, the quoted add-on rate for a 135-day BA is 4.17%. The sale price for the BA can be calculated using Equation 11 for FV = 10,216,000, Days = 135, Year = 365, and AOR = 0.0417. The sale price is CAD10,060,829.

$$PV = \frac{10,216,000}{\left(1 + \frac{135}{365} \times 0.0417\right)} = 10,060,829$$

The characteristics of an add-on rate can be examined with Equation 12, which transforms Equation 11 algebraically to isolate the AOR term.

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{PV}\right) \qquad (12)$$

This equation indicates that an add-on rate is a reasonable yield measure for a money market investment. The first term, Year/Days, is the periodicity of the annual rate. The second term is the interest earned, $FV - PV$, divided by PV, the amount invested.

The pension fund's rate of return on its 45-day investment in the banker's acceptance can be calculated with Equation 12. Enter Year = 365, Days = 45, FV = 10,060,829, and PV = 10,000,000. Notice that FV here is the sale price, not the redemption amount.

$$AOR = \left(\frac{365}{45}\right) \times \left(\frac{10,060,829 - 10,000,000}{10,000,000}\right) = 0.04934$$

The rate of return, stated on a 365-day add-on rate basis, is 4.934%. This result is an annual rate for a periodicity of 8.11 (= 365/45). Implicitly, this assumes that the investment can be replicated 8.11 times over the year.

Investment analysis is made difficult for money market securities because (1) some instruments are quoted on a discount rate basis and others on an add-on rate basis and (2) some are quoted for a 360-day year and others for a 365-day year. Another difference is that the "amount" of a money market instrument quoted on a discount rate basis typically is the face value paid at maturity. However, the "amount" when quoted on an add-on rate basis usually is the principal, the price at issuance. To make money market investment decisions, it is essential to compare instruments on a common basis. An example illustrates this point.

Suppose that an investor is comparing two money market instruments: (A) 90-day commercial paper quoted at a discount rate of 5.76% for a 360-day year and (B) a 90-day bank time deposit quoted at an add-on rate of 5.90% for a 365-day year. Which offers the higher expected rate of return assuming that the credit risks are the same? The price of the commercial paper is 98.560 per 100 of face value, calculated using Equation 9 and entering $FV = 100$, Days = 90, Year = 360, and $DR = 0.0576$.

$$PV = 100 \times \left(1 - \frac{90}{360} \times 0.0576\right) = 98.560$$

Next, use Equation 12 to solve for the *AOR* for a 365-day year, whereby Year = 365, Days = 90, $FV = 100$, and $PV = 98.560$.

$$AOR = \left(\frac{365}{90}\right) \times \left(\frac{100 - 98.560}{98.560}\right) = 0.05925$$

The 90-day commercial paper discount rate of 5.76% converts to an add-on rate for a 365-day year of 5.925%. This converted rate is called a **bond equivalent yield**, or sometimes just an "investment yield." A bond equivalent yield is a money market rate stated on a 365-day add-on rate basis. If the risks are the same, the commercial paper offers 2.5 bps more in annual return than the bank time deposit.

EXAMPLE 10 Comparing Money Market Instruments Based on Bond Equivalent Yields

Suppose that a money market investor observes quoted rates on the following four 180-day money market instruments:

Money Market Instrument	Quotation Basis	Assumed Number of Days in the Year	Quoted Rate
A	Discount Rate	360	4.33%
B	Discount Rate	365	4.36%
C	Add-On Rate	360	4.35%
D	Add-On Rate	365	4.45%

Calculate the bond equivalent yield for each instrument. Which instrument offers the investor the highest rate of return if the credit risks are the same?

Solution:

A. Use Equation 9 to get the price per 100 of par value, where $FV = 100$, Days = 180, Year = 360, and $DR = 0.0433$.

$$PV = 100 \times \left(1 - \frac{180}{360} \times 0.0433\right) = 97.835$$

Use Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, $FV = 100$, and $PV = 97.835$.

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{100 - 97.835}{97.835}\right) = 0.04487$$

The bond equivalent yield for Bond A is 4.487%.

B. Use Equation 9 to get the price per 100 of face value, where $FV = 100$, Days = 180, Year = 365, and $DR = 0.0436$.

$$PV = 100 \times \left(1 - \frac{180}{365} \times 0.0436\right) = 97.850$$

Use Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, $FV = 100$, and $PV = 97.850$.

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{100 - 97.850}{97.850}\right) = 0.04456$$

The bond equivalent yield for Bond B is 4.456%.

C. First, determine the redemption amount per 100 of principal ($PV = 100$), where Days = 180, Year = 360, and $AOR = 0.0435$.

$$FV = 100 + \left(100 \times \frac{180}{360} \times 0.0435\right) = 102.175$$

Use Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, $FV = 102.175$, and $PV = 100$.

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{102.175 - 100}{100}\right) = 0.04410$$

The bond equivalent yield for Bond C is 4.410%.

Another way to get the bond equivalent yield for Bond C is to observe that the *AOR* of 4.35% for a 360-day year can be obtained using Equation 12 for Year = 360, Days = 180, $FV = 102.175$, and $PV = 100$.

$$AOR = \left(\frac{360}{180}\right) \times \left(\frac{102.175 - 100}{100}\right) = 0.0435$$

Therefore, an add-on rate for a 360-day year only needs to be multiplied by the factor of 365/360 to get the 365-day year bond equivalent yield.

$$\frac{365}{360} \times 0.0435 = 0.04410$$

D. The quoted rate for Bond D of 4.45% is a bond equivalent yield, which is defined as an add-on rate for a 365-day year.

If the risks of these money market instruments are the same, Bond A offers the highest rate of return on a bond equivalent yield basis, 4.487%.

The third difference between yield measures in the money market and the bond market is the periodicity of the annual rate. Because bond yields-to-maturity are computed using interest rate compounding, there is a well-defined periodicity. For instance, bond yields-to-maturity for semiannual compounding are annualized for a periodicity of two. Money market rates are computed using simple interest without compounding. In the money market, the periodicity is the number of days in the year divided by the number of days to maturity. Therefore, money market rates for different times-to-maturity have different periodicities.

Suppose that an analyst prefers to convert money market rates to a semiannual bond basis so that the rates are directly comparable to yields on bonds that make semiannual coupon payments. The quoted rate for a 90-day money market instrument is 10%, quoted as a bond equivalent yield, which means its periodicity is 365/90. Using Equation 7, the conversion is from $m = 365/90$ to $n = 2$ for $APR_{365/90} = 0.10$.

$$\left(1 + \frac{0.10}{365/90}\right)^{365/90} = \left(1 + \frac{APR_2}{2}\right)^2; APR_2 = 0.10127$$

Therefore, 10% for a periodicity of 365/90 corresponds to 10.127% for a periodicity of two. The difference is significant—12.7 bps. In general, the difference depends on the level of the annual percentage rate. When interest rates are lower, the difference between the annual rates for any two periodicities is reduced.

4. THE MATURITY STRUCTURE OF INTEREST RATES

- **define and compare the spot curve, yield curve on coupon bonds, par curve, and forward curve**
- **define forward rates and calculate spot rates from forward rates, forward rates from spot rates, and the price of a bond using forward rates**

There are many reasons why the yields-to-maturity on any two bonds are different. Suppose that the yield-to-maturity is higher on Bond X than on Bond Y. The following are some possible reasons for the difference between the yields:

- Currency—Bond X could be denominated in a currency with a higher expected rate of inflation than the currency in which Bond Y is denominated.
- Credit risk—Bond X could have a non-investment-grade rating of BB, and Bond Y could have an investment-grade rating of AA.
- Liquidity—Bond X could be illiquid, and Bond Y could be actively traded.
- Tax status—Interest income on Bond X could be taxable, whereas interest income on Bond Y could be exempt from taxation.
- Periodicity—Bond X could make a single annual coupon payment, and its yield-to-maturity could be quoted for a periodicity of one. Bond Y could make monthly coupon payments, and its yield-to-maturity could be annualized for a periodicity of 12.

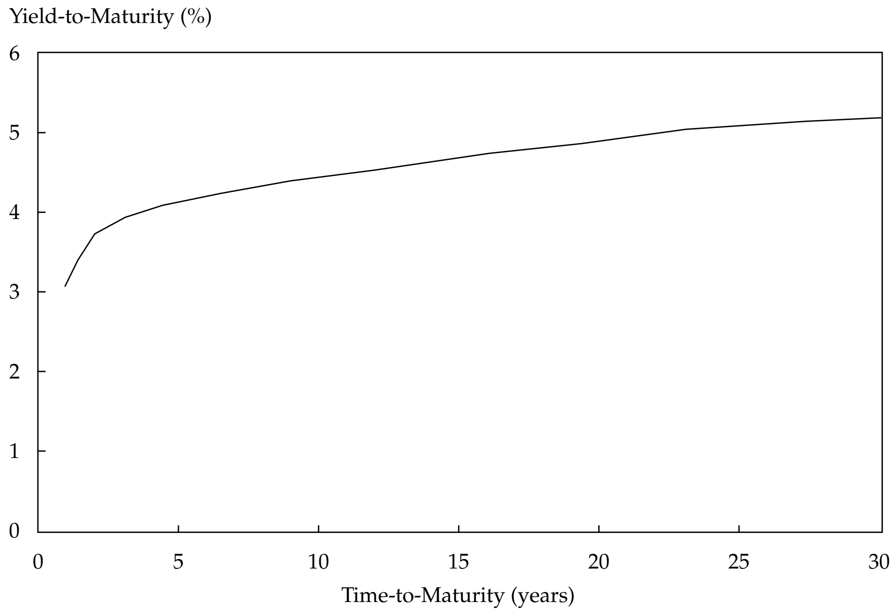
Obviously, another reason is that Bond X and Bond Y could have different times-to-maturity. This factor explaining the differences in yields is called the **maturity structure**, or **term structure**, of interest rates. It involves the analysis of yield curves, which are relationships between yields-to-maturity and times-to-maturity. There are different types of yield curves, depending on the characteristics of the underlying bonds.

In theory, maturity structure should be analyzed for bonds that have the same properties other than time-to-maturity. The bonds should be denominated in the same currency and have the same credit risk, liquidity, and tax status. Their annual rates should be quoted for the same periodicity. Also, they should have the same coupon rate so that they each have the same degree of coupon reinvestment risk. In practice, maturity structure is analyzed for bonds for which these strong assumptions rarely hold.

The ideal dataset would be yields-to-maturity on a series of *zero-coupon* government bonds for a full range of maturities. This dataset is the government bond **spot curve**, sometimes called the zero or "strip" curve (because the coupon payments are "stripped" off the bonds). The spot, zero, or strip curve is a sequence of yields-to-maturity on zero-coupon bonds. Often, these government spot rates are interpreted as the "risk-free" yields; in this context, "risk-free" refers only to default risk. There still could be a significant amount of inflation risk to the investor, as well as liquidity risk.

A government bond spot curve is illustrated in Exhibit 5 for maturities ranging from 1 to 30 years. The annual yields are stated on a semiannual bond basis, which facilitates comparison to coupon-bearing bonds that make semiannual payments.

EXHIBIT 5 A Government Bond Spot Curve



This spot curve is upward sloping and flattens for longer times-to-maturity. Longer-term government bonds usually have higher yields than shorter-term bonds. This pattern is typical under normal market conditions. Sometimes, a spot curve is downward sloping in that shorter-term yields are higher than longer-term yields. Such downward-sloping spot curves are called inverted yield curves. The theories that attempt to explain the shape of the yield curve and its implications for future financial market conditions are covered later.

This hypothetical spot curve is ideal for analyzing maturity structure because it best meets the "other things being equal" assumption. These government bonds presumably have the same currency, credit risk, liquidity, and tax status. Most importantly, they have no coupon reinvestment risk because there are no coupons to reinvest. However, most actively traded government and corporate bonds make coupon payments. Therefore, analysis of maturity structure usually is based on price data on government bonds that make coupon payments. These coupon bonds might not have the same liquidity and tax status. Older ("seasoned") bonds tend to be less liquid than newly issued debt because they are owned by "buy-and-hold" institutional and retail investors. Governments issue new debt for regular times-to-maturity—for instance, 5-year and 10-year bonds. The current 6-year bond could be a 10-year bond that was issued four years ago. Also, as interest rates fluctuate, older bonds are priced at a discount or premium to par value, which can lead to tax differences. In some countries, capital gains have different tax treatment than capital losses and interest income.

Analysts usually use only the most recently issued and actively traded government bonds to build a yield curve. These bonds have similar liquidity, and because they are priced closer to par value, they have fewer tax effects. A problem is that there are limited data for the full range of maturities. Therefore, it is necessary to *interpolate* between observed yields. Exhibit 6 illustrates a yield curve for a government that issues 2-year, 3-year, 5-year, 7-year, 10-year, and 30-year bonds that make semiannual coupon payments. Straight-line interpolation is used between those points on the yield curve for coupon bonds.

EXHIBIT 6 A Government Bond Yield Curve

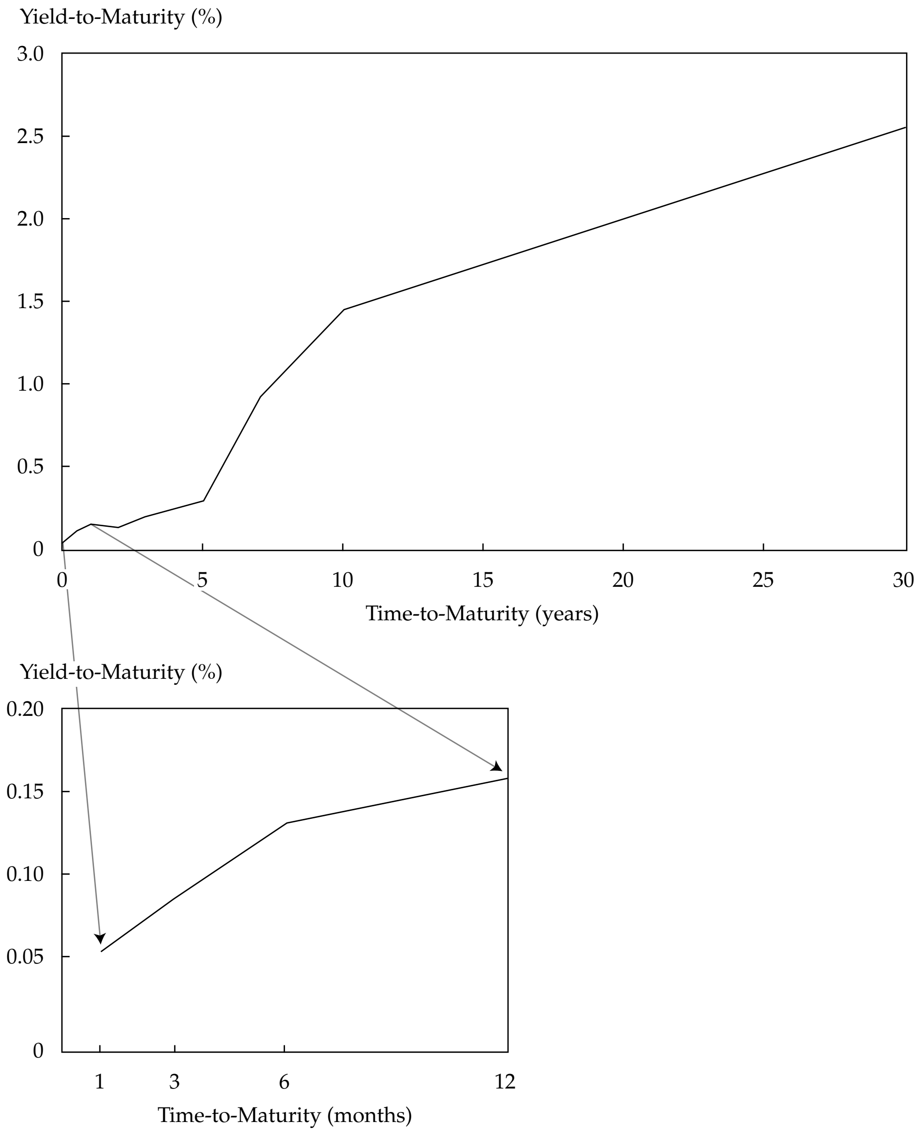


Exhibit 6 also includes yields for short-term government securities having 1 month, 3 months, 6 months, and 12 months to maturity. Although these money market instruments might have been issued and traded on a discount rate basis, they typically are reported as bond equivalent yields. It is important for the analyst to know whether they have been converted to the same periodicity as the longer-term government bonds. If not, the observed yield curve can be misleading because the number of periods in the year is not the same.

In addition to the yield curve on coupon bonds and the spot curve on zero-coupon bonds, maturity structure can be assessed using a **par curve**. A par curve is a sequence of yields-to-maturity such that each bond is priced at par value. The bonds, of course, are assumed to have the same currency, credit risk, liquidity, tax status, and annual yields stated for the same periodicity. Between coupon payment dates, the flat price (not the full price) is assumed to be equal to par value.

The par curve is obtained from a spot curve. On a coupon payment date, the following equation can be used to calculate a par rate given the sequence of spot rates.

$$100 = \frac{PMT}{(1 + z_1)^1} + \frac{PMT}{(1 + z_2)^2} + \cdots + \frac{PMT + 100}{(1 + z_N)^N} \tag{13}$$

This equation is very similar to Equation 2, whereby $PV = FV = 100$. The problem is to solve for PMT algebraically. Then, $PMT/100$ is equal to the par rate *per period*.

An example illustrates the calculation of the par curve given a spot curve. Suppose the spot rates on government bonds are 5.263% for one year, 5.616% for two years, 6.359% for three years, and 7.008% for four years. These are effective annual rates. The one-year par rate is 5.263%.

$$100 = \frac{PMT + 100}{(1.05263)^1};\ PMT = 5.263$$

The two-year par rate is 5.606%.

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT + 100}{(1.05616)^2};\ PMT = 5.606$$

The three-year and four-year par rates are 6.306% and 6.899%, respectively.

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT}{(1.05616)^2} + \frac{PMT + 100}{(1.06359)^3};\ PMT = 6.306$$

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT}{(1.05616)^2} + \frac{PMT}{(1.06359)^3} + \frac{PMT + 100}{(1.07008)^4};\ PMT = 6.899$$

The fixed-income securities covered so far have been **cash market securities**. Money market securities often are settled on a "same day," or "cash settlement," basis. Other securities have a difference between the trade date and the settlement date. For instance, if a government bond trades on a $T + 1$ basis, there is a one-day difference between the trade date and the settlement date. If a corporate bond trades on a $T + 3$ basis, the seller delivers the bond and the buyer makes payment in three business days. Cash markets are also called spot markets, which can be confusing because "spot rate" can have two meanings. It can mean the rate on a bond traded in the spot, or cash, market. It can also mean yield on a zero-coupon bond, which is the meaning used in our discussion.

A **forward market** is for future delivery, beyond the usual settlement time period in the cash market. Agreement to the terms for the transaction is on the trade date, but delivery of the security and payment for it are deferred to a future date. A **forward rate** is the interest rate on a bond or money market instrument traded in a forward market. For example, suppose that in the cash market, a five-year zero-coupon bond is priced at 81 per 100 of par value. Its yield-to-maturity is 4.2592%, stated on a semiannual bond basis.

$$81 = \frac{100}{(1+r)^{10}};\ r = 0.021296;\ \times 2 = 0.042592$$

Suppose that a dealer agrees to deliver a five-year bond two years into the future for a price of 75 per 100 of par value. The credit risk, liquidity, and tax status of this bond traded in the forward market are the same as those for this bond in the cash market. The forward rate is 5.8372%.

$$75 = \frac{100}{(1+r)^{10}};\ r = 0.029186;\ \times 2 = 0.058372$$

The notation for forward rates is important to understand. Although finance textbook authors use varying notation, the most common market practice is to name this forward rate "the 2y5y." This term is pronounced "the two-year into five-year rate," or simply "the 2's, 5's." The idea is that the first number (two years) refers to the length of the forward period in years from today and the second number (five years) refers to the **tenor** of the underlying bond. The tenor is the remaining time-to-maturity for a bond (or a derivative contract). Therefore, 5.8372% is the "2y5y" forward rate for the zero-coupon bond—the five-year yield two years into the future. Note that the bond that will be a five-year zero in two years currently has seven years to maturity. In the money market, the forward rate usually refers to months. For instance, an analyst might inquire about the "1m6m" forward rate on Euribor, which is the rate on six-month Euribor one month into the future.

Implied forward rates (also known as forward yields) are calculated from spot rates. An implied forward rate is a breakeven reinvestment rate. It links the return on an investment in a shorter-term zero-coupon bond to the return on an investment in a longer-term zero-coupon bond. Suppose that the shorter-term bond matures in A periods and the longer-term bond matures in B periods. The yields-to-maturity per period on these bonds are denoted z_A and z_B. The first is an A-period zero-coupon bond trading in the cash market. The second is a B-period zero-coupon cash market bond. The implied forward rate between period A and period B is denoted $IFR_{A,B-A}$. It is a forward rate on a security that starts in period A and ends in period B. Its tenor is $B - A$ periods.

Equation 14 is a general formula for the relationship between the two spot rates and the implied forward rate.

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B \tag{14}$$

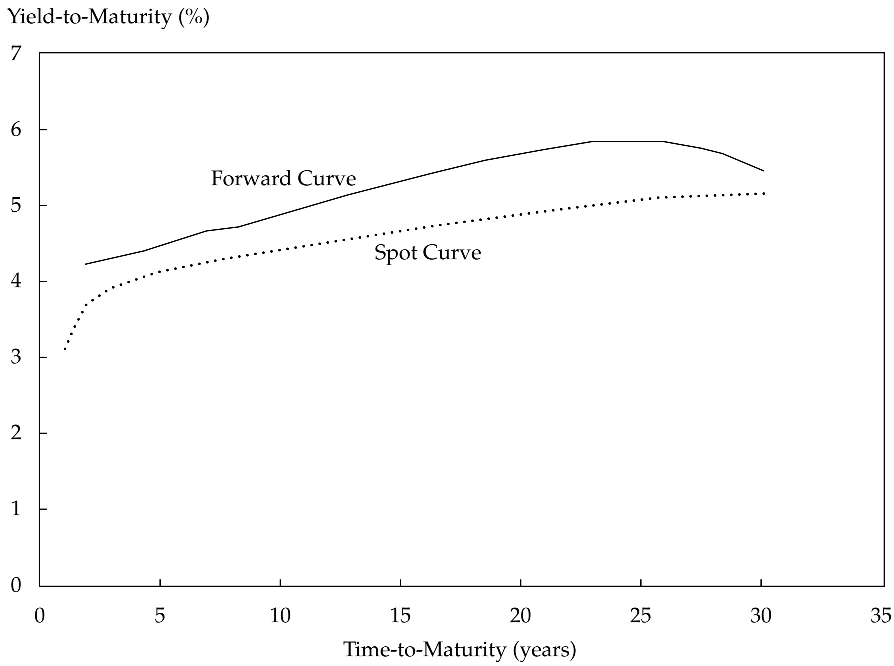
Suppose that the yields-to-maturity on three-year and four-year zero-coupon bonds are 3.65% and 4.18%, respectively, stated on a semiannual bond basis. An analyst would like to know the "3y1y" implied forward rate, which is the implied one-year forward yield three years into the future. Therefore, $A = 6$ (periods), $B = 8$ (periods), $B - A = 2$ (periods), $z_6 = 0.0365/2$ (per period), and $z_8 = 0.0418/2$ (per period).

$$\left(1+\frac{0.0365}{2}\right)^6 \times (1+IFR_{6,2})^2 = \left(1+\frac{0.0418}{2}\right)^8;\ IFR_{6,2} = 0.02889;\ \times 2 = 0.05778$$

The "3y1y" implied forward yield is 5.778%, annualized for a periodicity of two.

Equation 14 can be used to construct a **forward curve**. A forward curve is a series of forward rates, each having the same time frame. These forward rates might be observed on transactions in the derivatives market. Often, the forward rates are implied from transactions in the cash market. Exhibit 7 displays the forward curve that is calculated from the government bond spot curve shown in Exhibit 5. These are one-year forward rates stated on a semiannual bond basis.

EXHIBIT 7 A Government Bond Spot Curve and Forward Curve



A forward rate can be interpreted as an incremental, or marginal, return for extending the time-to-maturity for an additional time period. Suppose an investor has a four-year investment horizon and is choosing between buying a three-year zero-coupon bond that is priced to yield 3.65% and a four-year zero that is priced to yield 4.18%. The incremental, or marginal, return for the fourth year is 5.778%, the "3y1y" implied forward rate. If the investor's view on future bond yields is that the one-year yield in three years is likely to be less than 5.778%, the investor might prefer to buy the four-year bond. However, if the investor's view is that the one-year yield will be more than the implied forward rate, the investor might prefer the three-year bond and the opportunity to reinvest at the expected higher rate. That explains why an implied forward rate is the *breakeven reinvestment rate*. Implied forward rates are very useful to investors as well as bond issuers in making maturity decisions.

EXAMPLE 11 Computing Forward Rates

Suppose that an investor observes the following prices and yields-to-maturity on zero-coupon government bonds:

Maturity	Price	Yield-to-Maturity
1 year	97.50	2.548%
2 years	94.25	2.983%
3 years	91.75	2.891%

The prices are per 100 of par value. The yields-to-maturity are stated on a semiannual bond basis.

1. Compute the "1y1y" and "2y1y" implied forward rates, stated on a semiannual bond basis.
2. The investor has a three-year investment horizon and is choosing between (1) buying the two-year zero and reinvesting in another one-year zero in two years and (2) buying and holding to maturity the three-year zero. The investor decides to buy the two-year bond. Based on this decision, which of the following is the minimum yield-to-maturity the investor expects on one-year zeros two years from now?
   A. 2.548%
   B. 2.707%
   C. 2.983%

Solution to 1: The "1y1y" implied forward rate is 3.419%. In Equation 14, $A = 2$ (periods), $B = 4$ (periods), $B - A = 2$ (periods), $z_2 = 0.02548/2$ (per period), and $z_4 = 0.02983/2$ (per period).

$$\left(1 + \frac{0.02548}{2}\right)^2 \times (1 + IFR_{2,2})^2 = \left(1 + \frac{0.02983}{2}\right)^4;\ IFR_{2,2} = 0.017095$$
$$\times 2 = 0.03419$$

The "2y1y" implied forward rate is 2.707%. In Equation 14, A = 4 (periods), $B = 6$ (periods), $B - A = 2$ (periods), $z_4 = 0.02983/2$ (per period), and $z_6 = 0.02891/2$ (per period).

$$\left(1 + \frac{0.02983}{2}\right)^4 \times (1 + IFR_{4,2})^2 = \left(1 + \frac{0.02891}{2}\right)^6;\ IFR_{4,2} = 0.013536$$
$$\times 2 = 0.02707$$

Solution to 2: B is correct. The investor's view is that the one-year yield in two years will be greater than or equal to 2.707%.

The "2y1y" implied forward rate of 2.707% is the breakeven reinvestment rate. If the investor expects the one-year rate in two years to be less than that, the investor would prefer to buy the three-year zero. If the investor expects the one-year rate in two years to be greater than 2.707%, the investor might prefer to buy the two-year zero and reinvest the cash flow.

The forward curve has many applications in fixed-income analysis. Forward rates are used to make maturity decisions. They are used to identify arbitrage opportunities between transactions in the cash market for bonds and in derivatives markets. Forward rates are important in the valuation of derivatives, especially interest rate swaps and options. Those applications for the forward curve are covered elsewhere.

Forward rates can be used to value a fixed-income security in the same manner as spot rates because they are interconnected. The spot curve can be calculated from the forward curve, and the forward curve can be calculated from the spot curve. Either curve can be used to value a fixed-rate bond. An example will illustrate this process.

Suppose the current forward curve for one-year rates is the following:

Time Period	Forward Rate
0y1y	1.88%
1y1y	2.77%
2y1y	3.54%
3y1y	4.12%

These are annual rates stated for a periodicity of one. They are effective annual rates. The first rate, the "0y1y," is the one-year spot rate. The others are one-year forward rates. Given these rates, the spot curve can be calculated as the *geometric average* of the forward rates.

The two-year implied spot rate is 2.3240%.

$$(1.0188 \times 1.0277) = (1 + z_2)2;\ z_2 = 0.023240$$

The following are the equations for the three-year and four-year implied spot rates.

$$(1.0188 \times 1.0277 \times 1.0354) = (1 + z_3)^3;\ z_3 = 0.027278$$
$$(1.0188 \times 1.0277 \times 1.0354 \times 1.0412) = (1 + z_4)^4;\ z_4 = 0.030741$$

The three-year implied spot rate is 2.7278%, and the four-year spot rate is 3.0741%.

Suppose that an analyst needs to value a four-year, 3.75% annual coupon payment bond that has the same risks as the bonds used to obtain the forward curve. Using the implied spot rates, the value of the bond is 102.637 per 100 of par value.

$$\frac{3.75}{(1.0188)^1} + \frac{3.75}{(1.023240)^2} + \frac{3.75}{(1.027278)^3} + \frac{103.75}{(1.030741)^4} = 102.637$$

The bond also can be valued using the forward curve.

$$\frac{3.75}{(1.0188)} + \frac{3.75}{(1.0188 \times 1.0277)} + \frac{3.75}{(1.0188 \times 1.0277 \times 1.0354)} + \frac{103.75}{(1.0188 \times 1.0277 \times 1.0354 \times 1.0412)} = 102.637$$

5. YIELD SPREADS

- **compare, calculate, and interpret yield spread measures**

A yield spread, in general, is the difference in yield between different fixed-income securities. This section describes a number of yield spread measures.

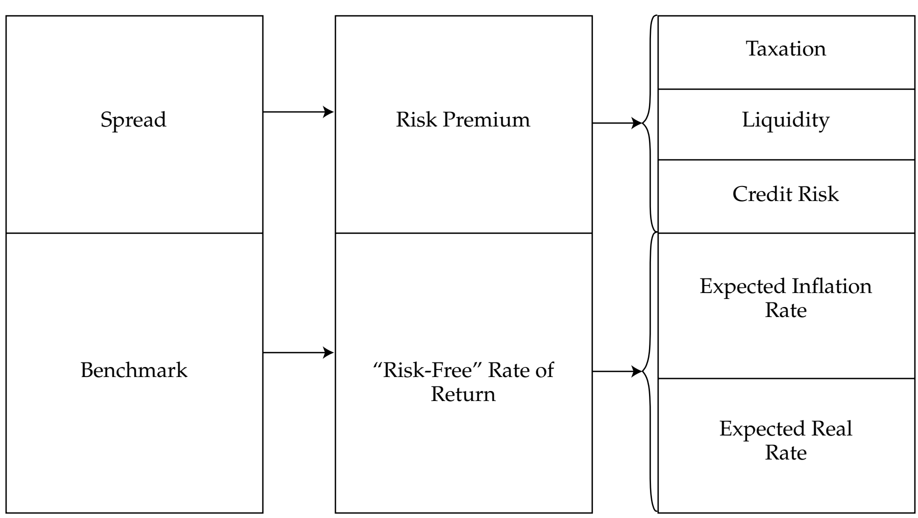
5.1. Yield Spreads over Benchmark Rates

In fixed-income security analysis, it is important to understand *why* bond prices and yields-to-maturity change. To do this, it is useful to separate a yield-to-maturity into two components: the **benchmark** and the **spread**. The benchmark yield for a fixed-income security with a given time-to-maturity is the base rate, often a government bond yield. The spread is the difference between the yield-to-maturity and the benchmark.

The reason for this separation is to distinguish between macroeconomic and microeconomic factors that affect the bond price and, therefore, its yield-to-maturity. The benchmark captures the macroeconomic factors: the expected rate of inflation in the currency in which the bond is denominated, general economic growth and the business cycle, foreign exchange rates, and the impact of monetary and fiscal policy. Changes in those factors impact all bonds in the market, and the effect is seen mostly in changes in the benchmark yield. The spread captures the microeconomic factors specific to the bond issuer and the bond itself: credit risk of the issuer and changes in the quality rating on the bond, liquidity and trading in comparable securities, and the tax status of the bond. It should be noted, however, that general yield spreads across issuers can widen and narrow with changes in macroeconomic factors.

Exhibit 8 illustrates the building blocks of the yield-to-maturity, starting with the benchmark and the spread. The benchmark is often called the risk-free rate of return. Also, the benchmark can be broken down into the expected real rate and the expected inflation rate in the economy. The yield spread is called the risk premium over the "risk-free" rate of return. The risk premium provides the investor with compensation for the credit and liquidity risks and possibly the tax impact of holding a specific bond.

EXHIBIT 8 Yield-to-Maturity Building Blocks



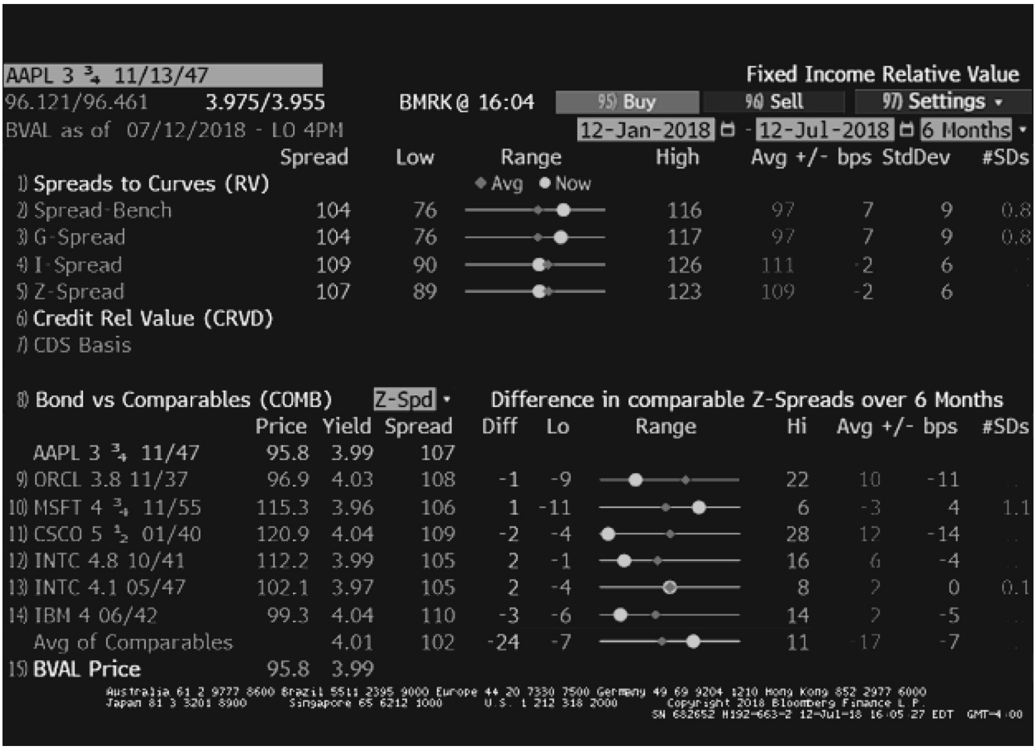
The benchmark varies across financial markets. Fixed-rate bonds often use a government benchmark security with the same time-to-maturity as, or the closest time-to-maturity to, the specified bond. This benchmark is usually the most recently issued government bond and is called the **on-the-run** security. The on-the-run government bond is the most actively traded security and has a coupon rate closest to the current market discount rate for that maturity. That implies that it is priced close to par value. Seasoned government bonds are called **off-the-run**. On-the-run bonds typically trade at slightly lower yields-to-maturity than off-the-run bonds having the same or similar times-to-maturity because of differences in demand for the securities and, sometimes, differences in the cost of financing the government security in the repo market.

A frequently used benchmark for floating-rate notes has long been Libor, which is due to be phased out, with the Market Reference Rate to take its place. As a composite interbank rate, it is not a risk-free rate. The yield spread over a specific benchmark is referred to as the **benchmark spread** and is usually measured in basis points. If no benchmark exists for a specific bond's tenor or a bond has an unusual maturity, interpolation is used to derive an implied benchmark. Also, bonds with very long tenors are priced over the longest available benchmark bond. For example, 100-year bonds (often called "century bonds") in the United States are priced over the 30-year US Treasury benchmark rate.

In the United Kingdom, the United States, and Japan, the benchmark rate for fixed-rate bonds is a government bond yield. The yield spread in basis points over an actual or interpolated government bond is known as the **G-spread**. The spread over a government bond is the return for bearing greater credit, liquidity, and other risks relative to the sovereign bond. Euro-denominated corporate bonds are priced over a EUR interest rate swap benchmark. For example, a newly issued five-year EUR bond might be priced at a rate of "mid-swaps" plus 150 bps, where "mid-swaps" is the average of the bid and offered swap rates. The yield spread is over a five-year EUR swap rate rather than a government benchmark. Note that the government bond yield or swap rate used as the benchmark for a specific corporate bond will change over time as the remaining time-to-maturity changes.

The yield spread of a specific bond over the standard swap rate in that currency of the same tenor is known as the **I-spread** or **interpolated spread** to the swap curve. This yield spread over MRR allows comparison of bonds with differing credit and liquidity risks against an interbank lending benchmark. Issuers will use the spread above MRR to determine the relative cost of fixed-rate bonds versus floating-rate alternatives, such as an FRN or commercial paper. Investors use the spread over MRR as a measure of a bond's credit risk. Whereas a standard interest rate swap involves an exchange of fixed for floating cash flows based on a floating index, an **asset swap** converts the periodic fixed coupon of a specific bond to an MRR plus or minus a spread. If the bond is priced close to par, this conversion approximates the price of a bond's credit risk over the MRR index. Exhibit 9 illustrates these yield spreads using the Bloomberg Fixed Income Relative Value (FIRV) page.

EXHIBIT 9 Bloomberg FIRV Page for the 3.75% Apple Bond



This example is for the 3.75% Apple bond that matures on 13 November 2047. The spreads are in the top-left corner of the page. The bond's flat asked price was 96.461 per 100 of par value on 12 July 2018, and its yield-to-maturity was 3.955%. On that date, the yield spread over a particular Treasury benchmark was 104 bps. Its G-spread over an interpolated government bond yield was also 104 bps. These two spreads sometimes differ by a few basis points, especially if the benchmark maturity differs from that of the underlying bond. The bond's I-spread was 109 bps. That Libor spread (to be replaced by MRR in the future) was a little larger than the G-spread because 30-year Treasury yields were slightly higher than 30-year Libor swap rates at that time. The use of these spreads in investor strategies will be covered in more detail later. In general, an analyst will track these spreads relative to their averages and historical highs and lows in an attempt to identify relative value.

5.2. Yield Spreads over the Benchmark Yield Curve

A yield curve shows the relationship between yields-to-maturity and times-to-maturity for securities with the same risk profile. For example, the government bond yield curve is the relationship between the yields of on-the-run government bonds and their times-to-maturity. The swap yield curve shows the relationship between fixed MRR swap rates and their times-to-maturity.

Each of these yield curves represents the term structure of benchmark interest rates, whether for "risk-free" government yields or "risky" fixed swap rates. Benchmark yield curves tend to be upward-sloping because investors typically demand a premium for holding longer-term securities. In general, investors face greater price risk for a given change in yield for longer-term bonds. This concept is covered further in the discussion of the topic of "Fixed-Income Risk and Return." The term structure of interest rates is dynamic, with short-term rates driven by central bank policy and longer-term rates affected by long-term growth and inflation expectations.

Isolating credit risk over varying times-to-maturity gives rise to a term structure of credit spreads that is distinct for each borrower. The G-spread and I-spread each use the same discount rate for each cash flow. Another approach is to calculate a constant yield spread over a government (or interest rate swap) spot curve instead. This spread is known as the **zero-volatility spread (Z-spread)** of a bond over the benchmark rate. In Exhibit 9, the Z-spread for the Apple bond was reported to be 107 bps.

The Z-spread over the benchmark spot curve can be calculated with Equation 15:

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \cdots + \frac{PMT+FV}{(1+z_N+Z)^N} \quad (15)$$

The benchmark spot rates—$z_1, z_2, \ldots, z_N$—are derived from the government yield curve (or from fixed rates on interest rate swaps). Z is the Z-spread per period and is the same for all time periods. In Equation 15, N is an integer, so the calculation is on a coupon date when the accrued interest is zero. Sometimes, the Z-spread is called the "static spread" because it is constant (and has zero volatility). In practice, the Z-spread is usually calculated in a spreadsheet using a goal seek function or similar solver function.

The Z-spread is also used to calculate the **option-adjusted spread** (OAS) on a callable bond. The OAS, like the option-adjusted yield, is based on an option-pricing model and an assumption about future interest rate volatility. Then, the value of the embedded call option, which is stated in basis points per year, is subtracted from the yield spread. In particular, it is subtracted from the Z-spread:

$$\text{OAS} = \text{Z-spread} - \text{Option value (in basis points per year)}$$

This important topic is covered later.

EXAMPLE 12 The G-Spread and the Z-Spread

A 6% annual coupon corporate bond with two years remaining to maturity is trading at a price of 100.125. The two-year, 4% annual payment government benchmark bond is trading at a price of 100.750. The one-year and two-year government spot rates are 2.10% and 3.635%, respectively, stated as effective annual rates.

1. Calculate the G-spread, the spread between the yields-to-maturity on the corporate bond and the government bond having the same maturity.
2. Demonstrate that the Z-spread is 234.22 bps.

Solution to 1: The yield-to-maturity for the corporate bond is 5.932%.

$$100.125 = \frac{6}{(1+r)^1} + \frac{106}{(1+r)^2};\ r = 0.05932$$

The yield-to-maturity for the government benchmark bond is 3.605%.

$$100.750 = \frac{4}{(1+r)^1} + \frac{104}{(1+r)^2};\ r = 0.03605$$

The G-spread is 232.7 bps: 0.05932 – 0.03605 = 0.02327.

Solution to 2: Solve for the value of the corporate bond using $z_1 = 0.0210$, $z_2 = 0.03635$, and $Z = 0.023422$:

$$\frac{6}{(1+0.0210+0.023422)^1} + \frac{106}{(1+0.03635+0.023422)^2}$$
$$= \frac{6}{(1.044422)^1} + \frac{106}{(1.059772)^2} = 100.125$$

SUMMARY

We have covered the principles and techniques that are used in the valuation of fixed-rate bonds, as well as floating-rate notes and money market instruments. These building blocks are used extensively in fixed-income analysis. The following are the main points made:

- The market discount rate is the rate of return required by investors given the risk of the investment in the bond.
- A bond is priced at a premium above par value when the coupon rate is greater than the market discount rate.

- A bond is priced at a discount below par value when the coupon rate is less than the market discount rate.
- The amount of any premium or discount is the present value of the "excess" or "deficiency" in the coupon payments relative to the yield-to-maturity.
- The yield-to-maturity, the internal rate of return on the cash flows, is the implied market discount rate given the price of the bond.
- A bond price moves inversely with its market discount rate.
- The relationship between a bond price and its market discount rate is convex.
- The price of a lower-coupon bond is more volatile than the price of a higher-coupon bond, other things being equal.
- Generally, the price of a longer-term bond is more volatile than the price of a shorter-term bond, other things being equal. An exception to this phenomenon can occur on low-coupon (but not zero-coupon) bonds that are priced at a discount to par value.
- Assuming no default, premium and discount bond prices are "pulled to par" as maturity nears.
- A spot rate is the yield-to-maturity on a zero-coupon bond.
- A yield-to-maturity can be approximated as a weighted average of the underlying spot rates.
- Between coupon dates, the full (or invoice, or "dirty") price of a bond is split between the flat (or quoted, or "clean") price and the accrued interest.
- Flat prices are quoted to not misrepresent the daily increase in the full price as a result of interest accruals.
- Accrued interest is calculated as a proportional share of the next coupon payment using either the actual/actual or 30/360 methods to count days.
- Matrix pricing is used to value illiquid bonds by using prices and yields on comparable securities having the same or similar credit risk, coupon rate, and maturity.
- The periodicity of an annual interest rate is the number of periods in the year.
- A yield quoted on a semiannual bond basis is an annual rate for a periodicity of two. It is the yield per semiannual period times two.
- The general rule for periodicity conversions is that compounding more frequently at a lower annual rate corresponds to compounding less frequently at a higher annual rate.
- Street convention yields assume payments are made on scheduled dates, neglecting weekends and holidays.
- The current yield is the annual coupon payment divided by the flat price, thereby neglecting as a measure of the investor's rate of return the time value of money, any accrued interest, and the gain from buying at a discount or the loss from buying at a premium.
- The simple yield is like the current yield but includes the straight-line amortization of the discount or premium.
- The yield-to-worst on a callable bond is the lowest of the yield-to-first-call, yield-to-second-call, and so on, calculated using the call price for the future value and the call date for the number of periods.
- The option-adjusted yield on a callable bond is the yield-to-maturity after adding the theoretical value of the call option to the price.
- A floating-rate note (floater, or FRN) maintains a more stable price than a fixed-rate note because interest payments adjust for changes in market interest rates.
- The quoted margin on a floater is typically the specified yield spread over or under the reference rate, which we refer to as the Market Reference Rate.
- The discount margin on a floater is the spread required by investors, and to which the quoted margin must be set, for the FRN to trade at par value on a rate reset date.

- Money market instruments, having one year or less time-to-maturity, are quoted on a discount rate or add-on rate basis.
- Money market discount rates understate the investor's rate of return (and the borrower's cost of funds) because the interest income is divided by the face value or the total amount redeemed at maturity, and not by the amount of the investment.
- Money market instruments need to be converted to a common basis for analysis.
- A money market bond equivalent yield is an add-on rate for a 365-day year.
- The periodicity of a money market instrument is the number of days in the year divided by the number of days to maturity. Therefore, money market instruments with different times-to-maturity have annual rates for different periodicities.
- In theory, the maturity structure, or term structure, of interest rates is the relationship between yields-to-maturity and times-to-maturity on bonds having the same currency, credit risk, liquidity, tax status, and periodicity.
- A spot curve is a series of yields-to-maturity on zero-coupon bonds.
- A frequently used yield curve is a series of yields-to-maturity on coupon bonds.
- A par curve is a series of yields-to-maturity assuming the bonds are priced at par value.
- In a cash market, the delivery of the security and cash payment is made on a settlement date within a customary time period after the trade date—for example, "$T + 3$."
- In a forward market, the delivery of the security and cash payment are made on a predetermined future date.
- A forward rate is the interest rate on a bond or money market instrument traded in a forward market.
- An implied forward rate (or forward yield) is the breakeven reinvestment rate linking the return on an investment in a shorter-term zero-coupon bond to the return on an investment in a longer-term zero-coupon bond.
- An implied forward curve can be calculated from the spot curve.
- Implied spot rates can be calculated as geometric averages of forward rates.
- A fixed-income bond can be valued using a market discount rate, a series of spot rates, or a series of forward rates.
- A bond yield-to-maturity can be separated into a benchmark and a spread.
- Changes in benchmark rates capture macroeconomic factors that affect all bonds in the market—inflation, economic growth, foreign exchange rates, and monetary and fiscal policy.
- Changes in spreads typically capture microeconomic factors that affect the particular bond—credit risk, liquidity, and tax effects.
- Benchmark rates are usually yields-to-maturity on government bonds or fixed rates on interest rate swaps.
- A G-spread is the spread over or under a government bond rate, and an I-spread is the spread over or under an interest rate swap rate.
- A G-spread or an I-spread can be based on a specific benchmark rate or on a rate interpolated from the benchmark yield curve.
- A Z-spread (zero-volatility spread) is based on the entire benchmark spot curve. It is the constant spread that is added to each spot rate such that the present value of the cash flows matches the price of the bond.
- An option-adjusted spread (OAS) on a callable bond is the Z-spread minus the theoretical value of the embedded call option.

PRACTICE PROBLEMS

1. A portfolio manager is considering the purchase of a bond with a 5.5% coupon rate that pays interest annually and matures in three years. If the required rate of return on the bond is 5%, the price of the bond per 100 of par value is *closest* to:
   A. 98.65.
   B. 101.36.
   C. 106.43.
2. A bond with two years remaining until maturity offers a 3% coupon rate with interest paid annually. At a market discount rate of 4%, the price of this bond per 100 of par value is *closest* to:
   A. 95.34.
   B. 98.00.
   C. 98.11.
3. An investor who owns a bond with a 9% coupon rate that pays interest semiannually and matures in three years is considering its sale. If the required rate of return on the bond is 11%, the price of the bond per 100 of par value is *closest* to:
   A. 95.00.
   B. 95.11.
   C. 105.15.
4. A bond offers an annual coupon rate of 4%, with interest paid semiannually. The bond matures in two years. At a market discount rate of 6%, the price of this bond per 100 of par value is *closest* to:
   A. 93.07.
   B. 96.28.
   C. 96.33.
5. A bond offers an annual coupon rate of 5%, with interest paid semiannually. The bond matures in seven years. At a market discount rate of 3%, the price of this bond per 100 of par value is *closest* to:
   A. 106.60.
   B. 112.54.
   C. 143.90.
6. A zero-coupon bond matures in 15 years. At a market discount rate of 4.5% per year and assuming annual compounding, the price of the bond per 100 of par value is *closest* to:
   A. 51.30.
   B. 51.67.
   C. 71.62.
7. Consider the following two bonds that pay interest annually:

Bond	Coupon Rate	Time-to-Maturity
A	5%	2 years
B	3%	2 years

   At a market discount rate of 4%, the price difference between Bond A and Bond B per 100 of par value is *closest* to:
   A. 3.70.
   B. 3.77.
   C. 4.00.

The following information relates to Questions 8 and 9

Bond	Price	Coupon Rate	Time-to-Maturity
A	101.886	5%	2 years
B	100.000	6%	2 years
C	97.327	5%	3 years

8. Which bond offers the lowest yield-to-maturity?
   A. Bond A
   B. Bond B
   C. Bond C
9. Which bond will *most likely* experience the smallest percent change in price if the market discount rates for all three bonds increase by 100 bps?
   A. Bond A
   B. Bond B
   C. Bond C

---

10. Suppose a bond's price is expected to increase by 5% if its market discount rate decreases by 100 bps. If the bond's market discount rate increases by 100 bps, the bond price is *most likely* to change by:
    A. 5%.
    B. less than 5%.
    C. more than 5%.

The following information relates to Questions 11 and 12

Bond	Coupon Rate	Maturity (years)
A	6%	10
B	6%	5
C	8%	5

All three bonds are currently trading at par value.

11. Relative to Bond C, for a 200 bp decrease in the required rate of return, Bond B will *most likely* exhibit a(n):
    A. equal percentage price change.
    B. greater percentage price change.
    C. smaller percentage price change.
12. Which bond will *most likely* experience the greatest percentage change in price if the market discount rates for all three bonds increase by 100 bps?
    A. Bond A
    B. Bond B
    C. Bond C

---

13. An investor considers the purchase of a two-year bond with a 5% coupon rate, with interest paid annually. Assuming the sequence of spot rates shown below, the price of the bond is *closest* to:

Time-to-Maturity	Spot Rates
1 year	3%
2 years	4%

    A. 101.93.
    B. 102.85.
    C. 105.81.

14. A three-year bond offers a 10% coupon rate with interest paid annually. Assuming the following sequence of spot rates, the price of the bond is *closest* to:

Time-to-Maturity	Spot Rates
1 year	8.0%
2 years	9.0%
3 years	9.5%

    A. 96.98.
    B. 101.46.
    C. 102.95.

The following information relates to Questions 15–17

Bond	Coupon Rate	Time-to-Maturity	Time-to-Maturity	Spot Rates
X	8%	3 years	1 year	8%
Y	7%	3 years	2 years	9%
Z	6%	3 years	3 years	10%

All three bonds pay interest annually.

15. Based on the given sequence of spot rates, the price of Bond X is *closest* to:
    A. 95.02.
    B. 95.28.
    C. 97.63.
16. Based on the given sequence of spot rates, the price of Bond Y is *closest* to:
    A. 87.50.
    B. 92.54.
    C. 92.76.
17. Based on the given sequence of spot rates, the yield-to-maturity of Bond Z is *closest* to:
    A. 9.00%.
    B. 9.92%.
    C. 11.93%.

18. Bond dealers *most* often quote the:
    A. flat price.
    B. full price.
    C. full price plus accrued interest.

The following information relates to Questions 19–21

Bond G, described in the exhibit below, is sold for settlement on 16 June 2020.

Annual Coupon	5%
Coupon Payment Frequency	Semiannual
Interest Payment Dates	10 April and 10 October
Maturity Date	10 October 2022
Day-Count Convention	30/360
Annual Yield-to-Maturity	4%

19. The full price that Bond G settles at on 16 June 2020 is *closest* to:
    A. 102.36.
    B. 103.10.
    C. 103.65.
20. The accrued interest per 100 of par value for Bond G on the settlement date of 16 June 2020 is *closest* to:
    A. 0.46.
    B. 0.73.
    C. 0.92.
21. The flat price for Bond G on the settlement date of 16 June 2020 is *closest* to:
    A. 102.18.
    B. 103.10.
    C. 104.02.

---

22. Matrix pricing allows investors to estimate market discount rates and prices for bonds:
    A. with different coupon rates.
    B. that are not actively traded.
    C. with different credit quality.
23. When underwriting new corporate bonds, matrix pricing is used to get an estimate of the:
    A. required yield spread over the benchmark rate.
    B. market discount rate of other comparable corporate bonds.
    C. yield-to-maturity on a government bond having a similar time-to-maturity.
24. A bond with 20 years remaining until maturity is currently trading for 111 per 100 of par value. The bond offers a 5% coupon rate with interest paid semiannually. The bond's annual yield-to-maturity is *closest* to:
    A. 2.09%.
    B. 4.18%.
    C. 4.50%.

25. The annual yield-to-maturity, stated for with a periodicity of 12, for a four-year, zero-coupon bond priced at 75 per 100 of par value is *closest* to:
   A. 6.25%.
   B. 7.21%.
   C. 7.46%.
26. A five-year, 5% semiannual coupon payment corporate bond is priced at 104.967 per 100 of par value. The bond's yield-to-maturity, quoted on a semiannual bond basis, is 3.897%. An analyst has been asked to convert to a monthly periodicity. Under this conversion, the yield-to-maturity is *closest to*:
   A. 3.87%.
   B. 4.95%.
   C. 7.67%.

The following information relates to Questions 27–30

A bond with five years remaining until maturity is currently trading for 101 per 100 of par value. The bond offers a 6% coupon rate with interest paid semiannually. The bond is first callable in three years and is callable after that date on coupon dates according to the following schedule:

End of Year	Call Price
3	102
4	101
5	100

27. The bond's annual yield-to-maturity is *closest* to:
   A. 2.88%.
   B. 5.77%.
   C. 5.94%.
28. The bond's annual yield-to-first-call is *closest* to:
   A. 3.12%.
   B. 6.11%.
   C. 6.25%.
29. The bond's annual yield-to-second-call is *closest* to:
   A. 2.97%.
   B. 5.72%.
   C. 5.94%.
30. The bond's yield-to-worst is *closest* to:
   A. 2.88%.
   B. 5.77%.
   C. 6.25%.

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31. A two-year floating-rate note pays six-month Libor plus 80 bps. The floater is priced at 97 per 100 of par value. The current six-month MRR is 1.00%. Assume a 30/360 day-count convention and evenly spaced periods. The discount margin for the floater in basis points is *closest* to:
   A. 180 bps.
   B. 236 bps.
   C. 420 bps.

32. An analyst evaluates the following information relating to floating-rate notes (FRNs) issued at par value that have three-month MRR as a reference rate:

Floating-Rate Note	Quoted Margin	Discount Margin
X	0.40%	0.32%
Y	0.45%	0.45%
Z	0.55%	0.72%

Based only on the information provided, the FRN that will be priced at a premium on the next reset date is:
A. FRN X.
B. FRN Y.
C. FRN Z.

33. A 365-day year bank certificate of deposit has an initial principal amount of USD96.5 million and a redemption amount due at maturity of USD100 million. The number of days between settlement and maturity is 350. The bond equivalent yield is *closest* to:
A. 3.48%.
B. 3.65%.
C. 3.78%.

34. The bond equivalent yield of a 180-day banker's acceptance quoted at a discount rate of 4.25% for a 360-day year is *closest* to:
A. 4.31%.
B. 4.34%.
C. 4.40%.

35. Which of the following statements describing a par curve is *incorrect*?
A. A par curve is obtained from a spot curve.
B. All bonds on a par curve are assumed to have different credit risk.
C. A par curve is a sequence of yields-to-maturity such that each bond is priced at par value.

36. A yield curve constructed from a sequence of yields-to-maturity on zero-coupon bonds is the:
A. par curve.
B. spot curve.
C. forward curve.

37. The rate interpreted to be the incremental return for extending the time-to-maturity of an investment for an additional time period is the:
A. add-on rate.
B. forward rate.
C. yield-to-maturity.

The following information relates to Questions 38 and 39

Time Period	Forward Rate
"0y1y"	0.80%
"1y1y"	1.12%
"2y1y"	3.94%
"3y1y"	3.28%
"4y1y"	3.14%

All rates are annual rates stated for a periodicity of one (effective annual rates).

38. The three-year implied spot rate is *closest* to:
    A. 1.18%.
    B. 1.94%.
    C. 2.28%.
39. The value per 100 of par value of a two-year, 3.5% coupon bond with interest payments paid annually is *closest* to:
    A. 101.58.
    B. 105.01.
    C. 105.82.

---

40. The spread component of a specific bond's yield-to-maturity is *least likely* impacted by changes in:
    A. its tax status.
    B. its quality rating.
    C. inflation in its currency of denomination.
41. The yield spread of a specific bond over the standard swap rate in that currency of the same tenor is *best* described as the:
    A. I-spread.
    B. Z-spread.
    C. G-spread.

The following information relates to Question 42

Bond	Coupon Rate	Time-to-Maturity	Price
UK Government Benchmark Bond	2%	3 years	100.25
UK Corporate Bond	5%	3 years	100.65

Both bonds pay interest annually. The current three-year EUR interest rate swap benchmark is 2.12%.

42. The G-spread in basis points on the UK corporate bond is *closest* to:
    A. 264 bps.
    B. 285 bps.
    C. 300 bps.

---

43. A corporate bond offers a 5% coupon rate and has exactly three years remaining to maturity. Interest is paid annually. The following rates are from the benchmark spot curve:

Time-to-Maturity	Spot Rate
1 year	4.86%
2 years	4.95%
3 years	5.65%

The bond is currently trading at a Z-spread of 234 bps. The value of the bond is *closest to*:
A. 92.38.
B. 98.35.
C. 106.56.

44. An option-adjusted spread (OAS) on a callable bond is the Z-spread:
A. over the benchmark spot curve.
B. minus the standard swap rate in that currency of the same tenor.
C. minus the value of the embedded call option expressed in basis points per year.

CHAPTER 4

INTRODUCTION TO ASSET-BACKED SECURITIES

Frank J. Fabozzi, PhD, CPA, CFA

LEARNING OUTCOMES

The candidate should be able to:

- explain benefits of securitization for economies and financial markets;
- describe securitization, including the parties involved in the process and the roles they play;
- describe typical structures of securitizations, including credit tranching and time tranching;
- describe types and characteristics of residential mortgage loans that are typically securitized;
- describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type;
- define prepayment risk and describe the prepayment risk of mortgage-backed securities;
- describe characteristics and risks of commercial mortgage-backed securities;
- describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and risks of each type;
- describe collateralized debt obligations, including their cash flows and risks;
- describe characteristics and risks of covered bonds and how they differ from other asset-backed securities.

1. INTRODUCTION: BENEFITS OF SECURITIZATION

- **explain benefits of securitization for economies and financial markets**

Previous chapters examined the risk characteristics of various fixed-income instruments and the relationships between maturity, coupon, and interest rate changes. This chapter introduces fixed-income instruments created through a process known as **securitization**. The securitization process transfers ownership of assets such as loans or receivables from the original owners into a special legal entity. The special legal entity then issues securities, using the asset cash flows to pay interest and repay the principal to investors. These securities are referred to generically as **asset-backed securities** (ABS), and the pool of assets from which their cash flows are generated is called collateral or **securitized assets**. These loans and receivables typically include residential mortgage loans (mortgages), commercial mortgages, automobile (auto) loans, student loans, bank loans, accounts receivable, or credit card receivables. While the ABS market in the United States remains the largest in the world, securitization has recently expanded in both Asia and Europe as well as to other income-yielding assets, such as airport landing slots, toll roads, and cell tower leases. The growing depth and breadth of the global ABS market underscores the importance of a solid understanding of securitization among issuers, investors, and financial analysts.

This chapter describes the securitization process, discusses its benefits for both issuers and investors, and explains the investment characteristics of different types of ABS. Although investors can invest in some loan types via private credit funds or through secondary markets, securitization creates a more direct link between investors and borrowers for many types of loans and receivables. The terminology regarding ABS varies by jurisdiction. **Mortgage-backed securities** (MBS) are ABS backed by a pool of mortgages, and a distinction is sometimes made between MBS and ABS backed by non-mortgage assets. This distinction is common in the United States, for example, where typically the term "mortgage-backed securities" refers to securities backed by high-quality real estate mortgages and the term "asset-backed securities" refers to securities backed by other types of assets. **Covered bonds** date back to 18th century Europe and are similar to ABS, but they offer investors recourse to both the issuing financial institution and an underlying asset pool.

To underline the importance of securitization from a macroeconomic perspective, we discuss the benefits of securitization for economies and financial markets. Then, we describe the structure of a securitization, identifying the parties involved and their roles as well as typical structures. We further discuss securities backed by mortgages for real estate property, including residential MBS and commercial MBS, and review two common types of non-mortgage ABS—namely, those for auto loans and credit card receivables. The chapter concludes with a description of collateralized debt obligations and covered bonds.

1.1. Benefits of Securitization for Economies and Financial Markets

The securitization of pools of loans and receivables into multiple securities provides economies and financial markets with several benefits.

Home or auto purchases have been traditionally financed by loans originated by financial institutions, such as commercial banks. For investors to gain exposure to these relatively illiquid loans, they must hold some combination of deposits, debt, or common equity issued by banks. This situation creates an additional intermediary (that is, the bank) between the borrowers and the investors. In addition, by being constrained to hold bank deposits and securities, investors cannot gain direct exposure to loans but are also affected by economic risks undertaken in other bank activities.

Securitization solves a number of these problems. It allows investors to achieve more direct legal claims on loan and receivables portfolios, enabling them to tailor interest rate and credit risk exposures to suit their specific needs. Disintermediation (that is, lessening the role of intermediaries) can effectively reduce borrower costs and enhance risk-adjusted investor returns. At the same time, banks can separate loan origination from financing, improving their profitability via origination fees and reducing capital requirements for loans that are sold.

Securitization enables banks to expand lending origination beyond their balance sheets, ultimately benefiting individuals, governments, and companies that need to borrow.

Securitization also benefits investors by creating access to securities with profiles that match their risk, return, and maturity needs that are otherwise not directly available. For example, a pension fund with a long-term horizon can gain access to long-term real estate loans by investing in residential MBS without having to invest in bank bonds or stocks. Although few institutional or individual investors are willing to make or purchase real estate loans, auto loans, or credit card receivables directly, they may invest in a security backed by such loans or receivables. The ABS that are created by pooling these loans and receivables have characteristics similar to those of a standard bond and do not require the specialized resources and expertise needed to originate, monitor, and collect the payments from the underlying loans and receivables. As a result, investors can increase exposure to the risk–return characteristics of a wider range of underlying assets. Note that in many countries, the sale of ABS and similar instruments is restricted to investors who meet certain qualifications, such as those pertaining to net worth.

Securitization allows for the creation of tradable securities with better liquidity than that of the original loans on the bank's balance sheet. In making loans and receivables tradable, securitization makes financial markets more efficient. It also improves liquidity, which reduces liquidity risk in the financial system, as described later.

An important benefit of securitization for companies is that ABS provide an alternative means of funding operations that can be considered alongside bond, preferred equity, and common equity issuance. Companies that originate loans and receivables that can be securitized often compare and optimize the funding costs associated with each source of financing. As we will see, securitization is often less costly than a corporate bond issue secured by the same collateral.

For these reasons, securitization is beneficial to economies and financial markets and has been embraced by many sovereign governments. For example, the Italian government has used securitization since the late 1990s for privatizing public assets. In emerging markets, securitization is widely used. For example, in South America, companies and banks with high credit ratings have used securitization to sell receivables on exports, such as oil, to lower their funding costs.

Although securitization brings many benefits to economies, it is not without risks, and some of these risks are widely attributed to have precipitated the turmoil in financial markets during 2007–2009. Broadly, those risks fall into two categories: risks that relate primarily to the timing of the ABS's cash flows, such as contraction risk and extension risk, and risks related to the inherent credit risk of the loans and receivables backing the ABS. This chapter describes these risks and discusses some of the structures used to mitigate them as well as redistribute them.

2. HOW SECURITIZATION WORKS

- **describe securitization, including the parties involved in the process and the roles they play**

When assets are securitized, several legal and regulatory conditions must be satisfied. A number of parties participate in the process to facilitate the transaction and ensure these conditions are met. In this section, a typical securitization is described by way of a hypothetical example. The example describes the parties involved in a securitization and their roles. It also introduces the typical structures of securitizations, such as credit tranching and time tranching.

2.1. An Example of a Securitization

Mediquip, a hypothetical company, is a manufacturer of medical equipment that ranges in cost from US$50,000 to US$300,000. The majority of Mediquip's sales are made through loans granted by the company to its customers, and the medical equipment serves as collateral for the loans. These loans, which represent an asset to Mediquip, have maturities of five years and carry a fixed interest rate. They are fully amortizing with monthly payments; that is, the borrowers make equal payments each month consisting of interest payment and principal repayment. The total principal repaid from the 60 loan payments (12 months × 5 years) is such that the amount borrowed is fully repaid at the end of the term.

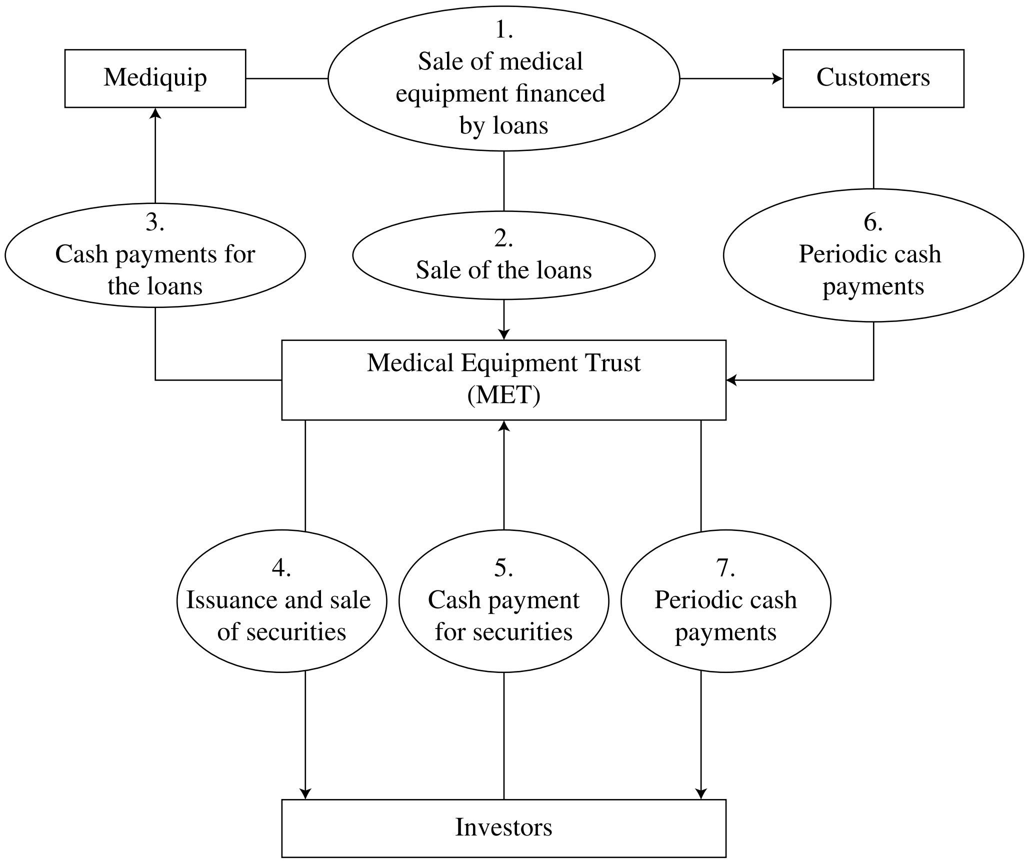
Mediquip's credit department makes the decision about whether to extend credit to customers and services the loans that are made. Loan servicing refers to administering any aspect of a loan, including collecting payments from borrowers, notifying borrowers who may be delinquent, and recovering and disposing of the medical equipment if the borrower does not make the scheduled payments by a specified time. If one of its customers defaults, Mediquip can seize the medical equipment and sell it to try to recoup the remaining principal on the loan. Although the servicer of such loans need not be the originator of the loans, the assumption in this example is that Mediquip is the servicer.

The following is an illustration of how these loans can be securitized. Assume that Mediquip has US$200 million of loans. This amount is shown on Mediquip's balance sheet as an asset. Assume also that Mediquip wants to raise US$200 million, which happens to be the amount of the loans. Because Mediquip's treasurer is aware of the potentially lower funding costs of securitization, he decides to raise the US$200 million by securitizing the loans on the medical equipment rather than by issuing corporate bonds.

To do so, Mediquip sets up a separate legal entity called Medical Equipment Trust (MET), to which it sells the loans on the medical equipment. Such a legal entity is referred to as a **special purpose entity** (SPE) and sometimes also called a special purpose vehicle (SPV) or a special purpose company. The legal form of the SPE varies by jurisdiction, but in almost all cases, the ultimate owner of the loans—MET in our example—is legally independent and is considered bankruptcy remote from the seller of the loans. Setting up a separate legal entity ensures that if Mediquip, the originator of the loans, files for bankruptcy, the loans backing the ABS that are issued by MET are secure within the SPE and creditors of Mediquip have no claim on them. Note that in some jurisdictions, the SPE may, in turn, transfer the loans to a trust or a limited company.

A securitization is diagrammed in Exhibit 1. The SPE set up by Mediquip is called MET. The top of Exhibit 1 reflects Mediquip's business model as described above—that is, the sale of medical equipment financed by loans (first oval). Mediquip sells to MET US$200 million of loans (second oval) and receives from MET US$200 million in cash (third oval); in this simplified example, the costs associated with the securitization are ignored. MET issues and sells securities that are backed by the pool of securitized loans (fourth oval) and receives cash (fifth oval). These securities are the ABS mentioned earlier, and the US$200 million of loans represent the collateral. The periodic cash payments that are received from the collateral—that is, the monthly payments made by Mediquip's customers that include both interest payment and principal repayment (sixth oval)—are used to make the periodic cash payments to the security holders—the investors who bought the ABS (seventh oval).

EXHIBIT 1 Mediquip's Securitization



2.2. Parties to a Securitization and Their Roles

Securitization requires the publication of a prospectus, a document that contains information about the securitization. The three main parties to a securitization are

- the seller of the collateral, sometimes called the depositor (Mediquip in our example);
- the SPE that purchases the loans or receivables and uses them as collateral to issue the ABS—MET in our example (the SPE is often referred to as the issuer in the prospectus because it is the entity that issues the securities; it may also be called the trust if the SPE is set up as a trust); and
- the servicer of the loans (Mediquip in our example).

Other parties are also involved in a securitization: independent accountants, lawyers/attorneys, trustees, underwriters, rating agencies, and financial guarantors. All these parties, including the servicer when it is different from the seller of the collateral, are referred to as third parties to the securitization.

A significant amount of legal documentation is involved in a securitization. Lawyers/attorneys are responsible for preparing the legal documents. An important legal document is the purchase agreement between the seller of the collateral and the SPE, which sets forth the representations and warranties that the seller makes about the assets sold. These representations and warranties assure investors about the quality of the assets, an important consideration when assessing the risks associated with the ABS.

Another important legal document is the prospectus, which describes the structure of the securitization, including the priority and amount of payments to be made to the servicer, administrators, and the ABS holders. Securitizations often use several forms of credit enhancements, which are documented in the prospectus. Credit enhancements are provisions that are used to reduce the credit risk of a bond issue. They include (1) internal credit enhancements, such as subordination, overcollateralization, and reserve accounts, and (2) external credit enhancements, such as financial guarantees by banks or insurance companies, letters of credit, and cash collateral accounts. Securitizations often use subordination, which will be discussed later. Prior to the 2007–09 credit crisis, many securitizations included financial guarantees by a third party. The most common third-party financial guarantors are monoline insurance companies, or monoline insurers. A monoline insurer is a private insurance company whose business is restricted to providing guarantees for financial instruments such as ABS. Following the financial difficulties and downgrading of the major monoline insurers as a result of the financial crisis that began in the mortgage market in mid-2007, few structures have used financial guarantees from a monoline insurer.

A trustee or trustee agent is typically a financial institution with trust powers that safeguards the assets after they have been sold to the SPE, holds the funds due to the ABS holders until they are paid, and provides periodic information to the ABS holders. The information is provided in the form of remittance reports, which may be issued monthly, quarterly, or as agreed to in the terms of the prospectus.

Underwriters and rating agencies perform the same functions in a securitization as they do in a standard bond offering

EXAMPLE 1 A Used Luxury Auto Securitization

Used Luxury Auto (ULA) is a hypothetical company that has established a nationwide business in buying used luxury autos and then refurbishing them with the latest in electronic equipment (for instance, UBS ports and rear-view cameras). ULA Corp then sells these autos in the retail market, often financing the sales with promissory notes from the buyers via its ULA Credit Corp.

The following information is taken from a theoretical filing by ULA with the Securities and Exchange Commission for a securitization:

Issuer: ULA Trust 2020
Seller and Servicer: ULA Credit Corp
Notes:

$500,000,000 4.00% ULA Trust contract-backed Class A notes, rated AAA

$250,000,000 4.80% ULA Trust contract-backed Class B notes, rated A

Contracts: The assets underlying the notes are fixed-rate promissory notes relating to the purchase of used automobiles refurbished by ULA Corp.

1. The collateral for this securitization is:
   A. ULA Trust contract-backed Class A and Class B notes.
   B. Used automobiles refurbished by ULA Corp.
   C. Fixed-rate promissory notes relating to the purchase of used automobiles refurbished by ULA Corp.
2. The special purpose entity in this securitization is:
   A. ULA Corp.
   B. ULA Credit Corp.
   C. ULA Trust 2020.
3. ULA Credit Corp is responsible for:
   A. selling the collateral to the SPE and collecting payments from borrowers on the underlying promissory notes.
   B. refurbishing the used motorcycles and collecting payments from borrowers on the underlying promissory notes.
   C. selling the contract-backed Class A and Class B notes to investors and making the cash interest and principal payments to them.

Solution to 1: C is correct. The collateral is the pool of securitized assets from which the cash flows will be generated. It is the debt obligations that have been securitized. These contracts are the loans, called promissory notes, provided to purchasers of the used automobiles that were refurbished by ULA Corp.

Solution to 2: C is correct. ULA Trust 2020 is the issuer of the ABS and, thus, the SPE. The SPE purchases the contracts that become the collateral from ULA Corp, the automobile refurbisher, but it is ULA Credit Corp that originates the loans and is, therefore, the seller of the collateral. ULA Credit Corp is also the servicer of the debt obligations.

Solution to 3: A is correct. ULA Credit Corp is the seller of the collateral, the promissory notes. As the servicer, it is responsible for collecting payments from borrowers, notifying borrowers who may be delinquent, and if necessary, recovering and disposing of the automobile if the borrower defaults.

3. STRUCTURE OF A SECURITIZATION

- **describe typical structures of securitizations, including credit tranching and time tranching**

A simple securitization may involve the sale of only one class of bond or ABS. Let us call this class Bond Class A. Returning to the Mediquip and MET example, MET may raise US$200 million by issuing 200,000 certificates for Bond Class A with a par value of US$1,000 per certificate. Thus, each certificate holder is entitled to 1/200,000 of the payments from the collateral after payment of servicing and other administrative fees.

The structure of the securitization is often more complicated than a single class of ABS. As mentioned earlier, it is common for securitizations to include a form of internal credit enhancement called **subordination**, also referred to as **credit tranching**. In such a structure, there is more than one bond class or tranche, and the bond classes differ as to how they will share any losses resulting from defaults of the borrowers whose loans are in the collateral. The bond classes are classified as senior bond classes or subordinated bond classes—hence, the reason this structure is also referred to as a senior/subordinated structure. The subordinated bond classes are sometimes called "non-senior bond classes" or "junior bond classes." They function as credit protection for the more senior bond classes; that is, losses are realized by the subordinated bond classes before any losses are realized by the senior bond classes. This type of protection is also commonly referred to as a "waterfall" structure because of the cascading flow of payments between bond classes in the event of default.

For example, suppose MET issues two bond classes with a total par value of US$200 million: Bond Class A, the senior bond class, with a par value of US$120 million, and Bond Class B, the subordinated bond class, with a par value of US$80 million. In this senior/subordinated structure, also referred to as credit tranching, Bond Class B will absorb losses up to US$80 million. Thus, if defaults by Mediquip's customers do not exceed US$80 million, Bond Class A will be fully repaid its US$120 million. The purpose of this structure is to redistribute the credit risk associated with the collateral. The creation of a set of bond classes allows investors to choose the level of credit risk that they prefer to bear.

More than one subordinated bond class may be created. Suppose MET issues the following structure:

Bond Class	Par Value (US$ millions)
A (senior)	180
B (subordinated)	14
C (subordinated)	6
Total	200

In this structure, Bond Class A is the senior bond class whereas both Bond Class B and Bond Class C are subordinated bond classes from the perspective of Bond Class A. The rules for the distribution of losses are as follows. All losses on the collateral are absorbed by Bond Class C before any losses are realized by Bond Class B and then Bond Class A. Consequently, if the losses on the collateral do not exceed US$6 million, no losses will be realized by Bond Class A or Bond Class B. If the losses exceed US$6 million, Bond Class B must absorb the losses up to an additional US$14 million. For example, if the total loss on the collateral is US$16 million, Bond Class C loses its entire par value of US$6 million and Bond Class B realizes a loss of US$10 million of its par value of US$14 million. Bond Class A does not realize any loss in this scenario. Clearly, Bond Class A realizes a loss only if the total loss on the collateral exceeds US$20 million.

The structure of a securitization may also allow the redistribution of another type of risk, called "prepayment risk," among bond classes. **Prepayment risk** is the uncertainty that the cash flows will be different from the scheduled cash flows as set forth in the loan agreement because of the borrowers' ability to alter payments, usually to take advantage of interest rate movements. For example, when interest rates decline, borrowers tend to pay off part or all of their loans and refinance at lower interest rates. The creation of bond classes that possess different expected maturities is referred to as **time tranching** and is discussed later in this chapter.

It is possible, and quite common, for a securitization to have structures with both credit tranching and time tranching.

EXAMPLE 2 Bond Classes and Tranching

Return to the ULA securitization described in Example 1. Based on the information provided, the form of credit enhancement that the transaction *most likely* includes is:

A. time tranching.
B. credit tranching.
C. a financial guarantee.

Solution: B is correct. The ULA securitization includes two bond classes: Class A and Class B. Each bond class has a fixed but different interest rate; the interest rate increases from 4.00% for Class A notes to 4.80% for Class B notes. Thus, it is likely that the transaction has credit tranching and that the two bond classes display a senior/subordinated structure, with Class A notes being senior to Class B notes, the subordinated bond class. As the credit risk increases from the Class A notes to the Class B notes, so does the interest rate to reflect the additional compensation investors require for bearing the additional credit risk. The information provided does not give any indication of either time tranching or a financial guarantee.

3.1. Key Role of the Special Purpose Entity

The SPE plays a pivotal role in securitization. In fact, the setup of a legal entity that plays the same role as an SPE in terms of protecting the rights of ABS holders is a prerequisite in any country that wants to allow securitization. Indeed, without a provision in a country's legal system for the equivalent of an SPE, the benefits of using securitization by an entity seeking to raise funds would not exist. Let us explain why by returning to our example involving Mediquip and MET.

Assume that Mediquip has a credit rating from a credit-rating agency, such as Standard & Poor's, Moody's Investors Service, or Fitch Ratings. A credit rating reflects the agency's opinion about the creditworthiness of an entity and/or the debt securities the entity issues. Suppose that the credit rating assigned to Mediquip is BB or Ba2. Such a credit rating means that Mediquip is below what is referred to as an investment-grade credit rating.

Assume again that Mediquip's treasurer wants to raise US$200 million and is contemplating doing so by issuing a five-year corporate bond rather than by securitizing the loans. The treasurer is, of course, concerned about the funding cost and would like the lowest possible interest rate available relative to some benchmark interest rate. The difference between the interest rate paid on the five-year corporate bond and the benchmark interest rate is the spread. The spread reflects the compensation investors require for buying the corporate bond, which is riskier than the bonds issued at the benchmark interest rate. The major factor affecting the spread is the issuer's credit rating—hence, the reason the spread is called a "credit spread."

Another factor that will influence the credit spread is whether the bond is backed by collateral. A corporate bond that has collateral is often referred to as a secured bond. The collateral usually reduces the credit spread, making the credit spread of a secured bond lower than that of an otherwise identical unsecured bond. In our example, Mediquip's treasurer can use the loans on the medical equipment as collateral for the secured corporate bond issue. Thus, if Mediquip issues a five-year corporate bond to raise US$200 million, the credit spread will primarily reflect its credit rating, with a slight benefit for the use of collateral as explained below.

Now suppose that instead of using the loans as collateral for a secured corporate bond issue, Mediquip sells the loan contracts in an arm's length transaction to MET, the SPE. After the sale is completed, it is MET, not Mediquip, that legally owns them. As a result, if Mediquip is forced into bankruptcy while the loans are still outstanding, Mediquip's creditors cannot recover them because they are legally owned by another entity. Note that it is possible, however, that transfers made to bankruptcy-remote vehicles can be challenged as fraudulent conveyances and potentially unwound. The legal implication of setting up MET is that investors contemplating the purchase of any bond class backed by the cash flows from the pool of loans on the medical equipment will evaluate the credit risk associated with collecting the payments due on the receivables independently of Mediquip's credit rating.

Credit ratings are assigned to each bond class created in the securitization. They depend on the quality of the collateral—that is, how the rating agencies evaluate the credit risk of the pool of securitized loans or receivables. Depending on the structure of the securitization, each bond class receives a credit rating that reflects its credit risk, and some of the bond classes may have a better credit rating than the company that is seeking to raise funds. As a result, in aggregate, the funding cost of a securitization may be lower than that of a corporate bond issue. Access to lower funding cost is a key role of the SPE in a securitization.

You may ask why a securitization can be cheaper than a corporate bond issue secured by the same collateral. The reason is that the SPE would not be affected by the bankruptcy of the seller of the collateral. As mentioned above, the assets belong to the SPE, not to the entity that sold the assets to the SPE. In the United States and other countries, when a company is liquidated, creditors receive distributions based on the absolute priority rule to the extent that assets are available. The absolute priority rule is the principle that senior creditors are paid in full before subordinated creditors are paid anything. The absolute priority rule also guarantees the seniority of creditors relative to equityholders.

Whereas the absolute priority rule generally holds in liquidations, it has not always been upheld by the courts in reorganizations. Thus, although investors in the debt of a company may believe they have priority over the equityholders and priority over other classes of creditors, the actual outcome of a reorganization may be far different from the terms stated in the debt agreement; that is, there is no assurance that if the corporate bond has collateral, the rights of the bondholders will be respected. For this reason, the credit spread for a corporate bond backed by collateral does not decrease dramatically.

In the case of a securitization, the courts have in most jurisdictions no discretion to change seniority because the bankruptcy of a company does not affect the SPE. The rules set forth in the legal document, which describes how losses are to be absorbed by each bond class, are unaffected by the company's bankruptcy. This important decoupling of the credit risk of the entity needing funds from the bond classes issued by the SPE explains why the SPE's legal role is critical.

The SPE is bankruptcy remote from the seller of the collateral, which means that the bankruptcy of the seller of the collateral will not affect the holders of securities issued by the SPE and backed by the collateral. The security holders face credit risk only to the extent that the borrowers whose claims the SPE has purchased default on their loans. The SPE's ability to make cash payments to the security holders remains intact if the borrowers make the interest payments and/or the principal repayments on their loans.

In many EU countries, the creditors are protected in the recognition of the securitization as a true sale. The SPE has full legal ownership of the securitized assets, which are de-recognized from the seller's balance sheet. However, it is important to note that not all countries have the same legal framework. Impediments with respect to ABS issuance have arisen in some countries because the concept of trust law is less well developed globally than it is in the United States and other developed countries. Thus, investors should be aware of the legal considerations that apply in the jurisdictions where they purchase ABS.

EXAMPLE 3 Special Purpose Entity and Bankruptcy

Agnelli Industries (Agnelli), a manufacturer of industrial machine tools based in Bergamo, Italy, has €500 million of corporate bonds outstanding. These bonds have a credit rating below investment grade. Agnelli has €400 million of receivables on its balance sheet that it would like to securitize. The receivables represent payments Agnelli expects to receive for machine tools it has sold to various customers in Europe. Agnelli sells the receivables to Agnelli Trust, a special purpose entity. Agnelli Trust then issues ABS, backed by the pool of receivables, with the following structure:

Bond Class	Par Value (€ millions)
A (senior)	280
B (subordinated)	60
C (subordinated)	60
Total	400

Bond Class A is given an investment-grade credit rating by the credit-rating agencies.

1. Why does Bond Class A have a higher credit rating than the corporate bonds?
2. If Agnelli Industries files for bankruptcy after the issuance of the asset-backed security:
   A. Bond Classes A, B, and C will be unaffected.
   B. Bond Classes A, B, and C will lose their entire par value.
   C. Losses will be realized by Bond Class C first, then by Bond Class B, and then by Bond Class A.
3. If one of Agnelli's customers defaults on its €60 million loan:
   A. Bond Classes A, B, and C will realize losses of €20 million each.
   B. Bond Class C will realize losses of €60 million, but Bond Classes A and B will be unaffected.
   C. Bond Classes B and C will realize losses of €30 million each, but Bond Class A will be unaffected.

Solution to 1: Bond Class A is issued by Agnelli Trust, an SPE that is bankruptcy remote from Agnelli. Thus, the investors who hold Agnelli's bonds and/or common shares have no legal claim on the cash flows from the securitized receivables that are the collateral for the ABS. As long as Agnelli's customers make the interest payments and/or principal repayments on their loans, Agnelli Trust will be able to make cash payments to the ABS investors. Because of the credit tranching, even if some of Agnelli's customers were to default on their loans, the losses would be realized by the subordinated Bond Classes B and C before any losses are realized by the senior Bond Class A. The credit risk associated with Bond Class A is, therefore, lower than that of Bond Classes B and C and the corporate bonds, justifying the investment-grade credit rating.

Solution to 2: A is correct. The ABS have been issued by Agnelli Trust, an SPE that is bankruptcy remote from Agnelli. If the securitization is viewed as resulting in a true sale, the fact that Agnelli files for bankruptcy does not affect the ABS holders. These ABS holders face credit risk only to the extent that Agnelli's customers who bought the machine tools do not make the obligatory payments on their loans. As long as the customers continue to make payments, all three bond classes will receive their expected cash flows. These cash flows are completely and legally independent of anything that happens to Agnelli itself.

Solution to 3: B is correct. The rules for the distribution of losses are as follows. All losses on the collateral are absorbed by Bond Class C before any losses are realized by Bond Class B and then Bond Class A. Consequently, if the losses on the collateral are €60 million, which is the par value of Bond Class C, Bond Class C loses its entire par value, but Bond Classes A and B are unaffected.

4. RESIDENTIAL MORTGAGE LOANS

- **describe types and characteristics of residential mortgage loans that are typically securitized**

Before describing the various types of residential mortgage-backed securities, this section briefly discusses the fundamental features of the underlying assets: residential mortgage loans. The mortgage designs described in this section are those that are typically securitized.

A **mortgage loan**, or simply mortgage, is a loan secured by the collateral of some specified real estate property that obliges the borrower (often someone wishing to buy a home) to make a predetermined series of payments to the lender (often initially a bank or mortgage company). The mortgage gives the lender the right to foreclose on the loan if the borrower defaults; that is, a **foreclosure** allows the lender to take possession of the mortgaged property and then sell it in order to recover funds toward satisfying the debt obligation.

Typically, the amount of the loan advanced to buy the property is less than the property's purchase price. The borrower makes a down payment, and the amount borrowed is the difference between the property's purchase price and the down payment. When the loan is first taken out, the borrower's equity in the property is equal to the down payment. Over time, as the market value of the property changes, the borrower's equity also changes. It also changes as the borrower makes mortgage payments that include principal repayment.

The ratio of the amount of the mortgage to the property's value is called the **loan-to-value ratio** (LTV). The lower the LTV, the higher the borrower's equity. From the lender's perspective, the higher the borrower's equity, the less likely the borrower is to default. Moreover, the lower the LTV, the more protection the lender has for recovering the amount loaned if the borrower does default and the lender repossesses and sells the property.

In the United States, market participants typically identify two types of mortgages based on the credit quality of the borrower: prime loans and subprime loans. Prime loans generally have borrowers of high credit quality with strong employment and credit histories, income sufficient to pay the loan obligation, and substantial equity in the underlying property. Subprime loans have borrowers with lower credit quality and/or are loans without a first lien on the property (that is, another party has a prior claim on the underlying property).

Throughout the world, there are a considerable number of mortgage designs. Mortgage design means the specification of (1) the maturity of the loan, (2) how the interest rate is determined, (3) how the principal is to be repaid (that is, the amortization schedule), (4) whether the borrower has the option to prepay and, in such cases, whether any prepayment penalties might be imposed, and (5) the rights of the lender in a foreclosure.

4.1. Maturity

In the United States, the typical term or number of years to maturity of a mortgage ranges from 15 to 30 years. For most European countries, a residential mortgage typically has a maturity between 20 and 40 years, whereas in France and Spain, it can be as long as 50 years. Japan is an extreme case, with mortgage maturities of 100 years.

4.2. Interest Rate Determination

The interest rate on a mortgage is called the **mortgage rate**, **contract rate**, or **note rate**. Determination of mortgage rates varies considerably among countries but may be categorized in four basic ways:

- *Fixed rate:* The mortgage rate remains the same during the life of the mortgage. The United States and France have a high proportion of this type of interest rate determination. Although fixed-rate mortgages are not the dominant form in Germany, they do exist there.
- *Adjustable or variable rate:* The mortgage rate is reset periodically—daily, weekly, monthly, or annually. The determination of the new mortgage rate for an adjustable-rate mortgage (ARM) at the reset date may be based on some reference rate or index (in which case, it is called an indexed-referenced ARM) or a rate determined at the lender's discretion (in which case, it is called a reviewable ARM). Residential mortgages in Australia, Ireland, South Korea, Spain, and the United Kingdom are dominated by adjustable-rate mortgages. In Australia, Ireland, and the United Kingdom, the reviewable ARM is standard. In South Korea and Spain, the indexed-referenced ARM is the norm. Canada and the United States have ARMs that are typically tied to an index or reference rate, although this type of mortgage rate is not the dominant form of interest rate determination. An important feature of an ARM is that it will usually have a maximum interest rate by which the mortgage rate can change at a reset date and a maximum interest rate that the mortgage rate can reach during the mortgage's life.
- *Initial period fixed rate:* The mortgage rate is fixed for some initial period and is then adjusted. The adjustment may call for a new fixed rate or for a variable rate. When the adjustment calls for a fixed rate, the mortgage is referred to as a rollover or renegotiable mortgage. This mortgage design is dominant in Canada, Denmark, Germany, the Netherlands, and Switzerland. When the mortgage starts out with a fixed rate and then switches to an adjustable rate after a specified initial term, the mortgage is referred to as a hybrid mortgage. Hybrid mortgages are popular in the United Kingdom.

- *Convertible:* The mortgage rate is initially either a fixed rate or an adjustable rate. At some point, the borrower has the option to convert the mortgage to a fixed rate or an adjustable rate for the remainder of the mortgage's life. Almost half of the mortgages in Japan are convertible.

4.3. Amortization Schedule

In most countries, residential mortgages are **amortizing loans**. The amortization of a loan means the gradual scheduled reduction of the principal over time. Assuming no prepayments are made by the borrower, the periodic mortgage payments made by the borrower consist of interest payments and scheduled principal repayments. As discussed in a previous chapter, there are two types of amortizing loans: fully amortizing loans and partially amortizing loans. In a fully amortizing loan, the sum of all the scheduled principal repayments during the mortgage's life is such that when the last mortgage payment is made, the loan is fully repaid. Most residential mortgages in the United States are fully amortizing loans. In a partially amortizing loan, the sum of all the scheduled principal repayments is less than the amount borrowed. The final payment then includes the unpaid mortgage balance, sometimes referred to as a "balloon" payment.

If no scheduled principal repayment is specified for a certain number of years, the loan is said to be an **interest-only mortgage**. Interest-only mortgages have been available in Australia, Denmark, Finland, France, Germany, Greece, Ireland, the Netherlands, Portugal, South Korea, Spain, Switzerland, and the United Kingdom. Interest-only mortgages also have been available to a limited extent in the United States. A special type of interest-only mortgage is one in which there are no scheduled principal repayments over the entire life of the loan. In this case, the balloon payment is equal to the original loan amount. These mortgages, referred to as "interest-only lifetime mortgages" or "bullet mortgages," have been available in Denmark, the Netherlands, and the United Kingdom.

4.4. Prepayment Options and Prepayment Penalties

A prepayment is any payment of principal that exceeds the scheduled principal repayment. A mortgage may entitle the borrower to prepay all or part of the outstanding mortgage principal prior to the scheduled due date. This contractual provision is referred to as a **prepayment option** or an **early repayment option**. From the lender's or investor's viewpoint, the effect of a prepayment option is that the cash flow amounts and timing from a mortgage cannot be known with certainty. This risk was referred to earlier as prepayment risk. Prepayment risk affects all mortgages that allow prepayment, not just the level-payment, fixed-rate, fully amortizing mortgages.

The mortgage may stipulate a monetary penalty when a borrower prepays within a certain time period after the mortgage is originated, which may extend for the full life of the loan. Such mortgage designs are referred to as **prepayment penalty mortgages**. The purpose of the prepayment penalty is to compensate the lender for the difference between the contract rate and the prevailing mortgage rate if the borrower prepays when interest rates decline. Hence, the prepayment penalty is effectively a mechanism that provides for yield maintenance for the lender. The method for calculating the penalty varies. Prepayment penalty mortgages are common in Europe. Although the proportion of prepayment penalty mortgages in the United States is small, they do exist.

4.5. Rights of the Lender in a Foreclosure

A mortgage can be a recourse loan or a non-recourse loan. When the borrower fails to make the contractual loan payments, the lender can repossess the property and sell it, but the proceeds received from the sale of the property may be insufficient to recoup the losses. In a **recourse loan**, the lender has a claim against the borrower for the shortfall between the amount of the outstanding mortgage balance and the proceeds received from the sale of the property. In a **non-recourse loan**, the lender does not have such a claim and thus can look only to the property to recover the outstanding mortgage balance. In the United States, recourse is typically determined by the state, and residential mortgages are non-recourse loans in many states. In contrast, residential mortgages in most European countries are recourse loans.

The recourse/non-recourse feature of a mortgage has implications for projecting the likelihood of defaults by borrowers, particularly in the case of what is sometimes called "underwater mortgages"—that is, mortgages for which the value of the property has declined below the amount owed by the borrower. For example, in the United States, where mortgages are typically non-recourse, the borrower may have an incentive to default on an underwater mortgage and allow the lender to foreclose on the property, even if resources are available to continue to make mortgage payments. This type of default by a borrower is referred to as a "strategic default." A strategic default, however, has negative consequences for the borrower, who will then have a lower credit score and a reduced ability to borrow in the future. Thus, not all borrowers faced with underwater mortgages will default. In countries where residential mortgages are recourse loans, a strategic default is less likely because the lender can seek to recover the shortfall from the borrower's other assets and/or income.

Now that the basics of residential mortgages have been set out, we can turn our attention to how these mortgages are securitized—that is, transformed into MBS. In the following sections, we focus on the US residential mortgage sector because it is the largest in the world and many non-US investors hold US MBS in their portfolios.

EXAMPLE 4 Residential Mortgage Designs

1. In an interest-only mortgage, the borrower:
   A. does not have to repay the principal as long as she pays the interest.
   B. does not have to make principal repayments for a certain number of years, after which she starts paying down the original loan amount.
   C. does not have to make principal repayments over the entire life of the mortgage and pays down the original loan amount as a balloon payment.
2. A bank advertises a mortgage with the following interest rate: 2.99% (12-month Euribor + 2.50%), resetting once a year. The mortgage is *most likely*:
   A. a hybrid mortgage.
   B. an adjustable-rate mortgage.
   C. an initial period fixed-rate mortgage.
3. If the borrower fails to make the contractual mortgage payments on a non-recourse mortgage, the lender:
   A. cannot foreclose the property.
   B. can recover the outstanding mortgage balance only through the sale of the property.
   C. can recover the outstanding mortgage balance through the sale of the property and the borrower's other assets and/or income.

Solution to 1: B is correct. In an interest-only mortgage, there is no scheduled principal repayment for a certain number of years, so the borrower starts paying down the original loan amount only after an initial period of interest-only payments. Some, but not all, interest-only mortgages do not have scheduled principal repayments over the entire life of the loan. These mortgages are called interest-only lifetime mortgages or bullet mortgages, and they require the borrower to pay back the original loan amount at maturity.

Solution to 2: B is correct. An adjustable-rate mortgage is one for which the mortgage rate is typically based on some reference rate or index (indexed-referenced ARM) or a rate determined at the lender's discretion (reviewable ARM) and is reset periodically. A mortgage rate of 12-month Euribor + 2.50%, resetting once per year, suggests that the mortgage is an indexed-referenced ARM. The 2.99% rate is the current mortgage rate (that is, 12-month Euribor of 0.49% + 2.50%) and should not be taken as an indication that it is a fixed-rate, initial period fixed-rate, or hybrid mortgage.

Solution to 3: B is correct. In the case of a non-recourse mortgage, the lender can foreclose the property if the borrower fails to make the contractual mortgage payments. However, the lender can use only the proceeds from the property to recover the outstanding mortgage balance.

5. MORTGAGE PASS-THROUGH SECURITIES

- **describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type**
- **define prepayment risk and describe the prepayment risk of mortgage-backed securities**

The bonds created from the securitization of mortgages related to the purchase of residential properties are residential mortgage-backed securities (RMBS). In such countries as the United States, Canada, Japan, and South Korea, a distinction is often made between securities that are guaranteed by the government or a quasi-government entity and securities that are not. Quasi-government entities are usually created by governments to perform various functions for them. Examples of quasi-government entities include government-sponsored enterprises (GSEs) such as Fannie Mae (previously the Federal National Mortgage Association) and Freddie Mac (previously the Federal Home Loan Mortgage Corporation) in the United States and the Japan Housing Finance Agency (JHF).

In the United States, securities backed by residential mortgages are divided into three sectors: (1) those guaranteed by a federal agency, (2) those guaranteed by a GSE, and (3) those issued by private entities and that are not guaranteed by a federal agency or a GSE. The first two sectors are referred to as **agency RMBS**, and the third sector is referred to as **non-agency RMBS**. We devote significant attention to US agency and non-agency RMBS because these securities represent a large sector of the investment-grade bond market and are included in the portfolios of many global investors.

Agency RMBS include securities issued by federal agencies, such as the Government National Mortgage Association, popularly referred to as Ginnie Mae. This entity is a federally related institution because it is part of the US Department of Housing and Urban Development. As a result, the RMBS that it guarantees carry the full faith and credit of the US government with respect to timely payment of interest and repayment of principal.

Agency RMBS also include RMBS issued by GSEs, such as Fannie Mae and Freddie Mac. RMBS issued by GSEs do not carry the full faith and credit of the US government. Agency RMBS issued by GSEs differ from non-agency RMBS in two ways. First, the credit risk of the RMBS issued by Fannie Mae and Freddie Mac is reduced by the guarantee of the GSE itself, which charges a fee for insuring the issue. In contrast, non-agency RMBS use credit enhancements to reduce credit risk. The pool of securitized loans is another way in which RMBS issued by GSEs differ from non-agency RMBS. Loans included in agency RMBS must meet specific underwriting standards established by various government agencies. These standards set forth the maximum size of the loan, the loan documentation required, the maximum loan-to-value ratio, and whether insurance is required. Loans satisfying the underwriting standards for inclusion as collateral for an agency RMBS are called "conforming" mortgages; otherwise, loans are categorized as "non-conforming" mortgages.

This section starts with a discussion of agency RMBS, which include mortgage pass-through securities and collateralized mortgage obligations. Note that the popular Bloomberg Barclays US Aggregate Bond Index includes only agency RMBS that are mortgage pass-through securities in its mortgage sector definition. We then discuss non-agency RMBS.

5.1. Mortgage Pass-Through Securities

A **mortgage pass-through security** is a security created when one or more holders of mortgages form a pool of mortgages and sell shares or participation certificates in the pool. A pool can consist of several thousand or only a few mortgages. When a mortgage is used as collateral for a mortgage pass-through security, the mortgage is said to be securitized.

5.1.1. Characteristics

The cash flows of a mortgage pass-through security depend on the cash flows of the underlying pool of mortgages. The cash flows consist of monthly mortgage payments representing interest, the scheduled repayment of principal, and any prepayments. Cash payments are made to security holders each month. Neither the amount nor the timing of the cash flows from the pool of mortgages, however, is necessarily identical to that of the cash flow passed through to the security holders. In fact, the monthly cash flows of a mortgage pass-through security are less than the monthly cash flows of the underlying pool of mortgages by an amount equal to the servicing and other administrative fees.

The servicing fee is the charge related to administrative tasks, such as collecting monthly payments from borrowers, forwarding proceeds to owners of the loan, sending payment notices to borrowers, reminding borrowers when payments are overdue, maintaining records of the outstanding mortgage balance, initiating foreclosure proceedings if necessary, and providing tax information to borrowers when applicable. The servicing fee is typically a portion of the mortgage rate. The other administrative fees are those charged by the issuer or financial guarantor of the mortgage pass-through security for guaranteeing the issue.

A mortgage pass-through security's coupon rate is called the **pass-through rate**. The pass-through rate is lower than the mortgage rate on the underlying pool of mortgages by an amount equal to the servicing and other administrative fees. The pass-through rate that the investor receives is said to be "net interest" or "net coupon."

Not all mortgages included in a pool of securitized mortgages have the same mortgage rate and the same maturity. Consequently, for each mortgage pass-through security, a **weighted average coupon rate** (WAC) and a **weighted average maturity** (WAM) are determined. The WAC is calculated by weighting the mortgage rate of each mortgage in the pool by the percentage of the outstanding mortgage balance relative to the outstanding amount of all the mortgages in the pool. Similarly, the WAM is calculated by weighting the remaining number of months to maturity of each mortgage in the pool by the outstanding mortgage balance relative to the outstanding amount of all the mortgages in the pool. Example 5 illustrates the calculation of the WAC and WAM.

EXAMPLE 5 Weighted Average Coupon Rate and Weighted Average Maturity

Assume that a pool includes three mortgages with the following characteristics:

Mortgage	Outstanding Mortgage Balance (US$)	Coupon Rate (%)	Number of Months to Maturity
1	1,000	5.1	34
2	3,000	5.7	76
3	6,000	5.3	88

The outstanding amount of three mortgages is US$10,000. Thus, the weights of Mortgages 1, 2, and 3 are 10%, 30%, and 60%, respectively.

The WAC is

$$10\% \times 5.1\% + 30\% \times 5.7\% + 60\% \times 5.3\% = 5.4\%$$

The WAM is

$$10\% \times 34 + 30\% \times 76 + 60\% \times 88 = 79 \text{ months}$$

5.1.2. Prepayment Risk

Mortgage pass-through security cash flows are uncertain because they depend on actual prepayments. As we noted earlier, this risk is called prepayment risk. Prepayment risk has two components: contraction risk and extension risk, both of which largely reflect changes in the general level of interest rates.

Contraction risk is the risk that when interest rates decline, actual prepayments will be higher than forecasted because homeowners will refinance at now-available lower interest rates. Thus, a security backed by mortgages will have a shorter maturity than was anticipated at the time of purchase. Holding a security whose maturity becomes shorter when interest rates decline has two adverse consequences for investors. First, investors must reinvest the proceeds at lower interest rates. Second, if the security is prepayable or callable, its price appreciation is not as great as that of an otherwise identical bond without a prepayment or call option.

In contrast, **extension risk** is the risk that when interest rates rise, prepayments will be lower than forecasted because homeowners are reluctant to give up the benefits of a contractual interest rate that now looks low. As a result, a security backed by mortgages will typically have a longer maturity than was anticipated at the time of purchase. From the investor's perspective, the value of the security has fallen because the higher interest rates translate into a lower price for the security, and the income investors receive (and can potentially reinvest) is typically limited to the interest payment and scheduled principal repayments.

5.1.3. Prepayment Rate Measures

In describing prepayments, market participants refer to the prepayment rate or prepayment speed. The two key prepayment rate measures are the single monthly mortality rate (SMM), a monthly measure, and its corresponding annualized rate, the conditional prepayment rate (CPR).

The SMM reflects the dollar amount of prepayment for the month as a fraction of the balance on the mortgage after accounting for the scheduled principal repayment for the month. It is calculated as follows:

$$\text{SMM} = \frac{\text{Prepayment for the month}}{\begin{pmatrix}\text{Beginning outstanding mortgage balance for the month} \\ - \text{ Scheduled principal repayment for the month}\end{pmatrix}} \quad (1)$$

Note that the SMM is typically expressed as a percentage.

When market participants describe the assumed prepayment for a pool of residential mortgages, they refer to the annualized SMM, which is the CPR. A CPR of 6%, for example, means that approximately 6% of the outstanding mortgage balance at the beginning of the year is expected to be prepaid by the end of the year.

A key factor in the valuation of a mortgage pass-through security and other products derived from a pool of mortgages is forecasting the future prepayment rate. This task involves prepayment modeling. Prepayment modeling uses characteristics of the mortgage pool and other factors to develop a statistical model for forecasting future prepayments.

In the United States, market participants describe prepayment rates in terms of a prepayment pattern or benchmark over the life of a mortgage pool. This pattern is the Public Securities Association (PSA) prepayment benchmark, which is produced by the Securities Industry and Financial Markets Association (SIFMA). The PSA prepayment benchmark is expressed as a series of monthly prepayment rates. Based on historical patterns, it assumes that prepayment rates are low for newly originated mortgages and then speed up as the mortgages become seasoned. Slower or faster prepayment rates are then referred to as some percentage of the PSA prepayment benchmark. Rather than going into the details of the PSA prepayment benchmark, this discussion will rely on some PSA assumptions. What is important to remember is that the standard for the PSA model is 100 PSA; that is, at 100 PSA, investors can expect prepayments to follow the PSA prepayment benchmark—for example, an increase of prepayment rates of 0.20% for the first 30 months until they peak at 6% in Month 30. A PSA assumption greater than 100 PSA means that prepayments are assumed to be faster than the standard model. In contrast, a PSA assumption lower than 100 PSA means that prepayments are assumed to be slower than the standard model.

5.1.4. Cash Flow Construction

Let us see how to construct the monthly cash flow for a hypothetical mortgage pass-through security. We assume the following:

- The underlying pool of mortgages has a par value of US$800 million.
- The mortgages are fixed-rate, level-payment, and fully amortizing loans.
- The WAC for the mortgages in the pool is 6%.
- The WAM for the mortgages in the pool is 357 months.
- The pass-through rate is 5.5%.

Exhibit 2 shows the cash flows to the mortgage pass-through security holders for selected months assuming a prepayment rate of 165 PSA. The SMM is given in Column 3, and mortgage payments are given in Column 4. The net interest payment in Column 5 is the amount available to pay security holders after servicing and other administrative fees. This amount is equal to the beginning outstanding mortgage balance in Column 2 multiplied by the pass-through rate of 5.5% and then divided by 12. The scheduled principal repayment in Column 6 is the difference between the mortgage payment in Column 4 and the gross interest payment. The gross interest payment is equal to the beginning outstanding mortgage balance in Column 2 multiplied by the WAC of 6% and then divided by 12. The prepayment in Column 7 is calculated by applying Equation 1, using the SMM provided in Column 3, the beginning outstanding mortgage balance in Column 2, and the scheduled principal repayment in Column 6. The total principal repayment in Column 8 is the sum of the scheduled principal repayment in Column 6 and the prepayments in Column 7. Subtracting this amount from the beginning outstanding mortgage balance for the month gives the beginning outstanding mortgage balance for the following month. Finally, the projected cash flow for this mortgage pass-through security in Column 9 is the sum of the net interest payment in Column 5 and the total principal repayment in Column 8.

EXHIBIT 2 Monthly Cash Flow to Bondholders for a US$800 Million Mortgage Pass-Through Security with a WAC of 6.0%, a WAM of 357 Months, and a Pass-Through Rate of 5.5%, Assuming a Prepayment Rate of 165 PSA

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Month	Beginning Outstanding Mortgage Balance (US$)	SMM (%)	Mortgage Payment (US$)	Net Interest Payment (US$)	Scheduled Principal Repayment (US$)	Prepayment (US$)	Total Principal Repayment (US$)	Projected Cash Flow (US$)
1	800,000,000	0.111	4,810,844	3,666,667	810,844	884,472	1,695,316	5,361,982
2	798,304,684	0.139	4,805,520	3,658,896	813,996	1,104,931	1,918,927	5,577,823
3	796,385,757	0.167	4,798,862	3,650,101	816,933	1,324,754	2,141,687	5,791,788
⋮								
29	674,744,235	0.865	4,184,747	3,092,578	811,026	5,829,438	6,640,464	9,733,042
30	668,103,771	0.865	4,148,550	3,062,142	808,031	5,772,024	6,580,055	9,642,198
⋮								
100	326,937,929	0.865	2,258,348	1,498,466	623,659	2,822,577	3,446,236	4,944,702
101	323,491,693	0.865	2,238,814	1,482,670	621,355	2,792,788	3,414,143	4,896,814
⋮								
200	103,307,518	0.865	947,322	473,493	430,784	889,871	1,320,655	1,794,148
201	101,986,863	0.865	939,128	467,440	429,193	878,461	1,307,654	1,775,094
⋮								
300	19,963,930	0.865	397,378	91,501	297,559	170,112	467,670	559,172
301	19,496,260	0.865	393,941	89,358	296,460	166,076	462,536	551,893
⋮								
356	484,954	0.865	244,298	2,223	241,873	2,103	243,976	246,199
357	240,978	0.865	242,185	1,104	240,980	0	240,980	242,084

Note: The SMM in Column 3 is rounded, which results in some rounding error in the calculation of the prepayments in Column 7 and, thus, of the total principal repayments and the projected cash flows in Columns 8 and 9, respectively. Since the WAM is 357 months, the underlying mortgage pool is seasoned an average of three months, and therefore, based on a 165 PSA, the CPR is 0.132% in Month 1 (seasoned Month 4), and the pool seasons at 6% in Month 27.

5.1.5. Weighted Average Life

A standard practice in the bond market is to refer to the maturity of a bond. This practice is not followed for MBS because principal repayments (scheduled principal repayments and prepayments) are made over the life of the security. Although an MBS has a "legal maturity," which is the date when the last scheduled principal repayment is due, the legal maturity does not reveal much about the actual principal repayments and the interest rate risk associated with the MBS. For example, a 30-year, option-free, corporate bond and an MBS with a 30-year legal maturity with the same coupon rate are not equivalent in terms of interest rate risk. Effective duration can be calculated for both the corporate bond and the MBS to assess the sensitivity of the securities to interest rate movements. But a measure widely used by market participants for MBS is the **weighted average life**, or simply the **average life** of the MBS. This measure gives investors an indication of how long they can expect to hold the MBS before it is paid off assuming interest rates stay at current levels and, thus, expected prepayments are realized. In other words, the average life of the MBS is the convention-based average time to receipt of all the projected principal repayments (scheduled principal repayments and projected prepayments).

A mortgage pass-through security's average life depends on the prepayment assumption, as illustrated in the following table. This table provides the average life of the mortgage pass-through security used in Exhibit 2 for various prepayment rates. Note that at the assumed prepayment rate of 165 PSA, the mortgage pass-through security has an average life of 8.6 years. The average life extends when the prepayment rate goes down and contracts rapidly as the prepayment rate goes up. So, at a prepayment rate of 600 PSA, the average life of the mortgage pass-through security is only 3.2 years.

PSA assumption	100	125	165	250	400	600
Average life (years)	11.2	10.1	8.6	6.4	4.5	3.2

Environmental, Social, and Governance (ESG) Risks for RMBS

Beyond traditional factors that affect RMBS prepayment risk, such as loan-to-value ratios and FICO scores of residential borrowers, rating agencies have more recently incorporated ESG factors into their assessment of these securities. In the first example, the potential loss of mortgage principal in the event of a natural disaster is incorporated into the ratings process for a new securitization, whereas in the second example, several long outstanding RMBS backed by subprime mortgages were downgraded because of a potential decline in interest income as a result of the adverse social impact of the COVID-19 pandemic.

Fitch Ratings Example

As the physical risk of climate change increases, Fitch Ratings has incorporated natural disaster risk into its loan loss expectations by factoring in the impact of potential weather-related damage on the value of residential properties that collateralize the mortgages underlying an RMBS transaction. Fitch Ratings has created ESG Relevance Scores that are integrated into the existing research process (they range from 1 to 5, with 5 representing ESG risk that is "highly relevant" to the credit decision and 1 representing ESG risk that is considered "irrelevant"). In the case of RMBS, Fitch assesses the geographic asset location and concentration within a given pool and compares it to regions that face greater climate risk. Fitch then assesses concentration penalties and adjusts stress assumptions for projected property losses from storm surges in cases of heightened environmental risk. For example, in 2019, Fitch published an ESG Relevance Score of 5 for an RMBS transaction with a large pool concentration in the US Gulf Coast region. Because this region is particularly susceptible to hurricane-related damage and over 75% of pool properties were concentrated in Louisiana and Texas, it faced a 1.16× probability of default adjustment with an increase in expected loan loss of over 100 bps due to the greater natural disaster and catastrophe risk in this pool versus other transactions.

Moody's Investors Service Example

In a 2020 report, Moody's Investor Service announced that ESG factors were a material credit consideration in one-third of its rating actions for private-sector issuers in 2019. The vast majority (88%) cited governance issues, followed by social (20%) and environmental (16%) issues, and many actions referenced more than one ESG consideration. The COVID-19 pandemic has wide-ranging political, economic, and social consequences that can adversely affect the creditworthiness of residential borrowers. For example, in June 2020, Moody's downgraded the ratings of over 40 bonds from 25 different US RMBS transactions. These securities, originally issued between 2004 and 2006 and backed by subprime mortgages, were largely downgraded to B1. In its rating action, Moody's noted the heightened risk of interest loss due to the 2020 slowdown in US economic activity and rising unemployment due to the coronavirus outbreak. Moody's cited unusually high downside risks to its forecast regarding the COVID-19 pandemic as a social risk under its ESG framework, with significant risks to public health and safety. Because many residential borrowers have been offered relief in the form of mortgage payment forbearance owing to the loss of income during the lockdown, these loans also face the possibility of modification in the future depending on the timing and pace of economic recovery.

EXAMPLE 6 Mortgage Pass-Through Securities

1. A non-conforming mortgage:
   A. cannot be used as collateral in a mortgage-backed security.
   B. does not satisfy the underwriting standards for inclusion as collateral for an agency residential mortgage-backed security.
   C. does not give the lender a claim against the borrower for the shortfall between the amount of the outstanding mortgage balance and the proceeds from the sale of the property if the borrower defaults on the mortgage.
2. The monthly cash flows of a mortgage pass-through security *most likely*:
   A. are constant.
   B. change when interest rates decline.
   C. are equal to the cash flows of the underlying pool of mortgages.
3. A prepayment rate of 80 PSA means that investors can expect:
   A. 80% of the par value of the mortgage pass-through security to be repaid prior to the security's maturity.
   B. 80% of the borrowers whose mortgages are included in the collateral backing the mortgage pass-through security to prepay their mortgages.

   C. the prepayment rate of the mortgages included in the collateral backing the mortgage pass-through security to be 80% of the monthly prepayment rates forecasted by the PSA model.
4. All else being equal, when interest rates decline:
   A. investors in mortgage pass-through securities face extension risk.
   B. the weighted average maturity of a mortgage pass-through security lengthens.
   C. the increase in the price of a mortgage pass-through security is less than the increase in the price of an otherwise identical bond with no prepayment option.

Solution to 1: B is correct. A non-conforming mortgage is one that does not satisfy the underwriting standards for inclusion as collateral for an agency RMBS. The standards specify the maximum size of the loan, the loan documentation required, the maximum loan-to-value ratio, and whether or not insurance is required for the loans in the pool.

Solution to 2: B is correct. The monthly cash flows of a mortgage pass-through security are not equal to but rather depend on the cash flows of the underlying pool of mortgages. That said, their amount and timing cannot be known with certainty because of prepayments. When interest rates decline, borrowers are likely to prepay all or part of their outstanding mortgage balance, which will affect the monthly cash flows of the mortgage pass-through security. Remember that the fees related to servicing and guaranteeing the mortgages reduce the monthly cash flows of a mortgage pass-through security relative to those of the underlying pool of mortgages.

Solution to 3: C is correct. A prepayment rate of 80 PSA means that investors can expect the prepayment rate of the mortgages included in the collateral backing the mortgage pass-through security to be 80% of the monthly prepayment rates forecasted by the PSA model. For example, if the PSA model forecasts an increase in prepayment rates of 0.20% for the first 30 months until they peak at 6% in Month 30, 80 PSA would assume an increase in prepayment rates of 0.16% (80% × 0.20%) for the first 30 months until they peak at 4.80% (80% × 6%) in Month 30. Thus, investors can expect slower prepayments than the PSA prepayment benchmark.

Solution to 4: C is correct. When interest rates decline, the prepayment rate on a mortgage pass-through security goes up because homeowners refinance at now-available lower interest rates. As a result, investors face contraction risk; that is, they receive payments faster than anticipated. Investors who decide to retain the security face the prospect of having to reinvest those payments at relatively low interest rates. Investors who decide to sell the security would have to do so at a price lower than that of an otherwise identical bond with no prepayment option and thus no prepayment risk.

6. COLLATERALIZED MORTGAGE OBLIGATIONS AND NON-AGENCY RMBS

- **describe types and characteristics of residential mortgage-backed securities, including mortgage pass-through securities and collateralized mortgage obligations, and explain the cash flows and risks for each type**

As noted in the previous section, prepayment risk is an important consideration when investing in mortgage pass-through securities. Some institutional investors are concerned with extension risk, and others, with contraction risk. The structuring of a securitization can help redistribute the cash flows of mortgage-related products (mortgage pass-through securities or pools of loans) to different bond classes or tranches, which leads to the creation of securities that have different exposures to prepayment risk and thus different risk–return patterns relative to the mortgage-related product from which they were created.

When the cash flows of mortgage-related products are redistributed to various tranches, the resulting securities are called **collateralized mortgage obligations** (CMOs). The mortgage-related products from which the cash flows are obtained are considered the collateral. Note that in contrast to a mortgage pass-through security, the collateral is not a pool of mortgages but a mortgage pass-through security. In fact, in practice, the collateral is usually a pool of mortgage pass-through securities—hence the reason market participants sometimes use the terms "collateral" and "mortgage pass-through securities" interchangeably.

The creation of a CMO cannot eliminate or change prepayment risk; it can only distribute the various forms of this risk among different bond classes. The CMO's major financial innovation is that securities can be created to closely satisfy the asset/liability needs of institutional investors, thereby broadening the appeal of mortgage-backed products.

A wide range of CMO structures exists. The major ones are reviewed in the following subsections.

6.1. Sequential-Pay CMO Structures

The first CMOs were structured so that each tranche would be retired sequentially. Such structures are called "sequential-pay CMOs." The rule for the monthly distribution of the principal repayments (scheduled principal repayment plus prepayments) to the tranches in this structure is as follows. First, distribute all principal payments to Tranche 1 until the principal balance for Tranche 1 is zero. After Tranche 1 is paid off, distribute all principal payments to Tranche 2 until the principal balance for Tranche 2 is zero, and so on.

To illustrate a sequential-pay CMO, let us use a hypothetical transaction called CMO-01. Assume that the collateral for CMO-01 is the mortgage pass-through security described in Exhibit 2. Recall that the total par value of the collateral is US$800 million, the pass-through coupon rate is 5.5%, the WAC is 6%, and the WAM is 357 months. From this US$800 million of collateral, four tranches are created, as shown in Exhibit 3. In this simple structure, the coupon rate is the same for each tranche and the same as the mortgage pass-through security's coupon rate. This feature is for simplicity; typically, the coupon rate varies by tranche based on the term structure of interest rates, among other factors.

EXHIBIT 3 CMO-01: Sequential-Pay CMO Structure with Four Tranches

Tranche	Par Amount (US$ millions)	Coupon Rate (%)
A	389	5.5
B	72	5.5
C	193	5.5
D	146	5.5
Total	800	

Payment rules: *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding for each tranche at the beginning of the month. *For disbursement of principal payments:* Disburse principal payments to Tranche A until it is completely paid off. After Tranche A is completely paid off, disburse principal payments to Tranche B until it is completely paid off. After Tranche B is completely paid off, disburse principal payments to Tranche C until it is completely paid off. After Tranche C is completely paid off, disburse principal payments to Tranche D until it is completely paid off.

Remember that a CMO is created by redistributing the cash flows—interest payments and principal repayments—to the various tranches based on a set of payment rules. The payment rules at the bottom of Exhibit 3 describe how the cash flows from the mortgage pass-through security are to be distributed to the four tranches. CMO-01 has separate rules for the interest payment and the principal repayment, the latter being the sum of the scheduled principal repayment and the prepayments.

Although the payment rules for the distribution of the principal repayments are known, the precise principal repayment amount in each month is not. This amount will depend on the cash flow of the collateral, which depends on the actual prepayment rate of the collateral. The assumed prepayment rate (165 PSA in Exhibit 2) allows determining only the projected, not the actual, cash flow.

Consider what has been accomplished by creating the sequential-pay CMO-01 structure. Earlier, we saw that with a prepayment rate of 165 PSA, the mortgage pass-through security's average life was 8.6 years. Exhibit 4 reports the average life of the collateral and the four tranches assuming various actual prepayment rates. Note that the four tranches have average lives that are shorter or longer than the collateral, thereby attracting investors who have preferences for different average lives. For example, a pension fund that needs cash only after a few years because it expects a significant increase in the number of retirements after that time may opt for a tranche with a longer average life.

EXHIBIT 4. Average Life of the Collateral and the Four Tranches of CMO-01 for Various Actual Prepayment Rates

Prepayment	Average Life (years)				
Rate	Collateral	Tranche A	Tranche B	Tranche C	Tranche D
100 PSA	11.2	4.7	10.4	15.1	24.0
125 PSA	10.1	4.1	8.9	13.2	22.4
165 PSA	8.6	3.4	7.3	10.9	19.8
250 PSA	6.4	2.7	5.3	7.9	15.2
400 PSA	4.5	2.0	3.8	5.3	10.3
600 PSA	3.2	1.6	2.8	3.8	7.0

A major problem that remains is the considerable variability of the average lives of the tranches. How this problem can be handled is shown in the next section, but at this point, note that some protection against prepayment risk is provided for each tranche. The protection arises because prioritizing the distribution of principal (that is, establishing the payment rule for the principal repayment) effectively protects the shorter-term tranche (A in this structure) against extension risk. This protection actually comes from the longer-term tranches. Similarly, Tranches C and D provide protection against extension risk for Tranches A and B. At the same time, Tranches C and D benefit because they are provided protection against contraction risk; this protection comes from Tranches A and B. Thus, the sequential-pay CMO-01 structure allows investors concerned about extension risk to invest in Tranches A or B and those concerned about contraction risk to invest in Tranches C or D.

6.2. CMO Structures Including Planned Amortization Class and Support Tranches

A common structure in CMOs is to include planned amortization class (PAC) tranches, which offer greater predictability of the cash flows if the prepayment rate is within a specified band over the collateral's life. Remember that the creation of an MBS, whether it is a mortgage pass-through or a CMO, cannot make prepayment risk disappear. So where does the reduction of prepayment risk (both extension risk and contraction risk) that PAC tranches offer investors come from? The answer is that it comes from the existence of non-PAC tranches, called **support tranches** or companion tranches. The structure of the CMO makes the support tranches absorb prepayment risk first. Because PAC tranches have limited (but not complete) protection against both extension risk and contraction risk, they are said to provide two-sided prepayment protection.

The greater predictability of the cash flows for the PAC tranches occurs because a principal repayment schedule must be satisfied. When the prepayment rate is within the specified band, called the PAC band, all prepayment risk is absorbed by the support tranche. If the collateral prepayments are slower than forecasted, the support tranches do not receive any principal repayment until the PAC tranches receive their scheduled principal repayment. This rule reduces the extension risk of the PAC tranches. Similarly, if the collateral prepayments are faster than forecasted, the support tranches absorb any principal repayments in excess of the scheduled principal repayments. This rule reduces the contraction risk of the PAC tranches. Even if the prepayment rate is outside the PAC band, prepayment risk is first absorbed by the support tranche. Thus, the key to the prepayment protection that PAC tranches offer investors is the amount of support tranches outstanding. If the support tranches are paid off quickly because of faster-than-expected prepayments, they no longer provide any protection for the PAC tranches.

Support tranches expose investors to the highest level of prepayment risk. Therefore, investors must be particularly careful in assessing the cash flow characteristics of support tranches in order to reduce the likelihood of adverse portfolio consequences resulting from prepayments.

To illustrate how to create CMO structures including PAC and support tranches, we use again the US$800 million mortgage pass-through security described in Exhibit 2, with a pass-through coupon rate of 5.5%, a WAC of 6%, and a WAM of 357 months as collateral. The creation of PAC tranches requires the specification of two PSA prepayment rates: a *lower* PSA prepayment assumption and an *upper* PSA prepayment assumption. The lower and upper PSA prepayment assumptions are called the "initial PAC collar" or the "initial PAC band." The PAC collar for a CMO is typically dictated by market conditions. In our example, we assume that the lower and upper PSA prepayment assumptions are 100 PSA and 250 PSA, respectively, so the initial PAC collar is 100–250 PSA.

Exhibit 5 shows a CMO structure called CMO-02 that contains only two tranches: a 5.5% coupon PAC tranche created assuming an initial PAC collar of 100–250 PSA and a support tranche.

EXHIBIT 5 CMO-02: CMO Structure with One PAC Tranche and One Support Tranche

Tranche	Par Amount (US$ millions)	Coupon Rate (%)
P (PAC)	487.6	5.5
S (support)	312.4	5.5
Total	800.0	

Payment rules: *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche based on the amount of principal outstanding for each tranche at the beginning of the month. *For disbursement of principal payments:* Disburse principal payments to Tranche P based on its schedule of principal repayments. Tranche P has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule for Tranche P are paid to Tranche S. When Tranche S is completely paid off, all principal payments are to be made to Tranche P regardless of the schedule.

Exhibit 6 reports the average life of the PAC and support tranches in CMO-02 assuming various actual prepayment rates. Note that between 100 PSA and 250 PSA, the average life of the PAC tranche is constant at 7.7 years. At slower or faster PSA rates, however, the schedule is broken and the average life changes—extending when the prepayment rate is less than 100 PSA and contracting when it is greater than 250 PSA. Even so, there is much less variability for the average life of the PAC tranche compared with that of the support tranche.

EXHIBIT 6 Average Life of the PAC Tranche and the Support Tranche of CMO-02 for Various Actual Prepayment Rates and an Initial PAC Collar of 100–250 PSA

Prepayment	Average Life (years)	
Rate	PAC Tranche (P)	Support Tranche (S)
50 PSA	10.2	24.9
75 PSA	8.6	22.7
100 PSA	7.7	20.0
165 PSA	7.7	10.7
250 PSA	7.7	3.3
400 PSA	5.5	1.9
600 PSA	4.0	1.4

Most CMO structures including PAC and support tranches have more than one PAC tranche. A sequence of six PAC tranches (that is, PAC tranches paid off in sequence as specified by a principal repayment schedule) is not uncommon. For example, consider CMO-03 in Exhibit 7, which contains four sequential PAC tranches (P-A, P-B, P-C, and P-D) and one support tranche. The total par amount of the PAC and support tranches is the same as for CMO-02 in Exhibit 5. The difference is that instead of one PAC tranche with a schedule, there are four PAC tranches with schedules. As described in the payment rules, the PAC tranches are paid off in sequence.

EXHIBIT 7 CMO-03: CMO Structure with Sequential PAC Tranches and One Support Tranche

Tranche	Par Amount (US$ million)	Coupon Rate (%)
P-A (PAC)	287.6	5.5
P-B (PAC)	90.0	5.5
P-C (PAC)	60.0	5.5
P-D (PAC)	50.0	5.5
S (support)	312.4	5.5
Total	800.0	

Payment rules: *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche based on the amount of principal outstanding for each tranche at the beginning of the month. *For disbursement of principal payments:* Disburse principal payments to Tranche P-A based on its schedule of principal repayments. Tranche P-A has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule while P-A is outstanding is paid to Tranche S. Once P-A is paid off, disburse principal payments to Tranche P-B based on its schedule of principal repayments. Tranche P-B has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule while P-B is outstanding are paid to Tranche S. The same rule applies for P-C and P-D. When Tranche S is completely paid off, all principal payments are to be made to the outstanding PAC tranches regardless of the schedule.

6.3. Other CMO Structures

Often, there is a demand for tranches that have a floating rate. Although the collateral pays a fixed rate, it is possible to create a tranche with a floating rate. This is done by constructing a floater and an inverse floater combination from any of the fixed-rate tranches in the CMO structure. Because the floating-rate tranche pays a higher rate when interest rates go up and the inverse floater pays a lower rate when interest rates go up, they offset each other. Thus, a fixed-rate tranche can be used to satisfy the demand for a floating-rate tranche.

In a similar vein, other types of tranches to satisfy the various needs of investors are possible.

EXAMPLE 7 Collateralized Mortgage Obligations

1. A collateralized mortgage obligation:
   A. eliminates prepayment risk.
   B. is created from a pool of conforming loans.
   C. redistributes various forms of prepayment risk among different bond classes.
2. The variability in the average life of the PAC tranche of a CMO relative to the average life of the mortgage pass-through securities from which the CMO is created is:
   A. lower.
   B. the same.
   C. higher.
3. Referring to Exhibit 7, the tranche of CMO-03 that is *most suitable* for an investor concerned about contraction risk is:
   A. P-A (PAC).
   B. P-D (PAC).
   C. S (support).
4. The tranche of a collateralized mortgage obligation that is *most suitable* for an investor who expects a fall in interest rates is:
   A. a fixed-rate tranche.
   B. an inverse floating-rate tranche.
   C. a PAC tranche.
5. The investment that is *most suitable* for an investor who is willing and able to accept significant prepayment risk is:
   A. a mortgage pass-through security.
   B. the support tranche of a collateralized mortgage obligation.
   C. the inverse floating-rate tranche of a collateralized mortgage obligation.

Solution to 1: C is correct. CMOs are created by redistributing the cash flows of mortgage-related products, including mortgage pass-through securities, to different bond classes or tranches on the basis of a set of payment rules.

Solution to 2: A is correct. The purpose of creating different bond classes in a CMO is to provide a risk–return profile that is more suitable to investors than the risk–return profile of the mortgage pass-through securities from which the CMO is created. The PAC tranche has considerably less variability in average life than the mortgage pass-through securities. In contrast, the support tranche has more variability in average life than the mortgage pass-through securities.

Solution to 3: B is correct. Contraction risk is the risk that when interest rates decline, prepayments will be higher than expected and the security's maturity will become shorter than was anticipated at the time of purchase. PAC tranches offer investors protection against contraction risk (and extension risk). The PAC tranche that is most suitable for an investor concerned about contraction risk is P-D because it is the latest-payment PAC tranche; that is, any principal repayments in excess of the scheduled principal repayments are absorbed sequentially by the support tranche, then P-A, P-B, and, finally, P-D.

Solution to 4: B is correct. The tranche of a CMO that is most suitable for an investor who expects a fall in interest rates is an inverse floating-rate tranche. The inverse floater pays a coupon rate that is inversely related to prevailing interest rates. Thus, if interest rates fall, the CMO's coupon rate will rise.

Solution to 5: B is correct. The investment that is most suitable to an investor who is willing and able to accept significant prepayment risk is the support tranche of a collateralized mortgage obligation. Because the PAC tranche has a stable average life at prepayment rates within the PAC band, all prepayment risk is absorbed by the support tranche for prepayment rates within the band. Even at rates outside the PAC band, prepayment risk is first absorbed by the support tranche. Investors will be compensated for bearing prepayment risk in the sense that, if properly priced, the support tranche will have a higher expected rate of return than the PAC tranche.

6.4. Non-Agency Residential Mortgage-Backed Securities

Agency RMBS are those issued by Ginnie Mae, Fannie Mae, and Freddie Mac. RMBS issued by any other entity are non-agency RMBS. Entities that issue non-agency RMBS are typically thrift institutions, commercial banks, and private conduits. Private conduits may purchase non-conforming mortgages, pool them, and then sell mortgage pass-through securities whose collateral is the underlying pool of non-conforming mortgages. Because they are not guaranteed by the government or by a GSE, credit risk is an important consideration when investing in non-agency RMBS.

Non-agency RMBS share many features and structuring techniques with agency CMOs. However, because non-agency RMBS are not guaranteed by the US government or by a GSE that can provide protection against losses in the pool, some form of internal or external credit enhancement is necessary to make these securities attractive to investors. These credit enhancements allow investors to reduce credit risk or transfer credit risk between bond classes, thus enabling investors to choose the risk–return profile that best suits their needs. Credit enhancements also play an important role in obtaining favorable credit ratings, which make non-agency RMBS more marketable to investors. The level of credit enhancement is usually determined relative to a specific credit rating desired by the issuer for a security. Note that one of the consequences of the 2007–09 credit crisis has been an overall increase in the level of credit enhancement.

As mentioned earlier, subordination, or credit tranching, is a common form of credit enhancement. The subordination levels (that is, the amount of credit protection for a bond class) are set at the time of issuance. However, the subordination levels change over time, as voluntary prepayments and defaults occur. To protect investors in non-agency RMBS, a securitization is designed to keep the amount of credit enhancement from deteriorating over time. If the credit enhancement for senior tranches deteriorates because of poor performance of the collateral, a mechanism called the "shifting interest mechanism" locks out subordinated bond classes from receiving payments for a period. Many non-agency RMBS also include other credit enhancements, such as overcollateralization and reserve accounts.

When forecasting the future cash flows of non-agency RMBS, investors must consider two important components. The first is the assumed default rate for the collateral. The second is the recovery rate, because even though the collateral may default, not all of the outstanding mortgage balance may be lost. The repossession and subsequent sale of the recovered property may provide cash flows that will be available to pay bondholders. That amount is based on the assumed amount that will be recovered.

Now that we have discussed securities backed by a pool of residential mortgages, we turn to securities backed by a pool of commercial mortgages.

7. COMMERCIAL MORTGAGE-BACKED SECURITIES

- **describe characteristics and risks of commercial mortgage-backed securities**

Commercial mortgage-backed securities (CMBS) are backed by a pool of commercial mortgages on income-producing property, such as multifamily properties (e.g., apartment buildings), office buildings, industrial properties (including warehouses), shopping centers, hotels, and health care facilities (e.g., senior housing care facilities). The collateral is a pool of commercial loans that were originated either to finance a commercial purchase or to refinance a prior mortgage obligation.

7.1. Credit Risk

In the United States and other countries where commercial mortgages are non-recourse loans, the lender can look only to the income-producing property backing the loan for interest payments and principal repayments. If a default occurs, the lender can foreclose the commercial property but it can use only the proceeds from the sale of that property to recover the principal outstanding, and it has no recourse to the borrower's other assets and/or income for any unpaid balance. The lender must view each property individually, and lenders evaluate each property using measures that have been found useful in assessing credit risk.

Two key indicators of potential credit performance are the loan-to-value ratio (LTV), which was discussed earlier, and the debt-service-coverage (DSC) ratio, sometimes referred to as DSCR. The DSC ratio is equal to the property's annual net operating income (NOI) divided by the debt service (that is, the annual amount of interest payments and principal repayments). The NOI is defined as the rental income reduced by cash operating expenses and a non-cash replacement reserve reflecting the depreciation of the property over time. A DSC ratio that exceeds 1.0 indicates that the cash flows from the property are sufficient to cover the debt service while maintaining the property in its initial state of repair. The higher the DSC ratio, the more likely it is that the borrower will be able to meet debt-servicing requirements from the property's cash flows.

7.2. CMBS Structure

A credit-rating agency determines the level of credit enhancement necessary to achieve a desired credit rating. For example, if specific loan-to-value and DSC ratios are needed and those ratios cannot be met at the loan level, subordination is used to achieve the desired credit rating.

Interest on the principal outstanding is paid to all tranches. Losses arising from loan defaults are charged against the outstanding principal balance of the CMBS tranche with the lowest priority. This tranche may not be rated by credit-rating agencies; in this case, this unrated tranche is called the "first-loss piece," "residual tranche," or "equity tranche." The total loss charged includes the amount previously advanced and the actual loss incurred in the sale of the loan's underlying property.

Two characteristics that are usually specific to CMBS structures are the presence of call protection and a balloon maturity provision.

7.2.1. Call Protection

A critical investment feature that distinguishes CMBS from RMBS is the protection against early prepayments available to investors known as call protection. An investor in an RMBS is exposed to considerable prepayment risk because the borrower has the right to prepay a loan, in whole or in part, before the scheduled principal repayment date. As explained previously, a borrower in the United States usually does not pay any penalty for prepayment. The discussion of CMOs highlighted how investors can purchase certain types of tranches (e.g., sequential-pay and PAC tranches) to modify or reduce prepayment risk.

CMBS investors have considerable call protection, which results in CMBS trading more like corporate bonds than RMBS. The call protection comes either at the structure level or at the loan level. Structural call protection is achieved when CMBS are structured to have sequential-pay tranches, by credit rating. A lower-rated tranche cannot be paid down until the higher-rated tranche is completely retired, so the AAA rated bonds must be paid off before the AA rated bonds are, and so on. Principal losses resulting from defaults, however, are affected from the bottom of the structure upward.

At the loan level, four mechanisms offer investors call protection:

- A prepayment lockout, which is a contractual agreement that prohibits any prepayments during a specified period.
- Prepayment penalty points, which are predetermined penalties that a borrower who wants to refinance must pay to do so; a point is equal to 1% of the outstanding loan balance.
- A yield maintenance charge, also called a "make-whole charge," which is a penalty paid by the borrower that makes refinancing solely to get a lower mortgage rate uneconomical for

the borrower. In its simplest terms, a yield maintenance charge is designed to make the lender indifferent as to the timing of prepayments.

- Defeasance, for which the borrower provides sufficient funds for the servicer to invest in a portfolio of government securities that replicates the cash flows that would exist in the absence of prepayments. The cash payments that must be met by the borrower are projected based on the terms of the loan. Then, a portfolio of government securities is constructed in such a way that the interest payments and the principal repayments from the portfolio will be sufficient to pay off each obligation when it comes due. When the last obligation is paid off, the value of the portfolio is zero (that is, there are no funds remaining). The cost of assembling such a portfolio is the cost of defeasing the loan that must be repaid by the issuer.

7.2.2. Balloon Maturity Provision

Many commercial loans backing CMBS are balloon loans that require a substantial principal repayment at maturity of the loan. If the borrower fails to make the balloon payment, the borrower is in default. The lender may extend the loan over a period known as the "workout period." In doing so, the lender may modify the original terms of the loan and charge a higher interest rate, called the "default interest rate," during the workout period.

The risk that a borrower will not be able to make the balloon payment because either the borrower cannot arrange for refinancing or cannot sell the property to generate sufficient funds to pay off the outstanding principal balance is called "balloon risk." Because the life of the loan is extended by the lender during the workout period, balloon risk is a type of extension risk.

EXAMPLE 8 An Example of a Commercial Mortgage-Backed Security

The following information is taken from a filing with the US Securities and Exchange Commission about a CMBS issued by a special purpose entity established by a major US commercial bank. The collateral for this CMBS was a pool of 72 fixed-rate mortgages secured by first liens (first claims) on various types of commercial, multifamily, and manufactured housing community properties.

Classes of Offered Certificates	Initial Principal Amount (US$)	Initial Pass-Through Rate (%)
A-1	75,176,000	0.754
A-2	290,426,000	1.987
A-3	150,000,000	2.815
A-4	236,220,000	3.093

Classes of Offered Certificates	Initial Principal Amount (US$)	Initial Pass-Through Rate (%)
A-AB	92,911,000	2.690
X-A	948,816,000	1.937
A-S	104,083,000	3.422
B	75,423,000	3.732
C	42,236,000	

The filing included the following statements:

If you acquire Class B certificates, then your rights to receive distributions of amounts collected or advanced on or in respect of the mortgage loans will be subordinated to those of the holders of the Class A-1, Class A-2, Class A-3, Class A-4, Class A-AB, Class X-A, and Class A-S certificates. If you acquire Class C certificates, then your rights to receive distributions of amounts collected or advanced on or in respect of the mortgage loans will be subordinated to those of the holders of the Class B certificates and all other classes of offered certificates.

"Prepayment Penalty Description" or "Prepayment Provision" means the number of payments from the first due date through and including the maturity date for which a mortgage loan, as applicable, (i) is locked out from prepayment, (ii) provides for payment of a prepayment premium or yield maintenance charge in connection with a prepayment, (iii) or permits defeasance.

1. Based on the information provided, this CMBS:
   A. did not include any credit enhancement.
   B. included an internal credit enhancement.
   C. included an external credit enhancement.
2. Based on the information provided, investors in this CMBS had prepayment protection at:
   A. the loan level.
   B. the structure level.
   C. both the loan and structure levels.
3. Defeasance can be *best* described as:
   A. a predetermined penalty that a borrower who wants to refinance must pay to do so.
   B. a contractual agreement that prohibits any prepayments during a specified period of time.
   C. funds that the borrower must provide to replicate the cash flows that would exist in the absence of prepayments.
4. A risk that investors typically face when holding CMBS is:
   A. call risk.
   B. balloon risk.
   C. contraction risk.

5. The credit risk of a commercial mortgage-backed security is lower:
   A. the lower the DSC ratio and the lower the LTV.
   B. the lower the DSC ratio and the higher the LTV.
   C. the higher the DSC ratio and the lower the LTV.

Solution to 1: B is correct. The CMBS included a senior/subordinated structure, which is a form of internal credit enhancement. Class B provided protection for all of the bond classes listed above it. Similarly, Class C provided protection for all other bond classes, including Class B; it was the first-loss piece, also called the residual tranche or equity tranche. Note that because it was the residual tranche, Class C had no specific pass-through rate. Investors in Class C will have priced it on the basis of some expected residual rate of return, but they could have done better or worse than expected depending on how interest rate movements and default rates affected the performance of the other tranches.

Solution to 2: C is correct. This CMBS offered investors prepayment protection at both the structure and loan levels. The structural call protection was achieved thanks to the sequential-pay tranches. At the loan level, the CMBS included three of the four types of call protection—namely, a prepayment lockout, a yield maintenance charge, and defeasance.

Solution to 3: C is correct. Defeasance is a call protection at the loan level that requires the borrower to provide sufficient funds for the servicer to invest in a portfolio of government securities that replicates the cash flows that would exist in the absence of prepayments.

Solution to 4: B is correct. Because many commercial loans backing CMBS require a balloon payment, investors in CMBS typically face balloon risk—that is, the risk that if the borrower cannot arrange for refinancing or cannot sell the property to make the balloon payment, the CMBS may extend in maturity because the lender has to wait to obtain the outstanding principal until the borrower can make the balloon amount. Balloon risk is a type of extension risk.

Solution to 5: C is correct. The DSC ratio and the LTV are key indicators of potential credit performance and thus allow investors to assess the credit risk of a CMBS. The DSC ratio is equal to the property's annual NOI divided by the annual amount of interest payments and principal repayments. So the higher the DSC ratio, the lower the CMBS's credit risk. The LTV is equal to the amount of the mortgage divided by the property's value. So the lower the LTV, the lower the CMBS's credit risk.

To this point, this chapter has addressed the securitization of real estate property, both residential and commercial. We now discuss the securitization of debt obligations in which the underlying asset is not real estate.

8. NON-MORTGAGE ASSET-BACKED SECURITIES

- **describe types and characteristics of non-mortgage asset-backed securities, including the cash flows and risks of each type**

Numerous types of non-mortgage assets have been used as collateral in securitization. The largest non-mortgage assets in most countries are auto loan and lease receivables, credit card receivables, personal loans, and commercial loans. What is important to keep in mind is that, regardless of the type of asset, ABS that are not guaranteed by a government or a quasi-government entity are subject to credit risk.

ABS can be categorized based on the way the collateral repays—that is, whether the collateral is amortizing or non-amortizing. Traditional residential mortgages and auto loans are examples of amortizing loans. The cash flows for an amortizing loan include interest payments, scheduled principal repayments, and any prepayments, if permissible. If the loan has no schedule for paying down the principal, it is a non-amortizing loan. Because a non-amortizing loan does not involve scheduled principal repayments, an ABS backed by non-amortizing loans is not affected by prepayment risk. Credit card receivable ABS are an example of ABS backed by non-amortizing loans.

Consider an ABS backed by a pool of 1,000 amortizing loans with a total par value of US$100 million. Over time, some of the loans will be paid off; the amounts received from the scheduled principal repayment and any prepayments will be distributed to the bond classes based on the payment rule. Consequently, over time, the number of loans in the collateral will drop from 1,000 and the total par value will fall to less than US$100 million.

Now, what happens if the collateral of the ABS is 1,000 non-amortizing loans? Some of these loans will be paid off in whole or in part before the maturity of the ABS. When those loans are paid off, what happens depends on whether the loans were paid off during or after the lockout period. The lockout or revolving period is the period during which the principal repaid is reinvested to acquire additional loans with a principal equal to the principal repaid. The reinvestment in new loans can result in the collateral including more or less than 1,000 loans, but the loans will still have a total par value of US$100 million. When the lockout period is over, any principal that is repaid will not be used to reinvest in new loans but will instead be distributed to the bond classes.

This chapter cannot cover all types of non-mortgage ABS. It focuses on the two popular non-mortgage ABS in most countries: auto loan ABS and credit card receivable ABS.

8.1. Auto Loan ABS

Auto loan ABS are backed by auto loans and lease receivables. The focus in this section is on the largest type of auto securitizations—that is, auto loan-backed securities. In some countries, auto loan-backed securities represent the largest or second largest sector of the securitization market.

The cash flows for auto loan-backed securities consist of scheduled monthly payments (that is, interest payments and scheduled principal repayments) and any prepayments. For securities backed by auto loans, prepayments result from sales and trade-ins requiring full payoff of the loan, repossession and subsequent resale of autos, insurance proceeds received upon loss or destruction of autos, and early payoffs of the loans.

All auto loan-backed securities have some form of credit enhancement, often a senior/subordinated structure. In addition, many auto loan-backed securities come with overcollateralization and a reserve account, often an excess spread account. Recall from a previous chapter that the excess spread, sometimes called excess interest cash flow, is an amount that can be retained and deposited into a reserve account and that can serve as a first line of protection against losses.

To illustrate the typical structure of auto loan-backed securities, let us use the example of securities issued by Fideicomiso Financiero Autos VI. The collateral was a pool of 827 auto loans denominated in Argentine pesos (ARS). The loans were originated by BancoFinansur. The structure of the securitization included three bond classes:

Bond Class	Outstanding Principal Balance (ARS)
Class A Floating-Rate Debt Securities	22,706,000
Class B Floating-Rate Debt Securities	1,974,000
Certificates	6,008,581
Total	30,688,581

The certificates provided credit protection for Class B, and Class B provided credit protection for Class A. Further credit enhancement came from overcollateralization and the presence of an excess spread account. The reference rate for the floating-rate debt securities was BADLAR (Buenos Aires Deposits of Large Amount Rate), the benchmark rate for loans in Argentina. This reference rate is the average rate on 30-day deposits of at least ARS1 million. For Class A, the interest rate was BADLAR plus 450 bps, with a minimum rate of 18% and a maximum rate of 26%; for Class B, it was BADLAR plus 650 bps, with 20% and 28% as the minimum and maximum rates, respectively.

EXAMPLE 9 An Example of an Auto Loan ABS

The following information is from the prospectus supplement for US$877,670,000 of auto loan ABS issued by XYZ Credit Automobile Receivables Trust 2019:

> The collateral for this securitization is a pool of subprime automobile loan contracts secured for new and used automobiles and light-duty trucks and vans.
>
> The issuing entity will issue seven sequential-pay classes of asset-backed notes pursuant to the indenture. The notes are designated as the "Class A-1 Notes," the "Class A-2 Notes," the "Class A-3 Notes," the "Class B Notes," the "Class C Notes," the "Class D Notes," and the "Class E Notes." The Class A-1 Notes, the Class A-2 Notes, and the Class A-3 Notes are the "Class A Notes." The Class A Notes, the Class B Notes, the Class C Notes, and the Class D Notes are being offered by this prospectus supplement and are sometimes referred to as the publicly offered notes. The Class E Notes are not being offered by this prospectus supplement and will initially be retained by the depositor or an affiliate of the depositor. The Class E Notes are sometimes referred to as the privately placed notes.

Each class of notes will have the initial note principal balance, interest rate, and final scheduled distribution date listed in the following tables:

Publicly Offered Notes			
Class	Initial Note Principal Balance (US$)	Interest Rate (%)	Final Scheduled Distribution Date
A-1 (senior)	168,000,000	0.25	8 August 2020
A-2 (senior)	279,000,000	0.74	8 November 2022
A-3 (senior)	192,260,000	0.96	9 April 2024
B (subordinated)	68,870,000	1.66	10 September 2024
C (subordinated)	85,480,000	2.72	9 September 2025
D (subordinated)	84,060,000	3.31	8 October 2025

Privately Placed Notes			
Class	Initial Note Principal Balance (US$)	Interest Rate (%)	Final Scheduled Distribution Date
E (subordinated)	22,330,000	4.01	8 January 2027

Interest on each class of notes will accrue during each interest period at the applicable interest rate.

The overcollateralization amount represents the amount by which the aggregate principal balance of the automobile loan contracts exceeds the principal balance of the notes. On the closing date, the initial amount of overcollateralization is approximately US$49,868,074, or 5.25% of the aggregate principal balance of the automobile loan contracts as of the cutoff date.

On the closing date, 2.0% of the expected initial aggregate principal balance of the automobile loan contracts will be deposited into the reserve account, which is approximately US$18,997,361.

1. The reference to subprime meant that:
   A. the asset-backed notes were rated below investment grade.
   B. the automobile (auto) loan contracts were made to borrowers who did not have or could not document strong credit.
   C. some of the auto loan contracts were secured by autos of low quality that may have been difficult to sell in case the borrower defaults.
2. Based on the information provided, if on the first distribution date there were losses on the loans of US$10 million:
   A. none of the classes of notes will have incurred losses.
   B. Class E notes will have incurred losses of US$10 million.
   C. Classes B, C, D, and E will have incurred losses pro rata of their initial note principal balances.
3. Based on the information provided, if the first loss on the loans was US$40 million over and above the protection provided by the internal credit enhancements and occurred in January 2020, which class(es) of notes realized losses?

A. Class E and then Class D
B. Each class of subordinated notes in proportion to its principal balance
C. Class E and then each class of subordinated notes in proportion to its principal balance

Solution to 1: B is correct. A subprime loan is one granted to borrowers with lower credit quality, who have typically experienced prior credit difficulties or who cannot otherwise document strong credit.

Solution to 2: A is correct. The amount of the loss (US$10 million) was lower than the combined amount of overcollateralization and the reserve account (US$49,868,074 + US$18,997,361 = US$68,865,435). Therefore, none of the classes of notes will have incurred losses.

Solution to 3: A is correct. Once the amount of losses exceeds the amount of protection provided by the overcollateralization and the reserve account, losses are absorbed by the bond classes. Because it was a sequential-pay structure, Class E notes were the first ones to absorb losses, up to the principal amount of US$22,330,000. It meant that there was still US$17,670,000 to be absorbed by another bond class, which would have been the Class D notes.

8.2. Credit Card Receivable ABS

When a purchase is made on a credit card, the issuer of the credit card (the lender) extends credit to the cardholder (the borrower). Credit cards are issued by banks, credit card companies, retailers, and travel and entertainment companies. At the time of purchase, the cardholder agrees to repay the amount borrowed (that is, the cost of the item purchased) plus any applicable finance charges. The amount that the cardholder agrees to pay the issuer of the credit card is a receivable from the perspective of the issuer of the credit card. Credit card receivables are used as collateral for the issuance of credit card receivable ABS.

For a pool of credit card receivables, the cash flows consist of finance charges collected, fees, and principal repayments. Finance charges collected represent the periodic interest the credit card borrower is charged on the unpaid balance after the grace period, which may be fixed or floating. The floating rate may be capped; that is, it may have an upper limit because some countries have usury laws that impose a maximum interest rate. Fees include late payment fees and any annual membership fees.

Interest is paid to holders of credit card receivable ABS periodically (e.g., monthly, quarterly, or semiannually). As noted earlier, the collateral of credit card receivable ABS is a pool of non-amortizing loans. These loans have lockout periods during which the cash flows that are paid out to security holders are based only on finance charges collected and fees. When the lockout period is over, the principal that is repaid by the cardholders is no longer reinvested but instead is distributed to investors.

Some provisions in credit card receivable ABS require early principal amortization if specific events occur. Such provisions are referred to as "early amortization" or "rapid amortization" provisions and are included to safeguard the credit quality of the issue. The only way the principal cash flows can be altered is by the triggering of the early amortization provision.

To illustrate the typical structure of credit card receivable ABS, consider the Master Note Trust Series 20XX issued in March 20XX by a large, well-known financial institution. The originator of the credit card receivables was the issuer's retail bank, and the servicer was its finance company. The collateral was a pool of credit card receivables from several private-label and co-branded credit card issuers, including JCPenney, Lowe's Home Improvement, Sam's Club, Walmart, Gap, and Chevron. The structure of the US$969,085,000 securitization was as follows: Class A notes for US$800,000,000, Class B notes for US$100,946,373, and Class C notes for US$68,138,802. Thus, the issue had a senior/subordinate structure. The Class A notes were the senior notes and were rated Aaa by Moody's and AAA by Fitch. The Class B notes were rated A2 by Moody's and A+ by Fitch. The Class C notes were rated Baa2 by Moody's and BBB+ by Fitch.

EXAMPLE 10 Credit Card Receivable ABS vs. Auto Loan ABS

Credit card receivable asset-backed securities differ from auto loan ABS in the following way:

A. credit card loans are recourse loans, whereas auto loans are non-recourse loans.
B. the collateral for credit card receivable-backed securities is a pool of non-amortizing loans, whereas the collateral for auto loan ABS is a pool of amortizing loans.
C. credit card receivable-backed securities have regular principal repayments, whereas auto loan ABS include a lockout period during which the cash proceeds from principal repayments are reinvested in additional loan receivables.

Solution: B is correct. A main difference between credit card receivable ABS and auto loan ABS is the type of loans that back the securities. For credit card receivable ABS, the collateral is a pool of non-amortizing loans. During the lockout period, the cash proceeds from principal repayments are reinvested in additional credit card receivables. When the lockout period is over, principal repayments are used to pay off the outstanding principal. For auto loan-backed securities, the collateral is a pool of amortizing loans. Security holders receive regular principal repayments. As a result, the outstanding principal balance declines over time.

9. COLLATERALIZED DEBT OBLIGATIONS

- **describe collateralized debt obligations, including their cash flows and risks**

Collateralized debt obligation (CDO) is a generic term used to describe a security backed by a diversified pool of one or more debt obligations: CDOs backed by corporate and emerging market bonds are collateralized bond obligations (CBOs); CDOs backed by leveraged bank loans are collateralized loan obligations (CLOs); CDOs backed by ABS, RMBS, CMBS, and other CDOs are structured finance CDOs; CDOs backed by a portfolio of credit default swaps for other structured securities are synthetic CDOs.

9.1. CDO Structure

A CDO involves the creation of an SPE. In a CDO, there is a need for a CDO manager, also called "**collateral manager**," to buy and sell debt obligations for and from the CDO's collateral (that is, the portfolio of assets) to generate sufficient cash flows to meet the obligations to the CDO bondholders.

The funds to purchase the collateral assets for a CDO are obtained from the issuance of debt obligations. These debt obligations are bond classes or tranches and include senior bond classes, mezzanine bond classes (that is, bond classes with credit ratings between senior and subordinated bond classes), and subordinated bond classes, often referred to as the residual or equity tranches. The motivation for investors to invest in senior or mezzanine bond classes is to earn a potentially higher yield than that on a comparably rated corporate bond by gaining exposure to debt products that they may not otherwise be able to purchase. Investors in equity tranches have the potential to earn an equity-type return, thereby offsetting the increased risk from investing in the subordinated class. The key to whether a CDO is viable depends upon whether a structure can be created that offers a competitive return for the subordinated tranche.

The basic economics of the CDO is that the funds are raised by the sale of the bond classes and the CDO manager invests those funds in assets. The CDO manager seeks to earn a rate of return higher than the aggregate cost of the bond classes. The return in excess of what is paid out to the bond classes accrues to the holders of the equity tranche and to the CDO manager. In other words, a CDO is a leveraged transaction in which those who invest in the equity tranche use borrowed funds (the bond classes issued) to generate a return above the funding cost.

As with ABS, each CDO bond class is structured to provide a specific level of risk for investors. CDOs impose restrictions on the CDO manager via various tests and limits that must be satisfied for the CDO to meet investors' varying risk appetites while still providing adequate protection for the senior bond class. If the CDO manager fails pre-specified tests, a provision is triggered that requires the payoff of the principal to the senior bond class until the tests are satisfied. This process effectively deleverages the CDO because the cheapest funding source for the CDO, the senior bond class, is reduced.

The ability of the CDO manager to make the interest payments and principal repayments depends on collateral performance. The proceeds to meet the obligations to the CDO bond classes can come from one or more of the following sources: interest payments from collateral assets, maturing of collateral assets, and sale of collateral assets. The cash flows and credit risks of CDOs are best illustrated by an example.

9.2. An Example of a CDO Transaction

Although various motivations may prompt a sponsor to create a CDO, the following example uses a CDO for which the purpose is to capture what market participants mistakenly label a CDO arbitrage transaction. The term "arbitrage" is not used here in the traditional sense—that is, a risk-free transaction that earns an expected positive net profit but requires no net investment of money. In this context, arbitrage is used in a loose sense to describe a transaction in which the motivation is to capture a spread between the potential collateral return and the funding cost.

To understand the structure of a CDO transaction and its risks, consider the following US$100 million issue:

Tranche	Par Value (US$ million)	Coupon Rate
Senior	80	MRR[a] + 70 bps
Mezzanine	10	10-year US Treasury rate + 200 bps
Equity	10	—

[a] MRR is the Market Reference Rate.

Suppose that the collateral consists of bonds that all mature in 10 years and that the coupon rate for every bond is the 10-year US Treasury rate plus 400 bps. Because the collateral pays a fixed rate (the 10-year US Treasury rate plus 400 bps) but the senior tranche requires a floating-rate payment (MRR plus 70 bps), the CDO manager enters into an interest rate swap agreement with another party. This interest rate swap is simply an agreement to periodically exchange fixed for floating interest payments based on a notional amount used to determine the interest payment for each party. The notional amount of the interest rate swap is the par value of the senior tranche—that is, US$80 million in this example. Let us suppose that through the interest rate swap, the CDO manager agrees to do the following: (1) Pay a fixed rate each year equal to the 10-year US Treasury rate plus 100 bps, and (2) receive MRR.

Assume that the 10-year US Treasury rate at the time this CDO is issued is 7%. Now, consider the annual cash flow for the first year. First, let us look at the collateral. Assuming no default, the collateral will pay an interest rate equal to the 10-year US Treasury rate of 7% plus 400 bps—that is, 11%. So, the interest payment is 11% × US$100,000,000 = US$11,000,000.

Now, let us determine the interest that must be paid to the senior and mezzanine tranches. For the senior tranche, the interest payment is US$80,000,000 × (MRR + 70 bps). For the mezzanine tranche, the coupon rate is the 10-year US Treasury rate plus 200 bps—that is, 9%. So, the interest payment for the mezzanine tranche is 9% × US$10,000,000 = US$900,000.

Finally, consider the interest rate swap. In this agreement, the CDO manager agreed to pay the swap counterparty the 10-year US Treasury rate plus 100 bps—that is, 8%—based on a notional amount of US$80 million. So, the amount paid to the swap counterparty the first year is 8% × US$80,000,000 = US$6,400,000. The amount received from the swap counterparty is MRR based on a notional amount of US$80 million—that is, MRR × US$80,000,000.

Combining this information, the CDO cash inflows are as follows:

Interest from collateral	$11,000,000
Interest from swap counterparty	$80,000,000 × MRR
Total interest received	$11,000,000 + $80,000,000 × MRR

The cash outflows for the CDO are as follows:

Interest to senior tranche	$80,000,000 × (MRR + 70 bps)
Interest to mezzanine tranche	$900,000
Interest to swap counterparty	$6,400,000
Total interest paid	$7,300,000 + $80,000,000 × (MRR + 70 bps)

Netting the total interest received ($11,000,000 + $80,000,000 × MRR) and the total interest paid—$7,300,000 + $80,000,000 × (MRR + 70 bps)—leaves a net interest of $3,700,000 – $80,000,000 × 70 bps = US$3,140,000. From this amount, any fees—including the CDO manager's fees—must be paid. The balance is then the amount available to pay the equity tranche. Suppose the CDO manager's fees are US$640,000. The cash flow available to the equity tranche for the first year is US$2.5 million ($3,140,000 – $640,000). Because the equity tranche has a par value of US$10 million and is assumed to be sold at par, the annual return is 25%.

This example includes some simplifying assumptions. For instance, it is assumed that no defaults would occur. Furthermore, it is assumed that all securities purchased by the CDO manager are non-callable and, thus, that the coupon rate would not decline because of securities being called. Despite these simplifying assumptions, the example demonstrates the economics of an arbitrage CDO transaction, the need for an interest rate swap, and how the equity tranche realizes a return.

In practice, CDOs are subject to additional risks. For example, in the case of collateral defaults, the manager may fail to earn a return sufficient to pay off the investors in the senior and mezzanine tranches, resulting in a loss. Equity tranche investors may lose their entire investment. Even if payments are made to these investors, the realized return may be below the return expected at the time of purchase.

Moreover, after some period, the CDO manager must begin repaying principal to the senior and mezzanine tranches. The interest rate swap must be structured to take this requirement into account because the entire amount of the senior tranche is not outstanding for the life of the collateral.

EXAMPLE 11 Collateralized Debt Obligations

An additional risk of an investment in an arbitrage collateralized debt obligation relative to an investment in an asset-backed security is:

A. the default risk on the collateral assets.
B. the risk that the CDO manager will fail to earn a return sufficient to pay off the investors in the senior and the mezzanine tranches.
C. the risk due to the mismatch between the collateral making fixed-rate payments and the bond classes making floating-rate payments.

Solution: B is correct. In addition to the risks associated with investments in ABS, such as the default risk on the collateral assets and the risk due to the potential mismatch between the collateral making fixed-rate payments and the bond classes making floating-rate payments, investors in CDOs face the risk that the CDO manager will fail to earn a return sufficient to pay off the investors in the senior and the mezzanine tranches. With an ABS, the cash flows from the collateral are used to pay off the holders of the bond classes without the active management of the collateral—that is, without a manager altering the composition of the debt obligations in the pool that is backing the securitization. In contrast, in an arbitrage CDO, a CDO manager buys and sells debt obligations with the dual purpose of not only paying off the holders of the bond classes but also generating an attractive/competitive return for the equity tranche and for the manager.

10. COVERED BONDS

- **describe characteristics and risks of covered bonds and how they differ from other asset-backed securities**

As outlined in an earlier chapter, covered bonds are senior debt obligations issued by a financial institution and backed by a segregated pool of assets that typically consist of commercial or residential mortgages or public sector assets. The covered bond (or *Pfandbrief* in German) originated over 250 years ago in Prussia and has been adopted by issuers in Europe, Asia, and Australia. Each country or jurisdiction specifies eligible collateral and structures permissible in the covered bond market, and the European Union is taking steps to harmonize these elements among its member states.

Covered bonds are similar to ABS but offer bondholders dual recourse—that is, to both the issuing financial institution and the underlying asset pool. In the case of ABS, the financial institution that originates loans transfers securitized assets to a bankruptcy-remote special legal entity, but the pool of underlying assets in a covered bond (or "cover pool") remains on the financial institution's balance sheet, against which covered bondholders retain a top-priority claim.

Whereas ABS often use credit tranching to create bond classes with different borrower default exposures, covered bonds usually consist of one bond class per cover pool. Another difference lies in the dynamic nature of the cover pool. In contrast to a static pool of mortgage loans that expose investors to prepayment risk as in the case of US mortgage-backed securities, cover pool sponsors must replace any prepaid or non-performing assets (i.e., assets that do not generate the promised cash flows) in the cover pool to ensure sufficient cash flows until the maturity of the covered bond.

Redemption regimes exist to align the covered bond's cash flows as closely as possible with the original maturity schedule in the event of default of a covered bond's financial sponsor. For example, in the case of **hard-bullet covered bonds**, if payments do not occur according to the original schedule, a bond default is triggered and bond payments are accelerated. **Soft-bullet covered bonds** delay the bond default and payment acceleration of bond cash flows until a new final maturity date, which is usually up to a year after the original maturity date. **Conditional pass-through covered bonds**, in contrast, convert to pass-through securities after the original maturity date if all bond payments have not yet been made.

Covered bonds have remained a relatively stable and reliable source of funding over time because of their dual recourse nature, strict eligibility criteria, dynamic cover pool, and redemption regimes in the event of sponsor default. As a result, covered bonds usually carry lower credit risks and offer lower yields than otherwise similar ABS.

EXAMPLE 12 Covered Bonds

Which of the following statements about covered bonds and asset-backed securities is *most* accurate?

A. Both covered bonds and ABS pass prepayment and extension risk of the underlying asset pool through to investors.
B. Both covered bonds and ABS offer investors recourse to both the bond's issuer and the underlying asset pool.
C. Covered bonds have a dynamic cover pool in which sponsors must replace prepaid or non-performing assets, whereas ABS, such as mortgage-backed securities, pass through default and prepayment risk to investors.

Solution: C is correct. In contrast to a static pool of mortgage loans that expose investors to prepayment risk as in the case of US mortgage-backed securities, cover pool sponsors must replace any prepaid or non-performing assets in the cover pool to ensure sufficient cash flows until the covered bond matures. A is incorrect because the dynamic cover pool insulates investors from prepayment risk, whereas the static mortgage loan pool does not protect ABS investors from prepayment risk. B is incorrect since only covered bonds offer investors recourse to both the bond's issuer and the underlying asset pool.

SUMMARY

- Securitization involves pooling debt obligations, such as loans or receivables, and creating securities backed by the pool of debt obligations called asset-backed securities (ABS). The cash flows of the debt obligations are used to make interest payments and principal repayments to the holders of the ABS.
- Securitization has several benefits. It allows investors direct access to liquid investments and payment streams that would be unattainable if all the financing were performed through banks. It enables banks to increase loan originations at economic scales greater than if they used only their own in-house loan portfolios. Thus, securitization contributes to lower costs of borrowing for entities raising funds, higher risk-adjusted returns to investors, and greater efficiency and profitability for the banking sector.
- The parties to a securitization include the seller of the collateral (pool of loans), the servicer of the loans, and the special purpose entity (SPE). The SPE is bankruptcy remote, which plays a pivotal role in the securitization.
- A common structure in a securitization is subordination, which leads to the creation of more than one bond class or tranche. Bond classes differ as to how they will share any losses resulting from defaults of the borrowers whose loans are in the collateral. The credit ratings assigned to the various bond classes depend on how the credit-rating agencies evaluate the credit risks of the collateral and any credit enhancements.
- The motivation for the creation of different types of structures is to redistribute prepayment risk and credit risk efficiently among different bond classes in the securitization. Prepayment risk is the uncertainty that the actual cash flows will be different from the scheduled cash flows as set forth in the loan agreements because borrowers may choose to repay the principal early to take advantage of interest rate movements.
- Because of the SPE, the securitization of a company's assets may include some bond classes that have better credit ratings than the company itself or its corporate bonds. Thus, the company's funding cost is often lower when raising funds through securitization than when issuing corporate bonds.
- A mortgage is a loan secured by the collateral of some specified real estate property that obliges the borrower to make a predetermined series of payments to the lender. The cash flow of a mortgage includes (1) interest, (2) scheduled principal payments, and (3) prepayments (any principal repaid in excess of the scheduled principal payment).
- The various mortgage designs throughout the world specify (1) the maturity of the loan; (2) how the interest rate is determined (i.e., fixed rate versus adjustable or variable rate); (3) how the principal is repaid (i.e., whether the loan is amortizing and if it is, whether it is fully amortizing or partially amortizing with a balloon payment); (4) whether the

borrower has the option to prepay and if so, whether any prepayment penalties might be imposed; and (5) the rights of the lender in a foreclosure (i.e., whether the loan is a recourse or non-recourse loan).

- In the United States, there are three sectors for securities backed by residential mortgages: (1) those guaranteed by a federal agency (Ginnie Mae) whose securities are backed by the full faith and credit of the US government, (2) those guaranteed by a GSE (e.g., Fannie Mae and Freddie Mac) but not by the US government, and (3) those issued by private entities that are not guaranteed by a federal agency or a GSE. The first two sectors are referred to as agency residential mortgage-backed securities (RMBS), and the third sector, as non-agency RMBS.
- A mortgage pass-through security is created when one or more holders of mortgages form a pool of mortgages and sell shares or participation certificates in the pool. The cash flow of a mortgage pass-through security depends on the cash flow of the underlying pool of mortgages and consists of monthly mortgage payments representing interest, the scheduled repayment of principal, and any prepayments, net of servicing and other administrative fees.
- Market participants measure the prepayment rate using two measures: the single monthly mortality rate (SMM) and its corresponding annualized rate—namely, the conditional prepayment rate (CPR). For MBS, a measure widely used by market participants to assess effective duration is the weighted average life or simply the average life of the MBS.
- Market participants use the Public Securities Association (PSA) prepayment benchmark to describe prepayment rates. A PSA assumption greater than 100 PSA means that prepayments are assumed to occur faster than the benchmark, whereas a PSA assumption lower than 100 PSA means that prepayments are assumed to occur slower than the benchmark.
- Prepayment risk includes two components: contraction risk and extension risk. The former is the risk that when interest rates decline, the security will have a shorter maturity than was anticipated at the time of purchase because homeowners will refinance at the new, lower interest rates. The latter is the risk that when interest rates rise, fewer prepayments will occur than what was anticipated at the time of purchase because homeowners are reluctant to give up the benefits of a contractual interest rate that now looks low.
- The creation of a collateralized mortgage obligation (CMO) can help manage prepayment risk by distributing the various forms of prepayment risk among different classes of bondholders. The CMO's major financial innovation is that the securities created more closely satisfy the asset/liability needs of institutional investors, thereby broadening the appeal of mortgage-backed products.
- The most common types of CMO tranche are sequential-pay tranches, planned amortization class (PAC) tranches, support tranches, and floating-rate tranches.
- Non-agency RMBS share many features and structuring techniques with agency CMOs. However, they typically include two complementary mechanisms. First, the cash flows are distributed by rules that dictate the allocation of interest payments and principal repayments to tranches with various degrees of priority/seniority. Second, there are rules for the allocation of realized losses that specify that subordinated bond classes have lower payment priority than senior classes.
- In order to obtain favorable credit ratings, non-agency RMBS and non-mortgage ABS often require one or more credit enhancements. The most common forms of internal credit enhancement are senior/subordinated structures, reserve funds, and overcollateralization. In external credit enhancement, credit support in the case of defaults resulting in losses in the pool of loans is provided in the form of a financial guarantee by a third party to the transaction.

- Commercial mortgage-backed securities (CMBS) are securities backed by a pool of commercial mortgages on income-producing property.
- Two key indicators of the potential credit performance of CMBS are the debt-service-coverage (DSC) ratio and the loan-to-value ratio (LTV). The DSC ratio is the property's annual net operating income divided by the debt service.
- CMBS have considerable call protection, which allows CMBS to trade in the market more like corporate bonds than like RMBS. This call protection comes in two forms: at the structure level and at the loan level. The creation of sequential-pay tranches is an example of call protection at the structure level. At the loan level, four mechanisms offer investors call protection: prepayment lockouts, prepayment penalty points, yield maintenance charges, and defeasance.
- ABS are backed by a wide range of asset types. The most popular non-mortgage ABS are auto loan ABS and credit card receivable ABS. The collateral is amortizing for auto loan ABS and non-amortizing for credit card receivable ABS. As with non-agency RMBS, these ABS must offer credit enhancement to be appealing to investors.
- A collateralized debt obligation (CDO) is a generic term used to describe a security backed by a diversified pool of one or more debt obligations (e.g., corporate and emerging market bonds, leveraged bank loans, ABS, RMBS, and CMBS).
- A CDO involves the creation of an SPE. The funds necessary to pay the bond classes come from a pool of loans that must be serviced. A CDO requires a collateral manager to buy and sell debt obligations for and from the CDO's portfolio of assets to generate sufficient cash flows to meet the obligations of the CDO bondholders and to generate a fair return for the equityholders.
- The structure of a CDO includes senior, mezzanine, and subordinated/equity bond classes.
- Covered bonds are similar to ABS, but they differ because of their dual recourse nature, strict eligibility criteria, dynamic cover pool, and redemption regimes in the event of sponsor default.

PRACTICE PROBLEMS

1. Securitization is beneficial for banks because it:
   A. repackages bank loans into simpler structures.
   B. increases the funds available for banks to lend.
   C. allows banks to maintain ownership of their securitized assets.
2. Securitization benefits financial markets by:
   A. increasing the role of intermediaries.
   B. establishing a barrier between investors and originating borrowers.
   C. allowing investors to tailor credit risk and interest rate risk exposures to meet their individual needs.
3. A benefit of securitization is the:
   A. reduction in disintermediation.
   B. simplification of debt obligations.
   C. creation of tradable securities with greater liquidity than the original loans.
4. Securitization benefits investors by:
   A. providing more direct access to a wider range of assets.
   B. reducing the inherent credit risk of pools of loans and receivables.
   C. eliminating cash flow timing risks of an ABS, such as contraction and extension risks.

5. In a securitization, the special purpose entity (SPE) is responsible for the:
   A. issuance of the asset-backed securities.
   B. collection of payments from the borrowers.
   C. recovery of underlying assets from delinquent borrowers.
6. In a securitization, the collateral is initially sold by the:
   A. issuer.
   B. depositor.
   C. underwriter.
7. A special purpose entity issues asset-backed securities in the following structure.

Bond Class	Par Value (€ millions)
A (senior)	200
B (subordinated)	20
C (subordinated)	5

   At which of the following amounts of default in par value would Bond Class A experience a loss?
   A. €20 million
   B. €25 million
   C. €26 million
8. In a securitization, time tranching provides investors with the ability to choose between:
   A. extension and contraction risks.
   B. senior and subordinated bond classes.
   C. fully amortizing and partially amortizing loans.
9. The creation of bond classes with a waterfall structure for sharing losses is referred to as:
   A. time tranching.
   B. credit tranching.
   C. overcollateralization.
10. Which of the following statements related to securitization is correct?
    A. Time tranching addresses the uncertainty of a decline in interest rates.
    B. Securitizations are rarely structured to include both credit tranching and time tranching.
    C. Junior and senior bond classes differ in that junior classes can be paid off only at the bond's set maturity.
11. A goal of securitization is to:
    A. separate the seller's collateral from its credit ratings.
    B. uphold the absolute priority rule in bankruptcy reorganizations.
    C. account for collateral's primary influence on corporate bond credit spreads.
12. The last payment in a partially amortizing residential mortgage loan is *best* referred to as a:
    A. waterfall.
    B. principal repayment.
    C. balloon payment.
13. If a mortgage borrower makes prepayments without penalty to take advantage of falling interest rates, the lender will *most likely* experience:
    A. extension risk.
    B. contraction risk.
    C. yield maintenance.

14. Which of the following characteristics of a residential mortgage loan would *best* protect the lender from a strategic default by the borrower?
    A. Recourse
    B. A prepayment option
    C. Interest-only payments
15. William Marolf obtains a EUR5 million mortgage loan from Bank Nederlandse. A year later, the principal on the loan is EUR4 million and Marolf defaults on the loan. Bank Nederlandse forecloses, sells the property for EUR2.5 million, and is entitled to collect the EUR1.5 million shortfall from Marolf. Marolf *most likely* had a:
    A. bullet loan.
    B. recourse loan.
    C. non-recourse loan.
16. Fran Martin obtains a non-recourse mortgage loan for $500,000. One year later, when the outstanding balance of the mortgage is $490,000, Martin cannot make his mortgage payments and defaults on the loan. The lender forecloses on the loan and sells the house for $315,000. What amount is the lender entitled to claim from Martin?
    A. $0.
    B. $175,000.
    C. $185,000.
17. A balloon payment equal to a mortgage's original loan amount is a characteristic of a:
    A. bullet mortgage.
    B. fully amortizing mortgage.
    C. partially amortizing mortgage.
18. Which of the following statements is correct concerning mortgage loan defaults?
    A. A non-recourse jurisdiction poses higher default risks for lenders.
    B. In a non-recourse jurisdiction, strategic default will not affect the defaulting borrower's future access to credit.
    C. When a recourse loan defaults, the mortgaged property is the lender's sole source for recovery of the outstanding mortgage balance.
19. Which of the following describes a typical feature of a non-agency residential mortgage-backed security (RMBS)?
    A. Senior/subordinated structure
    B. A pool of conforming mortgages as collateral
    C. A guarantee by a government-sponsored enterprise
20. If interest rates increase, an investor who owns a mortgage pass-through security is *most likely* affected by:
    A. credit risk.
    B. extension risk.
    C. contraction risk.
21. Which of the following is *most likely* an advantage of collateralized mortgage obligations (CMOs)? CMOs can
    A. eliminate prepayment risk.
    B. be created directly from a pool of mortgage loans.
    C. meet the asset/liability requirements of institutional investors.
22. The longest-term tranche of a sequential-pay CMO is *most likely* to have the lowest:
    A. average life.
    B. extension risk.
    C. contraction risk.

23. The tranches in a collateralized mortgage obligation that are *most likely* to provide protection for investors against both extension and contraction risk are:
    A. planned amortization class (PAC) tranches.
    B. support tranches.
    C. sequential-pay tranches.
24. Support tranches are *most* appropriate for investors who are:
    A. concerned about their exposure to extension risk.
    B. concerned about their exposure to concentration risk.
    C. willing to accept prepayment risk in exchange for higher returns.
25. In the context of mortgage-backed securities, a conditional prepayment rate (CPR) of 8% means that approximately 8% of the outstanding mortgage pool balance at the beginning of the year is expected to be prepaid:
    A. in the current month.
    B. by the end of the year.
    C. over the life of the mortgages.
26. For a mortgage pass-through security, which of the following risks *most likely* increases as interest rates decline?
    A. Balloon
    B. Extension
    C. Contraction
27. Compared with the weighted average coupon rate of its underlying pool of mortgages, the pass-through rate on a mortgage pass-through security is:
    A. lower.
    B. the same.
    C. higher.
28. The single monthly mortality rate (SMM) *most likely:*
    A. increases as extension risk rises.
    B. decreases as contraction risk falls.
    C. stays fixed over time when the standard prepayment model remains at 100 PSA.
29. Credit risk is an important consideration for commercial mortgage-backed securities (CMBS) if the CMBS are backed by mortgage loans that:
    A. are non-recourse.
    B. have call protection.
    C. have prepayment penalty points.
30. Which commercial mortgage-backed security characteristic causes a CMBS to trade more like a corporate bond than a residential mortgage-backed security?
    A. Call protection
    B. Internal credit enhancement
    C. Debt-service-coverage ratio level
31. A commercial mortgage-backed security does not meet the debt-to-service coverage at the loan level necessary to achieve a desired credit rating. Which of the following features would *most likely* improve the credit rating of the CMBS?
    A. Subordination
    B. Call protection
    C. Balloon payments

32. If a default occurs in a non-recourse commercial mortgage-backed security, the lender will *most likely*:
    A. recover prepayment penalty points paid by the borrower to offset losses.
    B. use only the proceeds received from the sale of the property to recover losses.
    C. initiate a claim against the borrower for any shortfall resulting from the sale of the property.
33. Which of the following investments is least subject to prepayment risk?
    A. Auto loan receivable–backed securities
    B. Commercial mortgage-backed securities
    C. Non-agency residential mortgage-backed securities
34. An excess spread account incorporated into a securitization is designed to limit:
    A. credit risk.
    B. extension risk.
    C. contraction risk.
35. Which of the following *best* describes the cash flow that owners of credit card receivable asset-backed securities receive during the lockout period?
    A. No cash flow
    B. Only principal payments collected
    C. Only finance charges collected and fees
36. Which type of asset-backed security is not affected by prepayment risk?
    A. Auto loan ABS
    B. Residential MBS
    C. Credit card receivable ABS
37. In auto loan ABS, the form of credit enhancement that *most likely* serves as the first line of loss protection is the:
    A. excess spread account.
    B. sequential-pay structure.
    C. proceeds from repossession sales.
38. In credit card receivable ABS, principal cash flows can be altered only when the:
    A. lockout period expires.
    B. excess spread account is depleted.
    C. early amortization provision is triggered.
39. The CDO tranche with a credit-rating status between senior and subordinated bond classes is called the:
    A. equity tranche.
    B. residual tranche.
    C. mezzanine tranche.
40. The key to a CDO's viability is the creation of a structure with a competitive return for the:
    A. senior tranche.
    B. mezzanine tranche.
    C. subordinated tranche.
41. When the collateral manager fails pre-specified risk tests, a CDO is:
    A. deleveraged by reducing the senior bond class.
    B. restructured to reduce its most expensive funding source.
    C. liquidated by paying off the bond classes in order of seniority.

42. Collateralized mortgage obligations are designed to:
    A. eliminate contraction risk in support tranches.
    B. distribute prepayment risk to various tranches.
    C. eliminate extension risk in planned amortization tranches.
43. Which statement about covered bonds is *least* accurate?
    A. Covered bonds provide investors with dual recourse, to the cover pool and also to the issuer.
    B. Covered bonds usually carry higher credit risks and offer higher yields than otherwise similar ABS.
    C. Covered bonds have a dynamic cover pool, meaning sponsors must replace any prepaid or non-performing assets.

CHAPTER 5

UNDERSTANDING FIXED-INCOME RISK AND RETURN

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LEARNING OUTCOMES

The candidate should be able to:

- calculate and interpret the sources of return from investing in a fixed-rate bond;
- define, calculate, and interpret Macaulay, modified, and effective durations;
- explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options;
- define key rate duration and describe the use of key rate durations in measuring the sensitivity of bonds to changes in the shape of the benchmark yield curve;
- explain how a bond's maturity, coupon, and yield level affect its interest rate risk;
- calculate the duration of a portfolio and explain the limitations of portfolio duration;
- calculate and interpret the money duration of a bond and price value of a basis point (PVBP);
- calculate and interpret approximate convexity and compare approximate and effective convexity;
- calculate the percentage price change of a bond for a specified change in yield, given the bond's approximate duration and convexity;
- describe how the term structure of yield volatility affects the interest rate risk of a bond;
- describe the relationships among a bond's holding period return, its duration, and the investment horizon;
- explain how changes in credit spread and liquidity affect yield-to-maturity of a bond and how duration and convexity can be used to estimate the price effect of the changes.
- describe the difference between empirical duration and analytical duration.

1. INTRODUCTION

Successful analysts must develop a solid understanding of the risk and return characteristics of fixed-income investments. Beyond the vast global market for public and private fixed-rate bonds, many financial assets and liabilities with known future cash flows you will encounter throughout your career are evaluated using similar principles. This analysis starts with the yield-to-maturity, or internal rate of return on future cash flows, introduced in the fixed-income valuation chapter. Fixed-rate bond returns are affected by many factors, the most important of which is the full receipt of all interest and principal payments on scheduled dates. Assuming no default, return is also affected by interest rate changes that affect coupon reinvestment and the bond price if it is sold prior to maturity. Price change measures may be derived from the mathematical relationship used to calculate a bond's price. Specifically, duration estimates the price change for a given change in interest rates, and convexity improves on the duration estimate by considering that the price and yield-to-maturity relationship of a fixed-rate bond is non-linear.

Sources of return on a fixed-rate bond investment include the receipt and reinvestment of coupon payments and either the redemption of principal if the bond is held to maturity or capital gains (or losses) if the bond is sold earlier. Fixed-income investors holding the same bond may have different interest rate risk exposures if their investment horizons differ.

We introduce bond duration and convexity, showing how these statistics are calculated and used as interest rate risk measures. Although procedures and formulas exist to calculate duration and convexity, these statistics can be approximated using basic bond-pricing techniques and a financial calculator. Commonly used versions of the statistics are covered, including Macaulay, modified, effective, and key rate durations, and we distinguish between risk measures based on changes in the bond's yield-to-maturity (i.e., *yield* duration and convexity) and on benchmark yield curve changes (i.e., *curve* duration and convexity).

We then return to the investment time horizon. When an investor has a short-term horizon, duration and convexity are used to estimate the change in the bond price. Note that yield volatility matters, because bonds with varying times-to-maturity have different degrees of yield volatility. When an investor has a long-term horizon, the interaction between coupon reinvestment risk and market price risk matters. The relationship among interest rate risk, bond duration, and the investment horizon is explored.

Finally, we discuss how duration and convexity may be extended to credit and liquidity risks and highlight how these factors can affect a bond's return and risk. In addition, we highlight the use of statistical methods and historical data to establish empirical as opposed to analytical duration estimates.

2. SOURCES OF RETURN

- **calculate and interpret the sources of return from investing in a fixed-rate bond**

Fixed-rate bond investors have three sources of return: (1) receipt of promised coupon and principal payments on the scheduled dates, (2) reinvestment of coupon payments, and (3) potential capital gains or losses on the sale of the bond prior to maturity. In this section, it is assumed that the issuer makes the coupon and principal payments as scheduled. Here, the focus is primarily on how interest rate changes affect the reinvestment of coupon payments and a bond's market price if sold prior to maturity. Credit risk is considered later and is also the primary subject of a subsequent chapter.

When a bond is purchased at a premium or a discount, it adds another aspect to the rate of return. Recall from the fixed-income valuation chapter that a discount bond offers the investor a "deficient" coupon rate below the market discount rate. The amortization of this discount in each period brings the return in line with the market discount rate as the bond's carrying value is "pulled to par." For a premium bond, the coupon rate exceeds the market discount rate and the amortization of the premium adjusts the return to match the market discount rate. Through amortization, the bond's carrying value reaches par value at maturity.

A series of examples will demonstrate the effect of a change in interest rates on two investors' realized rate of returns. Interest rates are the rates at which coupon payments are reinvested and the market discount rates at the time of purchase and at the time of sale if the bond is not held to maturity. In Examples 1 and 2, interest rates are unchanged. The two investors, however, have different time horizons for holding the bond. Examples 3 and 4 show the impact of higher interest rates on the two investors' total return. Examples 5 and 6 show the impact of lower interest rates. In each of the six examples, an investor initially buys a 10-year, 8% annual coupon payment bond at a price of 85.503075 per 100 of par value. The bond's yield-to-maturity is 10.40%.

$$85.503075 = \frac{8}{(1+r)^1} + \frac{8}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{8}{(1+r)^4} + \frac{8}{(1+r)^5} + \frac{8}{(1+r)^6} + \frac{8}{(1+r)^7} + \frac{8}{(1+r)^8} + \frac{8}{(1+r)^9} + \frac{108}{(1+r)^{10}}$$

$$r = 0.1040$$

EXAMPLE 1

A "buy-and-hold" investor purchases a 10-year, 8% annual coupon payment bond at 85.503075 per 100 of par value and holds it until maturity. The investor receives the series of 10 coupon payments of 8 (per 100 of par value) for a total of 80, plus the redemption of principal (100) at maturity. In addition to collecting the coupon interest and the principal, the investor may reinvest the cash flows. If the coupon payments are reinvested at 10.40%, the future value of the coupons on the bond's maturity date is 129.970678 per 100 of par value.

$$[8 \times (1.1040)^9] + [8 \times (1.1040)^8] + [8 \times (1.1040)^7] + [8 \times (1.1040)^6] + [8 \times (1.1040)^5] + [8 \times (1.1040)^4] + [8 \times (1.1040)^3] + [8 \times (1.1040)^2] + [8 \times (1.1040)^1] + 8 = 129.970678$$

The first coupon payment of 8 is reinvested at 10.40% for nine years until maturity, the second is reinvested for eight years, and so forth. The future value of the annuity is obtained easily on a financial calculator, using 8 for the payment that is received at the end of each of the 10 periods. The amount in excess of the coupons, 49.970678 (= 129.970678 − 80), is the "interest-on-interest" gain from compounding.

The investor's total return is 229.970678, the sum of the reinvested coupons (129.970678) and the redemption of principal at maturity (100). The realized rate of return is 10.40%.

$$85.503075 = \frac{229.970678}{(1 + r)^{10}}, \quad r = 0.1040$$

Example 1 demonstrates that the yield-to-maturity at the time of purchase measures the investor's rate of return under three assumptions: (1) The investor holds the bond to maturity, (2) there is no default by the issuer, and (3) the coupon interest payments are reinvested at that same rate of interest.

Example 2 considers another investor who buys the 10-year, 8% annual coupon payment bond and pays the same price. This investor, however, has a four-year investment horizon. Therefore, coupon interest is only reinvested for four years, and the bond is sold immediately after receiving the fourth coupon payment.

EXAMPLE 2

A second investor buys the 10-year, 8% annual coupon payment bond and sells the bond after four years. Assuming that the coupon payments are reinvested at 10.40% for four years, the future value of the reinvested coupons is 37.347111 per 100 of par value.

$$[8 \times (1.1040)^3] + [8 \times (1.1040)^2] + [8 \times (1.1040)^1] + 8 = 37.347111$$

The interest-on-interest gain from compounding is 5.347111 (= 37.347111 − 32). After four years, when the bond is sold, it has six years remaining until maturity. If the yield-to-maturity remains 10.40%, the sale price of the bond is 89.668770.

$$\frac{8}{(1.1040)^1} + \frac{8}{(1.1040)^2} + \frac{8}{(1.1040)^3} + \frac{8}{(1.1040)^4} + \frac{8}{(1.1040)^5} + \frac{8}{(1.1040)^6} = 89.668770$$

The total return is 127.015881 (= 37.347111 + 89.668770), and the realized rate of return is 10.40%.

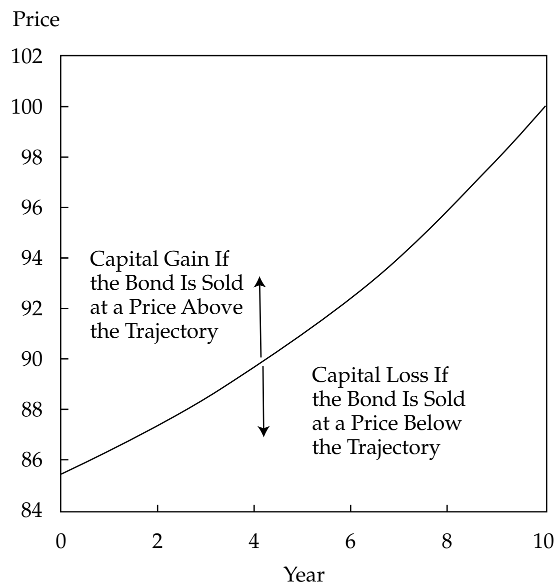
$$85.503075 = \frac{127.015881}{(1 + r)^4}, \quad r = 0.1040$$

In Example 2, the investor's **horizon yield** is 10.40%. A horizon yield is the internal rate of return between the total return (the sum of reinvested coupon payments and the sale price or redemption amount) and the purchase price of the bond. The horizon yield on a bond investment is the annualized holding-period rate of return.

Example 2 demonstrates that the realized horizon yield matches the original yield-to-maturity if: (1) coupon payments are reinvested at the same interest rate as the original yield-to-maturity, and (2) the bond is sold at a price on the constant-yield price trajectory, which implies that the investor does not have any capital gains or losses when the bond is sold.

Capital gains arise if a bond is sold at a price above its constant-yield price trajectory and capital losses occur if a bond is sold at a price below its constant-yield price trajectory. This trajectory is based on the yield-to-maturity when the bond is purchased. The trajectory is shown in Exhibit 1 for a 10-year, 8% annual payment bond purchased at a price of 85.503075 per 100 of par value.

EXHIBIT 1 Constant-Yield Price Trajectory for a 10-Year, 8% Annual Payment Bond



Note: Price is price per 100 of par value.

A point on the trajectory represents the **carrying value** of the bond at that time. The carrying value is the purchase price plus the amortized amount of the discount if the bond is purchased at a price below par value. If the bond is purchased at a price above par value, the carrying value is the purchase price minus the amortized amount of the premium.

The amortized amount for each year is the change in the price between two points on the trajectory. The initial price of the bond is 85.503075 per 100 of par value. Its price (the carrying value) after one year is 86.395394, calculated using the original yield-to-maturity of 10.40%. Therefore, the amortized amount for the first year is 0.892320 (= 86.395394 − 85.503075). The bond price in Example 2 increases from 85.503075 to 89.668770, and that increase over the four years is movement *along* the constant-yield price trajectory. At the time the bond is sold, its carrying value is also 89.668770, so there is no capital gain or loss.

Examples 3 and 4 demonstrate the impact on investors' realized horizon yields if interest rates go up by 100 basis points (bps). The market discount rate on the bond increases from 10.40% to 11.40%. Coupon reinvestment rates go up by 100 bps as well.

EXAMPLE 3

The buy-and-hold investor purchases the 10-year, 8% annual payment bond at 85.503075. After the bond is purchased and before the first coupon is received, interest rates go up to 11.40%. The future value of the reinvested coupons at 11.40% for 10 years is 136.380195 per 100 of par value.

$$[8 \times (1.1140)^9] + [8 \times (1.1140)^8] + [8 \times (1.1140)^7] + [8 \times (1.1140)^6] + [8 \times (1.1140)^5] + [8 \times (1.1140)^4] + [8 \times (1.1140)^3] + [8 \times (1.1140)^2] + [8 \times (1.1140)^1] + 8 = 136.380195$$

The total return is 236.380195 (= 136.380195 + 100). The investor's realized rate of return is 10.70%.

$$85.503075 = \frac{236.380195}{(1+r)^{10}}, \quad r = 0.1070$$

In Example 3, the buy-and-hold investor benefits from the higher coupon reinvestment rate. The realized horizon yield is 10.70%, 30 bps higher than the outcome in Example 1, when interest rates are unchanged. There is no capital gain or loss because the bond is held until maturity. The carrying value at the maturity date is par value, the same as the redemption amount.

EXAMPLE 4

The second investor buys the 10-year, 8% annual payment bond at 85.503075 and sells it in four years. After the bond is purchased, interest rates go up to 11.40%. The future value of the reinvested coupons at 11.40% after four years is 37.899724 per 100 of par value.

$$[8 \times (1.1140)^3] + [8 \times (1.1140)^2] + [8 \times (1.1140)^1] + 8 = 37.899724$$

The sale price of the bond after four years is 85.780408.

$$\frac{8}{(1.1140)^1} + \frac{8}{(1.1140)^2} + \frac{8}{(1.1140)^3} + \frac{8}{(1.1140)^4} + \frac{8}{(1.1140)^5} + \frac{108}{(1.1140)^6} = 85.780408$$

The total return is 123.680132 (= 37.899724 + 85.780408), resulting in a realized four-year horizon yield of 9.67%.

$$85.503075 = \frac{123.680132}{(1+r)^4}, \quad r = 0.0967$$

In Example 4, the second investor has a lower realized rate of return compared with the investor in Example 2, in which interest rates are unchanged. The future value of reinvested coupon payments goes up by 0.552613 (= 37.899724 – 37.347111) per 100 of par value because of the higher interest rates. But there is a *capital loss* of 3.888362 (= 89.668770 – 85.780408) per 100 of par value. Notice that the capital loss is measured from the bond's carrying value, the point on the constant-yield price trajectory, and not from the original purchase price. The bond is now sold at a price below the constant-yield price trajectory. The reduction in the realized four-year horizon yield from 10.40% to 9.67% is a result of the capital loss being greater than the gain from reinvesting coupons at a higher rate, which reduces the investor's total return.

Examples 5 and 6 complete the series of rate-of-return calculations for the two investors. Interest rates decline by 100 bps. The required yield on the bond falls from 10.40% to 9.40% after the purchase of the bond. The interest rates at which the coupon payments are reinvested fall as well.

EXAMPLE 5

The buy-and-hold investor purchases the 10-year bond at 85.503075 and holds the security until it matures. After the bond is purchased and before the first coupon is received, interest rates go down to 9.40%. The future value of reinvesting the coupon payments at 9.40% for 10 years is 123.888356 per 100 of par value.

$$[8 \times (1.0940)^9] + [8 \times (1.0940)^8] + [8 \times (1.0940)^7] + [8 \times (1.0940)^6] + [8 \times (1.0940)^5] + [8 \times (1.0940)^4] + [8 \times (1.0940)^3] + [8 \times (1.0940)^2] + [8 \times (1.0940)^1] + 8 = 123.888356$$

The total return is 223.888356, the sum of the future value of reinvested coupons and the redemption of par value. The investor's realized rate of return is 10.10%.

$$85.503075 = \frac{223.888365}{(1+r)^{10}}, \quad r = 0.1010$$

In Example 5, the buy-and-hold investor suffers from the lower coupon reinvestment rates. The realized horizon yield is 10.10%, 30 bps lower than the result in Example 1, when interest rates are unchanged. There is no capital gain or loss because the bond is held until maturity. Examples 1, 3, and 5 indicate that the interest rate risk for a buy-and-hold investor arises entirely from changes in coupon reinvestment rates.

EXAMPLE 6

The second investor buys the 10-year bond at 85.503075 and sells it in four years. After the bond is purchased, interest rates go down to 9.40%. The future value of the reinvested coupons at 9.40% is 36.801397 per 100 of par value.

$$[8 \times (1.0940)^3] + [8 \times (1.0940)^2] + [8 \times (1.0940)^1] + 8 = 36.801397$$

This reduction in future value is offset by the higher sale price of the bond, which is 93.793912 per 100 of par value.

$$\frac{8}{(1.0940)^1}+\frac{8}{(1.0940)^2}+\frac{8}{(1.0940)^3}+\frac{8}{(1.0940)^4}+\frac{8}{(1.0940)^5}+\frac{108}{(1.0940)^6}=93.793912$$

The total return is 130.595309 (= 36.801397 + 93.793912), and the realized yield is 11.17%.

$$85.503075=\frac{130.595309}{(1+r)^4}, \quad r=0.1117$$

The investor in Example 6 has a capital gain of 4.125142 (= 93.793912 − 89.668770). The capital gain is measured from the carrying value, the point on the constant-yield price trajectory. That gain offsets the reduction in the future value of reinvested coupons of 0.545714 (= 37.347111 − 36.801397). The total return is higher than that in Example 2, in which the interest rate remains at 10.40%.

In these examples, interest income for the investor is the return associated with the *passage of time*. Therefore, interest income includes the receipt of coupon interest, the reinvestment of those cash flows, and the amortization of the discount from purchase at a price below par value (or the premium from purchase at a price above par value) to bring the return back in line with the market discount rate. A capital gain or loss is the return to the investor associated with the *change in the value* of the security. On the fixed-rate bond, a change in value arises from a change in the yield-to-maturity, which is the implied market discount rate. In practice, the way interest income and capital gains and losses are calculated and reported on financial statements depends on financial and tax accounting rules.

This series of examples illustrates an important point about fixed-rate bonds: The *investment horizon* is at the heart of understanding interest rate risk and return. There are two offsetting types of interest rate risk that affect the bond investor: coupon reinvestment risk and market price risk. The future value of reinvested coupon payments (and, in a portfolio, the principal on bonds that mature before the horizon date) *increases* when interest rates rise and *decreases* when rates fall. The sale price on a bond that matures after the horizon date (and thus needs to be sold) *decreases* when interest rates rise and *increases* when rates fall. Coupon reinvestment risk matters more when the investor has a long-term horizon relative to the time-to-maturity of the bond. For instance, a buy-and-hold investor only has coupon reinvestment risk. Market price risk matters more when the investor has a short-term horizon relative to the time-to-maturity. For example, an investor who sells the bond before the first coupon is received has only market price risk. Therefore, two investors holding the same bond (or bond portfolio) can have different exposures to interest rate risk if they have different investment horizons.

EXAMPLE 7

An investor buys a four-year, 10% annual coupon payment bond priced to yield 5.00%. The investor plans to sell the bond in two years once the second coupon payment is received. Calculate the purchase price for the bond and the horizon yield assuming that the coupon reinvestment rate after the bond purchase and the yield-to-maturity at the time of sale are (1) 3.00%, (2) 5.00%, and (3) 7.00%.

Solution: The purchase price is 117.729753.

$$\frac{10}{(1.0500)^1}+\frac{10}{(1.0500)^2}+\frac{10}{(1.0500)^3}+\frac{110}{(1.0500)^4}=117.729753$$

1. 3.00%: The future value of reinvested coupons is 20.300.

$$(10 \times 1.0300) + 10 = 20.300$$

   The sale price of the bond is 113.394288.

$$\frac{10}{(1.0300)^1}+\frac{110}{(1.0300)^2}=113.394288$$

   Total return: 20.300 + 113.394288 = 133.694288.

   If interest rates go down from 5.00% to 3.00%, the realized rate of return over the two-year investment horizon is 6.5647%, higher than the original yield-to-maturity of 5.00%.

$$117.729753=\frac{133.694288}{(1+r)^2}, \quad r=0.065647$$

2. 5.00%: The future value of reinvested coupons is 20.500.

$$(10 \times 1.0500) + 10 = 20.500$$

   The sale price of the bond is 109.297052.

$$\frac{10}{(1.0500)^1}+\frac{110}{(1.0500)^2}=109.297052$$

   Total return: 20.500 + 109.297052 = 129.797052.

   If interest rates remain 5.00% for reinvested coupons and for the required yield on the bond, the realized rate of return over the two-year investment horizon is equal to the yield-to-maturity of 5.00%.

$$117.729753=\frac{129.797052}{(1+r)^2}, \quad r=0.050000$$

3. 7.00%: The future value of reinvested coupons is 20.700.

$$(10 \times 1.0700) + 10 = 20.700$$

The bond is sold at 105.424055.

$$\frac{10}{(1.0700)^1} + \frac{110}{(1.0700)^2} = 105.424055$$

Total return: 20.700 + 105.424055 = 126.124055.

$$117.729753 = \frac{126.124055}{(1+r)^2}, \quad r = 0.035037$$

If interest rates go up from 5.00% to 7.00%, the realized rate of return over the two-year investment horizon is 3.5037%, lower than the yield-to-maturity of 5.00%.

3. MACAULAY AND MODIFIED DURATION

- **define, calculate, and interpret Macaulay, modified, and effective durations**

This section covers two commonly used measures of interest rate risk: duration and convexity. It distinguishes between risk measures based on changes in a bond's own yield-to-maturity (yield duration and convexity) and those that affect the bond based on changes in a benchmark yield curve (curve duration and convexity).

3.1. Macaulay, Modified, and Approximate Duration

The duration of a bond measures the sensitivity of the bond's full price (including accrued interest) to changes in the bond's yield-to-maturity or, more generally, to changes in benchmark interest rates. Duration estimates changes in the bond price assuming that variables other than the yield-to-maturity or benchmark rates are held constant. Most importantly, the time-to-maturity is unchanged. Therefore, duration measures the *instantaneous* (or, at least, same-day) change in the bond price. The accrued interest is the same, so it is the flat price that goes up or down when the full price changes. Duration is a useful measure because it represents the approximate amount of time a bond would have to be held for the market discount rate at purchase to be realized if there is a single change in interest rate. If the bond is held for the duration period, an increase from reinvesting coupons is offset by a decrease in price if interest rates increase and a decrease from reinvesting coupons is offset by an increase in price if interest rates decrease.

There are several types of bond duration. In general, these can be divided into **yield duration** and **curve duration**. Yield duration is the sensitivity of the bond price with respect to the bond's own yield-to-maturity. Curve duration is the sensitivity of the bond price (or more generally, the market value of a financial asset or liability) with respect to a benchmark yield curve. The benchmark yield curve could be the government yield curve on coupon bonds, the spot curve, or the forward curve, but in practice, the government par curve is often used. Yield duration statistics used in fixed-income analysis include Macaulay duration, modified duration, money duration, and the price value of a basis point (PVBP). A curve duration statistic often used is effective duration. Effective duration is covered later in this chapter.

Macaulay duration is named after Frederick Macaulay, the Canadian economist who first wrote about the statistic in 1938. Equation 1 is a general formula to calculate the Macaulay duration (MacDur) of a traditional fixed-rate bond.

$$\text{MacDur} = \left[\frac{\dfrac{(1-t/T)\times PMT}{(1+r)^{1-t/T}} + \dfrac{(2-t/T)\times PMT}{(1+r)^{2-t/T}} + \cdots + \dfrac{(N-t/T)\times (PMT+FV)}{(1+r)^{N-t/T}}}{\dfrac{PMT}{(1+r)^{1-t/T}} + \dfrac{PMT}{(1+r)^{2-t/T}} + \cdots + \dfrac{PMT+FV}{(1+r)^{N-t/T}}}\right] \quad (1)$$

where

t = the number of days from the last coupon payment to the settlement date
T = the number of days in the coupon period
t/T = the fraction of the coupon period that has gone by since the last payment
PMT = the coupon payment per period
FV = the future value paid at maturity, or the par value of the bond
r = the yield-to-maturity, or the market discount rate, per period
N = the number of evenly spaced periods to maturity as of the beginning of the current period

The denominator in Equation 1 is the full price (PV^{Full}) of the bond including accrued interest. It is the present value of the coupon interest and principal payments, with each cash flow discounted by the same market discount rate, r.

$$PV^{Full} = \frac{PMT}{(1+r)^{1-t/T}} + \frac{PMT}{(1+r)^{2-t/T}} + \cdots + \frac{PMT+FV}{(1+r)^{N-t/T}} \quad (2)$$

Equation 3 combines Equations 1 and 2 to reveal an important aspect of the Macaulay duration: Macaulay duration is a weighted average of the time to receipt of the bond's promised payments, where the weights are the shares of the full price that correspond to each of the bond's promised future payments.

$$\text{MacDur} = \left\{ \begin{aligned} &(1-t/T)\left[\frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{Full}}\right] + (2-t/T)\left[\frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{Full}}\right] + \cdots + \\ &(N-t/T)\left[\frac{\frac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{Full}}\right] \end{aligned} \right\} \quad (3)$$

The times to receipt of cash flow measured in terms of time periods are $1 - t/T$, $2 - t/T$, . . . , $N - t/T$. The weights are the present values of the cash flows divided by the full price. Therefore, Macaulay duration is measured in terms of time periods. A couple of examples will clarify this calculation.

Consider first the 10-year, 8% annual coupon payment bond used in Examples 1–6. The bond's yield-to-maturity is 10.40%, and its price is 85.503075 per 100 of par value. This bond has 10 evenly spaced periods to maturity. Settlement is on a coupon payment date so that $t/T = 0$. Exhibit 2 illustrates the calculation of the bond's Macaulay duration.

EXHIBIT 2 Macaulay Duration of a 10-Year, 8% Annual Payment Bond

Period	Cash Flow	Present Value	Weight	Period × Weight
1	8	7.246377	0.08475	0.0847
2	8	6.563747	0.07677	0.1535
3	8	5.945423	0.06953	0.2086
4	8	5.385347	0.06298	0.2519
5	8	4.878032	0.05705	0.2853
6	8	4.418507	0.05168	0.3101
7	8	4.002271	0.04681	0.3277
8	8	3.625245	0.04240	0.3392
9	8	3.283737	0.03840	0.3456
10	108	40.154389	0.46963	4.6963
		85.503075	1.00000	7.0029

The first two columns of Exhibit 2 show the number of periods to the receipt of the cash flow and the amount of the payment per 100 of par value. The third column is the present value of the cash flow. For example, the final payment is 108 (the last coupon payment plus the redemption of principal) and its present value is 40.154389.

$$\frac{108}{(1.1040)^{10}} = 40.154389$$

The sum of the present values is the full price of the bond. The fourth column is the weight, the share of total market value corresponding to each cash flow. The final payment of 108 per 100 of par value is 46.963% of the bond's market value.

$$\frac{40.154389}{85.503075} = 0.46963$$

The sum of the weights is 1.00000. The fifth column is the number of periods to the receipt of the cash flow (the first column) multiplied by the weight (the fourth column). The sum of that column is 7.0029, which is the Macaulay duration of this 10-year, 8% annual coupon payment bond. This statistic is sometimes reported as 7.0029 *years,* although the time frame is not needed in most applications.

Now consider an example *between* coupon payment dates. A 6% semiannual payment corporate bond that matures on 14 February 2027 is purchased for settlement on 11 April 2019. The coupon payments are 3 per 100 of par value, paid on 14 February and 14 August of each year. The yield-to-maturity is 6.00% quoted on a street-convention semiannual bond basis. The full price of this bond comprises the flat price plus accrued interest. The flat price for the bond is 99.990423 per 100 of par value. The accrued interest is calculated using the 30/360 method to count days. This settlement date is 57 days into the 180-day semiannual period, so $t/T = 57/180$. The accrued interest is 0.950000 (= 57/180 × 3) per 100 of par value. The full price for the bond is 100.940423 (= 99.990423 + 0.950000). Exhibit 3 shows the calculation of the bond's Macaulay duration.

EXHIBIT 3 Macaulay Duration of an Eight-Year, 6% Semiannual Payment Bond Priced to Yield 6.00%

Period	Time to Receipt	Cash Flow	Present Value	Weight	Time × Weight
1	0.6833	3	2.940012	0.02913	0.019903
2	1.6833	3	2.854381	0.02828	0.047601
3	2.6833	3	2.771244	0.02745	0.073669
4	3.6833	3	2.690528	0.02665	0.098178
5	4.6833	3	2.612163	0.02588	0.121197
6	5.6833	3	2.536080	0.02512	0.142791
7	6.6833	3	2.462214	0.02439	0.163025
8	7.6833	3	2.390499	0.02368	0.181959
9	8.6833	3	2.320873	0.02299	0.199652
10	9.6833	3	2.253275	0.02232	0.216159
11	10.6833	3	2.187645	0.02167	0.231536
12	11.6833	3	2.123927	0.02104	0.245834
13	12.6833	3	2.062065	0.02043	0.259102
14	13.6833	3	2.002005	0.01983	0.271389
15	14.6833	3	1.943694	0.01926	0.282740
16	15.6833	103	64.789817	0.64186	10.066535
			100.940423	1.00000	12.621268

There are 16 semiannual periods to maturity between the last coupon payment date of 14 February 2019 and maturity on 14 February 2027. The time to receipt of cash flow in semiannual periods is in the second column: 0.6833 = 1 − 57/180, 1.6833 = 2 − 57/180, etc. The cash flow for each period is in the third column. The annual yield-to-maturity is 6.00%, so the yield per semiannual period is 3.00%. When that yield is used to get the present value of each cash flow, the full price of the bond is 100.940423, the sum of the fourth column. The weights, which are the shares of the full price corresponding to each cash flow, are in the fifth column. The Macaulay duration is the sum of the items in the sixth column, which is the weight multiplied by the time to receipt of each cash flow. The result, 12.621268, is the Macaulay duration on an eight-year, 6% semiannual payment bond for settlement on 11 April 2019 measured in *semiannual periods*. Similar to coupon rates and yields-to-maturity, duration statistics invariably are annualized in practice. Therefore, the Macaulay duration typically is reported as 6.310634 *years* (= 12.621268/2). (Such precision for the duration statistic is not needed in practice. Typically, "6.31 years" is enough. The full precision is shown here to illustrate calculations.) Microsoft Excel users can obtain the Macaulay duration using the DURATION financial function—DURATION(DATE(2019,4,11),DATE(2027,2,14),0.06,0.06,2,0)—and inputs that include the settlement date, maturity date, annual coupon rate as a decimal, annual yield-to-maturity as a decimal, periodicity, and day count code (0 for 30/360, 1 for actual/actual).

Another approach to calculating the Macaulay duration is to use a closed-form equation derived using calculus and algebra (see Smith 2014). Equation 4 is a general closed-form formula for determining the Macaulay duration of a fixed-rate bond, where c is the coupon rate per period (PMT/FV).

$$\text{MacDur} = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - (t/T) \quad (4)$$

The Macaulay duration of the 10-year, 8% annual payment bond is calculated by entering $r = 0.1040$, $c = 0.0800$, $N = 10$, and $t/T = 0$ into Equation 4.

$$\text{MacDur} = \frac{1+0.1040}{0.1040} - \frac{1+0.1040+[10 \times (0.0800 - 0.1040)]}{0.0800 \times [(1+0.1040)^{10} - 1] + 0.1040} = 7.0029$$

Therefore, the weighted average time to receipt of the interest and principal payments that will result in realization of the initial market discount rate on this 10-year bond is 7.00 years.

The Macaulay duration of the 6% semiannual payment bond maturing on 14 February 2027 is obtained by entering $r = 0.0300$, $c = 0.0300$, $N = 16$, and $t/T = 57/180$ into Equation 4.

$$\begin{aligned}\text{MacDur} &= \left[\frac{1+0.0300}{0.0300} - \frac{1+0.0300+[16 \times (0.0300 - 0.0300)]}{0.0300 \times [(1+0.0300)^{16} - 1] + 0.0300} \right] - (57/180) \\ &= 12.621268\end{aligned}$$

Equation 4 uses the yield-to-maturity *per period*, the coupon rate *per period*, the number of *periods* to maturity, and the fraction of the current *period* that has gone by. Its output is the Macaulay duration in terms of *periods*. It is converted to annual duration by dividing by the number of periods in the year.

The calculation of the **modified duration** (ModDur) statistic of a bond requires a simple adjustment to Macaulay duration. It is the Macaulay duration statistic divided by one plus the yield per period.

$$\text{ModDur} = \frac{\text{MacDur}}{1+r} \quad (5)$$

For example, the modified duration of the 10-year, 8% annual payment bond is 6.3432.

$$\text{ModDur} = \frac{7.0029}{1.1040} = 6.3432$$

The modified duration of the 6% semiannual payment bond maturing on 14 February 2027 is 12.253658 semiannual periods.

$$\text{ModDur} = \frac{12.621268}{1.0300} = 12.253658$$

The annualized modified duration of the bond is 6.126829 (= 12.253658/2).

Microsoft Excel users can obtain the modified duration using the MDURATION financial function using the same inputs as for the Macaulay duration: MDURATION (DATE(2019,4,11),DATE(2027,2,14),0.06,0.06,2,0). Although modified duration might seem to be just a Macaulay duration with minor adjustments, it has an important application in risk measurement: Modified duration provides an estimate of the percentage price change for a bond given a change in its yield-to-maturity.

$$\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta\text{Yield} \quad (6)$$

The percentage price change refers to the full price, including accrued interest. The AnnModDur term in Equation 6 is the *annual* modified duration, and the ΔYield term is the change in the *annual* yield-to-maturity. The ≈ sign indicates that this calculation is an estimation. The minus sign indicates that bond prices and yields-to-maturity move inversely.

If the annual yield on the 6% semiannual payment bond that matures on 14 February 2027 jumps by 100 bps, from 6.00% to 7.00%, the estimated loss in value for the bond is 6.1268%.

$$\%\Delta PV^{Full} \approx -6.126829 \times 0.0100 = -0.061268$$

If the yield-to-maturity were to drop by 100 bps to 5.00%, the estimated gain in value is also 6.1268%.

$$\%\Delta PV^{Full} \approx -6.126829 \times -0.0100 = 0.061268$$

Modified duration provides a *linear* estimate of the percentage price change. In terms of absolute value, the change is the same for either an increase or a decrease in the yield-to-maturity. Recall that for a given coupon rate and time-to-maturity, the percentage price change is greater (in absolute value) when the market discount rate goes down than when it goes up. Later in this chapter, a "convexity adjustment" to duration is introduced. It improves the accuracy of this estimate, especially when a large change in yield-to-maturity (such as 100 bps) is considered.

4. APPROXIMATE MODIFIED AND MACAULAY DURATION

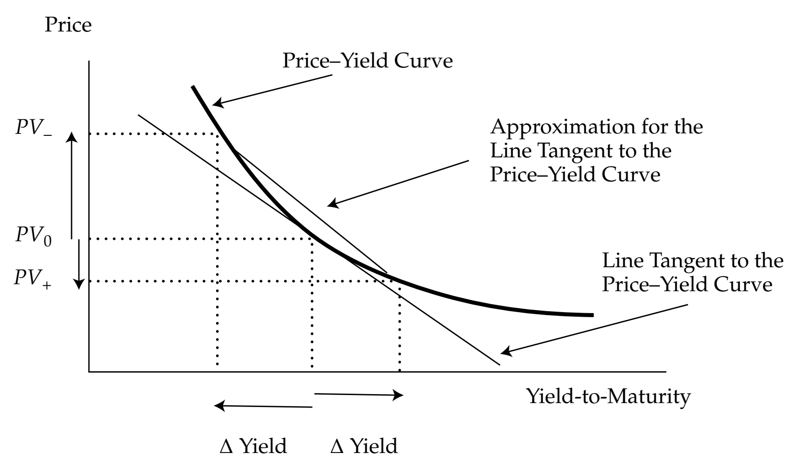
- **define, calculate, and interpret Macaulay, modified, and effective durations**

The modified duration statistic for a fixed-rate bond is easily obtained if the Macaulay duration is already known. An alternative approach is to *approximate* modified duration directly. Equation 7 is the approximation formula for annual modified duration.

$$\text{ApproxModDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Yield}) \times (PV_0)} \tag{7}$$

The objective of the approximation is to estimate the slope of the line tangent to the price–yield curve. The slope of the tangent and the approximated slope are shown in Exhibit 4.

EXHIBIT 4 Approximate Modified Duration



To estimate the slope, the yield-to-maturity is changed up and down by the same amount—the ΔYield. Then the bond prices given the new yields-to-maturity are calculated. The price when the yield is increased is denoted PV_+. The price when the yield-to-maturity is reduced is denoted PV_-. The original price is PV_0. These prices are the full prices, including accrued interest. The slope of the line based on PV_+ and PV_- is the approximation for the slope of the line tangent to the price–yield curve. The following example illustrates the remarkable accuracy of this approximation. In fact, as ΔYield approaches zero, the approximation approaches AnnModDur.

Consider the 6% semiannual coupon payment corporate bond maturing on 14 February 2027. For settlement on 11 April 2019, the full price (PV_0) is 100.940423 given that the yield-to-maturity is 6.00%.

$$PV_0 = \left[\frac{3}{(1.03)^1} + \frac{3}{(1.03)^2} + \cdots + \frac{103}{(1.03)^{16}}\right] \times (1.03)^{57/180} = 100.940423$$

Raise the annual yield-to-maturity by five bps, from 6.00% to 6.05%. This increase corresponds to an increase in the yield-to-maturity per semiannual period of 2.5 bps, from 3.00% to 3.025% per period. The new full price (PV_+) is 100.631781.

$$PV_+ = \left[\frac{3}{(1.03025)^1} + \frac{3}{(1.03025)^2} + \cdots + \frac{103}{(1.03025)^{16}}\right] \times (1.03025)^{57/180} = 100.631781$$

Lower the annual yield-to-maturity by 5 bps, from 6.00% to 5.95%. This decrease corresponds to a decrease in the yield-to-maturity per semiannual period of 2.5 bps, from 3.00% to 2.975% per period. The new full price (PV_-) is 101.250227.

$$PV_- = \left[\frac{3}{(1.02975)^1} + \frac{3}{(1.02975)^2} + \cdots + \frac{103}{(1.02975)^{16}}\right] \times (1.02975)^{57/180} = 101.250227$$

Enter these results into Equation 7 for the 5 bp change in the annual yield-to-maturity, or ΔYield = 0.0005:

$$\text{ApproxModDur} = \frac{101.250227 - 100.631781}{2 \quad 0.0005 \times 100.940423} = 6.126842$$

The "exact" annual modified duration for this bond is 6.126829 and the "approximation" is 6.126842—virtually identical results. Therefore, although duration can be calculated using the approach in Exhibits 2 and 3—basing the calculation on the weighted average time to receipt of each cash flow—or using the closed-form formula as in Equation 4, it can also be estimated quite accurately using the basic bond-pricing equation and a financial calculator. The Macaulay duration can be approximated as well—the approximate modified duration multiplied by one plus the yield per period.

$$\text{ApproxMacDur} = \text{ApproxModDur} \times (1 + r) \tag{8}$$

The approximation formulas produce results for *annualized* modified and Macaulay durations. The frequency of coupon payments and the periodicity of the yield-to-maturity are included in the bond price calculations.

EXAMPLE 8

Assume that the 3.75% US Treasury bond that matures on 15 August 2041 is priced to yield 5.14% for settlement on 15 October 2020. Coupons are paid semiannually on 15 February and 15 August. The yield-to-maturity is stated on a street-convention semi-annual bond basis. This settlement date is 61 days into a 184-day coupon period, using the actual/actual day-count convention. Compute the approximate modified duration and the approximate Macaulay duration for this Treasury bond assuming a 5 bp change in the yield-to-maturity.

Solution: The yield-to-maturity per semiannual period is 0.0257 (= 0.0514/2). The coupon payment per period is 1.875 (= 3.75/2). At the beginning of the period, there are 21 years (42 semiannual periods) to maturity. The fraction of the period that has passed is 61/184. The full price at that yield-to-maturity is 82.967530 per 100 of par value.

$$PV_0 = \left[\frac{1.875}{(1.0257)^1} + \frac{1.875}{(1.0257)^2} + \cdots + \frac{101.875}{(1.0257)^{42}}\right] \times (1.0257)^{61/184} = 82.96753$$

Raise the yield-to-maturity from 5.14% to 5.19%—therefore, from 2.57% to 2.595% per semiannual period—and the price becomes 82.411395 per 100 of par value.

$$PV_+ = \left[\frac{1.875}{(1.02595)^1} + \frac{1.875}{(1.02595)^2} + \cdots + \frac{101.875}{(1.02595)^{42}}\right] \times (1.02595)^{61/184}$$
$$= 82.411395$$

Lower the yield-to-maturity from 5.14% to 5.09%—therefore, from 2.57% to 2.545% per semiannual period—and the price becomes 83.528661 per 100 of par value.

$$PV_- = \left[\frac{1.875}{(1.02545)^1} + \frac{1.875}{(1.02545)^2} + \cdots + \frac{101.875}{(1.02545)^{42}}\right] \times (1.02545)^{61/184}$$
$$= 83.528661$$

The approximate annualized modified duration for the Treasury bond is 13.466.

$$\text{ApproxModDur} = \frac{83.528661 - 82.411395}{2 \times 0.0005 \times 82.967530} = 13.466$$

The approximate annualized Macaulay duration is 13.812.

$$\text{ApproxMacDur} = 13.466 \times 1.0257 = 13.812$$

Therefore, from these statistics, the investor knows that the weighted average time to receipt of interest and principal payments is 13.812 years (the Macaulay duration) and that the estimated loss in the bond's market value is 13.466% (the modified duration) if the market discount rate were to suddenly go up by 1% from 5.14% to 6.14%.

5. EFFECTIVE AND KEY RATE DURATION

- **define, calculate, and interpret Macaulay, modified, and effective durations**
- **explain why effective duration is the most appropriate measure of interest rate risk for bonds with embedded options**

Another approach to assess the interest rate risk of a bond is to estimate the percentage change in price given a change in a benchmark yield curve—for example, the government par curve. This estimate, which is very similar to the formula for approximate modified duration, is called the **effective duration**. The effective duration of a bond is the sensitivity of the bond's price to a change in a benchmark yield curve. The formula to calculate effective duration (EffDur) is Equation 9.

$$\text{EffDur} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\text{Curve}) \times (PV_0)} \tag{9}$$

The difference between approximate modified duration and effective duration is in the denominator. Modified duration is a *yield duration* statistic in that it measures interest rate risk in terms of a change in the bond's own yield-to-maturity (ΔYield). Effective duration is a *curve duration* statistic in that it measures interest rate risk in terms of a parallel shift in the benchmark yield curve (ΔCurve).

Effective duration is essential to the measurement of the interest rate risk of a complex bond, such as a bond that contains an embedded call option. The duration of a callable bond is *not* the sensitivity of the bond price to a change in the yield-to-worst (i.e., the lowest of the yield-to-maturity, yield-to-first-call, yield-to-second-call, and so forth). The problem is that future cash flows are uncertain because they are contingent on future interest rates. The issuer's decision to call the bond depends on the ability to refinance the debt at a lower cost of funds. In brief, a callable bond does not have a well-defined internal rate of return (yield-to-maturity). Therefore, yield duration statistics, such as modified and Macaulay durations, do not apply; effective duration is the appropriate duration measure.

The specific option-pricing models that are used to produce the inputs to effective duration for a callable bond are covered in later chapters. However, as an example, suppose that the full price of a callable bond is 101.060489 per 100 of par value. The option-pricing model inputs include (1) the length of the call protection period, (2) the schedule of call prices and call dates, (3) an assumption about credit spreads over benchmark yields (which includes any liquidity spread as well), (4) an assumption about future interest rate volatility, and (5) the level of market interest rates (e.g., the government par curve). The analyst then holds the first four inputs constant and raises and lowers the fifth input. Suppose that when the government par curve is raised and lowered by 25 bps, the new full prices for the callable bond from the model are 99.050120 and 102.890738, respectively. Therefore, $PV_0 = 101.060489$, $PV_{+} = 99.050120$, $PV_{-} = 102.890738$, and $\Delta\text{Curve} = 0.0025$. The effective duration for the callable bond is 7.6006.

$$\text{EffDur} = \frac{102.890738 - 99.050120}{2 \times 0.0025 \times 101.060489} = 7.6006$$

This curve duration measure indicates the bond's sensitivity to the benchmark yield curve—in particular, the government par curve—assuming no change in the credit spread. In practice, a callable bond issuer might be able to exercise the call option and obtain a lower cost of funds if (1) benchmark yields fall and the credit spread over the benchmark is unchanged or (2) benchmark yields are unchanged and the credit spread is reduced (e.g., because of an upgrade in the issuer's rating). A pricing model can be used to determine a "credit duration" statistic—that is, the sensitivity of the bond price to a change in the credit spread. On a traditional fixed-rate bond, modified duration estimates the percentage price change for a change in the benchmark yield and/or the credit spread. For bonds that do not have a well-defined internal rate of return because the future cash flows are not fixed—for instance, callable bonds and floating-rate notes—pricing models are used to produce different statistics for changes in benchmark interest rates and for changes in credit risk.

Another fixed-income security for which yield duration statistics, such as modified and Macaulay durations, are not relevant is a mortgage-backed bond. These securities arise from a residential (or commercial) loan portfolio securitization. The key point for measuring interest rate risk on a mortgage-backed bond is that the cash flows are contingent on homeowners' ability to refinance their debt at a lower rate. In effect, the homeowners have call options on their mortgage loans.

A practical consideration in using effective duration is in setting the change in the benchmark yield curve. With approximate modified duration, accuracy is improved by choosing a smaller yield-to-maturity change. But the pricing models for more-complex securities, such as callable and mortgage-backed bonds, include assumptions about the behavior of the corporate issuers, businesses, or homeowners. Rates typically need to change by a minimum amount to affect the decision to call a bond or refinance a mortgage loan because issuing new debt involves transaction costs. Therefore, estimates of interest rate risk using effective duration are not necessarily improved by choosing a smaller change in benchmark rates. Effective duration has become an important tool in the financial analysis of not only traditional bonds but also financial liabilities. Example 9 demonstrates such an application of effective duration.

EXAMPLE 9

Defined-benefit pension schemes typically pay retirees a monthly amount based on their wage level at the time of retirement. The amount could be fixed in nominal terms or indexed to inflation. These programs are referred to as "defined-benefit pension plans" when US GAAP or IFRS accounting standards are used. In Australia, they are called "superannuation funds."

A British defined-benefit pension scheme seeks to measure the sensitivity of its retirement obligations to market interest rate changes. The pension scheme manager hires an actuarial consultancy to model the present value of its liabilities under three interest rate scenarios: (1) a base rate of 5%, (2) a 100 bp increase in rates, up to 6%, and (3) a 100 bp drop in rates, down to 4%.

The actuarial consultancy uses a complex valuation model that includes assumptions about employee retention, early retirement, wage growth, mortality, and longevity. The following chart shows the results of the analysis.

Interest Rate Assumption	Present Value of Liabilities
4%	GBP973.5 million
5%	GBP926.1 million
6%	GBP871.8 million

Compute the effective duration of the pension scheme's liabilities.

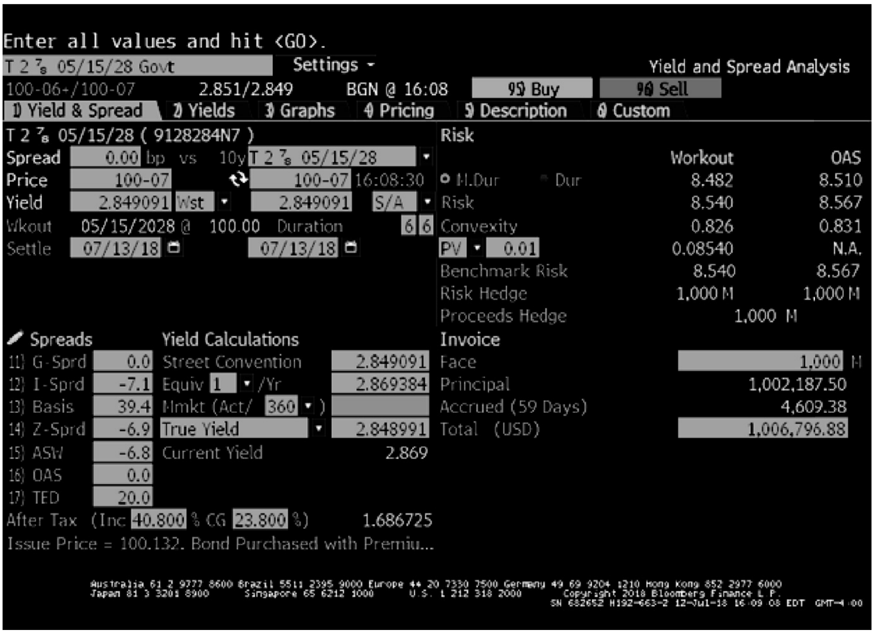
Solution: $PV_0 = 926.1$, $PV_+ = 871.8$, $PV_- = 973.5$, and $\Delta\text{Curve} = 0.0100$. The effective duration of the pension scheme's liabilities is 5.49.

$$\text{EffDur} = \frac{973.5 - 871.8}{2 \times 0.0100 \times 926.1} = 5.49$$

This effective duration statistic for the pension scheme's liabilities might be used in asset allocation decisions to decide the mix of equity, fixed income, and alternative assets.

Although effective duration is the most appropriate interest rate risk measure for bonds with embedded options, it also is useful with traditional bonds to supplement the information provided by the Macaulay and modified yield durations. Exhibit 5 displays the Bloomberg Yield and Spread (YAS) Analysis page for the 2.875% US Treasury note that matures on 15 May 2028.

EXHIBIT 5 Bloomberg YAS Page for the 2.875% US Treasury Note



In Exhibit 5, the quoted (flat) asked price for the bond is 100-07, which is equal to 100 and 7/32nds per 100 of par value for settlement on 13 July 2018. Most bond prices are stated in decimals, but US Treasuries are usually quoted in fractions. As a decimal, the flat price is 100.21875. The accrued interest uses the actual/actual day-count method. That settlement date is 59 days into a 184-day semiannual coupon payment period. The accrued interest is 0.4609375 per 100 of par value (= 59/184 × 0.02875/2 × 100). The full price of the bond is 100.679688. The yield-to-maturity of the bond is 2.849091%, stated on a street-convention semiannual bond basis.

The modified duration for the bond is shown in Exhibit 5 to be 8.482, which is the conventional *yield* duration statistic. Its *curve* duration, however, is 8.510, which is the price sensitivity with respect to changes in the US Treasury par curve. On Bloomberg, the effective duration is called the "OAS duration" because it is based on the option-pricing model that is also used to calculate the option-adjusted spread. The small difference arises because the government yield curve is not flat. When the par curve is shifted in the model, the government spot curve is also shifted, although not in the same "parallel" manner. Therefore, the change in the bond price is not the same as it would be if its own yield-to-maturity changed by the same amount as the change in the par curve. In general, the modified duration and effective duration on a traditional option-free bond are not identical. The difference narrows when the yield curve is flatter, the time-to-maturity is shorter, and the bond is priced closer to par value (so that the difference between the coupon rate and the yield-to-maturity is smaller).

The modified duration and effective duration on an option-free bond are identical only in the rare circumstance of a flat yield curve.

5.1. Key Rate Duration

- **define key rate duration and describe the use of key rate durations in measuring the sensitivity of bonds to changes in the shape of the benchmark yield curve**

The effective duration for a sample callable bond was calculated previously as

$$\text{EffDur} = \frac{102.890738 - 99.050120}{2 \times 0.0025 \times 101.060489} = 7.6006$$

This duration measure indicates the bond's sensitivity to the benchmark yield curve if all yields change by the same amount. "Key rate" duration provides further insight into a bond's sensitivity to non-parallel benchmark yield curve changes. A **key rate duration** (or **partial duration**) is a measure of a bond's sensitivity to a change in the benchmark yield at a specific maturity. Key rate durations define a security's price sensitivity over a set of maturities along the yield curve, with the sum of key rate durations being identical to the effective duration:

$$\begin{gathered}\text{KeyRateDur}^k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r^k} \\ \sum_{k=1}^{n} \text{KeyRateDur}^k = \text{EffDur}\end{gathered} \tag{10}$$

where r^k represents the *k*th key rate. In contrast to effective duration, key rate durations help identify "shaping risk" for a bond—that is, a bond's sensitivity to changes in the shape of the benchmark yield curve (e.g., the yield curve becoming steeper or flatter).

The previous illustration of effective duration assumed a parallel shift of 25 bps at all maturities. However, the analyst may want to know how the price of the callable bond is expected to change if short-term benchmark rates (say, for a current two-year Treasury note with modified duration of 1.9) rise by 25 bps but longer-maturity benchmark rates remain unchanged. This scenario would represent a flattening of the yield curve, given that the yield curve is upward sloping. Using key rate durations, the expected price change would be approximately equal to minus the key rate duration for the short-maturity segment (−1.9) times the 0.0025 interest rate shift at that segment, or −0.475% based on the following formula:

$$\frac{\Delta PV}{PV} = -\text{KeyRateDur}^k \times \Delta r^k$$

Of course, for parallel shifts in the benchmark yield curve, key rate durations will indicate the same interest rate sensitivity as effective duration.

6. PROPERTIES OF BOND DURATION

- **explain how a bond's maturity, coupon, and yield level affect its interest rate risk**

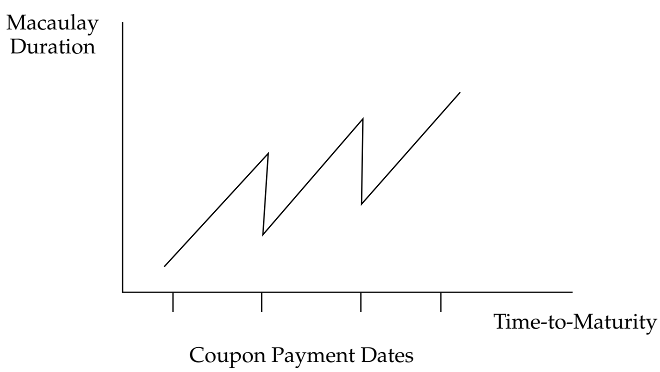
The Macaulay and modified yield duration statistics for a traditional fixed-rate bond are functions of the input variables: the coupon rate or payment per period, the yield-to-maturity per period, the number of periods to maturity (as of the beginning of the period), and the fraction of the period that has gone by. The properties of bond duration are obtained by changing one of these variables while holding the others constant. Because duration is the basic measure of interest rate risk on a fixed-rate bond, these properties are important to understand.

The closed-form formula for Macaulay duration, presented as Equation 4 and again here, is useful in demonstrating the characteristics of the bond duration statistic.

$$\text{MacDur} = \left\{ \frac{1+r}{r} - \frac{1 + r + [N \times (c - r)]}{c \times \left[(1+r)^N - 1\right] + r} \right\} - (t/T)$$

The same characteristics hold for modified duration. Consider first the fraction of the period that has gone by (t/T). Macaulay and modified durations depend on the day-count basis used to obtain the yield-to-maturity. The duration of a bond that uses the actual/actual method to count days is slightly different from that of an otherwise comparable bond that uses the 30/360 method. The key point is that for a constant yield-to-maturity (r), the expression in braces is unchanged as time passes during the period. Therefore, the Macaulay duration decreases smoothly as t goes from $t = 0$ to $t = T$, which creates a "saw-tooth" pattern. This pattern for a typical fixed-rate bond is illustrated in Exhibit 6.

EXHIBIT 6 Macaulay Duration between Coupon Payments with a Constant Yield-to-Maturity



As times passes during the coupon period (moving from right to left in the diagram), the Macaulay duration declines smoothly and then jumps upward after the coupon is paid.

The characteristics of bond duration related to changes in the coupon rate, the yield-to-maturity, and the time-to-maturity are illustrated in Exhibit 7.

EXHIBIT 7 Properties of the Macaulay Yield Duration

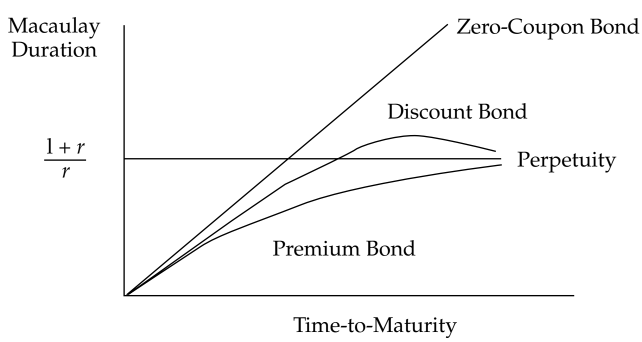


Exhibit 7 shows the graph for coupon payment dates when $t/T = 0$, thus not displaying the saw-tooth pattern between coupon payments. The relationship between the Macaulay duration and the time-to-maturity for a zero-coupon bond is the 45-degree line: MacDur = N when $c = 0$ (and $t/T = 0$). Therefore, the Macaulay duration of a zero-coupon bond is its time-to-maturity.

A **perpetuity** or perpetual bond, which also is called a consol, is a bond that does not mature. There is no principal to redeem. The investor receives a fixed coupon payment forever unless the bond is callable. Non-callable perpetuities are rare, but they have an interesting Macaulay duration: MacDur = $(1 + r)/r$ as N approaches infinity. In effect, the second expression within the braces approaches zero as the number of periods to maturity increases because N in the numerator is a coefficient but N in the denominator is an exponent and the denominator increases faster than the numerator as N grows larger.

Typical fixed-rate coupon bonds with a stated maturity date are portrayed in Exhibit 7 as the premium and discount bonds. The usual pattern is that longer times-to-maturity correspond to higher Macaulay duration statistics. This pattern always holds for bonds trading at par value or at a premium above par. In Equation 4, the second expression within the braces is a positive number for premium and par bonds. The numerator is positive because the coupon rate (c) is greater than or equal to the yield-to-maturity (r), whereas the denominator is always positive. Therefore, the Macaulay duration is always less than $(1 + r)/r$, and it approaches that threshold from below as the time-to-maturity increases.

The curious result displayed in Exhibit 7 is in the pattern for discount bonds. Generally, the Macaulay duration increases for a longer time-to-maturity. But at some point when the time-to-maturity is high enough, the Macaulay duration exceeds $(1 + r)/r$, reaches a maximum, and then approaches the threshold from above. In Equation 4, such a pattern develops when the number of periods (N) is large and the coupon rate (c) is below the yield-to-maturity (r). Then the numerator of the second expression within the braces can become negative. The implication is that on long-term discount bonds, the interest rate risk can actually be less than on a shorter-term bond, which explains why the word "generally" is needed in describing the maturity effect for the relationship between bond prices and yields-to-maturity. Generally, for the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their yields-to-maturity change by the same amount. The exception is when the longer-term bond has a lower duration statistic.

Coupon rates and yields-to-maturity are both inversely related to the Macaulay duration. In Exhibit 7, for the same time-to-maturity and yield-to-maturity, the Macaulay duration is higher for a zero-coupon bond than for a low-coupon bond trading at a discount. Also, the low-coupon bond trading at a discount has a higher duration than a high-coupon bond trading at a premium. Therefore, all else being equal, a lower-coupon bond has a higher duration and more interest rate risk than a higher-coupon bond. The same pattern holds for the yield-to-maturity. A higher yield-to-maturity reduces the weighted average of the time to receipt of cash flow. More weight is on the cash flows received in the near term, and less weight is on the cash flows received in the more-distant future periods if those cash flows are discounted at a higher rate.

In summary, the Macaulay and modified duration statistics for a fixed-rate bond depend primarily on the coupon rate, yield-to-maturity, and time-to-maturity. A higher coupon rate or a higher yield-to-maturity reduces the duration measures. A longer time-to-maturity *usually* leads to a higher duration. It *always* does so for a bond priced at a premium or at par value. But if the bond is priced at a discount, a longer time-to-maturity *might* lead to a lower duration. This situation only occurs if the coupon rate is low (but not zero) relative to the yield and the time-to-maturity is long.

EXAMPLE 10

A hedge fund specializes in investments in emerging market sovereign debt. The fund manager believes that the implied default probabilities are too high, which means that the bonds are viewed as "cheap" and the credit spreads are too high. The hedge fund plans to take a position on one of these available bonds.

Bond	Time-to-Maturity	Coupon Rate	Price	Yield-to-Maturity
(A)	10 years	10%	58.075279	20%
(B)	20 years	10%	51.304203	20%
(C)	30 years	10%	50.210636	20%

The coupon payments are annual. The yields-to-maturity are effective annual rates. The prices are per 100 of par value.

1. Compute the approximate modified duration of each of the three bonds using a 1 bp change in the yield-to-maturity and keeping precision to six decimals (because approximate duration statistics are very sensitive to rounding).
2. Which of the three bonds is expected to have the highest percentage price increase if the yield-to-maturity on each decreases by the same amount—for instance, by 10 bps from 20% to 19.90%?

Solution to 1:

Bond A

$$PV_0 = 58.075279$$

$$PV_+ = 58.047598$$

$$\frac{10}{(1.2001)^1} + \frac{10}{(1.2001)^2} + \cdots + \frac{110}{(1.2001)^{10}} = 58.047598$$

$$PV_- = 58.102981$$

$$\frac{10}{(1.1999)^1} + \frac{10}{(1.1999)^2} + \cdots + \frac{110}{(1.1999)^{10}} = 58.102981$$

The approximate modified duration of Bond A is 4.768.

$$\text{ApproxModDur} = \frac{58.102981 - 58.047598}{2 \times 0.0001 \times 58.075279} = 4.768$$

Bond B

$$PV_0 = 51.304203$$

$$PV_+ = 51.277694$$

$$\frac{10}{(1.2001)^1}+\frac{10}{(1.2001)^2}+\cdots+\frac{110}{(1.2001)^{20}}=51.277694$$

$$PV_-=51.330737$$

$$\frac{10}{(1.1999)^1}+\frac{10}{(1.1999)^2}+\cdots+\frac{110}{(1.1999)^{20}}=51.330737$$

The approximate modified duration of Bond B is 5.169.

$$\text{ApproxModDur}=\frac{51.330737-51.277694}{2\times 0.0001\times 51.304203}=5.169$$

Bond C

$$PV_0=50.210636$$

$$PV_+=50.185228$$

$$\frac{10}{(1.2001)^1}+\frac{10}{(1.2001)^2}+\cdots+\frac{110}{(1.2001)^{30}}=50.185228$$

$$PV_-=50.236070$$

$$\frac{10}{(1.1999)^1}+\frac{10}{(1.1999)^2}+\cdots+\frac{110}{(1.1999)^{30}}=50.236070$$

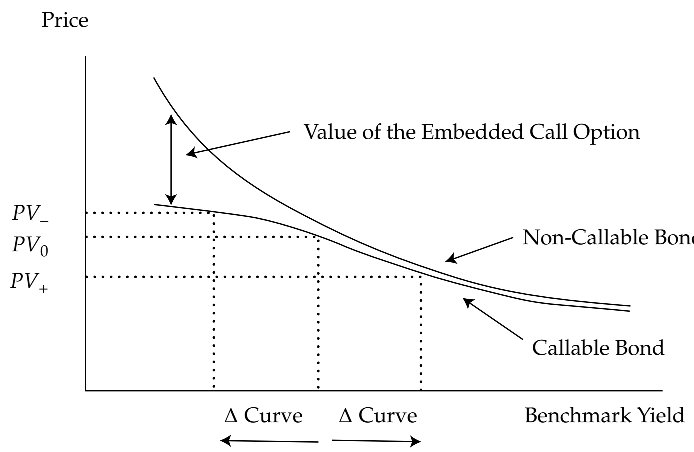
The approximate modified duration of Bond C is 5.063.

$$\text{ApproxModDur}=\frac{50.236070-50.185228}{2\times 0.0001\times 50.210636}=5.063$$

Solution to 2: Despite the significant differences in times-to-maturity (10, 20, and 30 years), the approximate modified durations on the three bonds are fairly similar (4.768, 5.169, and 5.063). Because the yields-to-maturity are so high, the additional time to receipt of interest and principal payments on the 20- and 30-year bonds have low weight. Nevertheless, Bond B, with 20 years to maturity, has the highest modified duration. If the yield-to-maturity on each is decreased by the same amount—for instance, by 10 bps, from 20% to 19.90%—Bond B would be expected to have the highest percentage price increase because it has the highest modified duration. This example illustrates the relationship between the Macaulay duration and the time-to-maturity on discount bonds in Exhibit 7. The 20-year bond has a higher duration than the 30-year bond.

Callable bonds require the use of effective duration because Macaulay and modified yield duration statistics are not relevant. The yield-to-maturity for callable bonds is not well-defined because future cash flows are uncertain. Exhibit 8 illustrates the impact of the change in the benchmark yield curve (ΔCurve) on the price of a callable bond price compared with that on a comparable non-callable bond. The two bonds have the same credit risk, coupon rate, payment frequency, and time-to-maturity. The vertical axis is the bond price. The horizontal axis is a benchmark yield—for instance, a point on the par curve for government bonds.

EXHIBIT 8 Interest Rate Risk Characteristics of a Callable Bond

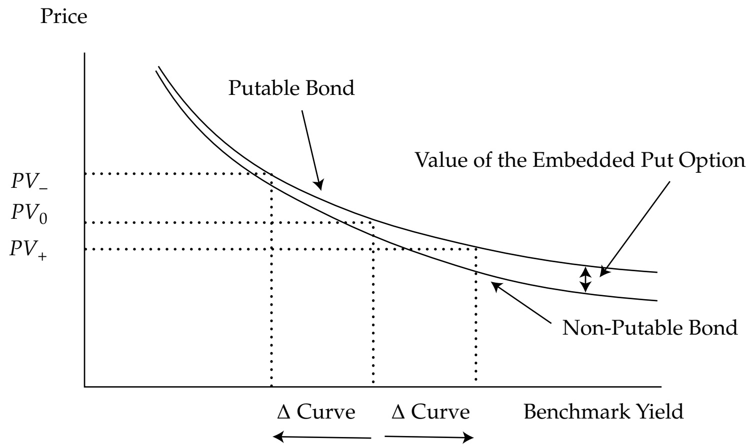


As shown in Exhibit 8, the price of the non-callable bond is always greater than that of the callable bond with otherwise identical features. The difference is the value of the embedded call option. Recall that the call option is an option to the issuer and not the holder of the bond. When interest rates are high compared with the coupon rate, the value of the call option is low. When rates are low, the value of the call option is much greater because the issuer is more likely to exercise the option to refinance the debt at a lower cost of funds. The investor bears the “call risk” because if the bond is called, the investor must reinvest the proceeds at a lower interest rate.

Exhibit 8 shows the inputs for calculating the effective duration of the callable bond. The entire benchmark curve is raised and lowered by the same amount, ΔCurve. The key point is that when benchmark yields are high, the effective durations of the callable and non-callable bonds are very similar. Although the exhibit does not illustrate it, the slopes of the lines tangent to the price–yield curve are about the same in such a situation. But when interest rates are low, the effective duration of the callable bond is lower than that of the otherwise comparable non-callable bond. That is because the callable bond price does not increase as much when benchmark yields fall. The slope of the line tangent to the price–yield curve would be flatter. The presence of the call option limits price appreciation. Therefore, an embedded call option reduces the effective duration of the bond, especially when interest rates are falling and the bond is more likely to be called. The lower effective duration can also be interpreted as a shorter expected life—the weighted average of time to receipt of cash flow is reduced.

Exhibit 9 considers another embedded option—a put option.

EXHIBIT 9 Interest Rate Risk Characteristics of a Putable Bond



A putable bond allows the investor to sell the bond back to the issuer prior to maturity, usually at par value, which protects the investor from higher benchmark yields or credit spreads that otherwise would drive the bond to a discounted price. Therefore, the price of a putable bond is always higher than that of an otherwise comparable non-putable bond. The price difference is the value of the embedded put option.

An embedded put option reduces the effective duration of the bond, especially when rates are rising. If interest rates are low compared with the coupon rate, the value of the put option is low and the impact of a change in the benchmark yield on the bond's price is very similar to the impact on the price of a non-putable bond. But when benchmark interest rates rise, the put option becomes more valuable to the investor. The ability to sell the bond at par value limits the price depreciation as rates rise. In summary, the presence of an embedded option reduces the sensitivity of the bond price to changes in the benchmark yield curve, assuming no change in credit risk.

7. DURATION OF A BOND PORTFOLIO

- **calculate the duration of a portfolio and explain the limitations of portfolio duration**

Similar to equities, bonds are typically held in a portfolio. There are two ways to calculate the duration of a bond portfolio: (1) the weighted average of time to receipt of the *aggregate* cash flows, and (2) the weighted average of the individual bond durations that comprise the portfolio. The first method is the theoretically correct approach, but it is difficult to use in practice. The second method is commonly used by fixed-income portfolio managers, but it has its own limitations. The differences in these two methods to compute portfolio duration can be examined with a numerical example.

Suppose an investor holds the following portfolio of two *zero-coupon* bonds:

Bond	Maturity	Price	Yield	Macaulay Duration	Modified Duration	Par Value	Market Value	Weight
(X)	1 year	98.00	2.0408%	1	0.980	10,000,000	9,800,000	0.50
(Y)	30 years	9.80	8.0503%	30	27.765	100,000,000	9,800,000	0.50

The prices are per 100 of par value. The yields-to-maturity are effective annual rates. The total market value for the portfolio is 19,600,000. The portfolio is evenly weighted in terms of market value between the two bonds.

The first approach views the portfolio as a series of aggregated cash flows. Its **cash flow yield** is 7.8611%. A cash flow yield is the internal rate of return on a series of cash flows, usually used on a complex security such as a mortgage-backed bond (using projected cash flows based on a model of prepayments as a result of refinancing) or a portfolio of fixed-rate bonds. It is the solution for *r* in the following equation.

$$19{,}600{,}000 = \frac{10{,}000{,}000}{(1+r)^1} + \frac{0}{(1+r)^2} + \cdots + \frac{0}{(1+r)^{29}} + \frac{100{,}000{,}000}{(1+r)^{30}}, \qquad r = 0.078611$$

The Macaulay duration of the portfolio in this approach is the weighted average of the times to receipt of aggregated cash flow. The cash flow yield is used to obtain the weights. This calculation is similar to Equation 1, and the portfolio duration is 16.2825.

$$\text{MacDur} = \left[\frac{\dfrac{1 \times 10{,}000{,}000}{(1.078611)^1} + \dfrac{30 \times 100{,}000{,}000}{(1.078611)^{30}}}{\dfrac{10{,}000{,}000}{(1.078611)^1} + \dfrac{100{,}000{,}000}{(1.078611)^{30}}}\right] = 16.2825$$

There are just two future cash flows in the portfolio—the redemption of principal on the two zero-coupon bonds. In more complex portfolios, a series of coupon and principal payments may occur on some dates, with an aggregated cash flow composed of coupon interest on some bonds and principal on those that mature.

The modified duration of the portfolio is the Macaulay duration divided by one plus the cash flow yield per period (here, the periodicity is 1).

$$\text{ModDur} = \frac{16.2825}{1.078611} = 15.0958$$

The modified duration for the portfolio is 15.0958. That statistic indicates the percentage change in the market value given a change in the cash flow yield. If the cash flow yield increases or decreases by 100 bps, the market value of the portfolio is expected to decrease or increase by about 15.0958%.

Although this approach is theoretically correct, it is difficult to use in practice. First, the cash flow yield is not commonly calculated for bond portfolios. Second, the amount and timing of future coupon and principal payments are uncertain if the portfolio contains callable or putable bonds or floating-rate notes. Third, interest rate risk is usually expressed as a change in benchmark interest rates, not as a change in the cash flow yield. Fourth, the change in the cash flow yield is not necessarily the same amount as the change in the yields-to-maturity on the individual bonds. For instance, if the yields-to-maturity on the two zero-coupon bonds in this portfolio both increase or decrease by 10 bps, the cash flow yield increases or decreases by only 9.52 bps.

In practice, the second approach to portfolio duration is commonly used. The Macaulay and modified durations for the portfolio are calculated as the weighted average of the statistics for the individual bonds. The shares of overall portfolio market value are the weights. This weighted average approximates the theoretically correct portfolio duration, which is obtained using the first approach. This approximation becomes more accurate when the differences in the yields-to-maturity on the bonds in the portfolio are smaller. When the yield curve is flat, the two approaches produce the same portfolio duration.

Given the equal "50/50" weights in this simple numerical example, this version of portfolio duration is easily computed.

$$\text{Average Macaulay duration} = (1 \times 0.50) + (30 \times 0.50) = 15.50$$

$$\text{Average modified duration} = (0.980 \times 0.50) + (27.765 \times 0.50) = 14.3725$$

Note that 0.980 = 1/1.020404 and 27.765 = 30/1.080503. An advantage of the second approach is that callable bonds, putable bonds, and floating-rate notes can be included in the weighted average using the effective durations for these securities.

The main advantage to the second approach is that it is easily used as a measure of interest rate risk. For instance, if the yields-to-maturity on the bonds in the portfolio increase by 100 bps, the estimated drop in the portfolio value is 14.3725%. However, this advantage also indicates a limitation: This measure of portfolio duration implicitly assumes a **parallel shift** in the yield curve. A parallel yield curve shift implies that all rates change by the same amount in the same direction. In reality, interest rate changes frequently result in a steeper or flatter yield curve. Yield volatility is discussed later in this chapter.

EXAMPLE 11

An investment fund owns the following portfolio of three fixed-rate government bonds:

	Bond A	Bond B	Bond C
Par value	EUR25,000,000	EUR25,000,000	EUR50,000,000
Coupon rate	9%	11%	8%
Time-to-maturity	6 years	8 years	12 years
Yield-to-maturity	9.10%	9.38%	9.62%
Market value	EUR24,886,343	EUR27,243,887	EUR44,306,787
Macaulay duration	4.761	5.633	7.652

The total market value of the portfolio is EUR96,437,017. Each bond is on a coupon date so that there is no accrued interest. The market values are the full prices given the par value. Coupons are paid semiannually. The yields-to-maturity are stated on a semiannual bond basis, meaning an annual rate for a periodicity of 2. The Macaulay durations are annualized.

1. Calculate the average (annual) modified duration for the portfolio using the shares of market value as the weights.
2. Estimate the percentage loss in the portfolio's market value if the (annual) yield-to-maturity on each bond goes up by 20 bps.

Solution to 1: The average (annual) modified duration for the portfolio is 6.0495.

$$\left(\frac{4.761}{1+\frac{0.0910}{2}}\times\frac{24{,}886{,}343}{96{,}437{,}017}\right)+\left(\frac{5.633}{1+\frac{0.0938}{2}}\times\frac{27{,}243{,}887}{96{,}437{,}017}\right)+$$
$$\left(\frac{7.652}{1+\frac{0.0962}{2}}\times\frac{44{,}306{,}787}{96{,}437{,}017}\right)=6.0495$$

Note that the annual modified duration for each bond is the annual Macaulay duration, which is given, divided by one plus the yield-to-maturity per semiannual period.

Solution to 2: The estimated decline in market value if each yield rises by 20 bps is 1.21%: $-6.0495 \times 0.0020 = -0.0121$.

8. MONEY DURATION AND THE PRICE VALUE OF A BASIS POINT

- **calculate and interpret the money duration of a bond and price value of a basis point (PVBP)**

Modified duration is a measure of the *percentage price change* of a bond given a change in its yield-to-maturity. A related statistic is **money duration**. The money duration of a bond is a measure of the *price change* in units of the currency in which the bond is denominated. The money duration can be stated per 100 of par value or in terms of the actual position size of the bond in the portfolio. In the United States, money duration is commonly called "dollar duration."

Money duration (MoneyDur) is calculated as the annual modified duration times the full price (PV^{Full}) of the bond, including accrued interest.

$$\text{MoneyDur} = \text{AnnModDur} \times PV^{Full} \quad (11)$$

The estimated change in the bond price in currency units is calculated using Equation 12, which is very similar to Equation 6. The difference is that for a given change in the annual yield-to-maturity (ΔYield), modified duration estimates the percentage price change and money duration estimates the change in currency units.

$$\Delta PV^{Full} \approx -\text{MoneyDur} \times \Delta\text{Yield} \quad (12)$$

For a theoretical example of money duration, consider the 6% semiannual coupon payment bond that matures on 14 February 2027 and is priced to yield 6.00% for settlement on 11 April 2019. The full price of the bond is 100.940423 per 100 of par value, and the annual modified duration is 6.1268. Suppose that a Nairobi-based life insurance company has a position in the bond for a par value of KES100,000,000. The market value of the investment is KES 100,940,423. The money duration of this bond is KES 618,441,784 (= 6.1268 × KES 100,940,423). Therefore, if the yield-to-maturity rises by 100 bps—from 6.00% to 7.00%—the expected loss is approximately –KES 6,184,418 (= –KES 618,441,784 × 0.0100). On a percentage basis, that expected loss is approximately 6.1268%. The "convexity adjustment" introduced in the next section makes these estimates more accurate.

Another version of money duration is the value of one basis point in price terms. The **price value of a basis point** (or PVBP) is an estimate of the change in the full bond price given a 1 bp change in the yield-to-maturity. The PVBP can be calculated using a formula similar to that for the approximate modified duration. Equation 13 is the formula for the PVBP.

$$\text{PVBP} = \frac{(PV_-) - (PV_+)}{2} \tag{13}$$

PV_- and PV_+ are the full prices calculated by decreasing and increasing the yield-to-maturity by 1 bp. The PVBP is also called the "PV01," standing for the "price value of an 01" or "present value of an 01," where "01" means 1 bp. In the United States, it is commonly called the "DV01," or the "dollar value of a 01." The PVBP is particularly useful for bonds where future cash flows are uncertain, such as callable bonds. A related statistic called a "basis point value" (BPV) is simply the money duration times 0.0001 (1 bp).

For a numerical example of the PVBP calculation, consider the 2.875% semiannual coupon payment US Treasury note that matures on 15 May 2028. In Exhibit 5, the PVBP for the Treasury note is shown to be 0.08540. Its yield-to-maturity is 2.849091%, and the settlement date is 59 days into a 184-day period. To confirm this result, calculate the new prices by increasing and decreasing the yield-to-maturity. First, increase the yield by 1 bp (0.01%), from 2.849091% to 2.859091%, to solve for a PV_+ of 100.594327.

$$PV_+ = \left[\frac{1.4375}{\left(1+\frac{0.02859091}{2}\right)^1} + \cdots + \frac{101.4375}{\left(1+\frac{0.02859091}{2}\right)^{20}}\right] \times \left(1+\frac{0.02859091}{2}\right)^{59/184}$$
$$= 101.594327$$

Then, decrease the yield-to-maturity by 1 bp, from 2.849091% to 2.839091%, to solve for a PV_- of 100.765123.

$$PV_- = \left[\frac{1.4375}{\left(1+\frac{0.02839091}{2}\right)^1} + \cdots + \frac{101.4375}{\left(1+\frac{0.02839091}{2}\right)^{20}}\right] \times \left(1+\frac{0.02839091}{2}\right)^{59/184}$$
$$= 100.765123$$

The PVBP is obtained by substituting these results into Equation 13.

$$\text{PVBP} = \frac{100.765123 - 100.594327}{2} = 0.08540$$

Another money duration statistic reported on the Bloomberg YAS page is "risk." It is shown to be 8.540. Bloomberg's risk statistic is simply the PVBP (or PV01) times 100.

EXAMPLE 12

A life insurance company holds a USD10 million (par value) position in a 5.95% Dominican Republic bond that matures on 25 January 2027. The bond is priced (flat) at 101.996 per 100 of par value to yield 5.6511% on a street-convention semiannual bond basis for settlement on 24 July 2018. The total market value of the position, including accrued interest, is USD10,495,447, or 101.95447 per 100 of par value. The bond's (annual) Macaulay duration is 6.622.

1. Calculate the money duration per 100 in par value for the sovereign bond.
2. Using the money duration, estimate the loss on the position for each 1 bp increase in the yield-to-maturity for that settlement date.

Solution to 1: The money duration is the annual modified duration times the full price of the bond per 100 of par value.

$$\left(\frac{6.622}{1+\frac{0.056511}{2}}\right) \times \text{USD}104.95447 = \text{USD}675.91$$

Solution to 2: For each 1 bp increase in the yield-to-maturity, the loss is estimated to be USD 0.067591 per 100 of par value: USD 675.91 × 0.0001 = USD 0.067591.

Given a position size of USD 10 million in par value, the estimated loss per basis-point increase in the yield is USD 6,759.10. The money duration is per 100 of par value, so the position size of USD10 million is divided by USD 100.

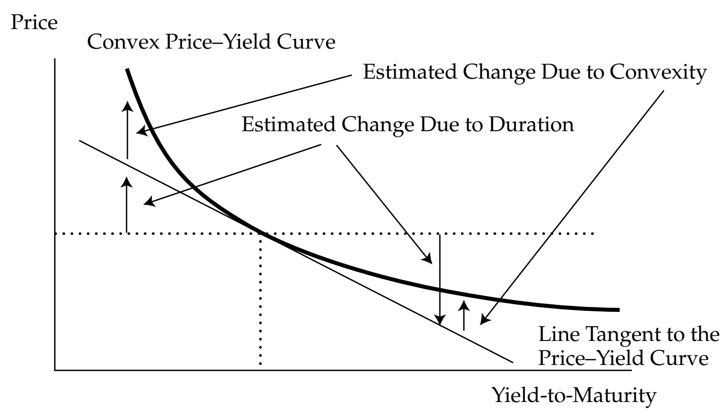
$$\text{USD}0.067591 \times \frac{\text{USD}10{,}000{,}000}{\text{USD}100} = \text{USD}6{,}759.10$$

9. BOND CONVEXITY

- **calculate and interpret approximate convexity and compare approximate and effective convexity**
- **calculate the percentage price change of a bond for a specified change in yield, given the bond's approximate duration and convexity**

Modified duration measures the primary effect on a bond's percentage price change given a change in the yield-to-maturity. A secondary effect is measured by the convexity statistic, which is illustrated in Exhibit 10 for a traditional (option-free) fixed-rate bond.

EXHIBIT 10 Convexity of a Traditional (Option-Free) Fixed-Rate Bond



The true relationship between the bond price and the yield-to-maturity is the curved (convex) line shown in Exhibit 10. This curved line shows the actual bond price given its market discount rate. Duration (in particular, money duration) estimates the change in the bond price along the straight line that is tangent to the curved line. For small yield-to-maturity changes, there is little difference between the lines. But for larger changes, the difference becomes significant.

The convexity statistic for the bond is used to improve the estimate of the percentage price change provided by modified duration alone. Equation 14 is the convexity-adjusted estimate of the percentage change in the bond's full price.

$$\%\Delta PV^{Full} \approx (-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right] \quad (14)$$

The first bracketed expression, the "first-order" effect, is the same as Equation 6. The (annual) modified duration, AnnModDur, is multiplied by the change in the (annual) yield-to-maturity, ΔYield. The second bracketed expression, the "second-order" effect, is the **convexity adjustment**. The convexity adjustment is the annual convexity statistic, AnnConvexity, times one-half, multiplied by the change in the yield-to-maturity *squared*. This additional term is a positive amount on a traditional (option-free) fixed-rate bond for either an increase or decrease in the yield. In Exhibit 10, this amount adds to the linear estimate provided by the duration alone, which brings the adjusted estimate very close to the actual price on the curved line. But it still is an estimate, so the ≈ sign is used.

Similar to the Macaulay and modified durations, the annual convexity statistic can be calculated in several ways. It can be calculated using tables, such as Exhibits 2 and 3. It also is possible to derive a closed-form equation for the convexity of a fixed-rate bond on and between coupon payment dates using calculus and algebra (see D. Smith, 2014). But like modified duration, convexity can be approximated with accuracy. Equation 15 is the formula for the approximate convexity statistic, ApproxCon.

$$\text{ApproxCon} = \frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta\text{Yield})^2 \times (PV_0)} \quad (15)$$

This equation uses the same inputs as Equation 7 for ApproxModDur. The new price when the yield-to-maturity is increased is PV_+. The new price when the yield is decreased by the same amount is PV_-. The original price is PV_0. These are the full prices, including accrued interest, for the bond.

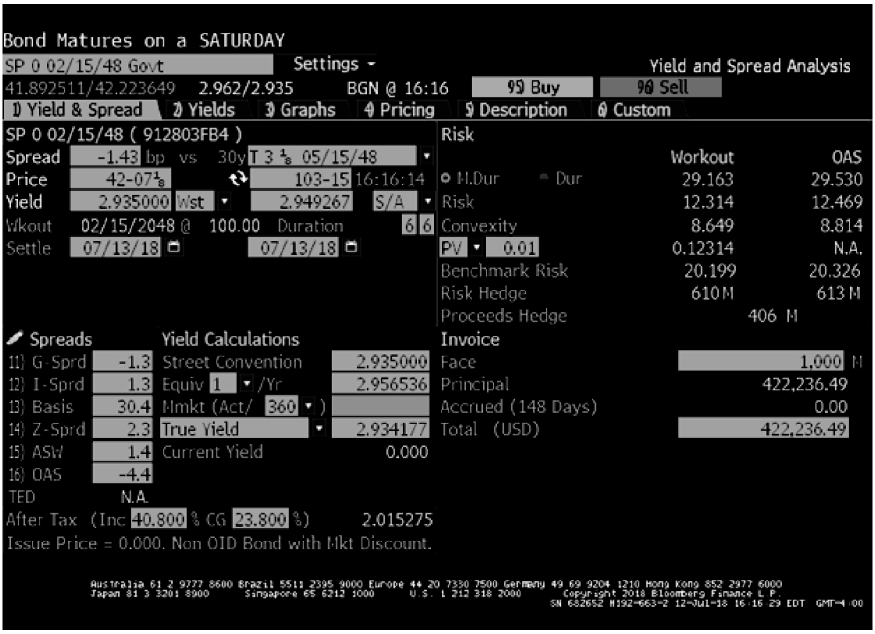
The accuracy of this approximation can be demonstrated with the special case of a zero-coupon bond. The absence of coupon payments simplifies the interest rate risk measures. The Macaulay duration of a zero-coupon bond is $N - t/T$ in terms of periods to maturity. The exact convexity statistic of a zero-coupon bond, also in terms of periods, is calculated with Equation 16.

$$\text{Convexity (of a zero-coupon bond)} = \frac{[N - (t/T)] \times [N + 1 - (t/T)]}{(1 + r)^2} \quad (16)$$

N is the number of periods to maturity as of the beginning of the current period, t/T is the fraction of the period that has gone by, and r is the yield-to-maturity per period.

For an example of this calculation, consider a long-term, zero-coupon US Treasury bond. The bond's Bloomberg YAS page is shown in Exhibit 11.

EXHIBIT 11 Bloomberg YAS Page for the Zero-Coupon US Treasury Bond



The bond matures on 15 February 2048 and its asked price was 42.223649 per 100 of par value for settlement on 13 July 2018. Its yield-to-maturity was 2.935% stated on a street-convention semiannual bond basis. Even though it is a zero-coupon bond, its yield-to-maturity is based on the actual/actual day-count convention. That settlement date was 148 days into a 181-day period. The annual modified duration was 29.163.

For this bond, $N = 60$, $t/T = 148/181$, and $r = 0.02935/2$. Entering these variables into Equation 16 produces a convexity of 3,459.45 in terms of semiannual periods.

$$\frac{[60 - (148/181)] \times [60 + 1 - (148/181)]}{\left(1 + \frac{0.02935}{2}\right)^2} = 3{,}459.45$$

As with the other statistics, convexity is annualized in practice and for use in the convexity adjustment in Equation 14. It is divided by the periodicity *squared*. The yield-to-maturity on this zero-coupon bond is stated on a semiannual bond basis, meaning a periodicity of 2. Therefore, the annualized convexity statistic is 864.9.

$$\frac{3{,}459.45}{4} = 864.9$$

For example, suppose that the yield-to-maturity is expected to fall by 10 bps, from 2.935% to 2.835%. Given the (annual) modified duration of 29.163 and (annual) convexity of 864.9, the expected percentage price gain is 2.9595%.

$$\begin{aligned}\%\Delta PV^{Full} &\approx [-29.163 \times -0.0010] + \left[\frac{1}{2} \times 864.9 \times (-0.0010)^2\right] \\ &= 0.029163 + 0.000432 \\ &= 0.029595\end{aligned}$$

Modified duration alone (under)estimates the gain to be 2.9163%. The convexity adjustment adds 4.32 bps.

The long-term, zero-coupon bond of Exhibit 11 demonstrates the difference between *yield* duration and convexity and *curve* duration and convexity, even on an option-free bond. Its modified duration is 29.163, whereas its effective duration is 29.530. Its yield convexity is reported on the Bloomberg page to be 8.649, and its effective convexity is 8.814. (Note that although Bloomberg scales the convexity statistics by dividing by 100, either raw or scaled convexity figures are acceptable in practice.) In general, the differences are heightened when the benchmark yield curve is not flat, when the bond has a long time-to-maturity, and the bond is priced at a significant discount or premium.

To obtain the ApproxCon for this long-term, zero-coupon bond, calculate PV_0, PV_+, and PV_- for yields-to-maturity of 2.935%, 2.945%, and 2.925%, respectively. For this exercise, $\Delta\text{Yield} = 0.0001$.

$$PV_0 = \frac{100}{\left(1 + \frac{0.02935}{2}\right)^{60}} \times \left(1 + \frac{0.02935}{2}\right)^{148/181} = 42.223649$$

$$PV_+ = \frac{100}{\left(1 + \frac{0.02945}{2}\right)^{60}} \times \left(1 + \frac{0.02945}{2}\right)^{148/181} = 42.100694$$

$$PV_- = \frac{100}{\left(1 + \frac{0.02925}{2}\right)^{60}} \times \left(1 + \frac{0.02925}{2}\right)^{148/181} = 42.346969$$

Using these results, first calculate ApproxModDur using Equation 7 to confirm that these inputs are correct. In Exhibit 11, modified duration is stated to be 29.163.

$$\text{ApproxModDur} = \frac{42.346969 - 42.100694}{2 \times 0.0001 \times 42.223649} = 29.163$$

Using Equation 15, ApproxCon is 864.9.

$$\text{ApproxCon} = \frac{42.346969 + 42.100694 - (2 \times 42.223649)}{(0.0001)^2 \times 42.223649} = 864.9$$

This result, 864.9, is an approximation for *annualized* convexity. The number of periods in the year is included in the price calculations. This approximation in this example is the same as the "exact" result using the closed-form equation for the special case of the zero-coupon bond. Any small difference is not likely to be meaningful for practical applications.

Because this is an individual zero-coupon bond, it is easy to calculate the new price if the yield-to-maturity does go down by 50 bps, to 2.435%.

$$\frac{100}{\left(1 + \frac{0.02435}{2}\right)^{60}} \times \left(1 + \frac{0.02435}{2}\right)^{148/181} = 48.860850$$

Therefore, the actual percentage price increase is 15.7192%.

$$\frac{48.860850 - 42.223649}{42.223649} = 0.157192$$

The convexity-adjusted estimate is 15.6626%.

$$
\begin{aligned}
\%\Delta PV^{Full} &\approx (-29.163 \times -0.0050) + \left[\frac{1}{2} \times 864.9 \times (-0.0050)^2\right] \\
&= 0.145815 + 0.010811 \\
&= 0.156626
\end{aligned}
$$

EXAMPLE 13

A Dutch bank holds a large position in a zero-coupon Federal Republic of Germany government bond maturing on 11 April 2025. The yield-to-maturity is –0.72% for settlement on 11 May 2020, stated as an effective annual rate on an Actual/Actual basis. That settlement date is 30 days into the 365-day year using this day count method.

1. Calculate the full price of the bond per 100 of par value.
2. Calculate the approximate modified duration and approximate convexity using a 1 bp increase and decrease in the yield-to-maturity.
3. Calculate the estimated convexity-adjusted percentage price change resulting from a 100 bp increase in the yield-to-maturity.
4. Compare the estimated percentage price change with the actual change, assuming the yield-to-maturity jumps 100 bps to 0.28% on that settlement date.

Solution to 1: There are five years from the beginning of the current period on 11 April 2020 to maturity on 11 April 2025.

The full price of the bond is 103.617526 per 100 of par value. Note that

$$1 + r = 1 + (-0.0072) = 0.9928$$

$$PV_0 = \left[\frac{100}{(0.9928)^5}\right] \times (0.9928)^{30/365} = 103.617526$$

Solution to 2: $PV_+ = 103.566215$, and $PV_- = 103.668868$.

$$PV_+ = \left[\frac{100}{(0.9929)^5}\right] \times (0.9929)^{30/365} = 103.566215$$

$$PV_- = \left[\frac{100}{(0.9927)^5}\right] \times (0.9927)^{30/365} = 103.668868$$

The approximate modified duration is 4.9535.

$$\text{ApproxModDur} = \frac{103.668868 - 103.566215}{2 \times 0.0001 \times 103.617526} = 4.9535$$

The approximate convexity is 29.918.

$$\text{ApproxCon} = \frac{103.668868 + 103.566215 - (2 \times 103.617526)}{(0.0001)^2 \times 103.617526} = 29.918$$

Solution to 3: The convexity-adjusted percentage price drop resulting from a 100 bp increase in the yield-to-maturity is estimated to be 4.80391%. Modified duration alone estimates the percentage drop to be 4.9535%. The convexity adjustment adds 14.96 bps.

$$\begin{aligned}\%\Delta PV^{Full} &\approx -(4.9535 \times 0.0100) + \left[\frac{1}{2} \times 29.918 \times (-0.0100)^2\right] \\ &= -0.049535 + 0.001496 \\ &= -0.0480391\end{aligned}$$

Solution to 4: The new full price if the yield-to-maturity goes from −0.72% to 0.28% on that settlement date is 98.634349.

$$PF^{Full} = \left[\frac{100}{(1.0028)^5}\right] \times (1.0028)^{30/365} = 98.634349$$

$$\%\Delta PV^{Full} = \frac{98.634349 - 103.617526}{103.617526} = -0.04809203$$

The actual percentage change in the bond price is −4.809203%. The convexity-adjusted estimate is −4.80391%, whereas the estimated change using modified duration alone is −4.9535%.

The money duration of a bond indicates the first-order effect on the full price of a bond in units of currency given a change in the yield-to-maturity. The **money convexity** statistic (MoneyCon) is the second-order effect. The money convexity of the bond is the annual convexity multiplied by the full price, such that

$$\Delta PV^{Full} \approx -(\text{MoneyDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{MoneyCon} \times (\Delta\text{Yield})^2\right] \tag{17}$$

For a money convexity example, consider again the Nairobi-based life insurance company that has a KES100 million position in the 6.00% bond that matures on 14 February 2027. Previously, using the money duration alone, the estimated loss was KES6,184,418 if the yield-to-maturity increased by 100 bps. The money duration for the position is KES618,441,784. That estimation is improved by including the convexity adjustment. Given the approximate modified duration of 6.1268 for a 5 bp change in the yield-to-maturity (ΔYield = 0.0005) and given that PV_0 = 100.940423, PV_+ = 100.631781, and PV_- = 101.250227, we use Equation 15 to calculate the approximate convexity:

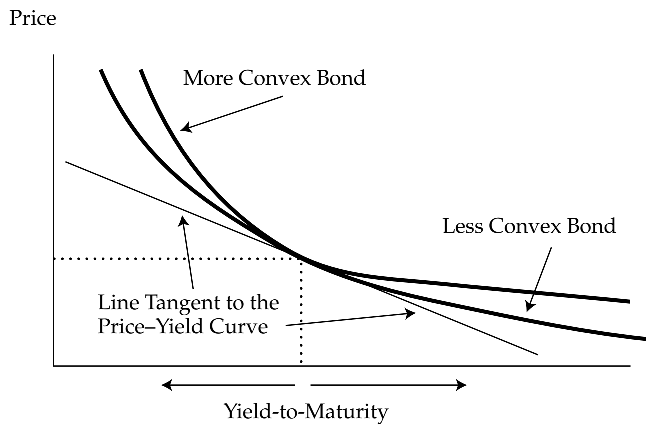
$$\text{ApproxCon} = \frac{101.250227 + 100.631781 - (2 \times 100.940423)}{(0.0005)^2 \times 100.940423} = 46.047$$

The money convexity is 46.047 times the market value of the position, KES100,940,423. The convexity-adjusted loss given a 100 bp jump in the yield-to-maturity is KES5,952,018:

$$\begin{aligned}&-[(6.1268 \times \text{KES}100{,}940{,}423) \times 0.0100] + \\ &\left[\frac{1}{2} \times (46.047 \times \text{KES}100{,}940{,}423) \times (0.0100)^2\right] \\ &= -\text{KES}6{,}184{,}418 + \text{KES}232{,}400 \\ &= -\text{KES}5{,}952{,}018\end{aligned}$$

The factors that lead to greater convexity are the same as for duration. A fixed-rate bond with a longer time-to-maturity, a lower coupon rate, and a lower yield-to-maturity has greater convexity than a bond with a shorter time-to-maturity, a higher coupon rate, and a higher yield-to-maturity. Another factor is the dispersion of cash flows, meaning the degree to which payments are spread out over time. If two bonds have the same duration, the one that has the greater dispersion of cash flows has the greater convexity. The positive attributes of greater convexity for an investor are shown in Exhibit 12.

EXHIBIT 12 The Positive Attributes of Greater Bond Convexity on a Traditional (Option-Free) Bond



The two bonds in Exhibit 12 are assumed to have the same price, yield-to-maturity, and modified duration. Therefore, they share the same line tangent to their price–yield curves. The benefit of greater convexity occurs when their yields-to-maturity change. For the same decrease in yield-to-maturity, the more convex bond *appreciates more* in price. And for the same increase in yield-to-maturity, the more convex bond *depreciates less* in price. The conclusion is that the more convex bond outperforms the less convex bond in both bull (rising price) and bear (falling price) markets. This conclusion assumes, however, that this positive attribute is not "priced into" the bond. To the extent that it is included, the more convex bond would have a higher price (and lower yield-to-maturity). That does not diminish the value of convexity. It only suggests that the investor has to pay for it. As economists say, "There is no such thing as a free lunch."

EXAMPLE 14

The investment manager for a UK defined-benefit pension scheme is considering two bonds about to be issued by a large life insurance company. The first is a 30-year, 4% semiannual coupon payment bond. The second is a 100-year, 4% semiannual coupon payment "century" bond. Both bonds are expected to trade at par value at issuance.

Calculate the approximate modified duration and approximate convexity for each bond using a 5 bp increase and decrease in the annual yield-to-maturity. Retain accuracy to six decimals per 100 of par value.

Solution: In the calculations, the yield per semiannual period goes up by 2.5 bps to 2.025% and down by 2.5 bps to 1.975%. The 30-year bond has an approximate modified duration of 17.381 and an approximate convexity of 420.80.

$$PV_+ = \frac{2}{(1.02025)^1} + \cdots + \frac{102}{(1.02025)^{60}} = 99.136214$$

$$PV_- = \frac{2}{(1.01975)^1} + \cdots + \frac{102}{(1.01975)^{60}} = 100.874306$$

$$\text{ApproxModDur} = \frac{100.874306 - 99.136214}{2 \times 0.0005 \times 100} = 17.381$$

$$\text{ApproxCon} = \frac{100.874306 + 99.136214 - (2 \times 100)}{(0.0005)^2 \times 100} = 420.80$$

The 100-year century bond has an approximate modified duration of 24.527 and an approximate convexity of 1,132.88.

$$PV_+ = \frac{2}{(1.02025)^1} + \cdots + \frac{102}{(1.02025)^{200}} = 98.787829$$

$$PV_- = \frac{2}{(1.01975)^1} + \cdots + \frac{102}{(1.01975)^{200}} = 101.240493$$

$$\text{ApproxModDur} = \frac{101.240493 - 98.787829}{2 \times 0.0005 \times 100} = 24.527$$

$$\text{ApproxCon} = \frac{101.240493 + 98.787829 - (2 \times 100)}{(0.0005)^2 \times 100} = 1{,}132.88$$

The century bond offers a higher modified duration, 24.527 compared with 17.381, and a much greater degree of convexity, 1,132.88 compared with 420.80.

In the same manner that the primary, or first-order, effect of a shift in the benchmark yield curve is measured by effective duration, the secondary, or second-order, effect is measured by **effective convexity**. The effective convexity of a bond is a *curve convexity* statistic that measures the secondary effect of a change in a benchmark yield curve. A pricing model is used to determine the new prices when the benchmark curve is shifted upward (PV_+) and downward (PV_-) by the same amount (ΔCurve). These changes are made holding other factors constant—for example, the credit spread. Then, Equation 18 is used to calculate the effective convexity (EffCon) given the initial price (PV_0).

$$\text{EffCon} = \frac{[(PV_-) + (PV_+)] - [2 \times (PV_0)]}{(\Delta\text{Curve})^2 \times (PV_0)} \tag{18}$$

This equation is very similar to Equation 15, for approximate *yield* convexity. The difference is that in Equation 15, the denominator includes the change in the yield-to-maturity squared, $(\Delta\text{Yield})^2$. Here, the denominator includes the change in the benchmark yield curve squared, $(\Delta\text{Curve})^2$.

Consider again the callable bond example in our initial discussion of effective duration. It is assumed that an option-pricing model is used to generate these callable bond prices: PV_0 = 101.060489, PV_+ = 99.050120, PV_- = 102.890738, and ΔCurve = 0.0025. The effective duration for the callable bond is 7.6006.

$$\text{EffDur} = \frac{102.890738 - 99.050120}{2 \times 0.0025 \times 101.060489} = 7.6006$$

Using these inputs in Equation 18, the effective convexity is −285.17.

$$\text{EffCon} = \frac{102.890738 + 99.050120 - (2 \times 101.060489)}{(0.0025)^2 \times 101.060489} = -285.17$$

Negative convexity, which could be called "concavity," is an important feature of callable bonds. Putable bonds, on the other hand, always have positive convexity. As a second-order effect, effective convexity indicates the change in the first-order effect (i.e., effective duration) as the benchmark yield curve is changed. In Exhibit 8, as the benchmark yield goes down, the slope of the line tangent to the curve for the non-callable bond steepens, which indicates positive convexity. But the slope of the line tangent to the callable bond flattens as the benchmark yield goes down. Technically, it reaches an inflection point, which is when the effective convexity shifts from positive to negative.

In summary, when the benchmark yield is high and the value of the embedded call option is low, the callable and the non-callable bonds experience very similar effects from interest rate changes. They both have positive convexity. But as the benchmark yield is reduced, the curves diverge. At some point, the callable bond moves into the range of negative convexity, which indicates that the embedded call option has more value to the issuer and is more likely to be exercised. This situation limits the potential price appreciation of the bond arising from lower interest rates, whether because of a lower benchmark yield or a lower credit spread.

Another way to understand why a callable bond can have negative convexity is to rearrange Equation 18.

$$\text{EffCon} = \frac{[(PV_-) - (PV_0)] - [(PV_0) - (PV_+)]}{(\Delta\text{Curve})^2 \times (PV_0)}$$

In the numerator, the first bracketed expression is the increase in price when the benchmark yield curve is lowered. The second expression is the decrease in price when the benchmark yield curve is raised. On a non-callable bond, the increase is always larger than the decrease (in absolute value). This result is the "convexity effect" for the relationship between bond prices and yields-to-maturity. On a callable bond, the increase can be smaller than the decrease (in absolute value). That creates negative convexity, as illustrated in Exhibit 8.

10. INVESTMENT HORIZON, MACAULAY DURATION, AND INTEREST RATE RISK

- **describe how the term structure of yield volatility affects the interest rate risk of a bond**
- **describe the relationships among a bond's holding period return, its duration, and the investment horizon**

This section explores the effect of yield volatility on the investment horizon, and on the interaction between the investment horizon, market price risk, and coupon reinvestment risk.

10.1. Yield Volatility

An important aspect in understanding the interest rate risk and return characteristics of an investment in a fixed-rate bond is the time horizon. This section considers a short-term horizon. A primary concern for the investor is the change in the price of the bond given a sudden (i.e., same-day) change in its yield-to-maturity. The accrued interest does not change, so the impact of the change in the yield is on the flat price of the bond. Next, we consider a long-term horizon. The reinvestment of coupon interest then becomes a key factor in the investor's horizon yield.

Bond duration is the primary measure of risk arising from a change in the yield-to-maturity. Convexity is the secondary risk measure. In the discussion of the impact on the bond price, the phrase "for a *given* change in the yield-to-maturity" is used repeatedly. For instance, the given change in the yield-to-maturity could be 1 bp, 25 bps, or 100 bps. In comparing two bonds, it is assumed that the "given change" is the same for both securities. When the government bond par curve is shifted up or down by the same amount to calculate effective duration and effective convexity, the events are described as "parallel" yield curve shifts. Because yield curves are rarely (if ever) straight lines, this shift may also be described as a "shape-preserving" shift to the yield curve. The key assumption is that all yields-to-maturity under consideration rise or fall by the same amount across the curve.

Although the assumption of a parallel shift in the yield curve is common in fixed-income analysis, it is not always realistic. The shape of the yield curve changes based on factors affecting the supply and demand of shorter-term versus longer-term securities. In fact, the term structure of bond yields (also called the "term structure of interest rates") is typically upward sloping. However, the **term structure of yield volatility** may have a different shape depending on a number of factors. The term structure of yield volatility is the relationship between the volatility of bond yields-to-maturity and times-to-maturity.

For example, a central bank engaging in expansionary monetary policy might cause the yield curve to steepen by reducing short-term interest rates. But this policy might cause greater *volatility* in short-term bond yields-to-maturity than in longer-term bonds, resulting in a downward-sloping term structure of yield volatility. Longer-term bond yields are mostly determined by future inflation and economic growth expectations. Those expectations often tend to be less volatile.

The importance of yield volatility in measuring interest rate risk is that bond price changes are products of two factors: (1) the impact *per* basis-point change in the yield-to-maturity and (2) the *number* of basis points in the yield-to-maturity change. The first factor is duration or the combination of duration and convexity, and the second factor is the yield volatility. For example, consider a 5-year bond with a modified duration of 4.5 and a 30-year bond with a modified duration of 18.0. Clearly, for a *given* change in yield-to-maturity, the 30-year bond represents much more interest rate risk to an investor who has a short-term horizon. In fact, the 30-year bond appears to have *four times* the risk given the ratio of the modified durations. But that assumption neglects the possibility that the 30-year bond might have half the yield volatility of the 5-year bond.

Equation 14, restated here, summarizes the two factors.

$$\%\Delta PV^{Full} \approx (-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right]$$

The estimated percentage change in the bond price depends on the modified duration and convexity as well as on the yield-to-maturity change. Parallel shifts between two bond yields and along a benchmark yield curve are common assumptions in fixed-income analysis. However, an analyst must be aware that non-parallel shifts frequently occur in practice.

EXAMPLE 15

A fixed-income analyst is asked to rank three bonds in terms of interest rate risk. Interest rate risk here means the potential price decrease on a percentage basis given a sudden change in financial market conditions. The increases in the yields-to-maturity represent the "worst case" for the scenario being considered.

Bond	Modified Duration	Convexity	ΔYield
A	3.72	12.1	25 bps
B	5.81	40.7	15 bps
C	12.39	158.0	10 bps

The modified duration and convexity statistics are annualized. ΔYield is the increase in the annual yield-to-maturity. Rank the bonds in terms of interest rate risk.

Solution: Calculate the estimated percentage price change for each bond:

Bond A

$$(-3.72 \times 0.0025) + \left[\frac{1}{2} \times 12.1 \times (0.0025)^2\right] = -0.009262$$

Bond B

$$(-5.81 \times 0.0015) + \left[\frac{1}{2} \times 40.7 \times (0.0015)^2\right] = -0.008669$$

Bond C

$$(-12.39 \times 0.0010) + \left[\frac{1}{2} \times 158.0 \times (0.0010)^2\right] = -0.012311$$

Based on these assumed changes in the yield-to-maturity and the modified duration and convexity risk measures, Bond C has the highest degree of interest rate risk (a potential loss of 1.2311%), followed by Bond A (a potential loss of 0.9262%) and Bond B (a potential loss of 0.8669%).

10.2. Investment Horizon, Macaulay Duration, and Interest Rate Risk

Although short-term interest rate risk is a concern to some investors, other investors have a long-term horizon. Day-to-day changes in bond prices cause *unrealized* capital gains and losses. Those unrealized gains and losses might need to be accounted for in financial statements. This section considers a long-term investor concerned only with the total return over the investment horizon. Therefore, interest rate risk is important to this investor. The investor faces coupon reinvestment risk as well as market price risk if the bond needs to be sold prior to maturity.

Earlier, we discussed examples of interest rate risk using a 10-year, 8% annual coupon payment bond that is priced at 85.503075 per 100 of par value. The bond's yield-to-maturity is 10.40%. A key result in Example 3 is that an investor with a 10-year time horizon is concerned only with coupon reinvestment risk. This situation assumes, of course, that the issuer makes all of the coupon and principal payments as scheduled. The buy-and-hold investor has a higher total return if interest rates rise (see Example 3) and a lower total return if rates fall (see

Example 5). The investor in Examples 4 and 6 has a four-year horizon. This investor faces market price risk in addition to coupon reinvestment risk. In fact, the market price risk dominates because this investor has a higher total return if interest rates fall (see Example 6) and a lower return if rates rise (see Example 4).

Now, consider a third investor who has a seven-year time horizon. If interest rates remain at 10.40%, the future value of reinvested coupon interest is 76.835787 per 100 of par value.

$$[8 \times (1.1040)^6] + [8 \times (1.1040)^5] + [8 \times (1.1040)^4] + [8 \times (1.1040)^3] +$$
$$[8 \times (1.1040)^2] + [8 \times (1.1040)^1] + 8 = 76.835787$$

The bond is sold for a price of 94.073336, assuming that the bond stays on the constant-yield price trajectory and continues to be "pulled to par."

$$\frac{8}{(1.1040)^1} + \frac{8}{(1.1040)^2} + \frac{108}{(1.1040)^3} = 94.073336$$

The total return is 170.909123 (= 76.835787 + 94.073336) per 100 of par value, and the horizon yield, as expected, is 10.40%.

$$85.503075 = \frac{170.909123}{(1+r)^7}, \quad r = 0.1040$$

Following Examples 3 and 4, assume that the yield-to-maturity on the bond rises to 11.40%. Also, coupon interest is now reinvested each year at 11.40%. The future value of reinvested coupons becomes 79.235183 per 100 of par value.

$$[8 \times (1.1140)^6] + [8 \times (1.1140)^5] + [8 \times (1.1140)^4] + [8 \times (1.1140)^3] +$$
$$[8 \times (1.1140)^2] + [8 \times (1.1140)^1] + 8 = 79.235183$$

After receiving the seventh coupon payment, the bond is sold. There is a capital loss because the price, although much higher than at purchase, is below the constant-yield price trajectory.

$$\frac{8}{(1.1140)^1} + \frac{8}{(1.1140)^2} + \frac{108}{(1.1140)^3} = 91.748833$$

The total return is 170.984016 (= 79.235183 + 91.748833) per 100 of par value and the holding-period rate of return is 10.407%.

$$85.503075 = \frac{170.984016}{(1+r)^7}, \quad r = 0.10407$$

Following Examples 5 and 6, assume that the coupon reinvestment rates and the bond yield-to-maturity fall to 9.40%. The future value of reinvested coupons is 74.512177.

$$[8 + (1.0940)^6] + [8 + (1.0940)^5] + [8 + (1.0940)^4] + [8 + (1.0940)^3] +$$
$$[8 + (1.0940)^2] + [8 + (1.0940)^1] + 8 = 74.52177$$

The bond is sold at a capital gain because the price is above the constant-yield price trajectory.

$$\frac{8}{(1.0940)^1} + \frac{8}{(1.0940)^2} + \frac{108}{(1.0940)^3} = 96.481299$$

The total return is 170.993476 (= 74.512177 + 96.481299) per 100 of par value, and the horizon yield is 10.408%.

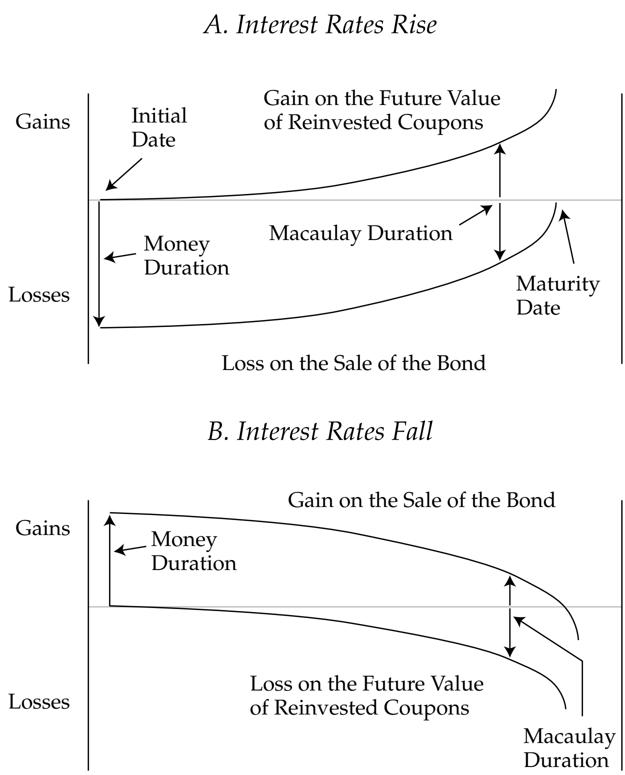
$$85.503075 = \frac{170.993476}{(1+r)^7}, \quad r = 0.10408$$

These results are summarized in the following table to reveal the remarkable outcome: The total returns and horizon yields are virtually the same. The investor with the 7-year horizon, unlike those having a 4- or 10-year horizon, achieves the same holding-period rate of return whether interest rates rise, fall, or remain the same. Note that the terms "horizon yield" and "holding-period rate of return" are used interchangeably in this chapter. Sometimes "horizon yield" refers to yields on bonds that need to be sold at the end of the investor's holding period.

Interest Rate	Future Value of Reinvested Coupon	Sale Price	Total Return	Horizon Yield
9.40%	74.512177	96.481299	170.993476	10.408%
10.40%	76.835787	94.073336	170.909123	10.400%
11.40%	79.235183	91.748833	170.984016	10.407%

This particular bond was chosen as an example to demonstrate an important property of Macaulay duration: For a particular assumption about yield volatility, Macaulay duration indicates the investment horizon for which coupon reinvestment risk and market price risk offset each other. In Exhibit 2, the Macaulay duration of this 10-year, 8% annual payment bond is calculated to be 7.0029 years. This is one of the applications for duration in which "years" is meaningful and in which Macaulay duration is used rather than modified duration. The particular assumption about yield volatility is that there is a one-time "parallel" shift in the yield curve that occurs before the next coupon payment date. Exhibit 13 illustrates this property of bond duration, assuming that the bond is initially priced at par value.

EXHIBIT 13 Interest Rate Risk, Macaulay Duration, and the Investment Horizon



As demonstrated in Panel A of Exhibit 13, when interest rates rise, duration measures the immediate drop in value. In particular, the money duration indicates the change in price. Then as time passes, the bond price is "pulled to par." The gain in the future value of reinvested coupons starts small but builds over time as more coupons are received. The curve indicates the additional future value of reinvested coupons because of the higher interest rate. At some point in the lifetime of the bond, those two effects offset each other and the gain on reinvested coupons is equal to the loss on the sale of the bond. That point in time is the Macaulay duration statistic.

The same pattern is displayed in the Panel B when interest rates fall, which leads to a reduction in the bond yield and the coupon reinvestment rate. There is an immediate jump in the bond price, as measured by the money duration, but then the "pull to par" effect brings the price down as time passes. The impact from reinvesting at a lower rate starts small but then becomes more significant over time. The loss on reinvested coupons is with respect to the future value if interest rates had not fallen. Once again, the bond's Macaulay duration indicates the point in time when the two effects offset each other and the gain on the sale of the bond matches the loss on coupon reinvestment.

The earlier numerical example and Exhibit 13 allow for a statement of the general relationships among interest rate risk, the Macaulay duration, and the investment horizon.

1. When the investment horizon is greater than the Macaulay duration of a bond, coupon reinvestment risk dominates market price risk. The investor's risk is to lower interest rates.
2. When the investment horizon is equal to the Macaulay duration of a bond, coupon reinvestment risk offsets market price risk.
3. When the investment horizon is less than the Macaulay duration of the bond, market price risk dominates coupon reinvestment risk. The investor's risk is to higher interest rates.

In the numerical example, the Macaulay duration of the bond is 7.0 years. Statement 1 reflects the investor with the 10-year horizon; Statement 2, the investor with the 7-year horizon; and Statement 3, the investor with the 4-year horizon.

The difference between the Macaulay duration of a bond and the investment horizon is called the **duration gap**. The duration gap is a bond's Macaulay duration minus the investment horizon. The investor with the 10-year horizon has a negative duration gap and currently is at risk of lower rates. The investor with the 7-year horizon has a duration gap of zero and currently is hedged against interest rate risk. The investor with the 4-year horizon has a positive duration gap and currently is at risk of higher rates. The word "currently" is important because interest rate risk is connected to an *immediate* change in the bond's yield-to-maturity and the coupon reinvestment rates. As time passes, the investment horizon is reduced and the Macaulay duration of the bond also changes. Therefore, the duration gap changes as well.

EXAMPLE 16

An investor plans to retire in 10 years. As part of the retirement portfolio, the investor buys a newly issued, 12-year, 8% annual coupon payment bond. The bond is purchased at par value, so its yield-to-maturity is 8.00% stated as an effective annual rate.

1. Calculate the approximate Macaulay duration for the bond, using a 1 bp increase and decrease in the yield-to-maturity and calculating the new prices per 100 of par value to six decimal places.

2. Calculate the duration gap at the time of purchase.
3. Does this bond at purchase entail the risk of higher or lower interest rates? Interest rate risk here means an immediate, one-time, parallel yield curve shift.

Solution to 1: The approximate modified duration of the bond is 7.5361. $PV_0 = 100$, $PV_+ = 99.924678$, and $PV_- = 100.075400$.

$$PV_+ = \frac{8}{(1.0801)^1} + \cdots + \frac{108}{(1.0801)^{12}} = 99.924678$$

$$PV_- = \frac{8}{(1.0799)^1} + \cdots + \frac{108}{(1.0799)^{12}} = 100.075400$$

$$\text{ApproxModDur} = \frac{100.075400 - 99.924678}{2 \times 0.0001 \times 100} = 7.5361$$

The approximate Macaulay duration is 8.1390 (= 7.5361 × 1.08).

Solution to 2: Given an investment horizon of 10 years, the duration gap for this bond at purchase is negative: 8.1390 − 10 = −1.8610.

Solution to 3: A negative duration gap entails the risk of lower interest rates. To be precise, the risk is an immediate, one-time, parallel, downward yield curve shift because coupon reinvestment risk dominates market price risk. The loss from reinvesting coupons at a rate lower than 8% is larger than the gain from selling the bond at a price above the constant-yield price trajectory.

11. CREDIT AND LIQUIDITY RISK

- **explain how changes in credit spread and liquidity affect yield-to-maturity of a bond and how duration and convexity can be used to estimate the price effect of the changes**

The focus of this chapter is to demonstrate how bond duration and convexity estimate the bond price change, either in percentage terms or in units of currency, given an assumed yield-to-maturity change. This section addresses the *source* of the change in the yield-to-maturity. In general, the yield-to-maturity on a corporate bond is composed of a government *benchmark* yield and a *spread* over that benchmark. A change in the bond's yield-to-maturity can originate in either component or a combination of the two.

The key point is that for a traditional (option-free) fixed-rate bond, the same duration and convexity statistics apply for a change in the benchmark yield as for a change in the spread. The "building blocks" approach covered in an earlier chapter shows that these yield-to-maturity changes can be broken down further. A change in the benchmark yield can arise from a change in either the expected inflation rate or the expected real rate of interest. A change in the spread can arise from a change in the credit risk of the issuer or in the liquidity of the bond. Therefore, for a fixed-rate bond, the "inflation duration," the "real rate duration," the "credit duration," and the "liquidity duration" are all the same number. The inflation duration would indicate the change in the bond price if expected inflation were to change by a certain amount. In the same manner, the real rate duration would indicate the bond price change if the real rate were to go up or down. The credit duration and liquidity duration would indicate the price sensitivity that would arise from changes in those building blocks in the yield-to-maturity. A bond with a modified duration of 5.00 and a convexity of 32.00 will appreciate in value by about 1.26% if its yield-to-maturity goes down by 25 bps: $(-5.00 \times -0.0025) + [1/2 \times 32.00 \times (-0.0025)^2] = +0.0126$, regardless of the *source* of the yield-to-maturity change.

Suppose that the yield-to-maturity on a corporate bond is 6.00%. If the benchmark yield is 4.25%, the spread is 1.75%. An analyst believes that credit risk makes up 1.25% of the spread and liquidity risk, the remaining 0.50%. Credit risk includes the probability of default as well as the recovery of assets if default does occur. A credit rating downgrade or an adverse change in the ratings outlook for a borrower reflects a higher risk of default. Liquidity risk refers to the transaction costs associated with selling a bond. In general, a bond with greater frequency of trading and a higher volume of trading provides fixed-income investors with more opportunity to purchase or sell the security and thus has less liquidity risk. In practice, there is a difference between the *bid* (or purchase) and the *offer* (or sale) price. This difference depends on the type of bond, the size of the transaction, and the time of execution, among other factors. For instance, government bonds often trade at just a few basis points between the purchase and sale prices. More thinly traded corporate bonds can have a much wider difference between the bid and offer prices.

The problem for a fixed-income analyst is that it is rare for the changes in the components of the overall yield-to-maturity to occur in isolation. In practice, the analyst is concerned with the *interaction* between changes in benchmark yields and spreads, between changes in expected inflation and the expected real rate, and between changes in credit and liquidity risk. For example, during a financial crisis, a "flight to quality" can cause government benchmark yields to fall as credit spreads widen. An unexpected credit downgrade on a corporate bond can result in greater credit as well as liquidity risk.

EXAMPLE 17

The (flat) price on a fixed-rate corporate bond falls one day from 92.25 to 91.25 per 100 of par value because of poor earnings and an unexpected ratings downgrade of the issuer. The (annual) modified duration for the bond is 7.24. Which of the following is *closest* to the estimated change in the credit spread on the corporate bond, assuming benchmark yields are unchanged?

A. 15 bps
B. 100 bps
C. 108 bps

Solution: Given that the price falls from 92.25 to 91.25, the percentage price decrease is 1.084%.

$$\frac{91.25 - 92.25}{92.25} = -0.01084$$

Given an annual modified duration of 7.24, the change in the yield-to-maturity is 14.97 bps.

$$-0.01084 \approx -7.24 \times \Delta\text{Yield},\ \Delta\text{Yield} = 0.001497$$

Therefore, the answer is A. The change in price reflects a credit spread increase on the bond of about 15 bps.

12. EMPIRICAL DURATION

- **describe the difference between empirical duration and analytical duration**

The approach taken in this chapter to estimate duration and convexity statistics using mathematical formulas is often referred to as **analytical duration**. These estimates of the impact of benchmark yield changes on bond prices implicitly assume that government bond yields and spreads are independent variables that are uncorrelated with one another. Analytical duration offers a reasonable approximation of the price–yield relationship in many situations, but fixed-income professionals often use historical data in statistical models that incorporate various factors affecting bond prices to calculate **empirical duration** estimates. These estimates calculated over time and in different interest rate environments inform the fixed-income portfolio decision-making process and will be addressed in detail in later chapters.

For instance, in the "flight to quality" example cited earlier in which investors sell risky assets during market turmoil and purchase default-risk-free government bonds, we might expect analytical and empirical duration estimates to differ among bond types. For example, on the one hand, for a government bond with little or no credit risk, we would expect analytical and empirical duration to be similar because benchmark yield changes largely drive bond prices. On the other hand, the same macroeconomic factors driving government bond yields lower in a market stress scenario will cause high-yield bond credit spreads to widen because of an increase in expected default risk. Since credit spreads and benchmark yields are negatively correlated under this scenario, wider credit spreads will partially or fully offset the decline in government benchmark yields, resulting in lower empirical duration estimates than analytical duration estimates. Importantly, analysts must consider the correlation between benchmark yields and credit spreads when deciding whether to use empirical or analytical duration estimates.

EXAMPLE 18

AFC Investment Ltd. is a fixed-income investment firm that actively manages a government bond fund and a corporate bond fund. Holdings of the government bond fund are mainly medium-term US Treasury securities but also include debt of highly rated developed-market sovereign issuers. About half of the corporate bond fund is invested in investment-grade issues, and the other half consists of high-yield issues, all with a mix of maturities and from a mix of North American, European, and Asian companies.

Explain why empirical duration is likely to be a more accurate risk measure for AFC's corporate bond fund than for its government bond fund.

Solution: The government bond fund includes debt securities of the US government and other highly rated developed-market sovereign issuers. Since benchmark yields are the primary driver of changes in overall bond yields in this fund, the results of analytical duration and empirical duration should be broadly similar.

The corporate bond fund includes a wide variety of debt securities with varying levels of credit quality and liquidity and, therefore, different credit and liquidity spreads. Interactions between benchmark yield changes and credit and liquidity spreads would tend to offset each other, particularly during stressed market conditions, making empirical duration significantly lower than analytical duration. As a result, empirical duration may be the more accurate risk measure for the corporate bond fund.

SUMMARY

This chapter covers the risk and return characteristics of fixed-rate bonds. The focus is on the widely used measures of interest rate risk—duration and convexity. These statistics are used extensively in fixed-income analysis. The following are the main points made in the chapter:

- The three sources of return on a fixed-rate bond purchased at par value are: (1) receipt of the promised coupon and principal payments on the scheduled dates, (2) reinvestment of coupon payments, and (3) potential capital gains, as well as losses, on the sale of the bond prior to maturity.
- For a bond purchased at a discount or premium, the rate of return also includes the effect of the price being "pulled to par" as maturity nears, assuming no default.
- The total return is the future value of reinvested coupon interest payments and the sale price (or redemption of principal if the bond is held to maturity).
- The horizon yield (or holding period rate of return) is the internal rate of return between the total return and purchase price of the bond.
- Coupon reinvestment risk increases with a higher coupon rate and a longer reinvestment time period.
- Capital gains and losses are measured from the carrying value of the bond and not from the purchase price. The carrying value includes the amortization of the discount or premium if the bond is purchased at a price below or above par value. The carrying value is any point on the constant-yield price trajectory.
- Interest income on a bond is the return associated with the passage of time. Capital gains and losses are the returns associated with a change in the value of a bond as indicated by a change in the yield-to-maturity.
- The two types of interest rate risk on a fixed-rate bond are coupon reinvestment risk and market price risk. These risks offset each other to a certain extent. An investor gains from higher rates on reinvested coupons but loses if the bond is sold at a capital loss because the price is below the constant-yield price trajectory. An investor loses from lower rates on reinvested coupon but gains if the bond is sold at a capital gain because the price is above the constant-yield price trajectory.
- Market price risk dominates coupon reinvestment risk when the investor has a short-term horizon (relative to the time-to-maturity on the bond).
- Coupon reinvestment risk dominates market price risk when the investor has a long-term horizon (relative to the time-to-maturity)—for instance, a buy-and-hold investor.
- Bond duration, in general, measures the sensitivity of the full price (including accrued interest) to a change in interest rates.
- Yield duration statistics measuring the sensitivity of a bond's full price to the bond's own yield-to-maturity include the Macaulay duration, modified duration, money duration, and price value of a basis point.
- Curve duration statistics measuring the sensitivity of a bond's full price to the benchmark yield curve are usually called "effective durations."
- Macaulay duration is the weighted average of the time to receipt of coupon interest and principal payments, in which the weights are the shares of the full price corresponding to each payment. This statistic is annualized by dividing by the periodicity (number of coupon payments or compounding periods in a year).
- Modified duration provides a linear estimate of the percentage price change for a bond given a change in its yield-to-maturity.

- Approximate modified duration approaches modified duration as the change in the yield-to-maturity approaches zero.
- Effective duration is very similar to approximate modified duration. The difference is that approximate modified duration is a yield duration statistic that measures interest rate risk in terms of a change in the bond's own yield-to-maturity, whereas effective duration is a curve duration statistic that measures interest rate risk assuming a parallel shift in the benchmark yield curve.
- Key rate duration is a measure of a bond's sensitivity to a change in the benchmark yield curve at specific maturity segments. Key rate durations can be used to measure a bond's sensitivity to changes in the shape of the yield curve.
- Bonds with an embedded option do not have a meaningful internal rate of return because future cash flows are contingent on interest rates. Therefore, effective duration is the appropriate interest rate risk measure, not modified duration.
- The effective duration of a traditional (option-free) fixed-rate bond is its sensitivity to the benchmark yield curve, which can differ from its sensitivity to its own yield-to-maturity. Therefore, modified duration and effective duration on a traditional (option-free) fixed-rate bond are not necessarily equal.
- During a coupon period, Macaulay and modified durations decline smoothly in a "sawtooth" pattern, assuming the yield-to-maturity is constant. When the coupon payment is made, the durations jump upward.
- Macaulay and modified durations are inversely related to the coupon rate and the yield-to-maturity.
- Time-to-maturity and Macaulay and modified durations are *usually* positively related. They are *always* positively related on bonds priced at par or at a premium above par value. They are *usually* positively related on bonds priced at a discount below par value. The exception is on long-term, low-coupon bonds, on which it is possible to have a lower duration than on an otherwise comparable shorter-term bond.
- The presence of an embedded call option reduces a bond's effective duration compared with that of an otherwise comparable non-callable bond. The reduction in the effective duration is greater when interest rates are low and the issuer is more likely to exercise the call option.
- The presence of an embedded put option reduces a bond's effective duration compared with that of an otherwise comparable non-putable bond. The reduction in the effective duration is greater when interest rates are high and the investor is more likely to exercise the put option.
- The duration of a bond portfolio can be calculated in two ways: (1) the weighted average of the time to receipt of *aggregate* cash flows and (2) the weighted average of the durations of individual bonds that compose the portfolio.
- The first method to calculate portfolio duration is based on the cash flow yield, which is the internal rate of return on the aggregate cash flows. It cannot be used for bonds with embedded options or for floating-rate notes.
- The second method is simpler to use and quite accurate when the yield curve is relatively flat. Its main limitation is that it assumes a parallel shift in the yield curve in that the yields on all bonds in the portfolio change by the same amount.
- Money duration is a measure of the price change in terms of units of the currency in which the bond is denominated.
- The price value of a basis point (PVBP) is an estimate of the change in the full price of a bond given a 1 bp change in the yield-to-maturity.

- Modified duration is the primary, or first-order, effect on a bond's percentage price change given a change in the yield-to-maturity. Convexity is the secondary, or second-order, effect. It indicates the change in the modified duration as the yield-to-maturity changes.
- Money convexity is convexity times the full price of the bond. Combined with money duration, money convexity estimates the change in the full price of a bond in units of currency given a change in the yield-to-maturity.
- Convexity is a positive attribute for a bond. Other things being equal, a more convex bond appreciates in price more than a less convex bond when yields fall and depreciates less when yields rise.
- Effective convexity is the second-order effect on a bond price given a change in the benchmark yield curve. It is similar to approximate convexity. The difference is that approximate convexity is based on a yield-to-maturity change and effective convexity is based on a benchmark yield curve change.
- Callable bonds have negative effective convexity when interest rates are low. The increase in price when the benchmark yield is reduced is less in absolute value than the decrease in price when the benchmark yield is raised.
- The change in a bond price is the product of: (1) the impact per basis-point change in the yield-to-maturity and (2) the number of basis points in the yield change. The first factor is estimated by duration and convexity. The second factor depends on yield volatility.
- The investment horizon is essential in measuring the interest rate risk on a fixed-rate bond.
- For a particular assumption about yield volatility, the Macaulay duration indicates the investment horizon for which coupon reinvestment risk and market price risk offset each other. The assumption is a one-time parallel shift to the yield curve in which the yield-to-maturity and coupon reinvestment rates change by the same amount in the same direction.
- When the investment horizon is greater than the Macaulay duration of the bond, coupon reinvestment risk dominates price risk. The investor's risk is to lower interest rates. The duration gap is negative.
- When the investment horizon is equal to the Macaulay duration of the bond, coupon reinvestment risk offsets price risk. The duration gap is zero.
- When the investment horizon is less than the Macaulay duration of the bond, price risk dominates coupon reinvestment risk. The investor's risk is to higher interest rates. The duration gap is positive.
- Credit risk involves the probability of default and degree of recovery if default occurs, whereas liquidity risk refers to the transaction costs associated with selling a bond.
- For a traditional (option-free) fixed-rate bond, the same duration and convexity statistics apply if a change occurs in the benchmark yield or a change occurs in the spread. The change in the spread can result from a change in credit risk or liquidity risk.
- In practice, there often is interaction between changes in benchmark yields and in the spread over the benchmark.
- Empirical duration uses statistical methods and historical bond prices to derive the price–yield relationship for specific bonds or bond portfolios.

REFERENCE

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PRACTICE PROBLEMS

1. A "buy-and-hold" investor purchases a fixed-rate bond at a discount and holds the security until it matures. Which of the following sources of return is *least likely* to contribute to the investor's total return over the investment horizon, assuming all payments are made as scheduled?
   A. Capital gain
   B. Principal payment
   C. Reinvestment of coupon payments
2. Which of the following sources of return is *most likely* exposed to interest rate risk for an investor of a fixed-rate bond who holds the bond until maturity?
   A. Capital gain or loss
   B. Redemption of principal
   C. Reinvestment of coupon payments
3. An investor purchases a bond at a price above par value. Two years later, the investor sells the bond. The resulting capital gain or loss is measured by comparing the price at which the bond is sold to the:
   A. carrying value.
   B. original purchase price.
   C. original purchase price value plus the amortized amount of the premium.

The following information relates to Questions 4–6

An investor purchases a nine-year, 7% annual coupon payment bond at a price equal to par value. After the bond is purchased and before the first coupon is received, interest rates increase to 8%. The investor sells the bond after five years. Assume that interest rates remain unchanged at 8% over the five-year holding period.

4. Per 100 of par value, the future value of the reinvested coupon payments at the end of the holding period is *closest* to:
   A. 35.00.
   B. 40.26.
   C. 41.07.
5. The capital gain/loss per 100 of par value resulting from the sale of the bond at the end of the five-year holding period is *closest* to a:
   A. loss of 8.45.
   B. loss of 3.31.
   C. gain of 2.75.
6. Assuming that all coupons are reinvested over the holding period, the investor's five-year horizon yield is *closest* to:
   A. 5.66%.
   B. 6.62%.
   C. 7.12%.

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7. An investor buys a three-year bond with a 5% coupon rate paid annually. The bond, with a yield-to-maturity of 3%, is purchased at a price of 105.657223 per 100 of par value. Assuming a 5-basis point change in yield-to-maturity, the bond's approximate modified duration is *closest* to:
   A. 2.78.
   B. 2.86.
   C. 5.56.

8. Which of the following statements about duration is correct? A bond's:
   A. effective duration is a measure of yield duration.
   B. modified duration is a measure of curve duration.
   C. modified duration cannot be larger than its Macaulay duration (assuming a positive yield-to-maturity).
9. An investor buys a 6% annual payment bond with three years to maturity. The bond has a yield-to-maturity of 8% and is currently priced at 94.845806 per 100 of par. The bond's Macaulay duration is *closest* to:
   A. 2.62.
   B. 2.78.
   C. 2.83.
10. The interest rate risk of a fixed-rate bond with an embedded call option is *best* measured by:
    A. effective duration.
    B. modified duration.
    C. Macaulay duration.
11. Which of the following is *most* appropriate for measuring a bond's sensitivity to shaping risk?
    A. Key rate duration
    B. Effective duration
    C. Modified duration
12. A Canadian pension fund manager seeks to measure the sensitivity of her pension liabilities to market interest rate changes. The manager determines the present value of the liabilities under three interest rate scenarios: a base rate of 7%, a 100 basis point increase in rates up to 8%, and a 100 basis point drop in rates down to 6%. The results of the manager's analysis are presented below:

Interest Rate Assumption	Present Value of Liabilities
6%	CAD510.1 million
7%	CAD455.4 million
8%	CAD373.6 million

    The effective duration of the pension fund's liabilities is *closest* to:
    A. 1.49.
    B. 14.99.
    C. 29.97.
13. Which of the following statements about Macaulay duration is correct?
    A. A bond's coupon rate and Macaulay duration are positively related.
    B. A bond's Macaulay duration is inversely related to its yield-to-maturity.
    C. The Macaulay duration of a zero-coupon bond is less than its time-to-maturity.
14. Assuming no change in the credit risk of a bond, the presence of an embedded put option:
    A. reduces the effective duration of the bond.
    B. increases the effective duration of the bond.
    C. does not change the effective duration of the bond.

15. A bond portfolio consists of the following three fixed-rate bonds. Assume annual coupon payments and no accrued interest on the bonds. Prices are per 100 of par value.

Bond	Maturity	Market Value	Price	Coupon	Yield-to-Maturity	Modified Duration
A	6 years	170,000	85.0000	2.00%	4.95%	5.42
B	10 years	120,000	80.0000	2.40%	4.99%	8.44
C	15 years	100,000	100.0000	5.00%	5.00%	10.38

The bond portfolio's modified duration is *closest* to:
A. 7.62.
B. 8.08.
C. 8.20.

16. A limitation of calculating a bond portfolio's duration as the weighted average of the yield durations of the individual bonds that compose the portfolio is that it:
A. assumes a parallel shift to the yield curve.
B. is less accurate when the yield curve is less steeply sloped.
C. is not applicable to portfolios that have bonds with embedded options.

17. Using the information below, which bond has the *greatest* money duration per 100 of par value assuming annual coupon payments and no accrued interest?

Bond	Time-to-Maturity	Price Per 100 of Par Value	Coupon Rate	Yield-to-Maturity	Modified Duration
A	6 years	85.00	2.00%	4.95%	5.42
B	10 years	80.00	2.40%	4.99%	8.44
C	9 years	85.78	3.00%	5.00%	7.54

A. Bond A
B. Bond B
C. Bond C

18. A bond with exactly nine years remaining until maturity offers a 3% coupon rate with annual coupons. The bond, with a yield-to-maturity of 5%, is priced at 85.784357 per 100 of par value. The estimated price value of a basis point for the bond is *closest* to:
A. 0.0086.
B. 0.0648.
C. 0.1295.

19. The "second-order" effect on a bond's percentage price change given a change in yield-to-maturity can be *best* described as:
A. duration.
B. convexity.
C. yield volatility.

20. A bond is currently trading for 98.722 per 100 of par value. If the bond's yield-to-maturity (YTM) rises by 10 basis points, the bond's full price is expected to fall to 98.669. If the bond's YTM decreases by 10 basis points, the bond's full price is expected to increase to 98.782. The bond's approximate convexity is *closest* to:
A. 0.071.
B. 70.906.
C. 1,144.628.

21. A bond has an annual modified duration of 7.020 and annual convexity of 65.180. If the bond's yield-to-maturity decreases by 25 basis points, the expected percentage price change is *closest* to:
    A. 1.73%.
    B. 1.76%.
    C. 1.78%.
22. A bond has an annual modified duration of 7.140 and annual convexity of 66.200. The bond's yield-to-maturity is expected to increase by 50 basis points. The expected percentage price change is *closest* to:
    A. −3.40%.
    B. −3.49%.
    C. −3.57%.
23. Which of the following statements relating to yield volatility is *most* accurate? If the term structure of yield volatility is downward sloping, then:
    A. short-term rates are higher than long-term rates.
    B. long-term yields are more stable than short-term yields.
    C. short-term bonds will always experience greater price fluctuation than long-term bonds.
24. The holding period for a bond at which the coupon reinvestment risk offsets the market price risk is *best* approximated by:
    A. duration gap.
    B. modified duration.
    C. Macaulay duration.
25. When the investor's investment horizon is less than the Macaulay duration of the bond she owns:
    A. the investor is hedged against interest rate risk.
    B. reinvestment risk dominates, and the investor is at risk of lower rates.
    C. market price risk dominates, and the investor is at risk of higher rates.
26. An investor purchases an annual coupon bond with a 6% coupon rate and exactly 20 years remaining until maturity at a price equal to par value. The investor's investment horizon is eight years. The approximate modified duration of the bond is 11.470 years. The duration gap at the time of purchase is *closest* to:
    A. −7.842.
    B. 3.470.
    C. 4.158.
27. A manufacturing company receives a ratings upgrade and the price increases on its fixed-rate bond. The price increase was *most likely* caused by a(n):
    A. decrease in the bond's credit spread.
    B. increase in the bond's liquidity spread.
    C. increase of the bond's underlying benchmark rate.
28. Empirical duration is likely the best measure of the impact of yield changes on portfolio value, especially under stressed market conditions, for a portfolio consisting of:
    A. 100% sovereign bonds of several AAA rated euro area issuers.
    B. 100% covered bonds of several AAA rated euro area corporate issuers.
    C. 25% AAA rated sovereign bonds, 25% AAA rated corporate bonds, and 50% high-yield (i.e., speculative-grade) corporate bonds, all from various euro area sovereign and corporate issuers.

CHAPTER 6

FUNDAMENTALS OF CREDIT ANALYSIS

Christopher L. Gootkind, CFA

LEARNING OUTCOMES

The candidate should be able to:

- describe credit risk and credit-related risks affecting corporate bonds;
- describe default probability and loss severity as components of credit risk;
- describe seniority rankings of corporate debt and explain the potential violation of the priority of claims in a bankruptcy proceeding;
- compare and contrast corporate issuer credit ratings and issue credit ratings and describe the rating agency practice of "notching";
- explain risks in relying on ratings from credit rating agencies;
- explain the four Cs (Capacity, Collateral, Covenants, and Character) of traditional credit analysis;
- calculate and interpret financial ratios used in credit analysis;
- evaluate the credit quality of a corporate bond issuer and a bond of that issuer, given key financial ratios of the issuer and the industry;
- describe macroeconomic, market, and issuer-specific factors that influence the level and volatility of yield spreads;
- explain special considerations when evaluating the credit of high-yield, sovereign, and non-sovereign government debt issuers and issues.

1. INTRODUCTION

- **describe credit risk and credit-related risks affecting corporate bonds**
- **describe default probability and loss severity as components of credit risk**

The author would like to thank several of his Fixed Income Research colleagues at Loomis, Sayles & Company for their assistance with this chapter: Paul Batterton, CFA, Diana Moskowitz, CFA, *Diana* Monteith, Shannon O'Mara, CFA, and Laura Sarlo, CFA.

With bonds outstanding worth many trillions of US dollars, the debt markets play a critical role in the global economy. Companies and governments raise capital in the debt market to fund current operations; buy equipment; build factories, roads, bridges, airports, and hospitals; acquire assets; and so on. By channeling savings into productive investments, the debt markets facilitate economic growth. Credit analysis has a crucial function in the debt capital markets—efficiently allocating capital by properly assessing credit risk, pricing it accordingly, and repricing it as risks change. How do fixed-income investors determine the riskiness of that debt, and how do they decide what they need to earn as compensation for that risk?

In the sections that follow, we cover basic principles of credit analysis, which may be broadly defined as the process by which credit risk is evaluated. Readers will be introduced to the definition of credit risk, the interpretation of credit ratings, the four Cs of traditional credit analysis, and key financial measures and ratios used in credit analysis. We explain, among other things, how to compare bond issuer creditworthiness within a given industry as well as across industries and how credit risk is priced in the bond market.

Our coverage focuses primarily on analysis of corporate debt; however, credit analysis of sovereign and nonsovereign, particularly municipal, government bonds will also be addressed. Structured finance, a segment of the debt markets that includes securities backed by such pools of assets as residential and commercial mortgages as well as other consumer loans, will not be covered here.

We first introduce the key components of credit risk—default probability and loss severity—along with such credit-related risks as spread risk, credit migration risk, and liquidity risk. We then discuss the relationship between credit risk and the capital structure of the firm before turning attention to the role of credit rating agencies. We also explore the process of analyzing the credit risk of corporations and examine the impact of credit spreads on risk and return. Finally, we look at special considerations applicable to the analysis of (i) high-yield (low-quality) corporate bonds and (ii) government bonds.

2. CREDIT RISK

Credit risk is the risk of loss resulting from the borrower (issuer of debt) failing to make full and timely payments of interest and/or principal. Credit risk has two components. The first is known as **default risk**, or **default probability**, which is the probability that a borrower defaults—that is, fails to meet its obligation to make full and timely payments of principal and interest according to the terms of the debt security. The second component is **loss severity** (also known as "loss given default") in the event of default—that is, the portion of a bond's value (including unpaid interest) an investor loses. A default can lead to losses of various magnitudes. In most instances, in the event of default, bondholders will recover some value, so there will not be a total loss on the investment. Thus, credit risk is reflected in the distribution of potential losses that may arise if the investor is not paid in full and on time. Although it is sometimes important to consider the entire distribution of potential losses and their respective probabilities—for instance, when losses have a disproportionate impact on the various tranches of a securitized pool of loans—it is often convenient to summarize the risk with a single default probability and loss severity and focus on the **expected loss**:

$$\text{Expected loss} = \text{Default probability} \times \text{Loss severity given default}$$

The loss severity, and hence the expected loss, can be expressed as either a monetary amount (e.g., €450,000) or as a percentage of the principal amount (e.g., 45%). The latter form of expression is generally more useful for analysis because it is independent of the amount of investment. Loss severity is often expressed as (1 – Recovery rate), where the recovery rate is the percentage of the principal amount recovered in the event of default.

Because default risk (default probability) is quite low for most high-quality debt issuers, bond investors tend to focus primarily on assessing this probability and devote less effort to assessing the potential loss severity arising from default. However, as an issuer's default risk rises, investors will focus more on what the recovery rate might be in the event of default. This issue will be discussed in more detail later. Important credit-related risks include the following:

- **Spread risk**. Corporate bonds and other "credit-risky" debt instruments typically trade at a yield premium, or spread, to bonds that have been considered "default-risk free," such as US Treasury bonds or German government bonds. Yield spreads, expressed in basis points, widen based on factors specific to the issuer, such as a decline in creditworthiness, sometimes referred to as credit migration or downgrade risk, or factors associated with the market as a whole, such as an increase in **market liquidity risk** or a general aversion to risk during periods of financial distress.
- **Credit migration risk** or **downgrade risk**. This is the risk that a bond issuer's creditworthiness deteriorates, or migrates lower, leading investors to believe the risk of default is higher and thus causing the yield spreads on the issuer's bonds to widen and the price of its bonds to fall. The term "downgrade" refers to action by the major bond rating agencies, whose role will be covered in more detail later.
- **Market liquidity risk**. This is the risk that the price at which investors can actually transact—buying or selling—may differ from the price indicated in the market. To compensate investors for the risk that there may not be sufficient market liquidity for them to buy or sell bonds in the quantity they desire, the spread or yield premium on corporate bonds includes a market liquidity component, in addition to a credit risk component. Unlike stocks, which trade on exchanges, most markets bonds trade primarily over the counter through broker–dealers trading for their own accounts. Their ability and willingness to make markets, as reflected in the bid–ask spread, is an important determinant of market liquidity risk. The two main issuer-specific factors that affect market liquidity risk are (1) the size of the issuer (that is, the amount of publicly traded debt an issuer has outstanding) and (2) the credit quality of the issuer. In general, the less debt an issuer has outstanding, the less frequently its debt trades and thus the higher the market liquidity risk. And the lower the quality of the issuer, the higher the market liquidity risk.

During times of financial stress or crisis, such as in late 2008, market liquidity can decline sharply, causing yield spreads on corporate bonds and other credit-risky debt to widen and their prices to drop.

EXAMPLE 1 Defining Credit Risk

1. Which of the following *best* defines credit risk?
   A. The probability of default times the severity of loss given default
   B. The loss of principal and interest payments in the event of bankruptcy
   C. The risk of not receiving full interest and principal payments on a timely basis
2. Which of the following is the *best* measure of credit risk?
   A. The expected loss
   B. The severity of loss
   C. The probability of default

3. Which of the following is not credit or credit-related risk?
   A. Default risk
   B. Interest rate risk
   C. Downgrade or credit migration risk

Solution to 1: C is correct. Credit risk is the risk that the borrower will not make full and timely payments.

Solution to 2: A is correct. The expected loss captures both of the key components of credit risk: (the product of) the probability of default and the loss severity in the event of default. Neither component alone fully reflects the risk.

Solution to 3: B is correct. Bond price changes due to general interest rate movements are not considered credit risk.

3. CAPITAL STRUCTURE, SENIORITY RANKING, AND RECOVERY RATES

- **describe seniority rankings of corporate debt and explain the potential violation of the priority of claims in a bankruptcy proceeding**

The various debt obligations of a given borrower will not necessarily all have the same **seniority ranking**, or priority of payment. In this section, we will introduce the topic of an issuer's capital structure and discuss the various types of debt claims that may arise from that structure, as well as their ranking and how those rankings can influence recovery rates in the event of default. The term "creditors" is used throughout our coverage to mean holders of debt instruments, such as bonds and bank loans. Unless specifically stated, it does not include such obligations as trade credit, tax liens, or employment-related obligations.

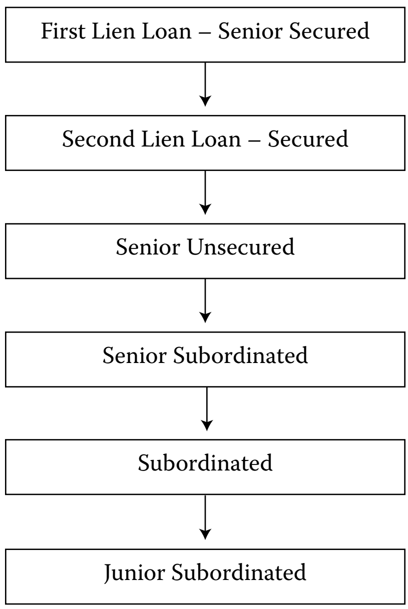
3.1. Capital Structure

The composition and distribution across operating units of a company's debt and equity—including bank debt, bonds of all seniority rankings, preferred stock, and common equity—is referred to as its **capital structure**. Some companies and industries have straightforward capital structures, with all the debt equally ranked and issued by one main operating entity. Other companies and industries, due to their frequent acquisitions and divestitures (e.g., media companies or conglomerates) or high levels of regulation (e.g., banks and utilities), tend to have more complicated capital structures. Companies in these industries often have many different subsidiaries, or operating companies, that have their own debt outstanding and parent holding companies that also issue debt, with different levels or rankings of seniority. Similarly, the cross-border operations of multinational corporations tend to increase the complexity of their capital structures.

3.2. Seniority Ranking

Just as borrowers can issue debt with many different maturity dates and coupons, they can also have many different rankings in terms of seniority. The ranking refers to the priority of payment, with the most senior or highest-ranking debt having the first claim on the cash flows and assets of the issuer. This level of seniority can affect the value of an investor's claim in the event of default and restructuring. Broadly, there is **secured debt** and **unsecured debt**. Secured debt means the debtholder has a direct claim—a pledge from the issuer—on certain assets and their associated cash flows. Unsecured bondholders have only a general claim on an issuer's assets and cash flows. In the event of default, unsecured debtholders' claims rank below (i.e., get paid after) those of secured creditors under what's known as the **priority of claims**.

EXHIBIT 1 Seniority Ranking



Within each category of debt are finer gradations of types and rankings. Within secured debt, there is first mortgage and first lien debt, which are the highest-ranked debt in terms of priority of repayment. **First mortgage debt** or loan refers to the pledge of a specific property (e.g., a power plant for a utility or a specific casino for a gaming company). **First lien debt** or loan refers to a pledge of certain assets that could include buildings but might also include property and equipment, licenses, patents, brands, and so on. There can also be **second lien**, or even third lien, secured debt, which, as the name implies, has a secured interest in the pledged assets but ranks below first lien debt in both collateral protection and priority of payment.

Within unsecured debt, there can also be finer gradations and seniority rankings. The highest-ranked unsecured debt is senior unsecured debt. It is the most common type of all corporate bonds outstanding. Other, lower-ranked debt includes **subordinated debt** and junior subordinated debt. Among the various creditor classes, these obligations have among the lowest priority of claims and frequently have little or no recovery in the event of default. That is, their loss severity can be as high as 100%. (See Exhibit 1 for a sample seniority ranking.) For regulatory and capital purposes, banks in Europe and the United States have issued debt and debt-like securities that rank even lower than subordinated debt, typically referred to as hybrids or trust preferred, and are intended to provide a capital cushion in times of financial distress. Many of them did not work as intended during the global financial crisis that began in 2008, and most were phased out, potentially to be replaced by more effective instruments that automatically convert to equity in certain circumstances.

Companies issue—and investors buy—debt with different seniority rankings for many reasons. Issuers are interested in optimizing their cost of capital—finding the right mix of the various types of both debt and equity—for their industry and type of business. Issuers may offer secured debt because that is what the market (i.e., investors) may require—given a company's perceived riskiness or because secured debt is generally lower cost due to the reduced credit risk inherent in its higher priority of claims. Or, issuers may offer subordinated debt because (1) they believe it is less expensive than issuing equity, as debtholders require a lower rate of return due to their superior place in line in the event of default; (2) doing so prevents dilution to existing shareholders; (3) it is typically less restrictive than issuing senior debt; and (4) investors are willing to buy it because they believe the yield being offered is adequate compensation for the risk they perceive.

EXAMPLE 2 Seniority Ranking

The Acme Company has senior unsecured bonds as well as both first and second lien debt in its capital structure. Which ranks higher with respect to priority of claims: senior unsecured bonds or second lien debt?

Solution: Second lien debt ranks higher than senior unsecured bonds because of its secured position.

3.3. Recovery Rates

All creditors at the same level of the capital structure are treated as one class; thus, a senior unsecured bondholder whose debt is due in 30 years has the same pro rata claim in bankruptcy as one whose debt matures in six months. This provision is referred to as bonds ranking **pari passu** ("on an equal footing") in right of payment.

Defaulted debt will often continue to be traded by investors and broker–dealers based on their assessment that either in liquidation of the bankrupt company's assets or in reorganization, the bonds will have some recovery value. In the case of reorganization or restructuring (whether through formal bankruptcy or on a voluntary basis), new debt, equity, cash, or some combination thereof could be issued in exchange for the original defaulted debt.

As discussed, recovery rates vary by seniority of ranking in a company's capital structure, under the priority of claims treatment in bankruptcy. Over many decades, there have been enough defaults to generate statistically meaningful historical data on recovery rates by seniority ranking. Exhibit 2 provides recovery rates by seniority ranking for North American non-financial companies. For example, as shown in Exhibit 2, investors on average recovered 46.9% of the value of senior secured debt that defaulted in 2016 but only 29.2% of the value of senior unsecured issues that defaulted that year.

EXHIBIT 2 Average Corporate Debt Recovery Rates Measured by Ultimate Recoveries

	Emergence Year*			Default Year		
Seniority Ranking	2017	2016	1987–2017	2017	2016	1987–2017
Bank loans	81.3%	72.6%	80.4%	80.2%	78.3%	80.4%
Senior secured bonds	52.3%	35.9%	62.3%	57.5%	46.9%	62.3%
Senior unsecured bonds	54.1%	11.7%	47.9%	47.4%	29.2%	47.9%
Subordinated bonds	4.5%	6.6%	28.0%	NA	8.0%	28.0%

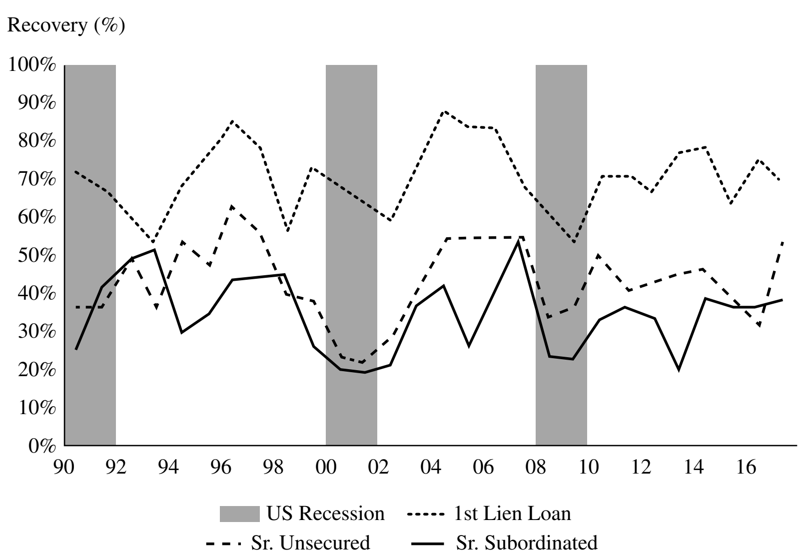
NA = not available.

* Emergence year is typically the year the defaulted company emerges from bankruptcy. Default year data refer to the recovery rate of debt that defaulted in that year (i.e., 2016 and 2017) or range of years (i.e., 1987–2017). Data are for North American nonfinancial companies.
Source: Moody's Investors Service's Ultimate Recovery Database.

A few things are worth noting:

1. **Recovery rates can vary widely by industry**. Companies that go bankrupt in industries that are in secular decline (e.g., newspaper publishing) will most likely have lower recovery rates than those that go bankrupt in industries merely suffering from a cyclical economic downturn.
2. **Recovery rates can also vary depending on when they occur in a credit cycle**. Credit cycles describe the changing availability—and pricing—of credit. When the economy is strong or improving, the willingness of lenders to extend credit on favorable terms is high. Conversely, when the economy is weak or weakening, lenders pull back, or "tighten" credit, by making it less available and more expensive. As shown in Exhibit 3, at or near the bottom of a credit cycle—which is almost always closely linked with an economic cycle—recoveries will tend to be lower than at other times in the credit cycle.

EXHIBIT 3 Global Recovery Rates by Seniority Ranking, 1990–2017



Source: Based on data from Moody's Investors Service's Ultimate Recovery Database.

3. **These recovery rates are averages**. In fact, there can be large variability, both across industries, as noted, as well as across companies within a given industry. Factors might include composition and proportion of debt across an issuer's capital structure. An abundance of secured debt will lead to smaller recovery rates on lower-ranked debt.

Understanding recovery rates is important because they are a key component of credit analysis and risk. Recall that the best measure of credit risk is expected loss—that is, probability of default times loss severity given default. And loss severity equals (1 − Recovery rate). Having an idea how much one can lose in the event of default is a critical factor in valuing credit, particularly lower-quality credit, as the default risk rises.

Priority of claims: Not always absolute. The priority of claims in bankruptcy—the idea that the highest-ranked creditors get paid out first, followed by the next level, and on down, like a waterfall—is well established and is often described as "absolute." In principle, in the event of bankruptcy or liquidation:

- Creditors with a secured claim have the right to the value of that specific property before any other claim. If the value of the pledged property is less than the amount of the claim, then the difference becomes a senior unsecured claim.
- Unsecured creditors have a right to be paid in full before holders of equity interests (common and preferred shareholders) receive value on their interests.
- Senior unsecured creditors take priority over all subordinated creditors. A creditor is senior unsecured unless expressly subordinated.

In practice, however, creditors with lower seniority and even shareholders may receive some consideration without more senior creditors being paid in full. Why might this be the case? In bankruptcy, there are different classes of claimants, and all classes that are impaired (that is, receive less than full claim) get to vote to confirm the plan of reorganization. This vote is subject to the absolute priority of claims. Either by consent of the various parties or by the judge's order; however, absolute priority may not be strictly enforced in the final plan. There may be disputes over the value of various assets in the bankruptcy estate (e.g., what is a plant, or a patent portfolio, worth?) or the present value or timing of payouts. For example, what is the value of the new debt I'm receiving for my old debt of a reorganized company before it emerges from bankruptcy?

Resolution of these disputes takes time, and cases can drag on for months and years. In the meantime, during bankruptcy, substantial expenses are being incurred for legal and accounting fees, and the value of the company may be declining as key employees leave, customers go elsewhere, and so on. Thus, to avoid the time, expense, and uncertainty over disputed issues, such as the value of property in the estate and the legality of certain claims, the various claimants have an incentive to negotiate and compromise. This frequently leads to creditors with lower seniority and other claimants (e.g., even shareholders) receiving more consideration than they are legally entitled to.

It's worth noting that in the United States, the bias is toward reorganization and recovery of companies in bankruptcy, whereas in other jurisdictions, such as the United Kingdom, the bias is toward liquidation of companies in bankruptcy and maximizing value to the banks and other senior creditors. It's also worth noting that bankruptcy and bankruptcy laws are very complex and can vary greatly by country, so it is difficult to generalize about how creditors will fare. As shown in the earlier chart, there is huge variability in recovery rates for defaulted debt. Every case is different.

EXAMPLE 3 Priority of Claims

1. Under which circumstance is a subordinated bondholder *most likely* to recover some value in a bankruptcy without a senior creditor getting paid in full? When:
   A. absolute priority rules are enforced.
   B. the various classes of claimants agree to it.
   C. the company is liquidated rather than reorganized.
2. In the event of bankruptcy, claims at the same level of the capital structure are:
   A. on an equal footing, regardless of size, maturity, or time outstanding.
   B. paid in the order of maturity from shortest to longest, regardless of size or time outstanding.
   C. paid on a first-in, first-out (FIFO) basis so that the longest-standing claims are satisfied first, regardless of size or maturity.

Solution to 1: B is correct. All impaired classes get to vote on the reorganization plan. Negotiation and compromise are often preferable to incurring huge legal and accounting fees in a protracted bankruptcy process that would otherwise reduce the value of the estate for all claimants. This process may allow junior creditors (e.g., subordinated bondholders) to recover some value even though more senior creditors do not get paid in full.

Solution to 2: A is correct. All claims at the same level of the capital structure are pari passu (on an equal footing).

4. RATING AGENCIES, CREDIT RATINGS, AND THEIR ROLE IN THE DEBT MARKETS

- **compare and contrast corporate issuer credit ratings and issue credit ratings and describe the rating agency practice of "notching"**

The major credit rating agencies—Moody's Investors Service ("Moody's"), Standard & Poor's ("S&P"), and Fitch Ratings ("Fitch")—play a central, if somewhat controversial, role in the credit markets. For the vast majority of outstanding bonds, at least two of the agencies provide ratings: a symbol-based measure of the potential risk of default of a particular bond or issuer of debt. In the public and quasi-public bond markets, underwritten by investment banks as opposed to privately placed, issuers won't offer, and investors won't buy, bonds that do not carry ratings from Moody's, S&P, or Fitch. This practice applies for all types of bonds: government or sovereign; entities with implicit or explicit guarantees from the government, such as Ginnie Mae in the United States and *Pfandbriefe* in Germany; supranational entities, such as the World Bank, which are owned by several governments; corporate; non-sovereign government; and mortgage- and asset-backed debt. How did the rating agencies attain such a dominant position in the credit markets? What are credit ratings, and what do they mean? How does the market use credit ratings? What are the risks of relying solely or excessively on credit ratings?

The history of the major rating agencies goes back more than 100 years. John Moody began publishing credit analysis and opinions on US railroads in 1909. S&P published its first ratings in 1916. They have grown in size and prominence since then. Many bond investors like the fact that there are independent analysts who meet with the issuer and often have access to material, non-public information, such as financial projections that investors cannot receive, to aid in the analysis. What has also proven very attractive to investors is that credit ratings provide direct and easy comparability of the relative credit riskiness of all bond issuers, within and across industries and bond types, although there is some debate about ratings comparability across the types of bonds. For instance, investigations conducted after the 2008–2009 global financial crisis suggested that, for a given rating category, municipal bonds have experienced a lower historical incidence of default than corporate debt.

Several factors have led to the near universal use of credit ratings in the bond markets and the dominant role of the major credit rating agencies. These factors include the following:

- Independent assessment of credit risk
- Ease of comparison across bond issuers, issues, and market segments
- Regulatory and statutory reliance and usage
- Issuer payment for ratings
- Huge growth of debt markets
- Development and expansion of bond portfolio management and the accompanying bond indexes

However, in the aftermath of the global financial crisis of 2008–2009, when the rating agencies were blamed for contributing to the crisis with their overly optimistic ratings on securities backed by subprime mortgages, attempts were made to reduce the role and dominant positions of the major credit rating agencies. New rules, regulations, and legislation were passed to require the agencies to be more transparent, reduce conflicts of interest, and stimulate more competition. The "issuer pay" model allows the distribution of ratings to a broad universe of investors and undoubtedly facilitated widespread reliance on ratings. Challenging the dominance of Moody's, S&P, and Fitch, additional credit rating agencies have emerged. Some credit rating agencies that are well-established in their home markets but are not so well known globally, such as Dominion Bond Rating Service (DBRS) in Canada and Japan Credit Rating Agency (JCR) in Japan, have tried to raise their profiles. The market dominance of the biggest credit rating agencies, however, remains largely intact.

4.1. Credit Ratings

The three major global credit rating agencies—Moody's, S&P, and Fitch—use similar, symbol-based ratings that are basically an assessment of a bond issue's risk of default. Exhibit 4 shows their long-term ratings ranked from highest to lowest. Ratings on short-term debt, although available, are not shown here.

Bonds rated triple-A (Aaa or AAA) are said to be "of the highest quality, subject to the lowest level of credit risk" (Moody's Investors Service, see "Rating Symbols and Definitions," https://www.moodys.com/Pages/amr002002.aspx) and thus have extremely low probabilities of default. Double-A (Aa or AA) rated bonds are referred to as "high-quality grade" and are also regarded as having very low default risk. Bonds rated single-A are referred to as "upper-medium grade." Baa (Moody's) or BBB (S&P and Fitch) are called "low-medium grade." Bonds rated Baa3/BBB– or higher are called "investment grade." Bonds rated Ba1 or lower by Moody's and BB+ or lower by S&P and Fitch, respectively, have speculative credit characteristics and increasingly higher default risk. As a group, these bonds are referred to in a variety of ways:

EXHIBIT 4 Long-Term Rating Matrix: Investment Grade vs. Non-Investment Grade

		Moody's	S&P	Fitch
Investment Grade	High-Quality Grade	Aaa	AAA	AAA
		Aa1	AA+	AA+
		Aa2	AA	AA
		Aa3	AA–	AA–
	Upper-Medium Grade	A1	A+	A+
		A2	A	A
		A3	A–	A–
	Low-Medium Grade	Baa1	BBB+	BBB+
		Baa2	BBB	BBB
		Baa3	BBB–	BBB–
Non-Investment Grade ("Junk" or "High Yield")	Low Grade or Speculative Grade	Ba1	BB+	BB+
		Ba2	BB	BB
		Ba3	BB–	BB–
		B1	B+	B+
		B2	B	B
		B3	B–	B–
		Caa1	CCC+	CCC+
		Caa2	CCC	CCC
		Caa3	CCC–	CCC–
		Ca	CC	CC
		C	C	C
	Default	C	D	D

"low grade," "speculative grade," "non-investment grade," "below investment grade," "high yield," and, in an attempt to reflect the extreme level of risk, some observers refer to these bonds as "junk bonds." The D rating is reserved for securities that are already in default in S&P's and Fitch's scales. For Moody's, bonds rated C are likely, but not necessarily, in default. Generally, issuers of bonds rated investment grade are more consistently able to access the debt markets and can borrow at lower interest rates than those rated below investment grade.

In addition, rating agencies will typically provide outlooks on their respective ratings—positive, stable, or negative—and may provide other indicators on the potential direction of their ratings under certain circumstances, such as "On Review for Downgrade" or "On CreditWatch for an Upgrade." It should also be noted that, in support of the ratings they publish, the rating agencies also provide extensive written commentary and financial analysis on the obligors they rate, as well as summary industry statistics.

4.2. Issuer vs. Issue Ratings

Rating agencies will typically provide both issuer and issue ratings, particularly as they relate to corporate debt. Terminology used to distinguish between issuer and issue ratings includes corporate family rating (CFR) and corporate credit rating (CCR) or issuer credit rating and issue credit rating. An issuer credit rating is meant to address an obligor's overall creditworthiness—its ability and willingness to make timely payments of interest and principal on its debt. The issuer credit rating usually applies to its senior unsecured debt.

Issue ratings refer to specific financial obligations of an issuer and take into consideration such factors as ranking in the capital structure (e.g., secured or subordinated). Although **cross-default provisions**, whereby events of default (such as non-payment of interest) on one bond trigger default on all outstanding debt, imply the same default probability for all issues, specific issues may be assigned different credit ratings—higher or lower—due to a rating adjustment methodology known as **notching**.

Notching. For the rating agencies, likelihood of default—default risk—is the primary factor in assigning their ratings. However, there are secondary factors as well. These factors include the priority of payment in the event of a default (e.g., secured versus senior unsecured versus subordinated) as well as potential loss severity in the event of default. Another factor considered by rating agencies is **structural subordination**, which can arise when a corporation with a holding company structure has debt at both its parent holding company and operating subsidiaries. Debt at the operating subsidiaries will get serviced by the cash flow and assets of the subsidiaries before funds can be passed ("upstreamed") to the holding company to service debt at that level.

Recognizing these different payment priorities, and thus the potential for higher (or lower) loss severity in the event of default, the rating agencies have adopted a notching process whereby their credit ratings on issues can be moved up or down from the issuer rating, which is usually the rating applied to its senior unsecured debt. As a general rule, the higher the senior unsecured rating, the smaller the notching adjustment. The reason behind this is that the higher the rating, the lower the perceived risk of default; so, the need to "notch" the rating to capture the potential difference in loss severity is greatly reduced. For lower-rated credits, however, the risk of default is greater and thus the potential difference in loss from a lower (or higher) priority ranking is a bigger consideration in assessing an issue's credit riskiness. Thus, the rating agencies will typically apply larger rating adjustments. For example, S&P applies the following notching guidelines:

> A key principle is that investment-grade ratings focus more on timeliness, while non-investment-grade ratings give additional weight to recovery. For example, subordinated debt can be rated up to two notches below a non-investment-grade corporate credit rating, but one notch at most if the corporate credit rating is investment grade. Conversely,… the "AAA" rating category need not be notched at all, while at the "CCC" level the gaps may widen.
>
> The rationale for this convention is straightforward: as default risk increases, the concern over what can be recovered takes on greater relevance and, therefore, greater rating significance. Accordingly, the ultimate recovery aspect of ratings is given more weight as one moves down the rating spectrum. (Standard & Poor's, "Rating the Issue," in *Corporate Ratings Criteria 2008* [New York: Standard & Poor's, 2008]: 65)

Exhibit 5 is an example of S&P's notching criteria, as applied to Infor Software Parent LLC, Inc. (Infor). Infor is a US-based global software and services company whose corporate credit rating from S&P is B–. Note how the company's senior secured bonds are rated B, whereas its senior unsecured bonds are rated two notches lower at CCC+ and its holding company debt is rated even one notch lower at CCC.

EXHIBIT 5 Infor S&P Ratings Detail (as of December 2018)

Corporate credit rating	B−/Stable
Senior secured (3 issues)	B
Senior unsecured (2 issues)	CCC+
Holding company debt (1 issue)	CCC

Source: Standard & Poor's Financial Services, LLC.

4.3. ESG Ratings

ESG investing has received a great deal of attention and acceptance as a viable investment strategy, even as a source of alpha. ESG investing involves asset selection and investment decision making based on disciplined evaluation of one or all of these considerations:

- Environmental themes, such as investing in companies that are responding to consumer demand for sustainable practices, the reduction of carbon emissions, and other environmental agendas.
- Social themes, such as investing in companies committed to a diverse and inclusive workplace, minimizing salary gap, and community involvement programs.
- Governance themes, such as investing in companies committed to diverse board composition and strong oversight.

MSCI Inc., a global provider of market indexes and analytic tools, has launched a set of ratings that aim to measure a company's attitudes, practices, and advances related to ESG. Their rules-based methodology aims to identify and track leaders and laggards in the space. Companies are evaluated according to their exposure to ESG risks and how well they manage those risks relative to peers. The MSCI ESG rating scale has seven tiers and spans AAA to CCC. From best to worst, leaders receive AAA or AA; the next echelon constitutes an average rating of A, BBB, or BB; while laggards receive B or CCC. MSCI has also added countries, mutual funds, and ETFs to their ESG rating system. Traditional rating agencies, such as Moody's and S&P, are evaluating ESG for borrowers under these criteria too.

ESG investing allows investors to align their investment objectives with long-term trends considered important to the overall sustainability of a business. Some evidence suggests that funds that value "ethics," by construction, also outperform key market indexes.

4.4. Risks in Relying on Agency Ratings

- **explain risks in relying on ratings from credit rating agencies**

The dominant position of the rating agencies in the global debt markets, and the near-universal use of their credit ratings on debt securities, suggests that investors believe they do a good job assessing credit risk. In fact, with a few exceptions (e.g., too high ratings on US subprime mortgage-backed securities issued in the mid-2000s, which turned out to be much riskier than expected), their ratings have proved quite accurate as a relative measure of default risk. For example, Exhibit 6 shows historical S&P one-year global corporate default rates by rating category for the 20-year period from 1998 to 2017. It measures the percentage of issues that defaulted in a given calendar year based on how they were rated at the beginning of the year.

EXHIBIT 6 Global Corporate Annual Default Rates by Rating Category (%)

	AAA	AA	A	BBB	BB	B	CCC/C
1998	0.00	0.00	0.00	0.41	0.82	4.63	42.86
1999	0.00	0.17	0.18	0.20	0.95	7.29	33.33
2000	0.00	0.00	0.27	0.37	1.16	7.70	35.96
2001	0.00	0.00	0.27	0.34	2.96	11.53	45.45
2002	0.00	0.00	0.00	1.01	2.89	8.21	44.44
2003	0.00	0.00	0.00	0.23	0.58	4.07	32.73
2004	0.00	0.00	0.08	0.00	0.44	1.45	16.18
2005	0.00	0.00	0.00	0.07	0.31	1.74	9.09
2006	0.00	0.00	0.00	0.00	0.30	0.82	13.33
2007	0.00	0.00	0.00	0.00	0.20	0.25	15.24
2008	0.00	0.38	0.39	0.49	0.81	4.09	27.27
2009	0.00	0.00	0.22	0.55	0.75	10.94	49.46
2010	0.00	0.00	0.00	0.00	0.58	0.86	22.62
2011	0.00	0.00	0.00	0.07	0.00	1.67	16.30
2012	0.00	0.00	0.00	0.00	0.03	1.57	27.52
2013	0.00	0.00	0.00	0.00	0.10	1.64	24.50
2014	0.00	0.00	0.00	0.00	0.00	0.78	17.42
2015	0.00	0.00	0.00	0.00	0.16	2.40	26.51
2016	0.00	0.00	0.00	0.00	0.47	3.70	33.17
2017	0.00	0.00	0.00	0.00	0.08	0.98	26.23
Mean	0.00	0.03	0.07	0.19	0.69	3.82	27.98
Max	0.00	0.38	0.39	1.01	2.96	11.53	45.45
Min	0.00	0.00	0.00	0.00	0.00	0.25	9.09

Source: Based on data from Standard & Poor's Financial Services, LLC.

As Exhibit 6 shows, the highest-rated bonds have extremely low default rates. With very few exceptions, the lower the rating, the higher the annual rate of default, with bonds rated CCC and lower experiencing the highest default rates by far.

Relying on credit rating agency ratings, however, also has limitations and risks, including the following:

- **Credit ratings can change over time**. Over a long time period (e.g., many years), credit ratings can migrate—move up or down—significantly from what they were at the time of bond issuance. Using Standard & Poor's data, Exhibit 7 shows the average three-year migration (or "transition") by rating from 1981 to 2017. Note that the higher the credit rating, the greater the ratings stability. Even for AAA rated credits, however, only about

65% of the time did ratings remain in that rating category over a three-year period. (Of course, AAA rated credits can have their ratings move in only one direction—down.) A very small fraction of AAA rated credits became non-investment grade or defaulted within three years. For single-B rated credits, only 41% of the time did ratings remain in that rating category over three-year periods. This observation about how credit ratings can change over time isn't meant to be a criticism of the rating agencies. It is meant to demonstrate that creditworthiness can and does change—up or down—and that bond investors should not assume an issuer's credit rating will remain the same from time of purchase through the entire holding period.

EXHIBIT 7 Average Three-Year Global Corporate Transition Rates, 1981–2017 (%)

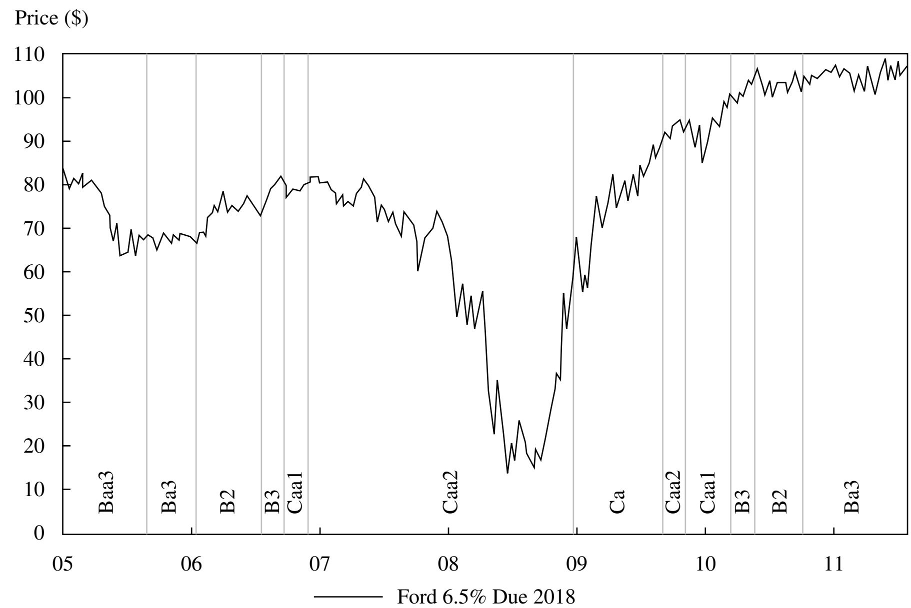
From/To	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	65.48	22.09	2.35	0.32	0.19	0.08	0.11	0.13	9.24
AA	1.21	66.14	18.53	2.06	0.35	0.22	0.03	0.12	11.33
A	0.06	4.07	68.85	11.72	1.30	0.44	0.09	0.25	13.21
BBB	0.02	0.28	8.42	64.66	7.11	1.64	0.30	0.87	16.70
BB	0.01	0.06	0.51	11.08	47.04	11.58	1.25	3.96	24.51
B	0.00	0.03	0.21	0.78	10.23	41.46	4.67	12.57	30.05
CCC/C	0.00	0.00	0.14	0.61	1.63	16.86	10.54	40.65	29.57

Notes: D = default. NR = not rated—that is, certain corporate issuers were no longer rated by S&P. This could occur for a variety of reasons, including issuers paying off their debt and no longer needing ratings.
Source: S&P Global Ratings, "2017 Annual Global Corporate Default Study and Rating Transitions" (5 April 2018): 53.

- **Credit ratings tend to lag the market's pricing of credit risk**. Bond prices and credit spreads frequently move more quickly because of changes in perceived creditworthiness than rating agencies change their ratings (or even outlooks) up or down. Bond prices and relative valuations can move every day, whereas bond ratings, appropriately, don't change that often. Even over long time periods, however, credit ratings can badly lag changes in bond prices. Exhibit 8 shows the price and Moody's rating of a bond from US automaker Ford Motor Company before, during, and after the global financial crisis in 2008. Note how the bond's price moved down sharply well before Moody's downgraded its credit rating—multiple times—and also how the bond's price began to recover—and kept recovering—well before Moody's upgraded its credit rating on Ford debt.

  Moreover, particularly for certain speculative-grade credits, two bonds with similar ratings may trade at very different valuations. This is partly a result of the fact that credit ratings primarily try to assess the risk of default, whereas for low-quality credits, the market begins focusing more on expected loss (default probability times loss severity). So, bonds from two separate issuers with comparable (high) risk of default but different recovery rates may have similar ratings but trade at significantly different dollar prices.

EXHIBIT 8 Historical Example: Ford Motor Company Senior Unsecured Debt: Price vs. Moody's Rating 2005–2011



Sources: Data based on Bloomberg Finance LP and Moody's Investors Service, Inc.

Thus, bond investors who wait for rating agencies to change their ratings before making buy and sell decisions in their portfolios may be at risk of underperforming other investors who make portfolio decisions in advance of—or not solely based on—rating agency changes.

- **Rating agencies may make mistakes**. The mis-rating of billions of dollars of subprime-backed mortgage securities is one example. Other historical examples include the mis-ratings of US companies Enron and WorldCom and European issuer Parmalat. Like many investors, the rating agencies did not understand the accounting fraud being committed in those companies.
- **Some risks are difficult to capture in credit ratings**. Examples include litigation risk, such as that which can affect tobacco companies, or environmental and business risks faced by chemical companies and utility power plants. This would also include the impact from natural disasters. Leveraged transactions, such as debt-financed acquisitions and large stock buybacks (share repurchases), are often difficult to anticipate and thus to capture in credit ratings.

As described, there are risks in relying on credit rating agency ratings when investing in bonds. Thus, while the credit rating agencies will almost certainly continue to play a significant role in the bond markets, it is important for investors to perform their own credit analyses and draw their own conclusions regarding the credit risk of a given debt issue or issuer.

EXAMPLE 4 Credit Ratings

1. Using the S&P rating scale, investment-grade bonds carry which of the following ratings?
   A. AAA to EEE
   B. BBB– to CCC
   C. AAA to BBB–
2. Using both Moody's and S&P ratings, which of the following pairs of ratings is considered high yield, also known as "below investment grade," "speculative grade," or "junk"?
   A. Baa1/BBB–
   B. B3/CCC+
   C. Baa3/BB+
3. What is the difference between an issuer rating and an issue rating?
   A. The issuer rating applies to all of an issuer's bonds, whereas the issue rating considers a bond's seniority ranking.
   B. The issuer rating is an assessment of an issuer's overall creditworthiness, whereas the issue rating is always higher than the issuer rating.
   C. The issuer rating is an assessment of an issuer's overall creditworthiness, typically reflected as the senior unsecured rating, whereas the issue rating considers a bond's seniority ranking (e.g., secured or subordinated).
4. Based on the practice of notching by the rating agencies, a subordinated bond from a company with an issuer rating of BB would likely carry what rating?
   A. B+
   B. BB
   C. BBB–
5. The fixed-income portfolio manager you work with asked you why a bond from an issuer you cover didn't rise in price when it was upgraded by Fitch from B+ to BB. Which of the following is the *most likely* explanation?
   A. Bond prices never react to rating changes.
   B. The bond doesn't trade often, so the price hasn't adjusted to the rating change yet.
   C. The market was expecting the rating change, and so it was already "priced in" to the bond.
6. Amalgamated Corp. and Widget Corp. each have bonds outstanding with similar coupons and maturity dates. Both bonds are rated B2, B–, and B by Moody's, S&P, and Fitch, respectively. The bonds, however, trade at very different prices—the Amalgamated bond trades at €89, whereas the Widget bond trades at €62. What is the *most likely* explanation of the price (and yield) difference?
   A. Widget's credit ratings are lagging the market's assessment of the company's credit deterioration.
   B. The bonds have similar risks of default (as reflected in the ratings), but the market believes the Amalgamated bond has a higher expected loss in the event of default.

C. The bonds have similar risks of default (as reflected in the ratings), but the market believes the Widget bond has a higher expected recovery rate in the event of default.

Solution to 1: C is correct.

Solution to 2: B is correct. Note that issuers with such ratings as Baa3/BB+ (answer C) are called "crossovers" because one rating is investment grade (the Moody's rating of Baa3) and the other is high yield (the S&P rating of BB+).

Solution to 3: C is correct.

Solution to 4: A is correct. The subordinated bond would have its rating notched lower than the company's BB rating, probably by two notches, reflecting the higher weight given to loss severity for below-investment-grade credits.

Solution to 5: C is correct. The market was anticipating the rating upgrade and had already priced it in. Bond prices often do react to rating changes, particularly multi-notch ones. Even if bonds don't trade, their prices adjust based on dealer quotations given to bond pricing services.

Solution to 6: A is correct. Widget's credit ratings are probably lagging behind the market's assessment of its deteriorating creditworthiness. Answers B and C both state the situation backwards. If the market believed that the Amalgamated bond had a higher expected loss given default, then that bond would be trading at a lower, not a higher, price. Similarly, if the market believed that the Widget bond had a higher expected recovery rate in the event of default, then that bond would be trading at a higher, not a lower, price.

5. TRADITIONAL CREDIT ANALYSIS: CORPORATE DEBT SECURITIES

- **explain the four Cs (Capacity, Collateral, Covenants, and Character) of traditional credit analysis**

The goal of credit analysis is to assess an issuer's ability to satisfy its debt obligations, including bonds and other indebtedness, such as bank loans. These debt obligations are contracts, the terms of which specify the interest rate to be paid, the frequency and timing of payments, the maturity date, and the covenants that describe the permissible and required actions of the borrower. Because corporate bonds are contracts, enforceable by law, credit analysts generally assume an issuer's willingness to pay and concentrate instead on assessing its ability to pay. Thus, the main focus in credit analysis is to understand a company's ability to generate cash flow over the term of its debt obligations. In so doing, analysts must assess both the credit quality of the company and the fundamentals of the industry in which the company operates. Traditional credit analysis considers the sources, predictability, and sustainability of cash generated by a company to service its debt obligations. This section will focus on corporate credit analysis; in particular, it will emphasize non-financial companies. Financial institutions have very different business models and funding profiles from industrial and utility companies.

5.1. Credit Analysis vs. Equity Analysis: Similarities and Differences

The description of credit analysis suggests credit and equity analyses should be very similar, and in many ways, they are. There are motivational differences, however, between equity and fixed-income investors that are an important aspect of credit analysis. Strictly speaking, management works for the shareholders of a company. Its primary objective is to maximize the value of the company for its owners. In contrast, management's legal duty to its creditors—including bondholders—is to meet the terms of the governing contracts. Growth in the value of a corporation from rising profits and cash flow accrues to the shareholders, while the best outcome for bondholders is to receive full, timely payment of interest and repayment of principal when due. Conversely, shareholders are more exposed to the decline in value if a company's earnings and cash flow decline because bondholders have a prior claim on cash flow and assets. But if a company's earnings and cash flow decline to the extent that it can no longer make its debt payments, then bondholders are at risk of loss as well.

In summary, in exchange for a prior claim on cash flow and assets, bondholders do not share in the growth in value of a company (except to the extent that its creditworthiness improves) but have downside risk in the event of default. In contrast, shareholders have theoretically unlimited upside opportunity, but in the event of default, their investment is typically wiped out before the bondholders suffer a loss. This is very similar to the type of payoff patterns seen in financial options. In fact, in recent years, credit risk models have been developed based on the insights of option pricing theory. Although it is beyond the scope of this present introduction to the subject, it is an expanding area of interest to both institutional investors and rating agencies.

Thus, although the analysis is similar in many respects for both equity and credit, equity analysts are interested in the strategies and investments that will increase a company's value and grow earnings per share. They then compare that earnings and growth potential with that of other companies in a given industry. Credit analysts will look more at the downside risk by measuring and assessing the sustainability of a company's cash flow relative to its debt levels and interest expense. Importantly for credit analysts, the balance sheet will show the composition of an issuer's debt—the overall amount, how much is coming due and when, and the distribution by seniority ranking. In general, equity analysts will focus more on income and cash flow statements, whereas credit analysts tend to focus more on the balance sheet and cash flow statements.

5.2. The Four Cs of Credit Analysis: A Useful Framework

Traditionally, many analysts evaluated creditworthiness based on what is often called the "four Cs of credit analysis":

- Capacity
- Collateral
- Covenants
- Character

Capacity refers to the ability of the borrower to make its debt payments on time; this is the focus of this section. **Collateral** refers to the quality and value of the assets supporting the issuer's indebtedness. **Covenants** are the terms and conditions of lending agreements that the issuer must comply with. **Character** refers to the quality of management. Each of these will now be covered in greater detail. Please note that the list of Cs is a convenient way to summarize the important aspects of the analysis; it is not a checklist to be applied mechanically, nor is it always exhaustive.

5.2.1. Capacity

Capacity is the ability of a borrower to service its debt. To determine that, credit analysis, in a process similar to equity analysis, starts with industry analysis and then turns to examination of the specific issuer (company analysis).

Industry structure. The Porter framework (Michael E. Porter, "The Five Competitive Forces That Shape Strategy," *Harvard Business Review* 86 [1; 2008]: 78–93) considers the effects of five competitive forces on an industry:

1. **Threat of entry.** Threat of entry depends on the extent of barriers to entry and the expected response from incumbents to new entrants. Industries with high entry barriers tend to be more profitable and have lower credit risk than industries with low entry barriers because incumbents do not need to hold down prices or take other steps to deter new entrants. High entry barriers can take many forms, including high capital investment, such as in aerospace; large, established distribution systems, such as in auto dealerships; patent protection, such as in technology or pharmaceutical industries; or a high degree of regulation, such as in utilities.
2. **Bargaining power of suppliers.** An industry that relies on just a few suppliers tends to be less profitable and to have greater credit risk than an industry that has multiple suppliers. Industries and companies with just a few suppliers have limited negotiating power to keep the suppliers from raising prices, whereas industries that have many suppliers can play them off against each other to keep prices in check.
3. **Bargaining power of customers.** Industries that rely heavily on just a few main customers have greater credit risk because the negotiating power lies with the buyers.
4. **Threat of substitutes.** Industries (and companies) that offer products and services that provide great value to their customers, and for which there are not good or cost-competitive substitutes, typically have strong pricing power, generate substantial cash flows, and represent less credit risk than other industries or companies. Certain (patent-protected) drugs are an example. Over time, however, disruptive technologies and inventions can increase substitution risk. For example, years ago, airplanes began displacing many trains and steamships. Newspapers were considered to have a nearly unassailable market position until television and then the internet became substitutes for how people received news and information. Over time, recorded music has shifted from records to tapes, to compact discs, to mp3s and other forms of digital media.
5. **Rivalry among existing competitors.** Industries with strong rivalry—because of numerous competitors, slow industry growth, or high barriers to exit—tend to have less cash flow predictability and, therefore, higher credit risk than industries with less competition. Regulation can affect the extent of rivalry and competition. For example, regulated utilities typically have a monopoly position in a given market, which results in relatively stable and predictable cash flows.

It is important to consider how companies in an industry generate revenues and earn profits. Is it an industry with high fixed costs and capital investment or one with modest fixed costs? These structures generate revenues and earn profits in very different ways. Two examples of industries with high fixed costs, also referred to as "having high operating leverage," are airlines and hotels. Many of their operating costs are fixed—running a hotel, flying a plane—so they cannot easily cut costs. If an insufficient number of people stay at a hotel or fly in a plane, fixed operating costs may not be covered and losses may result. With higher occupancy of a hotel or plane, revenues are higher, and it is more likely that fixed costs will be covered and profits earned.

Industry fundamentals. After understanding an industry's structure, the next step is to assess its fundamentals, including its sensitivity to macroeconomic factors, its growth prospects, its profitability, and its business need—or lack thereof—for high credit quality. Judgments about these can be made by looking at the following:

- *Cyclical or non-cyclical.* This is a crucial assessment because industries that are cyclical—that is, have greater sensitivity to broader economic performance—have more volatile revenues, margins, and cash flows and thus are inherently riskier than non-cyclical industries. Food producers, retailers, and health care companies are typically considered non-cyclical, whereas auto and steel companies can be very cyclical. Companies in cyclical industries should carry lower levels of debt relative to their ability to generate cash flow over an economic cycle than companies in less-cyclical or non-cyclical industries.
- *Growth prospects*. Although growth is typically a greater focus for equity analysts than for credit analysts, bond investors have an interest in growth as well. Industries that have little or no growth tend to consolidate via mergers and acquisitions. Depending upon how these are financed (e.g., using stock or debt) and the economic benefits (or lack thereof) of the merger, they may or may not be favorable to corporate bond investors. Weaker competitors in slow-growth industries may begin to struggle financially, adversely affecting their creditworthiness.
- *Published industry statistics*. Analysts can get an understanding of an industry's fundamentals and performance by researching statistics that are published by and available from a number of different sources, including the rating agencies, investment banks, industry publications, and frequently, government agencies.

Company fundamentals. Following analysis of an industry's structure and fundamentals, the next step is to assess the fundamentals of the company: the corporate borrower. Analysts should examine the following:

- Competitive position
- Track record/operating history
- Management's strategy and execution
- Ratios and ratio analysis

When assessing the company fundamentals, analysts should also explore how the fundamentals are impacted by environmental, social, and governance (ESG) factors.

Competitive position. Based on their knowledge of the industry structure and fundamentals, analysts assess a company's competitive position within the industry. What is its market share? How has it changed over time: Is it increasing, decreasing, holding steady? Is it well above (or below) its peers? How does it compare with respect to cost structure? How might it change its competitive position? What sort of financing might that require?

Track record/Operating history. How has the company performed over time? It's useful to go back several years and analyze the company's financial performance, perhaps during times of both economic growth and contraction. What are the trends in revenues, profit margins, and cash flow? Capital expenditures represent what percentage of revenues? What are the trends on the balance sheet—use of debt versus equity? Was this track record developed under the current management team? If not, when did the current management team take over?

Management's strategy and execution. What is management's strategy for the company—to compete and to grow? Does it make sense, and is it plausible? How risky is it, and how differentiated is it from its industry peers? Is it venturing into unrelated businesses? Does the analyst have confidence in management's ability to execute? What is management's track record, both at this company and at previous ones?

Credit analysts also want to know and understand how management's strategy will affect its balance sheet. Does management plan to manage the balance sheet prudently, in a manner that doesn't adversely affect bondholders? Analysts can learn about management's strategy from reading comments, discussion, and analysis that are included with financial statements filed with appropriate regulators, listening to conference calls about earnings or other big announcements (e.g., acquisitions), going to company websites to find earnings releases and copies of slides of presentations at various industry conferences, visiting and speaking with the company, and so on.

EXAMPLE 5 Industry and Company Analysis

1. Given a hotel company, a chemical company, and a food retail company, which is *most likely* to be able to support a high debt load over an economic cycle?
   A. The hotel company, because people need a place to stay when they travel.
   B. The chemical company, because chemicals are a key input to many products.
   C. The food retail company, because such products as food are typically resistant to recessions.
2. Heavily regulated monopoly companies, such as utilities, often carry high debt loads. Which of the following statements about such companies is *most* accurate?
   A. Regulators require them to carry high debt loads.
   B. They generate strong and stable cash flows, enabling them to support high levels of debt.
   C. They are not very profitable and need to borrow heavily to maintain their plant and equipment.
3. XYZ Corp. manufactures a commodity product in a highly competitive industry in which no company has significant market share and where there are low barriers to entry. Which of the following *best* describes XYZ's ability to take on substantial debt?
   A. Its ability is very limited because companies in industries with those characteristics generally cannot support high debt loads.
   B. Its ability is high because companies in industries with those characteristics generally have high margins and cash flows that can support significant debt.
   C. We don't have enough information to answer the question.

Solution to 1: C is correct. Food retail companies are considered non-cyclical, whereas hotel and chemical companies are more cyclical and thus more vulnerable to economic downturns.

Solution to 2: B is correct. Because such monopolies' financial returns are generally dictated by the regulators, they generate consistent cash flows and are thus able to support high debt levels.

Solution to 3: A is correct. Companies in industries with those characteristics typically have low margins and limited cash flow and thus cannot support high debt levels.

- **calculate and interpret financial ratios used in credit analysis**
- **evaluate the credit quality of a corporate bond issuer and a bond of that issuer, given key financial ratios of the issuer and the industry**

Ratios and ratio analysis. To provide context to the analysis and understanding of a company's fundamentals—based on the industry in which it operates, its competitive position, its strategy and execution—a number of financial measures derived from the company's principal financial statements are examined. Credit analysts calculate a number of ratios to assess the financial health of a company, identify trends over time, and compare companies across an industry to get a sense of relative creditworthiness. Note that typical values of these ratios vary widely from one industry to another because of different industry characteristics previously identified: competitive structure, economic cyclicality, regulation, and so on.

We will categorize the key credit analysis measures into three different groups:

- Profitability and cash flow
- Leverage
- Coverage

Profitability and cash flow measures. It is from profitability and cash flow generation that companies can service their debt. Credit analysts typically look at operating profit margins and operating income to get a sense of a company's underlying profitability and see how it varies over time. Operating income is defined as operating revenues minus operating expenses and is commonly referred to as "earnings before interest and taxes" (EBIT). Credit analysts focus on EBIT because it is useful to determine a company's performance prior to costs arising from its capital structure (i.e., how much debt it carries versus equity). And "before taxes" is used because interest expense is paid before income taxes are calculated.

Several measures of cash flow are used in credit analysis; some are more conservative than others because they make certain adjustments for cash that gets used in managing and maintaining the business or in making payments to shareholders. The cash flow measures and leverage and coverage ratios discussed next are non-IFRS in the sense that they do not have official IFRS definitions; the concepts, names, and definitions given should be viewed as one usage among several possible, in most cases.

- **Earnings before interest, taxes, depreciation, and amortization (EBITDA).** EBITDA is a commonly used measure of cash flow that takes operating income and adds back depreciation and amortization expense because those are non-cash items. This is a somewhat crude measure of cash flow because it excludes certain cash-related expenses of running a business, such as capital expenditures and changes in (non-cash) working capital. Thus, despite its popularity as a cash flow measure, analysts look at other measures in addition to EBITDA.

- **Funds from operations (FFO).** Standard & Poor's defines funds from operations as net income from continuing operations plus depreciation, amortization, deferred income taxes, and other non-cash items.
- **Free cash flow before dividends (FCF before dividends).** This measures excess cash flow generated by the company (excluding non-recurring items) before payments to shareholders or that could be used to pay down debt or pay dividends. It can be calculated as net income (excluding non-recurring items) plus depreciation and amortization minus increase (plus decrease) in non-cash working capital minus capital expenditures. This is, depending upon the treatment of dividends and interest in the cash flow statement, approximated by the cash flow from operating activities minus capital expenditures. Companies that have negative free cash flow before payments to shareholders will be consuming cash they have or will need to rely on additional financing—from banks, bond investors, or equity investors. This obviously represents higher credit risk.
- **Free cash flow after dividends (FCF after dividends).** This measure just takes free cash flow before dividends and subtracts dividend payments. If this number is positive, it represents cash that could be used to pay down debt or build up cash on the balance sheet. Either action may be viewed as deleveraging, which is favorable from a credit risk standpoint. Some credit analysts will calculate net debt by subtracting balance sheet cash from total debt, although they shouldn't assume the cash will be used to pay down debt. Actual debt paid down from free cash flow is a better indicator of deleveraging. Some analysts will also deduct stock buybacks to get the "truest" measure of free cash flow that can be used to de-lever on either a gross or net debt basis; however, others view stock buybacks (share repurchases) as more discretionary and as having less certain timing than dividends, and thus treat those two types of shareholder payments differently when calculating free cash flow.

Leverage ratios. A few measures of leverage are used by credit analysts. The most common are the debt/capital, debt/EBITDA, and measures of funds or cash flows/debt ratios. Note that many analysts adjust a company's reported debt levels for debt-like liabilities, such as underfunded pensions and other retiree benefits, as well as operating leases. When adjusting for leases, analysts will typically add back the imputed interest or rent expense to various cash flow measures.

- **Debt/capital.** Capital is calculated as total debt plus shareholders equity. This ratio shows the percentage of a company's capital base that is financed with debt. A lower percentage of debt indicates lower credit risk. This traditional ratio is generally used for investment-grade corporate issuers. Where goodwill or other intangible assets are significant (and subject to obsolescence, depletion, or impairment), it is often informative to also compute the debt to capital ratio after assuming a write-down of the after-tax value of such assets.
- **Debt/EBITDA.** This ratio is a common leverage measure. Analysts use it on a "snapshot" basis, as well as to look at trends over time and at projections and to compare companies in a given industry. Rating agencies often use it as a trigger for rating actions, and banks reference it in loan covenants. A higher ratio indicates more leverage and thus higher credit risk. Note that this ratio can be very volatile for companies with high cash flow variability, such as those in cyclical industries and with high operating leverage (fixed costs).
- **FFO/debt.** Credit rating agencies often use this leverage ratio. They publish key median and average ratios, such as this one, by rating category so analysts can get a sense of why an issuer is assigned a certain credit rating, as well as where that rating may migrate based on changes to such key ratios as this one. A higher ratio indicates greater ability to pay debt by funds from operations.

- **FCF after dividends/debt**. A higher ratio indicates that a greater amount of debt can be paid off from free cash flow after dividend payments.

Coverage ratios. Coverage ratios measure an issuer's ability to meet—to "cover"—its interest payments. The two most common are the EBITDA/interest expense and EBIT/interest expense ratios.

- **EBITDA/interest expense.** This measurement of interest coverage is a bit more liberal than the one that uses EBIT because it does not subtract out the impact of (non-cash) depreciation and amortization expense. A higher ratio indicates higher credit quality.
- **EBIT/interest expense.** Because EBIT does not include depreciation and amortization, it is considered a more conservative measure of interest coverage. This ratio is now used less frequently than EBITDA/interest expense.

Exhibit 9 is an example of key average credit ratios by rating category for industrial companies over the 12-month period 3Q2017–3Q2018, as calculated by Bloomberg Barclays Indices, using public company data. Note only a few AAA-rated corporations remain, so the small sample size can skew the average ratios of a few key credit metrics. That said, it should be clear that, overall, higher-rated issuers have stronger credit metrics.

EXHIBIT 9 Industrial Comparative Ratio Analysis

Credit Rating	EBITDA Margin (%)	Return on Capital (%)	EBIT Interest Coverage (×)	EBITDA Interest Coverage (×)	FFO/ Debt (%)	Free Operations Cash Flow/ Debt (%)	Debt/ EBITDA (×)	Debt/Debt plus Equity (%)
Aaa								
US	66.4	6.5	4.2	21.3	51.9	43.5	–0.2	43.3
Aa								
US	21.9	10.8	15.4	45.0	109.9	58.1	1.2	50.6
A								
US	26.0	13.5	13.3	18.9	49.1	31.8	1.8	51.2
Baa								
US	23.9	11.5	7.2	NA	40.7	20.3	3.9	49.4
Ba								
US	21.7	3.5	4.6	NA	27.7	11.0	4.1	64.0
B								
US	21.2	3.7	2.5	NA	20.3	1.8	5.2	69.3
Caa								
US	16.0	0.2	–0.6	1.3	10.0	–6.6	9.3	95.3

Note: As of 19 December 2018.
Source: Bloomberg Barclays Indices.

Comments on issuer liquidity. An issuer's access to liquidity is also an important consideration in credit analysis. Companies with high liquidity represent lower credit risk than those with weak liquidity, other factors being equal. The global financial crisis of 2008–2009 showed that access to liquidity via the debt and equity markets should not be taken for granted, particularly for companies that do not have strong balance sheets or steady operating cash flow.

When assessing an issuer's liquidity, credit analysts tend to look at the following:

- **Cash on the balance sheet.** Cash holdings provide the greatest assurance of having sufficient liquidity to make promised payments.
- **Net working capital.** The big US automakers used to have enormous negative working capital, despite having high levels of cash on the balance sheet. This proved disastrous when the global financial crisis hit in 2008 and the economy contracted sharply. Auto sales—and thus revenues—fell, the auto companies cut production, and working capital consumed billions of dollars in cash as accounts payable came due when the companies most needed liquidity.
- **Operating cash flow.** Analysts will project this figure out a few years and consider the risk that it may be lower than expected.
- **Committed bank lines.** Committed but untapped lines of credit provide contingent liquidity in the event that the company is unable to tap other, potentially cheaper, financing in the public debt markets.
- **Debt coming due and committed capital expenditures in the next one to two years.** Analysts will compare the sources of liquidity with the amount of debt coming due as well as with committed capital expenditures to ensure that companies can repay their debt and still invest in the business if the capital markets are somehow not available.

As will be discussed in more detail in the section on special considerations for high-yield credits, issuer liquidity is a bigger consideration for high-yield companies than for investment-grade companies.

EXAMPLE 6

Mallinckrodt PLC (Mallinckrodt) is an Ireland-incorporated specialty pharmaceutical company. As a credit analyst, you have been asked to assess its creditworthiness—on its own, compared to a competitor in its overall industry, and compared with a similarly rated company in a different industry. Using the financial statements provided in Exhibits 10 through 12 for the three years ending 31 December 2015, 2016, and 2017, address the following:

1. Calculate Mallinckrodt's operating profit margin, EBITDA, and free cash flow after dividends. Comment on what these measures indicate about Mallinckrodt's profitability and cash flow.
2. Determine Mallinckrodt's leverage ratios: debt/EBITDA, debt/capital, free cash flow after dividends/debt. Comment on what these leverage ratios indicate about Mallinckrodt's creditworthiness.
3. Calculate Mallinckrodt's interest coverage using both EBIT and EBITDA. Comment on what these coverage ratios indicate about Mallinckrodt's creditworthiness.

4. Using the credit ratios provided in Exhibit 11 on Johnson & Johnson, compare the creditworthiness of Mallinckrodt relative to Johnson & Johnson.
5. Compare the Exhibit 12 credit ratios of Luxembourg-based ArcelorMittal, one of the world's largest global steelmakers, with those of Mallinckrodt. Comment on the volatility of the credit ratios of the two companies. Which company looks to be more cyclical? What industry factors might explain some of the differences? In comparing the creditworthiness of these two companies, what other factors might be considered to offset greater volatility of credit ratios?

EXHIBIT 10A Mallinckrodt PLC Financial Statements

Consolidated Statements of Operations	Year End 30 Sept.*	Years Ended 31 Dec.	
(Dollars in millions, except per share amounts)	2015	2016	2017
Net revenues	**2,923.1**	**3,399.5**	**3,221.6**
Operating expenses:			
Cost of sales	1,300.2	1,549.6	1,565.3
Research and development	203.3	267.0	277.3
Selling, general, and administrative expenses	1,023.8	1,070.3	920.9
Restructuring charges, net	45.0	33.0	31.2
Non-restructuring impairment charges	—	231.2	63.7
Gain on divestiture and license	(3.0)	—	(56.9)
Total operating expenses	**2,569.3**	**3,151.1**	**2,801.5**
Operating income	**353.8**	**248.4**	**420.1**
Other (expense) income:			
Interest expense	(255.6)	(378.1)	(369.1)
Interest income	1.0	1.6	4.6
Other income (expense), net	8.1	(3.5)	6.0
Total other (expense) income, net	**(246.5)**	**(380.0)**	**(358.5)**
Income before income taxes and non-controlling interest	**107.3**	**(131.6)**	**61.6**
Provision (Benefit) for income taxes	(129.3)	(340.0)	(1,709.6)
Net income	**236.6**	**208.4**	**1,771.2**
Income from discontinued operations, net of income taxes	88.1	71.0	363.2
Net income attributable to common shareholders	**324.7**	**279.4**	**2,134.4**

*Mallinckrodt changed their fiscal year end from 30 September to 31 December.
Source: Company filings, Loomis, Sayles & Company.

EXHIBIT 10B Mallinckrodt PLC Financial Statements

Consolidated Balance Sheets (Dollars in millions)	Year End 30 Sept.* 2015	Years Ended 31 Dec. 2016	2017
ASSETS			
Current assets:			
Cash and cash equivalents	365.9	342.0	1,260.9
Accounts receivable	489.6	431.0	445.8
Inventories	262.1	350.7	340.4
Deferred income taxes	139.2	—	—
Prepaid expenses and other current assets	194.4	131.9	84.1
Notes receivable	—	—	154.0
Current assets held for sale	394.9	310.9	—
Total current assets	**1,846.1**	**1,566.5**	**2,285.2**
Property, plant, and equipment, net	793.0	881.5	966.8
Goodwill	3,649.4	3,498.1	3,482.7
Intangible assets, net	9,666.3	9,000.5	8,375.0
Other assets	225.7	259.7	171.2
Long-term assets held for sale	223.6	—	—
Total assets	**16,404.1**	**15,206.3**	**15,280.9**
LIABILITIES AND EQUITY			
Current liabilities:			
Current maturities of long-term debt	22.0	271.2	313.7
Accounts payable	116.8	112.1	113.3
Accrued payroll and payroll-related costs	95.0	76.1	98.5
Accrued interest	80.2	68.7	57.0
Income taxes payable	—	101.7	15.8
Accrued and other current liabilities	486.1	557.1	452.1
Current liabilities held for sale	129.3	120.3	—
Total current liabilities	**929.4**	**1,307.2**	**1,050.4**
Long-term debt	6,474.3	5,880.8	6,420.9
Pension and post-retirement benefits	114.2	136.4	67.1
Environmental liabilities	73.3	73.0	73.2
Deferred income taxes	3,117.5	2,398.1	689.0
Other income tax liabilities	121.3	70.4	94.1
Other liabilities	209.0	356.1	364.2
Long-term liabilities held for sale	53.9	—	—
Total liabilities	**11,092.9**	**10,222.0**	**8,758.9**

EXHIBIT 10B (Continued)

Consolidated Balance Sheets (Dollars in millions)	Year End 30 Sept.* 2015	Years Ended 31 Dec. 2016	2017
Shareholders' equity:			
Ordinary shares	23.5	23.6	18.4
Ordinary shares held in treasury at cost	(109.7)	(919.8)	(1,564.7)
Additional paid-in capital	5,357.6	5,424.0	5,492.6
Retained earnings	38.9	529.0	2,588.6
Accumulated other comprehensive income	0.9	(72.5)	(12.9)
Total shareholders' equity	5,311.2	4,984.3	6,522.0
Total liabilities and shareholders' equity	16,404.1	15,206.3	15,280.9

*Mallinckrodt changed their fiscal year end from 30 September to 31 December.
Source: Company filings, Loomis, Sayles & Company.

EXHIBIT 10C Mallinckrodt PLC Financial Statements

Consolidated Statements of Cash Flow (Dollars in millions)	Year End 30 Sept.* 2015	Years Ended 31 Dec. 2016	2017
Cash Flows from Operating Activities:			
Net income (loss)	324.7	279.4	2,134.4
Depreciation and amortization	672.5	831.7	808.3
Share-based compensation	117.0	45.4	59.2
Deferred income taxes	(191.6)	(528.3)	−1744.1
Non-cash impairment charges	—	231.2	63.7
Inventory provisions	—	8.5	34.1
Gain on disposal of discontinued operations	—	1.7	−418.1
Other non-cash items	(25.5)	45.5	−21.4
Change in working capital	33.4	153.7	−188.8
Net cash from operating activities	**930.5**	**1,068.8**	**727.3**
Cash Flows from Investing Activities:			
Capital expenditures	(148.0)	(199.1)	−186.1
Acquisitions and intangibles, net of cash acquired	(2,154.7)	(247.2)	−76.3
Proceeds from divestitures, net of cash	—	3.0	576.9
Other	3.0	(4.9)	3.9
Net cash from investing activities	**(2,299.7)**	**(448.2)**	**318.4**

(*continued*)

EXHIBIT 10C (Continued)

Consolidated Statements of Cash Flow (Dollars in millions)	Year End 30 Sept.* 2015	Years Ended 31 Dec. 2016	2017
Cash Flows from Financing Activities:			
Issuance of external debt	3,010.0	226.3	1,465
Repayment of external debt and capital leases	(1,848.4)	(525.7)	−917.2
Debt financing costs	(39.9)	—	−12.7
Proceeds from exercise of share options	34.4	10.8	4.1
Repurchase of shares	(92.2)	(536.3)	−651.7
Other	(28.1)	(21.8)	−17.7
Net cash from financing activities	**1,035.8**	**(846.7)**	**−130.2**
Effect of currency rate changes on cash	(11.6)	(1.2)	**2.5**
Net increase (decrease) in cash and cash equivalents	(345.0)	(227.3)	918
Cash and cash equivalents at beginning of period	777.6	588.4	361.1
Cash and cash equivalents at end of period	432.6	361.1	1,279.1

*Mallinckrodt changed their fiscal year end from 30 September to 31 December.
Source: Company filings, Loomis, Sayles & Company.

EXHIBIT 10D Mallinckrodt PLC Credit Ratios

	2015	2016	2017
Operating margin	12.1%	7.3%	13.0%
Debt/EBITDA	6.3x	5.7x	5.5x
EBITDA/Interest	4.0x	2.9x	3.3x
FCF/Debt	12.0%	14.1%	8.0%
Debt/Capital	55.0%	55.2%	50.8%

Source: Company filings, Loomis, Sayles & Company.

EXHIBIT 11 Johnson & Johnson's Credit Ratios

	2015	2016	2017
Operating profit margin	26.2%	29.5%	25.8%
Debt/EBITDA	0.9x	1.1x	1.4x
EBITDA/Interest	40.1x	34.4x	27.1x
FCF after dividends/Debt	81.1%	57.3%	51.4%
Debt/Capital	22.0%	28.1%	36.5%

Source: Company filings, Loomis, Sayles & Company.

EXHIBIT 12 ArcelorMittal Credit Ratios

	2015	2016	2017
Operating profit margin	0.3%	5.5%	7.7%
Debt/EBITDA	5.8x	2.3x	1.6x
EBITDA/Interest	2.5x	5.0x	9.2x
FCF after dividends/Debt	−2.8%	1.9%	13.5%
Debt/Capital	41.8%	29.7%	24.0%

Source: Company filings, Loomis, Sayles & Company.

Solutions:

1. Operating profit margin (%) = Operating income/Revenue
   2015: 353.8/2,923.1 = 0.121 or 12.1%
   2016: 248.4/3,399.5 = 0.073 or 7.3%
   2017: 420.1/3,221.6 = 0.130 or 13.0%
   EBITDA = Operating income + Depreciation and Amortization
   2015: 353.8 + 672.5 = 1,026.3
   2016: 248.4+ 831.7 = 1,080.1
   2017: 420.1 + 808.3 = 1,228.4
   FCF after dividends = Cash flow from operations – Capital expenditures – Dividends
   2015: 930.5 − 148 − 0 = 782.5
   2016: 1,068.8 − 199.1 − 0 = 869.7
   2017: 727.3 − 186.1 − 0 = 541.2

   Operating profit margin decreased from 2015 to 2016 but increased from 2016 to 2017. Conversely, FCF after dividends increased from 2015 to 2016 but decreased from 2016 to 2017. EBITDA increased from 2015 to 2017. From 2015 to 2016, sales increased by 16.3% and operating expenses increased by 22.6%. As a result, operating profit margin decreased even though EBITDA and FCF increased. However, from 2016 to 2017, sales decreased by 5.2% and operating expenses decreased by 11.1%. As a result, operating profit margin and EBITDA increased, while FCF after dividends decreased.
2. Debt/EBITDA
   Total debt = Short-term debt and Current portion of long-term debt + Long-term debt
   2015: Debt: 22.0 + 6,474.3 = 6,496.3
   Debt/EBITDA: 6,496.3/1,026.3 = 6.3x
   2016: Debt: 271.2 + 5,880.8 = 6,152.0
   Debt/EBITDA: 6,152.0/1,080.1 = 5.7x
   2017: Debt: 313.7 + 6,420.9 = 6,734.6
   Debt/EBITDA: 6,734.6/1,228. 4 = 5.5x
   Debt/Capital (%)
   Capital = Debt + Equity
   2015: Capital: 6,496.3 + 5,311.2 = 11,807.5
   Debt/Capital: 6,496.3/11,807.5 = 55.0%
   2016: Capital: 6,152.0 + 4,984.3 = 11,136.3
   Debt/Capital: 6,152.0/11,136.3 = 55.2%

2017: Capital: 6,734.6 + 6,522.0 = 13,256.6
Debt/Capital: 6,734.6/13,256.6 = 50.8%

FCF after dividends/Debt (%)
2015: 782.5/6,496.3 = 12.0%
2016: 869.7/6,152 = 14.1%
2017: 541.2/6,734.6 = 8.0%

Although debt/EBITDA and debt/capital improved between 2015 and 2017, the "FCF after dividends/Debt" deteriorated significantly as cash flow from operations declined as a result of the loss taken on disposal of discontinued operations. Given that the loss is most likely a non-recurring event, Mallinckrodt's creditworthiness likely improved over the 2015 to 2017 period.

3. EBIT/Interest expense
2015: 353.8/255.6 = 1.4x
2016: 248.4/378.1 = 0.7x
2017: 420.1/369.1 = 1.1x
EBITDA/Interest expense
2015: 1,026.3/255.6 = 4.0x
2016: 1,080.1/378.1= 2.9x
2017: 1,228.4/369.1 = 3.3x
Based on these coverage ratios, Mallinckrodt's creditworthiness declined from 2015 to 2016 and then showed modest improvement in 2017. The 2017 coverage ratios are still weaker than the 2015 coverage ratios, indicating that growth in EBIT and EBITDA are not keeping pace with the rising interest expense.
4. Johnson & Johnson (J&J) has a higher operating profit margin, better leverage ratios—lower debt/EBITDA, higher FCF after dividends/debt over the three years, lower debt/capital, and better interest coverage as measured by EBITDA/interest. Collectively, those ratios suggest J&J has higher credit quality than Mallinckrodt.
5. Mallinckrodt has both a higher and a less volatile operating profit margin than ArcelorMittal (Arcelor). However, while Mallinckrodt's leverage ratios have been deteriorating, Arcelor's have been improving. Based on the volatility of its cash flow and operating profit margin, Arcelor appears to be a much more cyclical credit. However, with its meaningfully lower debt levels, one could expect Arcelor to have a higher credit rating.

A steelmaker likely has a significant amount of long-term assets financed by debt. It is a highly competitive industry with little ability to distinguish products from other competitors. To mitigate the impact of its more volatile credit ratios, Arcelor might maintain high levels of liquidity. Its size and global diversity may also be a "plus." Given its size, it may be able to negotiate favorable supplier and customer contracts and keep costs down through economies of scale.

5.2.2. Collateral

Collateral, or asset value, analysis is typically emphasized more with lower credit quality companies. As discussed earlier, credit analysts focus primarily on probability of default, which is mostly about an issuer's ability to generate sufficient cash flow to support its debt payments, as well as its ability to refinance maturing debt. Only when the default probability rises to a sufficient level do analysts typically consider asset or collateral value in the context of loss severity in the event of default.

Analysts do think about the value and quality of a company's assets; however, these are difficult to observe directly. Factors to consider include the nature and amount of intangible assets on the balance sheet. Some assets, such as patents, are clearly valuable and can be sold if necessary to cover liabilities. Goodwill, on the other hand, is not considered a high-quality asset. In fact, sustained weak financial performance most likely implies that a company's goodwill will be written down, reinforcing its poor quality. Another factor to consider is the amount of depreciation an issuer takes relative to its capital expenditures: Low capital expenditures relative to depreciation expense could imply that management is insufficiently investing in its business, which will lead to lower-quality assets, potentially reduced future operating cash flow, and higher loss severity in the event of default.

A market-based signal that credit analysts use to impute the quality of a publicly traded company's assets, and its ability to support its debt, is equity market capitalization. For instance, a company whose stock trades below book value may have lower-quality assets than is suggested by the amount reported on the balance sheet.

As economies become more service- and knowledge-based and those types of companies issue debt, it's important to understand that these issuers rely more on human and intellectual capital than on "hard assets." In generating profits and cash flow, these companies are not as asset intensive. One example would be software companies. Another example would be investment management firms. Human- and intellectual-capital-based companies may generate a lot of cash flow, but their collateral value is questionable unless there are patents and other types of intellectual property and "intangible capital" that may not appear directly on the balance sheet but could be valuable in the event of financial distress or default.

Regardless of the nature of the business, the key point of collateral analysis is to assess the value of the assets relative to the issuer's level—and seniority ranking—of debt.

5.2.3. Covenants

Covenants are meant to protect creditors while also giving management sufficient flexibility to operate its business on behalf of and for the benefit of the shareholders. They are integral to credit agreements, whether they are bonds or bank loans, and they spell out what the issuer's management is (1) obligated to do and (2) limited in doing. The former are called "affirmative covenants," whereas the latter are called "negative" or "restrictive covenants." Obligations would include such duties as making interest and principal payments and filing audited financial statements on a timely basis. Covenants might also require a company to redeem debt in the event of the company being acquired, the change of control covenant, or to keep the ratio of debt to EBITDA below some prescribed amount. The limitations might include a cap on the amount of cash that can be paid out to shareholders relative to earnings, or perhaps on the amount of additional secured debt that can be issued. Covenant violations are a breach of contract and can be considered default events unless they are cured in a short time or a waiver is granted.

For corporate bonds, covenants are described in the bond **prospectus**, the document that is part of a new bond issue. The prospectus describes the terms of the bond issue, as well as supporting financial statements, to help investors perform their analyses and make investment decisions as to whether or not to submit orders to buy the new bonds. Actually, the **trust deed** or **bond indenture** is the governing legal credit agreement and is typically incorporated by reference in the prospectus.

Covenants are an important but underappreciated part of credit analysis. Strong covenants protect bond investors from the possibility of management taking actions that would hurt an issuer's creditworthiness. For example, without appropriate covenants, management might pay large dividends, undertake stock buybacks well in excess of free cash flow, sell the company in a leveraged buyout, or take on a lot of secured debt that structurally subordinates unsecured bondholders. All of these actions would enrich shareholders at the expense of bondholders. Recall that management works for the shareholders and that bonds are contracts, with management's only real obligation to creditors being to uphold the terms of the contract. The inclusion of covenants in the contract is intended to protect bondholders.

The bond-buying investor base is very large and diverse, particularly for investment-grade debt. It includes institutional investors, such as insurance companies, investment management firms, pension funds, mutual funds, hedge funds, sovereign wealth funds, and so on. Although there are some very large institutional investors, the buyer base is fragmented and does not—and legally cannot—act as a syndicate. Thus, bondholders are generally not able to negotiate strong covenants on most new bond issues. At the same time, issuers expect that borrowing from capital markets will carry lighter convenants than borrowing from banks. During weak economic or market conditions, however, covenants on new bond issues tend to be stronger because investors require additional incentive to lend money while seeking more protection. A few organized institutional investor groups are focused on strengthening covenants: the Credit Roundtable (thecreditroundtable.org) in the United States and the European Model Covenant Initiative in the United Kingdom.

Covenant language is often very technical and written in "legalese," so it can be helpful to have an in-house person with a legal background to review and interpret the specific covenant terms and wording. One might also use a third-party service specializing in covenant analysis, such as Covenant Review (www.covenantreview.com).

We will go into more detail on specific covenants in the section on special considerations for high-yield bonds.

5.2.4. Character

The character of a corporate borrower can be difficult to observe. The analysis of character as a factor in credit analysis dates to when loans were made to companies owned by individuals. Most corporate bond issuers are now publicly owned by shareholders or privately owned by pools of capital, such as private equity firms. Management often has little ownership in a corporation, so analysis and assessment of character is different than it would be for owner-managed firms. Credit analysts can make judgments about management's character in the following ways:

- An assessment of the soundness of management's strategy.
- Management's track record in executing past strategies, particularly if they led to bankruptcy or restructuring. A company run by executives whose prior positions/ventures resulted in significant distress might still be able to borrow in the debt markets, but it would likely have to borrow on a secured basis and/or pay a higher interest rate.
- Use of aggressive accounting policies and/or tax strategies. Examples might include using a significant amount of off-balance-sheet financing, capitalizing versus immediately expensing items, recognizing revenue prematurely, and/or frequently changing auditors. These are potential warning flags to other behaviors or actions that may adversely impact an issuer's creditworthiness.
- Any history of fraud or malfeasance—a major warning flag to credit analysts.
- Previous poor treatment of bondholders—for example, management actions that resulted in major credit rating downgrades. These actions might include a debt-financed acquisition, a large special dividend to shareholders, or a major debt-financed stock buyback program.

EXAMPLE 7 The Four Cs

1. Which of the following would not be a bond covenant?
   A. The issuer must file financial statements with the bond trustee on a timely basis.
   B. The company can buy back as much stock as it likes.
   C. If the company offers security to any creditors, it must offer security to this bond issue.
2. Why should credit analysts be concerned if a company's stock trades below book value?
   A. It means the company is probably going bankrupt.
   B. It means the company will probably incur lots of debt to buy back its undervalued stock.
   C. It's a signal that the company's asset value on its balance sheet may be impaired and have to be written down, suggesting less collateral protection for creditors.
3. If management is of questionable character, how can investors incorporate this assessment into their credit analysis and investment decisions?
   A. They can choose not to invest based on the increased credit risk.
   B. They can insist on getting collateral (security) and/or demand a higher return.
   C. They can choose not to invest or insist on additional security and/or higher return.

Solution to 1: B is correct. Covenants describe what the borrower is (1) obligated to do or (2) limited in doing. It's the absence of covenants that would permit a company to buy back as much stock as it likes. A requirement that the company offer security to this bond issue if it offers security to other creditors (answer C) is referred to as a "negative pledge."

Solution to 2: C is correct.

Solution to 3: C is correct. Investors can always say no if they are not comfortable with the credit risk presented by a bond or issuer. They may also decide to lend to a borrower with questionable character only on a secured basis and/or demand a higher return for the perceived higher risk.

6. CREDIT RISK VS. RETURN: YIELDS AND SPREADS

- **describe macroeconomic, market, and issuer-specific factors that influence the level and volatility of yield spreads**

The material in this section applies to all bonds subject to credit risk. For simplicity, in what follows all such bonds are sometimes referred to as "corporate" bonds.

As in other types of investing, taking more risk in credit offers higher potential return but with more volatility and less certainty of earning that return. Using credit ratings as a proxy for risk, Exhibit 13 shows the composite yield-to-maturity for bonds of all maturities within each rating category in the US and European bond markets according to Bloomberg Barclays, one of the largest providers of fixed-income market indexes. For non-investment-grade bonds, the yield-to-call (YTC) or yield-to-worst (YTW) is reported as many contain optionality.

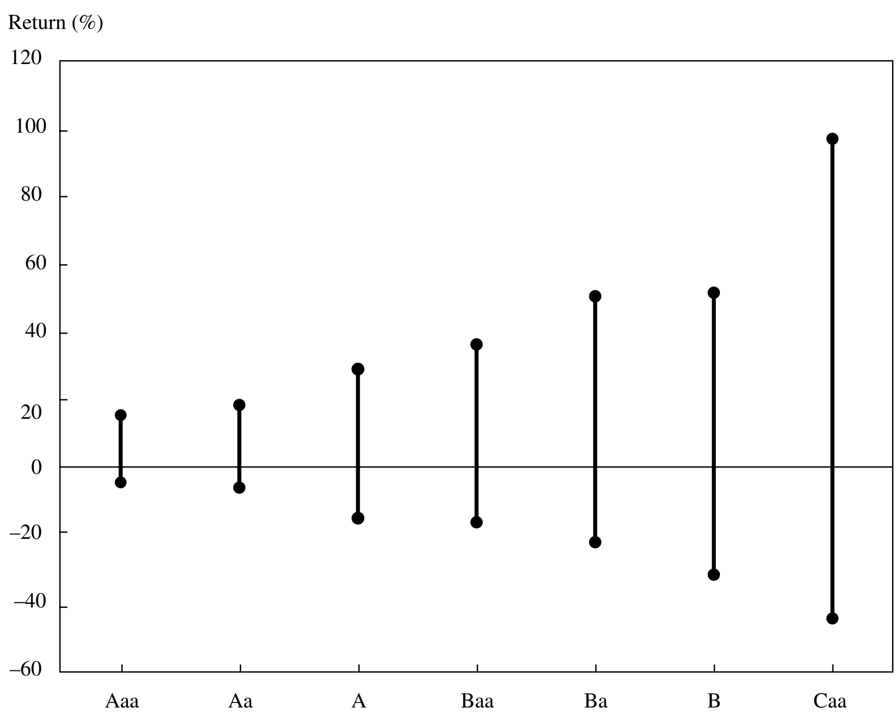
EXHIBIT 13 Corporate Yields by Rating Category (%)

Bloomberg Barclays Indices	Investment Grade				Non-Investment Grade			
	AAA	AA	A	BBB	BB	B	CCC	CC–D
US	3.63	3.52	3.86	4.35	5.14	6.23	8.87	19.51
Pan European*	1.25	0.76	1.18	1.67	2.92	5.63	8.78	54.95

* Pan European yields may be "artificially low" due to the ECB's extraordinary corporate bond Quantitative Easing (QE) Program.
Note: Data as of 30 September 2018.
Source: Bloomberg Barclays Indices.

Note that the lower the credit quality, the higher the quoted yield. The realized yield, or return, will almost always be different because of changes in interest rates, re-investment of coupons, holding period changes, and the credit-related risks discussed earlier. In addition to the absolute returns, the volatility of returns will also vary by rating. Trailing 12-month returns by credit rating category and the volatility (standard deviation) of those returns are shown in Exhibit 14.

EXHIBIT 14 US Credit Trailing 12-Month Returns by Rating Category, 31 December 1996–30 September 2018



	Aaa	Aa	A	Baa	Ba	B	Caa
Avg. Return:	4.82%	5.60%	5.78%	6.53%	7.98%	6.79%	8.26%
Max. Return:	16.08%	19.30%	29.39%	36.43%	50.80%	52.10%	97.47%
Min. Return:	–4.17%	–5.47%	–14.98%	–15.70%	–22.12%	–31.47%	–44.35%
Avg. Volatility:	3.43%	4.14%	4.76%	5.15%	5.75%	7.13%	11.00%

Sources: Bloomberg Barclays Indices and Loomis, Sayles & Company.

As shown in the exhibit, the higher the credit risk, the greater the return potential and the higher the volatility of that return. This pattern is consistent with other types of investing that involve risk and return (although average returns on single-B rated bonds appear anomalous in this example).

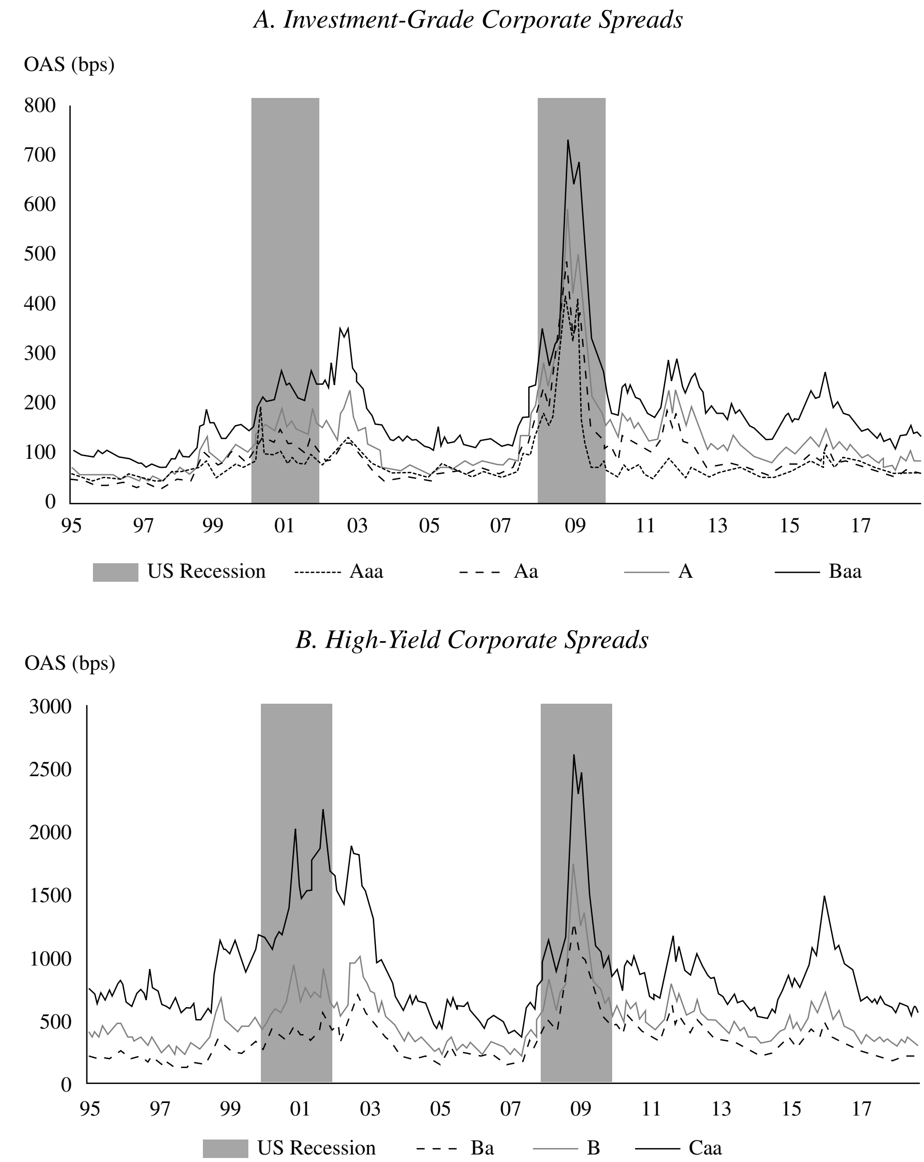
For extremely liquid bonds that are deemed to have virtually no default risk (e.g., German government bonds, or *Bunds*), the yield for a given maturity is a function of real interest rates plus an expected inflation rate premium. The yield on corporate bonds will include an additional risk premium that provides the investor with compensation for the credit and liquidity risks and possibly the tax impact of holding a specific bond. Changes in any of these components will alter the yield, price, and return on the bond. In general, however, it is not possible to directly observe the market's assessment of the components separately—analysts can only observe the total yield spread.

Spreads on all corporate bonds can be affected by a number of factors, with lower-quality issuers typically experiencing greater spread volatility. These factors, which are frequently linked, include the following:

- **Credit cycle.** As the credit cycle improves, credit spreads will narrow. Conversely, a deteriorating credit cycle will cause credit spreads to widen. Spreads are tightest at or near the top of the credit cycle, when financial markets believe risk is low; they are widest at or near the bottom of the credit cycle, when financial markets believe risk is high.
- **Broader economic conditions.** Not surprisingly, weakening economic conditions will push investors to desire a greater risk premium and drive overall credit spreads wider. Conversely, a strengthening economy will cause credit spreads to narrow because investors anticipate credit measures will improve due to rising corporate cash flow, thus reducing the risk of default. In a steady, low-volatility environment, credit spreads will typically also narrow as investors tend to "reach for yield."
- **Funding availability in financial sector.** Bonds trade primarily over the counter, so investors need broker–dealers to commit capital for market-making purposes. Episodes of financial and regulatory stress have the potential to greatly reduce the total capital available for making markets (to facilitate trading) and the willingness to buy/sell credit-risky bonds. Future regulatory reform may well lead to persistent or even permanent reductions in broker-provided capital. Funding stresses would naturally translate into wider spreads.
- **General market supply and demand.** In periods of heavy new issue supply, credit spreads will widen if there is insufficient demand. In periods of high demand for bonds, spreads will move tighter.
- **Financial performance of the issuer.** Corporate bond spreads will be impacted by earnings releases, news, and other developments associated with the issuer. Earlier we explained the "four Cs of credit analysis": capacity, collateral, covenants, and character. Announcement and disclosures by the issuer will impact investors' view of the issuer's financial performance and ability to service and repay its debt. Good news increases the attractiveness of buying and holding bonds issued by the corporation, which raises bond prices and narrows spreads; bad news will have the opposite effect.

A number of these factors played a role during the global financial crisis of 2008–2009, causing spreads to widen dramatically, as shown in Exhibit 15, before narrowing sharply as governments intervened and markets stabilized. This is shown in two panels—one for investment grade, another for high yield—because of the much greater spread volatility in high-yield bonds, particularly CCC rated credits. This spread volatility is reflected in the different spread ranges on the *y*-axes. OAS is option-adjusted spread, which incorporates the value of the embedded call option in certain corporate bonds that issuers have the right to exercise before maturity.

EXHIBIT 15 US Investment-Grade and High-Yield Corporate Spreads



Sources: Bloomberg Barclays Indices and Loomis, Sayles & Company.

EXAMPLE 8 Yield Spreads

1. Which bonds are likely to exhibit the greatest spread volatility?
   A. Bonds from issuers rated AA
   B. Bonds from issuers rated BB
   C. Bonds from issuers rated A
2. If investors become increasingly worried about the economy—say, as shown by declining stock prices—what is the *most likely* impact on credit spreads?
   A. There will be no change to credit spreads. They aren't affected by equity markets.
   B. Narrower spreads will occur. Investors will move out of equities into debt securities.
   C. Wider spreads will occur. Investors are concerned about weaker creditworthiness.

Solution to 1: B is correct. Lower-quality bonds exhibit greater spread volatility than higher-quality bonds. All of the factors that affect spreads—the credit cycle, economic conditions, financial performance, market-making capacity, and supply/demand conditions—will tend to have a greater impact on the pricing of lower-quality credits.

Solution to 2: C is correct. Investors will require higher yields as compensation for the credit deterioration—including losses—that is likely to occur in a weakening economy.

6.1. Credit Risk vs. Return: The Price Impact of Spread Changes

We have discussed how yield spreads on credit-risky debt obligations, such as corporate bonds, can fluctuate based on a number of factors, including changes in the market's view of issuer-specific or idiosyncratic risk. The next question to consider is how these spread changes affect the price of and return on these bonds.

Although bond investors do concern themselves with default risks, recall that the probability of default for higher-quality bonds is typically very low: For investment-grade bonds, annual defaults are nearly always well below 1% (recall Exhibit 6). On the other hand, default rates can be very high for lower-quality issuers, although they can vary widely depending upon the credit cycle, among other things. What most investors in investment-grade debt focus on more than default risk is spread risk—that is, the effect on prices and returns from changes in spreads.

The price impact from spread changes is driven by two main factors: (1) the modified duration (price sensitivity with respect to changes in interest rates) of the bond and (2) the magnitude of the spread change. The effect on return to the bondholder depends on the holding period used for calculating the return.

The simplest example is that of a small, instantaneous change in the yield spread. In this case, the price impact (i.e., the percentage change in price, including accrued interest) can be approximated by

$$\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta\text{Spread}$$

where AnnModDur is the annual modified duration. The negative sign in this equation reflects the fact that because bond prices and yields move in opposite directions, lower spreads have a positive impact on bond prices and thus returns, whereas higher spreads have a negative impact on bond returns. Note that if the spread change is expressed in basis points, then the price impact will also be in basis points, whereas if the spread change is expressed as a decimal, the price impact will also be expressed as a decimal.

For larger spread changes (and thus larger yield changes), the impact of convexity needs to be incorporated into the approximation:

$$\%\Delta PV^{Full} \approx -(\text{AnnModDur} \times \Delta\text{Spread}) + \tfrac{1}{2}\text{AnnConvexity} \times (\Delta\text{Spread})^2$$

In this case, one must be careful to ensure that convexity (denoted by AnnConvexity) is appropriately scaled to be consistent with the way the spread change is expressed. In general, for bonds without embedded options, one can scale convexity so that it has the same order of magnitude as the duration squared and then express the spread change as a decimal. For example, for a bond with duration of 5.0 and reported convexity of 0.235, one would re-scale convexity to 23.5 before applying the formula. For a 1% (i.e., 100 bps) increase in spread, the result would be

$$\%\Delta PV^{Full} = (-5.0 \times 0.01) + \tfrac{1}{2} \times 23.5 \times (0.01)^2 = -0.048825 \text{ or } -4.8825\%$$

The price impact of instantaneous spread changes is illustrated in Exhibit 16 using two bonds from British Telecom, the UK telecommunications company. The bonds, denominated in British pounds, are priced to provide a certain spread over British government bonds (gilts) of a similar maturity. From the starting spread, in increments of 25 bps and for both wider and narrower spreads, the new price and actual return for each spread change are calculated. In addition, the exhibit shows the approximate returns with and without the convexity term. As can be seen, the approximation using only duration is reasonably accurate for small spread changes; for larger changes, however, the convexity term generally provides a meaningful improvement.

EXHIBIT 16 Impact of Duration on Price for a Given Change in Spread

Issuer: British Telecom, 5.75%, 12/07/2028		
Price: £122.978	Modified Duration: 7.838	Spread to Gilt Curve: 150.7 b.p.
Accrued interest: 0.958	Convexity: 77.2	YTM (conv): 3.16

	Scenarios								
Spread Δ (b.p.)	−100	−75	−50	−25	0	25	50	75	100
Spread (b.p.)	50.7	75.7	100.7	125.7	150.7	175.7	200.7	225.7	250.7
Price (£)	131.62	129.12	126.68	124.29	122.98	119.69	117.47	115.30	113.18
Price + Accrued (£)	132.58	130.08	127.64	125.25	123.94	120.65	118.43	116.26	114.14
Price Δ (£)	8.64	6.14	3.70	1.31	0.00	−3.29	−5.51	−7.68	−9.80
Return (%)									
Actual	6.97%	4.96%	2.99%	1.06%	0.00%	−2.65%	−4.44%	−6.20%	−7.91%
Approx: Dur only	7.84%	5.88%	3.92%	1.96%	0.00%	−1.96%	−3.92%	−5.88%	−7.84%
Approx: Dur & Cvx	8.22%	6.10%	4.02%	1.98%	0.00%	−1.94%	−3.82%	−5.66%	−7.45%

EXHIBIT 16 (Continued)

Issuer: British Telecom, 3.625%, 21/11/2047		
Price: £94.244	Modified Duration: 17.144	Spread to Gilt Curve: 210.8 b.p.
Accrued interest: 2.185	Convexity: 408.4	YTM (conv): 4.11

	Scenarios								
Spread Δ (b.p.)	−100	−75	−50	−25	0	25	50	75	100
Spread (b.p.)	110.8	135.8	160.8	185.8	210.8	235.8	260.8	285.8	310.8
Price (£)	111.28	106.38	101.77	97.41	93.24	89.41	85.75	82.30	79.04
Price + Accrued (£)	113.47	108.57	103.96	99.60	95.43	91.60	87.94	84.48	81.22
Price Δ (£)	18.04	13.14	8.53	4.17	0.00	−3.83	−7.49	−10.95	−14.21
Return (%)									
Actual	18.90%	13.77%	8.93%	4.37%	0.00%	−4.02%	−7.85%	−11.47%	−14.89%
Approx: Dur only	17.14%	12.86%	8.57%	4.29%	0.00%	−4.29%	−8.57%	−12.86%	−17.14%
Approx: Dur & Cvx	19.19%	14.01%	9.08%	4.41%	0.00%	−4.16%	−8.06%	−11.71%	−15.10%

Note: Settle date is 13 December 2018.
Source: Bloomberg Finance, L.P.

Note that the price change for a given spread change is higher for the longer-duration bond—in this case, the 2047 maturity British Telecom bond—than for the shorter-duration, 2028 maturity British Telecom bond. Longer-duration corporate bonds are referred to as having "higher spread sensitivity"; that is, their prices, and thus returns, are more volatile with respect to changes in spread. It is essentially the same concept as duration for any bond: The longer the duration of a bond, the greater the price volatility for a given change in interest rates/yields.

In addition, investors want to be compensated for the fact that the further time one is from a bond's maturity (i.e., the longer the bond), the greater the uncertainty about an issuer's future creditworthiness. Based on credit analysis, an investor might be confident that an issuer's risk of default is relatively low in the near term; however, looking many years into the future, the investor's uncertainty grows because of factors that are increasingly difficult, if not impossible, to forecast (e.g., poor management strategy or execution, technological obsolescence, natural or man-made disasters, corporate leveraging events). This increase in credit risk over time can be seen in Exhibit 17. Note that in this Standard & Poor's study, one-year default rates for the 2017 issuance pool are 0% for all rating categories of BB or higher. The three-year default rates for bonds issued in 2015 are materially higher, and the observed defaults include bonds originally rated up to BBB (i.e., low investment grade). The 5-year default rates for bonds issued in 2013 are higher than the 3-year default rates, and the defaults also include bonds initially rated as high as BBB. In addition to the risk of default rising over time, the data also show quite conclusively that the lower the credit rating, the higher the risk of default.

Finally, note the very high risk of default for bonds rated CCC or lower over all time horizons. This is consistent with Exhibit 7 presented earlier, which showed significant three-year ratings variability ("migration"), with much of the migration to lower credit ratings (i.e., higher risk of default).

EXHIBIT 17 Default Rate by Rating Category (%) (Non-financials)

Credit Rating	1 Year (2017 pool)	3 Year (2015 pool)	10 Year (2013 pool)
AAA	0.00	0.00	0.00
AA	0.00	0.00	0.00
A	0.00	0.00	0.00
BBB	0.00	0.08	0.27
BB	0.10	2.46	3.33
B	0.95	10.11	12.90
CCC/C	27.15	41.43	44.70

Source: Based on data from S&P Global Ratings, "2017 Annual Global Corporate Default Study and Rating Transitions" (5 April 2018).

EXAMPLE 9 Price Impact

Calculate the price impact on a 10-year corporate bond with a 4.75% coupon priced at 100, with an instantaneous 50 bps widening in spread due to the issuer's announcement that it was adding substantial debt to finance an acquisition (which resulted in a two-notch downgrade by the rating agencies). The bond has a modified duration of 7.9, and its convexity is 74.9.

Solution: The impact from the 50 bps spread widening is:

$$\text{Price impact} \approx -(\text{AnnModDur} \times \Delta\text{Spread}) + \tfrac{1}{2}\,\text{AnnConvexity} \times (\Delta\text{Spread})^2$$
$$= -(0.0050 \times 7.9) + (0.5 \times 74.9) \times (0.0050)^2$$
$$= -0.0386, \text{ or } -3.86\%$$

Because yields and bond prices move in opposite directions, the wider spread caused the bond price to fall. Using a bond-pricing calculator, the exact return is −3.85%, so this approximation was very accurate.

In summary, spread changes can have a significant impact on the price and performance of credit-risky bonds over a given holding period, and the higher the modified duration of the bond(s), the greater the price impact from changes in spread. Wider spreads hurt bond performance, whereas narrower spreads help bond performance. For bond investors who actively manage their portfolios (i.e., don't just buy bonds and hold them to maturity), forecasting spread changes and expected credit losses on both individual bonds and their broader portfolios is an important strategy for enhancing investment performance.

7. HIGH-YIELD, SOVEREIGN, AND NON-SOVEREIGN CREDIT ANALYSIS

- **explain special considerations when evaluating the credit of high-yield, sovereign, and non-sovereign government debt issuers and issues**

Thus far, we have focused primarily on basic principles of credit analysis and investing with emphasis on higher-quality, investment-grade corporate bonds. Although many of these principles are applicable to other credit-risky segments of the bond market, some differences in credit analysis need to be considered. This section focuses on special considerations in evaluating the credit of debt issuers from the following three market segments: high-yield corporate bonds, sovereign bonds, and non-sovereign government bonds.

7.1. High Yield

Recall that high-yield, or non-investment-grade, corporate bonds are those rated below Baa3/BBB– by the major rating agencies. These bonds are sometimes referred to as "junk bonds" because of the higher risk inherent in their weak balance sheets and/or poor or less-proven business prospects.

Companies are rated below investment grade for many reasons, including the following:

- Highly leveraged capital structure
- Weak or limited operating history
- Limited or negative free cash flow
- Highly cyclical business
- Poor management
- Risky financial policies
- Lack of scale and/or competitive advantages
- Large off-balance-sheet liabilities
- Declining industry (e.g., newspaper publishing)

Companies with weak balance sheets and/or business profiles have lower margin for error and greater risk of default relative to higher-quality investment-grade names. And the higher risk of default means more attention must be paid to recovery analysis (or loss severity, in the event of default). Consequently, high-yield analysis typically is more in-depth than investment-grade analysis and thus has special considerations. This includes the following:

- Greater focus on issuer liquidity and cash flow
- Detailed financial projections
- Detailed understanding and analysis of the debt structure
- Understanding of an issuer's corporate structure
- Covenants
- Equity-like approach to high-yield analysis

Liquidity. Liquidity—that is, having cash and/or the ability to generate or raise cash—is important to all issuers. It is absolutely critical for high-yield companies. Investment-grade companies typically have substantial cash on their balance sheets, generate a lot of cash from operations relative to their debt (or else they wouldn't be investment grade!), and/or are presumed to have alternate sources of liquidity, such as bank lines and commercial paper. For these reasons, investment-grade companies can more easily roll over (refinance) maturing debt.

On the other hand, high-yield companies may not have those options available. For example, there is no high-yield commercial paper market, and bank credit facilities often carry tighter restrictions for high-yield companies. Both bad company-specific news and difficult financial market conditions can lead to high-yield companies being unable to access the debt markets. And although the vast majority of investment-grade corporate debt issuers have publicly traded equity and can thus use that equity as a financing option, many high-yield companies are privately held and thus don't have access to public equity markets.

Thus, issuer liquidity is a key focus in high-yield analysis. Sources of liquidity, from strongest to weakest, are the following:

1. Cash on the balance sheet
2. Working capital
3. Operating cash flow
4. Bank credit facilities
5. Equity issuance
6. Asset sales

Cash on the balance sheet is easy to see and self-evident as a source for repaying debt, although a portion of it may be "trapped" overseas for certain tax, business, accounting, or regulatory reasons and thus not easily accessible. As mentioned earlier, working capital can be a large source or use of liquidity, depending on its amount, its use in a company's cash-conversion cycle, and its role in a company's operations. Operating cash flow is a ready source of liquidity as sales turn to receivables, which turn to cash over a fairly short time period. Bank lines, or credit facilities, can be an important source of liquidity, though there may be some covenants relating to the use of the bank lines that are crucial to know and will be covered a little later. Equity issuance may not be a reliable source of liquidity because an issuer is private or because of poor market conditions if a company does have publicly traded equity. Asset sales are the least reliable source of liquidity because both the potential value and the actual time of closing can be highly uncertain.

The amount of these liquidity sources should be compared with the amount and timing of upcoming debt maturities. A large amount of debt coming due in the next 6–12 months alongside low sources of liquidity will be a warning flag for bond investors and could push an issuer into default because investors may choose not to buy new bonds intended to pay off the existing debt. Insufficient liquidity—that is, running out of cash or no longer having access to external financing to refinance or pay off existing debt—is the principal reason issuers default. Although liquidity is important for industrial companies, it is an absolute necessity for financial firms, as seen in the case of Lehman Brothers and other troubled firms during the global financial crisis of 2008. Financial institutions are highly levered and often highly dependent on funding longer-term assets with short-term liabilities.

Financial Projections. Because high-yield companies have less room for error, it's important to forecast, or project, future earnings and cash flow out several years, perhaps including several scenarios, to assess whether the issuer's credit profile is stable, improving, or declining and thus whether it needs other sources of liquidity or is at risk of default. Ongoing capital expenditures and working capital changes should be incorporated as well. Special emphasis should be given to realistic "stress" scenarios that could expose a borrower's vulnerabilities.

Debt Structure. High-yield companies tend to have many layers of debt in their capital structures, with varying levels of seniority and, therefore, different potential recovery rates in the event of default. (Recall the historical table of default recovery rates based on seniority in

Exhibit 2.) A high-yield issuer will often have at least some of the following types of obligations in its debt structure:

- (Secured) Bank debt
- Second lien debt
- Senior unsecured debt
- Subordinated debt, which may include convertible bonds
- Preferred stock

The lower the ranking in the debt structure, the lower the credit rating and the lower the expected recovery in the event of default. In exchange for these associated higher risks, investors will normally demand higher yields.

As discussed earlier, a standard leverage calculation used by credit analysts is debt/EBITDA and is quoted as a multiple (e.g., "5.2x levered"). For an issuer with several layers of debt with different expected recovery rates, high-yield analysts should calculate leverage at each level of the debt structure. Example 10 shows calculations of gross leverage, as measured by debt/EBITDA, at each level of the debt structure and net leverage for the entire debt structure. Gross leverage calculations do not adjust debt for cash on hand. Net leverage adjusts debt by subtracting cash from total debt.

EXAMPLE 10 Debt Structure and Leverage

Hexion Inc. is a specialty chemical company. It has a complicated, high-yield debt structure, consisting of first lien debt (loans and bonds), secured bonds, second lien bonds, and senior unsecured debt, due to a series of mergers as well as a leveraged buyout in 2005. Exhibit 18 is a simplified depiction of the company's debt structure, as well as some key credit-related statistics.

EXHIBIT 18 Hexion Inc. Debt and Leverage Structure as of Year-End 2017

Financial Information ($ millions)	
Cash	$115
Total debt	$3,668
Net debt	$3,553
Interest expense	$329
EBITDA	$365

Debt Structure ($ millions)	
First lien debt (loans and bonds)	$2,607
Secured bonds	$225
Second lien bonds	$574
Senior unsecured bonds	$263
TOTAL DEBT	$3,669

Sources: Company filings, Loomis, Sayles & Company.

Using the information provided, address the following:

1. Calculate gross leverage, as measured by debt/EBITDA, through each level of debt, including total debt.
2. Calculate the net leverage, as measured by (Debt – Cash)/EBITDA, for the total debt structure.
3. Why might Hexion have so much secured debt relative to unsecured debt (both senior and subordinated)? (*Note*: This question draws on concepts from earlier sections.)

Solutions to 1 and 2:

	Gross Leverage (Debt/EBITDA)	Net Leverage (Debt – Cash)/ EBITDA
Secured debt leverage		
(First lien + Secured debt)/EBITDA		
(2,607 + 225)/365	7.8x	
Second lien leverage		
(First lien + Secured debt + Second lien debt)/EBITDA		
(2,607 + 225 + 574)/365	9.3x	
Total leverage (includes unsecured)		
(Total debt/EBITDA)		
3,669/365	10.1x	
Net leverage (leverage net of cash through entire debt structure)		
(Total debt – Cash)/EBITDA		9.7x

Solution to 3: Hexion might have that much secured debt because (1) it was less expensive than issuing additional unsecured debt on which investors would have demanded a higher yield and/or (2) given the riskiness of the business (chemicals are a cyclical business), the high leverage of the business model, and the riskiness of the balance sheet (lots of debt from a leveraged buyout), investors would only be willing to lend the company money on a secured basis.

High-yield companies that have a lot of secured debt (typically bank debt) relative to unsecured debt are said to have a "top-heavy" capital structure. With this structure, there is less capacity to take on more bank debt in the event of financial stress. Along with the often more stringent covenants associated with bank debt and its generally shorter maturity compared with other types of debt, this means that these issuers are more susceptible to default, as well as to lower recovery for the various less secured creditors.

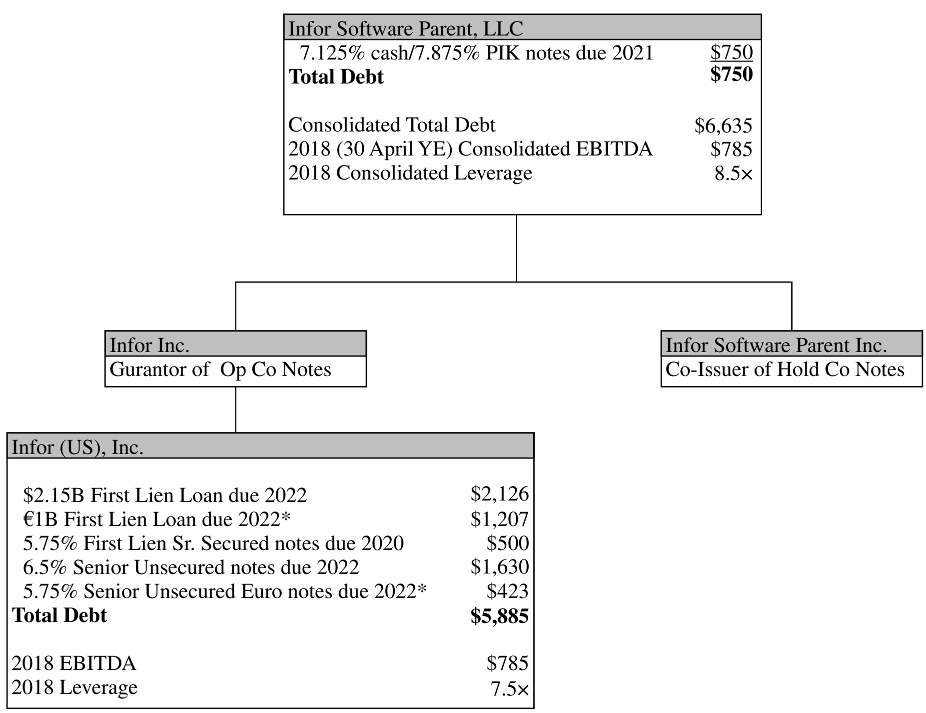
Corporate Structure. Many debt-issuing corporations, including high-yield companies, utilize a holding company structure with a parent and several operating subsidiaries. Knowing where an issuer's debt resides (parent versus subsidiaries) and how cash can move from subsidiary to parent ("upstream") and vice versa ("downstream") are critical to the analysis of high-yield issuers.

In a holding company structure, the parent owns stock in its subsidiaries. Typically, the parent doesn't generate much of its own earnings or cash flow but instead receives dividends from its subsidiaries. The subsidiaries' dividends are generally paid out of earnings after they satisfy all their other obligations, such as debt payments. To the extent that their earnings and cash flow are weak, subsidiaries may be limited in their ability to pay dividends to the parent. Moreover, subsidiaries that carry a lot of their own debt may have restrictions or limitations on how much cash they can provide to the parent via dividends or in another way, such as through an intercompany loan. These restrictions and limitations on cash moving between parent and subsidiaries can have a major impact on their respective abilities to meet their debt obligations. The parent's reliance on cash flow from its subsidiaries means the parent's debt is structurally subordinated to the subsidiaries' debt and thus will usually have a lower recovery rating in default.

For companies with very complex holding companies, there may also be one or more intermediate holding companies, each carrying their own debt, and in some cases, they may not own 100% of the subsidiaries' stock. Oftentimes a default in one subsidiary may not trigger a cross-default. This structure is sometimes seen in high-yield companies that have been put together through many mergers and acquisitions or that were part of a leveraged buyout.

Exhibit 19 shows the capital structure of Infor, Inc. (Infor), a high-yield software and services company highlighted earlier as an example of the credit rating agency notching process. Infor's capital structure consists of a parent company that has debt—in this case, convertible senior notes—as well as a subsidiary with multiple layers of outstanding debt by seniority.

EXHIBIT 19 Infor's Capital Structure



* Conversion from euro to US dollar.

Sources: Company filing, Loomis, Sayles & Company.

Thus, high-yield investors should analyze and understand an issuer's corporate structure, including the distribution of debt between the parent and its subsidiaries. Leverage ratios should be calculated at each of the debt-issuing entities, as well as on a consolidated basis.

Also important is that although the debt of an operating subsidiary may be "closer to" and better secured by particular assets of the subsidiary, the credit quality of a parent company might still be higher. The parent company could, while being less directly secured by any particular assets, still benefit from the diversity and availability of all the cash flows in the consolidated system. In short, credit quality is not simply an automatic analysis of debt provisions and liens.

Covenant Analysis. As discussed earlier, analysis of covenants is very important for all bonds. It is especially important for high-yield credits because of their reduced margin of safety. Key covenants for high-yield issuers may include the following:

- Change of control put
- Restricted payments
- Limitations on liens and additional indebtedness
- Restricted versus unrestricted subsidiaries

Under the **change of control put**, in the event of an acquisition (a "change of control"), bondholders have the right to require the issuer to buy back their debt (a "put option"), often at par or at some small premium to par value. This covenant is intended to protect creditors from being exposed to a weaker, more indebted borrower as a result of acquisition. For investment-grade issuers, this covenant typically has a two-pronged test: acquisition of the borrower and a consequent downgrade to a high-yield rating.

The **restricted payments** covenant is meant to protect creditors by limiting how much cash can be paid out to shareholders over time. The restricted payments "basket" is typically sized relative to an issuer's cash flow and debt outstanding—or is being raised—and is an amount that can grow with retained earnings or cash flow, giving management more flexibility to make payouts.

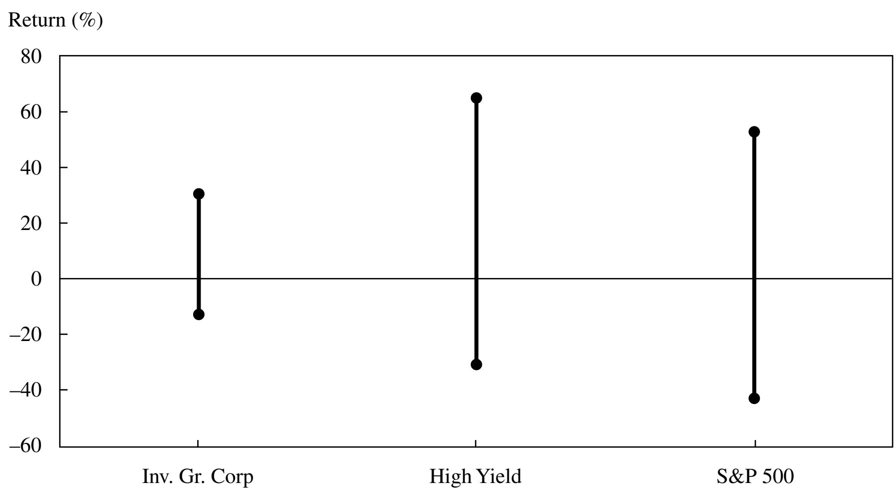
The **limitations on liens** covenant is meant to put limits on how much secured debt an issuer can have. This covenant is important to unsecured creditors who are structurally subordinated to secured creditors; the higher the amount of debt that is layered ahead of them, the less they stand to recover in the event of default.

With regard to restricted versus unrestricted subsidiaries, issuers may classify certain of their subsidiaries as restricted and others as unrestricted as it pertains to offering guarantees for their holding company debt. These subsidiary guarantees can be very useful to holding company creditors because they put their debt on equal standing (pari passu) with debt at the subsidiaries instead of with structurally subordinated debt. Restricted subsidiaries should be thought of as those that are designated to help service parent-level debt, typically through guarantees. They tend to be an issuer's larger subsidiaries and have significant assets, such as plants and other facilities, and/or cash flow. There may be tax or legal (e.g., country of domicile) reasons why certain subsidiaries are restricted while others are not. Analysts should carefully read the definitions of restricted versus unrestricted subsidiaries in the indenture because sometimes the language is so loosely written that the company can reclassify subsidiaries from one type to another with a simple vote by a board of directors or trustees.

For high-yield investors, it is also important to know what covenants are in an issuer's bank credit agreements. These agreements are typically filed with the securities commission in the country where the loan document was drafted. Bank covenants can be more restrictive than bond covenants and may include so-called **maintenance covenants**, such as leverage tests, whereby the ratio of, say, debt/EBITDA may not exceed "x" times. In the event a covenant is breached, the bank is likely to block further loans under the agreement until the covenant is cured. If not cured, the bank may accelerate full payment of the facility, triggering a default.

Equity-like approach to high-yield analysis. High-yield bonds are sometimes thought of as a "hybrid" between higher-quality bonds, such as investment-grade corporate debt, and equity securities. Their more volatile price and spread movements are less influenced by interest rate changes than are higher-quality bonds, and they show greater correlation with movements in equity markets. Indeed, as shown in Exhibit 20, historical returns on high-yield bonds and the standard deviation of those returns fall somewhere between investment-grade bonds and equities.

EXHIBIT 20 US Trailing 12-Month Returns by Asset Class, 31 December 1988–30 September 2018



	Inv. Gr. Corp	High Yield	S&P 500
Avg. Return:	7.00%	8.86%	11.39%
Max. Return:	31.07%	64.95%	53.62%
Min. Return:	–13.82%	–31.23%	–43.32%
Avg. Volatility:	4.71%	6.79%	12.91%

Sources: Bloomberg Barclays Indices; Haver Analytics; and Loomis, Sayles & Company.

Consequently, an equity market-like approach to analyzing a high-yield issuer can be useful. One approach is to calculate an issuer's enterprise value. Enterprise value (EV) is usually calculated by adding equity market capitalization and total debt and then subtracting excess cash. Enterprise value is a measure of what a business is worth (before any takeover premium) because an acquirer of the company would have to either pay off or assume the debt and it would receive the acquired company's cash.

Bond investors like using EV because it shows the amount of equity "cushion" beneath the debt. It can also give a sense of (1) how much more leverage management might attempt to put on a company in an effort to increase equity returns or (2) how likely—and how expensive—a credit-damaging leveraged buyout might be. Similar to how stock investors look at equity multiples, bond investors may calculate and compare EV/EBITDA and debt/EBITDA across several issuers as part of their analysis. Narrow differences between EV/EBITDA and debt/EBITDA for a given issuer indicate a small equity cushion and, therefore, potentially higher risk for bond investors.

7.2. Sovereign Debt

Governments around the world issue debt to help finance their general operations, including such current expenses as wages for government employees, and investments in such long-term assets as infrastructure and education. Government bonds in developed countries have traditionally been viewed as the default risk-free rate off of which all other credits are priced. Fiscal challenges in developed countries exacerbated by the 2008 global financial crisis and the 2011–2012 eurozone crisis, however, have called into question the notion of a "risk-free rate," even for some of the highest-quality government borrowers. As their capital markets have developed, an increasing number of sovereign governments have been able to issue debt in foreign markets (generally denominated in a currency other than that of the sovereign government, often the US dollar or euro) as well as debt in the domestic market (issued in the sovereign government's own currency). Generally, sovereign governments with weak currencies can access foreign debt markets only by issuing bonds in foreign currencies that are viewed to be safer stores of value. Debt issued in the domestic market is somewhat easier to service because the debt is typically denominated in the country's own currency, subject to its own laws, and money can be printed to service the sovereign government's domestic debt. Twenty years ago, many emerging market countries could only issue debt in foreign markets because a domestic market did not exist. Today, many are able to issue debt domestically and have successfully built yield curves of domestic bonds across the maturity spectrum. All sovereign governments are best able to service foreign and domestic debt if they run "twin surpluses"—that is, a government budget surplus as well as a current account surplus.

Despite ongoing financial globalization and the development of domestic bond markets, sovereign government defaults occur. Defaults are often precipitated by such events as war, political upheaval, major currency devaluation, a sharp deterioration in trade, or dramatic price declines in a country's key commodity exports. Default risks for some developed countries escalated after 2009 as government revenues dropped precipitously following the global financial crisis of 2008, expenditures surged, and financial markets focused on the long-term sustainability of public finances, given aging populations and rising social security needs. Some of the weaker and more highly indebted members of the eurozone became unable to access the debt markets at economical rates and had to seek loans from the International Monetary Fund (IMF) and the European Union. These weaker governments had previously been able to borrow at much lower rates because of their membership in the European Union and adoption of the euro. Intra-eurozone yield spreads widened and countries were shut out of markets, however, as the global financial crisis exacted a high toll on their public finances and, in some cases, their banking systems, which became contingent liabilities for the sovereigns. In Ireland, the government guaranteed most bank liabilities, undermining the country's own fiscal metrics.

Like corporate analysis, sovereign credit analysis is based on a combination of qualitative and quantitative factors. Ultimately, the two key issues for sovereign analysis are: (1) a government's ability to pay and (2) its willingness to pay. Willingness to pay is important because of the principle of sovereign immunity, where investors are generally unable to force a sovereign to pay its debts. Sovereign immunity prevents governments from being sued. To date, global initiatives aimed at creating a mechanism for orderly sovereign restructurings and defaults have not found traction.

To illustrate the most important considerations in sovereign credit analysis, we present a basic framework for evaluating sovereign credit and assigning sovereign debt ratings (this outline was developed from the detailed exposition of Standard & Poor's methodology given in S&P Global Ratings, "Sovereign Rating Methodology" [18 December 2017]). The framework highlights the specific characteristics analysts should expect in a high-quality sovereign credit. Some of these are self-explanatory (e.g., absence of corruption). For others, a brief rationale and/or range of values is included to clarify interpretation. Most, but not all, of these items are included in rating agency Standard & Poor's methodology.

Institutional and economic profile

- *Institutional assessment*
  - Institutions' ability to deliver sound public finances and balanced economic growth.
  - Effectiveness and predictability of policymaking institutions.
  - Track record of managing previous political, economic, and/or financial crises.
  - Ability and willingness to implement reforms to address fiscal challenges.
  - Transparent and accountable institutions with low perceived level of corruption.
  - Independence of statistical offices and media.
  - Checks and balances between institutions.
  - Unbiased enforcement of contracts and respect for rule of law and property rights.
  - Debt repayment culture.
  - Potential external and domestic security risks.

- *Economic assessment*
  - Income per capita: More prosperous countries generally have a broader and deeper tax base with which to support debt.
  - Trend growth prospects: Creditworthiness is supported by sustainable and durable trend growth across business cycles.
  - Diversity and stability of growth: Sovereigns exposed to economic concentration are more vulnerable. A narrow economy tends to show higher volatility in growth and can impair a government's balance sheet.

Flexibility and performance profile

- *External assessment*
  - Status of currency: Sovereigns that control a reserve currency or a very actively traded currency are able to use their own currency in many international transactions and are less vulnerable to adverse shifts in global investor portfolios.

    - External liquidity: Countries with a substantial supply of foreign currency (foreign exchange reserves plus current account receipts) relative to projected funding needs in foreign currency (current account payments plus debt maturities) are less vulnerable to interruption of external liquidity.
    - External debt: Countries with low foreign debt relative to current account receipts are better able to service their foreign debt. This is similar to a coverage ratio for a corporation.

- *Fiscal assessment*
    - Fiscal performance and flexibility: Trend change in net general government debt as a percentage of GDP. Stable or declining debt as a percentage of GDP indicates a strong credit; a rising ratio can prove unsustainable and is, therefore, a sign of diminishing creditworthiness.
    - Long-term fiscal trends: Perceived willingness and ability to increase revenue or cut expenditure to ensure debt service.
    - Debt burden and structure: Net general government debt of less than 30% is good; more than 100% is poor. General government interest expense as a percentage of revenue: Less than 5% is good; greater than 15% is poor.
    - Ability to access funding, manage both the amortization profile and contingent liabilities arising from the financial sector, public enterprises, and guarantees.

- *Monetary assessment*
    - Ability to use monetary policy tailored to domestic economic objectives (e.g., growth) to address imbalances or shocks.
    - Exchange rate regime: Sovereigns with a reserve currency have the most flexibility. A freely floating currency allows maximum effectiveness for monetary policy. A fixed-rate regime limits effectiveness and flexibility. A hard peg, such as a currency board or monetary union, affords no independent monetary policy.
    - Credibility of monetary policy: Measured by track record of low and stable inflation. Credible monetary policy is supported by an operationally and legally independent central bank with a clear mandate. The central bank's ability to be a lender of last resort to the financial system also enhances stability.
    - Confidence in the central bank provides a foundation for confidence in the currency as a store of value and for the central bank to effectively manage policy through a crisis.
    - Most effective policy transmission occurs in systems with sound banking systems and well-developed domestic capital markets, including active money market and corporate bond markets, such that policymakers credibly enact policy relying on market-based policy tools (e.g., open market operations) over administrative policy tools (e.g., reserve requirements).

In light of a sovereign government's various powers—taxation, regulation, monetary policy, and ultimately, the sovereign's ability to "print money" to repay debt—within its own economy, it is virtually always at least as good a credit in its domestic currency as it is in foreign currency. Thus, credit rating agencies often distinguish between domestic and foreign bonds, with domestic bond ratings sometimes one notch higher. Of course, if a sovereign government were to rely heavily on printing money to repay debt, it would fuel high inflation or hyperinflation and increase default risk on domestic debt as well.

EXAMPLE 11 Sovereign Debt

Exhibit 21 shows several key sovereign statistics for Portugal.

EXHIBIT 21 Key Sovereign Statistics for Portugal

€ (billions), except where noted	2006	2008	2010	2012	2014	2015	2016	2017
Nominal GDP	160.3	171.2	172.6	168.4	173.1	179.8	186.5	194.6
Population (millions)	10.6	10.6	10.6	10.5	10.4	10.3	10.3	10.3
Unemployment (%)	8.6	8.5	12	15.6	13.9	12.4	11.1	8.9
Exports as share GDP (%)	22.2	22.6	21.3	26.8	27.8	27.6	26.8	28.3
Current account as share GDP (%)	−10.7	−12.6	−10	−2.1	0.2	0.3	0.7	0.6
Government revenues	64.8	70.7	71.8	72.2	77.2	78.8	79.9	83.1
Government expenditures	71.4	77.1	88.7	81.7	89.6	86.7	83.5	88.9
Budget balance (surplus/deficit)	−6.5	−6.4	−16.9	−9.5	−12.4	−7.9	−3.6	−5.8
Government interest payments	4.2	5.3	5.2	8.2	8.5	8.2	7.8	7.4
Primary balance (surplus/deficit)	−2.2	−1.1	−11.7	−1.3	−3.9	0.3	4.2	1.6
Government debt	102.4	123.1	161.3	212.6	226	231.5	241	242.8
Interest rate on new debt (%)	3.9	4.5	5.4	3.4	3.8	2.4	3.2	3.1

Sources: Haver Analytics, Eurostat, and Instituto Nacional de Estatística (Portugal).

1. Calculate the government debt/GDP ratio for Portugal for the years 2014–2017 as well as for the years 2006, 2008, 2010, and 2012.
2. Calculate GDP/capita for the same periods.
3. Based on those calculations, as well as other data from Exhibit 21, what can you say about Portugal's credit trend?

Solutions to 1 and 2:

	2006	2008	2010	2012	2014	2015	2016	2017
Gross government debt/GDP	64%	72%	93%	126%	131%	129%	129%	125%
GDP/capita	15,123	16,151	16,283	16,038	16,644	17,456	18,107	18,893

Solution to 3: The credit trend is stabilizing. Government debt/GDP is declining ever so slightly after peaking in 2014. The government is running a modest budget deficit with a primary balance that is in surplus for the past three years. Portugal is running a very small current account surplus, reducing its reliance on external funding, and has increased its exports as a share of GDP. Unemployment, while still fairly high, has fallen in the past several years. Interest payments on government debt have started to decline, both as a percentage of GDP and in absolute terms. The interest rate on new government debt has stabilized, perhaps benefiting from the European Central Bank's quantitative easing policies. Taken together, there are strong indications that the Portuguese government's credit situation has stabilized and may be expected to improve further if current trends are sustained.

7.3. Non-Sovereign Government Debt

Sovereigns are the largest issuers of government debt, but non-sovereign—sometimes called sub-sovereign or local—governments and the quasi-government entities that are created by governments issue bonds as well. The non-sovereign or local governments include governments of states, provinces, regions, and cities. For example, the City of Tokyo (Tokyo Metropolitan Government) has debt outstanding, as does the Lombardy region in Italy, the City of Buenos Aires in Argentina, and the State of California in the United States. Local government bonds may be referred to as municipal bonds.

However, when people talk about municipal bonds, they are usually referring to US municipal bonds, which represent one of the largest bond markets. As of year-end 2017, the US municipal bond market was approximately $3.9 trillion in size, roughly 9% of the total US bond market (Securities Industry and Financial Markets Association [SIFMA], "Outstanding U.S. Bond Market Data," as of 2Q 2018). The US municipal bond market is composed of both tax-exempt and, to a lesser extent, taxable bonds issued by state and city governments and their agencies. Municipal borrowers may also issue bonds on behalf of private entities, such as non-profit colleges or hospitals. Historically, for any given rating category, these bonds have much lower default rates than corporate bonds with the same ratings. For example, according to Moody's Investors Service ("US Municipal Bond Defaults and Recoveries, 1970–2017"), the 10-year average cumulative default rate from 1970 through 2017 was 0.17% for municipal bonds, compared with a 10.24% 10-year average cumulative default rate for all corporate debt.

The majority of local government bonds, including municipal bonds, are either general obligation bonds or revenue bonds. General obligation (GO) bonds are unsecured bonds issued with the full faith and credit of the issuing non-sovereign government. These bonds are supported by the taxing authority of the issuer. Revenue bonds are issued for specific project financing (e.g., financing for a new sewer system, a toll road, bridge, hospital, or sports arena).

The credit analysis of GO bonds has some similarities to sovereign debt analysis (e.g., the ability to levy and collect taxes and fees to help service debt) but also some differences. For example, almost without exception, US municipalities must balance their operating budgets (i.e., exclusive of long-term capital projects) annually. Non-sovereign governments are unable to use monetary policy the way many sovereigns can.

The economic analysis of non-sovereign government GO bonds, including US municipal bonds, focuses on employment, per capita income (and changes in it over time), per capita debt (and changes in it over time), the tax base (depth, breadth, diversification, stability, etc.), demographics, and net population growth, as well as an analysis of whether the area represented by the non-sovereign government has the infrastructure and location to attract and support new jobs. Analysis should look at the volatility and variability of revenues during times of both economic strength and weakness. An overreliance on one or two types of tax revenue—particularly a volatile one, such as capital gains taxes or sales taxes—can signal increased credit risk. Pensions and other post-retirement obligations may not show up directly on the non-sovereign government's balance sheet, and many of these entities have underfunded pensions that need to be addressed. Adding the unfunded pension and post-retirement obligations to the debt reveals a more realistic picture of the issuer's debt and longer-term obligations. The relative ease or difficulty in managing the annual budgeting process and the government's ability to operate consistently within its budget are also important credit analysis considerations.

Disclosure by non-sovereign governments varies widely, with some of the smaller issuers providing limited financial information. Reporting requirements are inconsistent, so the financial reports may not be available for six months or more after the closing of a reporting period.

Exhibit 22 compares several key debt statistics from two of the larger states in the United States: Illinois and Texas. Illinois has the lowest credit ratings of any of the states, whereas Texas has one of the highest. Note the higher debt burden (and lower ranking) across several measures: Total debt, Debt/Capita, Debt/Personal income, and debt as a percentage of state GDP. When including net pension liabilities of government employees and retirees, the debt burdens are even greater, especially in the case of Illinois. What is not shown here is that Illinois also has a higher tax burden and greater difficulty balancing its budget on an annual basis than Texas does.

EXHIBIT 22 Municipal Debt Comparison: Illinois vs. Texas

	Illinois	Texas
Ratings:		
Moody's	Baa3	Aaa
S&P	BBB–	AAA
Fitch	BBB	AAA
Unemployment rate (%)*	4.20	3.70
Median household income ($)**	$61,229	$57,051
Debt burden, net ($/rank)***		
Total (millions)	37,374 (5)	11,603 (13)
Per capita	2,919 (6)	410 (42)
As a percentage of 2016 personal income	5.60 (5)	0.90 (42)
As a percentage of 2016 GDP	4.70 (6)	0.73 (42)
ANPL, net ($/rank)****		
Total (millions)	250,136 (1)	140,253 (3)
Per capita	19,539 (1)	4,955 (19)
As a percentage of 2017 personal income	37.00 (1)	10.60 (19)
As a percentage of 2017 GDP	30.50 (1)	8.30 (20)

* *Source:* Bureau of Labor Statistics, data as of October 2018.
** *Source:* US Census Bureau, data as of 2017.
*** *Source:* Moody's Investors Service, Inc., debt data as of 2017.
**** *Source:* Moody's Investors Service, Inc., adjusted net pension liability data as of 2017.

Revenue bonds, which are issued to finance a specific project, have a higher degree of risk than GO bonds because they are dependent on a single source of revenue. The analysis of these bonds is a combination of an analysis of the project and the finances around the particular project. The project analysis focuses on the need and projected utilization of the project, as well as on the economic base supporting the project. The financial analysis has some similarities to the analysis of a corporate bond in that it is focused on operating results, cash flow, liquidity, capital structure, and the ability to service and repay the debt. A key credit measure for revenue-backed non-sovereign government bonds is the debt-service-coverage (DSC) ratio, which measures how much revenue is available to cover debt payments (principal and interest) after operating expenses. Many revenue bonds have a minimum DSC ratio covenant; the higher the DSC ratio, the stronger the creditworthiness.

SUMMARY

We introduced basic principles of credit analysis. We described the importance of the credit markets and credit and credit-related risks. We discussed the role and importance of credit ratings and the methodology associated with assigning ratings, as well as the risks of relying on credit ratings. We covered the key components of credit analysis and the financial measure used to help assess creditworthiness.

We also discussed risk versus return when investing in credit and how spread changes affect holding period returns. In addition, we addressed the special considerations to take into account when doing credit analysis of high-yield companies, sovereign borrowers, and non-sovereign government bonds.

- Credit risk is the risk of loss resulting from the borrower failing to make full and timely payments of interest and/or principal.
- The key components of credit risk are risk of default and loss severity in the event of default. The product of the two is expected loss. Investors in higher-quality bonds tend not to focus on loss severity because default risk for those securities is low.
- Loss severity equals (1 − Recovery rate).
- Credit-related risks include downgrade risk (also called credit migration risk) and market liquidity risk. Either of these can cause yield spreads—yield premiums—to rise and bond prices to fall.
- Downgrade risk refers to a decline in an issuer's creditworthiness. Downgrades will cause its bonds to trade with wider yield spreads and thus lower prices.
- Market liquidity risk refers to a widening of the bid–ask spread on an issuer's bonds. Lower-quality bonds tend to have greater market liquidity risk than higher-quality bonds, and during times of market or financial stress, market liquidity risk rises.
- The composition of an issuer's debt and equity is referred to as its "capital structure." Debt ranks ahead of all types of equity with respect to priority of payment, and within the debt component of the capital structure, there can be varying levels of seniority.
- With respect to priority of claims, secured debt ranks ahead of unsecured debt, and within unsecured debt, senior debt ranks ahead of subordinated debt. In the typical case, all of an issuer's bonds have the same probability of default due to cross-default provisions in most indentures. Higher priority of claim implies higher recovery rate—lower loss severity—in the event of default.
- For issuers with more complex corporate structures—for example, a parent holding company that has operating subsidiaries—debt at the holding company is structurally subordinated to the subsidiary debt, although the possibility of more diverse assets and earnings streams from other sources could still result in the parent having higher effective credit quality than a particular subsidiary.
- Recovery rates can vary greatly by issuer and industry. They are influenced by the composition of an issuer's capital structure, where in the economic and credit cycle the default occurred, and what the market's view of the future prospects are for the issuer and its industry.

- The priority of claims in bankruptcy is not always absolute. It can be influenced by several factors, including some leeway accorded to bankruptcy judges, government involvement, or a desire on the part of the more senior creditors to settle with the more junior creditors and allow the issuer to emerge from bankruptcy as a going concern, rather than risking smaller and delayed recovery in the event of a liquidation of the borrower.
- Credit rating agencies, such as Moody's, Standard & Poor's, and Fitch, play a central role in the credit markets. Nearly every bond issued in the broad debt markets carries credit ratings, which are opinions about a bond issue's creditworthiness. Credit ratings enable investors to compare the credit risk of debt issues and issuers within a given industry, across industries, and across geographic markets.
- Bonds rated Aaa to Baa3 by Moody's and AAA to BBB– by Standard & Poor's (S&P) and/or Fitch (higher to lower) are referred to as "investment grade." Bonds rated lower than that—Ba1 or lower by Moody's and BB+ or lower by S&P and/or Fitch—are referred to as "below investment grade" or "speculative grade." Below-investment-grade bonds are also called "high-yield" or "junk" bonds.
- The rating agencies rate both issuers and issues. Issuer ratings are meant to address an issuer's overall creditworthiness—its risk of default. Ratings for issues incorporate such factors as their rankings in the capital structure.
- The rating agencies will notch issue ratings up or down to account for such factors as capital structure ranking for secured or subordinated bonds, reflecting different recovery rates in the event of default. Ratings may also be notched due to structural subordination.
- Rating agencies incorporate ESG factors into their ratings of firms. Some have launched a set of ratings that aim to measure a company's attitudes, practices, and advances related to ESG. They identify and track leaders and laggards in the space. Companies are evaluated according to their exposure to ESG risks and how well they manage those risks relative to peers.
- There are risks in relying too much on credit agency ratings. Creditworthiness may change over time, and initial/current ratings do not necessarily reflect the creditworthiness of an issuer or bond over an investor's holding period. Valuations often adjust before ratings change, and the notching process may not adequately reflect the price decline of a bond that is lower ranked in the capital structure. Because ratings primarily reflect the probability of default but not necessarily the severity of loss given default, bonds with the same rating may have significantly different expected losses (default probability times loss severity). And like analysts, credit rating agencies may have difficulty forecasting certain credit-negative outcomes, such as adverse litigation and leveraging corporate transactions, and such low probability/high severity events as earthquakes and hurricanes.
- The role of corporate credit analysis is to assess the company's ability to make timely payments of interest and to repay principal at maturity.
- Credit analysis is similar to equity analysis. It is important to understand, however, that bonds are contracts and that management's duty to bondholders and other creditors is limited to the terms of the contract. In contrast, management's duty to shareholders is to act in their best interest by trying to maximize the value of the company—perhaps even at the expense of bondholders at times.
- Credit analysts tend to focus more on the downside risk given the asymmetry of risk/return, whereas equity analysts focus more on upside opportunity from earnings growth, and so on.
- The "4 Cs" of credit—capacity, collateral, covenants, and character—provide a useful framework for evaluating credit risk.
- Credit analysis focuses on an issuer's ability to generate cash flow. The analysis starts with an industry assessment—structure and fundamentals—and continues with an analysis of an issuer's competitive position, management strategy, and track record.

- Credit measures are used to calculate an issuer's creditworthiness, as well as to compare its credit quality with peer companies. Key credit ratios focus on leverage and interest coverage and use such measures as EBITDA, free cash flow, funds from operations, interest expense, and balance sheet debt.
- An issuer's ability to access liquidity is also an important consideration in credit analysis.
- The higher the credit risk, the greater the offered/required yield and potential return demanded by investors. Over time, bonds with more credit risk offer higher returns but with greater volatility of return than bonds with lower credit risk.
- The yield on a credit-risky bond comprises the yield on a default risk–free bond with a comparable maturity plus a yield premium, or "spread," that comprises a credit spread and a liquidity premium. That spread is intended to compensate investors for credit risk—risk of default and loss severity in the event of default—and the credit-related risks that can cause spreads to widen and prices to decline—downgrade or credit migration risk and market liquidity risk.

$$\text{Yield spread} = \text{Liquidity premium} + \text{Credit spread}$$

- In times of financial market stress, the liquidity premium can increase sharply, causing spreads to widen on all credit-risky bonds, with lower-quality issuers most affected. In times of credit improvement or stability, however, credit spreads can narrow sharply as well, providing attractive investment returns.
- The impact of spread changes on holding period returns for credit-risky bonds is a product of two primary factors: the basis point spread change and the sensitivity of price to yield as reflected by (end-of-period) modified duration and convexity. Spread narrowing enhances holding period returns, whereas spread widening has a negative impact on holding period returns. Longer-duration bonds have greater price and return sensitivity to changes in spread than shorter-duration bonds.

$$\text{Price impact} \approx -(\text{AnnModDur} \times \Delta\text{Spread}) + \tfrac{1}{2}\text{AnnConvexity} \times (\Delta\text{Spread})^2$$

- For high-yield bonds, with their greater risk of default, more emphasis should be placed on an issuer's sources of liquidity and its debt structure and corporate structure. Credit risk can vary greatly across an issuer's debt structure depending on the seniority ranking. Many high-yield companies have complex capital structures, resulting in different levels of credit risk depending on where the debt resides.
- Covenant analysis is especially important for high-yield bonds. Key covenants include payment restrictions, limitation on liens, change of control, coverage maintenance tests (often limited to bank loans), and any guarantees from restricted subsidiaries. Covenant language can be very technical and legalistic, so it may help to seek legal or expert assistance.
- An equity-like approach to high-yield analysis can be helpful. Calculating and comparing enterprise value with EBITDA and debt/EBITDA can show a level of equity "cushion" or support beneath an issuer's debt.
- Sovereign credit analysis includes assessing both an issuer's ability and willingness to pay its debt obligations. Willingness to pay is important because, due to sovereign immunity, a sovereign government cannot be forced to pay its debts.
- In assessing sovereign credit risk, a helpful framework is to focus on five broad areas: (1) institutional effectiveness and political risks, (2) economic structure and growth prospects, (3) external liquidity and international investment position, (4) fiscal performance, flexibility, and debt burden, and (5) monetary flexibility.

- Among the characteristics of a high-quality sovereign credit are the absence of corruption and/or challenges to political framework; governmental checks and balances; respect for rule of law and property rights; commitment to honor debts; high per capita income with stable, broad-based growth prospects; control of a reserve or actively traded currency; currency flexibility; low foreign debt and foreign financing needs relative to receipts in foreign currencies; stable or declining ratio of debt to GDP; low debt service as a percentage of revenue; low ratio of net debt to GDP; operationally independent central bank; track record of low and stable inflation; and a well-developed banking system and active money market.
- Non-sovereign or local government bonds, including municipal bonds, are typically either general obligation bonds or revenue bonds.
- General obligation (GO) bonds are backed by the taxing authority of the issuing non-sovereign government. The credit analysis of GO bonds has some similarities to sovereign analysis—debt burden per capita versus income per capita, tax burden, demographics, and economic diversity. Underfunded and "off-balance-sheet" liabilities, such as pensions for public employees and retirees, are debt-like in nature.
- Revenue-backed bonds support specific projects, such as toll roads, bridges, airports, and other infrastructure. The creditworthiness comes from the revenues generated by usage fees and tolls levied.

PRACTICE PROBLEMS

1. The risk that a bond's creditworthiness declines is *best* described by:
   A. credit migration risk.
   B. market liquidity risk.
   C. spread widening risk.
2. Stedsmart Ltd and Fignermo Ltd are alike with respect to financial and operating characteristics, except that Stedsmart Ltd has less publicly traded debt outstanding than Fignermo Ltd. Stedsmart Ltd is *most likely* to have:
   A. no market liquidity risk.
   B. lower market liquidity risk.
   C. higher market liquidity risk.
3. In the event of default, the recovery rate of which of the following bonds would *most likely* be the highest?
   A. First mortgage debt
   B. Senior unsecured debt
   C. Junior subordinate debt
4. During bankruptcy proceedings of a firm, the priority of claims was not strictly adhered to. Which of the following is the *least likely* explanation for this outcome?
   A. Senior creditors compromised.
   B. The value of secured assets was less than the amount of the claims.
   C. A judge's order resulted in actual claims not adhering to strict priority of claims.
5. A fixed-income analyst is *least likely* to conduct an independent analysis of credit risk because credit rating agencies:
   A. may at times mis-rate issues.
   B. often lag the market in pricing credit risk.
   C. cannot foresee future debt-financed acquisitions.

6. If goodwill makes up a large percentage of a company's total assets, this *most likely* indicates that:
   A. the company has low free cash flow before dividends.
   B. there is a low likelihood that the market price of the company's common stock is below book value.
   C. a large percentage of the company's assets are not of high quality.
7. In order to analyze the **collateral** of a company, a credit analyst should assess the:
   A. cash flows of the company.
   B. soundness of management's strategy.
   C. value of the company's assets in relation to the level of debt.
8. In order to determine the **capacity** of a company, it would be *most* appropriate to analyze the:
   A. company's strategy.
   B. growth prospects of the industry.
   C. aggressiveness of the company's accounting policies.
9. A credit analyst is evaluating the credit worthiness of three companies: a construction company, a travel and tourism company, and a beverage company. Both the construction and travel and tourism companies are cyclical, whereas the beverage company is non-cyclical. The construction company has the highest debt level of the three companies. The highest credit risk is *most likely* exhibited by the:
   A. construction company.
   B. beverage company.
   C. travel and tourism company.
10. Based on the information provided in Exhibit 1, the EBITDA interest coverage ratio of Adidas AG is *closest* to:
   A. 16.02x.
   B. 23.34x.
   C. 37.98x.

EXHIBIT 1 Adidas AG Excerpt from Consolidated Income Statement in a given year (€ in millions)

Gross profit	12,293
Royalty and commission income	154
Other operating income	56
Other operating expenses	9,843
Operating profit	2,660
Interest income	64
Interest expense	166
Income before taxes	2,558
Income taxes	640
Net income	1,918
Additional information:	
Depreciation and amortization: €1,214 million	

Source: Adidas AG Annual Financial Statements, December 2019.

11. The following information is from the annual report of Adidas AG for December 2019:
    - Depreciation and amortization: €1,214 million
    - Total assets: €20,640 million
    - Total debt: €4,364 million
    - Shareholders' equity: €7,058 million

    The debt/capital of Adidas AG is *closest* to:
    A. 21.14%.
    B. 38.21%.
    C. 61.83%.
12. Funds from operations (FFO) of Pay Handle Ltd (a fictitious company) increased in 20X1. In 20X1, the total debt of the company remained unchanged while additional common shares were issued. Pay Handle Ltd's ability to service its debt in 20X1, as compared to 20X0, *most likely*:
    A. improved.
    B. worsened.
    C. remained the same.
13. Based on the information in Exhibit 2, GZ Group's (a hypothetical company) credit risk is *most likely*:
    A. lower than the industry.
    B. higher than the industry.
    C. the same as the industry.

EXHIBIT 2 European Food, Beverage, and Tobacco Industry and GZ Group Selected Financial Ratios for 20X0

	Total Debt/Total Capital (%)	FFO/Total Debt (%)	Return on Capital (%)	Total Debt/ EBITDA (x)	EBITDA Interest Coverage (x)
GZ Group	47.1	77.5	19.6	1.2	17.7
Industry median	**42.4**	**23.6**	**6.55**	**2.85**	**6.45**

14. Based on the information in Exhibit 3, the credit rating of DCM Group (a hypothetical company in the European food & beverage sector) is *most likely*:
    A. lower than AB plc.
    B. higher than AB plc.
    C. the same as AB plc.

EXHIBIT 3 DCM Group and AB plc Selected Financial Ratios for 20X0

Company	Total Debt/Total Capital (%)	FFO/Total Debt (%)	Return on Capital (%)	Total Debt/ EBITDA (x)	EBITDA Interest Coverage (x)
AB plc	0.2	84.3	0.1	1.0	13.9
DCM Group	42.9	22.9	8.2	3.2	3.2
European Food, Beverage, and Tobacco median	**42.4**	**23.6**	**6.55**	**2.85**	**6.45**

15. Holding all other factors constant, the *most likely* effect of low demand and heavy new issue supply on bond yield spreads is that yield spreads will:
    A. widen.
    B. tighten.
    C. not be affected.
16. Credit risk of a corporate bond is *best* described as the:
    A. risk that an issuer's creditworthiness deteriorates.
    B. probability that the issuer fails to make full and timely payments.
    C. risk of loss resulting from the issuer failing to make full and timely payments.
17. The risk that the price at which investors can actually transact differs from the quoted price in the market is called:
    A. spread risk.
    B. credit migration risk.
    C. market liquidity risk.
18. Loss severity is *best* described as the:
    A. default probability multiplied by the loss given default.
    B. portion of a bond's value recovered by bondholders in the event of default.
    C. portion of a bond's value, including unpaid interest, an investor loses in the event of default.
19. The two components of credit risk are default probability and:
    A. spread risk.
    B. loss severity.
    C. market liquidity risk.
20. For a high-quality debt issuer with a large amount of publicly traded debt, bond investors tend to devote *most* effort to assessing the issuer's:
    A. default risk.
    B. loss severity.
    C. market liquidity risk.
21. The expected loss for a given debt instrument is estimated as the product of default probability and:
    A. (1 + Recovery rate).
    B. (1 − Recovery rate).
    C. 1/(1 + Recovery rate).
22. The priority of claims for senior subordinated debt is:
    A. lower than for senior unsecured debt.
    B. the same as for senior unsecured debt.
    C. higher than for senior unsecured debt.
23. A senior unsecured credit instrument holds a higher priority of claims than one ranked as:
    A. mortgage debt.
    B. second lien loan.
    C. senior subordinated.
24. In a bankruptcy proceeding, when the absolute priority of claims is enforced:
    A. senior subordinated creditors rank above second lien holders.
    B. preferred equity shareholders rank above unsecured creditors.
    C. creditors with a secured claim have the first right to the value of that specific property.
25. In the event of default, which of the following is *most likely* to have the highest recovery rate?
    A. Second lien
    B. Senior unsecured
    C. Senior subordinated

26. The process of moving credit ratings of different issues up or down from the issuer rating in response to different payment priorities is *best* described as:
    A. notching.
    B. structural subordination.
    C. cross-default provisions.
27. The factor considered by rating agencies when a corporation has debt at both its parent holding company and operating subsidiaries is *best* referred to as:
    A. credit migration risk.
    B. corporate family rating.
    C. structural subordination.
28. Which type of security is *most likely* to have the same rating as the issuer?
    A. Preferred stock
    B. Senior secured bond
    C. Senior unsecured bond
29. Which of the following corporate debt instruments has the highest seniority ranking?
    A. Second lien
    B. Senior unsecured
    C. Senior subordinated
30. An issuer credit rating usually applies to a company's:
    A. secured debt.
    B. subordinated debt.
    C. senior unsecured debt.
31. The rating agency process whereby the credit ratings on issues are moved up or down from the issuer rating *best* describes:
    A. notching.
    B. pari passu ranking.
    C. cross-default provisions.
32. The notching adjustment for corporate bonds rated Aa2/AA is *most likely*:
    A. larger than the notching adjustment for corporate bonds rated B2/B.
    B. the same as the notching adjustment for corporate bonds rated B2/B.
    C. smaller than the notching adjustment for corporate bonds rated B2/B.
33. Which of the following statements about credit ratings is *most accurate*?
    A. Credit ratings can migrate over time.
    B. Changes in bond credit ratings precede changes in bond prices.
    C. Credit ratings are focused on expected loss rather than risk of default.
34. Which industry characteristic *most likely* has a positive effect on a company's ability to service debt?
    A. Low barriers to entry in the industry
    B. High number of suppliers to the industry
    C. Broadly dispersed market share among large number of companies in the industry
35. When determining the capacity of a borrower to service debt, a credit analyst should begin with an examination of:
    A. industry structure.
    B. industry fundamentals.
    C. company fundamentals.
36. Which of the following accounting issues should *mostly likely* be considered a character warning flag in credit analysis?
    A. Expensing items immediately
    B. Changing auditors infrequently
    C. Significant off-balance-sheet financing

37. In credit analysis, capacity is *best* described as the:
    A. quality of management.
    B. ability of the borrower to make its debt payments on time.
    C. quality and value of the assets supporting an issuer's indebtedness.
38. Among the four Cs of credit analysis, the recognition of revenue prematurely *most likely* reflects a company's:
    A. character.
    B. covenants.
    C. collateral.

The following information relates to Questions 39 and 40

EXHIBIT 4 Industrial Comparative Ratio Analysis, Year 20XX

	EBITDA Margin (%)	Return on Capital (%)	EBIT/Interest Expense (×)	EBITDA/ Interest Expense (×)	Debt/ EBITDA (×)	Debt/Capital (%)
Company A	25.1	25.0	15.9	19.6	1.6	35.2
Company B	29.6	36.3	58.2	62.4	0.5	15.9
Company C	21.8	16.6	8.9	12.4	2.5	46.3

39. Based on only the leverage ratios in Exhibit 4, the company with the *highest* credit risk is:
    A. Company A.
    B. Company B.
    C. Company C.
40. Based on only the coverage ratios in Exhibit 4, the company with the *highest* credit quality is:
    A. Company A.
    B. Company B.
    C. Company C.

The following information relates to Questions 41 and 42

EXHIBIT 5 Consolidated Income Statement (£ millions)

	Company X	Company Y
Net revenues	50.7	83.7
Operating expenses	49.6	70.4
Operating income	1.1	13.3
Interest income	0.0	0.0
Interest expense	0.6	0.8
Income before income taxes	0.5	12.5
Provision for income taxes	–0.2	–3.5
Net income	0.3	9.0

EXHIBIT 6 Consolidated Balance Sheets (£ millions)

	Company X	Company Y
ASSETS		
Current assets	10.3	21.9
Property, plant, and equipment, net	3.5	20.1
Goodwill	8.3	85.0
Other assets	0.9	5.1
Total assets	23.0	132.1
LIABILITIES AND SHAREHOLDERS' EQUITY		
Current liabilities		
Accounts payable and accrued expenses	8.4	16.2
Short-term debt	0.5	8.7
Total current liabilities	8.9	24.9
Long-term debt	11.7	21.1
Other non-current liabilities	1.1	22.1
Total liabilities	21.7	68.1
Total shareholders' equity	1.3	64.0
Total liabilities and shareholders' equity	23.0	132.1

EXHIBIT 7 Consolidated Statements of Cash Flow (£ millions)

	Company X	Company Y
CASH FLOWS FROM OPERATING ACTIVITIES		
Net income	0.3	9.0
Depreciation	1.0	3.8
Goodwill impairment	2.0	1.6
Changes in working capital	0.0	−0.4
Net cash provided by operating activities	3.3	14.0
CASH FLOWS FROM INVESTING ACTIVITIES		
Additions to property and equipment	−1.0	−4.0
Additions to marketable securities	−0.1	0.0
Proceeds from sale of property and equipment	0.2	2.9
Proceeds from sale of marketable securities	0.3	0.0
Net cash used in investing activities	−0.6	−1.1

(*continued*)

EXHIBIT 7 (Continued)

	Company X	Company Y
CASH FLOWS FROM FINANCING ACTIVITIES		
Repurchase of common stock	−1.5	−4.0
Dividends to shareholders	−0.3	−6.1
Change in short-term debt	0.0	−3.4
Additions to long-term debt	3.9	3.9
Reductions in long-term debt	−3.4	−2.5
Net cash–financing activities	−1.3	−12.1
NET INCREASE IN CASH AND CASH EQUIVALENTS	1.4	0.8

41. Based on Exhibits 5–7, in comparison to Company X, Company Y has a higher:
    A. debt/capital.
    B. debt/EBITDA.
    C. free cash flow after dividends/debt.
42. Based on Exhibits 5–7, in comparison to Company Y, Company X has greater:
    A. leverage.
    B. interest coverage.
    C. operating profit margin.

43. Credit yield spreads *most likely* widen in response to:
    A. high demand for bonds.
    B. weak performance of equities.
    C. strengthening economic conditions.
44. The factor that *most likely* results in corporate credit spreads widening is:
    A. an improving credit cycle.
    B. weakening economic conditions.
    C. a period of high demand for bonds.
45. Credit spreads are *most likely* to widen:
    A. in a strengthening economy.
    B. as the credit cycle improves.
    C. in periods of heavy new issue supply and low borrower demand.
46. Which of the following factors in credit analysis is more important for general obligation non-sovereign government debt than for sovereign debt?
    A. Per capita income
    B. Power to levy and collect taxes
    C. Requirement to balance an operating budget
47. In contrast to high-yield credit analysis, investment-grade analysis is *more likely* to rely on:
    A. spread risk.
    B. an assessment of bank credit facilities.
    C. matching of liquidity sources to upcoming debt maturities.
48. Which of the following factors would *best* justify a decision to avoid investing in a country's sovereign debt?
    A. Freely floating currency
    B. A population that is not growing
    C. Suitable checks and balances in policymaking

PART II

FIXED INCOME TERM STRUCTURE, ADVANCED VALUATION, AND CREDIT ANALYSIS

CHAPTER 7

THE TERM STRUCTURE AND INTEREST RATE DYNAMICS

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LEARNING OUTCOMES

The candidate should be able to:

- describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve;
- describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping;
- describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management;
- describe the strategy of rolling down the yield curve;
- explain the swap rate curve and why and how market participants use it in valuation;
- calculate and interpret the swap spread for a given maturity;
- describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk;
- explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve;
- explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks;
- explain the maturity structure of yield volatilities and their effect on price volatility;
- explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes.

1. SPOT RATES, FORWARD RATES, AND THE FORWARD RATE MODEL

- **describe relationships among spot rates, forward rates, yield-to-maturity, expected and realized returns on bonds, and the shape of the yield curve**
- **describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping**

Interest rates are both a barometer of the economy and an instrument for its control. The term structure of interest rates—market interest rates at various maturities—is a vital input into the valuation of many financial products. The quantification of interest rate risk is of critical importance to risk managers. Understanding the determinants of interest rates, and thus the drivers of bond returns, is imperative for fixed-income market participants. Here, we explore the tools necessary to understand the term structure and interest rate dynamics—that is, the process by which bond yields and prices evolve over time.

Section 1 explains how spot (or current) rates and forward rates, which are set today for a period starting in the future, are related, as well as how their relationship influences yield curve shape. Section 2 builds upon this foundation to show how forward rates impact the yield-to-maturity and expected bond returns. Section 3 explains how these concepts are put into practice by active fixed-income portfolio managers.

The swap curve is the term structure of interest rates derived from a periodic exchange of payments based on fixed rates versus short-term market reference rates rather than default-risk-free government bonds. Sections 4 and 5 describe the swap curve and its relationship to government yields, known as the swap spread, and explains their use in valuation.

Section 6 describes traditional theories of the term structure of interest rates. These theories outline several qualitative perspectives on economic forces that may affect the shape of the term structure.

Section 7 describes yield curve factor models. The focus is a popular three-factor term structure model in which the yield curve changes are described in terms of three independent movements: level, steepness, and curvature. These factors can be extracted from the variance-covariance matrix of historical interest rate movements.

Section 8 builds on the factor model and describes how to manage the risk of changing rates over different maturities. Section 9 concludes with a discussion of key variables known to influence interest rates, the development of interest rate views based on forecasts of those variables, and common trades tailored to capitalize on an interest rate view. A summary of key points concludes the chapter.

1.1. Spot Rates and Forward Rates

We first explain the relationships among spot rates, forward rates, yield-to-maturity, expected and realized returns on bonds, and the shape of the yield curve. We then discuss the assumptions made about forward rates in active bond portfolio management.

The price of a risk-free single-unit payment (e.g., \$1, €1, or £1) after N periods is called the **discount factor** with maturity N, denoted by PV_{N}. The yield-to-maturity of the payment is called a **spot rate**, denoted by Z_N. That is,

$$DF_N = \frac{1}{(1 + Z_N)^N} \tag{1}$$

The N-period discount factor, DF_N, and the N-period spot rate, Z_N, for a range of maturities in years $N > 0$ are called the **discount function** and the **spot yield curve** (or, more simply, **spot**

curve), respectively. This spot curve represents the term structure of interest rates. Note that the discount function completely identifies the spot curve and vice versa, because both contain the same set of information about the time value of money.

The spot curve shows, for various maturities, the annualized return on an option-free and default-risk-free **zero-coupon bond** (**zero** for short) with a single payment at maturity. For this reason, spot rates are also referred to as zero-coupon yields or zero rates. The spot rate as a yield concept avoids the need for a reinvestment rate assumption for coupon-paying securities.

As Equation 1 suggests, the spot curve is a benchmark for the time value of money received on a future date as determined by the market supply and demand for funds. It is viewed as the most basic term structure of interest rates because no reinvestment risk is involved; the stated yield equals the actual realized return if the zero is held to maturity. Thus, the yield on a zero-coupon bond maturing in year T is regarded as the most accurate representation of the T-year interest rate.

A **forward rate** is an interest rate determined today for a loan that will be initiated in a future period. The set of forward rates for loans of different maturities with the same future start date is called the **forward curve**. Forward rates and forward curves can be mathematically derived from the current spot curve.

Denote the forward rate of a loan initiated A periods from today with tenor (further maturity) of B periods by $f_{A,B-A}$. Consider a forward contract in which one party, the buyer, commits to pay another party, the seller, a forward contract price $f_{A,B-A}$ at time A for a zero-coupon bond with maturity $B - A$ and unit principal. Because this is an agreement to do something in the future, no money is exchanged at contract initiation. At A, the buyer will pay the seller the contracted forward price and will receive from the seller at time B a payment defined here as a single currency unit.

The **forward pricing model** describes the valuation of forward contracts. The no-arbitrage principle, which simply states that tradable securities with identical cash flow payments must have the same price, may be used to derive the model as shown in Equation 2:

$$DF_B = DF_A \times F_{A,B-A} \tag{2}$$

The discount factors DF_A and DF_B represent the respective prices for period A and a longer period B needed to derive the forward price, $F_{A,B-A}$, a contract that starts in the future at time A and ends at time B.

To understand the reasoning behind Equation 2, consider two alternative investments: (1) buying a two-year zero-coupon bond at a cost of $DF_2 = 0.93$ and (2) entering into a one-year forward contract to purchase a one-year zero-coupon bond for $DF_1 = 0.95$. Because the payoffs in two years are the same and the initial costs of the investments must be equal, the no-arbitrage forward price $F_{1,1}$ must equal 0.93/0.95, or 0.9789. Otherwise, any trader could sell the overvalued investment and buy the undervalued investment with the proceeds to generate risk-free profits with zero net investment.

Example 1 should help confirm your understanding of discount factors and forward prices. Please note that the solutions in the examples that follow may be rounded to two or four decimal places.

EXAMPLE 1 Spot and Forward Prices and Rates (1)

Consider a two-year loan beginning in one year ($A = 1$, $B = 3$). The one-year spot rate is $z_1 = z_A = 7\% = 0.07$. The three-year spot rate is $z_3 = z_B = 9\% = 0.09$.

1. Calculate the one-year discount factor: $DF_A = DF_1$.
2. Calculate the three-year discount factor: $DF_B = DF_3$.

3. Calculate the forward price of a two-year bond to be issued in one year: $F_{A,B-A} = F_{1,3}$.
4. Interpret your answer to Problem 3.

Solution to 1: Using Equation 1,

$$DF_1 = \frac{1}{(1+0.07)^1} = 0.9346$$

Solution to 2:

$$DF_3 = \frac{1}{(1+0.09)^3} = 0.7722$$

Solution to 3: Using Equation 2,

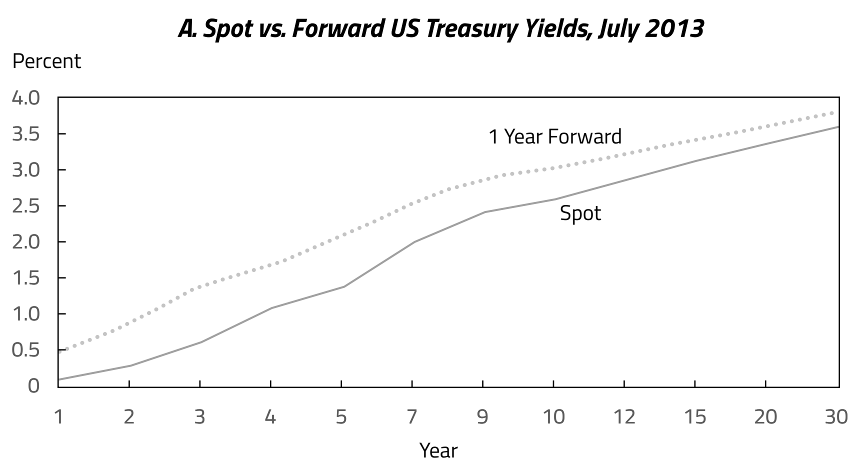
$$0.7722 = 0.9346 \times F_{1,3}$$
$$F_{1,3} = 0.7722 \div 0.9346 = 0.8262$$

Solution to 4: The forward contract price of $DF_{1,3} = 0.8262$ is the price agreed on today, to be paid one year from today for a bond with a two-year maturity and a risk-free unit-principal payment (e.g., \$1, €1, or £1) at maturity in three years. As shown in the solution to 3, it is calculated as the three-year discount factor, $DF_3 = 0.7722$, divided by the one-year discount factor, $DF_1 = 0.9346$.

1.1.1. The Forward Rate Model

This section uses the forward rate model to establish that forward rates are above spot rates when the spot curve is upward sloping and below spot rates when the spot curve slopes downward. Exhibit 1 shows these spot versus forward relationships for the US Treasury yield curve in July 2013 versus December 2006, respectively. As we illustrate later, the relationship between spot and forward rates is important for future rate expectations as well as valuing fixed-income instruments.

EXHIBIT 1 Spot and Forward Curves



In contrast to the forward price $F_{A,B-A}$, the forward *rate* $f_{A,B-A}$ is the discount rate for a risk-free unit-principal payment (e.g., \$1, €1, or £1) B periods from today, valued at time A, such that the present value equals the forward contract price, $DF_{A,B-A}$. Then, by definition,

$$DF_{A,B-A} = \frac{1}{(1 + F_{A,B-A})^{B-A}} \tag{3}$$

By substituting Equations 1 and 3 into Equation 2, the forward pricing model can be expressed in terms of rates as noted by Equation 4, which is the **forward rate model**:

$$(1 + z_B)^B = (1 + z_A)^A(1 + f_{A,B-A})^{B-A} \tag{4}$$

Thus, the spot rate for B periods, which is z_B, and the spot rate for A periods, which is z_A, imply a value for the (B-A)-period forward rate at A, $f_{A,B-A}$. Equation 4 is important because it shows how forward rates may be extrapolated from spot rates—that is, they are implicit in the spot rates at any given point in time.

Equation 4 suggests two ways to interpret forward rates. For example, suppose $f_{7,1}$, the rate agreed on today for a one-year loan to be made seven years from today, is 3%. Then 3% is the

- reinvestment rate that would make an investor indifferent between buying an eight-year zero-coupon bond or investing in a seven-year zero-coupon bond and at maturity reinvesting the proceeds for one year. In this sense, the forward rate can be viewed as a type of breakeven interest rate.
- one-year rate that can be locked in today by buying an eight-year zero-coupon bond rather than investing in a seven-year zero-coupon bond and, when it matures, reinvesting the proceeds in a zero-coupon instrument that matures in one year. In this sense, the forward rate can be viewed as a rate that can be locked in by extending maturity by one year.

Example 2 addresses forward rates and the relationship between spot and forward rates.

EXAMPLE 2 Spot and Forward Prices and Rates (2)

The spot rates for three hypothetical zero-coupon bonds (zeros) with maturities of one, two, and three years are given in the following table.

Maturity (T)	1	2	3
Spot rates	$z_1 = 9\%$	$z_2 = 10\%$	$z_3 = 11\%$

1. Calculate the forward rate for a one-year zero issued one year from today, $f_{1,1}$.
2. Calculate the forward rate for a one-year zero issued two years from today, $f_{2,1}$.
3. Calculate the forward rate for a two-year zero issued one year from today, $f_{1,2}$.
4. Based on your answers to 1 and 2, describe the relationship between the spot rates and the implied one-year forward rates.

Solution to 1: $f_{1,1}$ is calculated as follows (using Equation 4):

$$(1+z_2)^2 = (1+z_1)^1(1+f_{1,1})^1$$
$$(1+0.10)^2 = (1+0.09)^1(1+f_{1,1})^1$$
$$f_{1,1} = \frac{(1.10)^2}{1.09} - 1 = 11.01\%$$

Solution to 2: $f_{2,1}$ is calculated as follows:

$$(1+z_3)^3 = (1+z_2)^2(1+f_{2,1})^1$$
$$(1+0.11)^3 = (1+0.10)^2(1+f_{2,1})^1$$
$$f_{2,1} = \frac{(1.11)^3}{(1.10)^2} - 1 = 13.03\%$$

Solution to 3: $f_{1,2}$ is calculated as follows:

$$(1+z_3)^3 = (1+z_1)^1(1+f_{1,2})^2$$
$$(1+0.11)^3 = (1+0.09)^1(1+f_{1,2})^2$$
$$f_{1,2} = \sqrt[2]{\frac{(1.11)^3}{(1.09)^1}} - 1 = 12.01\%$$

Solution to 4: The upward-sloping zero-coupon yield curve is associated with an upward-sloping forward curve (a series of increasing one-year forward rates because 13.03% is greater than 11.01%). This dynamic is explained further in the following discussion.

The relationship between spot rates and one-period forward rates may be demonstrated using the forward rate model and successive substitution, resulting in Equations 5a and 5b:

$$(1+z_T)^T = (1+z_1)(1+f_{2,1})(1+f_{3,1})\ldots(1+f_{T-1,1}) \quad (5a)$$
$$z_T = \left\{(1+z_1)(1+f_{2,1})(1+f_{3,1})\ldots(1+f_{T-1,1})\right\}^{\frac{1}{T}} - 1 \quad (5b)$$

Equation 5b shows that the spot rate for a security with a maturity of $T > 1$ can be expressed as a geometric mean of the spot rate for a security with a maturity of $T = 1$ and a series of $T - 1$ forward rates.

Equation 5b is critical for active fixed-income portfolio managers. Although the question of whether forward rates are unbiased estimators of market consensus expectations remains open to debate, implied forward rates are generally the best available and most accessible proxy for market expectations of future spot rates. If an active trader can identify a series of short-term bonds whose actual returns exceed today's quoted forward rates, then the total return over her investment horizon would exceed the return on a maturity-matching, buy-and-hold strategy if the yield curve were to remain relatively stable. Later, we will apply this concept to dynamic hedging strategies and the local expectations theory.

Examples 3 and 4 explore the relationship between spot and forward rates.

EXAMPLE 3 Spot and Forward Prices and Rates (3)

Given the data and conclusions for z_1, $f_{1,1}$, and $f_{2,1}$ from Example 2:

$$z_1 = 9\%$$
$$f_{1,1} = 11.01\%$$
$$f_{2,1} = 13.03\%$$

Show that the two-year spot rate of $z_2 = 10\%$ and the three-year spot rate of $z_3 = 11\%$ are geometric averages of the one-year spot rate and the forward rates.

Solution: Using Equation 5a,

$$(1 + z_2)^2 = (1 + z_1)(1 + f_{1,1})$$
$$z_2 = \sqrt[2]{(1 + 0.09)(1 + 0.1101)} - 1 \approx 10\%$$

$$(1 + z_3)^3 = (1 + z_1)(1 + f_{1,1})(1 + f_{2,1})$$
$$z_3 = \sqrt[3]{(1 + 0.09)(1 + 0.1101)(1 + 0.1303)} - 1 \approx 11\%$$

We can now consolidate our knowledge of spot and forward rates to explain important relationships between the spot and forward rate curves. The forward rate model (Equation 4) can also be expressed as Equation 6.

$$\left\{\frac{1 + z_B}{1 + z_A}\right\}^{\frac{A}{B-A}}(1 + z_B) = 1 + f_{A,B-A} \tag{6}$$

To illustrate, suppose $A = 1$, $B = 5$, $z_1 = 2\%$, and $z_5 = 3\%$; the left-hand side of Equation 6 is

$$\left(\frac{1.03}{1.02}\right)^{\frac{1}{4}}(1.03) = (1.0024)(1.03) = 1.0325$$

so $f_{1,4} = 3.25\%$. Given that the yield curve is upward sloping—so, $z_B > z_A$—Equation 6 implies that the forward rate from A to B is greater than the long-term spot rate: $f_{A,B-A} > z_B$. This is the case in our example, because 3.25% > 3.00%. Conversely, when the yield curve is downward sloping, then $z_B < z_A$ and the forward rate from A to B is lower than the long-term spot rate: $f_{A,B-A} < z_B$. Equation 6 also shows that if the spot curve is flat, all one-period forward rates equal the spot rate. For an upward-sloping yield curve— $z_B > z_A$ —the forward rate rises as time periods increase. For a downward-sloping yield curve— $z_B < z_A$ —the forward rate declines as time periods increase.

EXAMPLE 4 Spot and Forward Prices and Rates (4)

Given the spot rates $z_1 = 9\%$, $z_2 = 10\%$, and $z_3 = 11\%$, as in Examples 2 and 3:

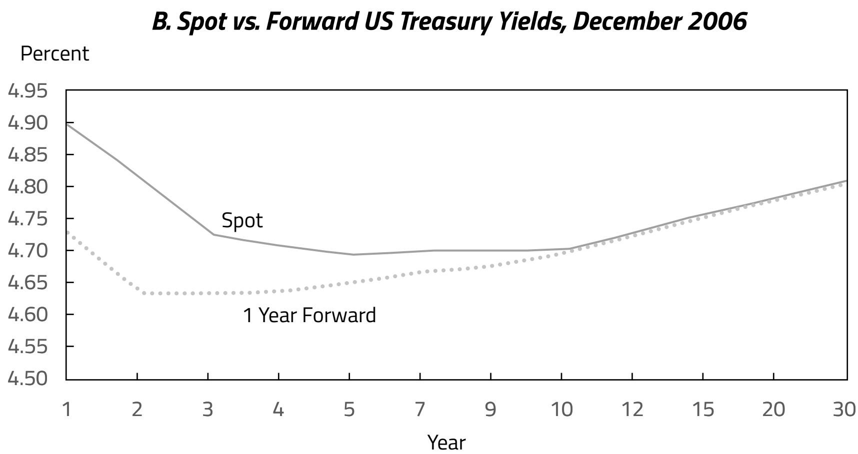
1. Determine whether the forward rate $f_{1,2}$ is greater than or less than the long-term rate, z_3.
2. Determine whether forward rates rise or fall as the initiation date, A, for the forward rate is later.

Solution to 1: The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$, or in general, $z_B > z_A$. Thus, the forward rate will be greater than the long-term rate, or $f_{A,B-A} > z_B$. Note from Example 2 that $f_{1,2} = 12.01\% > z_3 = 11\%$.

Solution to 2: The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$. Thus, the forward rates will rise with increasing A. This relationship was shown in Example 2, in which $f_{1,1} = 11.01\%$ and $f_{2,1} = 13.03\%$.

These relationships are illustrated in Exhibits 2 and 3 as an extension of Exhibit 1. The spot rates for US Treasuries as of 31 July 2013 constructed using interpolation are the lowest, as shown in the table following the exhibit. Note that the spot curve is upward sloping. The forward curves for the end of July 2014, 2015, 2016, and 2017 are also presented in Exhibit 2. Because the yield curve is upward sloping, these forward curves are all above the spot curve and become successively higher and steeper as the forward period increases, the highest of which is that for July 2017.

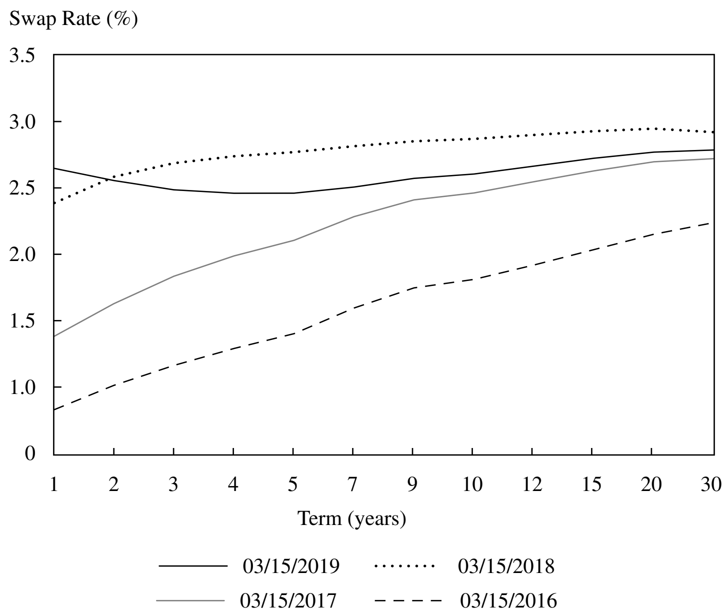
EXHIBIT 2 Historical Example: Upward-Sloping Spot Curve vs. Forward Curves, 31 July 2013



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	0.11	0.33	0.61	1.37	2.00	2.61	3.35	3.66

Exhibit 3 shows the opposite case of a downward sloping spot curve based on US Treasury rates as of 31 December 2006. This data also uses interpolation and is somewhat modified to make the yield curve more downward sloping for illustrative purposes. The spot curve and forward curves for the end of December 2007, 2008, 2009, and 2010 are presented in Exhibit 3.

EXHIBIT 3 Historical Example: Downward-Sloping Spot Curve vs. Forward Curves, 31 December 2006 (modified for illustrative purposes)



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	4.90	4.82	4.74	4.70	4.60	4.51	4.41	4.31

The highest curve is the spot yield curve, and it is downward sloping. The forward curves are below the spot curve, with longer forward periods associated with lower forward curves, the lowest of which is dated December 2010.

An important point that can be inferred from Exhibit 2 and Exhibit 3 is that forward rates do not extend beyond the longest maturity on today's yield curve. For example, if yields reach a 30-year maturity on today's yield curve, then a three-year forward model will extend just 27 years. Similarly, four years hence, the longest-maturity forward rate would be $f_{4,26}$.

In summary, when the spot curve slopes upward, the forward curve will lie above the spot curve. Conversely, when the spot curve slopes downward, the forward curve will lie below the spot curve. This dynamic reflects the basic mathematical truth that when an average is rising (falling), the marginal data point must be above (below) the average. In this case, the spot curve represents an average over an entire period and the forward rates represent the marginal changes between future periods.

We have thus far discussed the spot curve and the forward curve. Another curve important in practice is the government par curve. The **par curve** represents the yields to maturity on coupon-paying government bonds, priced at par, over a range of maturities. In practice, recently issued ("on the run") bonds are most often used to create the par curve, because these securities are most liquid and typically priced at or close to par.

The par curve is important for valuation in that it can be used to construct a zero-coupon yield curve. The process considers a coupon-paying bond as a portfolio of zero-coupon bonds. The zero-coupon rates are determined by using the par yields and solving for the zero-coupon rates one by one, from the shortest to longest maturities using a forward substitution process known as **bootstrapping**.

WHAT IS BOOTSTRAPPING?

Because the practical details of deriving the zero-coupon yield are beyond the scope of this chapter, the concept of bootstrapping may be best shown using a numerical illustration. Suppose the following yields are observed for annual coupon sovereign debt:

Par Rates:
One-year par rate = 5%, two-year par rate = 5.97%, three-year par rate = 6.91%, four-year par rate = 7.81%. From these data, we can bootstrap zero-coupon rates.

Zero-Coupon Rates:
Given annual coupons, the one-year zero-coupon rate equals the one-year par rate because it has one cash flow, whereas two-year and longer maturity bonds have coupon payments prior to maturity.

The derivation of zero-coupon rates begins with the two-year maturity. The two-year zero-coupon rate is determined by using $z_1 = 5\%$ and solving for z_2 in the following equation for one monetary unit of current market value:

$$1 = \frac{0.0597}{(1.05)} + \frac{1 + 0.0597}{(1 + z_2)^2}$$

In the equation, 0.0597 and 1.0597 represent payments from interest and principal and interest, respectively, per unit of principal value. The equation implies that $z_2 = 6\%$. We have bootstrapped the two-year spot rate. Continuing with forward substitution, the three-year zero-coupon rate can be bootstrapped by solving for z_3 using the known values of the one-year and two-year spot rates of 5% and 6%:

$$1 = \frac{0.0691}{(1.05)} + \frac{0.0691}{(1.06)^2} + \frac{1 + 0.0691}{(1 + z_3)^3}$$

Thus, $z_3 = 7\%$. Finally, we solve for the four-year zero-coupon rate, z_4:

$$1 = \frac{0.0781}{(1.05)} + \frac{0.0781}{(1.06)^2} + \frac{0.0781}{(1.07)^3} + \frac{1 + 0.0781}{(1 + z_4)^4}$$

In summary, $z_1 = 5\%$, $z_2 = 6\%$, $z_3 = 7\%$, and $z_4 = 8\%$.

In the preceding discussion, we considered an upward-sloping (spot) yield curve (Exhibit 2) and an inverted or downward-sloping (spot) yield curve (Exhibit 3). In developed markets, yield curves are most commonly upward sloping with diminishing marginal increases in yield for identical changes in maturity; that is, the yield curve "flattens" at longer maturities. Because nominal yields incorporate a premium for expected inflation, an upward-sloping yield curve is generally interpreted as reflecting a market expectation of rising or at least stable future inflation (associated with relatively strong economic growth). The existence of risk premiums (e.g., for the greater interest rate risk of longer-maturity bonds) also contributes to a positive slope.

An inverted yield curve (Exhibit 3) is less common. Such a term structure may reflect a market expectation of declining future inflation rates (because a nominal yield incorporates a premium for expected inflation) from a relatively high current level. Expectations of an economic slowdown may be one reason to anticipate a decline in inflation, and a downward-sloping yield curve is frequently observed before recessions. A flat yield curve typically occurs briefly in the transition from an upward-sloping to a downward-sloping yield curve, or vice versa. A humped yield curve, which is relatively rare, occurs when intermediate-term interest rates are higher than short- and long-term rates.

2. YIELD-TO-MATURITY IN RELATION TO SPOT AND FORWARD RATES

- **describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management**

Yield-to-maturity (YTM) is perhaps the most familiar pricing concept in bond markets. In this section, we clarify how it is related to spot rates and a bond's expected and realized returns.

How is the yield-to-maturity related to spot rates? In bond markets, most bonds outstanding have coupon payments and many have various options, such as a call provision. The YTM of these bonds with maturity T would not be the same as the spot rate at T but should be mathematically related to the spot curve. Because the principle of no arbitrage shows that a bond's value is the sum of the present values of payments discounted by their corresponding spot rates, the YTM of the bond should be some weighted average of spot rates used in the valuation of the bond.

Example 5 addresses the relationship between spot rates and YTM.

EXAMPLE 5 Spot Rate and Yield-to-Maturity

Recall from earlier examples the spot rates $z_1 = 9\%$, $z_2 = 10\%$, and $z_3 = 11\%$. Let y_T be the YTM.

1. Calculate the price of a two-year annual coupon bond using the spot rates. Assume the coupon rate is 6% and the face value is \$1,000. Next, state the formula for determining the price of the bond in terms of its YTM. Is z2 greater than or less than *y*2? Why?
2. Calculate the price of a three-year annual coupon-paying bond using the spot rates. Assume the coupon rate is 5% and the face value is £100. Next, write a formula for determining the price of the bond using the YTM. Is z_3 greater or less than y_3? Why?

Solution to 1: Using the spot rates,

$$\text{Price} = \frac{\$60}{(1 + 0.09)^1} + \frac{\$1{,}060}{(1 + 0.10)^2} = \$931.08$$

Using the YTM,

$$\text{Price} = \frac{\$60}{(1 + y_2)} + \frac{\$1{,}060}{(1 + y_2)^2} = \$931.08$$

Note that y_2 is used to discount both the first- and second-year cash flows. Because the bond can have only one price, it follows that $z_1 < y_2 < z_2$ because y_2 is a weighted average of z_1 and z_2 and the yield curve is upward sloping. Using a calculator, one can calculate the YTM as $y2 = 9.97\%$, which is less than $z_2 = 10\%$ and greater than $z_1 = 9\%$, just as we would expect. Note that y_2 is much closer to z_2 than to z_1 because the bond's largest cash flow occurs in Year 2, thereby giving z_2 a greater weight than z_1 in the determination of y_2.

Solution to 2: Using the spot rates,

$$\text{Price} = \frac{£5}{(1 + 0.09)^1} + \frac{£5}{(1 + 0.10)^2} + \frac{£105}{(1 + 0.11)^3} = £85.49$$

Using the yield-to-maturity,

$$\text{Price} = \frac{£5}{(1 + y_3)} + \frac{£5}{(1 + y_3)^2} + \frac{£105}{(1 + y_3)^3} = £85.49$$

Note that y_3 is used to discount all three cash flows. Because the bond can have only one price, y_3 must be a weighted average of z_1, z_2, and z_3. Given that the yield curve is upward sloping in this example, $y_3 < z_3$. Using a calculator to compute YTM, $y_3 = 10.93\%$, which is less than $z_3 = 11\%$ and greater than $z_1 = 9\%$—just as we would expect, because the weighted YTM must lie between the highest and lowest spot rates. Note that y_3 is much closer to z_3 than it is to z_2 or z_1 because the bond's largest cash flow occurs in Year 3, thereby giving z_3 a greater weight than z_1 and z_2 in the determination of y_3.

Investors can expect to earn the yield-to-maturity on a bond only under extremely restrictive assumptions. The YTM is the expected rate of return for a bond held to maturity, assuming that all promised coupon and principal payments are made in full when due and that coupons are reinvested at the original YTM. As interest rates change, the reinvestment of coupons at the original YTM is unlikely. The YTM can provide a poor estimate of expected return if (1) interest rates are volatile, (2) the yield curve is sloped either upward or downward, (3) there is significant risk of default, or (4) the bond has one or more embedded options (e.g., put, call, or conversion). If either (1) or (2) is the case, reinvestment of coupons would not be expected to be at the assumed rate (YTM). Case 3 implies that actual cash flows may differ from those assumed in the YTM calculation, and in Case 4, the exercise of an embedded option would result in a holding period shorter than the bond's original maturity.

The realized return is the actual bond return during an investor's holding period. It is based on actual reinvestment rates and the yield curve at the end of the holding period. If we had perfect foresight, the expected bond return would equal the realized bond return.

To illustrate these concepts, assume that $z_1 = 5\%$, $z_2 = 6\%$, $z_3 = 7\%$, $z_4 = 8\%$, and $z_5 = 9\%$. Consider a five-year annual coupon bond with a coupon rate of 10%. The forward rates extrapolated from the spot rates are $f_{1,1} = 7.0\%$, $f_{2,1} = 9.0\%$, $f_{3,1} = 11.1\%$, and $f_{4,1} = 13.1\%$. The price, determined as a percentage of par, is 105.43.

The yield-to-maturity of 8.62% can be determined by solving

$$105.43 = \frac{10}{(1 + y_5)} + \frac{10}{(1 + y_5)^2} + \frac{10}{(1 + y_5)^3} + \frac{10}{(1 + y_5)^4} + \frac{110}{(1 + y_5)^5}$$

The yield-to-maturity of 8.62% is the bond's expected return assuming no default, a holding period of five years, and a reinvestment rate of 8.62%. But what if the forward rates are assumed to be the future spot rates?

Using the forward rates as the expected reinvestment rates results in the following expected cash flow at the end of Year 5:

$$10(1 + 0.07)(1 + 0.09)(1 + 0.111)(1 + 0.131) + 10(1 + 0.09)(1 + 0.011)(1 + 0.131) \\ + 10(1 + 0.111)(1 + 0.131) + 10(1 + 0.131) + 110 \approx 162.22$$

Therefore, the expected bond return is (162.22 – 105.43)/105.43 = 53.87% and the expected annualized rate of return is 9.00% [solve $(1 + x)^5 = 1 + 0.5387$].

From this example, we can see that the expected rate of return is not equal to the YTM even if we make the generally unrealistic assumption that the forward rates are the future spot rates. The YTM is generally a realistic estimate of expected return only if the yield curve is flat. Note that in the foregoing formula, all cash flows were discounted at 8.62% regardless of maturity.

Example 6 will reinforce your understanding of various yield and return concepts.

EXAMPLE 6 Yield and Return Concepts

1. When the spot curve is upward sloping, the forward curve:
   A. lies above the spot curve.
   B. lies below the spot curve.
   C. is coincident with the spot curve.
2. Which of the following statements concerning the YTM of a default-risk-free bond is *most* accurate? The YTM of such a bond:
   A. equals the expected return on the bond if the bond is held to maturity.
   B. can be viewed as a weighted average of the spot rates applying to its cash flows.
   C. will be closer to the realized return if the spot curve is upward sloping rather than flat through the life of the bond.

3. When the spot curve is downward sloping, a later initiation date results in a forward curve that is:
   A. closer to the spot curve.
   B. a greater distance above the spot curve.
   C. a greater distance below the spot curve.

Solution to 1: A is correct. Points on a spot curve can be viewed as an average of single-period rates over given maturities, whereas forward rates reflect the marginal changes between future periods.

Solution to 2: B is correct. The YTM is the discount rate that, when applied to a bond's promised cash flows, equates those cash flows to the bond's market price and the fact that the market price should reflect discounting promised cash flows at appropriate spot rates.

Solution to 3: C is correct. This answer follows from the forward rate model as expressed in Equation 6. If the spot curve is downward sloping (upward sloping), a later initiation date will result in a forward curve that is a greater distance below (above) the spot curve. See Exhibit 2 and Exhibit 3.

2.1. Yield Curve Movement and the Forward Curve

This section establishes several important results concerning forward prices and the spot yield curve to demonstrate the relevance of the forward curve to active bond investors.

The forward contract price remains unchanged as long as future spot rates evolve as predicted by today's forward curve. If a trader expects the future spot rate to be below what is predicted by the prevailing forward rate, the forward contract value is expected to increase and the trader would buy the forward contract. Conversely, if the trader expects the future spot rate to be above that predicted by the existing forward rate, then the forward contract value is expected to decrease and the trader would sell the forward contract.

Using the forward pricing model defined by Equation 2, we can determine the forward contract price that delivers a $(B - A)$-period-maturity bond at time A, $F_{A,B-A}$, using Equation 7 (which is Equation 2 solved for the forward price):

$$F_{A,B-A} = \frac{DF_B}{DF_A} \tag{7}$$

Now suppose that after t periods, the new discount function for some maturity time T period, denoted as DF_T^{new}, is the same as the forward discount function implied by today's discount function, as shown by Equation 8.

$$DF_T^{new} = \frac{DF_{t+T}}{DF_t} \tag{8}$$

Next, after a lapse of t periods, the time to expiration of the contract is $A - t$, and the forward contract price at time t is $F_{A-t,B-A}^{new}$. Equation 7 can be rewritten as Equation 9:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} \tag{9}$$

Substituting Equation 8 into Equation 9 and adjusting for the lapse of time *t* results in Equation 10:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} = \frac{\frac{DF_B}{DF_t}}{\frac{DF_A}{DF_t}} = \frac{DF_B}{DF_A} = F_{A,B-A} \tag{10}$$

Equation 10 shows that the forward contract price remains unchanged as long as future spot rates are equal to what is predicted by today's forward curve. Therefore, a change in the forward price is the result of a deviation of the spot curve from what is predicted by today's forward curve.

To make these observations concrete, consider a flat yield curve for which the interest rate is 4%. Using Equation 1, the discount factors for the one-year, two-year, and three-year terms are, to four decimal places, as follows:

$$DF_1 = \frac{1}{(1 + 0.04)} = 0.9615$$

$$DF_2 = \frac{1}{(1 + 0.04)^2} = 0.9246$$

$$DF_3 = \frac{1}{(1 + 0.04)^3} = 0.8890$$

Therefore, using Equation 7, the forward contract price that delivers a one-year bond at Year 2 is

$$F_{2,1} = \frac{DF_3}{DF_2} = \frac{0.8890}{0.9246} = 0.9615$$

Suppose the future discount function at Year 1 is the same as the forward discount function implied by the Year 0 spot curve. The lapse of time is $t = 1$. Using Equation 8, the discount factors for the one-year and two-year terms one year from today are as follows:

$$DF_1^{new} = \frac{DF_2}{DF_1} = \frac{0.9246}{0.9615} = 0.9616$$

$$DF_2^{new} = \frac{DF_3}{DF_1} = \frac{0.8890}{0.9615} = 0.9246$$

Using Equation 9, the price of the forward contract one year from today is

$$F_{2,1}^{new} = \frac{DF_2^{new}}{DF_1^{new}} = \frac{0.9246}{0.9615} = 0.9616$$

The price of the forward contract is nearly unchanged. This will be the case as long as future discount functions are the same as those based on today's forward curve.

From this numerical example, we can see that if the spot rate curve is unchanged, then each bond "rolls down" the curve and earns the current one-period spot rate and subsequent forward rates. Specifically, when one year passes, a three-year bond will return (0.9246 − 0.8890)/0.8890 = 4%, which is equal to the spot rate. Furthermore, if another year passes, the bond will return (0.9615 − 0.9246)/0.9246 = 4%, which is equal to the implied forward rate for a one-year security one year from today.

3. ACTIVE BOND PORTFOLIO MANAGEMENT

- **describe the strategy of rolling down the yield curve**

One way that active bond portfolio managers attempt to outperform the bond market's return is by anticipating changes in interest rates relative to the projected evolution of spot rates reflected in today's forward curves.

The forward rate model (Equation 4) provides insight into these issues. By rearranging terms in Equation 4 and setting the time horizon to one period, $A = 1$, we obtain

$$\frac{(1+z_B)^B}{(1+f_{A,B-A})^{B-A}} = (1+z_A)^A \tag{11}$$

The numerator of the left-hand side of Equation 11 is for a bond with an initial maturity of B periods and a remaining maturity of $B - A$ periods after A periods pass. Suppose the prevailing spot yield curve after one period ($A = 1$) is the current forward curve; then, Equation 11 shows that the total return on the bond is the one-period risk-free rate. The following sidebar shows that returns on bonds of varying tenor over a one-year period always equal the one-year rate (the risk-free rate over the one-year period) if the spot rates evolve as implied by the current forward curve at the end of the first year.

WHEN SPOT RATES EVOLVE AS IMPLIED BY THE CURRENT FORWARD CURVE

As in earlier examples, assume the following:

$$z_1 = 9\%$$

$$z_2 = 10\%$$

$$z_3 = 11\%$$

$$f_{1,1} = 11.01\%$$

$$f_{1,2} = 12.01\%$$

If the spot curve one year from today reflects the current forward curve, the return on a zero-coupon bond for the one-year holding period is 9%, regardless of the bond's maturity. The following computations assume a par amount of 100 and represent the percentage change in price. Given the rounding of price and the forward rates to the nearest hundredth, the returns all approximate 9%. With no rounding, however, all answers would be precisely 9%.

The return of the one-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 91.74 and is worth the par amount of 100 at maturity.

$$\left(100 \div \frac{100}{1+z_1}\right) - 1 = \left(100 \div \frac{100}{1+0.09}\right) - 1 = \frac{100}{91.74} - 1 = 9\%$$

The return of the two-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 82.64. One year from today, the two-year bond has a remaining maturity of one year. Its price one year from today is 90.08, determined as the par amount divided by 1 plus the forward rate for a one-year bond issued one year from today.

$$\left(\frac{100}{(1+f_{1,1})} \div \frac{100}{(1+z_2)^2}\right) - 1 = \left(\frac{100}{(1+0.1101)} \div \frac{100}{(1+0.10)^2}\right) - 1$$
$$= \frac{90.08}{82.64} - 1 = 9\%$$

The return of the three-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 73.12. One year from today, the three-year bond has a remaining maturity of two years. Its price one year from today of 79.71 reflects the forward rate for a two-year bond issued one year from today.

$$\left(\frac{100}{(1+f_{1,2})^2} \div \frac{100}{(1+z_3)^3}\right) - 1 =$$
$$\left(\frac{100}{(1+0.1201)^2} \div \frac{100}{(1+0.11)^3}\right) - 1 = \frac{79.71}{73.12} - 1 \approx 9\%$$

This numerical example shows that the return of a bond over a one-year period is always the one-year rate (the risk-free rate over the one period) if the spot rates evolve as implied by the current forward curve.

But if the spot curve one year from today differs from today's forward curve, the returns on each bond for the one-year holding period will not all be 9%. To show that the returns on the two-year and three-year bonds over the one-year holding period are not 9%, we assume that the spot rate curve at Year 1 is flat with yields of 10% for all maturities.

The return on a one-year zero-coupon bond over the one-year holding period is

$$\left(100 \div \frac{100}{1+0.09}\right) - 1 = 9\%$$

The return on a two-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{1+0.10} \div \frac{100}{(1+0.10)^2}\right) - 1 = 10\%$$

The return on a three-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{(1+0.10)^2} \div \frac{100}{(1+0.11)^3}\right) - 1 = 13.03\%$$

The bond returns are 9%, 10%, and 13.03%. The returns on the two-year and three-year bonds differ from the one-year risk-free interest rate of 9%.

Equation 11 provides a total return investor with a means to evaluate the cheapness or expensiveness of a bond of a certain maturity. If any of the investor's expected future spot rates is below a quoted forward rate for the same maturity, then (all else being equal) the investor would perceive the bond to be undervalued, in the sense that the market is effectively discounting the bond's payments at a higher rate than the investor and the bond's market price is below the intrinsic value perceived by the investor.

Another example will reinforce the point that if a portfolio manager's projected spot curve is above (below) the forward curve and his expectation turns out to be true, the return will be less (more) than the one-period risk-free interest rate.

For the sake of simplicity, assume a flat yield curve of 8% and that a trader holds a three-year bond paying an 8% annual coupon. Assuming a par value of 100, the current market price is also 100. If today's forward curve turns out to be the spot curve one year from today, the trader will earn an 8% return.

If the trader projects that the spot curve one year from today is above today's forward curve—for example, a flat yield curve of 9%—the trader's expected rate of return is 6.24%, which is less than 8%:

$$\frac{8 + \frac{8}{1 + 0.09} + \frac{108}{(1 + 0.09)^2}}{100} - 1 = 6.24\%$$

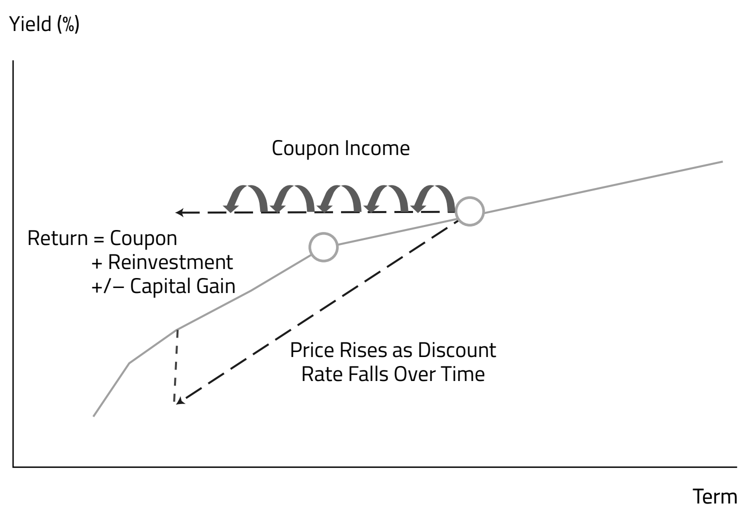
If the trader predicts a flat yield curve of 7%, the trader's expected return is 9.81%, which is greater than 8%:

$$\frac{8 + \frac{8}{1 + 0.07} + \frac{108}{(1 + 0.07)^2}}{100} - 1 = 9.81\%$$

As the gap between the projected future spot rate and the forward rate widens, so too will the difference between the trader's expected return and the original YTM of 8%.

This logic is the basis for a popular yield curve trade called **rolling down the yield curve**, also referred to as riding the yield curve. As we have noted, when a yield curve is upward sloping, the forward curve is always above the current spot curve. If the trader expects the yield curve to remain static over an investment horizon, then buying bonds with a maturity longer than the investment horizon would provide a total return greater than the return on a maturity-matching strategy. The bond's total return will depend on the spread between the forward rate and the spot rate as well as the maturity of the bond. The longer the bond's maturity, the more sensitive its total return is to the spread. This strategy is shown in Exhibit 4.

EXHIBIT 4 Rolling Down the Yield Curve



The return on a yield curve rolldown strategy may be demonstrated using a simple example. As stated earlier, the investment return on a fixed-rate (non-defaulted and non-callable) bond return may be defined as follows:

Bond return = Receipt of promised coupons (and principal)
+ Reinvestment of coupon payments (12)
+/– Capital gain/Loss on sale prior to maturity

Say we observe one-, three-, four-, five-, and six-year spot rates on annual coupon bonds trading at par of 2%, 4%, 5%, 6%, and 7%, respectively. An investor with a five-year maturity target decides to forgo a matched-maturity 6% five-year bond in favor of the 7%, six-year bond given her expectation of an unchanged yield curve over the next two years. We can compare the annualized return over two years for both bonds, assuming unchanged yields, as follows.

The 6% five-year bond purchased for 100 returns 120.61 in two years [(6 × 1.02) + 6 + 108.49], which consists of the first year's coupon reinvested at the one-year rate, the second annual coupon, and the capital gain on the sale of the 6% bond with three years to maturity at an unchanged three-year yield of 4% [$108.49 = 6/1.04 + 6/(1.04)^2 + 106/(1.04)^3$]. The annualized rate of return is 9.823% [solve for r, where $(120.61/100) = (1 + r)^2$].

The 7% six-year bond purchased at par returns 125.03 in two years [(7 × 1.02) + 7 + 110.89] with an annualized return of 11.817%. The excess return of nearly 2% results from both higher coupon income than the five-year matched maturity bond as well as a larger capital gain on the sale of the 7% bond with four years to maturity at an unchanged four-year yield of 5% [$110.89 = 7/1.05 + 7/(1.05)^2 + + 7/(1.05)^3 + 107/(1.05)^4$].

In the years following the 2008 financial crisis, many central banks acted to keep short-term interest rates very low. As a result, yield curves subsequently had a steep upward slope (see Exhibit 2). For active fixed-income managers, this situation provided an incentive to access short-term funding and invest in long-term bonds. This is just one form of a carry trade, referred to as a maturity spread carry trade, and is subject to significant interest rate risk, such as an unexpected increase in future spot rates (e.g., as a result of a spike in inflation). The maturity spread carry trade, in which the trader borrows short term and lends long term in the same currency, is common in an upward-sloping yield curve environment.

In summary, when the yield curve slopes upward, as a bond approaches maturity or "rolls down the yield curve," it is valued at successively lower yields and higher prices. Using this strategy, a bond can be held for a period of time as it appreciates in price and then sold before maturity to realize a higher return. As long as interest rates remain stable and the yield curve retains an upward slope, this strategy can continuously add to the total return of a bond portfolio.

Example 7 addresses how the preceding analysis relates to active bond portfolio management.

EXAMPLE 7 Active Bond Portfolio Management

1. The "rolling down the yield curve" strategy is executed by buying bonds whose maturities are:
   A. equal to the investor's investment horizon.
   B. longer than the investor's investment horizon.
   C. shorter than the investor's investment horizon.

2. A bond will be overvalued if the expected spot rate is:
   A. equal to the current forward rate.
   B. lower than the current forward rate.
   C. higher than the current forward rate.
3. Assume a flat yield curve of 6%. A three-year £100 bond is issued at par paying an annual coupon of 6%. What is the portfolio manager's expected return if he predicts that the yield curve one year from today will be a flat 7%?
   A. 4.19%
   B. 6.00%
   C. 8.83%
4. A forward contract price will increase if:
   A. future spot rates evolve as predicted by current forward rates.
   B. future spot rates are lower than what is predicted by current forward rates.
   C. future spot rates are higher than what is predicted by current forward rates.

Solution to 1: B is correct. A bond with a longer maturity than the investor's investment horizon is purchased but then sold prior to maturity at the end of the investment horizon. If the yield curve is upward sloping and yields do not change, the bond will be valued at successively lower yields and higher prices over time. The bond's total return will exceed that of a bond whose maturity is equal to the investment horizon.

Solution to 2: C is correct. If the expected discount rate is higher than the forward rate, then the bond will be overvalued. The expected price of the bond is lower than the price obtained from discounting using the forward rate.

Solution to 3: A is correct. Expected return will be less than the current YTM of 6% if yields increase to 7%. The expected return of 4.19% is computed as follows:

$$\frac{6 + \frac{6}{1 + 0.07} + \frac{106}{(1 + 0.07)^2}}{100} - 1 \approx 4.19\%$$

Solution to 4: B is correct. The forward rate model can be used to show that a change in the forward contract price requires a deviation of the spot curve from that predicted by today's forward curve. If the future spot rate is lower than what is predicted by the prevailing forward rate, the forward contract price will increase because it is discounted at an interest rate that is lower than the originally anticipated rate.

4. THE SWAP RATE CURVE

- **explain the swap rate curve and why and how market participants use it in valuation**

Earlier, we described the spot rate curve of default-risk-free bonds as a measure of the time value of money. The swap rate curve, or swap curve for short, is another important representation of the time value of money used in fixed-income markets. Here we will discuss how the swap curve is used in valuation, where the spread of swap rates over government benchmark rates is a proxy for perceived credit risk relative to risk-free debt.

4.1. Swap Rate Curve

Interest rate swaps are an integral part of the fixed-income market. These derivative contracts usually involve the net exchange, or swap, of fixed-rate for floating-rate interest payments, and these contracts are an essential tool for investors who use them to hedge, speculate on, or otherwise modify risk. The fixed and floating payments are determined by multiplying the respective rate by a principal (or notional) amount for each interest period over the swap maturity. The rate for the fixed leg of an interest rate swap is known as the **swap rate**. The swap rate is analogous to the YTM on a government bond, which as we saw earlier may be derived from zero rates using bootstrapping. The key difference between the swap rate and the government bond rate is that the swap rate is derived using short-term lending rates rather than default-risk-free rates. Swap floating rates historically referenced short-term survey-based interest rates, such as three- or six-month US dollar Libor (London Interbank Offered Rate) and are slated to transition to transaction-based market reference rates (MRR) based on secured overnight funding transactions. The yield curve of swap rates is called the **swap rate curve** or, more simply, the **swap curve**. Because it is based on so-called **par swaps**, in which the fixed rate is set so that no money is exchanged at contract initiation—the present values of the fixed-rate and benchmark floating-rate legs being equal—the swap curve is a type of par curve. When we refer to the "par curve" here, however, the reference is to the government par yield curve.

The swap market is a highly liquid market for two reasons. First, unlike bonds, a swap does not have multiple borrowers or lenders, only counterparties who exchange cash flows. Such arrangements offer significant flexibility and customization in the swap contract's design. Second, swaps provide one of the most efficient ways to hedge interest rate risk. The Bank for International Settlements (BIS) estimates that the notional amount outstanding on interest rate swaps was nearly $350 trillion as of June 2020.

Many countries do not have a liquid government bond market with maturities longer than one year. The swap curve is a necessary market benchmark for interest rates in these countries. In countries where the private sector is much bigger than the public sector, the swap curve is a far more relevant measure of the time value of money than is the government's cost of borrowing.

Swaps are frequently used as a benchmark in Europe, whereas in Asia, the swap markets and the government bond markets have developed in parallel, and both are used in valuation in credit and loan markets.

4.2. Why Do Market Participants Use Swap Rates When Valuing Bonds?

Government spot curves and swap rate curves are the chief reference curves in fixed-income valuation. The choice between them can depend on multiple factors, including the relative liquidity of these two markets. In the United States, where there is both an active Treasury security market and a swap market, the choice of a benchmark for the time value of money often depends on the interest rate exposure profile of the institution using the benchmark. On one hand, wholesale banks frequently use the swap curve to value assets and liabilities because they hedge their balance sheet with swaps. On the other hand, retail banks with little exposure to the swap market are more likely to use the government spot curve as their benchmark.

Let us illustrate how a financial institution uses the swap market for its internal operations. Consider the case of a bank raising funds using a certificate of deposit (CD).

Assume the bank can borrow \$10 million in the form of a CD that bears interest of 1.5% for a two-year term. Another \$10 million CD offers 1.70% for a three-year term. The bank can arrange two swaps: (1) The bank receives 1.50% fixed and pays MRR minus 10 bps with a two-year term and a notional amount of \$10 million, and (2) the bank receives 1.70% fixed and pays MRR minus 15 bps with a three-year term and a notional amount of \$10 million. After issuing the two CDs and committing to the two swaps, the bank has raised \$20 million with an annual funding cost for the first two years of MRR minus 12.5 bps applied to the total notional amount of \$20 million. The fixed interest payments received from the counterparty to the swap are paid to the CD investors; in effect, fixed-rate liabilities have been converted to floating-rate liabilities. The margins on the floating rates become the standard by which value is measured in assessing the bank's total funding cost.

By using the swap curve as a benchmark for the time value of money, the investor can adjust the swap spread so that the swap will be fairly priced given the spread. Conversely, given a swap spread, the investor can determine a fair price for the bond. We will use the swap spread in the following section to determine the value of a bond.

4.3. How Do Market Participants Use the Swap Curve in Valuation?

Although benchmark swap rates are quoted for specific maturities, swap contracts may be customized by two parties in the over-the-counter market. The fixed payment can be specified by an amortization schedule or involve a coupon with non-standard payment dates. In this section, we will focus on zero-coupon bonds. The yields on these bonds determine the swap curve, which, in turn, can be used to determine bond values.

Each forward date has an associated discount factor that represents the value today of a unit payment that one would hypothetically receive on the forward date expressed as a decimal fraction. For example, if we expect to receive ₩10,000 (10,000 South Korean won) in one year and the current price of the security is ₩9,259.30, then the discount factor for one year will be 0.92593 (= ₩9,259.30/₩10,000). Note that the rate associated with this discount factor is $1/0.92593 - 1 \approx 8.00\%$.

To price a swap using current market rates, as mentioned we must solve for a constant fixed rate that sets the present value of fixed-leg payments equal to the present value of floating-leg payments over the life of the swap. Once established, the fixed cash flows are specified by the coupon rate set at the time of the original agreement. Pricing the floating leg is more complex than pricing the fixed leg because, by definition, its cash flows change with future changes in interest rates. The forward rate for each floating payment date is calculated by using the forward curves.

Let s_T stand for the T-period swap rate. Because the value of a swap at origination is set to zero, the swap rates must satisfy Equation 13. Note that the swap rates can be determined from the spot rates and the spot rates can be determined from the swap rates.

$$\sum_{t=1}^{T} \frac{s_T}{(1+z_t)^t} + \frac{1}{(1+z_T)^T} = 1 \tag{12}$$

The right-hand side of Equation 12 is the value of the floating leg, which is always 1 at origination. The swap rate is determined by equating the value of the fixed leg, on the left-hand side, to the value of the floating leg.

Example 8 addresses the relationship between the swap rate curve and spot curve.

EXAMPLE 8 Determining the Swap Rate Curve

Suppose a government spot curve implies the following discount factors:

$$DF_1 = 0.9524$$
$$DF_2 = 0.8900$$
$$DF_3 = 0.8163$$
$$DF_4 = 0.7350$$

Given this information, determine the swap rate curve.

Solution: Recall from Equation 1 that $DF_N = \frac{1}{(1 + Z_N)^N}$. Therefore,

$$z_N = \left(\frac{1}{DF_N}\right)^{1/N} - 1$$
$$z_1 = \left(\frac{1}{0.9524}\right)^{1/1} - 1 = 5.00\%$$
$$z_2 = \left(\frac{1}{0.8900}\right)^{1/2} - 1 = 6.00\%$$
$$z_3 = \left(\frac{1}{0.8163}\right)^{1/3} - 1 = 7.00\%$$
$$z_4 = \left(\frac{1}{0.7350}\right)^{1/4} - 1 = 8.00\%$$

Using Equation 12, for $N = 1$,

$$\frac{s_1}{(1 + z_1)} + \frac{1}{(1 + z_1)} = \frac{s_1 + 1}{(1 + 0.05)} = 1$$

Therefore, $s_1 = 5\%$.

For $T = 2$,

$$\frac{s_2}{(1 + z_1)} + \frac{s_2}{(1 + z_2)^2} + \frac{1}{(1 + z_2)^2} = \frac{s_2}{(1 + 0.05)} + \frac{s_2 + 1}{(1 + 0.06)^2} = 1$$

Therefore, $s_2 = 5.97\%$.

For $T = 3$,

$$\frac{s_3}{(1 + z_1)} + \frac{s_3}{(1 + z_2)^2} + \frac{s_3}{(1 + z_3)^3} + \frac{1}{(1 + z_3)^3}$$
$$= \frac{s_3}{(1 + 0.05)} + \frac{s_3}{(1 + 0.06)^2} + \frac{s_3}{(1 + 0.07)^3} + \frac{1}{(1 + 0.07)^3} = 1$$

Therefore, $s_3 = 6.91\%$.

For $T = 4$,

$$\frac{s_4}{(1+z_1)} + \frac{s_4}{(1+z_2)^2} + \frac{s_4}{(1+z_3)^3} + \frac{s_4}{(1+z_4)^4} + \frac{1}{(1+z_4)^4}$$
$$= \frac{s_4}{(1+0.05)} + \frac{s_4}{(1+0.06)^2} + \frac{s_4}{(1+0.07)^3} + \frac{s_4}{(1+0.08)^4} + \frac{1}{(1+0.08)^4} = 1$$

Therefore, $s_4 = 7.81\%$.

Note that the swap rates, spot rates, and discount factors are all mathematically linked together. Having access to data for one of the series allows you to calculate the other two.

5. THE SWAP SPREAD AND SPREADS AS A PRICE QUOTATION CONVENTION

- **calculate and interpret the swap spread for a given maturity**
- **describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk**

The swap spread is a popular way to indicate credit spreads in a market. The **swap spread** is defined as the spread paid by the fixed-rate payer of an interest rate swap over the rate of the "on-the-run" (most recently issued) government security with the same maturity as the swap. The spread captures the yield premium required for credit relative to the benchmark government bond. Because swap rates are built from market rates for short-term risky debt, this spread is a barometer of the market's perceived credit risk relative to default-risk-free rates. This spread typically widens countercyclically, exhibiting greater values during recessions and lower values during economic expansions.

The term "swap spread" is sometimes also used as a reference to a bond's basis point spread over the interest rate swap curve and is a measure of the credit and/or liquidity risk of a bond. Here, a swap spread is an excess yield of swap rates over the yields on government bonds, and we use the terms I-spread, ISPRD, or interpolated spread to refer to bond yields net of the swap rates of the same maturities. In its simplest form, the I-spread can be measured as the difference between the yield-to-maturity of the bond and the swap rate given by a straight-line interpolation of the swap curve.

Often, fixed-income prices will be quoted as a swap rate plus (or minus) a spread, for which the yield is simply the yield on an equal-maturity government bond plus the swap spread. For example, if the fixed rate of a five-year fixed-for-float MRR swap is 2.00% and the five-year Treasury is yielding 1.70%, the swap spread is 2.00% − 1.70% = 0.30%, or 30 bps.

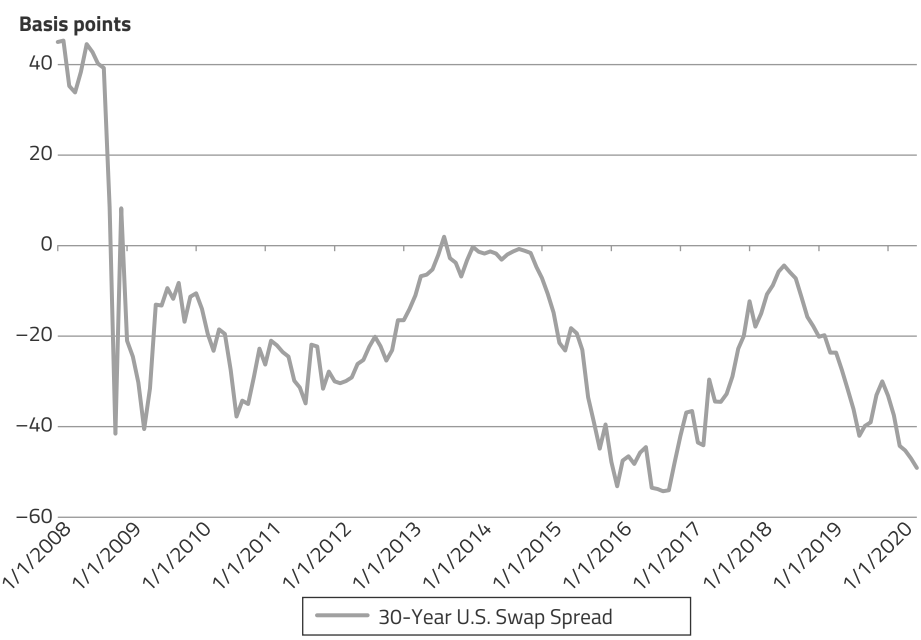
For euro-denominated swaps, the government yield used as a benchmark is most frequently Bunds (German government bonds) with the same maturity. Gilts (UK government bonds) are used as a benchmark in the United Kingdom.

Although the Libor swap curve is being phased out, it has historically been considered to reflect the default risk of A1/A+ rated commercial banks. The transition from Libor to MRR based on secured overnight funding rates will increase the influence of demand and supply conditions in government debt markets on swap rates. Another reason for the popularity of the swap market is that it is led by major financial institutions rather than controlled by governments, so swap rates are more comparable across different countries. The swap market also has more maturities with which to construct a yield curve than do government bond markets. Historically, cash or deposit rates such as Libor have been used for short-maturity yields; interest rate futures such as Eurodollar futures contracts have maturities of up to a year; and swap rates extend to maturities of up to 50 years in US dollars or euro. As the market transitions from Libor, the concept of this spread will be consistent with whichever market-based alternative to Libor emerges.

HISTORY OF THE US SWAP SPREAD SINCE 2008

The fact that governments generally pay less than private entities do in order to borrow suggests that swap spreads should always be positive. However, the 30-year Treasury swap spread turned negative following the collapse of Lehman Brothers Holdings Inc. in September 2008. Strong demand for duration combined with tighter liquidity and greater counterparty risk were widely cited as reasons for this phenomenon. For the period shown, the 30-year Treasury swap spread hit a record low (–62 bps intramonth) during November 2008. The 30-year Treasury swap spread was at or above zero for more than a year before becoming negative once again (see Exhibit 5). A recent study by the Federal Reserve Bank of New York (Boyarchenko, Gupta, Steele, and Yen, 2018) suggests that negative swap spreads have persisted because of increased regulatory capital requirements among swap dealers following the financial crisis.

EXHIBIT 5 US Swap Spread, January 2008–May 2020 (monthly data)



To illustrate the use of the swap spread in fixed-income pricing, consider a US$1 million investment in GE Capital (GECC) notes with a coupon rate of 1 5/8% (1.625%) that matures on 2 July 2024. Coupons are paid semiannually. The evaluation date is 12 July 2021, so the remaining maturity is 2.97 years [= 2 + (350/360)]. The Treasury rates for two-year and three-year maturities are 0.525% and 0.588%, respectively. By simple interpolation between these two rates, the US Treasury rate for 2.97 years is 0.586% [= 0.525% + (350/360)(0.588%

− 0.525%)]. If the swap spread for the same maturity is 0.918%, then the yield-to-maturity on the bond is 1.504% (= 0.918% + 0.586%). Given the yield-to-maturity, the invoice price (price including accrued interest) for US$1 million face value is as follows:

$$\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(1-\frac{10}{180}\right)}}+\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(2-\frac{10}{180}\right)}}+\cdots+$$

$$\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}}+\frac{1{,}000{,}000}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}}=\text{US\$1,003,954.12}$$

The left-hand side sums the present values of the semiannual coupon payments and the final principal payment of US$1,000,000. The accrued interest rate amount is US$451.39 [= 1,000,000 × (0.01625/2)(10/180)]. Therefore, the clean price (price not including accrued interest) is US$1,003,502.73 (= 1,003,954.12 – 451.39).

The swap spread helps an investor to identify the time value, credit, and liquidity components of a bond's YTM. If the bond is default free, then the swap spread could provide an indication of the bond's liquidity, or it could provide evidence of market mispricing. The higher the swap spread, the higher the return that investors require for credit and/or liquidity risks. Another approach introduced in an earlier chapter is to calculate a constant yield spread over a government (or interest rate swap) spot curve instead. This spread is known as the zero volatility spread (Z-spread) of a bond over the benchmark rate.

5.1. Spreads as a Price Quotation Convention

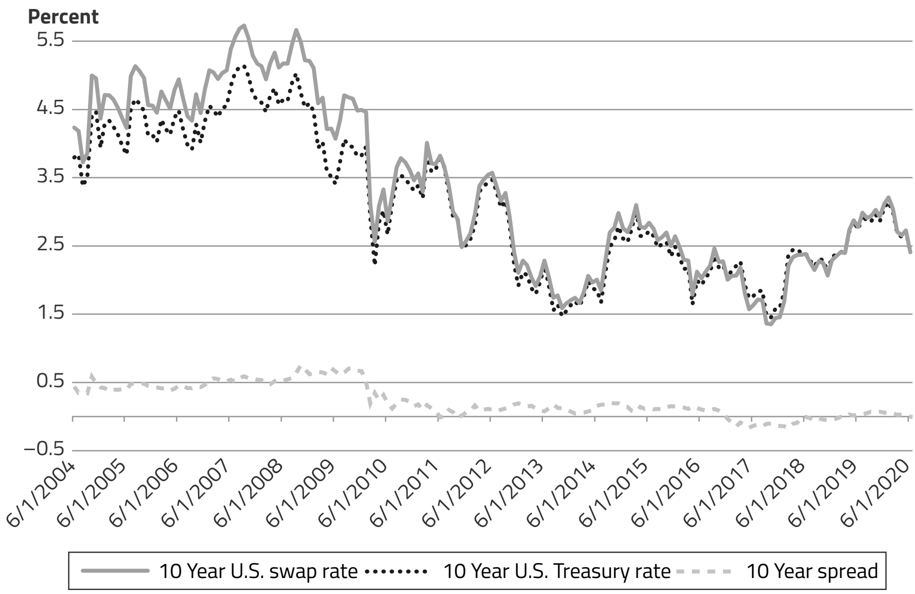
Treasury curves and swap curves represent different benchmarks for fixed-income valuation. It is therefore important to distinguish between a bond price quote that uses the bond yield net of a benchmark Treasury yield and one that uses a swap rate.

The Treasury rate can differ from the swap rate for the same term for several reasons. Unlike the cash flows from US Treasury bonds, the cash flows from swaps are subject to greater default risk. Market liquidity for specific maturities may differ. For example, some parts of the term structure of interest rates may be more actively traded with swaps than with Treasury bonds. Finally, arbitrage between these two markets cannot be perfectly executed.

Swap spreads to the Treasury rate (as opposed to **I-spreads**, which are bond rates net of the swap rates of the same maturities) are simply the differences between swap rates and government bond yields of a particular maturity. One problem in defining swap spreads is that, for example, a 10-year swap matures in exactly 10 years, whereas this condition is true for a 10-year government bond only at the time of issuance. By convention, therefore, the 10-year swap spread is defined as the difference between the 10-year swap rate and the 10-year on-the-run government bond. Swap spreads of other maturities are defined similarly.

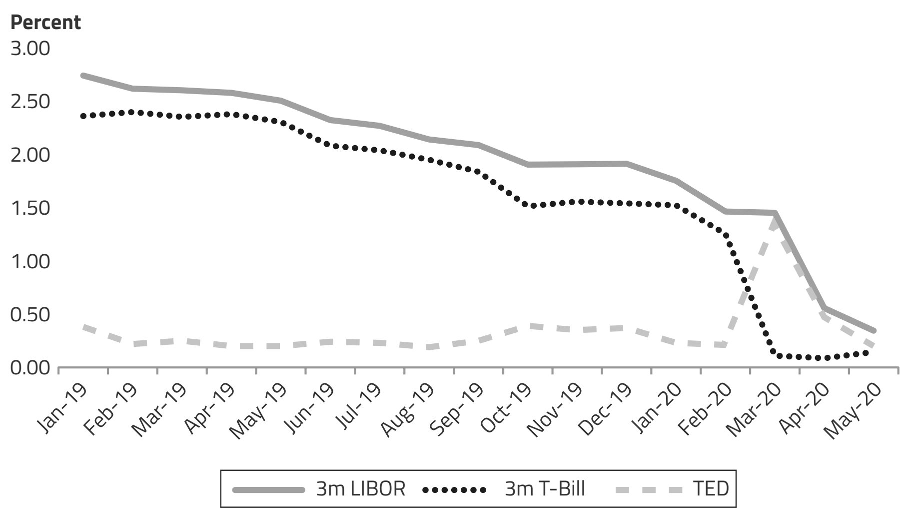
The curves in Exhibit 6 show the relationship between 10-year Treasury notes and 10-year swap rates. The 10-year swap spread is the 10-year swap rate less the 10-year Treasury note yield. Although positive swap spreads reflecting the difference between Libor-based rates and default-risk-free US government yields were historically the norm, these spreads have narrowed to zero or negative levels since the 2008 financial crisis because of higher swap dealer capital requirements and leverage constraints.

EXHIBIT 6 10-Year US Swap Rate vs. 10-Year US Treasury Rate



Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk. For example, the difference between Libor and the yield on a Treasury bill of the same maturity, or **TED spread**, has historically been a key indicator of perceived credit and liquidity risk. TED is an acronym formed from an abbreviation for the US T-bill (T) and the ticker symbol for the Libor-based Eurodollar futures contract (ED). Exhibit 7 shows the historical TED spread. An increase in the TED spread signals greater perceived credit and liquidity risk, as occurred in early 2020 amid market turmoil related to the COVID-19 pandemic.

EXHIBIT 7 TED Spread, January 2019–May 2020 (end-of-month data)



Another popular measure of such risk is the **Libor–OIS spread**, which is the difference between Libor and the **overnight indexed swap (OIS) rate**. An OIS is an interest rate swap in which the periodic floating rate of the swap equals the geometric average of a daily unsecured overnight rate (or overnight index rate). The index rate is typically the rate for overnight unsecured lending between banks, such as the federal funds rate for US dollars or Eonia (Euro OverNight Index Average) for euros. As market participants transition away from survey-based Libor to alternative benchmarks based on actual transaction data, the **secured overnight financing rate (SOFR)**, or overnight cash borrowing rate collateralized by US Treasuries, has gained prominence and is expected to replace Libor in the future. A barometer of the US Treasury repurchase (or repo) market, SOFR is a daily volume-weighted index of all qualified repo market transactions and is influenced by supply and demand conditions in secured funding markets. The shift to overnight secured funding benchmarks extends globally—for example, the secured European Short-Term Rate (ESTR) has been recommended to replace Eonia, and the Canadian Overnight Repo Rate Average (CORRA) is proposed to replace the survey-based unsecured Canadian Dollar Offered Rate (CDOR).

6. TRADITIONAL THEORIES OF THE TERM STRUCTURE OF INTEREST RATES

- **explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve**

This section presents four traditional theories of the underlying economic factors that affect the shape of the yield curve.

6.1. Expectations Theory

One branch of traditional term structure theory focuses on interpreting term structure shape in terms of investors' expectations. Historically, the first such theory is known as the **unbiased expectations theory**, also called **pure expectations theory**. It says that the forward rate is an unbiased predictor of the future spot rate; its broadest interpretation is that bonds of any maturity are perfect substitutes for one another. For example, buying a bond with a maturity of five years and holding it for three years has the same expected return as buying a three-year bond or buying a series of three one-year bonds.

The predictions of the unbiased expectations theory are consistent with the assumption of risk neutrality. In a risk-neutral world, investors are unaffected by uncertainty and risk premiums do not exist. Every security is risk free and yields the risk-free rate for that particular maturity. Although such an assumption leads to interesting results, it clearly is in conflict with the large body of evidence showing that investors are risk averse.

A theory that is similar but more rigorous than the unbiased expectations theory is the **local expectations theory**. Rather than asserting that every maturity strategy has the same expected return over a given investment horizon, this theory instead contends that the expected return for every bond over short periods is the risk-free rate. This conclusion results from an assumed no-arbitrage condition in which bond pricing does not allow for traders to earn arbitrage profits.

The primary way that the local expectations theory differs from the unbiased expectations theory is that it can be extended to a world characterized by risk. Although the theory requires that risk premiums be nonexistent for very short holding periods, no such restrictions are placed on longer-term investments. Thus, the theory is applicable to both risk-free as well as risky bonds.

Although the local expectations theory is economically appealing, it is often observed that short-holding-period returns on long-dated bonds in fact exceed those on short-dated bonds. The need for liquidity and the ability to hedge risk essentially ensure that the demand for short-term securities will exceed that for long-term securities. Thus, both the yields and the actual returns for short-dated securities are typically lower than those for long-dated securities.

6.2. Liquidity Preference Theory

Whereas expectations theories leave no room for risk aversion, liquidity preference theory attempts to account for it. **Liquidity preference theory** asserts that **liquidity premiums** exist to compensate investors for the added interest rate risk they face when lending long term and that these premiums increase with maturity. Thus, given an expectation of unchanging short-term spot rates, liquidity preference theory predicts an upward-sloping yield curve. The forward rate provides an estimate of the expected spot rate that is biased upward by the amount of the liquidity premium, which invalidates the unbiased expectations theory. The liquidity premium for each consecutive future period should be no smaller than that for the prior period.

For example, the US Treasury offers bonds that mature in 30 years. Most investors, however, have shorter investment horizons than 30 years. For investors to hold these bonds, they would demand a higher return for taking the risk that the yield curve changes and that they must sell the bond prior to maturity at an uncertain price. That incrementally higher return is the liquidity premium. Note that this premium is not to be confused with a yield premium for the lack of liquidity that thinly traded bonds may bear. Rather, it is a premium applying to all long-term bonds, including those with deep markets.

Liquidity preference theory fails to offer a complete explanation of the term structure. Rather, it simply argues for the existence of liquidity premiums. For example, a downward-sloping yield curve could still be consistent with the existence of liquidity premiums if one of the factors underlying the shape of the curve is an expectation of deflation (i.e., a negative rate of inflation resulting from monetary or fiscal policy actions). Expectations of sharply declining spot rates may also result in a downward-sloping yield curve if the expected decline in interest rates is severe enough to offset the effect of the liquidity premiums.

In summary, liquidity preference theory claims that lenders require a liquidity premium as an incentive to lend long term. Thus, forward rates derived from the current yield curve provide an upwardly biased estimate of expected future spot rates. Although downward-sloping or hump-shaped yield curves may sometimes occur, the existence of liquidity premiums implies that the yield curve will typically be upward sloping.

6.3. Segmented Markets Theory

Unlike expectations theory and liquidity preference theory, **segmented markets theory** allows for lender and borrower preferences to influence the shape of the yield curve. The result is that yields are not a reflection of expected spot rates or liquidity premiums. Rather, they are solely a function of the supply and demand for funds of a particular maturity. That is, each maturity sector can be thought of as a segmented market in which yield is determined independently from the yields that prevail in other maturity segments.

The theory is consistent with a world in which asset/liability management constraints exist, either regulatory or self-imposed. In such a world, investors might restrict their investment activity to a maturity sector that provides the best match for the maturity of their liabilities. Doing so avoids the risks associated with an asset/liability mismatch.

For example, because life insurers sell long-term liabilities against themselves in the form of life insurance contracts, they tend to be most active as buyers in the long end of the bond market. Similarly, because the liabilities of pension plans are long term, they typically invest in long-term securities. Why would they invest short term given that those returns might decline while the cost of their liabilities stays fixed? In contrast, money market funds would be limited to investing in debt with maturity of one year or less, in general.

In summary, the segmented markets theory assumes that market participants are either unwilling or unable to invest in anything other than securities of their preferred maturity. It follows that the yield of securities of a particular maturity is determined entirely by the supply and demand for funds of that particular maturity.

6.4. Preferred Habitat Theory

The **preferred habitat theory** is similar to the segmented markets theory in proposing that many borrowers and lenders have strong preferences for particular maturities, but it does not assert that yields at different maturities are determined independently of each other.

The theory contends, however, that if the expected additional returns to be gained become large enough, institutions will be willing to deviate from their preferred maturities or habitats. For example, if the expected returns on longer-term securities exceed those on short-term securities by a large enough margin, an intermediate-term bond fund might lengthen the maturities of their assets. And if the excess returns expected from buying short-term securities become large enough, life insurance companies might stop limiting themselves to long-term securities and place a larger part of their portfolios in shorter-term investments.

The preferred habitat theory is based on the realistic notion that agents and institutions will accept additional risk in return for additional expected returns. In accepting elements of both the segmented markets theory and the unbiased expectations theory, yet rejecting their extreme polar positions, the preferred habitat theory moves closer to explaining real-world phenomena. In this theory, both market expectations and the institutional factors emphasized in the segmented markets theory influence the term structure of interest rates.

PREFERRED HABITAT AND QE

The term "quantitative easing" (QE) refers to an unconventional monetary policy used by central banks to increase the supply of money in an economy when central bank and/or interbank interest rates are already close to zero. The first of several QE efforts by the US Federal Reserve began in late 2008, following the establishment of a near-zero target range for the federal funds rate. Since then, the Federal Reserve has greatly expanded its holdings of long-term securities via a series of asset purchase programs, with the goal of putting downward pressure on long-term interest rates and thereby making financial conditions even more accommodative. Exhibit 8 presents information regarding the securities held by the Federal Reserve on 20 September 2007 (when all securities held by the Fed were US Treasury issuance) and on 29 October 2014 (when the Federal Reserve ended its third round of QE).

EXHIBIT 8 Securities Held by the US Federal Reserve

(US$ billions)	20 Sep. 2007	29 Oct 2014
Securities held outright	780	4,219
US Treasury	780	2,462
Bills	267	0
Notes and bonds, nominal	472	2,347
Notes and bonds, inflation indexed	36	115
Inflation compensation	5	16
Federal agency	0	40
Mortgage-backed securities	0	1,718

As Exhibit 8 shows, the Federal Reserve's security holdings on 20 September 2007 consisted entirely of US Treasury securities, and about 34% of those holdings were short term in the form of T-bills. On 29 October 2014, only about 58% of the Federal Reserve's security holdings were Treasury securities, and none were T-bills. Furthermore, the Federal Reserve held well over US$1.7 trillion of mortgage-backed securities (MBS), which accounted for 41% of all securities held.

Prior to the QE efforts, the yield on MBS was typically in the 5%–6% range. It declined to less than 2% by the end of 2012. Concepts related to preferred habitat theory could possibly help explain that drop in yield.

The purchase of MBS by the Federal Reserve reduced the supply of these securities that was available for private purchase. Assuming that many MBS investors are either unwilling or unable to withdraw from the MBS market because of their comparative experience and expertise in managing interest rate and repayment risks of MBS versus option-free bonds, MBS investing institutions would have a "preferred habitat" in the MBS market. If they were unable to meet investor demand without bidding more aggressively, these buyers would drive down yields on MBS.

The Federal Reserve's purchase of MBS also resulted in a reduction in MBS yields. If a homeowner prepays on a mortgage, the payment is sent to MBS investors on a pro rata basis. Although investors are uncertain about when such a prepayment will be received, prepayment is more likely in a declining interest rate environment.

Use Example 9 to test your understanding of traditional term structure theories.

EXAMPLE 9 Traditional Term Structure Theories

1. Many fixed-income portfolio managers are limited in or prohibited from high-yield bond investments. When a bond is downgraded from an investment-grade to a high-yield (junk) rating, it is referred to as a *fallen angel*. Because of restrictions, many pension funds sell fallen angels when they are downgraded from investment grade to high yield (junk). This coordinated selling action often results in depressed

prices and attractive yields for the fallen angels. Which of the following reasons best explains why fallen angel yields often exceed otherwise identical bonds?
   A. The preferred habitat theory
   B. The segmented markets theory
   C. The local expectations theory
2. The term structure theory in which investors can be induced by relatively attractive yields to hold debt securities whose maturities do not match their investment horizon is *best* described as the:
   A. preferred habitat theory.
   B. segmented markets theory.
   C. unbiased expectations theory.
3. The unbiased expectations theory assumes investors are:
   A. risk averse.
   B. risk neutral.
   C. risk seeking.
4. Market evidence shows that forward rates are:
   A. unbiased predictors of future spot rates.
   B. upwardly biased predictors of future spot rates.
   C. downwardly biased predictors of future spot rates.
5. Market evidence shows that short holding-period returns on short-maturity bonds *most* often are:
   A. less than those on long-maturity bonds.
   B. about equal to those on long-maturity bonds.
   C. greater than those on long-maturity bonds.

Solution to 1: B is correct. Market segmentation in this example results from the requirement that some fixed-income fund managers are prohibited or limited in their capacity to hold high-yield bonds. The segmentation results in selling pressure on fallen angels that depresses their prices.

Solution to 2: A is correct. Preferred habitat theory asserts that investors are willing to deviate from their preferred maturities if yield differentials encourage the switch. Segmented markets theory is more rigid than preferred habitat in that asset/liability management constraints force investors to buy securities whose horizons match those of their liabilities. The unbiased expectations theory makes no assumptions about maturity preferences. Rather, it contends that forward rates are unbiased predictors of future spot rates.

Solution to 3: B is correct. The unbiased expectations theory asserts that different maturity strategies, such as rollover, maturity matching, and riding the yield curve, have the same expected return. By definition, a risk-neutral party is indifferent about choices with equal expected payoffs, even if one choice is riskier. Thus, the predictions of the theory are consistent with the existence of risk-neutral investors.

Solution to 4: B is correct. The existence of a liquidity premium ensures that the forward rate is an upwardly biased estimate of the future spot rate. Market evidence clearly shows that liquidity premiums exist, and this evidence effectively refutes the predictions of the unbiased expectations theory.

Solution to 5: A is correct. Although the local expectations theory predicts that the short-run return for all bonds will equal the risk-free rate, most of the evidence refutes that claim. Returns from long-dated bonds are generally higher than those from short-dated bonds, even over relatively short investment horizons. This market evidence is consistent with the risk–expected return trade-off that is central to finance and the uncertainty surrounding future spot rates.

7. YIELD CURVE FACTOR MODELS

- **explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks**

The effect of yield volatilities on price is an important consideration in fixed-income investment, particularly for risk management and portfolio evaluation. In this section, we describe measuring and managing the interest rate risk of bonds.

7.1. A Bond's Exposure to Yield Curve Movement

Shaping risk is defined as the sensitivity of a bond's price to the changing shape of the yield curve. The yield curve's shape changes continually, and yield curve shifts are rarely parallel. For active bond management, a bond investor may want to base trades on a forecasted yield curve shape or may want to hedge the yield curve risk on a bond portfolio using swaps. Shaping risk also affects the value of many options, which is very important because many fixed-income instruments have embedded options.

Exhibits 9 and 10 show historical yield curve movements for US and European swap rates from March 2006 until March 2020. The exhibits show the considerable swap yield curve changes over time. In both cases, the pre-financial-crisis March 2006 yield curves represent the highest swap yields and those from March 2020 (amid the COVID-19 pandemic-related market turmoil) the lowest. In the United States, however, the end of quantitative easing and tighter monetary policy resulted in a rebound in swap yields prior to 2020, whereas in Europe, yields remained low or negative because of continued accommodative monetary policy. Note that the vertical axis values of the three exhibits differ, and the horizontal axis is not to scale.

7.2. Factors Affecting the Shape of the Yield Curve

The previous section showed that the yield curve can take nearly any shape. The challenge for a fixed-income manager is to implement a process to manage the yield curve shape risk in her portfolio. One approach is to find a model that reduces most of the possible yield curve movements to a probabilistic combination of a few standardized yield curve movements. This section presents one of the best-known yield curve factor models.

A **yield curve factor model** is defined as a model or a description of yield curve movements that can be considered realistic when compared with historical data. Research has led to models that can describe these movements with some accuracy. One specific yield curve factor model is the three-factor model of Litterman and Scheinkman (1991), who found that yield curve movements are historically well described by a combination of three independent movements, which

EXHIBIT 9 Historical US Swap Yield Curve Movements

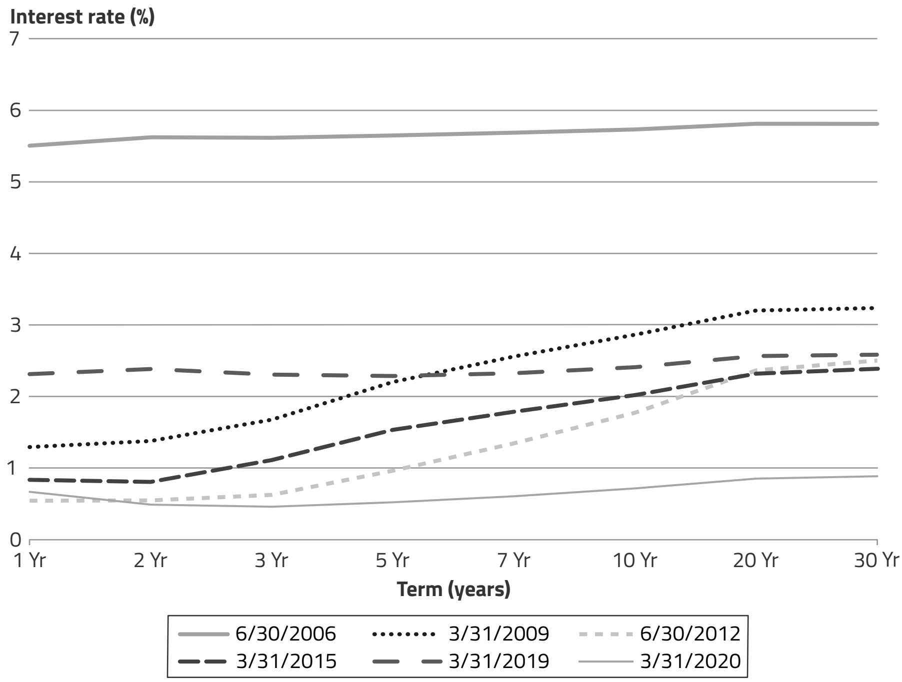
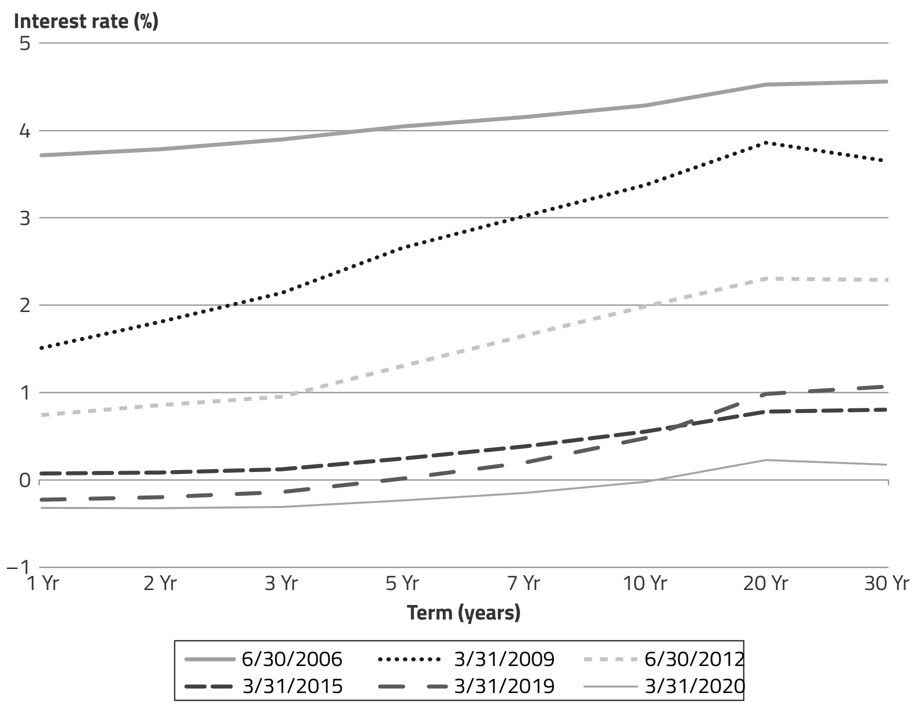
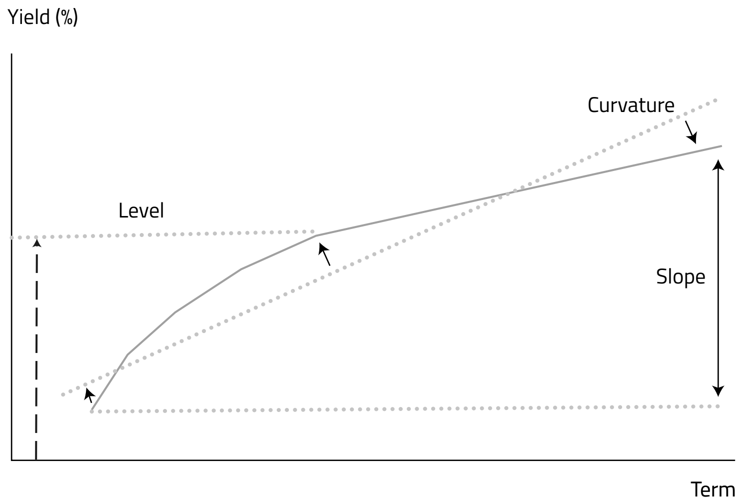


EXHIBIT 10 Historical European Swap Yield Curve Movements



they interpreted as **level**, **steepness**, and **curvature**. The level movement refers to an upward or downward shift in the yield curve. The steepness movement refers to a non-parallel shift in the yield curve when either short-term rates change more than long-term rates or long-term rates change more than short-term rates. The curvature movement is a reference to movement in three segments of the yield curve: The short-term and long-term segments rise while the middle-term segment falls, or vice versa. Exhibit 11 illustrates these factors.

EXHIBIT 11 Primary Yield Curve Factors: Level, Slope, and Curvature



In practice, the level movement factor explains most of the total changes in swap and bond market yields. This factor may be interpreted as a reflection of parallel yield curve moves in which rates move in the same direction and by a similar order of magnitude. The steepness factor addresses the shape of the curve, with short-term yields typically moving more than long-term yields. These changes take place over time and therefore explain less of the total variance in rates than the level factor. Finally, the third factor, curvature, tends to have a negative impact on intermediate yields and a positive impact on short- and long-term yields. This variable explaining the "twist" in the yield curve has the smallest impact of the three.

8. THE MATURITY STRUCTURE OF YIELD CURVE VOLATILITIES AND MANAGING YIELD CURVE RISKS

- **explain the maturity structure of yield volatilities and their effect on price volatility**

8.1. Yield Volatility

Quantifying interest rate volatilities is important for fixed income managers for at least two reasons. First, most fixed-income instruments and derivatives have embedded options. Option values, and hence the values of the fixed-income instrument, crucially depend on the level of interest rate volatilities. Second, fixed-income interest rate risk management is clearly an important part of any management process, and such risk management includes controlling the impact of interest rate volatilities on the instrument's price volatility.

The term structure of interest rate volatilities is a representation of the yield volatility of a zero-coupon bond for every maturity of security. This volatility curve (or "vol") or volatility term structure measures yield curve risk.

Interest rate volatility is not the same for all interest rates along the yield curve. On the basis of the typical assumption of a lognormal model, the uncertainty of an interest rate is measured by the annualized standard deviation of the proportional change in a bond yield over a specified interval. For example, if the interval is a one-month period, then the specified interval equals 1/12 years. This measure, called interest rate volatility, is denoted $\sigma(t, T)$, which is the volatility of the rate for a security with maturity T at time t. The term structure of volatilities is given by Equation 13:

$$\sigma(t,T) = \frac{\sigma[\Delta r(t,T)/r(t,T)]}{\sqrt{\Delta t}} \tag{13}$$

In Exhibit 12, to illustrate a term structure of volatility, the data series is deliberately chosen to end before the 2008 financial crisis, which was associated with some unusual volatility magnitudes.

EXHIBIT 12 Historical Example: US Treasuries, August 2005–December 2007

Maturity (years)	0.25	0.50	1	2	3	5	7	10	20	30
$\sigma(t,T)$	0.3515	0.3173	0.2964	0.2713	0.2577	0.2154	0.1885	0.1621	0.1332	0.1169

For example, the 35.15% standard deviation for the three-month T-bill in Exhibit 12 is based on a monthly standard deviation of 0.1015 = 10.15%, which annualizes as

$$0.1015 \div \sqrt{\frac{1}{12}} = 0.3515 = 35.15\%$$

The volatility term structure typically shows that short-term rates are more volatile than long-term rates. That said, long-term bond *prices* tend to vary more than short-term bond prices given the impact of duration. Research indicates that short-term volatility is most strongly linked to uncertainty regarding monetary policy, whereas long-term volatility is most strongly linked to uncertainty regarding the real economy and inflation. Furthermore, most of the co-movement between short-term and long-term volatilities appears to depend on the ever-changing correlations among these three determinants (monetary policy, the real economy, and inflation). During the period of August 2005–December 2007, long-term volatility was lower than short-term volatility, falling from 35.15% for the 0.25-year rate to 11.69% for the 30-year rate.

8.2. Managing Yield Curve Risks Using Key Rate Duration

Yield curve risk—the risk to portfolio value arising from unanticipated changes in the yield curve—can be managed on the basis of several measures of sensitivity to yield curve movements. Management of yield curve risk involves changing the identified exposures to desired values by trades in security or derivative markets (the details fall under the rubric of fixed-income portfolio management and thus are outside the scope of this chapter).

One available measure of yield curve sensitivity is effective duration, which measures the sensitivity of a bond's price to a small parallel shift in a benchmark yield curve. Another is based on **key rate duration**, which measures a bond's sensitivity to a small change in a benchmark yield curve at a specific maturity segment. A further measure can be developed on the basis of the factor model developed in Section 6.3. Using one of these last two measures allows identification and management of "shaping risk"—that is, sensitivity to changes in the shape of the benchmark yield curve—in addition to the risk associated with parallel yield curve changes, which is addressed adequately by effective duration.

To make the discussion more concrete, consider a portfolio of 1-year, 5-year, and 10-year zero-coupon bonds with \$100 value in each position; total portfolio value is therefore \$300. Also consider the hypothetical set of factor movements shown in the following table:

Year	1	5	10
Parallel	1	1	1
Steepness	−1	0	1
Curvature	1	0	1

In the table, a parallel movement or shift means that all the rates shift by an equal amount—in this case, by a unit of 1. A steepness movement means that the yield curve steepens with the long rate shifting up by one unit and the short rate shifting down by one unit. A curvature movement means that both the short rate and the long rate shift up by one unit, whereas the medium-term rate remains unchanged. These movements need to be defined, as they are here, such that none of the movements can be a linear combination of the other two movements. Next, we address the calculation of the various yield curve sensitivity measures.

Because the bonds are zero-coupon bonds, each bond's effective duration is the same as its maturity. The portfolio's effective duration is the weighted sum of the effective duration of each bond position; for this equally weighted portfolio, effective duration is 0.333(1 + 5 + 10) = 5.333.

To calculate key rate durations, consider various yield curve movements. First, suppose that the one-year rate changes by 100 bps while the other rates remain the same; the sensitivity of the portfolio to that shift is 1/[(300)(0.01)] = 0.3333. We conclude that the key rate duration of the portfolio ($KeyDur_{Full}$) to the one-year rate, denoted $KeyDur_1$, is 0.3333. Likewise, the key rate durations of the portfolio to the 5-year rate, $KeyDur_5$, and the 10-year rate, $KeyDur_{10}$, are 1.6667 and 3.3333, respectively. Note that the sum of the key rate durations is 5.333, which is the same as the effective duration of the portfolio. This fact can be explained intuitively. Key rate duration measures the portfolio risk exposure to each key rate. If all the key rates move by the same amount, then the yield curve has made a parallel shift, and as a result, the proportional change in value has to be consistent with effective duration. The related model for yield curve risk based on key rate durations (*KeyDur*) is as follows:

$$\begin{aligned} KeyDur_{Full} = \%\Delta P = \left(\frac{\Delta P}{P}\right) &\approx -KeyDur_1 \Delta z_1 - KeyDur_5 \Delta z_5 - KeyDur_{10} \Delta z_{10} \\ &= -0.3333\Delta z_1 - 1.6667\Delta z_5 - -3.3333\Delta z_{10} \end{aligned} \quad (14)$$

Next, we can calculate a measure based on the decomposition of yield curve movements into parallel, steepness, and curvature movements, as described earlier. Define D_L, D_S, and D_C as the sensitivities of portfolio value to small changes in the level, steepness, and curvature factors, respectively. Based on this factor model, Equation 15 shows the proportional change in portfolio value that would result from a small change in the level factor (Δx_L), the steepness factor (Δx_S), and the curvature factor (Δx_C).

$$KeyDur_{Full} = \%\Delta P = \left(\frac{\Delta P}{P}\right) \approx -KeyDur_L \Delta x_L - KeyDur_S \Delta z_S - KeyDur_C \Delta z_C \quad (15)$$

Because $KeyDur_L$ is by definition sensitivity to a parallel shift, the proportional change in the portfolio value per unit shift (the line for a parallel movement in the table) is 5.3333 = (1 + 5 + 10)/[(300)(0.01)]. The sensitivity for steepness movement can be calculated as follows (see the line for steepness movement in the table). When the steepness makes an upward shift of 100 bps, it would result in a downward shift of 100 bps for the 1-year rate, resulting in a gain of $1, and an upward shift for the 10-year rate, resulting in a loss of $10. The change in value is therefore (1 – 10). $KeyDur_S$ is the negative of the proportional change in price per unit change in this movement and in this case is 3.0 = −(1 – 10)/[(300)(0.01)]. Considering the line for curvature movement in the table, $KeyDur_C$ = 3.6667 = (1 + 10)/[(300)(0.01)]. Thus, for our hypothetical bond portfolio, we can analyze the portfolio's yield curve risk using the following equation:

$$KeyDur_{Full} = \%\Delta P = \left(\frac{\Delta P}{P}\right) \approx -5.3333\Delta x_L - 3.0\Delta z_S - 3.6667\Delta z_C$$

For example, if $\Delta x_L = -0.0050$, $\Delta x_S = 0.002$, and $\Delta x_C = 0.001$, the predicted change in portfolio value would be +1.7%. It can be shown that key rate durations are directly related to level, steepness, and curvature in this example and that one set of sensitivities can be derived from the other. One can use the numerical example to verify that relation by decomposing changes in the term structure into level, slope, and curvature factors:

$$KeyDur_L = KeyDur_1 + KeyDur_5 + KeyDur_{10}$$

$$KeyDur_S = -KeyDur_1 + KeyDur_{10}$$

$$KeyDur_C = KeyDur_1 + KeyDur_{10}$$

Example 10 reviews concepts from this section and the preceding sections.

EXAMPLE 10 Term Structure Dynamics

1. The most important factor in explaining changes in the yield curve has been found to be:
   A. level.
   B. curvature.
   C. steepness.
2. A movement of the yield curve in which the short rate decreases by 150 bps and the long rate decreases by 50 bps would *best* be described as a:
   A. flattening of the yield curve resulting from changes in level and steepness.
   B. steepening of the yield curve resulting from changes in level and steepness.
   C. steepening of the yield curve resulting from changes in steepness and curvature.

3. The yield curve starts off flat, and then intermediate-maturity yields decrease by 10 bps while short- and long-maturity yields remain constant. This movement is *best* described as involving a change in:
   A. level only.
   B. curvature only.
   C. level and curvature.
4. Typically, short-term interest rates:
   A. are less volatile than long-term interest rates.
   B. are more volatile than long-term interest rates.
   C. have about the same volatility as long-term interest rates.
5. Suppose for a given portfolio that key rate changes are considered to be changes in the yield on 1-year, 5-year, and 10-year securities. Estimated key rate durations are $KeyDur_1 = 0.50$, $KeyDur_2 = 0.70$, and $KeyDur_3 = 0.90$. What is the percentage change in the value of the portfolio if a parallel shift in the yield curve results in all yields declining by 50 bps?
   A. −1.05%.
   B. +1.05%.
   C. +2.10%.

Solution to 1: A is correct. Research shows that upward and downward shifts in the yield curve explain more than 75% of the total change in the yield curve.

Solution to 2: B is correct. Both the short-term and long-term rates have declined, indicating a change in the level of the yield curve. Short-term rates have declined more than long-term rates, indicating a change in the steepness of the yield curve.

Solution to 3: B is correct. The curve starts off flat, with identical short, intermediate, and long rates. Both the short-term and long-term rates remained constant, indicating no change in the level of the yield curve. Intermediate rates decreased, however, resulting in curvature.

Solution to 4: B is correct. A possible explanation is that expectations for long-term inflation and real economic activity affecting longer-term interest rates are slower to change than those related to shorter-term interest rates.

Solution to 5: B is correct. A decline in interest rates would lead to an increase in bond portfolio value: $-0.50(-0.005) - 0.70(-0.005) - 0.90(-0.005) = 0.0105 = 1.05\%$.

9. DEVELOPING INTEREST RATE VIEWS USING MACROECONOMIC VARIABLES

- **explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes**

Interest rate dynamics such as changes in spot versus forward rates and the level, steepness, and curvature of the yield curve are influenced by key economic variables and market events. Implied forward rates serve as market-neutral reference points for fixed income traders. As we illustrated earlier, if today's forward rates are realized in the future, then bond values will simply roll down the yield curve. In practice, active fixed-income market participants establish their own views on future interest rate developments and then position their portfolios in order to capitalize on differences between their own rate view and the market consensus. If their forecast is accurate, the portfolio generates greater returns than it would have otherwise.

This section reviews the key drivers of interest rates before moving on to establishing views and positioning fixed-income portfolios to capitalize on a specific interest rate view.

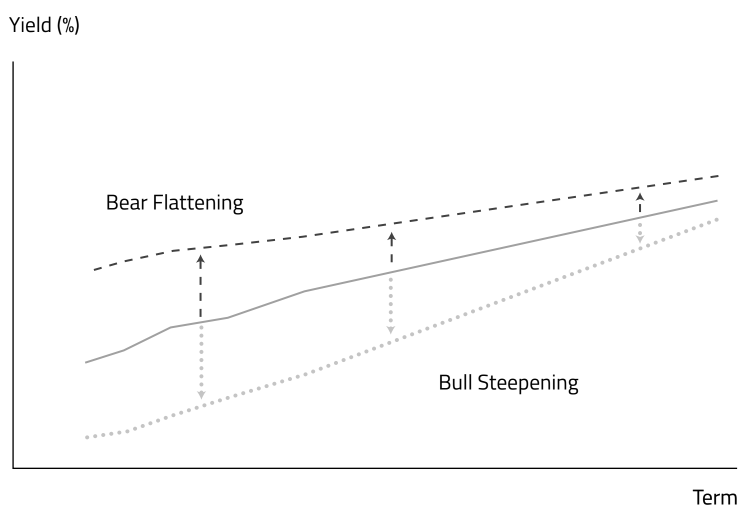
The term **bond risk premium** refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate. This premium is also referred to as the term (or duration) premium, and it is usually measured using government bonds to capture uncertainty of default-free rates, whereas credit, liquidity, and other risks may increase the overall risk premium for a specific bond. Unlike *ex post* observed historical returns, the bond risk premium is a forward-looking expectation and must be estimated.

Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.

Research shows that although inflation, GDP, and monetary policy explain most of the variance of bond yields, short- and intermediate-term bond yields are driven mostly by inflation, whereas other factors such as monetary policy are key drivers of long-term yields. Inflation explains about two-thirds of short- and intermediate-term bond yield variation, with the remaining third roughly equally attributable to economic growth and factors including monetary policy. In contrast, monetary policy explains nearly two-thirds of long-term yield variation, and the remaining third is largely attributable to inflation.

Monetary policy impacts the bond risk premium. Central banks such as the European Central Bank control the money supply and influence interest rates through policy tools in order to achieve stable prices and sustainable economic growth. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is often consistent with **bearish flattening**, or short-term bond yields rising more than long-term bond yields, resulting in a flatter yield curve. During economic recessions or anticipated recessions, the monetary authority cuts benchmark rates to help stimulate economic activity. The lowering of interest rates is associated with **bullish steepening**, in which short-term rates fall by more than long-term yields, resulting in a steeper term structure. These monetary policy actions lead to procyclical short-term interest rate changes. Exhibit 13 shows these two yield curve changes.

EXHIBIT 13 Examples of Yield Curve Flattening and Steepening



In recent years, central banks have increasingly used their balance sheets for large-sale asset purchases. For example, the Federal Reserve has bought large quantities of US Treasury bonds and mortgage-backed securities. The intended purpose is to stimulate economic activity by increasing the money supply through benchmark bond purchases and driving down the bond risk premium, encouraging capital allocation to incrementally higher-risk assets. Asset purchases impact the term structure by raising demand in a range of maturity segments.

Other factors that influence bond prices, yields, and the bond risk premium include fiscal policy, the maturity structure of debt, and investor demand.

Benchmark government bonds are the means by which nations fund their cumulative (current and past) budget deficits. Greater deficits require more borrowing, which influences both bond supply and required yield. Thus, fiscal supply-side effects affect bond prices and yields by increasing (decreasing) yields when budget deficits rise (fall). In the late 1990s, market participants believed the US government would run fiscal surpluses, leading to a reduction in government bond supply as the Treasury stopped issuing new 30-year bonds for four years. The expected reduction in supply drove long-maturity Treasury yields lower.

Longer government debt maturity structures predict greater excess bond returns. This is effectively a segmented market factor, wherein the greater supply of bonds of long-term maturity increases the yield in that market segment.

Domestic investor demand is a key driver of bond prices, especially among pension funds and insurance companies that use long-dated government bonds to match expected future liabilities. Greater domestic investor demand increases prices and reduces the bond risk premium.

Non-domestic investor demand influences government bond prices and may result either from holding reserves or from actions associated with currency exchange rate management. Non-domestic flows significantly influence bond prices because inflows (outflows) bid up (down) bond prices, lowering (raising) the bond risk premium.

During highly uncertain market periods, investors flock to government bonds in what is termed a **flight to quality**. This term refers to investors' selling off higher-risk asset classes such as stocks and commodities in favor of default-risk-free government bonds. A flight to quality is often associated with **bullish flattening**, in which the yield curve flattens as long-term rates fall by more than short-term rates.

Fixed-income trades based on interest rate forecasts can take a variety of forms, often using bond futures contracts to avoid significant portfolio turnover. Remember that any interest rate view must be evaluated relative to the current short rate and forward curve, because they reflect returns earned by investors rolling down the curve under the current set of implied forward rates.

Investors expecting interest rates to fall will generally extend portfolio duration relative to a benchmark to take advantage of bond price increases from falling rates, whereas investors expecting higher rates will shorten portfolio duration to reduce exposure to falling bond prices.

To capitalize on a steeper curve under which long-term rates rise relative to short-term rates, traders will short long-term bonds and purchase short-term bonds. If on the other hand a trader forecasts curve flattening, whereby short-term rates rise relative to long-term rates, she may capitalize on this trend by purchasing long-term bonds and selling short-term bonds short. In both the expected steepening and flattening trades, the position may be designed as duration neutral in order to insulate from changes in the level of the term structure. Fixed-income investors with long-only investment mandates may alternate between portfolios concentrated in a single maturity, known as a **bullet portfolio**, and those with similar duration that combine short and long maturities, known as a **barbell portfolio**. For example, an investor may seek to capitalize on an expected bullish flattening of the yield curve by shifting from a bullet to a barbell position.

EXAMPLE 11 Building a Rate View Based on Economic Forecasts and Monetary Policy

Morgan Salaz is a fixed income analyst responsible for advising fixed income clients about bond trading opportunities. In the current recessionary environment, the level of government bond yields is low and the term structure is nearly flat. Salaz's firm forecasts that after a brief recession, economic growth will return quickly during the coming 12 months.

1. Which of the following changes to the yield curve is consistent with Salaz's expectation of increasing economic growth over the coming year?
   A. Decrease in the level
   B. Decrease in the term spread of long-term rates over short-term rates
   C. Increase in the term spread of long-term rates over short-term rates

   Answer: C is correct. Economic growth forecasts impact long-term rates. The view that economic growth will return to robust levels is consistent with a shift to a positively sloped term structure.

2. Salaz also expects the Federal Reserve to decrease asset purchases of long-term bonds as the economic recovery continues. Which of the following scenarios is consistent with this view? The reduced asset purchases will likely:
   A. amplify the effect of increased economic activity on the term spread.
   B. dampen the effect of increased economic activity on the term spread.
   C. have no effect on the term spread.

   Answer: A. Reduced asset purchases constitute a negative shift in demand for longer-term bonds, which raises their yields. The reduced asset purchases of long-maturity bonds would add to the effect of greater economic activity, both of which will increase the term spread.

SUMMARY

- The spot rate for a given maturity can be expressed as a geometric average of the short-term rate and a series of forward rates.
- Forward rates are above (below) spot rates when the spot curve is upward (downward) sloping, whereas forward rates are equal to spot rates when the spot curve is flat.
- If forward rates are realized, then all bonds, regardless of maturity, will have the same one-period realized return, which is the first-period spot rate.
- If the spot rate curve is upward sloping and is unchanged, then each bond "rolls down" the curve and earns the forward rate that rolls out of its pricing (i.e., an N-period zero-coupon bond earns the N-period forward rate as it rolls down to be an $N - 1$ period security). This dynamic implies an expected return in excess of short-maturity bonds (i.e., a **term premium**) for longer-maturity bonds if the yield curve is upward sloping.

- Active bond portfolio management is consistent with the expectation that today's forward curve does not accurately reflect future spot rates.
- The swap curve provides another measure of the time value of money.
- Swaps are an essential tool frequently used by investors to hedge, take a position in, or otherwise modify interest rate risk.
- Bond quote conventions often use measures of spreads. Those quoted spreads can be used to determine a bond's price.
- Swap curves and Treasury curves can differ because of differences in their credit exposures, liquidity, and other supply/demand factors.
- Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk.
- The local expectations theory, liquidity preference theory, segmented markets theory, and preferred habitat theory provide traditional explanations for the shape of the yield curve.
- Historical yield curve movements suggest that they can be explained by a linear combination of three principal movements: level, steepness, and curvature.
- The volatility term structure can be measured using historical data and depicts yield curve risk.
- The sensitivity of a bond value to yield curve changes may make use of effective duration, key rate durations, or sensitivities to parallel, steepness, and curvature movements. Using key rate durations or sensitivities to parallel, steepness, and curvature movements allows one to measure and manage shaping risk.
- The term bond risk premium refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate
- Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.
- During highly uncertain market periods, investors flock to government bonds in a flight to quality that is often associated with bullish flattening, in which long-term rates fall by more than short-term rates.
- Investors expecting rates to fall will generally extend (shorten) portfolio duration to take advantage of expected bond price increases (decreases)
- When investors expect a steeper (flatter) curve under which long-term rates rise (fall) relative to short-term rates, they will sell (buy) long-term bonds and purchase (sell) short-term bonds.

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PRACTICE PROBLEMS

1. Given spot rates for one-, two-, and three-year zero coupon bonds, how many forward rates can be calculated?

2. Give two interpretations for the following forward rate: The two-year forward rate one year from now is 2%.
3. Describe the relationship between forward rates and spot rates if the yield curve is flat.
4. A. Define the yield-to-maturity for a coupon bond.
   B. Is it possible for a coupon bond to earn less than the yield-to-maturity if held to maturity?
5. If a bond trader believes that current forward rates overstate future spot rates, how might she profit from that conclusion?
6. Explain the strategy of rolling down the yield curve.
7. What are the advantages of using the swap curve as a benchmark of interest rates relative to a government bond yield curve?
8. What is the TED spread, and what type of risk does it measure?
9. What is the SOFR rate, and which market conditions does it reflect?
10. According to the local expectations theory, what would be the difference in the one-month total return if an investor purchased a five-year zero-coupon bond versus a two-year zero-coupon bond?
11. Compare the segmented market and the preferred habitat term structure theories.
12. A. List the three factors that have empirically been observed to affect Treasury security returns and explain how each of these factors affects returns on Treasury securities.
    B. What has been observed to be the most important factor in affecting Treasury returns?
    C. Which measures of yield curve risk can measure shaping risk?
13. Which forward rate cannot be computed from the one-, two-, three-, and four-year spot rates? The rate for a:
    A. one-year loan beginning in two years
    B. two-year loan beginning in two years
    C. three-year loan beginning in two years
14. Consider spot rates for three zero-coupon bonds: z(1) = 3%, z(2) = 4%, and z(3) = 5%. Which statement is correct? The forward rate for a one-year loan beginning in one year will be:
    A. less than the forward rate for a one-year loan beginning in two years.
    B. greater than the forward rate for a two-year loan beginning in one year.
    C. greater than the forward rate for a one-year loan beginning in two years.
15. If one-period forward rates are decreasing with maturity, the yield curve is *most likely*:
    A. flat.
    B. upward sloping.
    C. downward sloping.

The following information relates to Questions 16–19

A one-year zero-coupon bond yields 4.0%. The two- and three-year zero-coupon bonds yield 5.0% and 6.0%, respectively.

16. The rate for a one-year loan beginning in one year is *closest* to:
    A. 4.5%.
    B. 5.0%.
    C. 6.0%.
17. The forward rate for a two-year loan beginning in one year is *closest* to:
    A. 5.0%.
    B. 6.0%.
    C. 7.0%.

18. The forward rate for a one-year loan beginning in two years is *closest* to:
    A. 6.0%.
    B. 7.0%.
    C. 8.0%.
19. The five-year spot rate is not provided here; however, the forward price for a two-year zero-coupon bond beginning in three years is known to be 0.8479. The price today of a five-year zero-coupon bond is *closest* to:
    A. 0.7119.
    B. 0.7835.
    C. 0.9524.

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20. The one-year spot rate z_1 is 4%, the forward rate for a one-year loan beginning in one year is 6%, and the forward rate for a one-year loan beginning in two years is 8%. Which of the following rates is *closest* to the three-year spot rate?
    A. 4.0%
    B. 6.0%
    C. 8.0%
21. The one-year spot rate z_1 is 5%, and the forward price for a one-year zero-coupon bond beginning in one year is 0.9346. The spot price of a two-year zero-coupon bond is *closest* to:
    A. 0.87.
    B. 0.89.
    C. 0.93.
22. In a typical interest rate swap contract, the swap rate is *best* described as the interest rate for the:
    A. fixed-rate leg of the swap.
    B. floating-rate leg of the swap.
    C. difference between the fixed and floating legs of the swap.
23. A two-year fixed-for-floating MRR swap is 1.00%, and the two-year US Treasury bond is yielding 0.63%. The swap spread is *closest* to:
    A. 37 bps.
    B. 100 bps.
    C. 163 bps.
24. The swap spread is quoted as 50 bps. If the five-year US Treasury bond is yielding 2%, the rate paid by the fixed payer in a five-year interest rate swap is *closest* to:
    A. 0.50%.
    B. 1.50%.
    C. 2.50%.
25. If the three-month T-bill rate drops and Libor remains the same, the relevant TED spread:
    A. increases.
    B. decreases.
    C. does not change.
26. Given the yield curve for US Treasury zero-coupon bonds, which spread is *most* helpful pricing a corporate bond? The:
    A. Z-spread.
    B. TED spread.
    C. Libor–OIS spread.

The following information relates to Questions 27–33

Jane Nguyen is a senior bond trader for an investment bank, and Chris Alexander is a junior bond trader at the bank. Nguyen is responsible for her own trading activities and also for providing assignments to Alexander that will develop his skills and create profitable trade ideas. Exhibit 1 presents the current par and spot rates.

EXHIBIT 1 Current Par and Spot Rates

Maturity	Par Rate	Spot Rate
One year	2.50%	2.50%
Two years	2.99%	3.00%
Three years	3.48%	3.50%
Four years	3.95%	4.00%
Five years	4.37%	

Note: Par and spot rates are based on annual-coupon sovereign bonds.

Nguyen gives Alexander two assignments that involve researching various questions:

Assignment 1: What is the yield-to-maturity of the option-free, default-risk-free bond presented in Exhibit 2? Assume that the bond is held to maturity, and use the rates shown in Exhibit 1.

EXHIBIT 2 Selected Data for $1,000 Par Bond

Bond Name	Maturity (T)	Coupon
Bond Z	Three years	6.00%

Note: Terms are today for a T-year loan.

Assignment 2: Assuming that the projected spot curve two years from today will be below the current forward curve, is Bond Z fairly valued, undervalued, or overvalued?

After completing his assignments, Alexander asks about Nguyen's current trading activities. Nguyen states that she has a two-year investment horizon and will purchase Bond Z as part of a strategy to ride the yield curve. Exhibit 1 shows Nguyen's yield curve assumptions implied by the spot rates.

27. Based on Exhibit 1, the five-year spot rate is *closest* to:
    A. 4.40%.
    B. 4.45%.
    C. 4.50%.
28. Based on Exhibit 1, the market is *most likely* expecting:
    A. deflation.
    B. inflation.
    C. no risk premiums.
29. Based on Exhibit 1, the forward rate of a one-year loan beginning in three years is *closest* to:
    A. 4.17%.
    B. 4.50%.
    C. 5.51%.

30. Based on Exhibit 1, which of the following forward rates can be computed?
    A. A one-year loan beginning in five years
    B. A three-year loan beginning in three years
    C. A four-year loan beginning in one year
31. For Assignment 1, the yield-to-maturity for Bond Z is *closest* to the:
    A. one-year spot rate.
    B. two-year spot rate.
    C. three-year spot rate.
32. For Assignment 2, Alexander should conclude that Bond Z is currently:
    A. undervalued.
    B. fairly valued.
    C. overvalued.
33. By choosing to buy Bond Z, Nguyen is *most likely* making which of the following assumptions?
    A. Bond Z will be held to maturity.
    B. The three-year forward curve is above the spot curve.
    C. Future spot rates do not accurately reflect future inflation.

The following information relates to Questions 34–38

Laura Mathews recently hired Robert Smith, an investment adviser at Shire Gate Advisers, to assist her in investing. Mathews states that her investment time horizon is short, approximately two years or less. Smith gathers information on spot rates for on-the-run annual-coupon government securities and swap spreads, as presented in Exhibit 1. Shire Gate Advisers recently published a report for its clients stating its belief that, based on the weakness in the financial markets, interest rates will remain stable, the yield curve will not change its level or shape for the next two years, and swap spreads will also remain unchanged.

EXHIBIT 1 Government Spot Rates and Swap Spreads

	Maturity (years)			
	1	2	3	4
Government spot rate	2.25%	2.70%	3.30%	4.05%
Swap spread	0.25%	0.30%	0.45%	0.70%

Smith decides to examine the following three investment options for Mathews:

Investment 1: Buy a government security that would have an annualized return that is nearly risk free. Smith is considering two possible implementations: a two-year investment or a combination of two one-year investments.

Investment 2: Buy a four-year, zero-coupon corporate bond and then sell it after two years. Smith illustrates the returns from this strategy using the swap rate as a proxy for corporate yields.

Investment 3: Buy a lower-quality, two-year corporate bond with a coupon rate of 4.15% and a Z-spread of 65 bps.

When Smith meets with Mathews to present these choices, Mathews tells him that she is somewhat confused by the various spread measures. She is curious to know whether there is one spread measure that could be used as a good indicator of the risk and liquidity of money market securities during the recent past.

34. In his presentation of Investment 1, Smith could show that under the no-arbitrage principle, the forward price of a one-year government bond to be issued in one year is *closest* to:
    A. 0.9662.
    B. 0.9694.
    C. 0.9780.
35. In presenting Investment 1, using Shire Gate Advisers' interest rate outlook, Smith could show that riding the yield curve provides a total return that is *most likely*:
    A. lower than the return on a maturity-matching strategy.
    B. equal to the return on a maturity-matching strategy.
    C. higher than the return on a maturity-matching strategy.
36. In presenting Investment 2, Smith should show an annual return *closest* to:
    A. 4.31%.
    B. 5.42%.
    C. 6.53%.
37. The bond in Investment 3 is *most likely* trading at a price of:
    A. 100.97.
    B. 101.54.
    C. 104.09.
38. The *most* appropriate response to Mathews' question regarding a spread measure is the:
    A. Z-spread.
    B. TED spread.
    C. Libor–OIS spread.

The following information relates to Questions 39–42

Rowan Madison is a junior analyst at Cardinal Capital. Sage Winter, a senior portfolio manager and Madison's supervisor, meets with Madison to discuss interest rates and review two bond positions in the firm's fixed-income portfolio.

Winter begins the meeting by asking Madison to state her views on the term structure of interest rates. Madison responds:

> "Yields are a reflection of expected spot rates and risk premiums. Investors demand risk premiums for holding long-term bonds, and these risk premiums increase with maturity."

Winter tells Madison that, based on recent changes in spreads, she is concerned about a perceived increase in counterparty risk in the economy and its effect on the portfolio. Madison asks Winter:

> "Which spread measure should we use to assess changes in counterparty risk in the economy?"

Winter is also worried about the effect of yield volatility on the portfolio. She asks Madison to identify the economic factors that affect short-term and long-term rate volatility. Madison responds:

"Short-term rate volatility is mostly linked to uncertainty regarding monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation."

Finally, Winter asks Madison to analyze the interest rate risk portfolio positions in a 5-year and a 20-year bond. Winter requests that the analysis be based on level, slope, and curvature as term structure factors. Madison presents her analysis in Exhibit 1.

EXHIBIT 1 Three-Factor Model of Term Structure

	Time to Maturity (years)	
Factor	5	20
Level	−0.4352%	−0.5128%
Steepness	−0.0515%	−0.3015%
Curvature	0.3963%	0.5227%

Note: Entries indicate how yields would change for a one standard deviation increase in a factor.

Winter asks Madison to perform two analyses:

Analysis 1: Calculate the expected change in yield on the 20-year bond resulting from a two-standard-deviation increase in the steepness factor.

Analysis 2: Calculate the expected change in yield on the five-year bond resulting from a one-standard-deviation decrease in the level factor and a one-standard-deviation decrease in the curvature factor.

39. Madison's views on the term structure of interest rates are *most* consistent with the:
    A. local expectations theory.
    B. segmented markets theory.
    C. liquidity preference theory.
40. Is Madison's response regarding the factors that affect short-term and long-term rate volatility correct?
    A. Yes
    B. No, she is incorrect regarding factors linked to long-term rate volatility
    C. No, she is incorrect regarding factors linked to short-term rate volatility
41. Based on Exhibit 1, the results of Analysis 1 should show the yield on the 20-year bond decreasing by:
    A. 0.3015%.
    B. 0.6030%.
    C. 0.8946%.
42. Based on Exhibit 1, the results of Analysis 2 should show the yield on the five-year bond:
    A. decreasing by 0.8315%.
    B. decreasing by 0.0389%.
    C. increasing by 0.0389%.

The following information relates to Questions 43–50

Liz Tyo is a fund manager for an actively managed global fixed-income fund that buys bonds issued in Countries A, B, and C. She and her assistant are preparing the quarterly markets update. Tyo begins the meeting by distributing the daily rates sheet, which includes the current government spot rates for Countries A, B, and C as shown in Exhibit 1.

EXHIBIT 1 Today's Government Spot Rates

Maturity	Country A	Country B	Country C
One year	0.40%	−0.22%	14.00%
Two years	0.70	−0.20	12.40
Three years	1.00	−0.12	11.80
Four years	1.30	−0.02	11.00
Five years	1.50	0.13	10.70

Tyo asks her assistant how these spot rates were obtained. The assistant replies, "Spot rates are determined through the process of bootstrapping. It entails backward substitution using par yields to solve for zero-coupon rates one by one, in order from latest to earliest maturities."

Tyo then provides a review of the fund's performance during the last year and comments, "The choice of an appropriate benchmark depends on the country's characteristics. For example, although Countries A and B have both an active government bond market and a swap market, Country C's private sector is much bigger than its public sector, and its government bond market lacks liquidity."

Tyo further points out, "The fund's results were mixed; returns did not benefit from taking on additional risk. We are especially monitoring the riskiness of the corporate bond holdings. For example, our largest holdings consist of three four-year corporate bonds (Bonds 1, 2, and 3) with identical maturities, coupon rates, and other contract terms. These bonds have Z-spreads of 0.55%, 1.52%, and 1.76%, respectively."

Tyo continues, "We also look at risk in terms of the swap spread. We considered historical three-year swap spreads for Country B, which reflect that market's credit and liquidity risks, at three different points in time." Tyo provides the information in Exhibit 2.

EXHIBIT 2 Selected Historical Three-Year Rates for Country B

Period	Government Bond Yield (%)	Fixed-for-Floating Libor Swap (%)
1 month ago	−0.10	0.16
6 months ago	−0.08	0.01
12 months ago	−0.07	0.71

Tyo then suggests that the firm was able to add return by riding the yield curve. The fund plans to continue to use this strategy but only in markets with an attractive yield curve for this strategy.

She moves on to present her market views on the respective yield curves for a five-year investment horizon.

Country A: "The government yield curve has changed little in terms of its level and shape during the last few years, and I expect this trend to continue. We assume that future spot rates reflect the current forward curve for all maturities."

Country B: "Because of recent economic trends, I expect a reversal in the slope of the current yield curve. We assume that future spot rates will be higher than current forward rates for all maturities."

Country C: "To improve liquidity, Country C's central bank is expected to intervene, leading to a reversal in the slope of the existing yield curve. We assume that future spot rates will be lower than today's forward rates for all maturities."

Tyo's assistant asks, "Assuming investors require liquidity premiums, how can a yield curve slope downward? What does this imply about forward rates?"

Tyo answers, "Even if investors require compensation for holding longer-term bonds, the yield curve can slope downward—for example, if there is an expectation of severe deflation. Regarding forward rates, it can be helpful to understand yield curve dynamics by calculating implied forward rates. To see what I mean, we can use Exhibit 1 to calculate the forward rate for a two-year Country C loan beginning in three years."

43. Did Tyo's assistant accurately describe the process of bootstrapping?
    A. Yes
    B. No, with respect to par yields
    C. No, with respect to backward substitution
44. The swap curve is a better benchmark than the government spot curve for:
    A. Country A.
    B. Country B.
    C. Country C.
45. Based on Exhibit 2, the implied credit and liquidity risks as indicated by the historical three-year swap spreads for Country B were the lowest:
    A. 1 month ago.
    B. 6 months ago.
    C. 12 months ago.
46. Based on Exhibit 1 and Tyo's expectations, which country's term structure is currently best for traders seeking to ride the yield curve?
    A. Country A
    B. Country B
    C. Country C
47. Based on Exhibit 1 and assuming Tyo's market views on yield curve changes are realized, the forward curve of which country will lie below its spot curve?
    A. Country A
    B. Country B
    C. Country C
48. Based on Exhibit 1 and Tyo's expectations for the yield curves, Tyo *most likely* perceives the bonds of which country to be fairly valued?
    A. Country A
    B. Country B
    C. Country C
49. With respect to their discussion of yield curves, Tyo and her assistant are *most likely* discussing which term structure theory?
    A. Pure expectations theory
    B. Local expectations theory
    C. Liquidity preference theory
50. Tyo's assistant should calculate a forward rate *closest* to:
    A. 9.07%.
    B. 9.58%.
    C. 9.97%.

51. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is *most often* consistent with:
    A. bearish flattening.
    B. bullish steepening.
    C. bearish steepening.
52. When government budget deficits fall, fiscal supply-side effects are *most likely* to result in:
    A. higher bond yields.
    B. a steeper yield curve.
    C. lower bond yields.
53. A flight to quality is *most often* associated with:
    A. a general rise in the level of interest rates.
    B. bullish flattening.
    C. bearish flattening.

CHAPTER 8

THE ARBITRAGE-FREE VALUATION FRAMEWORK

Steven V. Mann, PhD

LEARNING OUTCOMES

The candidate should be able to:

- explain what is meant by arbitrage-free valuation of a fixed-income instrument;
- calculate the arbitrage-free value of an option-free, fixed-rate coupon bond;
- describe a binomial interest rate tree framework;
- describe the process of calibrating a binomial interest rate tree to match a specific term structure;
- describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node;
- compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice;
- describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path;
- describe a Monte Carlo forward-rate simulation and its application;
- describe term structure models and how they are used.

1. INTRODUCTION TO ARBITRAGE-FREE VALUATION

- **explain what is meant by arbitrage-free valuation of a fixed-income instrument**

The idea that market prices adjust until there are no arbitrage opportunities forms the basis for valuing fixed-income securities, derivatives, and other financial assets. If both the net proceeds (e.g., buying and selling the same value of an asset) and the risk of an investment are zero, the return on that investment should also be zero.

This chapter is designed to equip candidates with a set of bond valuation tools that are consistent with this idea. The remainder of Section 1 further defines the concept of no arbitrage, and Section 2 provides a framework for an arbitrage-free valuation of fixed-income securities. Section 3 introduces the binomial interest rate tree framework based on a lognormal random walk, which is used to value an option-free bond. The binomial tree model is calibrated to the current yield curve in Section 4. This step ensures that the interest rate tree is consistent with pricing using the zero-coupon (i.e., spot) curve as illustrated in Section 5. The chapter next turns to an introduction of pathwise valuation, in Section 6. Section 7 describes a Monte Carlo forward-rate simulation and its application. Section 8 goes beyond the lognormal random walk approach to introduce common term structure models. Building on principles established earlier in the chapter, these models incorporate assumptions about changes in interest rates and volatility to capture term structure dynamics and are used by practitioners to price and hedge fixed-income securities and derivatives.

1.1. The Meaning of Arbitrage-Free Valuation

Arbitrage-free valuation refers to an approach to security valuation that determines security values that are consistent with the absence of an **arbitrage opportunity**, which is an opportunity for trades that earn riskless profits without any net investment of money. In well-functioning markets, prices adjust until there are no arbitrage opportunities, which is the **principle of no arbitrage** that underlies the practical validity of arbitrage-free valuation. This principle itself can be thought of as an implication of the idea that identical assets should sell at the same price.

These concepts will be explained in greater detail shortly, but to indicate how they arise in bond valuation, consider first an imaginary world in which financial assets are free of risk and the benchmark yield curve is flat. In this chapter, the terms yield, interest rate, and discount rate will be used interchangeably. A flat yield curve implies that the relevant yield is the same for all cash flows regardless of when the cash flows are delivered in time. Accordingly, the value of a bond is the present value of its certain future cash flows. In discounting those cash flows—determining their present value—investors would use the risk-free interest rate because the cash flows are certain; because the yield curve is assumed to be flat, one risk-free rate would exist and apply to all future cash flows. This is the simplest case of bond valuation one can envision. When we exit this imaginary world and enter more realistic environs, bonds' cash flows are risky (i.e., there is some chance the borrower will default) and the benchmark yield curve is not flat. How would our approach change?

A fundamental principle of valuation is that the value of any financial asset is equal to the present value of its expected future cash flows. This principle holds for any financial asset, from zero-coupon bonds to interest rate swaps. Thus, the valuation of a financial asset involves the following three steps:

Step 1: Estimate the future cash flows.

Step 2: Determine the appropriate discount rate or discount rates that should be used to discount the cash flows.

Step 3: Calculate the present value of the expected future cash flows found in Step 1 by applying the appropriate discount rate or rates determined in Step 2.

The traditional approach to valuing bonds is to discount all cash flows with the same discount rate as if the yield curve were flat. However, a bond is properly thought of as a package or portfolio of zero-coupon bonds, also referred to as zeros or discount instruments. Each zero-coupon bond in such a package can be valued separately at a discount rate that depends on the shape of the yield curve and when its single cash flow is delivered in time. The term structure of these discount rates is referred to as the spot curve. Bond values derived by summing the present values of the individual zeros (cash flows) determined by such a procedure can be shown to be arbitrage free. Ignoring transaction costs for the moment, if the bond's value were much less than the sum of the values of its cash flows individually, a trader would perceive an arbitrage opportunity and buy the bond while selling claims to the individual cash flows and pocketing the excess value. Although the details bear further discussion, the valuation of a bond as a portfolio of zeros based on using the spot curve is an example of arbitrage-free valuation. Regardless of the complexity of the bond, each component must have an arbitrage-free value. A bond with embedded options can be valued in parts as the sum of the arbitrage-free bond without options (that is, a bond with no embedded options) and the arbitrage-free value of each of the options.

1.2. The Law of One Price

The central idea of financial economics is that market prices will adjust until there are no opportunities for arbitrage. We will define shortly what is meant by an arbitrage opportunity, but for now think of it as "free money." Prices will adjust until there is no free money to be acquired. Arbitrage opportunities arise from violations of the **law of one price**. The law of one price states that two goods that are perfect substitutes must sell for the same current price in the absence of transaction costs. Two goods that are identical, trading side by side, are priced the same. Otherwise, if it were costless to trade, one would simultaneously buy at the lower price and sell at the higher price. The riskless profit is the difference in the prices. An individual would repeat this transaction without limit until the two prices converge. An implication of these market forces is deceptively straightforward and basic. If you do not put up any of your own money and take no risk, your expected return should be zero.

1.3. Arbitrage Opportunity

With this background, let us define arbitrage opportunity more precisely. An arbitrage opportunity is a transaction that involves no cash outlay that results in a riskless profit. There are two types of arbitrage opportunities. The first type of arbitrage opportunity is often called **value additivity**; put simply, the value of the whole equals the sum of the values of the parts. Consider two risk-free investments with payoffs one year from today and the prices today provided in Exhibit 1. Asset A is a simple risk-free zero-coupon bond that pays off one dollar and is priced today at 0.952381 (= 1/1.05). Asset B is a portfolio of 105 units of Asset A that pays off 105 one year from today and is priced today at 97. The portfolio does not equal the sum of the parts. The portfolio (Asset B) is cheaper than buying 105 units of Asset A at a price of 100 and then combining. An astute investor would sell 105 units of Asset A for 105 × 0.952381 = 100 while simultaneously buying the portfolio, Asset B, for 97. This position generates a certain 3 today (100 – 97) and generates net 0 one year from today because cash inflow for Asset B matches the amount for the 105 units of Asset A sold. An investor would repeat this trade until the prices are equal.

The second type of arbitrage opportunity is often called **dominance**. A financial asset with a risk-free payoff in the future must have a positive price today. Consider two assets, C and D, that are risk-free zero-coupon bonds. Payoffs in one year and prices today are displayed in Exhibit 1. On careful review, it appears that Asset D is cheap relative to Asset C. If both assets are risk-free, they should have the same discount rate. To make money, sell two units of Asset C at a price of 200 and use the proceeds to purchase one unit of Asset D for 200. The construction of the portfolio involves no net cash outlay today. Although it requires zero dollars to construct today, the portfolio generates 10 one year from today. Asset D will generate a 220 cash inflow, whereas the two units of Asset C sold will produce a cash outflow of 210.

EXHIBIT 1 Price Today and Payoffs in One Year for Sample Assets

Asset	Price Today	Payoff in One Year
A	0.952381	1
B	97	105
C	100	105
D	200	220

This existence of both types of arbitrage opportunity is transitory. Investors aware of this mispricing will demand the securities in question in unlimited quantities. Something must change to restore stability. Prices will adjust until there are no arbitrage opportunities.

EXAMPLE 1 Arbitrage Opportunities

Which of the following investment alternatives includes an arbitrage opportunity?

Bond A: The yield for a 3% annual coupon 10-year bond is 2.5% in New York City. The same bond sells for $104.376 per $100 face value in Chicago.

Bond B: The yield for a 3% annual coupon 10-year bond is 3.2% in Hong Kong SAR. The same bond sells for RMB97.220 per RMB100 face value in Shanghai.

Solution: Bond B is correct. Bond B's arbitrage-free price may be solved for using a financial calculator or Microsoft Excel as $3/1.032 + 3/1.032^2 + \ldots + 103/1.032^{10} = 98.311$, which is higher than the price in Shanghai. Therefore, an arbitrage opportunity exists. Buy bonds in Shanghai for RMB97.220 and sell them in Hong Kong SAR for RMB98.311. You make RMB1.091 per RMB100 of bonds traded.

Bond A's arbitrage-free price is $3/1.025 + 3/1.025^2 + \ldots + 103/1.025^{10} = 104.376$, which matches the price in Chicago. Therefore, no arbitrage opportunity exists in this market.

1.4. Implications of Arbitrage-Free Valuation for Fixed-Income Securities

Using the arbitrage-free approach, any fixed-income security should be thought of as a package or portfolio of zero-coupon bonds. Thus, a five-year 2% coupon Treasury issue should be viewed as a package of 11 zero-coupon instruments (10 semiannual coupon payments, 1 of which is made at maturity, and 1 principal value payment at maturity). The market mechanism for US Treasuries that enables this approach is the dealer's ability to separate the bond's individual cash flows and trade them as zero-coupon securities. This process is called **stripping**. In addition, dealers can recombine the appropriate individual zero-coupon securities and reproduce the underlying coupon Treasury. This process is called **reconstitution**. Dealers in sovereign debt markets around the globe are free to engage in the same process.

Arbitrage profits are possible when value additivity does not hold. The arbitrage-free valuation approach does not allow a market participant to realize an arbitrage profit through stripping and reconstitution. By viewing any security as a package of zero-coupon securities, a consistent and coherent valuation framework can be developed. Viewing a security as a package of zero-coupon bonds means that two bonds with the same maturity and different coupon rates are viewed as different packages of zero-coupon bonds and valued accordingly. Moreover, two cash flows with identical risks delivered at the same time will be valued using the same discount rate even though they are attached to two different bonds.

2. ARBITRAGE-FREE VALUATION FOR AN OPTION-FREE BOND

- **calculate the arbitrage-free value of an option-free, fixed-rate coupon bond**

The goal of this section is to develop a method to produce an arbitrage-free value for an option-free bond and to provide a framework—based on interest rate trees—that is rich enough to be applied to the valuation of bonds with embedded options.

For bonds that are option-free, the simplest approach to arbitrage-free valuation involves determining the arbitrage-free value as the sum of the present values of expected future values using the benchmark spot rates. Benchmark securities are liquid, safe securities whose yields serve as building blocks for other interest rates in a country or currency. Sovereign debt is the benchmark in many countries. For example, on-the-run Treasuries serve as benchmark securities in the United States. Par rates derived from the Treasury yield curve can be used to obtain spot rates by means of bootstrapping. Gilts are the benchmark in the United Kingdom, while German bunds serve as the benchmark for euro-denominated bonds. In markets where the sovereign debt market is not sufficiently liquid, the swap curve is a viable alternative.

In this chapter, benchmark bonds are assumed to be correctly priced by the market. The valuation model we develop will be constructed to reproduce exactly the prices of the benchmark bonds.

EXAMPLE 2 The Arbitrage-Free Value of an Option-Free Bond

The yield-to-maturity ("par rate") for a benchmark one-year annual coupon bond is 2%, for a benchmark two-year annual coupon bond is 3%, and for a benchmark three-year annual coupon bond is 4%. A three-year, 5% annual coupon bond with the same risk and liquidity as the benchmarks is selling for 102.7751 today ($t = 0$) to yield 4%. Is this value correct for the bond given the current term structure?

Solution: The first step in the solution is to find the correct spot rate (zero-coupon rates) for each year's cash flow. The spot rates may be determined using bootstrapping, which is an iterative process. Using the bond valuation equation below, one can solve iteratively for the spot rates, z_t (rate on a zero-coupon bond of maturity t), given the periodic payment, *PMT*, on the relevant benchmark bond.

$$100 = \frac{PMT}{(1+z_1)^1} + \frac{PMT}{(1+z_2)^2} + \cdots + \frac{PMT + 100}{(1+z_N)^N}$$

A revised equation, which uses the par rate rather than *PMT*, may also be used to calculate the spot rates. The revised equation is

$$1 = \frac{\text{Par rate}}{(1+z_1)} + \frac{\text{Par rate}}{(1+z_2)^2} + \ldots + \frac{\text{Par rate} + 1}{(1+z_N)^N}$$

where par rate is *PMT* divided by 100 and represents the par rate on the benchmark bond and z_t is the t-period zero-coupon rate.

In this example, the one-year spot rate, z_1, is 2%, which is the same as the one-year par rate. To solve for z_2,

$$1 = \frac{0.03}{(1+z_1)} + \frac{0.03+1}{(1+z_2)^2} = \frac{0.03}{(1+0.02)} + \frac{0.03+1}{(1+z_2)^2}$$
$$z_2 = 3.015\%$$

To solve for z_3,

$$1 = \frac{0.04}{(1+z_1)} + \frac{0.04}{(1+z_2)^2} + \frac{0.04+1}{(1+z_3)^3} = \frac{0.04}{(1+0.02)} + \frac{0.04}{(1+0.03015)^2} + \frac{0.04+1}{(1+z_3)^3}$$
$$z_3 = 4.055\%$$

The spot rates are 2%, 3.015%, and 4.055%. The correct arbitrage-free price for the bond, then, is

$$P_0 = 5/1.02 + 5/1.03015^2 + 105/1.04055^3 = 102.8102$$

To be arbitrage free, each cash flow of a bond must be discounted by the spot rate for zero-coupon bonds maturing on the same date as the cash flow. Discounting early coupons by the bond's yield-to-maturity gives too much discounting with an upward sloping yield curve and too little discounting for a downward sloping yield curve. The bond is mispriced by 0.0351 per 100 of par value.

For option-free bonds, performing valuation discounting with spot rates produces an arbitrage-free valuation. For bonds that have embedded options, we need a different approach. The challenge one faces when developing a framework for valuing bonds with embedded options is that their expected future cash flows are interest rate dependent. If the bonds are option-free, changes in interest rates have no impact on the size and timing of the bond's cash flows. For bonds with options attached, changes in future interest rates impact the likelihood the option will be exercised and in so doing impact the cash flows. Therefore, to develop a framework that values bonds both without and with embedded options, we must allow interest rates to take on different potential values in the future based on some assumed level of volatility. The vehicle to portray this information is an interest rate "tree" representing possible future interest rates consistent with the assumed volatility. Because the interest rate tree resembles a lattice, these models are often called "lattice models." The interest rate tree performs two functions in the valuation process: (1) Generate the cash flows that are interest rate dependent, and (2) supply the interest rates used to determine the present value of the cash flows. This approach will be used in later chapters when considering learning outcome statements involving callable bonds.

An interest rate model seeks to identify the elements or *factors* that are believed to explain the dynamics of interest rates. These factors are random or *stochastic* in nature, so we cannot predict the path of any factor. An interest rate model must, therefore, specify a statistical process that describes the stochastic property of these factors to arrive at a reasonably accurate representation of the behavior of interest rates. What is important to understand is that the interest rate models commonly used are based on how short-term interest rates can evolve (i.e., change) over time. Consequently, these interest rate models are referred to as one-factor models because only one interest rate is being modeled over time. More complex models consider how more than one interest rate changes over time (e.g., the short rate and the long rate) and are referred to as two-factor models.

Our task at hand is to describe the binomial interest rate tree framework. The valuation model we are attempting to build is the binomial lattice model. It is so named because the short interest rate can take on one of two possible values consistent with the volatility assumption and an interest rate model. As we will soon discover, the two possible interest rates next period will be consistent with the following three conditions: (1) an interest rate model that governs the random process of interest rates, (2) the assumed level of interest rate volatility, and (3) the current benchmark yield curve. We take the prices of the benchmark bonds as given so that our model recovers the market values for each benchmark bond. In this way, we tie the model to the current yield curve that reflects the underlying economic reality.

2.1. The Binomial Interest Rate Tree

The first step for demonstrating the binomial valuation method is to present the benchmark par curve by using bonds of a country or currency. For simplicity in our illustration, we will use US dollars. The same principles hold with equal force regardless of the country or currency. The benchmark par curve is presented in Exhibit 2. For simplicity, we assume that all bonds have annual coupon payments. Benchmark bonds are conveniently priced at par so the yields-to-maturity and the coupon rates on the bonds are the same. From these par rates, we use the bootstrapping methodology to uncover the underlying spot rates shown in Exhibit 3. Because the par curve is upward sloping, it comes as no surprise that after Year 1 the spot rates are higher than the par rates. In Exhibit 4 we present the one-year implied forward rates derived from the spot curve using no arbitrage. Because the par, spot, and forward curves reflect the same information about interest rates, if one of the three curves is known, it is possible to generate the other two curves. The three curves are identical only if the yield curve is flat.

EXHIBIT 2 Benchmark Par Curve

Maturity (Years)	Par Rate	Bond Price
1	1.00%	100
2	1.20%	100
3	1.25%	100
4	1.40%	100
5	1.80%	100

EXHIBIT 3 Underlying One-Year Spot Rates of Par Rates

Maturity (Years)	One-Year Spot Rate
1	1.0000%
2	1.2012%
3	1.2515%
4	1.4045%
5	1.8194%

EXHIBIT 4 One-Year Implied Forward Rates

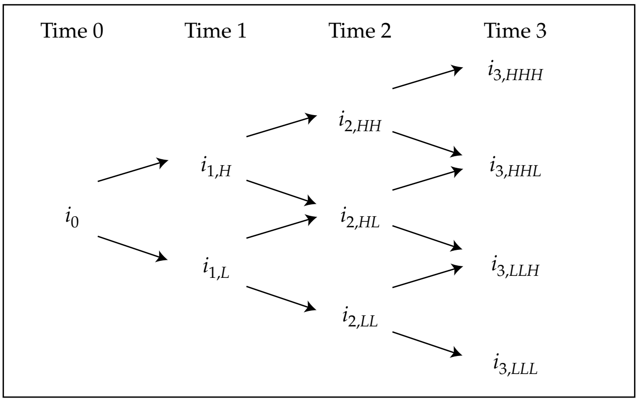
Maturity (Years)	Forward Rate
Current one-year rate	1.0000%
One-year rate, one year forward	1.4028%
One-year rate, two years forward	1.3521%
One-year rate, three years forward	1.8647%
One-year rate, four years forward	3.4965%

Recall from our earlier discussion that if we value the benchmark bonds using rates derived from these curves, we will recover the market price of par for all five bonds in Exhibit 2. Specifically, par rates represent the single interest applied to all the cash flows that will produce the market prices. Discounting each cash flow separately with the set of spot rates will also give the same answer. Finally, forward rates are the discount rates of a single cash flow over a single period. If we discount each cash flow with the appropriate discount rate for each period, the computed values will match the observed prices.

When we approach the valuation of bonds with cash flows that are interest rate dependent, we must explicitly allow interest rates to change. We accomplish this task by introducing interest rate volatility and generating an interest rate tree later in this chapter. An interest rate tree is simply a visual representation of the possible values of interest rates based on an interest rate model and an assumption about interest rate volatility.

A binomial interest rate tree is presented in Exhibit 5. Our goal is to learn how to populate this structure with interest rates. Notice the *i*'s, which represent different potential values the one-year interest rates may take over time. As we move from left to right on the tree, the number of possible interest rates increases. The first is the current time (in years), or formally, Time 0. The interest rate displayed at Time 0 is the discount rate that converts Time 1 payments to Time 0 present values. At the bottom of the graph, time is the unit of measurement. Notice that there is one year between possible interest rates. This is called the "time step," and in our illustration, it matches the frequency of the annual cash flows. The *i*'s in Exhibit 5 are called nodes. The first node is called the root of the tree and is simply the current one-year rate at Time 0. Each node thereafter is represented by both a time element and a rate change component.

EXHIBIT 5 Binomial Interest Rate Tree



We now turn to the question of how to obtain the two possible values for the one-year interest rate one year from today. Two assumptions are required: an interest rate model and a volatility of interest rates. Recall an interest rate model puts structure on the randomness. We are going to use the lognormal random walk, and the resulting tree structure is often referred to as a lognormal tree. A lognormal model of interest rates insures two appealing properties: (1) non-negativity of interest rates and (2) higher volatility at higher interest rates. At each node, there are two possible rates one year forward at Time 1. We will assume for the time being that each has an equal probability of occurring. The two possible rates we will calculate are going to be higher and lower than the one-year forward rate at Time 1 one year from now.

We denote i_L to be the rate lower than the implied forward rate and i_H to be the higher forward rate. The lognormal random walk posits the following relationship between $i_{1,L}$ and $i_{1,H}$:

$$i_{1,H} = i_{1,L}e^{2\sigma}$$

where σ is the standard deviation and *e* is Euler's number, the base of natural logarithms, which is a constant 2.7183. The random possibilities each period are (nearly) centered on the forward rates calculated from the benchmark curve. The intuition of this relationship is deceptively quick and simple. Think of the one-year forward implied interest rate from the yield curve as the average of possible values for the one-year rate at Time 1. The lower of the two rates, i_L, is one standard deviation below the mean (one-year implied forward rate), and i_H is one standard deviation above the mean. Thus, the higher and lower values (i_L and i_H) are multiples of each other, and the multiplier is $e^{2\sigma}$. Note that as the standard deviation (i.e., volatility) increases, the multiplier increases, and the two rates will grow farther apart but will still be (nearly) centered on the implied forward rate derived from the spot curve. We will demonstrate this soon.

We use the following notation to describe the tree at Time 1. Let

σ = assumed volatility of the one-year rate,
$i_{1,L}$ = the lower one-year forward rate one year from now at Time 1, and
$i_{1,H}$ = the higher one-year forward rate one year from now at Time 1.

For example, suppose that $i_{1,L}$ is 1.194% and σ is 15% per year; then $i_{1,H}$= 1.194%($e^{2\times0.15}$) = 1.612%.

At Time 2, there are three possible values for the one-year rate, which we will denote as follows:

$i_{2,LL}$ = one-year forward rate at Time 2 assuming the lower rate at Time 1 and the lower rate at Time 2.
$i_{2,HH}$ = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the higher rate at Time 2.
$i_{2,HL}$ = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the lower rate at Time 2, or equivalently, the lower rate at Time 1 and the higher rate at Time 2.

The middle rate will be close to the implied one-year forward rate two years from now derived from the spot curve, whereas the other two rates are two standard deviations above and below this value. (Recall that the multiplier for adjacent rates on the tree differs by a multiple of *e* raised to the 2σ.) This type of tree is called a recombining tree because there are two paths to get to the middle rate. This feature of the model results in faster computation because the number of possible outcomes each period grows linearly rather than exponentially.

The relationship between $i_{2,LL}$ and the other two one-year rates is as follows:

$$i_{2,HH} = i_{2,LL}(e^{4\sigma}), \text{ and } i_{2,HL} = i_{2,LL}(e^{2\sigma})$$

In a given period, adjacent possible outcomes in the tree are two standard deviations apart. So, for example, if $i_{2,LL}$ is 0.980%, and assuming once again that σ is 15%, we calculate

$$i_{2,HH} = 0.980\%(e^{4\times0.15}) = 1.786\%$$

and

$$i_{2,HL} = 0.980\%(e^{2\times0.15}) = 1.323\%$$

There are four possible values for the one-year forward rate at Time 3. These are represented as follows: $i_{3,HHH}$, $i_{3,HHL}$, $i_{3,LLH}$ and $i_{3,LLL}$. Once again, all the forward rates in the tree are multiples of the lowest possible rates each year. The lowest possible forward rate at Time 3 is $i_{3,LLL}$ and is related to the other three as given below:

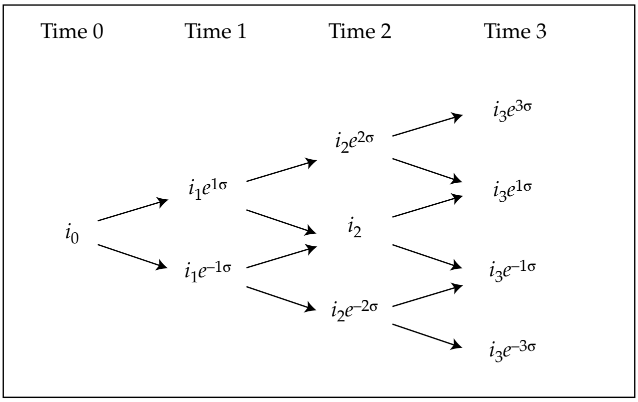
$$i_{3,HHH} = (e^{6\sigma})i_{3,LLL}$$

$$i_{3,HHL} = (e^{4\sigma})i_{3,LLL}$$

$$i_{3,LLH} = (e^{2\sigma})i_{3,LLL}$$

Exhibit 6 shows the notation for a four-year binomial interest rate tree. We can simplify the notation by centering the one-year rates on the tree on implied forward rates on the benchmark yield curve, so i_t is the one-year rate t years from now and the centering rate. The subscripts indicate the rates at the end of the year, so in the second year, it is the rate at the end of Time 2 to the end of Time 3. Exhibit 6 uses this uniform notation. Note that adjacent forward rates in the tree are two standard deviations (σ) apart.

EXHIBIT 6 Four-Year Binomial Tree



Before we attempt to build an interest rate tree, two additional tools are needed. These tools are introduced in the next two sections.

3. THE BASICS OF CREATING A BINOMIAL INTEREST RATE TREE

- **describe a binomial interest rate tree framework**

Recall that variance is a measure of dispersion of a probability distribution. The standard deviation is the square root of the variance and is measured in the same units as the mean. With a simple lognormal distribution, the changes in interest rates are proportional to the level of the one-period interest rates each period. Volatility is measured relative to the current level of rates. It can be shown that for a lognormal distribution the standard deviation of the one-year rate is equal to $i_0\sigma$. For example, if σ is 10% and the one-year rate (i_0) is 2%, then the standard deviation of the one-year rate is 2% × 10% = 0.2%, or 20 bps. As a result, interest rate moves are larger when interest rates are high and are smaller when interest rates are low. One of the characteristics of a lognormal distribution is that negative interest rates are not possible, since as rates approach zero, the absolute change in interest rates becomes smaller and smaller.

There are two methods commonly used to estimate interest rate volatility. The first method uses historical interest rate volatility based on data from the recent past, which is assumed to be indicative of the future. A second method to estimate interest rate volatility is that derived from observed market prices of interest rate derivatives (e.g., swaptions, caps, floors) known as implied volatility.

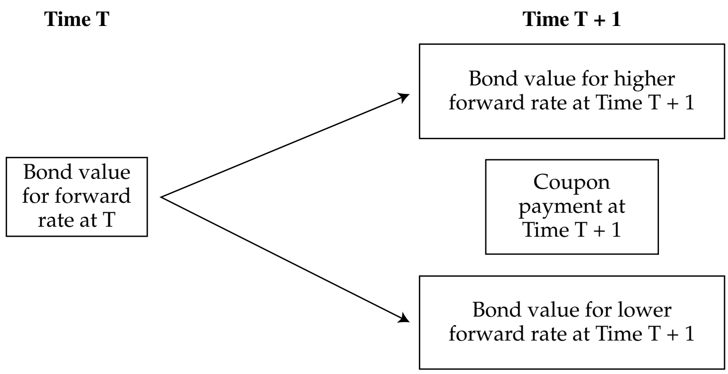
3.1. Determining the Value of a Bond at a Node

To find the value of the bond at a node, we use the backward induction valuation methodology. Barring default, we know that at maturity the bonds will be valued at par. So, we start at maturity, fill in those values, and work back from right to left to find the bond's value at the desired node. Suppose we want to determine the bond's value at the lowest node at Time 1.

To find this value, we must first calculate the bond's value at the two nodes to the right of the node we selected. The bond's value at the two nodes immediately to the right must be available.

A bond's value at any node will depend on the future coupon payment, *C*, and the expected future value for the bond. This expected value is the average of the value for the forward rate being higher, to be denoted below by *VH*, and the value for the forward rate being lower, *VL*. It is a simple average because in the lognormal model the probabilities for the rate going up or down are equal. This is illustrated in Exhibit 7. Notice that the coupon payment due at the end of the period, at Time T + 1, is placed directly to the right of the node for Time T. The arrows point to the two possible future bond values, one for the forward rate going up at Time T + 1 and the other for the rate going down.

EXHIBIT 7 Finding a Bond's Value at Any Node



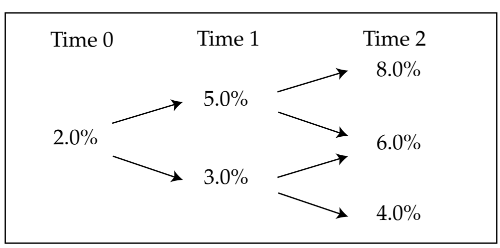
The next step is to determine the present value of the coupon payment and the expected future bond value. The relevant discount rate is the one-year forward rate prevailing at the beginning of the time period, *i*, at Time T. The bond's value at any node is determined by the following expression:

$$\text{Bond value at a node} = \frac{C + (0.5 \times VH + 0.5 \times VL)}{1 + i}$$

EXAMPLE 3 Pricing a Bond Using a Binomial Tree

Using the interest rate tree in Exhibit 8, find the correct price for a three-year, annual pay bond with a coupon rate of 5%.

EXHIBIT 8 Three-Year Binomial Interest Rate Tree



Solution: Exhibit 9 shows the binomial tree to value the three-year, 5% bond. We start with Time 3. The cash flow is 105, the redemption of par value (100) plus the final coupon payment (5), regardless of the level of the forward rate at Time 2. Using backward induction, we next calculate the present value of the bond as of Time 2 for the three possible forward rates:

$$105/1.08 = 97.2222$$
$$105/1.06 = 99.0566$$
$$105/1.04 = 100.9615$$

Working back to Time 1 requires the use of the general expression above for the value at any node. If the forward rate is 5.0% at Time 1, the bond value is 98.2280:

$$\frac{5 + (0.5 \times 97.2222 + 0.5 \times 99.0566)}{1.05} = 98.2280$$

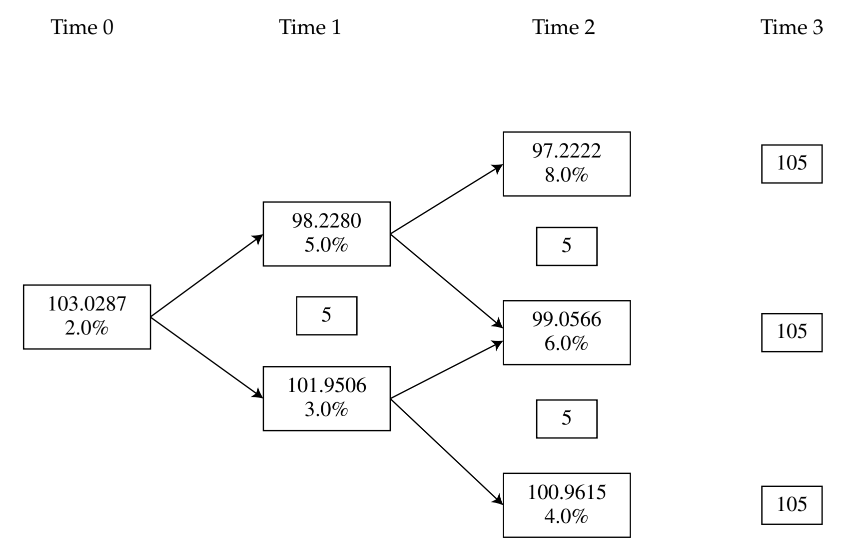
If the forward rate instead is 3.0%, the bond value is 101.9506:

$$\frac{5 + (0.5 \times 99.0566 + 0.5 \times 100.9615)}{1.03} = 101.9506$$

Finally, the value of the bond at Time 0 is 103.0287:

$$\frac{5 + (0.5 \times 98.2280 + 0.5 \times 101.9506)}{1.02} = 103.0287$$

EXHIBIT 9 Three-Year Binomial Tree



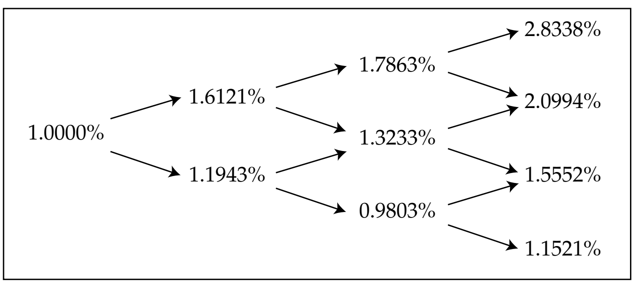
4. CALIBRATING THE BINOMIAL INTEREST RATE TREE TO THE TERM STRUCTURE

- **describe the process of calibrating a binomial interest rate tree to match a specific term structure**

The construction of a binomial interest rate tree requires multiple steps, but keep in mind what we are trying to accomplish. We assume a process that generates interest rates and volatility. The first step is to describe the calibration of a binomial interest rate tree to match a specific term structure. We do this to ensure that the model is arbitrage free. We fit the interest rate tree to the current yield curve by choosing interest rates such that the model produces the benchmark bond values reported earlier. By doing this, we tie the model to the underlying economic reality.

Recall from Exhibits 2, 3, and 4 the benchmark bond price information and the relevant par, spot, and forward curves. We will assume that volatility, σ, is 15% and construct a four-year tree starting with the two-year bond that carries a coupon rate of 1.20%. A complete four-year binomial interest rate tree is presented in Exhibit 10. We will demonstrate how these rates are determined. The current one-year rate is 1%, i_0.

EXHIBIT 10 Four-Year Binomial Interest Rate Tree



Finding the rates in the tree is an iterative process, and the interest rates are found numerically. There are two possible rates at Time 1—the higher rate and the lower rate. We observe these rates one year from today. These two rates must be consistent with the volatility assumption, the interest rate model, and the observed market value of the benchmark bond. Assume that the interest rate volatility is 15%. From our discussion earlier, we know that at Time 1 the lower one-year rate is lower than the implied one-year forward rate and the higher rate is a multiple of the lower rate. We iterate to a solution with constraints in mind. Once we select these rates, how will we know the rates are correct? The answer is when we discount the cash flows using the tree and produce a value that matches the price of the two-year benchmark bond. If the model does not produce the correct price with this result, we need to select another forward rate and repeat the process. The process of calibrating a binomial interest rate tree to match a specific term structure is illustrated in the following paragraphs.

The procedure starts with the selection of a trial rate for one of the Time 1 forward rates—for instance, $i_{1,L}$. This rate should be lower than the implied forward rate from Exhibit 4 of 1.4028%. Suppose that we select 1.2500%. The other forward rate will be 1.6873%

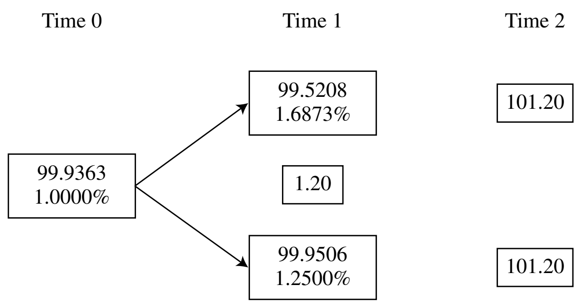
[= 1.2500% × ($e^{2\times0.15}$)]. Exhibit 11 shows that the Time 0 value for the 1.20%, two-year bond is 99.9363. The redemption of principal and the final interest payment are placed across from the two nodes for the forward rates. At Time 1, the interest payment due is placed across from the initial rate for Time 0. These are the calculations:

$$101.20/1.016873 = 99.5208$$

$$101.20/1.012500 = 99.9506$$

$$\frac{1.20 + (0.5 \times 99.5208 + 0.5 \times 99.9506)}{1.01} = 99.9363$$

EXHIBIT 11 Calibrating the Two-Year Binomial Tree



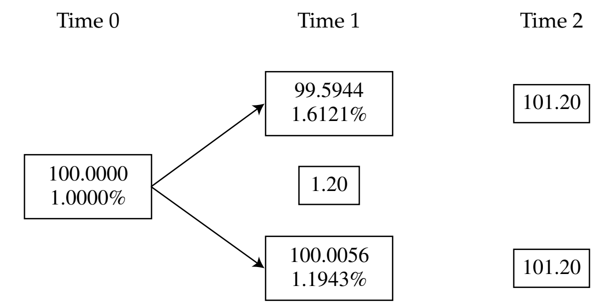
These two trial rates are clearly too high. They need to be lowered somewhat to raise the bond value to attain a Time 0 price for the bond of 100.0000. We could proceed with further trial-and-error search or use an analytic tool, such as Solver in Excel, to carry out this calculation. Essentially, we need to set the cell for the Time 0 bond price to a value of 100.0000 by changing the cell containing the initial lower forward rate for Time 1.

This procedure eventually obtains a value for $i_{1,L}$ of 1.1943%. This is the lower one-year rate. The higher one-year rate is 1.6121% [= 1.1943% × ($e^{2\times0.15}$)]. Notice that the average of these two forward rates is 1.4032% [= (1.6121% + 1.1943%)/2], slightly above the implied forward rate of 1.4028% from Exhibit 4. The binomial tree spreads out around the forward rate curve. The average is slightly higher than the implied forward rate because of the assumption of lognormality.

Recall from the information on the benchmark bonds that the two-year bond will pay its maturity value of 100 at Time 2 and an annual coupon payment of 1.20. The bond's value at Time 2 is 101.20. The present value of the coupon payment plus the bond's maturity value if the higher one-year rate is realized, *VH*, is 99.5944 (= 101.20/1.016121). Alternatively, the present value of the coupon payment plus the bond's maturity value if the lower one-year rate is realized, *VL*, is 100.0056 (= 101.20/1.011943). These two calculations determine the bond's value one year forward. Effectively, the forward rates move the bond's value from Time 2 to Time 1. Exhibit 12 demonstrates that the arbitrage-free forward rates for Time 1 are 1.6121% and 1.1943%. The value for the bond at Time 0 is 100.0000, confirming the calibration:

$$\frac{1.20 + (0.5 \times 99.5944 + 0.5 \times 100.0056)}{1.010000} = 100.0000$$

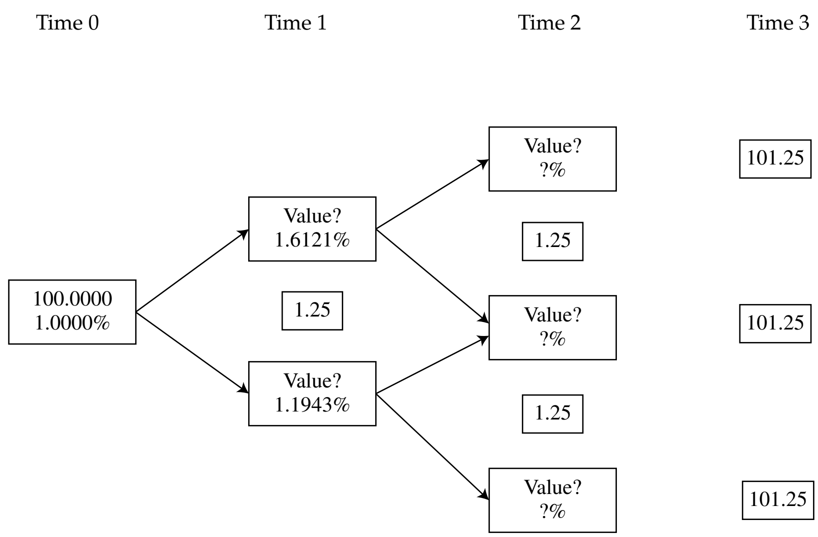
EXHIBIT 12 Building the Two-Year Binomial Tree



To build out the tree one more year, we repeat the same process, this time using a three-year benchmark bond with a coupon rate of 1.25%. Now, we are looking for three forward rates that are consistent with (1) the interest rate model assumed, (2) the assumed volatility of 15%, (3) a current one-year rate of 1.0%, and (4) the two possible forward rates one year from now (at Time 1) of 1.1943% (the lower rate) and 1.6121% (the higher rate).

At Time 3, we receive the final coupon payment and maturity value of 101.25. In Exhibit 13, we see the known coupon payments of 1.25 for Times 1 and 2. Also entered are the Time 1 forward rates and the target price of par value for the three-year bond. The unknown items to determine are the Time 1 and Time 2 bond values (Value?) and the Time 2 forward rates (?%).

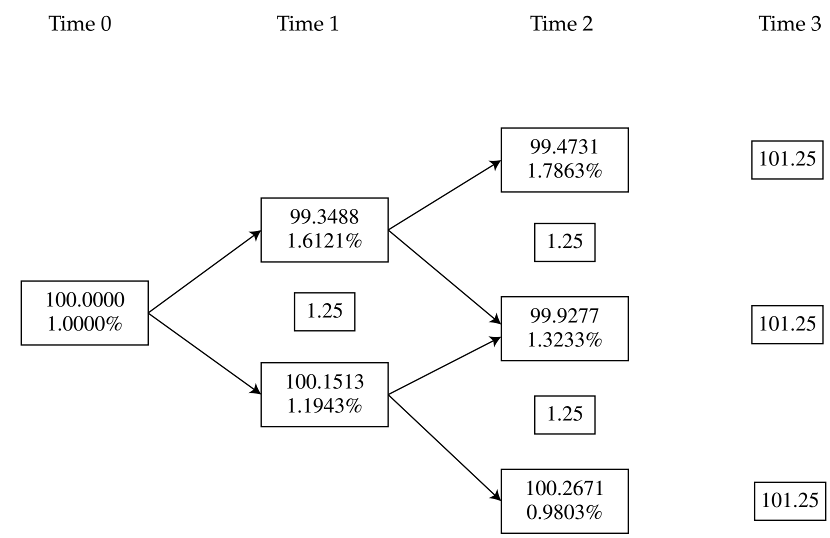
EXHIBIT 13 Finding the Time 2 Forward Rates



We need to select a trial value for the middle rate, $i_{2,HL}$. A good choice is the implied forward rate of 1.3521%. The trial value for the upper rate, $i_{2,HH}$, would need to be 1.3521% $\times$ ($e^{2\times0.15}$), and the trial value for the lower rate, $i_{2,LL}$, would need to be 1.3521%/($e^{2\times0.15}$). The middle rate is then changed, changing the others as well, until the value for the 1.25% three-year bond is 100.0000. It turns out that the three forward rates are 1.7863%, 1.3233%, and

0.9803%. To demonstrate that these are the correct values, we simply work backward from the cash flows at Time 3 of the tree in Exhibit 13. The same procedure is used to obtain the values at the other nodes. The completed tree is shown in Exhibit 14.

EXHIBIT 14 Completed Binomial Tree with Calculated Forward Rates



Let us focus on the impact of volatility on the possible forward rates in the tree. If we were to use a higher estimate of volatility—say, 20%—the possible forward rates should spread farther out around the forward curve. If we were to use a lower estimate of volatility—say, 0.01%—the rates should collapse to the implied forward rates from the current yield curve. Exhibits 15 and 16 depict the interest rate trees for the volatilities of 20% and 0.01%, respectively, and confirm the expected outcome. Notice that in Exhibit 16 for 0.01% volatility, the Time 1 forward rates are very close to the implied forward rate of 1.4028% shown in Exhibit 4. Likewise, the Time 2 and Time 3 rates are a small range around the forward rates of 1.3521% and 1.8647%, respectively. In fact, if $\sigma = 0$, the binomial tree is simply the implied forward curve.

EXHIBIT 15 Completed Tree with $\sigma = 20\%$

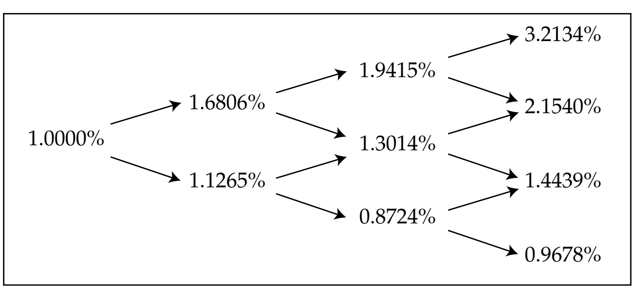
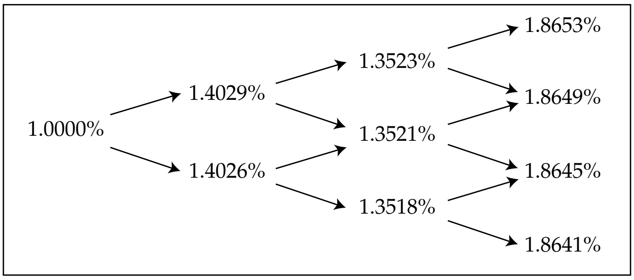


EXHIBIT 16 Completed Tree with $\sigma = 0.01\%$

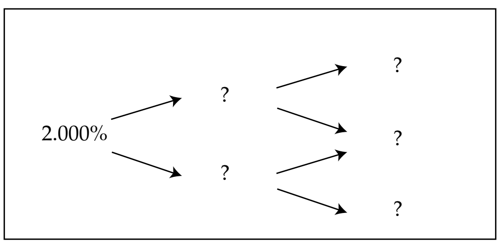


EXAMPLE 4 Calibrating a Binomial Tree to Match a Specific Term Structure

As in Example 2, the one-year par rate is 2.000%, the two-year par rate is 3.000%, and the three-year par rate is 4.000%. Consequently, the spot rates are S_0 = 2.000%, S_1 = 3.015%, and S_2 = 4.055%. The forward rates are F_0 = 2.000%, F_1 = 4.040%, and F_2 = 6.166%. Interest volatility is 15% for all years.

Calibrate the binomial tree in Exhibit 17.

EXHIBIT 17 Binomial Tree to Calibrate



Solution:

Time 0

The par, spot, and forward rates are all the same for the first period in a binomial tree. Consequently, $Y_0 = S_0 = F_0$ = 2.000%.

Time 1

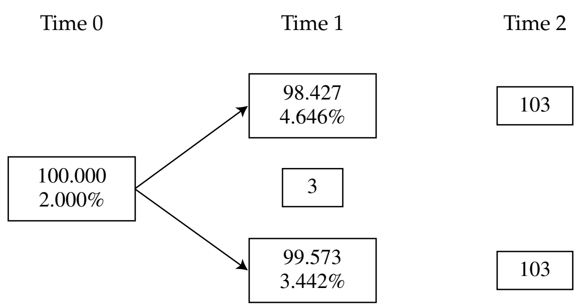
We need to use trial-and-error search (or Solver in Excel) to find the two forward rates that produce a value of 100.000 for the 3%, two-year bond. The lower trial rate needs to be lower than the implied forward rate of 4.040%—for instance, 3.500%. The higher trial rate would be 3.500% × ($e^{2\times0.15}$) = 4.725%. These lead to a Time 0 value for the bond of 99.936. Therefore, the next stage in the procedure lowers the trial rates. Finally, the calibrated forward rates are 4.646% and 3.442%. Exhibit 18 shows that these are the correct rates because the value of the bond at Time 0 is 100.000. These are the calculations:

$$103/1.04646 = 98.427$$

$$103/1.03442 = 99.573$$

$$\frac{3 + (0.5 \times 98.427 + 0.5 \times 99.573)}{1.02} = 100.0000$$

EXHIBIT 18 Calibration of Time 1 Forward Rates



Time 2

The initial trial rate for the middle node for Time 2 is the implied forward rate of 6.166%. The rate for the upper node is 8.323% [= 6.166% × ($e^{2\times0.15}$)], and the rate for the lower node is 4.568% [= 6.166%/($e^{2\times0.15}$)]. Exhibit 19 shows that these rates for Time 2 and the already calibrated rates for Time 1 lead to a value of 99.898 for the 4% three-year bond as of Time 0. These are not the arbitrage-free rates: The Time 2 rates need to be lowered slightly to get the price up to 100.000.

EXHIBIT 19 Calibration of Time 2 Forward Rates

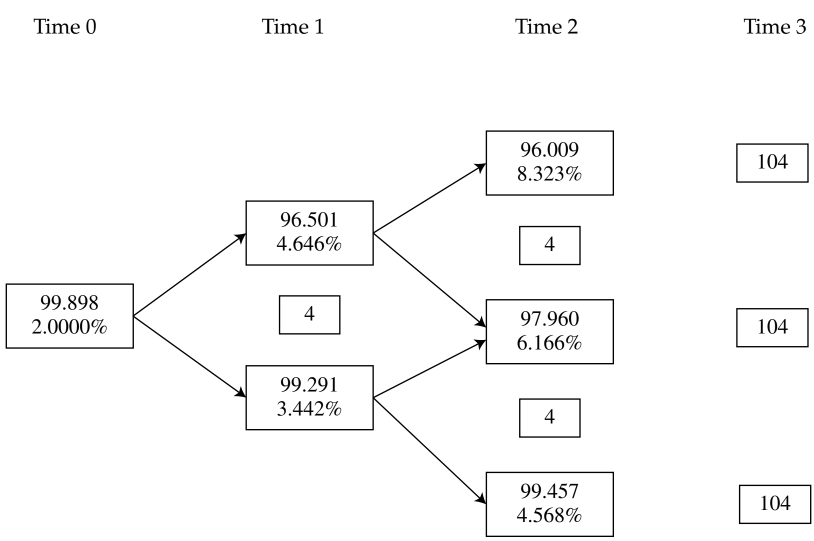


Exhibit 20 displays the completed binomial tree. The calibrated forward rates for Time 2 are 8.167%, 6.050%, and 4.482%. These are the calculations:

$$104/1.08167 = 96.148$$

$$104/1.06050 = 98.067$$

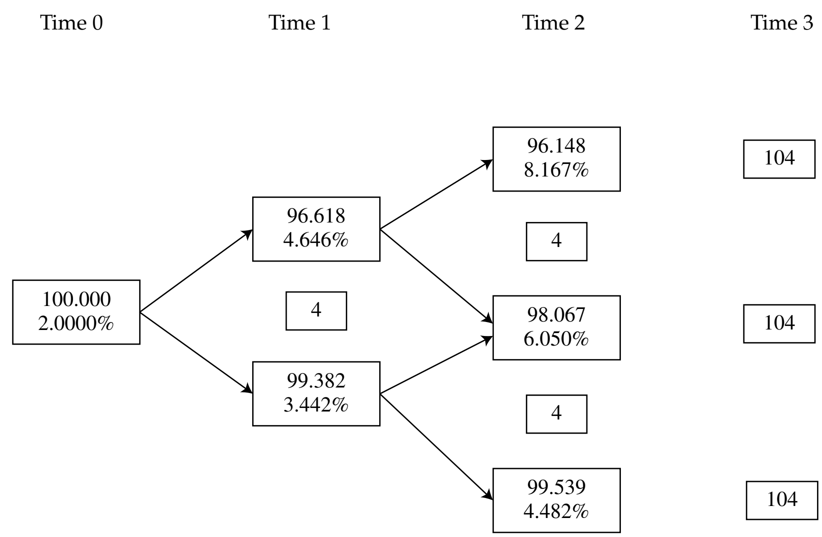
$$104/1.04482 = 99.538$$

$$\frac{4 + (0.5 \times 96.148 + 0.5 \times 98.067)}{1.04646} = 96.618$$

$$\frac{4 + (0.5 \times 98.067 + 0.5 \times 99.539)}{1.03442} = 99.382$$

$$\frac{4 + (0.5 \times 96.618 + 0.5 \times 99.382)}{1.02000} = 100.000$$

EXHIBIT 20 Completed Binomial Tree



Now that our tree gives the correct prices for the underlying par bonds maturing in one, two, and three years, we say that our tree is calibrated to be arbitrage free. It will price option-free bonds correctly, including prices for the zero-coupon bonds used to find the spot rates, and to the extent that we have chosen an appropriate interest rate process and interest rate volatility, it will provide insights into the value of bonds with embedded options and their risk parameters.

5. VALUING AN OPTION-FREE BOND WITH A BINOMIAL TREE

- **describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node**
- **compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice**

Our next task is twofold. First, we calculate the arbitrage-free value of an option-free, fixed-rate coupon bond. Second, we compare the pricing using the zero-coupon yield curve with the pricing using an arbitrage-free binomial lattice. Because these two valuation methods are arbitrage free, these two values must be the same.

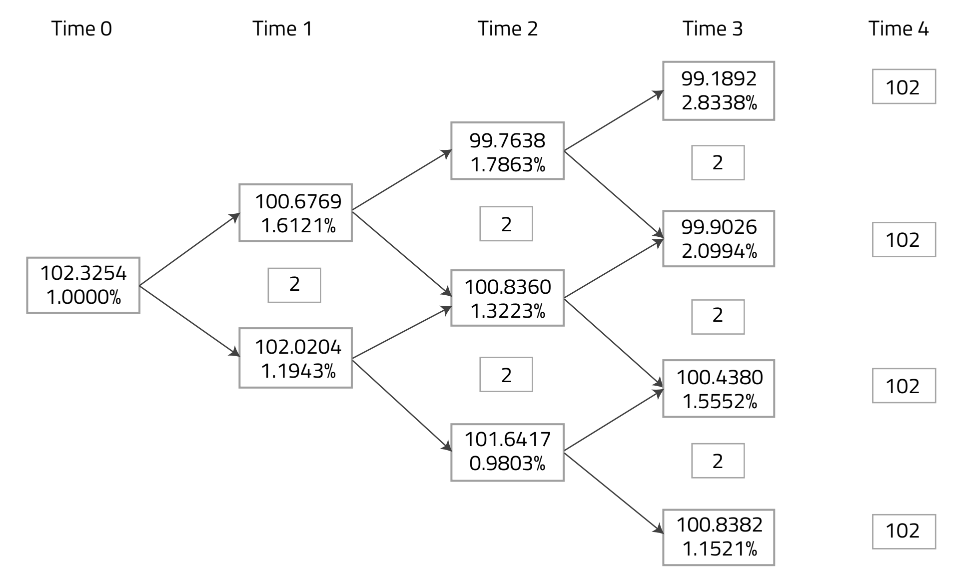
Now, consider an option-free bond with four years remaining to maturity and a coupon rate of 2%. Note that this is not a benchmark bond and it carries a higher coupon and price than the four-year benchmark bond, which is priced at par. The value of this bond can be calculated by discounting the cash flow at the spot rates in Exhibit 3 as shown in the following equation:

$$\frac{2}{(1.01)^1} + \frac{2}{(1.012012)^2} + \frac{2}{(1.012515)^3} + \frac{102}{(1.014044)^4} = 102.3254$$

The binomial interest rate tree should produce the same value as when discounting the cash flows with the spot rates. An option-free bond that is valued by using the binomial interest rate tree should have the same value as when discounting by the spot rates, which is true because the binomial interest rate tree is arbitrage free.

Let us give the tree a test run and use the 2% option-free bond with four years remaining to maturity. Also assume that the issuer's benchmark yield curve is the one given in Exhibit 2; hence the appropriate binomial interest rate tree is the one in Exhibit 10. Exhibit 21 shows the various values in the discounting process and obtains a bond value of 102.3254. The tree produces the same value for the bond as the spot rates produce and is therefore consistent with our standard valuation model.

EXHIBIT 21 Sample Valuation for an Option-Free Bond using a Binomial Tree



EXAMPLE 5 Confirming the Arbitrage-Free Value of a Bond

Using the par curve from Example 2 and Example 4, the yield-to-maturity for a one-year annual coupon bond is 2%, for a two-year annual coupon bond is 3%, and for a three-year annual coupon bond is 4%. Because this is the same curve as that used in Example 4, we can use the calibrated tree from that example to price a bond. Let us use a three-year annual coupon bond with a 5% coupon, just as we did in Example 2. We know that if the calibrated tree was built correctly and we perform calculations to value the bond with the tree shown in Exhibit 22, its price should be 102.8105.

EXHIBIT 22 Binomial Tree from Example 5

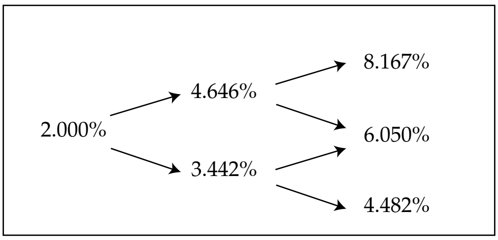
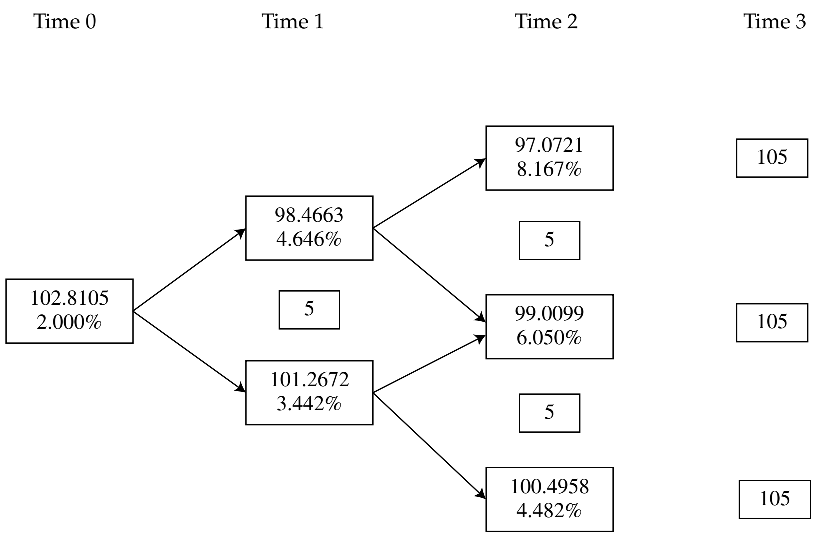


EXHIBIT 23 Valuing a 5%, Three-Year Bond



Because the tree was calibrated to the same par curve (and spot curve) that was used to price this option-free bond using spot rates only, the tree gives the same price as the spot rate pricing (the small difference is due to rounding).

6. VALUING AN OPTION-FREE BOND WITH PATHWISE VALUATION

- **describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path**

Pathwise valuation is an alternative approach to backward induction in a binomial tree. The binomial interest rate tree specifies all potential rate paths in the model, whereas an interest rate path is the route an interest rate takes from the current time to the security's maturity. Pathwise valuation calculates the present value of a bond for each possible interest rate path and takes the average of these values across paths. We will use the pathwise valuation approach to produce the same value as the backward induction method for an option-free bond. Pathwise valuation involves the following steps: (1) Specify a list of all potential paths through the tree, (2) determine the present value of a bond along each potential path, and (3) calculate the average across all possible paths.

Determining all potential paths is similar to the following experiment. Suppose you are tossing a fair coin and tracking how many ways heads and tails can be combined. We will use a device called Pascal's Triangle, displayed in Exhibit 24. Pascal's Triangle can be built as follows: Start with the number 1 at the top of the triangle. The numbers in the boxes below are the sum of the two numbers above it except that the edges on each side are all 1. The shaded numbers show that 3 is the sum of 2 and 1. Now toss the coin while keeping track of the possible outcomes. The possible groupings are listed in Exhibit 25, where H stands for heads and T stands for tails.

EXHIBIT 24 Pascal's Triangle

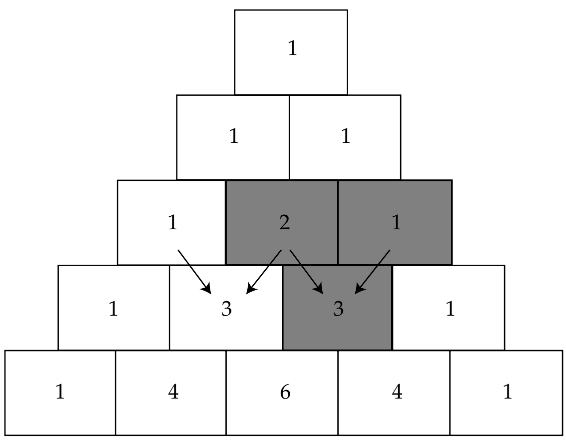


EXHIBIT 25 Possible Outcomes of Coin Tosses

Number of Tosses	Possible Outcomes	Pascal's Triangle
1	H T	1, 1
2	HH HT TH TT	1,2,1
3	HHH HHT HTH THH HTT THT TTH TTT	1, 3, 3, 1

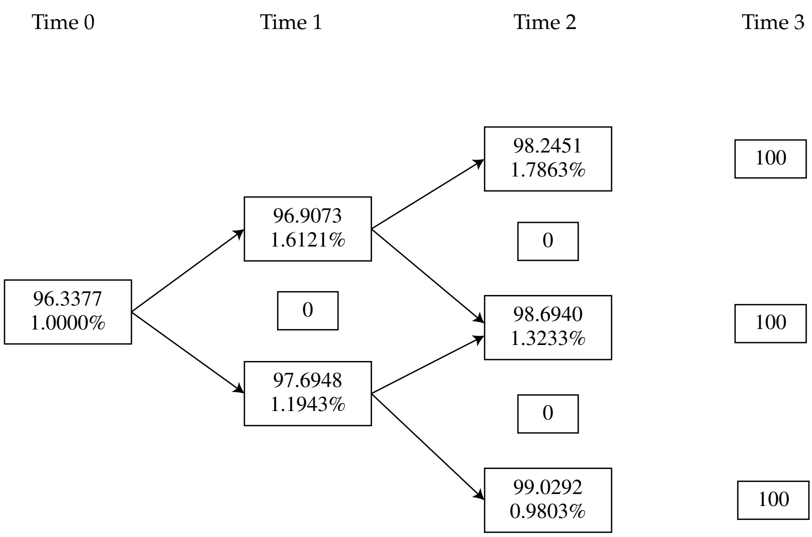
This experiment mirrors exactly the number of interest rate paths in our binomial interest rate tree. The total number of paths for each period/year can be easily determined by using Pascal's Triangle. Let us work through an example for a three-year zero-coupon bond. From Pascal's Triangle, there are four possible paths to arrive at Year 3: HH, HT, TH, TT. Using the same binomial tree from Exhibit 21, we specify the four paths as well as the possible forward rates along those paths. In Exhibit 26, the last column on the right shows the present value for each path. For example, 100/(1.01000 × 1.016121 × 1.017863) = 95.7291. In the bottom right corner is the average present value across all paths.

EXHIBIT 26 Four Interest Rate Paths for a Three-Year Zero-Coupon Bond

Path	Forward Rate Year 1	Forward Rate Year 2	Forward Rate Year 3	Present Value
1	1.0000%	1.6121%	1.7863%	95.7291
2	1.0000%	1.6121%	1.3233%	96.1665
3	1.0000%	1.1943%	1.3233%	96.5636
4	1.0000%	1.1943%	0.9803%	96.8916
				96.3377

Now, we can use the binomial tree to confirm our calculations for the three-year zero-coupon bond. The analysis is presented in Exhibit 27. The interest rate tree does indeed produce the same value.

EXHIBIT 27 Binomial Tree to Confirm Bond's Value



EXAMPLE 6 Pathwise Valuation Based on a Binomial Interest Rate Tree

Using the par curve from Example 2, Example 4, and Example 5, the yield-to-maturity for a one-year annual coupon bond is 2%, for a two-year annual coupon bond is 3%, and for a three-year annual coupon bond is 4%. We know that if we generate the paths in the tree correctly and discount the cash flows directly, the three-year, 5% annual coupon bond should still be priced at 102.8105, as calculated in Example 5.

There are four paths through the three-year tree. We discount the cash flows along each of the four paths and take their average, as shown in Exhibits 28, 29, and 30.

EXHIBIT 28 Cash Flows

Path	Time 0	Time 1	Time 2	Time 3
1	0	5	5	105
2	0	5	5	105
3	0	5	5	105
4	0	5	5	105

EXHIBIT 29 Discount Rates

Path	Time 0	Time 1	Time 2	Time 3
1	2.000%	4.646%	8.167%	
2	2.000%	4.646%	6.050%	
3	2.000%	3.442%	6.050%	
4	2.000%	3.442%	4.482%	

EXHIBIT 30 Present Values

Path	Time 0
1	100.5298
2	102.3452
3	103.4794
4	104.8877
Average	**102.8105**

The present values are calculated by discounting the cash flows in Exhibit 28 by the forward rates in Exhibit 29. For example, the present value for the bond along Path 1 is 100.5298:

$$\frac{5}{1.02} + \frac{5}{(1.02)(1.04646)} + \frac{105}{(1.02)(1.04646)(1.08167)} = 100.5298$$

The present value along Path 3 is 103.4794:

$$\frac{5}{1.02} + \frac{5}{(1.02)(1.03442)} + \frac{105}{(1.02)(1.03442)(1.06050)} = 103.4794$$

The average for the bond prices using pathwise valuation is 102.8105, which matches the result obtained using backward induction in Exhibit 23.

7. THE MONTE CARLO METHOD

- **describe a Monte Carlo forward-rate simulation and its application**

The Monte Carlo method is an alternative method for simulating a sufficiently large number of potential interest rate paths to discover how the value of a security is affected. This method involves randomly selecting paths to approximate the results of a complete pathwise valuation. Monte Carlo methods are often used when a security's cash flows are path dependent. Cash flows are path dependent when the cash flow to be received depends on the path followed to reach its current level as well as the current level itself. For example, the valuation of mortgage-backed securities depends to a great extent on the level of prepayments. As mentioned in an earlier chapter, prepayments tend to increase when interest rates fall, because borrowers are more likely to pay off mortgage loans and refinance at lower interest rates. Interest rate paths are generated on the basis of some probability distribution and a volatility assumption, and the model is fit to the current benchmark term structure of interest rates. The benchmark term structure is represented by the current spot rate curve such that the average present value across all scenario interest rate paths for each benchmark bond equals its actual market value. By using this approach, the model is rendered arbitrage free, which is equivalent to calibrating the interest rate tree as discussed in Section 3.

Suppose we intend to value with the Monte Carlo method a 30-year bond that has monthly coupon payments (e.g., mortgage-backed securities). The following steps are taken: (1) Simulate numerous (say, 500) paths of one-month interest rates under a volatility assumption and probability distribution, (2) generate spot rates from the simulated future one-month interest rates, (3) determine the cash flow along each interest rate path, (4) calculate the present value for each path, and (5) calculate the average present value across all interest rate paths.

Using the procedure just described, the model will produce benchmark bond values equal to the market prices only by chance. We want to ensure this is the case; otherwise the model will neither fit the current spot curve nor be arbitrage free. A constant is added to all interest rates on all paths such that the average present value for each benchmark bond equals its market value. The constant added to all short interest rates is called a drift term. When this technique is used, the model is said to be drift adjusted.

How many paths are appropriate for the Monte Carlo method? More paths increase the accuracy of the estimate in a statistical sense, but this does not mean the model is closer to the true fundamental value of the security. The Monte Carlo method is only as good as the valuation model used and the accuracy of the inputs.

Yield curve modelers also often include mean reversion in their Monte Carlo estimation. Mean reversion starts with the common-sense notion that history suggests that interest rates almost never get "too high" or "too low." What is meant by "too high" and "too low" is left to the discretion of the modeler. We implement mean reversion by implementing upper and lower bounds on the random process generating future interest rates. Mean reversion has the effect of moving the interest rate toward the implied forward rates from the yield curve.

EXAMPLE 7 The Application of Monte Carlo Simulation to Bond Pricing

Replace the interest rate paths from Example 6 with randomly generated paths calibrated to the same initial par and spot curves, as shown in Exhibit 31.

EXHIBIT 31 Discount Rates

Path	Time 0	Time 1	Time 2
1	2.000%	2.500%	4.548%
2	2.000%	3.600%	6.116%
3	2.000%	4.600%	7.766%
4	2.000%	5.500%	3.466%
5	2.000%	3.100%	8.233%
6	2.000%	4.500%	6.116%
7	2.000%	3.800%	5.866%
8	2.000%	4.000%	8.233%

EXHIBIT 32 Present Values

Path	Time 0
1	105.7459
2	103.2708
3	100.9104
4	103.8543
5	101.9075
6	102.4236
7	103.3020
8	101.0680
Average	**102.8103**

Because we continue to get 102.8103, as shown in Exhibit 32, as the price for our three-year, 5% annual coupon bond, we know that the Monte Carlo simulation has been calibrated correctly. The paths are now different enough such that path-dependent securities, such as mortgage-backed securities, can be analyzed in ways that provide insights not possible in binomial trees, because Monte Carlo techniques provide greater flexibility to change parameters over time.

8. TERM STRUCTURE MODELS

- **describe term structure models and how they are used**

Term structure models provide quantitatively precise descriptions of how interest rates evolve. A model provides a simplified description of a real-world phenomenon on the basis of a set of assumptions. These assumptions cannot be completely accurate in depicting the real world but are necessary for analytical tractability. Despite simplifying assumptions, models explain real-world phenomena sufficiently well to be useful for pricing and hedging.

The binomial tree and Monte Carlo simulation valuation approaches for complex fixed-income instruments described earlier rely on specific assumptions about the underlying asset properties. For example, how do we establish the node values in the binomial trees, and what determines the dispersion in rates from the top to the bottom nodes? This answer comes from term structure models, which make assumptions about the properties of rates over time and then use those properties to "fit," or determine the values of the rates at each node, binomial lattices used for pricing and risk management applications. The following section introduces common term structure models, with an emphasis on the underlying assumptions about the statistical properties of interest rates. Each of the models can be "fit" to lattice models for valuation and risk management applications.

Modeling the future path of interest rates is not only critical for scenario analysis and stress testing individual bonds and bond portfolio values but also important in the valuation of complex fixed-income instruments. A detailed description of these models depends on mathematical and statistical knowledge beyond the scope of this chapter, but fixed-income practitioners will often find that these or other term structure models are embedded in many of the desktop tools and data analytics software they may use during their investment industry career. Thus, we provide a broad overview of these models in this chapter.

8.1. Model Choice

Term structure models go beyond the lognormal random walk approach used earlier to describe the dynamics of the term structure for the purpose of pricing and hedging fixed-income securities and derivatives. All term structure models make simplifying assumptions about the evolution of rates over time. Many different interest rate models that differ in their assumptions exist. Arguably, there are many models, since no one model perfectly captures interest rate dynamics. Modelers face a trade-off between simplicity and accuracy when selecting a term structure model. Practitioners should be aware of the categories of models and their important features (which stem from their assumptions) as well as how those features affect pricing and hedging.

8.1.1. Interest Rate Factors

The valuation and hedging of fixed-income securities and their derivatives require information across the entire term structure. To develop a term structure model useful for pricing and hedging applications, we focus on modeling the factors that determine the term structure. The simplest class of models use one factor—the short rate, or the one-period rate—as the factor that drives the term structure. Although the use of one factor may seem limiting, because it implies all rates move in the same direction during any short time interval, it does not mean they have to move by the same amounts. Multi-factor models incorporate additional factors, such as the slope of the term structure, with the complexity of the models increasing in the number of factors.

8.1.2. Interest Rate Process

Term structure models use stochastic processes to describe interest rate dynamics. These stochastic processes have two components: a drift term and an uncertain, or stochastic, term. Although the stochastic processes are continuous time, the models can be "fit" to binomial or trinomial interest rate lattices using a discrete version of the models (integrating over time to obtain rates that span time intervals).

For a one-factor model, the general form of the process describing the short rate's (r) dynamics is

$$dr = \theta_t dt + \sigma_t dZ$$

The drift term, $\theta_t dt$, describes the expected (zero-volatility) rate path. For example, in a one-factor model of the short rate, the drift describes the expected evolution of the short rate over time. The drift term may be constant or mean reverting.

The second term, $\sigma_t dZ$, adds randomness, or volatility, to the process. This dispersion term allows for the pricing of bonds with option features as well as interest rate derivatives and may take a variety of forms. The term Z is a Weiner process that is distributed normally. Given the symmetry of the normal distribution, it is possible and quite common for these models to produce interest rate paths with negative rates.

Within classes of models, such as one-factor no-arbitrage models, the key differences between the various models involve the stochastic difference equation.

8.1.3. Class of Model

One class of models uses the arbitrage-free approach combined with assumptions about the statistical properties of interest rates. This class of models is referred to as no-arbitrage term structure models, where no-arbitrage is synonymous with arbitrage free. No-arbitrage term structure models begin with a set of assumptions about the term structure—a factor (or factors) and the stochastic process describing the factor evolution(s)—and take the term structure as given, assuming that both bond prices and the term structure bootstrapped from those prices are correct. The no-arbitrage models are "parameterized," which is the process of determining the values of the variables in the model such that those parameters produce bond prices that match current market prices. These models are used widely in practice and are often favored by practitioners since their pricing results are consistent with market prices.

Equilibrium term structure models seek to describe term structure dynamics using fundamental economic variables that are assumed to affect interest rates. The modeling process imposes restrictions that allow for the derivation of equilibrium prices for bonds and interest rate options.

Although equilibrium models use similar continuous stochastic difference equations to describe interest rate changes, equilibrium model parameters are not forced to values that produce bond prices consistent with current market prices. This property is seen by some market participants as a significant drawback in a static setting, such as pricing and hedging for the current time. However, other practitioners prefer equilibrium models since they capture not just the current market environment as reflected in the term structure but also the possibility of many different future paths. For more dynamic applications, equilibrium models may be preferred.

The best-known equilibrium models are the **Cox-Ingersoll-Ross model** (Cox, Ingersoll, and Ross 1985) and the **Vasicek model** (Vasicek 1977), discussed in the next two sections.

Both the Vasicek and Cox–Ingersoll–Ross (CIR) models assume a single factor, the short-term interest rate, r_t. This approach is plausible because empirically, parallel shifts are often found to explain more than 90% of yield changes. In contrast, multifactor models may be able to model the curvature of a yield curve more accurately, but at the cost of greater complexity.

The reason that no-arbitrage models fit the current term structure is their greater number of parameters. These added parameters increase the computational requirements for estimation, which some practitioners find to be undesirable.

Other contrasts are more technical. They include that equilibrium models use real probabilities, whereas arbitrage-free models use so-called risk-neutral probabilities. An excellent example of an equilibrium term structure model is the Cox–Ingersoll–Ross model, discussed next.

8.2. Equilibrium Models

This section introduces the Cox–Ingersoll–Ross and Vasicek interest rate models.

8.2.1. The Cox–Ingersoll–Ross Model

The Cox–Ingersoll–Ross (CIR) model assumes interest rates follow a mean-reverting process. However, the variance of rate changes differs depending on the level of rates. The CIR model uses the following formula to describe the interest rate process:

$$dr_t = k(\theta - r_t)\,dt + \sigma\sqrt{r_t}\,dZ$$

Note that the drift term has three components. The level of rates at time t is r_t, and θ is the long-run mean rate, so their difference is the distance of the rate from its mean. The drift term equals zero if the rate is at the long-run mean, or $r_t = \theta$. The remaining drift term parameter, k, modulates the speed at which the rate reverts to its mean.

Another important feature of the CIR model is that the random component varies as rates change. In other words, the short-rate volatility is a function of the short rate. Importantly, at low rates, r_t, the term becomes small, which prevents rates from turning negative.

8.2.2. The Vasicek Model

Although not developed in the context of a general equilibrium of individuals seeking to optimize consumption and investment decisions, as was the case for the CIR model, the Vasicek model is viewed as an equilibrium term structure model. Similar to the CIR model, the Vasicek model includes mean reversion. The Vasicek model uses the following equation to describe the interest rate process:

$$dr_t = k(\theta - r_t)\,dt + \sigma dZ$$

The Vasicek model has the same drift term as the CIR model and thus tends toward mean reversion in the short rate. The stochastic or volatility term follows a random normal distribution for which the mean is zero and the standard deviation is 1. Unlike the CIR model, interest rates are calculated assuming constant volatility over the period of analysis. As with the CIR model, there is only one stochastic driver of the interest rate process. A key characteristic of the Vasicek model worth noting is that it is theoretically possible for the interest rate to become negative.

8.3. Arbitrage-Free Models

We will next illustrate two foundational no-arbitrage term structure models. There are many additional no-arbitrage models, but the basic features are similar, with differences stemming from different assumed interest rate processes.

8.3.1. The Ho–Lee Model

In **arbitrage-free models**, the analysis begins with the current term structure, extrapolated from the market prices of a reference set of financial instruments. A maintained assumption is that the reference bonds are priced correctly. Unlike general equilibrium models, which have only a few parameters and can thus match only a few term structure points, arbitrage-free models allow the parameters to vary deterministically with time, creating a greater number of parameters and thus more points of match. As a result, the market yield curve can be modeled with the accuracy needed for such applications as valuing derivatives and bonds with embedded options.

The first arbitrage-free model was introduced by Ho and Lee (1986). The model is calibrated to market data and uses a binomial lattice approach to generate a distribution of possible future interest rates. In the **Ho–Lee model**, the short rate follows a normal process, as follows:

$$dr_t = \theta_t dt + \sigma dZ$$

We see that the drift term, θ_t, is time dependent. This time dependency means there is a value for θ_t at each time step, which is critical for the model to produce prices that match market prices.

The Ho–Lee model, similar to the Vasicek model, has constant volatility, and interest rates may become negative because of the symmetry of the normal distribution and the model's use of constant volatility.

8.3.2. The Kalotay–Williams–Fabozzi Model

The **Kalotay–Williams–Fabozzi (KWF) model** is analogous to the Ho–Lee model in that it assumes constant drift, no mean reversion, and constant volatility. However, the stochastic differential equation describes the dynamics of the log of the short rate, and as a result, the log of the short rate is distributed normally, meaning the short rate itself is distributed lognormally.

The differential process for the KWF model is

$$d\ln(r_t) = \theta_t dt + \sigma dZ$$

At first glance, the main implication of modeling the log of the short rate is that it will prevent negative rates. After further analysis, it becomes evident that there are pricing implications where interest rate option values are influenced by the tails of the rate distributions. Exhibit 33 summarizes the key differences between these term structure models.

EXHIBIT 33 Term Structure Model Summary

Model	Type	Short Rate	Drift Term	Volatility
CIR	Equilibrium	dr_t	Mean reversion at speed k	Varies with $\sqrt{r_t}$
Vasicek	Equilibrium	dr_t	Mean reversion at speed k	Constant
Ho–Lee	Arbitrage free	dr_t	Time dependent	Constant
KWF	Arbitrage free	$d\ln(r_t)$	Time dependent	Constant

8.4. Modern Models

The one-factor models presented thus far are the building blocks on which modern interest rate models rely. Some current models extend those models to include multiple factors, while others use sophisticated approaches that combine observed forward curves with volatilities extracted from interest rate option prices.

The Gauss+ model is a multi-factor interest rate model used extensively in valuation and hedging. The Gauss+ model incorporates short-, medium- and long-term rates. The long-term factor is mean reverting and reflects trends in macroeconomic variables. The medium-term rate also reverts to the long-run rate. The short-term rate does not exhibit a random component, which is consistent with the central bank controlling the short end of the rate curve. This results in a hump-shaped volatility curve across tenors, with medium-term rates being the most volatile.

Although there are many different term structure models, knowledge of the basic assumptions and design of the classic models helps professionals understand and adapt more sophisticated modern models.

Example 8 addresses several basic points about modern term structure models.

EXAMPLE 8 Term Structure Models

1. Which of the following would be expected to provide the *most* accurate modeling with respect to the observed term structure?
   A. CIR model
   B. Ho–Lee model
   C. Vasicek model
2. Which of the following statements about the Vasicek model is *most* accurate? It has:
   A. a single factor, the long rate.
   B. a single factor, the short rate.
   C. two factors, the short rate and the long rate.
3. The CIR model:
   A. assumes interest rates are not mean reverting.
   B. has a drift term that differs from that of the Vasicek model.
   C. assumes interest rate volatility increases with increases in the level of interest rates.

Solution to 1: B is correct. The CIR model and the Vasicek model are examples of equilibrium term structure models, whereas the Ho–Lee model is an example of an arbitrage-free term structure model. A benefit of arbitrage-free term structure models is that they are calibrated to the current term structure. In other words, the starting prices ascribed to securities are those currently found in the market. In contrast, equilibrium term structure models frequently generate term structures that are inconsistent with current market data.

Solution to 2: B is correct. Use of the Vasicek model requires assumptions for the short-term interest rate, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for the short-term rate, one can determine the yield on longer-term bonds by looking at the expected path of interest rates over time.

Solution to 3: C is correct. The drift term of the CIR model is identical to that of the Vasicek model, and both models assume that interest rates are mean reverting. The major difference between the two models is that the CIR model assumes a rise in interest rate volatility as rates increase, while the Vasicek model assumes interest rate volatility is constant.

SUMMARY

This chapter presents the principles and tools for arbitrage valuation of fixed-income securities. Much of the discussion centers on the binomial interest rate tree, which can be used extensively to value both option-free bonds and bonds with embedded options. The following are the main points made in the chapter:

- A fundamental principle of valuation is that the value of any financial asset is equal to the present value of its expected future cash flows.
- A fixed-income security is a portfolio of zero-coupon bonds, each with its own discount rate that depends on the shape of the yield curve and when the cash flow is delivered in time.
- In well-functioning markets, prices adjust until there are no opportunities for arbitrage, or a transaction that involves no cash outlay yet results in a riskless profit.
- Using the arbitrage-free approach, viewing a security as a package of zero-coupon bonds means that two bonds with the same maturity and different coupon rates are viewed as different packages of zero-coupon bonds and valued accordingly.
- For bonds that are option-free, an arbitrage-free value is simply the present value of expected future values using the benchmark spot rates.
- A binomial interest rate tree permits the short interest rate to take on one of two possible values consistent with the volatility assumption and an interest rate model based on a lognormal random walk.
- An interest rate tree is a visual representation of the possible values of interest rates (forward rates) based on an interest rate model and an assumption about interest rate volatility.
- The possible interest rates for any following period are consistent with the following three assumptions: (1) an interest rate model that governs the random process of interest rates, (2) the assumed level of interest rate volatility, and (3) the current benchmark yield curve.
- From the lognormal distribution, adjacent interest rates on the tree are multiples of e raised to the 2σ power, with the absolute change in interest rates becoming smaller and smaller as rates approach zero.
- We use the backward induction valuation methodology that involves starting at maturity, filling in those values, and working back from right to left to find the bond's value at the desired node.
- The interest rate tree is fit to the current yield curve by choosing interest rates that result in the benchmark bond value. By doing this, the bond value is arbitrage free.
- An option-free bond that is valued by using the binomial interest rate tree should have the same value as when discounting by the spot rates.
- Pathwise valuation calculates the present value of a bond for each possible interest rate path and takes the average of these values across paths.
- The Monte Carlo method is an alternative method for simulating a sufficiently large number of potential interest rate paths in an effort to discover how the value of a security is affected, and it involves randomly selecting paths in an effort to approximate the results of a complete pathwise valuation.

- Term structure models seek to explain the yield curve shape and are used to value bonds (including those with embedded options) and bond-related derivatives. General equilibrium and arbitrage-free models are the two major types of such models.
- Arbitrage-free models are frequently used to value bonds with embedded options. Unlike equilibrium models, arbitrage-free models begin with the observed market prices of a reference set of financial instruments, and the underlying assumption is that the reference set is correctly priced.

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PRACTICE PROBLEMS

The following information relates to Questions 1–6

Katrina Black, a portfolio manager at Coral Bond Management, Ltd., is conducting a training session with Alex Sun, a junior analyst in the fixed-income department. Black wants to explain to Sun the arbitrage-free valuation framework used by the firm. Black presents Sun with Exhibit 1, showing a fictitious bond being traded on three exchanges, and asks Sun to identify the arbitrage opportunity of the bond. Sun agrees to ignore transaction costs in his analysis.

EXHIBIT 1 Three-Year, €100 par, 3.00% Coupon, Annual Pay Option-Free Bond

	Eurex	NYSE Euronext	Frankfurt
Price	€103.7956	€103.7815	€103.7565

Black shows Sun some exhibits that were part of a recent presentation. Exhibit 3 presents most of the data of a binomial lognormal interest rate tree fit to the yield curve shown in Exhibit 2. Exhibit 4 presents most of the data of the implied values for a four-year, option-free, annual pay bond with a 2.5% coupon based on the information in Exhibit 3.

EXHIBIT 2 Yield-to-Maturity Par Rates for One-, Two-, and Three-Year Annual Pay Option-Free Bonds

One-year	Two-year	Three-year
1.25%	1.50%	1.70%

EXHIBIT 3 Binomial Interest Rate Tree Fit to the Yield Curve (Volatility = 10%)

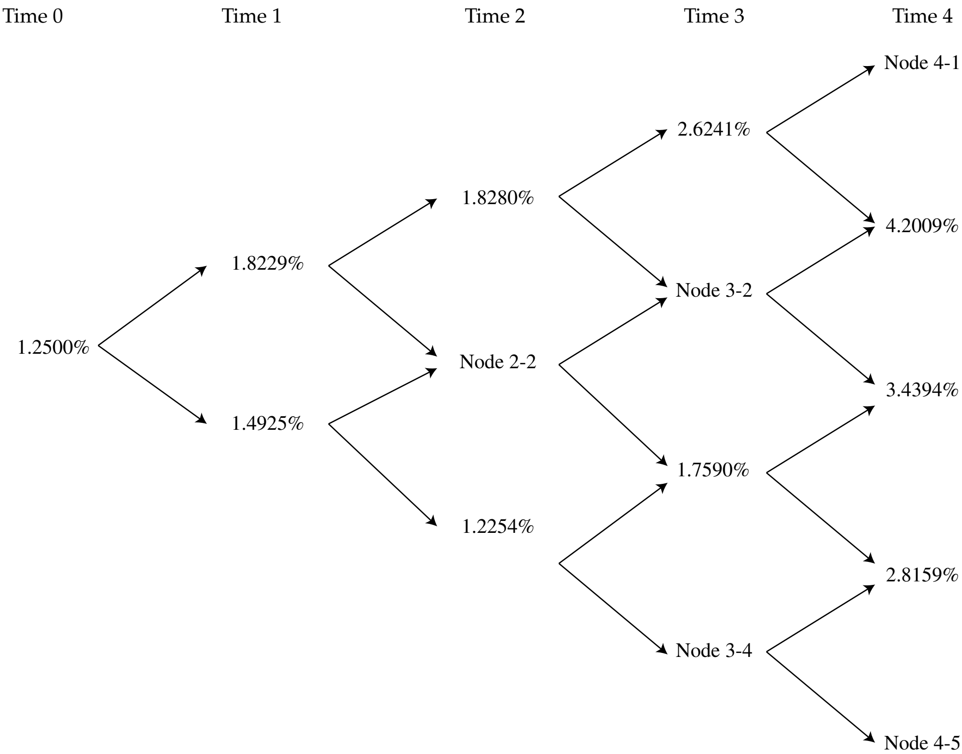
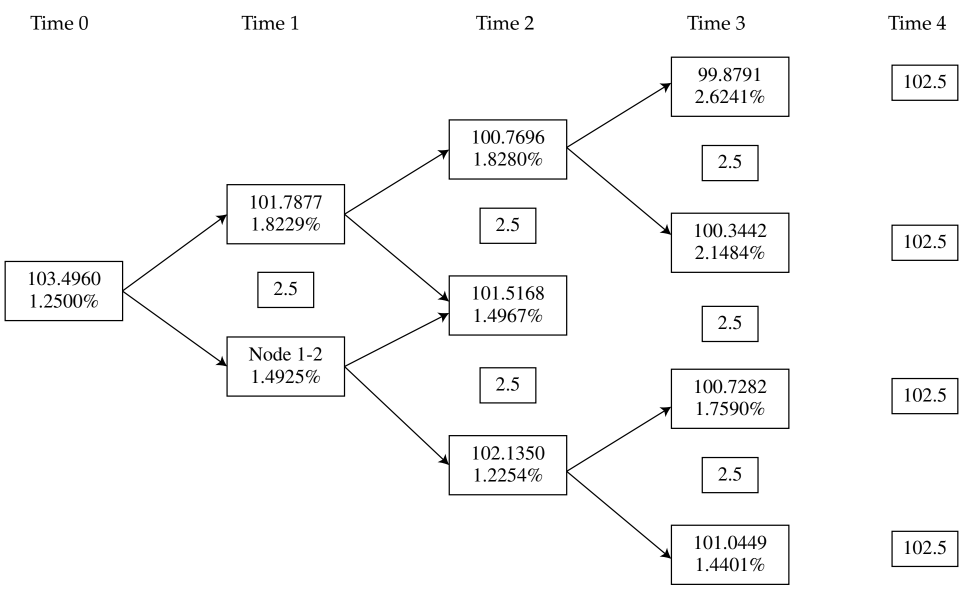


EXHIBIT 4 Implied Values (in Euros) for a 2.5%, Four-Year, Option-Free, Annual Pay Bond Based on Exhibit 3



Black asks about the missing data in Exhibits 3 and 4 and directs Sun to complete the following tasks related to those exhibits:

Task 1	Test that the binomial interest tree has been properly calibrated to be arbitrage free.
Task 2	Develop a spreadsheet model to calculate pathwise valuations. To test the accuracy of the spreadsheet, use the data in Exhibit 3 and calculate the value of the bond if it takes a path of lowest rates in Year 1 and Year 2 and the second lowest rate in Year 3.
Task 3	Identify a type of bond where the Monte Carlo calibration method should be used in place of the binomial interest rate method.
Task 4	Update Exhibit 3 to reflect the current volatility, which is now 15%.

1. Based on Exhibit 1, the *best* action that an investor should take to profit from the arbitrage opportunity is to:
   A. buy on Frankfurt, sell on Eurex.
   B. buy on NYSE Euronext, sell on Eurex.
   C. buy on Frankfurt, sell on NYSE Euronext.
2. Based on Exhibits 1 and 2, the exchange that reflects the arbitrage-free price of the bond is:
   A. Eurex.
   B. Frankfurt.
   C. NYSE Euronext.
3. Recall from the chapter that each node is represented by both a time element and a rate change component. Which of the following statements about the missing data in Exhibit 3 is correct?
   A. Node 3–2 can be derived from Node 2–2.
   B. Node 4–1 should be equal to Node 4–5 multiplied by $e^{0.4}$.
   C. Node 2–2 approximates the implied one-year forward rate two years from now.
4. Based on the information in Exhibits 3 and 4, the bond price in euros at Node 1–2 in Exhibit 4 is *closest* to:
   A. 102.7917.
   B. 104.8640.
   C. 105.2917.
5. A benefit of performing Task 1 is that it:
   A. enables the model to price bonds with embedded options.
   B. identifies benchmark bonds that have been mispriced by the market.
   C. allows investors to realize arbitrage profits through stripping and reconstitution.
6. If the assumed volatility is changed as Black requested in Task 4, the forward rates shown in Exhibit 3 will *most likely*:
   A. spread out.
   B. remain unchanged.
   C. converge to the spot rates.

The following information relates to Questions 7–10

Betty Tatton is a fixed-income analyst with the hedge fund Sailboat Asset Management (SAM). SAM invests in a variety of global fixed-income strategies, including fixed-income arbitrage. Tatton is responsible for pricing individual investments and analyzing market data to assess the opportunity for arbitrage. She uses two methods to value bonds:

Method 1: Discount each year's cash flow separately using the appropriate interest rate curve.
Method 2: Build and use a binomial interest rate tree.

Tatton compiles pricing data for a list of annual pay bonds (Exhibit 1). Each of the bonds will mature in two years, and Tatton considers the bonds risk-free; both the one-year and two-year benchmark spot rates are 2%. Tatton calculates the arbitrage-free prices and identifies an arbitrage opportunity to recommend to her team.

EXHIBIT 1 Market Data for Selected Bonds

Asset	Coupon	Market Price
Bond A	1%	98.0584
Bond B	3%	100.9641
Bond C	5%	105.8247

Next, Tatton uses the benchmark yield curve provided in Exhibit 2 to consider arbitrage opportunities of both option-free corporate bonds and corporate bonds with embedded options. The benchmark bonds in Exhibit 2 pay coupons annually, and the bonds are priced at par.

EXHIBIT 2 Benchmark Par Curve

Maturity (years)	Yield-to-Maturity (YTM)
1	3.0%
2	4.0%
3	5.0%

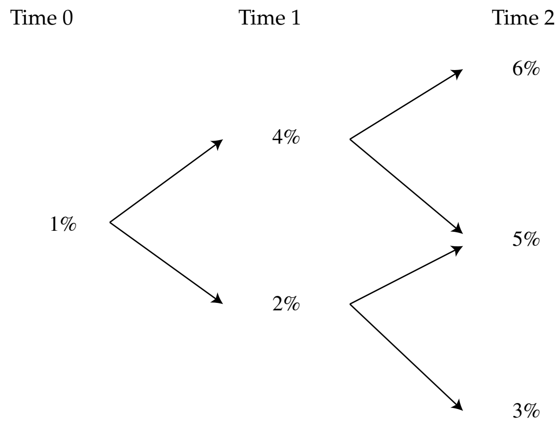
Tatton then identifies three mispriced three-year annual coupon bonds and compiles data on the bonds (see Exhibit 3).

EXHIBIT 3 Market Data of Annual Pay Corporate Bonds

Company	Coupon	Market Price	Yield	Embedded Option?
Hutto-Barkley Inc.	3%	94.9984	5.6%	No
Luna y Estrellas Intl.	0%	88.8996	4.0%	Yes
Peaton Scorpio Motors	0%	83.9619	6.0%	No

Lastly, Tatton identifies two mispriced Swiss bonds, Bond X, a three-year bond, and Bond Y, a five-year bond. Both are 6% annual coupon bonds. To calculate the bonds' values, Tatton devises the first three years of the interest rate lognormal tree presented in Exhibit 4 using historical interest rate volatility data. Tatton considers how these data would change if implied volatility, which is higher than historical volatility, were used instead.

EXHIBIT 4 Interest Rate Tree—Forward Rates Based on Swiss Market



7. Based on Exhibit 1, which of the following bonds *most likely* includes an arbitrage opportunity?
   A. Bond A
   B. Bond B
   C. Bond C
8. Based on Exhibits 2 and 3 and using Method 1, the amount (in absolute terms) by which the Hutto-Barkley Inc. corporate bond is mispriced is *closest* to:
   A. 0.3368 per 100 of par value.
   B. 0.4682 per 100 of par value.
   C. 0.5156 per 100 of par value.
9. Method 1 would *most likely* not be an appropriate valuation technique for the bond issued by:
   A. Hutto-Barkley Inc.
   B. Luna y Estrellas Intl.
   C. Peaton Scorpio Motors.
10. Based on Exhibit 4 and using Method 2, the correct price for Bond X is *closest* to:
   A. 97.2998.
   B. 109.0085.
   C. 115.0085.

The following information relates to Questions 11–18

Meredith Alvarez is a junior fixed-income analyst with Canzim Asset Management. Her supervisor, Stephanie Hartson, asks Alvarez to review the asset price and payoff data shown in Exhibit 1 to determine whether an arbitrage opportunity exists.

EXHIBIT 1 Price and Payoffs for Two Risk-Free Assets

Asset	Price Today	Payoff in One Year
Asset A	$500	$525
Asset B	$1,000	$1,100

Hartson also shows Alvarez data for a bond that trades in three different markets in the same currency. These data appear in Exhibit 2.

EXHIBIT 2 2% Coupon, Five-Year Maturity, Annual Pay Bond

	New York	Hong Kong	Mumbai
Yield-to-Maturity	1.9%	2.3%	2.0%

Hartson asks Alvarez to value two bonds (Bond C and Bond D) using the binomial tree in Exhibit 3. Exhibit 4 presents selected data for both bonds.

EXHIBIT 3 Binomial Interest Rate Tree with Volatility = 25%

Time 0	Time 1	Time 2
		2.7183%
	2.8853%	
1.500%		1.6487%
	1.7500%	
		1.0000%

EXHIBIT 4 Selected Data on Annual Pay Bonds

Bond	Maturity	Coupon Rate
Bond C	2 years	2.5%
Bond D	3 years	3.0%

Hartson tells Alvarez that she and her peers have been debating various viewpoints regarding the conditions underlying binomial interest rate trees. The following statements were made in the course of the debate.

Statement 1: The only requirements needed to create a binomial interest rate tree are current benchmark interest rates and an assumption about interest rate volatility.

Statement 2: Potential interest rate volatility in a binomial interest rate tree can be estimated using historical interest rate volatility or observed market prices from interest rate derivatives.

Statement 3: A bond value derived from a binomial interest rate tree with a relatively high volatility assumption will be different from the value calculated by discounting the bond's cash flows using current spot rates.

Based on data in Exhibit 5, Hartson asks Alvarez to calibrate a binomial interest rate tree starting with the calculation of implied forward rates shown in Exhibit 6.

EXHIBIT 5 Selected Data for a Binomial Interest Rate Tree

Maturity	Par Rate	Spot Rate
1	2.5000%	2.5000%
2	3.5000%	3.5177%

EXHIBIT 6 Calibration of Binomial Interest Rate Tree with Volatility = 25%

Time 0	Time 1
	5.8365%
2.500%	
	Lower one-period forward rate

Hartson mentions pathwise valuations as another method to value bonds using a binomial interest rate tree. Using the binomial interest rate tree in Exhibit 3, Alvarez calculates the possible interest rate paths for Bond D shown in Exhibit 7.

EXHIBIT 7 Interest Rate Paths for Bond D

Path	Time 0	Time 1	Time 2
1	1.500%	2.8853%	2.7183%
2	1.500	2.8853	1.6487
3	1.500	1.7500	1.6487
4	1.500	1.7500	1.0000

Before leaving for the day, Hartson asks Alvarez about the value of using the Monte Carlo method to simulate a large number of potential interest rate paths to value a bond. Alvarez makes the following statements.

Statement 4: Increasing the number of paths increases the estimate's statistical accuracy.

Statement 5: The bond value derived from a Monte Carlo simulation will be closer to the bond's true fundamental value.

11. Based on Exhibit 1, Alvarez finds that an arbitrage opportunity is:
    A. not available.
    B. available based on the dominance principle.
    C. available based on the value additivity principle.

12. Based on the data in Exhibit 2, the *most* profitable arbitrage opportunity would be to buy the bond in:
    A. Mumbai and sell it in Hong Kong.
    B. Hong Kong and sell it in New York.
    C. New York and sell it in Hong Kong.
13. Based on Exhibits 3 and 4, the value of Bond C at the upper node at Time 1 is *closest* to:
    A. 97.1957.
    B. 99.6255.
    C. 102.1255.
14. Based on Exhibits 3 and 4, the price for Bond D is *closest* to:
    A. 97.4785.
    B. 103.3230.
    C. 106.3230.
15. Which of the various statements regarding binomial interest rate trees is correct?
    A. Statement 1
    B. Statement 2
    C. Statement 3
16. Based on Exhibits 5 and 6, the value of the lower one-period forward rate is *closest* to:
    A. 3.5122%.
    B. 3.5400%.
    C. 4.8037%.
17. Based on Exhibits 4 and 7, the present value of Bond D's cash flows following Path 2 is *closest* to:
    A. 97.0322.
    B. 102.8607.
    C. 105.8607.
18. Which of the statements regarding Monte Carlo simulation is correct?
    A. Only Statement 4 is correct.
    B. Only Statement 5 is correct.
    C. Both Statement 4 and Statement 5 are correct.

---

19. Which term structure model can be calibrated to closely fit an observed yield curve?
    A. The Ho–Lee model
    B. The Vasicek model
    C. The Cox–Ingersoll–Ross model

The following information relates to Questions 20–21

Keisha Jones is a junior analyst at Sparling Capital. Julie Anderson, a senior partner and Jones's manager, meets with Jones to discuss interest rate models used for the firm's fixed-income portfolio.

Anderson begins the meeting by asking Jones to describe features of equilibrium and arbitrage-free term structure models. Jones responds by making the following statements:

Statement 1: Equilibrium term structure models are factor models that use the observed market prices of a reference set of financial instruments, assumed to be correctly priced, to model the market yield curve.

Statement 2: In contrast, arbitrage-free term structure models seek to describe the dynamics of the term structure by using fundamental economic variables that are assumed to affect interest rates.

Anderson then asks Jones about her preferences concerning term structure models. Jones states:

I prefer arbitrage-free models. Even though equilibrium models require fewer parameters to be estimated relative to arbitrage-free models, arbitrage-free models allow for time-varying parameters. In general, this allowance leads to arbitrage-free models being able to model the market yield curve more precisely than equilibrium models.

20. Which of Jones's statements regarding equilibrium and arbitrage-free term structure models is *incorrect*?
    A. Statement 1 only
    B. Statement 2 only
    C. Both Statement 1 and Statement 2
21. Is Jones correct in describing key differences in equilibrium and arbitrage-free models as they relate to the number of parameters and model accuracy?
    A. Yes
    B. No, she is incorrect about which type of model requires fewer parameter estimates.
    C. No, she is incorrect about which type of model is more precise at modeling market yield curves.

---

22. Which of the following statements comparing the Ho–Lee and Kalotay–Williams–Fabozzi (KWF) equilibrium term structure models is *correct*?
    A. The Ho–Lee model assumes constant volatility, while the KWF model does not.
    B. The KWF model incorporates the possibility of negative rates, while the Ho–Lee model does not.
    C. The KWF model describes the log of the dynamics of the short rate, while the Ho–Lee model does not.

CHAPTER 9

VALUATION AND ANALYSIS OF BONDS WITH EMBEDDED OPTIONS

Leslie Abreo, MFE
Ioannis Georgiou, CFA
Andrew Kalotay, PhD

LEARNING OUTCOMES

The candidate should be able to:

- describe fixed-income securities with embedded options;
- explain the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option;
- describe how the arbitrage-free framework can be used to value a bond with embedded options;
- explain how interest rate volatility affects the value of a callable or putable bond;
- explain how changes in the level and shape of the yield curve affect the value of a callable or putable bond;
- calculate the value of a callable or putable bond from an interest rate tree;
- explain the calculation and use of option-adjusted spreads;
- explain how interest rate volatility affects option-adjusted spreads;
- calculate and interpret effective duration of a callable or putable bond;
- compare effective durations of callable, putable, and straight bonds;
- describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options;
- compare effective convexities of callable, putable, and straight bonds;
- calculate the value of a capped or floored floating-rate bond;
- describe defining features of a convertible bond;
- calculate and interpret the components of a convertible bond's value;
- describe how a convertible bond is valued in an arbitrage-free framework;
- compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock.

The presentation of the binomial trees in this chapter was revised to conform with other chapters in 2018 & 2019 by Donald J. Smith, PhD, Boston University (USA).

1. INTRODUCTION AND OVERVIEW OF EMBEDDED OPTIONS

- **describe fixed-income securities with embedded options**

The valuation of a fixed-rate, option-free bond generally requires determining its future cash flows and discounting them at the appropriate rates. Valuation becomes more complicated when a bond has one or more embedded options because the values of embedded options are typically contingent on interest rates.

Understanding how to value and analyze bonds with embedded options is important for practitioners. Issuers of bonds often manage interest rate exposure with embedded options, such as call provisions. Investors in callable bonds must appreciate the risk of being called. The perception of this risk is collectively represented by the premium, in terms of increased coupon or yield, that the market demands for callable bonds relative to otherwise identical option-free bonds. Issuers and investors must also understand how other types of embedded options—such as put provisions, conversion options, caps, and floors—affect bond values and the sensitivity of these bonds to interest rate movements.

We first provide a brief overview of various types of embedded options. We then discuss bonds that include a call or put provision. Taking a building-block approach, we show how the arbitrage-free valuation framework discussed earlier can be applied to the valuation of callable and putable bonds—first in the absence of interest rate volatility, and then when interest rates fluctuate. We also discuss how option-adjusted spreads are used to value risky callable and putable bonds. We then turn to interest rate sensitivity. It highlights the need to use effective duration, including one-sided durations and key rate durations, as well as effective convexity to assess the effect of interest rate movements on the value of callable and putable bonds. We also explain the valuation of capped and floored floating-rate bonds (floaters) and convertible bonds.

1.1. Overview of Embedded Options

The term "embedded bond options" or **embedded options** refers to contingency provisions found in the bond's indenture or offering circular. These options represent rights that enable their holders to take advantage of interest rate movements. They can be exercised by the issuer or the bondholder, or they may be exercised automatically depending on the course of interest rates. For example, a call option allows the issuer to benefit from lower interest rates by retiring the bond issue early and refinancing at a lower cost. In contrast, a put option allows the bondholder to benefit from higher interest rates by putting back the bonds to the issuer and reinvesting the proceeds of the retired bond at a higher yield. These options are not independent of the bond and thus cannot be traded separately—hence the adjective "embedded." In this section, we provide a review of familiar embedded options.

Corresponding to every embedded option, or combination of embedded options, is an underlying bond with a specified issuer, issue date, maturity date, principal amount and repayment structure, coupon rate and payment structure, and currency denomination. We also refer to this underlying option-free bond as the **straight bond**. The coupon of an underlying bond can be fixed or floating. Fixed-coupon bonds may have a single rate for the life of the bond, or the rate may step up or step down according to a coupon schedule. The coupons of floaters are reset periodically according to a formula based on a reference rate plus a credit spread—for example, Market reference rate + 100 basis points (bps). Except when we discuss capped and floored floaters, our focus is on fixed-coupon, single-rate bonds, also referred to as fixed-rate bonds.

1.1.1. Simple Embedded Options

Call and put options are standard examples of embedded options. In fact, the vast majority of bonds with embedded options are callable, putable, or both. The call provision is by far the most prevalent type of embedded option.

1.1.1.1. Call Options A **callable bond** is a bond that includes an embedded call option. The call option is an issuer option; that is, the right to exercise the option is at the discretion of the bond's issuer. The call provision allows the issuer to redeem the bond issue prior to maturity. Early redemption usually happens when the issuer has the opportunity to replace a high-coupon bond with another bond that has more favorable terms, typically when interest rates have fallen or when the issuer's credit quality has improved.

Until the 1990s, most long-term corporate bonds in the United States were callable after either 5 or 10 years. The initial call price (exercise price) was typically at a premium above par, the premium depended on the coupon, and the call price gradually declined to par a few years prior to maturity. Today, most investment-grade corporate bonds are essentially non-refundable. They may have a "make-whole call," so named because the call price is such that the bondholders are more than "made whole" (compensated) in exchange for surrendering their bonds. The call price is calculated at a narrow spread to a benchmark security—usually an on-the-run sovereign bond, such as Treasuries in the United States or gilts in the United Kingdom. Thus, economical refunding is virtually out of the question. Investors need have no fear of receiving less than their bonds are worth.

Most callable bonds include a call **protection period** during which the issuer cannot call the bond. For example, a 10-year callable bond may have a call protection period of three years, meaning that the first potential call date is three years after the bond's issue date. Call protection periods may be as short as one month or extend to several years. For example, high-yield corporate bonds are often callable a few years after issuance. Holders of such bonds are usually less concerned about early redemption than about possible default. Of course, this perspective can change over the life of the bond—for example, if the issuer's credit quality improves.

Callable bonds include different types of call features. The issuer of a European-style callable bond can exercise the call option only once on the call date. An American-style callable bond is continuously callable at any time starting on the first call date. A Bermudan-style call option can be exercised only on a predetermined schedule on specified dates following the call protection period. These dates are specified in the bond's indenture or offering circular.

With a few exceptions, bonds issued by government-sponsored enterprises in the United States (e.g., Fannie Mae, Freddie Mac, Federal Home Loan Banks, and Federal Farm Credit Banks) are callable. These bonds tend to have relatively short maturities (5–10 years) and very short call protection periods (three months to one year). The call price is almost always at 100% of par, and the call option is often Bermudan style.

Tax-exempt municipal bonds (often called "munis"), a type of non-sovereign (local) government bond issued in the United States, are almost always callable at 100% of par any time after the end of the 10th year. They may also be eligible for advance refunding—a highly specialized topic that is not discussed here.

Although the bonds of US government-sponsored enterprises and municipal issuers account for most of the callable bonds issued and traded globally, bonds that include call provisions are also found in other countries in Asia Pacific, Europe, Canada, and Central and South America. The vast majority of callable bonds are denominated in US dollars or euros because of investors' demand for securities issued in these currencies. Australia, the United Kingdom, Japan, and Norway are examples of countries that have a market for callable bonds denominated in local currency.

1.1.1.2. Put Options and Extension Options A **putable bond** is a bond that includes an embedded put option. The put option is an investor option; that is, the right to exercise the option is at the discretion of the bondholder. The put provision allows the bondholders to put back the bonds to the issuer prior to maturity, usually at par. This usually happens when interest rates have risen and higher-yielding bonds are available.

Similar to callable bonds, most putable bonds include protection periods. They can be European or, rarely, Bermudan style, but there are no American-style putable bonds.

Another type of embedded option that resembles a put option is an extension option. At maturity, the holder of an **extendible bond** (sometimes spelled "extendable") has the right to keep the bond for a number of years after maturity, possibly with a different coupon. In this case, the terms of the bond's indenture or offering circular are modified, but the bond remains outstanding. An example of a corporate extendible is an offering from Heathrow Funding Ltd. It pays a 0.50% coupon and matures on 17 May 2024. However, it is extendible to 7 May 2026 as a floating-rate note paying 12-month MRR plus 4.00%. We will discuss the resemblance between a putable and an extendible bond later.

1.1.2. Complex Embedded Options

Although callable and putable bonds are the most common types of bonds with embedded options, there are bonds with other types of options or combinations of options. For instance, some bonds can be both callable and putable. These bonds can be either called by the issuer or put by the bondholders.

Convertible bonds are another type of bond with an embedded option. The conversion option allows bondholders to convert their bonds into the issuer's common stock. Convertible bonds are usually also callable by the issuer; the call provision enables the issuer to take advantage of lower interest rates or to force conversion.

Another layer of complexity is added when the option is contingent on some particular event. An example is the estate put or survivor's option that may be available to retail investors. In the event of the holder's death, this bond can be put at par by the heir(s). Because the estate put comes into play only in the event of the bondholder's death, the value of a bond with an estate put is contingent on the life expectancy of its holder, which is uncertain.

Bonds may contain several interrelated issuer options without any investor option. A prime example is a **sinking fund bond** (sinker). A sinker requires the issuer to set aside funds over time to retire the bond issue, thus reducing credit risk. Such a bond may be callable and may also include options unique to sinking fund bonds, such as an acceleration provision and a delivery option.

SINKING FUND BONDS

The underlying bond has an amortizing structure—for example, a 30-year maturity with level annual principal repayments beginning at the end of the 11th year. In this case, each payment is 5% of the original principal amount. A typical sinking fund bond may include the following options:

- A standard *call option* above par, with declining premiums, starting at the end of Year 10. Thus, the entire bond issue could be called from Year 10 onward.

- An *acceleration provision*, such as a "triple up." Such a provision allows the issuer to repurchase at par three times the mandatory amount, or in this case 15% of the original principal amount, on any scheduled sinking fund date. Assume that the issuer wants to retire the bonds at the end of Year 11. Instead of calling the entire outstanding amount at a premium, it would be more cost effective to "sink" 15% at par and call the rest at a premium. Thus, the acceleration provision provides an additional benefit to the issuer if interest rates decline.
- A *delivery option*, which allows the issuer to satisfy a sinking fund payment by delivering bonds to the bond's trustee in lieu of cash. The bond's trustee is appointed by the issuer but acts in a fiduciary capacity with the bondholders. If the bonds are currently trading below par, say at 90% of par, it is more cost effective for the issuer to buy back bonds from investors to meet the sinking fund requirements than to pay par. The delivery option benefits the issuer if interest rates rise. Of course, the benefit can be materialized only if there is a liquid market for the bonds. Investors can take defensive action by accumulating the bonds and refusing to sell them at a discount.

From the issuer's perspective, the combination of the call option and the delivery option is effectively a "long straddle"—an option strategy involving the purchase of a put option and a call option on the same underlying with the same exercise price and expiration date. At expiration, if the underlying price is above the exercise price, the put option is worthless but the call option is in the money. In contrast, if the underlying price is below the exercise price, the call option is worthless but the put option is in the money. Thus, a long straddle benefits the investor when the underlying price moves up or down. The greater the move up or down (i.e., the greater the volatility), the greater the benefit for the investor. As a consequence, a sinking fund bond benefits the issuer not only if interest rates decline but also if they rise. Determining the combined value of the underlying bond and the three options is quite challenging.

EXAMPLE 1 Types of Embedded Options

1. Investors in putable bonds *most likely* seek to take advantage of:
   A. higher interest rates.
   B. improvements in the issuer's credit rating.
   C. movements in the price of the issuer's common stock.
2. The conversion option in a convertible bond is a right held by:
   A. the issuer.
   B. the bondholders.
   C. the issuer and the bondholders jointly.

Solution to 1: A is correct. A putable bond offers the bondholder the ability to take advantage of a rise in interest rates by putting back the bond to the issuer and reinvesting the proceeds of the retired bond in a higher-yielding bond.

Solution to 2: B is correct. A conversion option is a call option that gives the bondholders the right to convert their bonds into the issuer's common stock.

The presence of embedded options affects a bond's value. To quantify this effect, financial theory and financial technology come into play. The following section presents basic valuation and analysis concepts for bonds with embedded options.

2. VALUATION AND ANALYSIS OF CALLABLE AND PUTABLE BONDS

- **explain the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option**
- **describe how the arbitrage-free framework can be used to value a bond with embedded options**

Under the arbitrage-free framework, the value of a bond with embedded options is equal to the sum of the arbitrage-free values of its parts. We first identify the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the call or put option. We then discuss how to value callable and putable bonds under different risk and interest rate volatility scenarios.

2.1. Relationships between the Values of a Callable or Putable Bond, Straight Bond, and Embedded Option

The value of a bond with embedded options is equal to the sum of the arbitrage-free value of the straight bond and the arbitrage-free values of the embedded options.

For a callable bond, the decision to exercise the call option is made by the issuer. Thus, the investor is long the bond but short the call option. From the investor's perspective, therefore, the value of the call option *decreases* the value of the callable bond relative to the value of the straight bond:

$$\text{Value of callable bond} = \text{Value of straight bond} - \text{Value of issuer call option}$$

The value of the straight bond can be obtained by discounting the bond's future cash flows at the appropriate rates. The hard part is valuing the call option because its value is contingent on future interest rates. Specifically, the issuer's decision to call the bond depends on its ability to refinance at a lower cost. In practice, the value of the call option is often calculated as the difference between the value of the straight bond and the value of the callable bond:

$$\text{Value of issuer call option} = \text{Value of straight bond} - \text{Value of callable bond} \qquad (1)$$

For a putable bond, the decision to exercise the put option is made by the investor. Thus, the investor has a long position in both the bond and the put option. As a consequence, the value of the put option *increases* the value of the putable bond relative to the value of the straight bond.

$$\text{Value of putable bond} = \text{Value of straight bond} + \text{Value of investor put option}$$

It follows that

$$\text{Value of investor put option} = \text{Value of putable bond} - \text{Value of straight bond} \qquad (2)$$

Although most investment professionals do not need to be experts in bond valuation, they should have a solid understanding of the basic analytical approach, which is presented in the following sections.

2.2. Valuation of Default-Free and Option-Free Bonds: A Refresher

An asset's value is the present value of the cash flows the asset is expected to generate in the future. In the case of a default-free and option-free bond, the future cash flows are, by definition, certain. Thus, the question is, at which rates should these cash flows be discounted? The answer is that each cash flow should be discounted at the spot rate corresponding to the cash flow's payment date. Although spot rates might not be directly observable, they can be inferred from readily available information, usually from the market prices of actively traded on-the-run sovereign bonds of various maturities. These prices can be transformed into spot rates, par rates (i.e., coupon rates of hypothetical bonds of various maturities selling at par), or forward rates. Recall from Level I that spot rates, par rates, and forward rates are equivalent ways of conveying the same information; knowing any one of them is sufficient to determine the others.

Suppose we want to value a three-year 4.25% annual coupon bond. Exhibit 1 provides the equivalent forms of a yield curve with maturities of one, two, and three years.

EXHIBIT 1 Equivalent Forms of a Yield Curve

Maturity (year)	Par Rate (%)	Spot Rate (%)	One-Year Forward Rate (%)	
1	2.500	2.500	0 years from now	2.500
2	3.000	3.008	1 year from now	3.518
3	3.500	3.524	2 years from now	4.564

We start with the par rates provided in the second column of Exhibit 1. Because we are assuming annual coupons and annual compounding, the one-year spot rate is simply the one-year par rate. The hypothetical one-year par bond implied by the given par rate has a single cash flow of 102.500 (principal plus coupon) in Year 1. In order to have a present value of par, this future cash flow must be divided by 1.025. Thus, the one-year spot rate or discount rate is 2.500%. (*Note:* All cash flows and values are expressed as a percentage of par.)

A two-year 3.000% par bond has two cash flows: 3 in Year 1 and 103 in Year 2. By definition, the sum of the two discounted cash flows must equal 100. We know that the discount rate appropriate for the first cash flow is the one-year spot rate (2.500%). We now solve the following equation to determine the two-year spot rate (z_2):

$$\frac{3}{(1.025)} + \frac{103}{(1 + z_2)^2} = 100$$

We can follow a similar approach to determine the three-year spot rate (z_3):

$$\frac{3.500}{(1.02500)} + \frac{3.500}{(1.03008)^2} + \frac{103.500}{(1 + z_3)^3} = 100$$

The one-year forward rates are determined by using indifference equations. Assume an investor has a two-year horizon. She could invest for two years either at the two-year spot rate or at the one-year spot rate for one year and then reinvest the proceeds at the one-year forward rate one year from now ($F_{1,1}$). The result of investing using either of the two approaches should be the same. Otherwise, there would be an arbitrage opportunity. Thus,

$$(1 + 0.03008)^2 = (1 + 0.02500) \times (1 + F_{1,1})$$

Similarly, the one-year forward rate two years from now ($F_{2,1}$) can be calculated using the following equation:

$$(1 + 0.03524)^3 = (1 + 0.03008)^2 \times (1 + F_{2,1})$$

The three-year 4.25% annual coupon bond can now be valued using the spot rates:

$$\frac{4.25}{(1.02500)} + \frac{4.25}{(1.03008)^2} + \frac{104.25}{(1.03524)^3} = 102.114$$

An equivalent way to value this bond is to discount its cash flows one year at a time using the one-year forward rates:

$$\frac{4.25}{(1.02500)} + \frac{4.25}{(1.02500)(1.03518)} + \frac{104.25}{(1.02500)(1.03518)(1.04564)} = 102.114$$

2.3. Valuation of Default-Free Callable and Putable Bonds in the Absence of Interest Rate Volatility

When valuing bonds with embedded options, the approach relying on one-period forward rates provides a better framework than that relying on the spot rates because we need to know the value of the bond at different points in time in the future to determine whether the embedded option will be exercised at those points in time.

2.3.1. Valuation of a Callable Bond at Zero Volatility

Let us apply this framework to the valuation of a Bermudan-style three-year 4.25% annual coupon bond that is callable at par one year and two years from now. The decision to exercise the call option is made by the issuer. Because the issuer borrowed money, it will exercise the call option when the value of the bond's future cash flows is higher than the call price (exercise price). Exhibit 2 shows how to calculate the value of this callable bond using the one-year forward rates calculated in Exhibit 1.

EXHIBIT 2 Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at Zero Volatility

	Today	Year 1	Year 2	Year 3
Cash Flow		4.250	4.250	104.250
Discount Rate		2.500%	3.518%	4.564%
Value of the Callable Bond	$\frac{100 + 4.250}{1.02500} = 101.707$	$\frac{99.700 + 4.250}{1.03518} =$ ~~100.417~~ Called at 100	$\frac{104.250}{1.04564} = 99.700$ Not called	

We start by discounting the bond's cash flow at maturity (104.250) to Year 2 using the one-year forward rate two years from now (4.564%). The present value at Year 2 of the bond's future cash flows is 99.700. This value is lower than the call price of 100, so a rational borrower will not call the bond at that point in time. Next, we add the cash flow in Year 2 (4.250) to the present value of the bond's future cash flows at Year 2 (99.700) and discount the sum to Year 1 using the one-year forward rate one year from now (3.518%). The present value at Year 1 of the bond's future cash flows is 100.417. Here, a rational borrower will call the bond at 100 because leaving it outstanding would be more expensive than redeeming it. Last, we add the cash flow in Year 1 (4.250) to the present value of the bond's future cash flows at Year 1 (100.000) then discount the sum to today at 2.500%. The result (101.707) is the value of the callable bond. (*Note:* For the purpose of coverage of this topic, all cash flows and values are expressed as a percentage of par.)

We can apply Equation 1 to calculate the value of the call option embedded in this callable bond. The value of the straight bond is the value of the default-free and option-free three-year 4.25% annual coupon bond calculated earlier (102.114). Thus,

$$\text{Value of issuer call option} = 102.114 - 101.707 = 0.407$$

Recall from the earlier discussion about the relationships between the value of a callable bond, straight bond, and call option that the investor is long the bond and short the call option. Thus, the value of the call option decreases the value of the callable bond relative to that of an otherwise identical option-free bond.

2.3.2. Valuation of a Putable Bond at Zero Volatility

We now apply this framework to the valuation of a Bermudan-style three-year 4.25% annual coupon bond that is putable at par one year and two years from now. The decision to exercise the put option is made by the investor. Because the investor lent money, he will exercise the put option when the value of the bond's future cash flows is lower than the put price (exercise price). Exhibit 3 shows how to calculate the value of the three-year 4.25% annual coupon bond putable at par one year and two years from today.

EXHIBIT 3 Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Putable at Par One Year and Two Years from Now at Zero Volatility

	Today	Year 1	Year 2	Year 3
Cash flow		4.250	4.250	104.250
Discount rate		2.500%	3.518%	4.564%
Value of the putable bond	$\frac{100.707 + 4.250}{1.02500} = 102.397$	$\frac{100 + 4.250}{1.03518} = 100.707$ Not put	$\frac{104.250}{1.04564} =$ ~~99.700~~ Put at 100	

We can apply Equation 2 to calculate the value of the put option:

$$\text{Value of investor put option} = 102.397 - 102.114 = 0.283$$

Because the investor is long the bond and the put option, the value of the put option increases the value of the putable bond relative to that of an otherwise identical option-free bond.

OPTIMAL EXERCISE OF OPTIONS

The holder of an embedded bond option can extinguish (or possibly modify the terms of) the bond. Assuming that the option is currently exercisable, the obvious question is, does it pay to exercise? Assuming that the answer is affirmative, the follow-up question is whether it is better to exercise the option at present or to wait.

Let us consider the first question: Would it be profitable to exercise the option? The answer is usually straightforward: Compare the value of exercising with the value of not exercising. For example, suppose that a bond is currently putable at 100. If the bond's market price is above 100, putting the bond makes no sense because the cash value from selling the bond would exceed 100. In contrast, if the bond's market price is 100, putting the bond should definitely be considered. Note that the market price of the bond cannot be less than 100 because such a situation creates an arbitrage opportunity: Buy the bond below 100 and immediately put it at 100.

The logic of a call decision by the issuer is similar. If a bond's market price is significantly less than the call price, calling is foolish because the bond could be simply repurchased in the market at a lower price. Alternatively, if the price is very close to the call price, calling may make sense.

Assume that we have determined that exercising the option would be profitable. If the option under consideration is European style, it is obvious that it should in fact be exercised: There is no justification for not doing so. But if it is an American-style or Bermudan-style option, the challenge is to determine whether it is better to act now or to wait for a better opportunity. The problem is that although circumstances may become more favorable, they may also get worse. So, option holders must consider the odds and decide to act or wait, depending on their risk preference.

The approach presented here for valuing bonds with embedded options assumes that the option holders, be they issuers or investors, are risk neutral. They exercise if, and only if, the benefit from exercise exceeds the expected benefit from waiting. In reality, option holders may be risk averse and may exercise early even if the option is worth more alive than dead.

EXAMPLE 2 Valuation of Default-Free Callable and Putable Bonds

George Cahill, a portfolio manager, has identified three five-year annual coupon bonds issued by a sovereign government. The three bonds have identical characteristics. The exceptions are that Bond A is an option-free bond; Bond B is callable at par two years and three years from today; and Bond C is also callable at par two years and three years from today as well as putable at par one year from today.

1. Relative to the value of Bond A, the value of Bond B is:
   A. lower.
   B. the same.
   C. higher.

2. Relative to the value of Bond B, the value of Bond C is:
   A. lower.
   B. the same.
   C. higher.
3. Given an anticipation of rising interest rates, Bond C will be expected to:
   A. be called by the issuer.
   B. be put by the bondholders.
   C. mature without exercise of any of the embedded options.

Solution to 1: A is correct. Bond B is a callable bond, and Bond A is the underlying option-free (straight) bond. The call option embedded in Bond B is an issuer option that decreases the bond's value for the investor. If interest rates decline, bond prices usually increase; however, the price appreciation of Bond B will be capped relative to the price appreciation of Bond A because the issuer will call the bond to refinance at a lower cost.

Solution to 2: C is correct. Relative to Bond B, Bond C includes a put option. A put option is an investor option that increases the bond's value for the investor. Thus, the value of Bond C is higher than that of Bond B.

Solution to 3: B is correct. As interest rates rise, bond prices decrease. Thus, the bondholders will have an incentive to exercise the put option so that they can reinvest the proceeds of the retired bond at a higher yield.

Exhibits 2 and 3 show how callable and putable bonds are valued in the absence of interest rate volatility. In real life, however, interest rates do fluctuate. Thus, the option holder must consider possible evolutions of the yield curve over time.

3. EFFECT OF INTEREST RATE VOLATILITY ON THE VALUE OF CALLABLE AND PUTABLE BONDS

- **explain how interest rate volatility affects the value of a callable or putable bond**
- **explain how changes in the level and shape of the yield curve affect the value of a callable or putable bond**

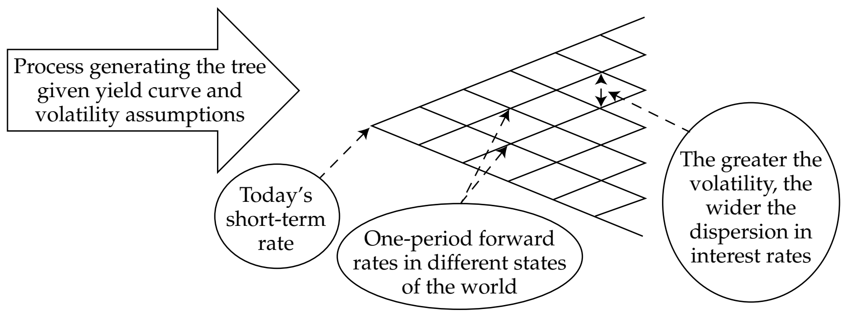
In this section, we discuss the effects of interest rate volatility as well as the level and shape of the yield curve on the value of embedded options.

3.1. Interest Rate Volatility

The value of any embedded option, regardless of the type of option, increases with interest rate volatility. The greater the volatility, the more opportunities for the embedded option to be exercised. Thus, it is critical for issuers and investors to understand the effect of interest rate volatility on the value of bonds with embedded options.

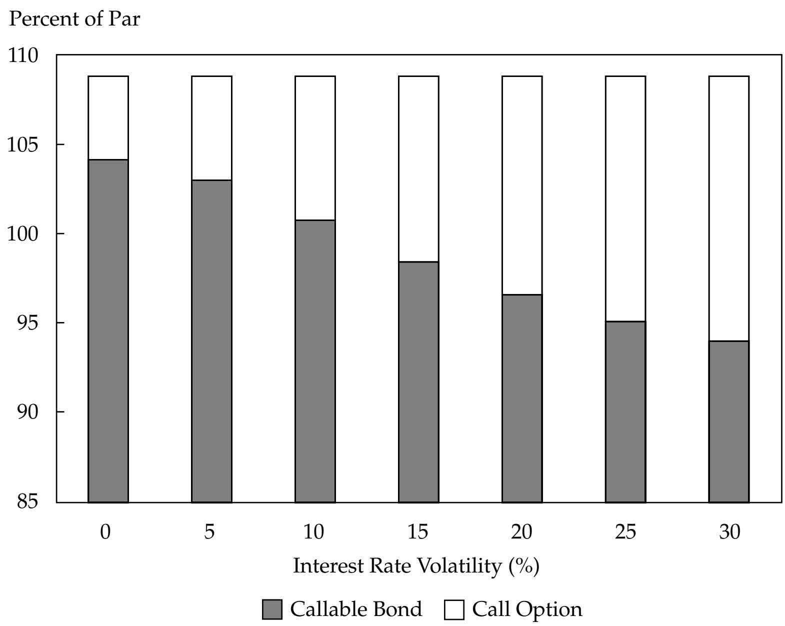
The effect of interest rate volatility is represented in an interest rate tree or lattice, as illustrated in Exhibit 4. From each node on the tree starting from today, interest rates could go up or down. From these two states, interest rates could again go up or down. The dispersion between these up and down states anywhere on the tree is determined by the process generating interest rates based on a given yield curve and interest rate volatility assumptions.

EXHIBIT 4 Building an Interest Rate Tree



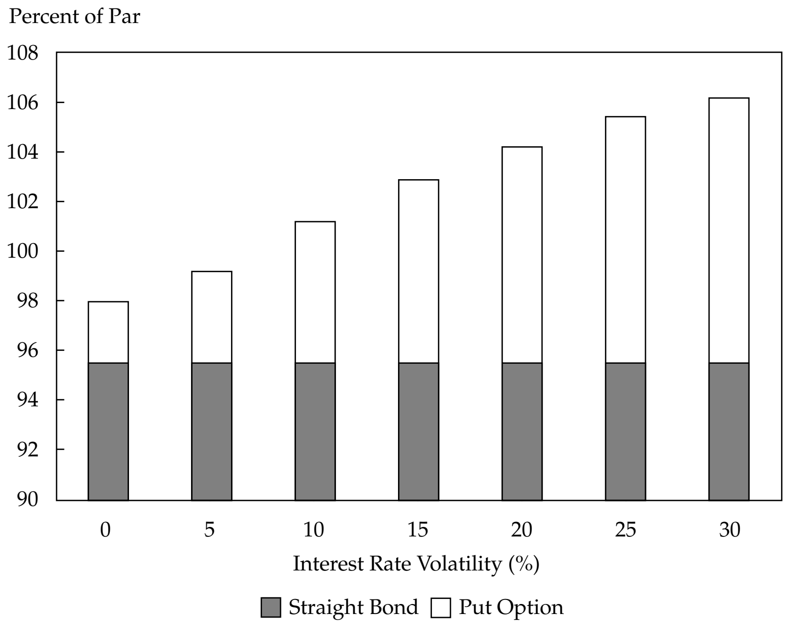
Exhibits 5 and 6 show the effect of interest rate volatility on the value of a callable bond and putable bond, respectively.

EXHIBIT 5 Value of a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Volatility Scenarios Assuming a 4% Flat Yield Curve



The stacked bars in Exhibit 5 represent the value of the straight bond, which is unaffected by interest rate volatility. The white component is the value of the call option; taking it away from the value of the straight bond gives the value of the callable bond—the shaded component. All else being equal, the call option increases in value with interest rate volatility. At zero volatility, the value of the call option is 4.60% of par; at 30% volatility, it is 14.78% of par. Thus, as interest rate volatility increases, the value of the callable bond decreases.

EXHIBIT 6 Value of a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Volatility Scenarios Assuming a 4% Flat Yield Curve



In Exhibit 6, the shaded component is the value of the straight bond, and the white component is the value of the put option; thus, the stacked bars represent the value of the putable bond. All else being equal, the put option increases in value with interest rate volatility. At zero volatility, the value of the put option is 2.30% of par; at 30% volatility, it is 10.54% of par. Thus, as interest rate volatility increases, the value of the putable bond increases.

3.2. Level and Shape of the Yield Curve

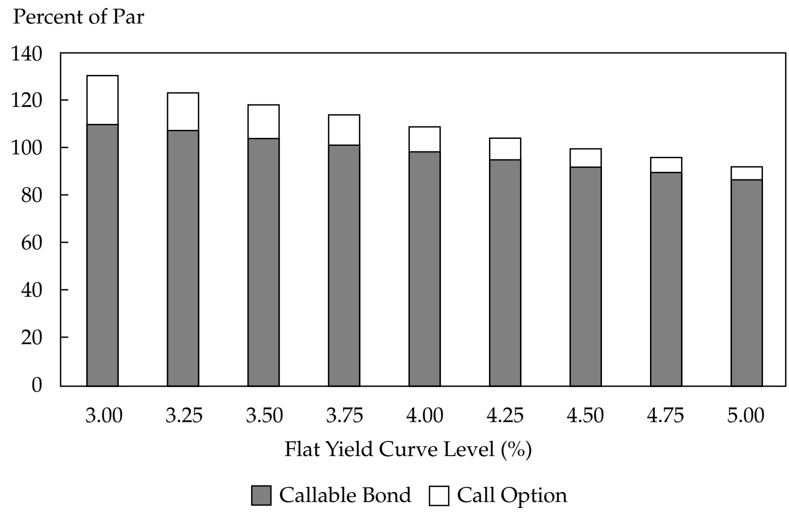
The value of a callable or putable bond is also affected by changes in the level and shape of the yield curve.

3.2.1. Effect on the Value of a Callable Bond

Exhibit 7 shows the value of the same callable bond as in Exhibit 5 under different flat yield curve levels assuming an interest rate volatility of 15%.

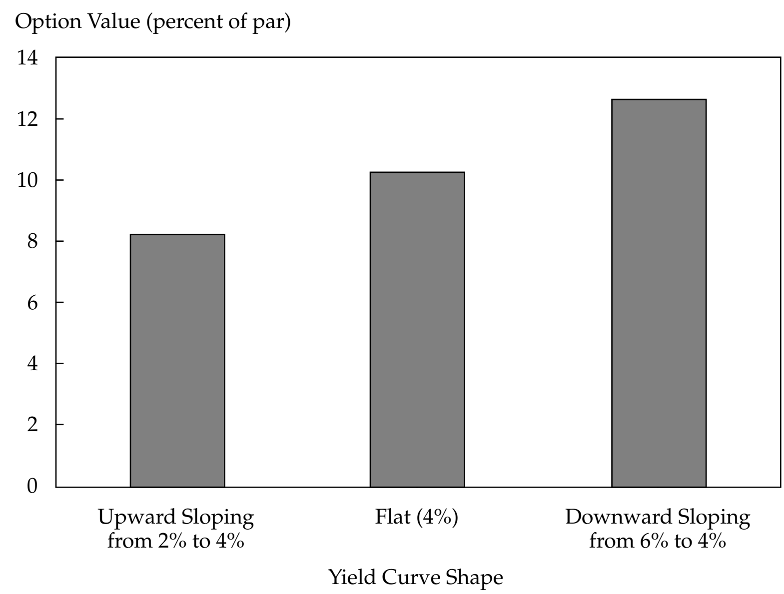
Exhibit 7 shows that as interest rates decline, the value of the straight bond rises; however, the rise is partially offset by the increase in the value of the call option. For example, if the yield curve is 5% flat, the value of the straight bond is 92.27% of par and the value of the call option is 5.37% of par; thus, the value of the callable bond is 86.90% of par. If the yield curve declines to 3% flat, the value of the straight bond rises by 40% to 129.54% of par, but the value of the callable bond increases by only 27% to 110.43% of par. Thus, the value of the callable bond rises less rapidly than the value of the straight bond, limiting the upside potential for the investor.

EXHIBIT 7 Value of a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Flat Yield Curve Levels at 15% Interest Rate Volatility



The value of a call option, and thus the value of a callable bond, is also affected by changes in the shape of the yield curve, as illustrated in Exhibit 8.

EXHIBIT 8 Value of a Call Option Embedded in a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Yield Curve Shapes at 15% Interest Rate Volatility



All else being equal, the value of the call option increases as the yield curve flattens. If the yield curve is upward sloping with short-term rates at 2% and long-term rates at 4% (the first bar), the value of the call option represents approximately 8% of par. It rises to approximately 10% of par if the yield curve flattens to 4% (the second bar). The value of the call option increases further if the yield curve actually inverts. Exhibit 8 shows that it exceeds 12% of par if the yield curve is downward sloping with short-term rates at 6% and long-term rates at 4% (the third bar). An inverted yield curve is rare but does happen from time to time.

The intuition to explain the effect of the shape of the yield curve on the value of the call option is as follows. When the yield curve is upward sloping, the one-period forward rates on the interest rate tree are high and opportunities for the issuer to call the bond are fewer. When the yield curve flattens or inverts, many nodes on the tree have lower forward rates that increase the opportunities to call.

Assuming a normal, upward-sloping yield curve at the time of issue, the call option embedded in a callable bond issued at par is out of the money. It would not be called if the arbitrage-free forward rates at zero volatility prevailed. Callable bonds issued at a large premium, as happens frequently in the municipal sector in the United States, are in the money. They will be called if the arbitrage-free forward rates prevail.

3.2.2. Effect on the Value of a Putable Bond

Exhibits 9 and 10 show how changes in the level and shape of the yield curve affect the value of the putable bond used in Exhibit 6.

EXHIBIT 9 Value of a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Flat Yield Curve Levels at 15% Interest Rate Volatility

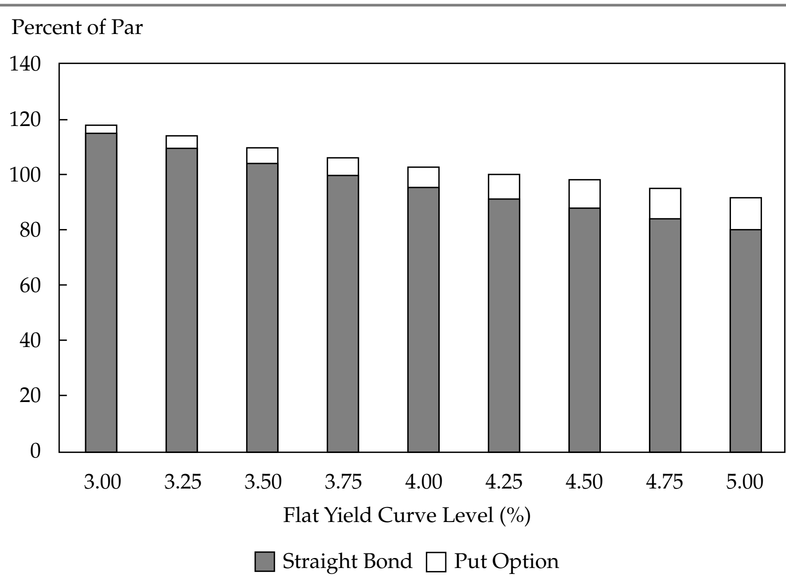
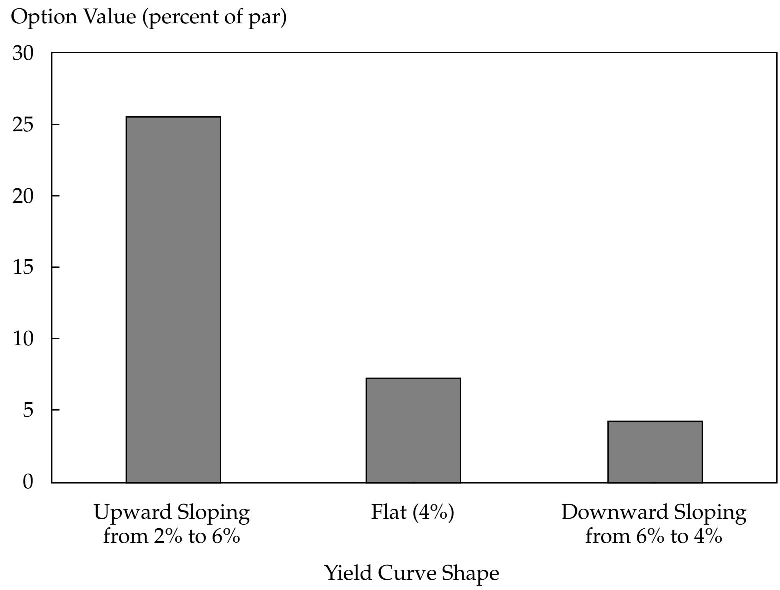


Exhibit 9 illustrates why the put option is considered a hedge against rising interest rates for investors. As interest rates rise, the value of the straight bond declines; however, the decline is partially offset by the increase in the value of the put option. For example, if the yield curve moves from 3% flat to 5% flat, the value of the straight bond falls by 30% while the fall in the value of the putable bond is limited to 22%.

EXHIBIT 10 Value of the Put Option Embedded in a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Yield Curve Shapes at 15% Interest Rate Volatility



All else being equal, the value of the put option decreases as the yield curve moves from being upward sloping, to flat, to downward sloping. When the yield curve is upward sloping, the one-period forward rates in the interest rate tree are high, which creates more opportunities for the investor to put the bond. As the yield curve flattens or inverts, the number of opportunities declines.

4. VALUATION OF DEFAULT-FREE CALLABLE AND PUTABLE BONDS IN THE PRESENCE OF INTEREST RATE VOLATILITY

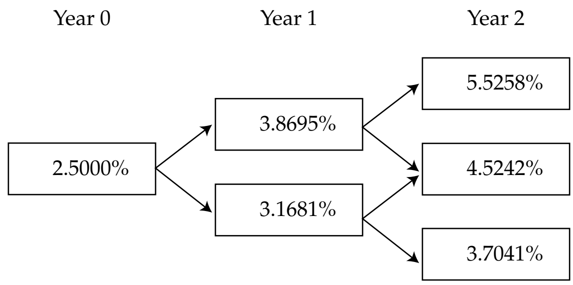
- **calculate the value of a callable or putable bond from an interest rate tree**

The procedure to value a bond with an embedded option in the presence of interest rate volatility is as follows:

- Generate a tree of interest rates based on the given yield curve and interest rate volatility assumptions.
- At each node of the tree, determine whether the embedded option will be exercised.
- Apply the backward induction valuation methodology to calculate the bond's present value. This methodology involves starting at maturity and working back from right to left to find the bond's present value.

Let us return to the default-free three-year 4.25% annual coupon bonds discussed earlier to illustrate how to apply this valuation procedure. The bonds' characteristics are identical. The yield curve given in Exhibit 1 remains the same—with one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively. But we now assume an interest rate volatility of 10% instead of 0%. The resulting binomial interest rate tree showing the one-year forward rates zero, one, and two years from now is shown in Exhibit 11. The branching from each node to an up state and a down state is assumed to occur with equal probability.

EXHIBIT 11 Binomial Interest Rate Tree at 10% Interest Rate Volatility



The calibration of a binomial interest rate tree was discussed in earlier coverage of fixed-income concepts. As mentioned before, the one-year par rate, the one-year spot rate, and the one-year forward rate zero years from now are identical (2.500%). Because there is no closed-form solution, the one-year forward rates one year from now in the two states are determined iteratively by meeting the following two constraints:

1. The rate in the up state (R_u) is given by

$$R_u = R_d \times e^{2\sigma\sqrt{t}}$$

   where R_d is the rate in the down state, σ is the interest rate volatility (10% here), and t is the time in years between "time slices" (a year, so here $t = 1$).
2. The discounted value of a two-year par bond (bearing a 3.000% coupon rate in this example) equals 100.

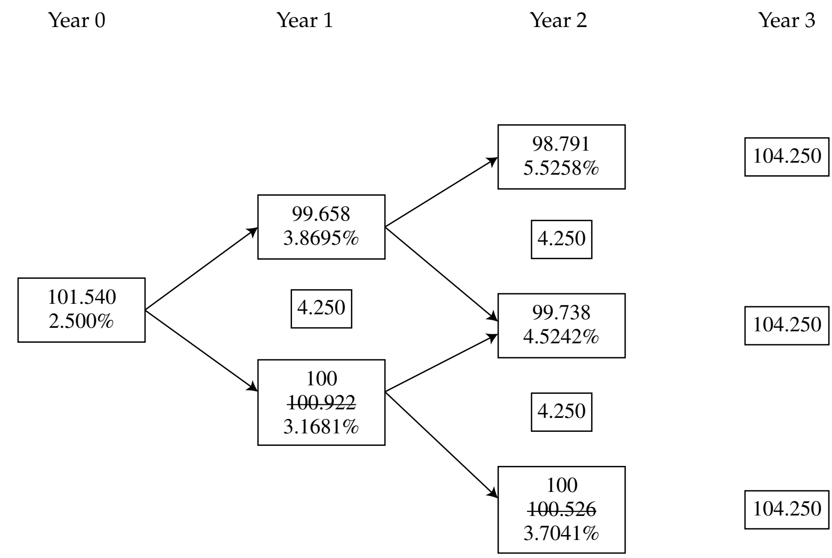
In Exhibit 11 at the one-year time slice, R_d is 3.1681% and R_u is 3.8695%. Having established the rates that correctly value the one-year and two-year par bonds implied by the given par yield curve, we freeze these rates and proceed to iterate the rates in the next time slice to determine the one-year forward rates in the three states two years from now. The same constraints as before apply: (1) Each rate must be related to its neighbor by the factor $e^{2\sigma\sqrt{t}}$, and (2) the rates must discount a three-year par bond (bearing a 3.500% coupon rate in this example) to a value of 100.

Now that we have determined all the one-year forward rates, we can value the three-year 4.25% annual coupon bonds that are either callable or putable at par one year and two years from now.

4.1. Valuation of a Callable Bond with Interest Rate Volatility

Exhibit 12 depicts the valuation of a callable bond at 10% volatility.

EXHIBIT 12 Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



The coupon and principal cash flows are placed directly to the right of the interest rate nodes. The calculated bond values at each node are placed above the interest rate. We start by calculating the bond values at Year 2 by discounting the cash flow for Year 3 with the three possible rates.

$$98.791 = \frac{104.250}{1.055258}$$

$$99.738 = \frac{104.250}{1.045242}$$

$$100.526 = \frac{104.250}{1.037041}$$

Because the bond is callable at par in Year 2, we check each scenario to determine whether the present value of the future cash flows is higher than the call price, in which case the issuer calls the bond. Exercise happens only at the bottom of the tree, where the rate is 3.7041%, and so we reset the value from 100.526 to 100 in that state.

The value in each state of Year 1 is calculated by discounting the values in the two future states emanating from the present state plus the coupon at the appropriate rate in the present state:

$$99.658 = \frac{4.250 + (0.5 \times 98.791 + 0.5 \times 99.738)}{1.038695}$$

The first term in the numerator is the coupon payment, and the second term is the expected bond value at Year 2. In this model, the probabilities for moving to the higher and lower node are the same (0.5):

$$100.922 = \frac{4.250 + (0.5 \times 99.738 + 0.5 \times 100)}{1.031681}$$

Notice that the reset value of 100 is used to get the expected bond value. Once again the bond will be callable at the lower node where the interest rate is 3.1681%.

At Year 0, the value of the callable bond is 101.540:

$$101.540 = \frac{4.250 + (0.5 \times 99.658 + 0.5 \times 100)}{1.025000}$$

The value of the call option, obtained by taking the difference between the value of the straight bond and the value of the callable bond, is now 0.574 (102.114 – 101.540). The fact that the value of the call option is larger at 10% volatility than at 0% volatility (0.407) is consistent with our earlier discussion that option value increases with interest rate volatility.

EXAMPLE 3 Valuation of a Callable Bond Assuming Interest Rate Volatility

Return to the valuation of the Bermudan-style three-year 4.25% annual coupon bond callable at par one year and two years from now as depicted in Exhibit 12. The one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively, and the interest rate volatility is 10%.

1. Assume that nothing changes relative to the initial setting except that the interest rate volatility is now 15% instead of 10%. The new value of the callable bond is:
   A. less than 101.540.
   B. equal to 101.540.
   C. more than 101.540.
2. Assume that nothing changes relative to the initial setting except that the bond is now callable at 102 instead of 100. The new value of the callable bond is *closest to*:
   A. 100.000.
   B. 102.000.
   C. 102.114.

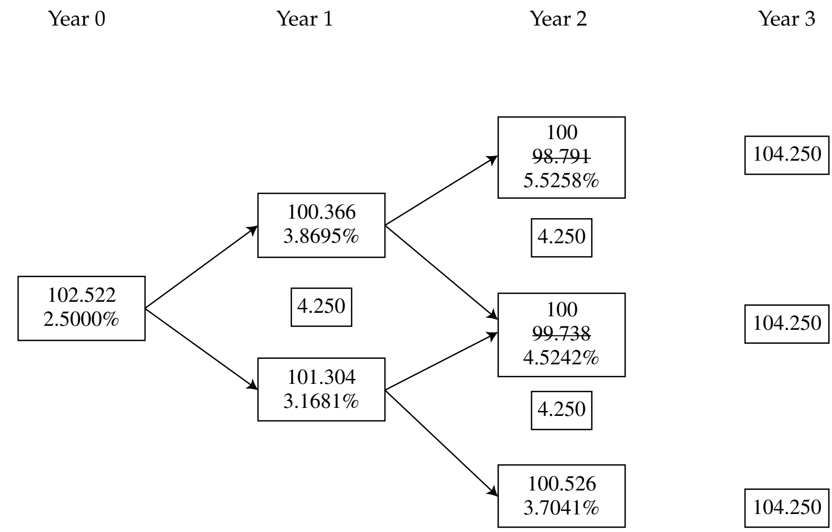
Solution to 1: A is correct. A higher interest rate volatility increases the value of the call option. Because the value of the call option is subtracted from the value of the straight bond to obtain the value of the callable bond, a higher value for the call option leads to a lower value for the callable bond. Thus, the value of the callable bond at 15% volatility is less than that at 10% volatility—that is, less than 101.540.

Solution to 2: C is correct. Looking at Exhibit 12, the call price is too high for the call option to be exercised in any scenario. Thus, the value of the call option is zero, and the value of the callable bond is equal to the value of the straight bond—that is, 102.114.

4.2. Valuation of a Putable Bond with Interest Rate Volatility

The valuation of the three-year 4.25% annual coupon bond putable at par one year and two years from now at 10% volatility is depicted in Exhibit 13. The procedure for valuing a putable bond is very similar to that described earlier for valuing a callable bond, except that in each state, the bond's value is compared with the put price. The investor puts the bond only when the present value of the bond's future cash flows is lower than the put price. In this case, the value is reset to the put price (100). It happens twice in Year 2, in the states where the interest rates are 5.5258% and 4.5242%. The investor would not exercise the put option in Year 1 because the values for the bond exceed the put price.

EXHIBIT 13 Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Putable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



The value of the putable bond is 102.522. The value of the put option, obtained by taking the difference between the value of the putable bond and the value of the straight bond, is now 0.408 (102.522 – 102.114). As expected, the value of the put option is larger at 10% volatility than at 0% volatility (0.283).

EXAMPLE 4 Valuation of a Putable Bond Assuming Interest Rate Volatility

Return to the valuation of the Bermudan-style three-year 4.25% annual coupon bond putable at par one year and two years from now, as depicted in Exhibit 13. The one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively, and the interest rate volatility is 10%.

1. Assume that nothing changes relative to the initial setting except that the interest rate volatility is now 20% instead of 10%. The new value of the putable bond is:
   A. less than 102.522.
   B. equal to 102.522.
   C. more than 102.522.

2. Assume that nothing changes relative to the initial setting except that the bond is now putable at 95 instead of 100. The new value of the putable bond is *closest* to:
   A. 97.522.
   B. 102.114.
   C. 107.522.

Solution to 1: C is correct. A higher interest rate volatility increases the value of the put option. Because the value of the put option is added to the value of the straight bond to obtain the value of the putable bond, a higher value for the put option leads to a higher value for the putable bond. Thus, the value of the putable bond at 20% volatility is more than that at 10% volatility—that is, more than 102.522.

Solution to 2: B is correct. Looking at Exhibit 13, the put price is too low for the put option to be exercised in any scenario. Thus, the value of the put option is zero, and the value of the putable bond is equal to the value of the straight bond—that is, 102.114.

PUTABLE VS. EXTENDIBLE BONDS

Putable and extendible bonds are equivalent, except that their underlying option-free bonds are different. Consider a three-year 3.30% bond putable in Year 2. Its value should be exactly the same as that of a two-year 3.30% bond extendible by one year. Otherwise, there would be an arbitrage opportunity. Clearly, the cash flows of the two bonds are identical up to Year 2. The cash flows in Year 3 are dependent on the one-year forward rate two years from now. These cash flows will also be the same for both bonds regardless of the level of interest rates at the end of Year 2.

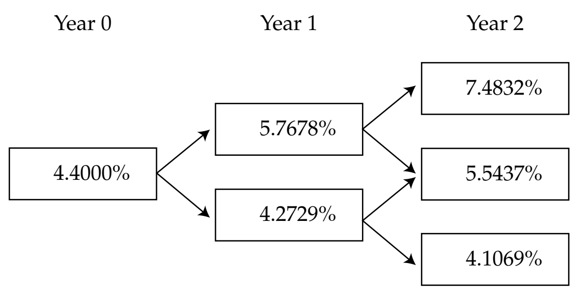
If the one-year forward rate at the end of Year 2 is higher than 3.30%, the putable bond will be put because the bondholder can reinvest the proceeds of the retired bond at a higher yield and the extendible bond will not be extended for the same reason. So, both bonds pay 3.30% for two years and are then redeemed. Alternatively, if the one-year forward rate at the end of Year 2 is lower than 3.30%, the putable bond will not be put because the bondholder would not want to reinvest at a lower yield and the extendible bond will be extended to hold onto the higher interest rate. Thus, both bonds pay 3.30% for three years and are then redeemed.

EXAMPLE 5 Valuation of Bonds with Embedded Options Assuming Interest Rate Volatility

Sidley Brown, a fixed-income associate at KMR Capital, is analyzing the effect of interest rate volatility on the values of callable and putable bonds issued by Weather Analytics (WA). WA is owned by the sovereign government, so its bonds are considered default free. Brown is currently looking at three of WA's bonds and has gathered the following information about them:

Characteristic	Bond X	Bond Y	Bond Z
Times to maturity	Three years from today	Three years from today	Three years from today
Coupon	5.2% annual	Not available	4.8% annual
Type of bond	Callable at par one year and two years from today	Callable at par one year and two years from today	Putable at par two years from today
Price (as a % of par)	Not available	101.325	Not available

The one-year, two-year, and three-year par rates are 4.400%, 4.700%, and 5.000%, respectively. Based on an estimated interest rate volatility of 15%, Brown has constructed the following binomial interest rate tree:



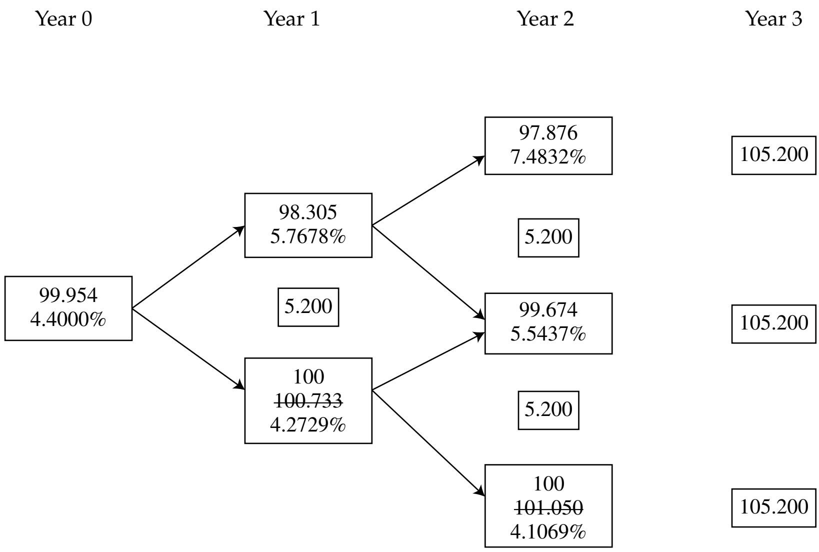
1. The price of Bond X is *closest* to:
   A. 96.057% of par.
   B. 99.954% of par.
   C. 100.547% of par.
2. The coupon rate of Bond Y is *closest* to:
   A. 4.200%.
   B. 5.000%.
   C. 6.000%.
3. The price of Bond Z is *closest* to:
   A. 99.638% of par.
   B. 100.340% of par.
   C. 100.778% of par.

Brown is now analyzing the effect of interest rate volatility on the price of WA's bonds.

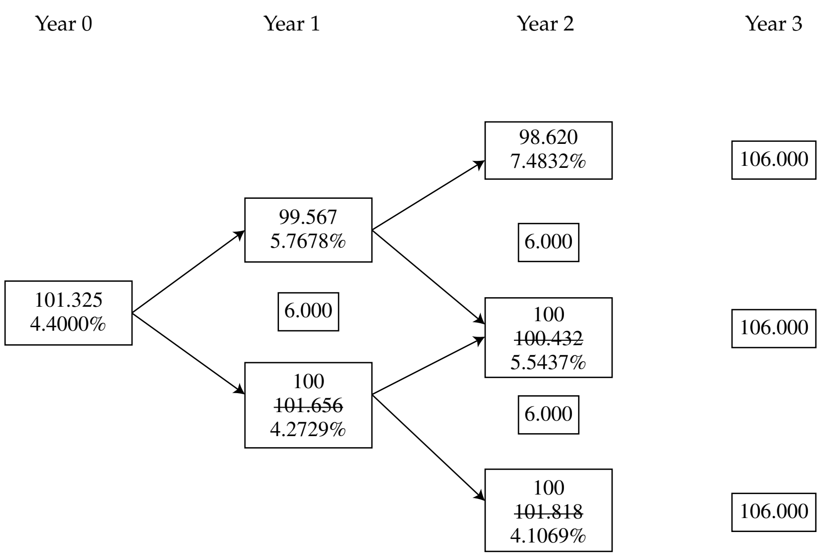
4. Relative to its price at 15% interest rate volatility, the price of Bond X at a lower interest rate volatility will be:
   A. lower.
   B. the same.
   C. higher.

5. Relative to its price at 15% interest rate volatility, the price of Bond Z at a higher interest rate volatility will be:
   A. lower.
   B. the same.
   C. higher.

Solution to 1: B is correct.



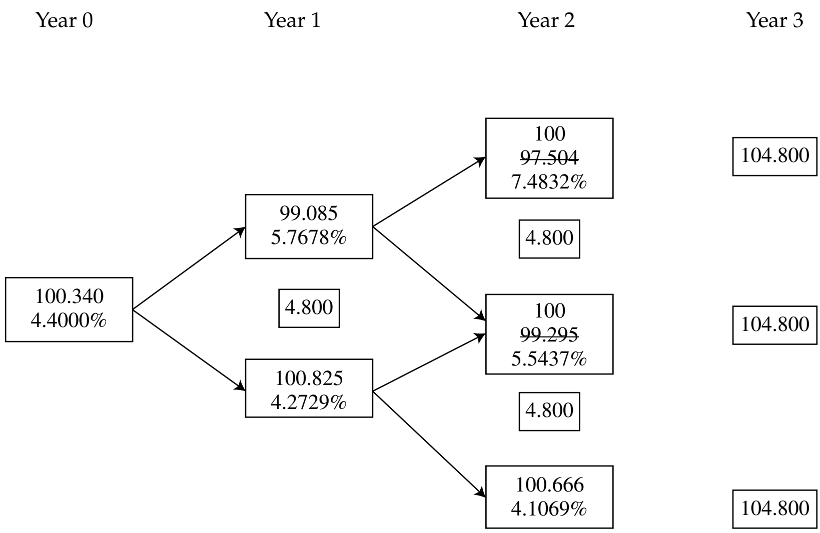
Solution to 2: C is correct.



Although the correct answer can be found by using the interest rate tree depicted, it is possible to identify it by realizing that the other two answers are clearly incorrect. The three-year 5% straight bond is worth par given that the three-year par rate is 5%.

Because the presence of a call option reduces the price of a callable bond, a three-year 5% bond callable at par can only be worth less than par—and certainly less than 101.325 given the yield curve and interest rate volatility assumptions—so B is incorrect. The value of a bond with a coupon rate of 4% is even less, so A is incorrect. Thus, C must be the correct answer.

Solution to 3: B is correct.



Solution to 4: C is correct. Bond X is a callable bond. As shown in Equation 1, the value of the call option decreases the value of Bond X relative to the value of the underlying option-free bond. As interest rate volatility decreases, the value of the call option decreases; thus, the value of Bond X increases.

Solution to 5: C is correct. Bond Z is a putable bond. As shown in Equation 2, the value of the put option increases the value of Bond Z relative to the value of the underlying option-free bond. As interest rate volatility increases, the value of the put option increases; thus, the value of Bond Z increases.

5. VALUATION OF RISKY CALLABLE AND PUTABLE BONDS

- **explain the calculation and use of option-adjusted spreads**
- **explain how interest rate volatility affects option-adjusted spreads**

Although the approach described earlier for default-free bonds may apply to securities issued by sovereign governments in their local currency, the fact is that most bonds are subject to default. Accordingly, we have to extend the framework to the valuation of risky bonds.

Two distinct approaches to valuing bonds are subject to default risk. The industry-standard approach is to increase the discount rates above the default-free rates to reflect default risk. Higher discount rates imply lower present values, and thus the value of a risky bond will be lower than that of an otherwise identical default-free bond.

The second approach to valuing risky bonds is to make the default probabilities explicit—that is, assigning a probability to each time period going forward. For example, the probability of default in Year 1 may be 1%; the probability of default in Year 2, conditional on surviving Year 1, may be 1.25%; and so on. This approach requires specifying the recovery value given default (e.g., 40% of par). Information about default probabilities and recovery values may be accessible from credit default swaps. This important topic is covered elsewhere.

5.1. Option-Adjusted Spread

Depending on available information, two standard approaches are used to construct a suitable yield curve for a risky bond. The more satisfactory but less convenient one is to use an issuer-specific curve, which represents the issuer's borrowing rates over the relevant range of maturities. Unfortunately, most bond professionals do not have access to such a level of detail. A more convenient and relatively satisfactory alternative is to uniformly raise the one-year forward rates derived from the default-free benchmark yield curve by a fixed spread, which is estimated from the market prices of suitable bonds of similar credit quality. This fixed spread is known as the zero-volatility spread, or Z-spread.

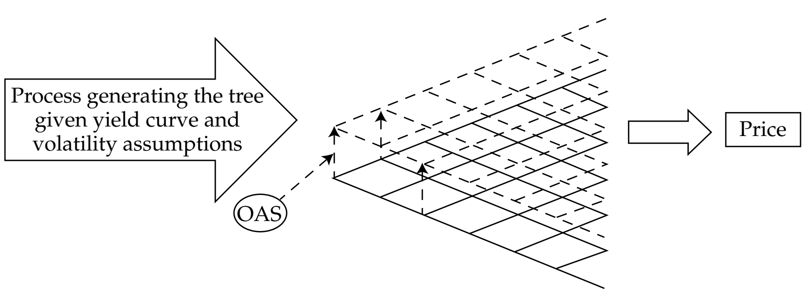
To illustrate, we return to the three-year 4.25% option-free bond introduced earlier, but now we assume that it is a risky bond and that the appropriate Z-spread is 100 bps. To calculate the arbitrage-free value of this bond, we have to increase each of the one-year forward rates given in Exhibit 1 by the Z-spread of 100 bps:

$$\frac{4.25}{(1.03500)}+\frac{4.25}{(1.03500)(1.04518)}+\frac{104.25}{(1.03500)(1.04518)(1.05564)}=99.326$$

As expected, the value of this risky bond (99.326) is considerably lower than the value of an otherwise identical but default-free bond (102.114).

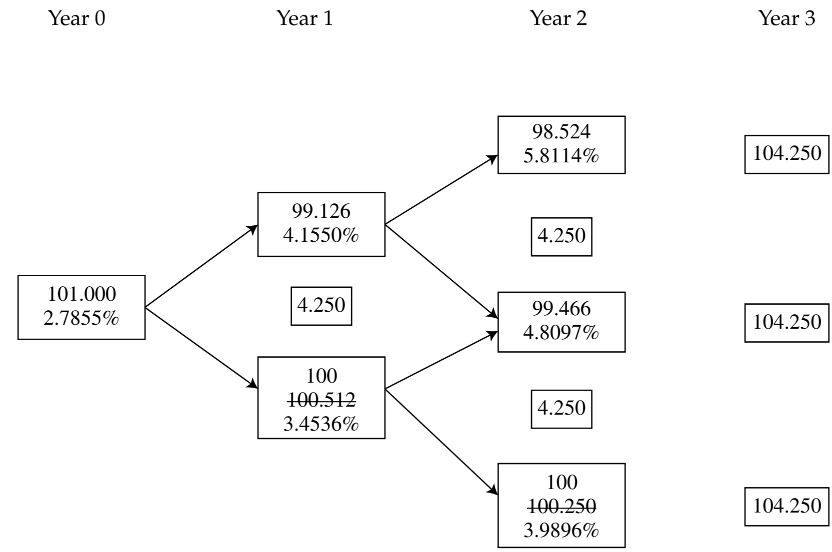
The same approach can be applied to the interest rate tree when valuing risky bonds with embedded options. In this case, an **option-adjusted spread** (OAS) is used. As depicted in Exhibit 14, the OAS is the constant spread that when added to all the one-period forward rates on the interest rate tree, makes the arbitrage-free value of the bond equal to its market price. Note that the Z-spread for an option-free bond is simply its OAS at zero volatility.

EXHIBIT 14 Interest Rate Tree and OAS



If the bond's price is given, the OAS is determined by trial and error. For example, suppose that the market price of a three-year 4.25% annual coupon bond callable in one year and two years from now (identical to the one valued in Exhibit 12 except that it is risky instead of default-free) is 101.000. To determine the OAS, we try shifting all the one-year forward rates in each state by adding a constant spread. For example, when we add 30 bps to all the one-year forward rates, we obtain a value for the callable bond of 100.973, which is lower than the bond's price. Because of the inverse relationship between a bond's price and its yield, this result means that the discount rates are too high, so we try a slightly lower spread. Adding 28 bps results in a value for the callable bond of 101.010, which is slightly too high. As illustrated in Exhibit 15, the constant spread added uniformly to all the one-period forward rates that justifies the given market price of 101.000 is 28.55 bps; this number is the OAS.

EXHIBIT 15 OAS of a Risky Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



As illustrated in Exhibit 15, the value at each node is adjusted based on whether the call option is exercised. Thus, the OAS removes the amount that results from the option risk, which is why this spread is called "option adjusted."

OAS is often used as a measure of value relative to the benchmark. An OAS lower than that for a bond with similar characteristics and credit quality indicates that the bond is likely overpriced (rich) and should be avoided. A larger OAS than that of a bond with similar characteristics and credit quality means that the bond is likely underpriced (cheap). If the OAS is close to that of a bond with similar characteristics and credit quality, the bond looks fairly priced. In our example, the OAS at 10% volatility is 28.55 bps. This number should be compared with the OAS of bonds with similar characteristics and credit quality to make a judgment about the bond's attractiveness.

5.2 Effect of Interest Rate Volatility on Option-Adjusted Spread

The dispersion of interest rates on the tree is volatility dependent, and so is the OAS. Exhibit 16 shows the effect of volatility on the OAS for a callable bond. The bond is a 5% annual coupon bond with 23 years left to maturity, callable in three years, priced at 95% of par, and valued assuming a flat yield curve of 4%.

EXHIBIT 16 Effect of Interest Rate Volatility on the OAS for a Callable Bond

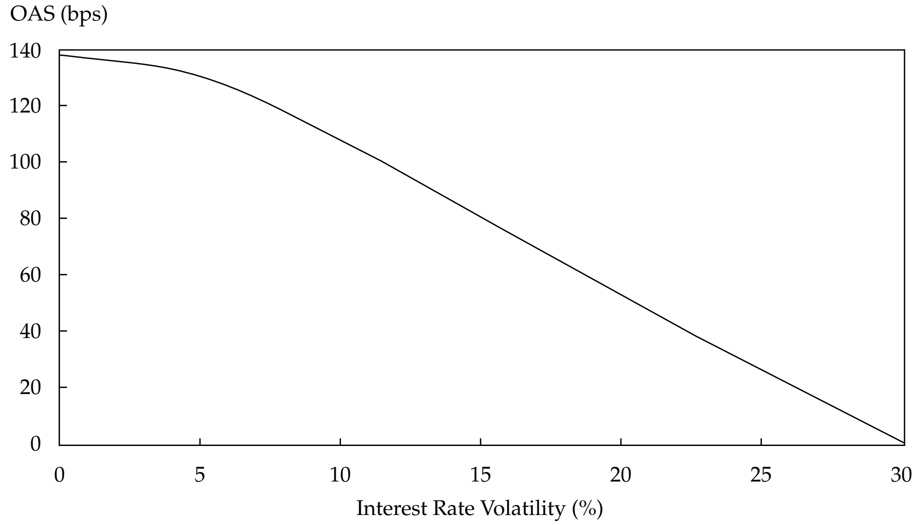
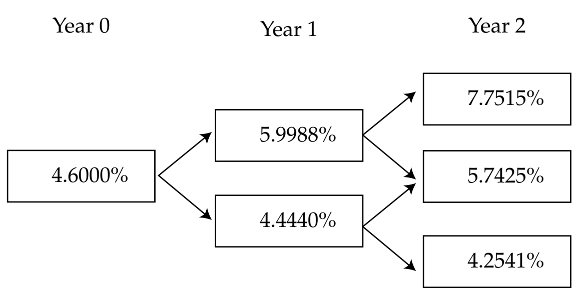


Exhibit 16 shows that as interest rate volatility increases, the OAS for the callable bond decreases. The OAS drops from 138.2 bps at 0% volatility to 1.2 bps at 30% volatility. This exhibit clearly demonstrates the importance of the interest rate volatility assumption. Returning to the example in Exhibit 15, the callable bond may look underpriced at 10% volatility. If an investor assumes a higher volatility, however, the OAS and thus relative cheapness will decrease.

EXAMPLE 6 Option-Adjusted Spread

Robert Jourdan, a portfolio manager, has just valued a 7% annual coupon bond that was issued by a French company and has three years remaining until maturity. The bond is callable at par one year and two years from now. In his valuation, Jourdan used the yield curve based on the on-the-run French government bonds. The one-year, two-year, and three-year par rates are 4.600%, 4.900%, and 5.200%, respectively. Based on an estimated interest rate volatility of 15%, Jourdan constructed the following binomial interest rate tree:

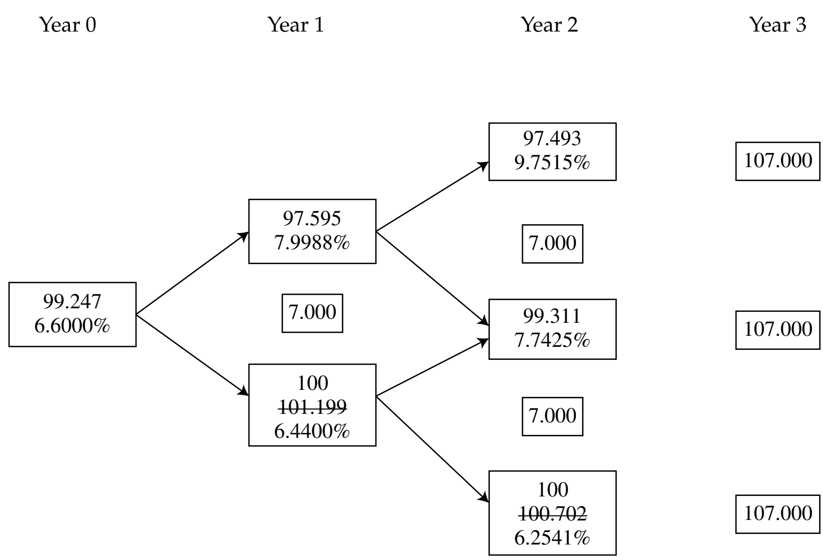


Jourdan valued the callable bond at 102.294% of par. However, Jourdan's colleague points out that because the corporate bond is riskier than French government bonds, the valuation should be performed using an OAS of 200 bps.

1. To update his valuation of the French corporate bond, Jourdan should:
   A. subtract 200 bps from the bond's annual coupon rate.
   B. add 200 bps to the rates in the binomial interest rate tree.
   C. subtract 200 bps from the rates in the binomial interest rate tree.
2. All else being equal, the value of the callable bond at 15% volatility is *closest* to:
   A. 99.198% of par.
   B. 99.247% of par.
   C. 104.288% of par.
3. Holding the price calculated in the previous question, the OAS for the callable bond at 20% volatility will be:
   A. lower.
   B. the same.
   C. higher.

Solution to 1: B is correct. The OAS is the constant spread that must be *added* to all the one-period forward rates given in the binomial interest rate tree to justify a bond's given market price.

Solution to 2: B is correct.



Solution to 3: A is correct. If interest rate volatility increases from 15% to 20%, the OAS for the callable bond will decrease.

SCENARIO ANALYSIS OF BONDS WITH OPTIONS

Another application of valuing bonds with embedded options is scenario analysis over a specified investment horizon. In addition to reinvestment of interest and principal, option valuation comes into play in that callable and putable bonds can be redeemed and their proceeds reinvested during the holding period. Making scenario-dependent, optimal option-exercise decisions involves computationally intensive use of OAS technology because the call or put decision must be evaluated considering the evolution of interest rate scenarios during the holding period.

Performance over a specified investment horizon entails a trade-off between reinvestment of cash flows and change in the bond's value. Let us take the example of a 4.5% bond with five years left to maturity and assume that the investment horizon is one year. If the bond is option free, higher interest rates increase the reinvestment income but result in lower principal value at the end of the investment horizon. Because the investment horizon is short, reinvestment income is relatively insignificant and performance will be dominated by the change in the value of the principal. Accordingly, lower interest rates will result in superior performance.

If the bond under consideration is callable, however, it is not at all obvious how the interest rate scenario affects performance. Suppose, for example, that the bond is first callable six months from now and that its current market price is 99.74. Steeply rising interest rates would depress the bond's price, and performance would definitely suffer. But steeply declining interest rates would also be detrimental because the bond would be called and *both interest and principal* would have to be reinvested at lower interest rates. Exhibit 17 shows the return over the one-year investment horizon for the 4.5% bond first callable in six months with five years left to maturity and valued on a 4% flat yield curve.

EXHIBIT 17 Effect of Interest Rate Changes on a Callable Bond's Total Return

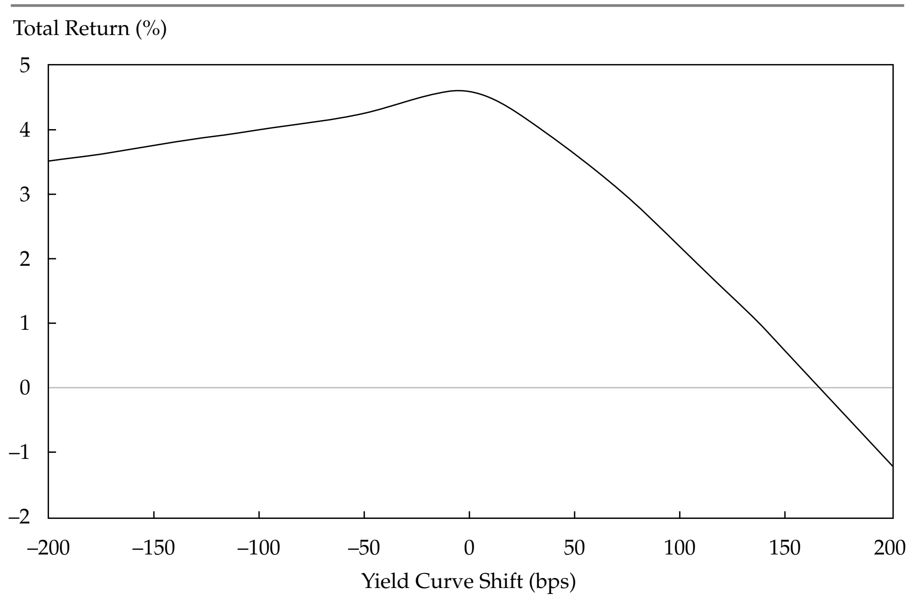


Exhibit 17 clearly shows that lower interest rates do not guarantee higher returns for callable bonds. The point to keep in mind is that the bond may be called long before the end of the investment horizon. Assuming that it is called on the horizon date would overestimate performance. Thus, a realistic prediction of option exercise is essential when performing scenario analysis of bonds with embedded options.

6. BONDS WITH EMBEDDED OPTIONS: EFFECTIVE DURATION

- **calculate and interpret effective duration of a callable or putable bond**
- **compare effective durations of callable, putable, and straight bonds**

Measuring and managing exposure to interest rate risk are two essential tasks of fixed-income portfolio management. Applications range from hedging a portfolio to asset–liability management of financial institutions. Portfolio managers, whose performance is often measured against a benchmark, also need to monitor the interest rate risk of both their portfolio and the benchmark. In this section, we cover two key measures of interest rate risk: duration and convexity.

6.1. Duration

The duration of a bond measures the sensitivity of the bond's full price (including accrued interest) to changes in the bond's yield to maturity (in the case of *yield* duration measures) or to changes in benchmark interest rates (in the case of yield-curve or *curve* duration measures). Yield duration measures, such as modified duration, can be used only for option-free bonds because these measures assume that a bond's expected cash flows do not change when the yield changes. This assumption is in general false for bonds with embedded options because the values of embedded options are typically contingent on interest rates. Thus, for bonds with embedded options, the only appropriate duration measure is the curve duration measure known as effective (or option-adjusted) duration. Because effective duration works for straight bonds as well as for bonds with embedded options, practitioners tend to use it regardless of the type of bond being analyzed.

6.1.1. Effective Duration

Effective duration indicates the sensitivity of the bond's price to a 100 bps parallel shift of the benchmark yield curve—in particular, the government par curve—assuming no change in the bond's credit spread. (*Note:* Although it is possible to explore how arbitrary changes in interest rates affect the bond's price, in practice the change is usually specified as a parallel shift of the benchmark yield curve.) The formula for calculating a bond's effective duration is

$$\text{EffDur} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\text{Curve}) \times (PV_0)} \tag{3}$$

where

ΔCurve = the magnitude of the parallel shift in the benchmark yield curve (in decimal)

PV_{-} = the full price of the bond when the benchmark yield curve is shifted down by ΔCurve

PV_{+} = the full price of the bond when the benchmark yield curve is shifted up by ΔCurve

PV_0 = the current full price of the bond (i.e., with no shift)

How is this formula applied in practice? Without a market price, we would need an issuer-specific yield curve to compute PV_0, PV_{-}, and PV_{+}. But practitioners usually have access to the bond's current price and thus use the following procedure:

1. Given a price (PV_0), calculate the implied OAS to the benchmark yield curve at an appropriate interest rate volatility.
2. Shift the benchmark yield curve down, generate a new interest rate tree, and then revalue the bond using the OAS calculated in Step 1. This value is PV_{-}.
3. Shift the benchmark yield curve up by the same magnitude as in Step 2, generate a new interest rate tree, and then revalue the bond using the OAS calculated in Step 1. This value is PV_{+}.
4. Calculate the bond's effective duration using Equation 3.

Let us illustrate using the same three-year 4.25% bond callable at par one year and two years from now, the same par yield curve (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively), and the same interest rate volatility (10%) as before. Also as before, we assume that the bond's current full price is 101.000. We apply the procedure just described:

1. As shown in Exhibit 15, given a price (PV_0) of 101.000, the OAS at 10% volatility is 28.55 bps.
2. We shift the par yield curve down by, say, 30 bps, generate a new interest rate tree, and then revalue the bond at an OAS of 28.55 bps. As shown in Exhibit 18, PV_{-} is 101.599.
3. We shift the par yield curve up by the same 30 bps, generate a new interest rate tree, and then revalue the bond at an OAS of 28.55 bps. As shown in Exhibit 19, PV_{+} is 100.407.
4. Thus,

$$\text{EffDur} = \frac{101.599 - 100.407}{2 \times 0.0030 \times 101.000} = 1.97$$

An effective duration of 1.97 indicates that a 100 bps increase in interest rate would reduce the value of the three-year 4.25% callable bond by 1.97%.

EXHIBIT 18 Valuation of a Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility with an OAS of 28.55 bps When Interest Rates Are Shifted Down by 30 bps

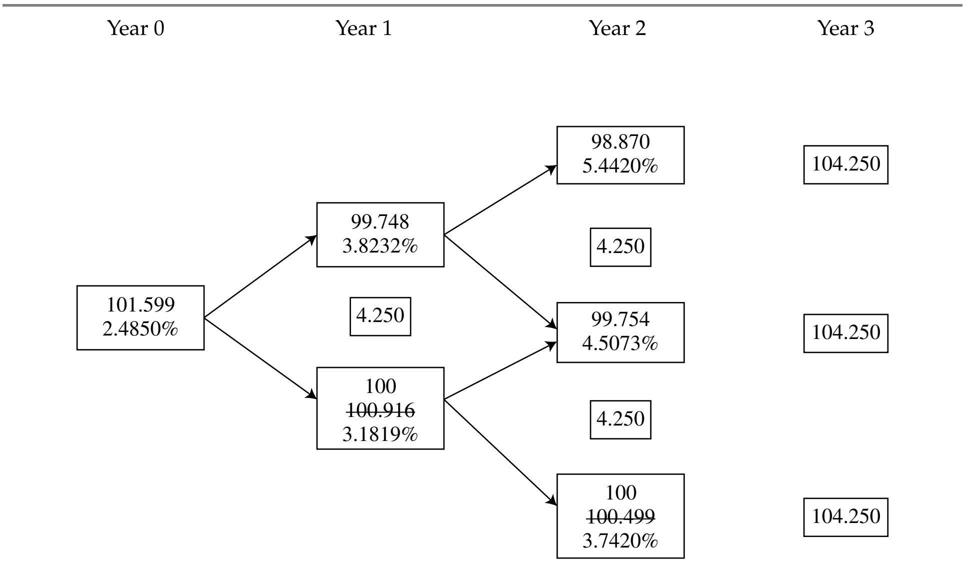
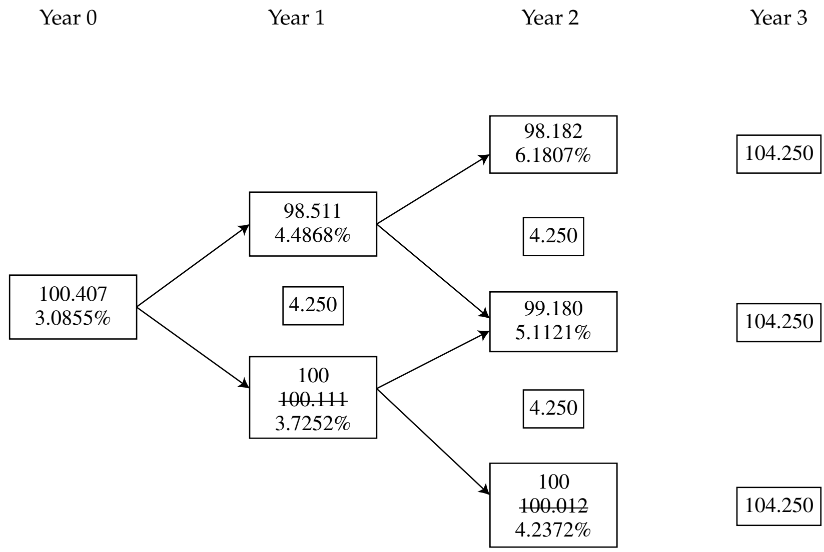


EXHIBIT 19 Valuation of a Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility with an OAS of 28.55 bps When Interest Rates Are Shifted Up by 30 bps



The effective duration of a callable bond cannot exceed that of the straight bond. When interest rates are high relative to the bond's coupon, the call option is out of the money so the bond is unlikely to be called. Thus, the effect of an interest rate change on the price of a callable bond is very similar to that on the price of an otherwise identical option-free bond; the callable and straight bonds have very similar effective durations. In contrast, when interest rates fall, the call option moves into the money. Remember that the call option gives the issuer the right to retire the bond at the call price and thus limits the price appreciation when interest rates decline. As a consequence, the call option reduces the effective duration of the callable bond relative to that of the straight bond.

The effective duration of a putable bond also cannot exceed that of the straight bond. When interest rates are low relative to the bond's coupon, the put option is out of the money so the bond is unlikely to be put. Thus, the effective duration of the putable bond is in this case very similar to that of an otherwise identical option-free bond. In contrast, when interest rates rise, the put option moves into the money and limits the price depreciation because the investor can put the bond and reinvest the proceeds of the retired bond at a higher yield. Thus, the put option reduces the effective duration of the putable bond relative to that of the straight bond.

When the embedded option (call or put) is deep in the money, the effective duration of the bond with an embedded option resembles that of the straight bond maturing on the first exercise date, reflecting the fact that the bond is highly likely to be called or put on that date.

Exhibit 20 compares the effective durations of option-free, callable, and putable bonds. All bonds are 4% annual coupon bonds with a maturity of 10 years. Both the call option and the put option are European-like and exercisable two months from now. The bonds are valued assuming a 4% flat yield curve and an interest rate volatility of 10%.

EXHIBIT 20 Comparison of the Effective Durations of Option-Free, Callable, and Putable Bonds

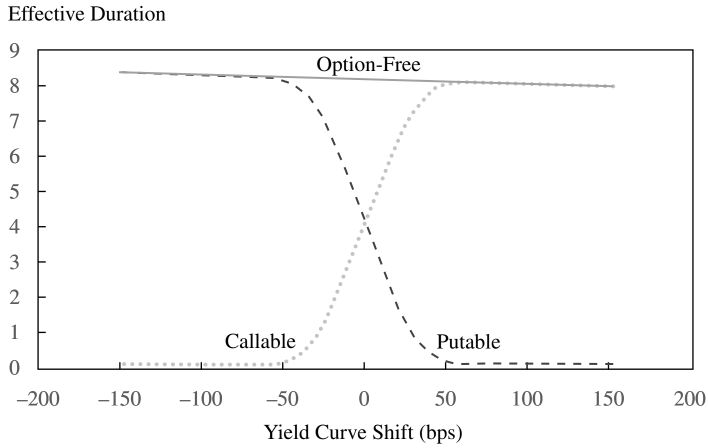


Exhibit 20 shows that the effective duration of an option-free bond changes very little in response to interest rate movements. As expected, when interest rates rise the put option moves into the money, which limits the price depreciation of the putable bond and shortens its effective duration. In contrast, the effective duration of the callable bond shortens when interest rates fall, which is when the call option moves into the money and thus limits the price appreciation of the callable bond.

EFFECTIVE DURATION IN PRACTICE

Effective duration is a concept most practically used in the context of a portfolio. Thus, an understanding of the effective durations of various types of instruments helps manage portfolio duration. In the following table, we show some properties of the effective duration of cash and the common types of bonds:

Type of Bond	Effective Duration
Cash	0
Zero-coupon bond	≈ Maturity
Fixed-rate bond	< Maturity
Callable bond	≤ Duration of straight bond
Putable bond	≤ Duration of straight bond
Floater (MRR flat)	≈ Time (in years) to next reset

In general, a bond's effective duration does not exceed its maturity. There are a few exceptions, however, such as tax-exempt bonds when analyzed on an after-tax basis.

Knowing the effective duration of each type of bond is useful when one needs to change portfolio duration. For example, a portfolio manager who wants to shorten the effective duration of a portfolio of fixed-rate bonds can add floaters. For the debt manager of a company or other issuing entity, another way of shortening effective duration is to issue callable bonds. The topic of changing portfolio duration is covered thoroughly in Level III.

7. ONE-SIDED AND KEY RATE DURATION

- **describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options**

Effective durations are normally calculated by averaging the changes resulting from shifting the benchmark yield curve up and down by the same amount. This calculation works well for option-free bonds, but the results can be misleading in the presence of embedded options. The problem is that when the embedded option is in the money, the price of the bond has limited upside potential if the bond is callable or limited downside potential if the bond is putable. Thus, the price sensitivity of bonds with embedded options is not symmetrical to positive and negative changes in interest rates of the same magnitude.

Consider, for example, a 4.5% bond maturing in five years, which is currently callable at 100. On a 4% flat yield curve at 15% volatility, the value of this callable bond is 99.75. If interest rates declined by 30 bps, the price would rise to 100. In fact, no matter how far interest rates decline, the price of the callable bond cannot exceed 100 because no investor will pay more than the price at which the bond can be immediately called. In contrast, the price decline has no limit if interest rates rise. Thus, the average price response to up- and down-shifts of interest rates (effective duration) is not as informative as the price responses to the up-shift (one-sided up-duration) and the down-shift (one-sided down-duration) of interest rates.

Exhibits 21 and 22 illustrate why **one-sided durations**—that is, the effective durations when interest rates go up or down—are better at capturing the interest rate sensitivity of a callable or putable bond than the (two-sided) effective durations, particularly when the embedded option is near the money.

EXHIBIT 21 Durations for a 4.5% Annual Coupon Bond Maturing in Five Years and Immediately Callable at Par on a 4% Flat Yield Curve at 15% Interest Rate Volatility

	At a 4% Flat Yield Curve	Interest Rate up by 30 bps	Interest Rate down by 30 bps
Value of the bond	99.75	99.17	100.00
Duration measure	Effective duration 1.39	One-sided up-duration 1.94	One-sided down-duration 0.84

Exhibit 21 shows that a 30 bps increase in the interest rate has a greater effect on the value of the callable bond than a 30 bps decrease in the interest rate. The fact that the one-sided up-duration is higher than the one-sided down-duration confirms that the callable bond is more sensitive to interest rate rises than to interest rate declines.

EXHIBIT 22 Durations for a 4.1% Annual Coupon Bond Maturing in Five Years and Immediately Putable at Par on a 4% Flat Yield Curve at 15% Interest Rate Volatility

	At a 4% Flat Yield Curve	Interest Rate up by 30 bps	Interest Rate down by 30 bps
Value of the bond	100.45	100.00	101.81
Duration measure	Effective duration 3.00	One-sided up-duration 1.49	One-sided down-duration 4.51

The one-sided durations in Exhibit 22 indicate that the putable bond is more sensitive to interest rate declines than to interest rate rises.

7.1. Key Rate Durations

Effective duration is calculated by assuming parallel shifts in the benchmark yield curve. In reality, however, interest rate movements are not as neat. Many portfolio managers and risk managers like to isolate the price responses to changes in the rates of key maturities on the benchmark yield curve. For example, how would the price of a bond be expected to change if only the two-year benchmark rate moved up by 5 bps? The answer is found by using **key rate durations** (also known as partial durations), which reflect the sensitivity of the bond's price to changes in specific maturities on the benchmark yield curve. Thus, key rate durations help portfolio managers and risk managers identify the "shaping risk" for bonds—that is, the bond's sensitivity to changes in the shape of the yield curve (e.g., steepening and flattening).

The valuation procedure and formula applied in the calculation of key rate durations are identical to those used in the calculation of effective duration, but instead of shifting the entire benchmark yield curve, only key points are shifted one at a time. Thus, the effective duration for each maturity point shift is calculated in isolation.

Exhibits 23, 24, and 25 show the key rate durations for bonds valued at a 4% flat yield curve. Exhibit 23 examines option-free bonds (assuming semi-annual coupons), and Exhibits 24 and 25 extend the analysis to callable and putable bonds, respectively.

EXHIBIT 23 Key Rate Durations of 10-Year Option-Free Bonds Valued at a 4% Flat Yield Curve

Coupon (%)	Price (% of par)	Key Rate Durations				
		Total	2-Year	3-Year	5-Year	10-Year
0	67.30	9.81	−0.07	−0.34	−0.93	11.15
2	83.65	8.83	−0.03	−0.13	−0.37	9.37
4	100.00	8.18	0.00	0.00	0.00	8.18
6	116.35	7.71	0.02	0.10	0.27	7.32
8	132.70	7.35	0.04	0.17	0.47	6.68
10	149.05	7.07	0.05	0.22	0.62	6.18

As shown in Exhibit 23, for option-free bonds not trading at par (the white rows), shifting any par rate has an effect on the value of the bond, but shifting the maturity-matched (10-year in this example) par rate has the greatest effect. This is simply because the largest cash flow of a fixed-rate bond occurs at maturity with the payment of both the final coupon and the principal.

For an option-free bond trading at par (the shaded row), the maturity-matched par rate is the only rate that affects the bond's value. It is a definitional consequence of "par" rates. If the 10-year par rate on a curve is 4%, then a 10-year 4% bond valued on that curve at zero OAS will be worth par regardless of the par rates of the other maturity points on the curve. In other words, shifting any rate other than the 10-year rate on the par yield curve will not change the value of a 10-year bond trading at par. Shifting a par rate up or down at a particular maturity point, however, respectively increases or decreases the *discount rate* at that maturity point. These facts will be useful to remember in the following paragraph.

As illustrated in Exhibit 23, key rate durations can sometimes be negative for maturity points that are shorter than the maturity of the bond being analyzed if the bond is a zero-coupon bond or has a very low coupon. We can explain why this is the case by using the zero-coupon bond (the first row of Exhibit 23). As discussed in the previous paragraph, if we increase the five-year par rate, the value of a 10-year bond trading at par must remain unchanged because the 10-year par rate has not changed. But the five-year zero-coupon rate has increased because of the increase in the five-year par rate. Thus, the value of the five-year coupon of the 10-year bond trading at par will be lower than before the increase. But because the value of the 10-year bond trading at par must remain par, the remaining cash flows, including the cash flow occurring in Year 10, must be discounted at slightly *lower* rates to compensate. This results in a lower 10-year zero-coupon rate, which makes the value of a 10-year zero-coupon bond (whose only cash flow is in Year 10) *rise* in response to an *upward* change in the five-year par rate. Consequently, the five-year key rate duration for a 10-year zero-coupon bond is negative (–0.93).

Unlike for option-free bonds, the key rate durations of bonds with embedded options depend not only on the *time to maturity* but also on the *time to exercise*. Exhibits 24 and 25 illustrate this phenomenon for 30-year callable and putable bonds. Both the call option and the put option are European-like exercisable 10 years from now, and the bonds are valued assuming a 4% flat yield curve and a volatility of 15%.

EXHIBIT 24 Key Rate Durations of 30-Year Bonds Callable in 10 Years Valued at a 4% Flat Yield Curve with 15% Interest Rate Volatility

Coupon (%)	Price (% of par)	Key Rate Durations					
		Total	2-Year	3-Year	5-Year	10-Year	30-Year
2	64.99	19.73	–0.02	–0.08	–0.21	–1.97	22.01
4	94.03	13.18	0.00	0.02	0.05	3.57	9.54
6	114.67	9.11	0.02	0.10	0.29	6.00	2.70
8	132.27	7.74	0.04	0.17	0.48	6.40	0.66
10	148.95	7.14	0.05	0.22	0.62	6.06	0.19

The bond with a coupon of 2% (the first row of Exhibit 24) is unlikely to be called, and thus it behaves more like a 30-year option-free bond, whose effective duration depends primarily on movements in the 30-year par rate. Therefore, the rate that has the highest effect on the value of the callable bond is the maturity-matched (30-year) rate. As the bond's coupon increases, however, so does the likelihood of the bond being called. Thus, the bond's total effective duration shortens, and the rate that has the highest effect on the callable bond's value gradually shifts from the 30-year rate to the 10-year rate. At the very high coupon of 10%, because of the virtual certainty of being called, the callable bond behaves like a 10-year option-free bond; the 30-year key rate duration is negligible (0.19) relative to the 10-year key rate duration (6.06).

EXHIBIT 25 Key Rate Durations of 30-Year Bonds Putable in 10 Years Valued at a 4% Flat Yield Curve with 15% Interest Rate Volatility

Coupon (%)	Price (% of par)	Key Rate Durations					
		Total	2-Year	3-Year	5-Year	10-Year	30-Year
2	83.89	9.24	−0.03	−0.14	−0.38	8.98	0.81
4	105.97	12.44	0.00	−0.01	−0.05	4.53	7.97
6	136.44	14.75	0.01	0.03	0.08	2.27	12.37
8	169.96	14.90	0.01	0.06	0.16	2.12	12.56
10	204.38	14.65	0.02	0.07	0.21	2.39	11.96

If the 30-year bond putable in 10 years has a high coupon, its price is more sensitive to the 30-year rate because it is unlikely to be put and thus behaves like an otherwise identical option-free bond. The 10% putable bond (the last row of Exhibit 25), for example, is most sensitive to changes in the 30-year rate, as illustrated by a 30-year key rate duration of 11.96. At the other extreme, a low-coupon bond is most sensitive to movements in the 10-year rate. It is almost certain to be put and so behaves like an option-free bond maturing on the put date.

8. EFFECTIVE CONVEXITY

- **compare effective convexities of callable, putable, and straight bonds**

Duration is an approximation of the expected bond price responses to changes in interest rates because actual changes in bond prices are not linear, particularly for bonds with embedded options. Thus, it is useful to measure **effective convexity**—that is, the sensitivity of duration to changes in interest rates—as well. The formula to calculate a bond's effective convexity is

$$\text{EffCon} = \frac{(PV_{-}) + (PV_{+}) - [2\times(PV_0)]}{(\Delta\text{Curve})^2 \times (PV_0)} \tag{4}$$

where

ΔCurve = the magnitude of the parallel shift in the benchmark yield curve (in decimal)
PV_{-} = the full price of the bond when the benchmark yield curve is shifted down by ΔCurve
PV_{+} = the full price of the bond when the benchmark yield curve is shifted up by ΔCurve
PV_0 = the current full price of the bond (i.e., with no shift)

Let us return to the three-year 4.25% bond callable at par one year and two years from now. We still use the same par yield curve (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively) and the same interest rate volatility (10%) as before, but we now assume that the bond's current full price is 100.785 instead of 101.000. Thus, the implied OAS is 40 bps. Given 30 bps shifts in the benchmark yield curve, the resulting PV_- and PV_+ are 101.381 and 100.146, respectively. Using Equation 4, the effective convexity is:

$$\text{EffCon} = \frac{101.381 + 100.146 - 2 \times 100.785}{(0.003)^2 \times 100.785} = -47.41$$

[Note that there are two different conventions for reporting convexity in practice; "raw" convexity figures, such as in this example, are sometimes scaled (divided) by 100.]

Exhibit 20, shown earlier, displays effective durations but also illustrates the effective convexities of callable and putable bonds. When interest rates are high and the value of the call option is low, the callable and straight bond experience very similar effects from changes in interest rates. They both have positive convexity. However, the effective convexity of the callable bond turns negative when the call option is near the money, as in the example just presented, which indicates that the upside for a callable bond is much smaller than the downside. The reason is because when interest rates decline, the price of the callable bond is capped by the price of the call option if it is near the exercise date.

Conversely, putable bonds always have positive convexity. When the option is near the money, the upside for a putable bond is much larger than the downside because the price of a putable bond is floored by the price of the put option if it is near the exercise date.

Compared side by side, putable bonds have more upside potential than otherwise identical callable bonds when interest rates decline. Putable bonds also have less downside risk than otherwise identical callable bonds when interest rates rise.

EXAMPLE 7 Interest Rate Sensitivity

Erna Smith, a portfolio manager, has two fixed-rate bonds in her portfolio: a callable bond (Bond X) and a putable bond (Bond Y). She wants to examine the interest rate sensitivity of these two bonds to a parallel shift in the benchmark yield curve. Assuming an interest rate volatility of 10%, her valuation software shows how the prices of these bonds change for 30 bps shifts up or down:

	Bond X	Bond Y
Time to maturity	Three years from today	Three years from today
Coupon	3.75% annual	3.75% annual
Type of bond	Callable at par one year from today	Putable at par one year from today
Current price (% of par)	100.594	101.330
Price (% of par) when shifting the benchmark yield curve down by 30 bps	101.194	101.882
Price (% of par) when shifting the benchmark yield curve up by 30 bps	99.860	100.924

1. The effective duration for Bond X is *closest* to:
   A. 0.67.
   B. 2.21.
   C. 4.42.
2. The effective duration for Bond Y is *closest* to:
   A. 0.48.
   B. 0.96.
   C. 1.58.
3. When interest rates rise, the effective duration of:
   A. Bond X shortens.
   B. Bond Y shortens.
   C. the underlying option-free (straight) bond corresponding to Bond X lengthens.
4. When the option embedded in Bond Y is in the money, the one-sided durations *most likely* show that the bond is:
   A. more sensitive to a decrease in interest rates.
   B. more sensitive to an increase in interest rates.
   C. equally sensitive to a decrease or to an increase in interest rates.
5. The price of Bond X is affected:
   A. only by a shift in the one-year par rate.
   B. only by a shift in the three-year par rate.
   C. by all par rate shifts but is most sensitive to shifts in the one-year and three-year par rates.
6. The effective convexity of Bond X:
   A. cannot be negative.
   B. turns negative when the embedded option is near the money.
   C. turns negative when the embedded option moves out of the money.
7. Which of the following statements is *most* accurate?
   A. Bond Y exhibits negative convexity.
   B. For a given decline in interest rate, Bond X has less upside potential than Bond Y.
   C. The underlying option-free (straight) bond corresponding to Bond Y exhibits negative convexity.

Solution to 1: B is correct. The effective duration for Bond X is

$$\text{EffDur} = \frac{101.194 - 99.860}{2 \times 0.003 \times 100.594} = 2.21$$

A is incorrect because the duration of a bond with a single cash flow one year from now is approximately one year, so 0.67 is too low—even assuming that the bond will be called in one year with certainty. C is incorrect because 4.42 exceeds the maturity of Bond X (three years).

Solution to 2: C is correct. The effective duration for Bond Y is

$$\text{EffDur} = \frac{101.882 - 100.924}{2 \times 0.003 \times 101.330} = 1.58$$

Solution to 3: B is correct. When interest rates rise, a put option moves into the money and the putable bond is more likely to be put. Thus, it behaves like a shorter-maturity bond, and its effective duration shortens. A is incorrect because when interest rates rise, a call option moves out of the money; so, the callable bond is less likely to be called. C is incorrect because the effective duration of an option-free bond goes down as interest rates rise.

Solution to 4: A is correct. If interest rates rise, the investor's ability to put the bond at par limits the price depreciation. In contrast, the increase in the bond's price has no limit when interest rates decline. Thus, the price of a putable bond whose embedded option is in the money is more sensitive to a decrease in interest rates.

Solution to 5: C is correct. The main driver of the call decision is the two-year forward rate one year from now. This rate is most significantly affected by changes in the one-year and three-year par rates.

Solution to 6: B is correct. The effective convexity of a callable bond turns negative when the call option is near the money because the price response of a callable bond to lower interest rates is capped by the call option. That is, in case of a decline in interest rates, the issuer will call the bonds and refund at lower rates, thus limiting the upside potential for the investor.

Solution to 7: B is correct. As interest rates decline, the value of a call option increases whereas the value of a put option decreases. The call option embedded in Bond X limits its price appreciation, but Bond Y has no such cap. Thus, Bond X has less upside potential than Bond Y. A is incorrect because a putable bond always has positive convexity; that is, Bond Y has more upside than downside potential. C is incorrect because an option-free bond exhibits low positive convexity.

9. VALUATION AND ANALYSIS OF CAPPED AND FLOORED FLOATING-RATE BONDS

- **calculate the value of a capped or floored floating-rate bond**

Options in floating-rate bonds (floaters) are exercised automatically depending on the course of interest rates; if the coupon rate rises or falls below the threshold, the cap or floor automatically applies. Similar to callable and putable bonds, capped and floored floaters can be valued by using the arbitrage-free framework.

9.1. Valuation of a Capped Floater

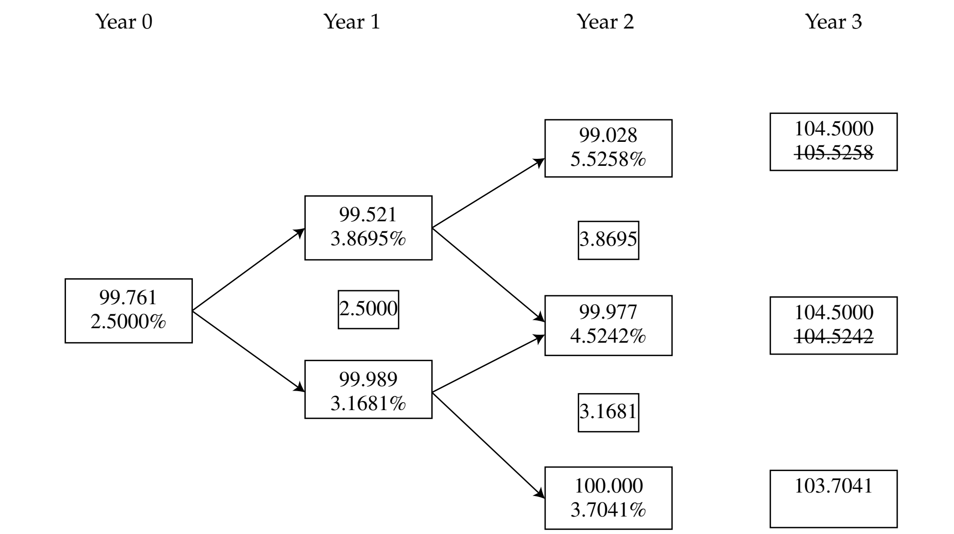
The cap provision in a floater prevents the coupon rate from increasing above a specified maximum rate. As a consequence, a **capped floater** protects the issuer against rising interest rates and is thus an issuer option. Because the investor is long the bond but short the embedded option, the value of the cap decreases the value of the capped floater relative to the value of the straight bond:

$$\text{Value of capped floater} = \text{Value of straight bond} - \text{Value of embedded cap} \tag{5}$$

To illustrate how to value a capped floater, consider a floating-rate bond that has a three-year maturity. The floater's coupon pays the one-year reference rate annually, set in arrears, and is capped at 4.500%. The term "set in arrears" means that the coupon rate is set at the *end* of the coupon period; the payment date and the setting date are one and the same. For simplicity, we assume that the issuer's credit quality closely matches the reference rate swap curve (i.e., there is no credit spread) and that the reference rate swap curve is the same as the par yield curve given in Exhibit 1 (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively). We also assume that the interest rate volatility is 10%.

The valuation of the capped floater is depicted in Exhibit 26.

EXHIBIT 26 Valuation of a Three-Year Reference Rate Floater Capped at 4.500% at 10% Interest Rate Volatility



Without a cap, the value of this floater would be 100 because in every scenario, the coupon paid would be equal to the discount rate. But because the coupon rate is capped at 4.500%, which is lower than the highest interest rates in the tree, the value of the capped floater will be lower than the value of the straight bond.

For each scenario, we check whether the cap applies; if it does, the cash flow is adjusted accordingly. For example, at the top of the tree at Year 2, the reference rate (5.5258%) is higher than the 4.500% cap. Thus, the coupon payment at Year 3 is capped at the 4.500 maximum amount, and the cash flow is adjusted downward from the uncapped amount (105.5258) to the capped amount (104.5000). The coupon is also capped when the reference rate is 4.5242% at Year 2.

As expected, the value of the capped floater is lower than 100 (99.761). The value of the cap can be calculated by using Equation 5:

$$\text{Value of embedded cap} = 100 - 99.761 = 0.239$$

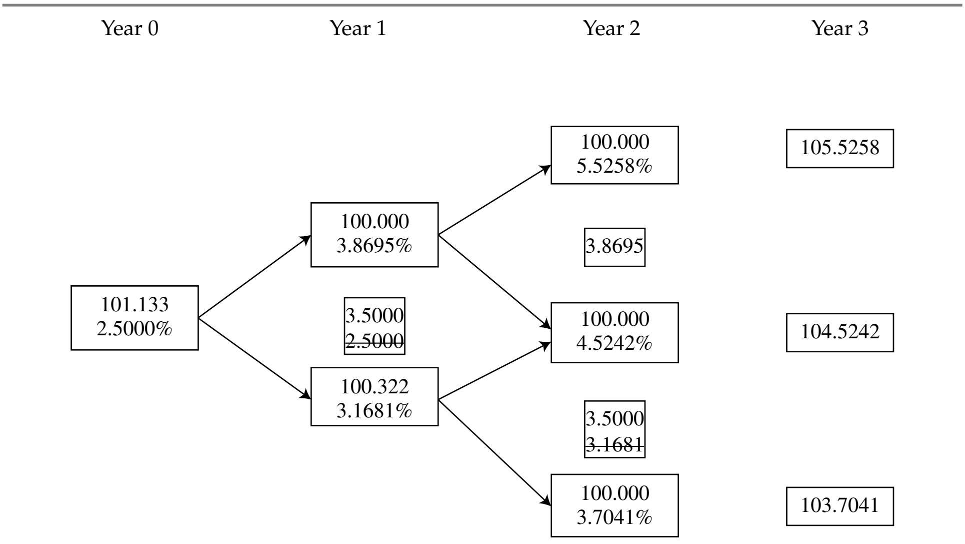
9.2. Valuation of a Floored Floater

The floor provision in a floater prevents the coupon rate from decreasing below a specified minimum rate. As a consequence, a **floored floater** protects the investor against declining interest rates and is thus an investor option. Because the investor is long both the bond and the embedded option, the value of the floor increases the value of the floored floater relative to the value of the straight bond:

$$\text{Value of floored floater} = \text{Value of straight bond} + \text{Value of embedded floor} \tag{6}$$

To illustrate how to value a floored floater, we return to the example we used for the capped floater but assume that the embedded option is now a 3.500% floor instead of a 4.500% cap. The other assumptions remain the same. The valuation of the floored floater is depicted in Exhibit 27.

EXHIBIT 27 Valuation of a Three-Year Reference Rate Floater Floored at 3.500% at 10% Interest Rate Volatility



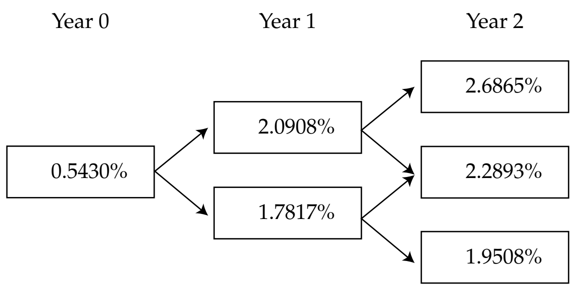
Recall from the discussion about the capped floater that if there were no cap, the value of the floater would be 100 because the coupon paid would equal the discount rate. The same principle applies here: If there were no floor, the value of this floater would be 100. Because the presence of the floor potentially increases the cash flows, however, the value of the floored floater must be equal to or higher than the value of the straight bond.

Exhibit 27 shows that the floor is binding at Year 0 because the reference rate (2.5000%) is less than the cap rate (3.5000%) and at Year 1 at the lower node where the reference rate is 3.1681%. Thus, the corresponding interest payments at Year 1 and 2 are increased to the minimum amount of 3.5000. As a consequence, the value of the floored floater exceeds 100 (101.133). The value of the floor can be calculated by using Equation 6:

$$\text{Value of embedded floor} = 101.133 - 100 = 1.133$$

EXAMPLE 8 Valuation of Capped and Floored Floaters

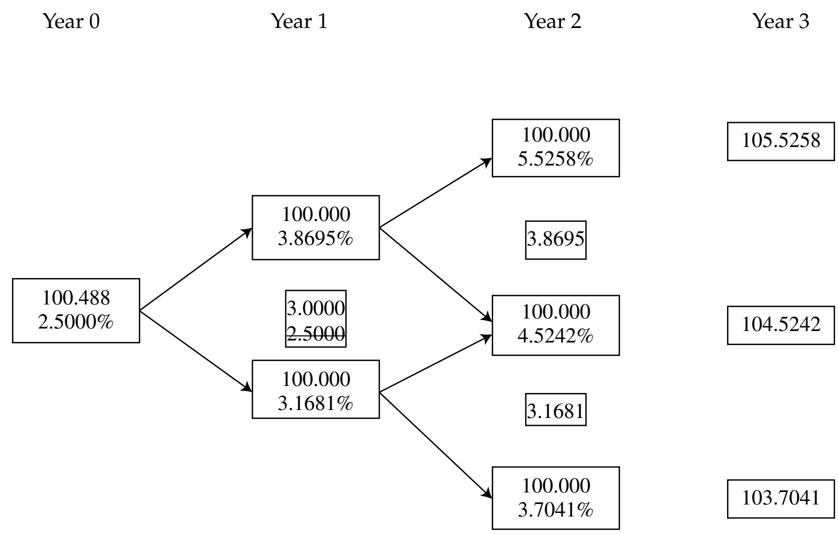
1. A three-year floating rate bond pays annual coupons of one-year reference rate (set in arrears) and is capped at 5.600%. The reference rate swap curve is as given in Exhibit 1 (i.e., the one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively), and interest rate volatility is 10%. The value of the capped floater is *closest* to:
   A. 100.000.
   B. 105.600.
   C. 105.921.
2. A three-year floating-rate bond pays annual coupons of one-year reference rate (set in arrears) and is floored at 3.000%. The reference swap curve is as given in Exhibit 1 (i.e., the one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively), and interest rate volatility is 10%. The value of the floored floater is *closest* to:
   A. 100.000.
   B. 100.488.
   C. 103.000.
3. An issuer in the eurozone wants to sell a three-year floating-rate note at par with an annual coupon based on the 12-month Euribor + 300 bps. Because the 12-month Euribor is currently at a historic low and the issuer wants to protect itself against a sudden increase in interest cost, the issuer's advisers recommend increasing the credit spread to 320 bps and capping the coupon at 5.50%. Assuming an interest rate volatility of 8%, the advisers have constructed the following binomial interest rate tree:



   The value of the capped floater is *closest* to:
   A. 92.929.
   B. 99.916.
   C. 109.265.

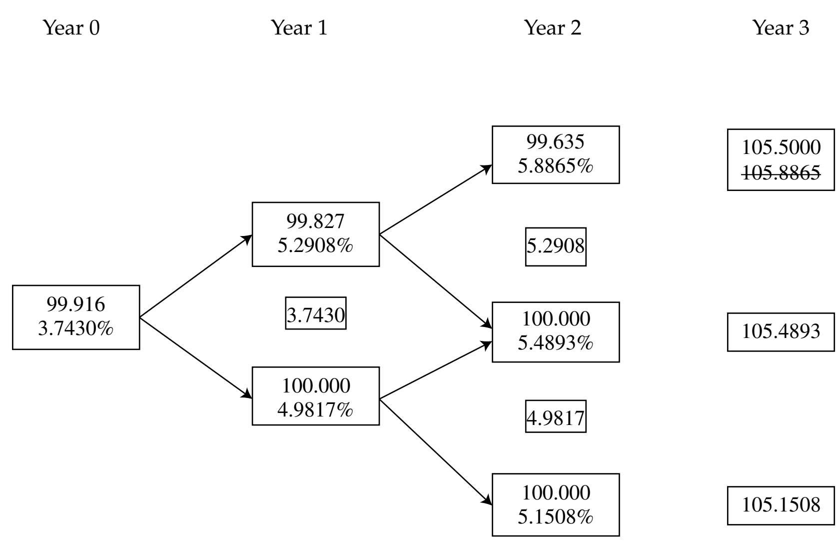
Solution to 1: A is correct. As illustrated in Exhibit 26, the cap is higher than any of the rates at which the floater is reset on the interest rate tree. Thus, the value of the bond is the same as if it had no cap—that is, 100.

Solution to 2: B is correct. One can eliminate C because as illustrated in Exhibit 27, all else being equal, the bond with a higher floor (3.500%) has a value of 101.133. The value of a bond with a floor of 3.000% cannot be higher. Intuitively, B is the likely correct answer because the straight bond is worth 100. However, it is still necessary to calculate the value of the floored floater because if the floor is low enough, it could be worthless.



Here, it turns out that the floor adds 0.488 in value to the straight bond. Had the floor been 2.500%, the floored floater and the straight bond would both be worth par.

Solution to 3: B is correct.



10. VALUATION AND ANALYSIS OF CONVERTIBLE BONDS: DEFINING FEATURES AND ANALYSIS OF A CONVERTIBLE BOND

- **describe defining features of a convertible bond**
- **calculate and interpret the components of a convertible bond's value**

So far, we have discussed bonds for which the exercise of the option is at the discretion of the issuer (callable bond), at the discretion of the bondholder (putable bond), or set through a pre-defined contractual arrangement (capped and floored floaters). What distinguishes a convertible bond from the bonds discussed earlier is that exercising the option results in the change of the security from a bond to a common stock. This section describes defining features of convertible bonds and discusses how to analyze and value these bonds.

10.1. Defining Features of a Convertible Bond

A **convertible bond** presents the characteristics of an option-free bond and an embedded conversion option, which gives bondholders the right to convert their debt into equity during the **conversion period** at a pre-determined **conversion price**.

Investors usually accept a lower coupon for convertible bonds than for otherwise identical non-convertible bonds because they can participate in the potential upside through the conversion mechanism that allows the bondholders to convert their bonds into shares at a cost lower than market value. The issuer benefits from paying a lower coupon. In case of conversion, an added benefit for the issuer is that it no longer has to repay the debt that was converted into equity.

However, what might appear as a win–win situation for both the issuer and the investors is not a "free lunch" because the issuer's existing shareholders face dilution in case of conversion. In addition, if the underlying share price remains below the conversion price and the bond is not converted, the issuer must repay the debt or refinance it, potentially at a higher cost. If conversion is not achieved, the bondholders will have lost interest income relative to an otherwise identical non-convertible bond that would have been issued with a higher coupon and would have thus offered investors an additional spread.

We will use the information provided in Exhibit 28 to describe the features of a convertible bond and then illustrate how to analyze it. Exhibit 28 is based on a $1 billion convertible bond issued in June 2018 by Twitter, Inc. (TWTR), a company listed on the New York Stock Exchange. Some features of the actual convertible bond, such as the presence of a make-whole call option, have been dropped for simplicity.

EXHIBIT 28 Twitter, Inc., $1 billion, 0.25% Convertible Bonds Due 15 June 2024

- **Issue Date:** 11 June 2018
- **Ranking:** Senior unsecured
- **Interest:** 0.25% per year. Interest will accrue from 11 June 2018 and will be payable semi-annually in arrears on 15 June and 15 December of each year, beginning on 15 December 2018.
- **Issue Price:** 100% of par value
- **Maturity:** 15 June 2024

(*continued*)

EXHIBIT 28 (Continued)

- **Conversion Rate:** Each bond of par value of $1,000 is convertible to 17.5 shares of common stock.
- **Conversion Price:** $57.14 per share
- **Share Price at Issuance:** $40.10
- (Assumed) **Share Price on 15 June 2019:** $35.14
- (Assumed) **Convertible Bond Price on 15 June 2019:** 95.225% of par value
- **Conversion Premium:** 42.5%

The applicable share price at which the investor can convert the bonds into ordinary (common) shares is called the conversion price. In the Twitter example provided in Exhibit 28, the conversion price is $57.14 per share.

The number of shares of common stock that the bondholder receives from converting the bonds into shares is called the **conversion rate (or ratio)**. In the Twitter example, bondholders who hold $10,000 in par value can convert their bonds into shares and receive 175 shares ($10,000/$57.14). The conversion rate is 17.5 per $1,000 in par value. The conversion may be exercised during a particular period or at set intervals during the life of the bond.

The conversion price in Exhibit 28 is referred to as the *initial* conversion price because it reflects the conversion price *at issuance*. Corporate actions—such as stock splits, bonus share issuances, and rights or warrants issuances—affect a company's share price and may reduce the benefit of conversion for the convertible bondholders. Thus, the terms of issuance of the convertible bond contain detailed information defining how the conversion price and conversion ratio are adjusted should such a corporate action occur during the life of the bond. For example, suppose that Twitter performs a 2:1 stock split to its common shareholders. In this case, the conversion price would be adjusted to $28.57 (i.e., $57.14/2) per share and the conversion rate adjusted to 35 (i.e., 17.5 × 2) shares per $1,000 of nominal value.

As long as the convertible bond is still outstanding and has not been converted, the bondholders receive interest payments (semiannually in the Twitter example). Meanwhile, if the issuer declares and pays dividends, common shareholders receive dividend payments. The terms of issuance may offer no compensation to convertible bondholders for dividends paid out during the life of the bond at one extreme, or they may offer full protection by adjusting the conversion price downward for any dividend payments at the other extreme. Typically, a threshold dividend is defined in the terms of issuance. Annual dividend payments below the threshold dividend have no effect on the conversion price. In contrast, the conversion price is adjusted downward for annual dividend payments above the threshold dividend to offer compensation to convertible bondholders.

Should the issuer be acquired by or merged with another company during the life of the bond, bondholders might no longer be willing to continue lending to the new entity. Change-of-control events are defined in the prospectus or offering circular, and if such an event occurs, convertible bondholders usually have the choice between

- a put option that can be exercised during a specified period following the change-of-control event and that provides full redemption of the nominal value of the bond; or
- an adjusted conversion price that is lower than the initial conversion price. This downward adjustment gives the convertible bondholders the opportunity to convert their bonds into shares earlier and at more advantageous terms—thus allowing them to participate in the announced merger or acquisition as common shareholders.

In addition to a put option in case of a change-of-control event, it is not unusual for a convertible bond to include a put option that convertible bondholders can exercise during specified periods. Put options can be classified as "hard" puts or "soft" puts. In the case of a hard put, the issuer must redeem the convertible bond for cash. In the case of a soft put, the investor has the right to exercise the put but the issuer chooses how the payment will be made. The issuer may redeem the convertible bond for cash, common stock, subordinated notes, or a combination of the three.

It is more frequent for convertible bonds to include a call option that gives the issuer the right to call the bond during a specified period and at specified times. As discussed earlier, the issuer may exercise the call option and redeem the bond early if interest rates are falling or if its credit rating is revised upward—thus enabling the issuance of debt at a lower cost. The issuer may also believe that its share price will increase significantly in the future because of its performance or because of events that will take place in the economy or in its sector. In this case, the issuer may try to maximize the benefit to its existing shareholders relative to convertible bondholders and call the bond. To offer convertible bondholders protection against early repayment, convertible bonds usually have a protection period. Subsequently, they can be called but at a premium, which decreases as the maturity of the bond approaches.

If a convertible bond is callable, the issuer has an incentive to call the bond when the underlying share price increases above the conversion price in order to avoid paying further coupons. Such an event is called **forced conversion** because it forces bondholders to convert their bonds into shares. Otherwise, the redemption value that bondholders would receive from the issuer calling the bond would result in a disadvantageous position and a loss compared with conversion. Even if interest rates have not fallen or the issuer's credit rating has not improved, thus not allowing refinancing at a lower cost, the issuer might still proceed with calling the bond when the underlying share price exceeds the conversion price. Doing so allows the issuer to take advantage of the favorable equity market conditions and force the bondholders to convert their bonds into shares. The forced conversion strengthens the issuer's capital structure and eliminates the risk that a subsequent correction in equity prices prevents conversion and requires redeeming the convertible bonds at maturity.

10.2. Analysis of a Convertible Bond

A number of investment metrics and ratios help analyze and value a convertible bond.

10.2.1. Conversion Value

The **conversion value**, or parity value, of a convertible bond indicates the value of the bond if it is converted at the market price of the shares.

$$\text{Conversion value} = \text{Underlying share price} \times \text{Conversion ratio}$$

Based on the information provided in Exhibit 28, we can calculate the conversion value for Twitter's convertible bonds at the issuance date and on 15 June 2019 (*Note:* The assumed prices actually pertain to 11 April 2019 to simplify the calculation of the straight bond values as there are then five full years to maturity.):

$$\text{Conversion value at the issuance date} = \$40.10 \times 17.5 = \$701.75$$

$$\text{Conversion value on 15 June 2019} = \$35.14 \times 17.5 = \$614.95$$

10.2.2 Minimum Value of a Convertible Bond

The minimum value of a convertible bond is equal to the greater of

- the conversion value and
- the value of the underlying option-free bond. Theoretically, the value of the straight bond (straight value) can be estimated by using the market value of a non-convertible bond of the issuer with the same characteristics as the convertible bond but without the conversion option. In practice, such a bond rarely exists. Thus, the straight value is found by using the arbitrage-free framework and by discounting the bond's future cash flows at the appropriate rates.

The minimum value of a convertible bond can also be described as a floor value. It is a *moving* floor, however, because the straight value is not fixed; it changes with fluctuations in interest rates and credit spreads. If interest rates rise, the value of the straight bond falls, making the floor fall. Similarly, if the issuer's credit spread increases—as a result, for example, of a downgrade of its credit rating from investment grade to non-investment grade—the floor value will fall too.

Using the conversion values calculated earlier, the minimum value of Twitter's convertible bonds at the issuance date is

$$\text{Minimum value at the issuance date} = \text{Maximum } (\$701.75; \$1{,}000) = \$1{,}000$$

The straight value at the issuance date is \$1,000 because the issue price is set at 100% of par. But after this date, this value will fluctuate. Thus, to calculate the minimum value of Twitter's convertible bond on 15 June 2019, it is first necessary to calculate the value of the straight bond that day using the arbitrage-free framework. From Exhibit 28, the coupon is 0.25%, paid semiannually. Assuming a 2.5% flat yield curve, the straight value on 15 June 2019 when five years remain until maturity is \$894.86 per \$1,000 in par value:

$$\frac{\$1.25}{\left(1+\frac{0.025}{2}\right)^{1}} + \frac{\$1.25}{\left(1+\frac{0.025}{2}\right)^{2}} + \ldots + \frac{\$1{,}001.25}{\left(1+\frac{0.025}{2}\right)^{10}} = \$894.86$$

It follows that the minimum value of Twitter's convertible bonds on 15 June 2019 is:

$$\text{Minimum value} = \text{Maximum } (\$614.95; \$894.86) = \$894.86$$

If the value of the convertible bond were lower than the greater of the conversion value and the straight value, an arbitrage opportunity would ensue. Two scenarios help illustrate this concept. Returning to the Twitter example, suppose that the convertible bond is selling for \$850.00 on 15 June 2019—that is, at a price that is lower than the straight value of \$894.86. In this scenario, the convertible bond is cheap relative to the straight bond; put another way, the convertible bond offers a higher yield than an otherwise identical non-convertible bond. Thus, investors will find the convertible bond attractive, buy it, and push its price up until the convertible bond price returns to the straight value and the arbitrage opportunity disappears.

Alternatively, assume that on 15 June 2019 the yield on otherwise identical non-convertible bonds is 12.00% instead of 2.50%. Using the arbitrage-free framework, the straight value is \$567.59 per \$1,000 in par value. Suppose that the convertible bond is selling at this straight value—that is, at a price that is lower than its conversion value of \$614.95. In this case, an arbitrageur can buy the convertible bond for \$567.59, convert it into 17.5 shares, and sell the shares at \$35.14 each or \$614.95 in total. The arbitrageur makes a profit equal to the difference between the conversion value and the straight value—that is, \$47.36 (\$614.95 − \$567.59). As more arbitrageurs follow the same strategy, the convertible bond price will increase until it reaches the conversion value and the arbitrage opportunity disappears.

10.2.3. Market Conversion Price, Market Conversion Premium per Share, and Market Conversion Premium Ratio

Many investors do not buy a convertible bond at issuance on the primary market but instead buy such a bond later in its life on the secondary market. The **market conversion premium per share** allows investors to identify the premium or discount payable when buying the convertible bond rather than the underlying common stock:

$$\text{Market conversion premium per share} = \text{Market conversion price} - \text{Underlying share price}$$

where

$$\text{Market conversion price} = \frac{\text{Convertible bond price}}{\text{Conversion ratio}}$$

The market conversion price represents the price that investors effectively pay for the underlying common stock if they buy the convertible bond and then convert it into shares. It can be viewed as a break-even price. Once the underlying share price exceeds the market conversion price, any further rise in the underlying share price is certain to increase the value of the convertible bond by at least the same percentage (we will discuss why at a later stage).

Based on the information provided in Exhibit 28,

$$\text{Market conversion price on 15 June 2019} = \frac{\$952.25}{17.5} = \$54.40$$

and

$$\text{Market conversion premium per share on 15 June 2019} = \$57.14 - \$54.40 = \$2.74$$

The **market conversion premium ratio** expresses the premium, or discount, investors have to pay as a percentage of the current market price of the shares:

$$\text{Market conversion premium ratio} = \frac{\text{Market conversion premium per share}}{\text{Underlying share price}}$$

In the Twitter example,

$$\text{Market conversion premium ratio on 15 June 2019} = \frac{\$2.74}{\$35.14} = 7.80\%$$

Why would investors be willing to pay a premium to buy the convertible bond? Recall that the straight value acts as a floor for the convertible bond price. Thus, as the underlying share price falls, the convertible bond price will not fall below the straight value. Viewed in this context, the market conversion premium per share resembles the price of a call option. Investors who buy a call option limit their downside risk to the price of the call option (premium). Similarly, the premium paid when buying a convertible bond allows investors to limit their downside risk to the straight value. There is a fundamental difference, however, between the buyers of a call option and the buyers of a convertible bond. The former know exactly the amount of the downside risk, whereas the latter know only that the most they can lose is the difference between the convertible bond price and the straight value because the straight value is not fixed.

Market conversion discounts per share are rare, but they can theoretically happen given that the convertible bond and the underlying common stock trade in different markets with different types of market participants. For example, highly volatile share prices may result in the market conversion price being lower than the underlying share price.

10.2.4 Downside Risk with a Convertible Bond

Many investors use the straight value as a measure of the downside risk of a convertible bond and calculate the following metric:

$$\text{Premium over straight value} = \frac{\text{Convertible bond price}}{\text{Straight value}} - 1$$

All else being equal, the higher the premium over straight value, the less attractive the convertible bond. In the Twitter example,

$$\text{Premium over straight value} = \frac{\$952.25}{\$894.86} = 6.41\%$$

Despite its use in practice, the premium over straight value is a flawed measure of downside risk because, as mentioned earlier, the straight value is not fixed but rather fluctuates with changes in interest rates and credit spreads.

10.2.5 Upside Potential of a Convertible Bond

The upside potential of a convertible bond depends primarily on the prospects of the underlying common stock. Thus, convertible bond investors should be familiar with the techniques used to value and analyze common stocks. These techniques are covered elsewhere.

11. VALUATION OF A CONVERTIBLE BOND AND COMPARISON OF RISK–RETURN CHARACTERISTICS

- **describe how a convertible bond is valued in an arbitrage-free framework**
- **compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock**

Historically, the valuation of convertible bonds has been challenging because these securities combine characteristics of bonds, stocks, and options—thus requiring an understanding of what affects the value of fixed income, equity, and derivatives. The complexity of convertible bonds has also increased over time as a result of market innovations and additions to the terms and conditions of these securities. For example, there are now contingent convertible bonds and convertible contingent convertible bonds, which are even more complex to value and analyze.

CONTINGENT CONVERTIBLES

Contingent convertible bonds, or "CoCos," pay a higher coupon than otherwise identical non-convertible bonds; however, they usually are deeply subordinated and may be converted into equity or face principal write-downs if regulatory capital ratios are breached. Convertible contingent convertible bonds, or "CoCoCos," combine a traditional convertible bond and a CoCo. They are convertible at the discretion of the investor, thus offering upside potential if the share price increases. They are also converted into equity or face principal write-downs in the event of a regulatory capital breach. CoCos and CoCoCos are usually issued by financial institutions, particularly in Europe.

The fact that many bond's prospectuses or offering circulars frequently provide for an independent financial valuer to determine the conversion price (and, in essence, the value of the convertible bond) under different scenarios is evidence of the complexity associated with valuing convertible bonds. Because of this complexity, convertible bonds in many markets come with selling restrictions. They are typically offered in very high denominations and only to professional or institutional investors. Regulators perceive them as securities that are too risky for retail investors to invest in directly.

As with any fixed-income instrument, convertible bond investors should perform a diligent risk–reward analysis of the issuer, including its ability to service the debt and repay the principal, as well as a review of the bond's terms of issuance (e.g., collateral, credit enhancements, covenants, and contingent provisions). In addition, convertible bond investors must analyze the factors that typically affect bond prices, such as interest rate movements. Because most convertible bonds have lighter covenants than otherwise similar non-convertible bonds and are frequently issued as subordinated securities, the valuation and analysis of some convertible bonds can be complex.

The investment characteristics of a convertible bond depend on the underlying share price, so convertible bond investors must also analyze factors that may affect the issuer's common stock, including dividend payments and the issuer's actions (e.g., acquisitions or disposals, rights issues). Even if the issuer is performing well, adverse market conditions might depress share prices and prevent conversion. Thus, convertible bond investors must also identify and analyze the exogenous reasons that might ultimately have a negative effect on convertible bonds.

Academics and practitioners have developed advanced models to value convertible bonds, but the most commonly used model remains the arbitrage-free framework. A traditional convertible bond can be viewed as a straight bond and a call option on the issuer's common stock, so

$$\text{Value of convertible bond} = \text{Value of straight bond} + \text{Value of call option on the issuer's stock}$$

Many convertible bonds include a call option that gives the issuer the right to call the bond during a specified period and at specified times. The value of such bonds is

Value of callable convertible bond = Value of straight bond + Value of call option on the issuer's stock − Value of issuer call option

Suppose that the callable convertible bond also includes a put option that gives the bondholder the right to require that the issuer repurchase the bond. The value of such a bond is

Value of callable putable convertible bond = Value of straight bond + Value of call option on the issuer's stock − Value of issuer call option + Value of investor put option

No matter how many options are embedded into a bond, the valuation procedure remains the same. It relies on generating a tree of interest rates based on the given yield curve and interest rate volatility assumptions, determining at each node of the tree whether the embedded options will be exercised, and then applying the backward induction valuation methodology to calculate the present value of the bond.

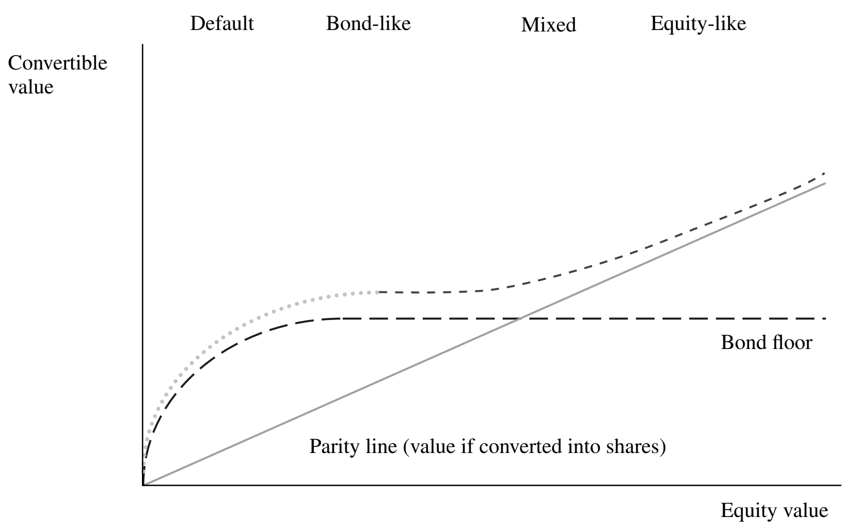
11.1. Comparison of the Risk–Return Characteristics of a Convertible Bond, the Straight Bond, and the Underlying Common Stock

In its simplest form, a convertible bond can be viewed as a straight bond and a call option on the issuer's common stock. When the underlying share price is well below the conversion price, the convertible bond is described as "busted convertible" and exhibits mostly bond risk–return characteristics. That is, the risk–return characteristics of the convertible bond resemble those of the underlying option-free (straight) bond. In this case, the call option is out of the money, so share price movements do not significantly affect the price of the call option and, thus, the price of the convertible bond. Consequently, the price movement of the convertible bond closely follows that of the straight bond, and such factors as interest rate movements and credit spreads significantly affect the convertible bond price. As the share price approaches zero, the value of the bond will fall to approach the present value of the recovery rate in bankruptcy. The convertible bond exhibits even stronger bond risk–return characteristics when the call option is out of the money and the conversion period is approaching its end because the time value component of the option decreases toward zero, making it highly likely that the conversion option will expire worthless. This scenario is shown in Exhibit 29 on the left.

In contrast, when the underlying share price is above the conversion price, a convertible bond exhibits mostly stock risk–return characteristics (see the right-hand side of Exhibit 29). That is, the risk–return characteristics of the convertible bond resemble those of the underlying common stock. In this case, the call option is in the money, so the price of the call option—and thus the price of the convertible bond—is significantly affected by share price movements but mostly unaffected by factors driving the value of an otherwise identical option-free bond, such as interest rate movements. When the call option is in the money, it is more likely to be exercised by the bondholder and the value of the shares resulting from the conversion is higher than the redemption value of the bond. Such convertible bonds trade at prices that closely follow the conversion value of the convertible bond, and their price exhibits similar movements to that of the underlying stock.

In between the bond and the stock extremes, the call option component increases in value as the underlying share price approaches the conversion price. The return on the convertible bond during such periods increases significantly but at a lower rate than the increase in the underlying share price because the conversion price has not yet been reached. When the share price exceeds the conversion price and goes higher, the change in the convertible bond price converges toward the change in the underlying share price. This is why we noted earlier that when the underlying share price exceeds the market conversion price, any further rise in the underlying share price is certain to increase the value of the convertible bond by at least the same percentage.

EXHIBIT 29 Price Behavior of a Convertible Bond and the Underlying Common Stock



Why would an investor not exercise the conversion option when the underlying share price is above the conversion price? The call option on the issuer's common stock may be a European-style option that cannot be exercised now but only at the end of a pre-determined period. Even if the call option is an American-style option, making it possible to convert the bond into equity, it may not be optimal for the convertible bondholder to exercise prior to the expiry of the conversion period. As discussed earlier, it is sometimes better to wait than to exercise an option that is in the money. The investor may also prefer to sell the convertible bond instead of exercising the conversion option.

Except for busted convertibles, the most important factor in the valuation of convertible bonds is the underlying share price. However, it is worth mentioning that large movements in interest rates or in credit spreads may significantly affect the value of convertible bonds. For a convertible bond with a fixed coupon, all else being equal, a significant fall in interest rates would result in an increase in its value and price, whereas a significant rise in interest rates would lead to a decrease in its value and price. Similarly, all else being equal, a significant improvement in the issuer's credit quality would result in an increase in the value and price of its convertible bonds, whereas a deterioration of the issuer's credit quality would lead to a decrease in the value and price of its convertible bonds.

EXAMPLE 9 Valuation of Convertible Bonds

Nick Andrews, a fixed-income investment analyst, has been asked by his supervisor to prepare an analysis of the convertible bond issued by Heavy Element Inc., a chemical industry company, for presentation to the investment committee. Andrews has gathered the following data from the convertible bond's prospectus and market information:

Issuer: Heavy Element Inc.
Issue Date: 15 September 2020
Maturity Date: 15 September 2025
Interest: 3.75% payable annually
Issue Size: $100,000,000
Issue Price: $1,000 at par
Conversion Ratio: 23.26
Convertible Bond Price on 16 September 2022: $1,230
Share Price on 16 September 2022: $52

1. The conversion price is *closest* to:
   A. $19.
   B. $43.
   C. $53.
2. The conversion value on 16 September 2022 is *closest* to:
   A. $24.
   B. $230.
   C. $1,209.
3. The market conversion premium per share on 16 September 2022 is *closest* to:
   A. $0.88.
   B. $2.24.
   C. $9.00.
4. The risk–return characteristics of the convertible bond on 16 September 2022 *most likely* resemble that of:
   A. a busted convertible.
   B. Heavy Element's common stock.
   C. a bond of Heavy Element that is identical to the convertible bond but without the conversion option.

5. As a result of favorable economic conditions, credit spreads for the chemical industry narrow, resulting in lower interest rates for the debt of such companies as Heavy Element. All else being equal, the price of Heavy Element's convertible bond will *most likely*:
   A. decrease significantly.
   B. not change significantly.
   C. increase significantly.
6. Suppose that on 16 September 2022 the convertible bond is available in the secondary market at a price of \$1,050. An arbitrageur can make a risk-free profit by:
   A. buying the underlying common stock and shorting the convertible bond.
   B. buying the convertible bond, exercising the conversion option, and selling the shares resulting from the conversion.
   C. shorting the convertible bond and buying a call option on the underlying common stock exercisable at the conversion price on the conversion date.
7. A few months have passed. Because of chemical spills in lake water at the site of a competing facility, the government has introduced very costly environmental legislation. As a result, share prices of almost all publicly traded chemical companies, including Heavy Element, have decreased sharply. Heavy Element's share price is now \$28. Now, the risk–return characteristics of the convertible bond *most likely* resemble that of:
   A. a bond.
   B. a hybrid instrument.
   C. Heavy Element's common stock.

Solution to 1: B is correct. The conversion price is equal to the par value of the convertible bond divided by the conversion ratio—that is, \$1,000/23.26 = \$43 per share.

Solution to 2: C is correct. The conversion value is equal to the underlying share price multiplied by the conversion ratio—that is, \$52 × 23.26 = \$1,209.

Solution to 3: A is correct. The market conversion premium per share is equal to the convertible bond price divided by the conversion ratio, minus the underlying share price—that is, (\$1,230/23.26) – \$52 = \$52.88 – \$52 = \$0.88.

Solution to 4: B is correct. The underlying share price (\$52) is well above the conversion price (\$43). Thus, the convertible bond exhibits risk–return characteristics that are similar to those of the underlying common stock. A is incorrect because a busted convertible is a convertible bond for which the underlying common stock trades at a significant discount relative to the conversion price. C is incorrect because it describes a busted convertible.

Solution to 5: B is correct. The underlying share price (\$52) is well above the conversion price (\$43). Thus, the convertible bond exhibits mostly stock risk–return characteristics, and its price is mainly driven by the underlying share price. Consequently, the decrease in credit spreads will have little effect on the convertible bond price.

Solution to 6: B is correct. The convertible bond price ($1,050) is lower than its minimum value ($1,209). Thus, the arbitrageur can buy the convertible bond for $1,050; convert it into 23.26 shares; and sell the shares at $52 each, or $1,209 in total, making a profit of $159. A and C are incorrect because in both scenarios, the arbitrageur is short the underpriced asset (convertible bond) and long an overpriced asset, resulting in a loss.

Solution to 7: A is correct. The underlying share price ($28) is now well below the conversion price ($43), so the convertible bond is a busted convertible and exhibits mostly bond risk–return characteristics. B is incorrect because the underlying share price would have to be close to the conversion price for the risk–return characteristics of the convertible bond to resemble that of a hybrid instrument. C is incorrect because the underlying share price would have to be in excess of the conversion price for the risk–return characteristics of the convertible bond to resemble that of the company's common stock.

SUMMARY

- An embedded option represents a right that can be exercised by the issuer, by the bondholder, or automatically depending on the course of interest rates. It is attached to, or embedded in, an underlying option-free bond called a straight bond.
- Simple embedded option structures include call options, put options, and extension options. Callable and putable bonds can be redeemed prior to maturity, at the discretion of the issuer in the former case and of the bondholder in the latter case. An extendible bond gives the bondholder the right to keep the bond for a number of years after maturity. Putable and extendible bonds are equivalent, except that their underlying option-free bonds are different.
- Complex embedded option structures include bonds with other types of options or combinations of options. For example, a convertible bond includes a conversion option that allows the bondholders to convert their bonds into the issuer's common stock. A bond with an estate put can be put by the heirs of a deceased bondholder. Sinking fund bonds make the issuer set aside funds over time to retire the bond issue and are often callable, may have an acceleration provision, and may also contain a delivery option. Valuing and analyzing bonds with complex embedded option structures is challenging.
- According to the arbitrage-free framework, the value of a bond with an embedded option is equal to the arbitrage-free values of its parts—that is, the arbitrage-free value of the straight bond and the arbitrage-free values of each of the embedded options.
- Because the call option is an issuer option, the value of the call option decreases the value of the callable bond relative to an otherwise identical but non-callable bond. In contrast, because the put option is an investor option, the value of the put option increases the value of the putable bond relative to an otherwise identical but non-putable bond.
- In the absence of default and interest rate volatility, the bond's future cash flows are certain. Thus, the value of a callable or putable bond can be calculated by discounting the bond's future cash flows at the appropriate one-period forward rates, taking into consideration the decision to exercise the option. If a bond is callable, the decision to exercise the option is made by the issuer, which will exercise the call option when the value of the bond's future cash flows is higher than the call price. In contrast, if the bond is putable, the decision to exercise the option is made by the bondholder, who will exercise the put option when the value of the bond's future cash flows is lower than the put price.

- In practice, interest rates fluctuate and interest rate volatility affects the value of embedded options. Thus, when valuing bonds with embedded options, it is important to consider the possible evolution of the yield curve over time.
- Interest rate volatility is modeled using a binomial interest rate tree. The higher the volatility, the lower the value of the callable bond and the higher the value of the putable bond.
- Valuing a bond with embedded options assuming an interest rate volatility requires three steps: (1) Generate a tree of interest rates based on the given yield curve and volatility assumptions; (2) at each node of the tree, determine whether the embedded options will be exercised; and (3) apply the backward induction valuation methodology to calculate the present value of the bond.
- The option-adjusted spread is the single spread added uniformly to the one-period forward rates on the tree to produce a value or price for a bond. OAS is sensitive to interest rate volatility: The higher the volatility, the lower the OAS for a callable bond.
- For bonds with embedded options, the best measure to assess the sensitivity of the bond's price to a parallel shift of the benchmark yield curve is effective duration. The effective duration of a callable or putable bond cannot exceed that of the straight bond.
- When the option is near the money, the convexity of a callable bond is negative, indicating that the upside for a callable bond is much smaller than the downside, whereas the convexity of a putable bond is positive, indicating that the upside for a putable bond is much larger than the downside.
- Because the prices of callable and putable bonds respond asymmetrically to upward and downward interest rate changes of the same magnitude, one-sided durations provide a better indication regarding the interest rate sensitivity of bonds with embedded options than (two-sided) effective duration.
- Key rate durations show the effect of shifting only key points, one at a time, rather than the entire yield curve.
- The arbitrage-free framework can be used to value capped and floored floaters. The cap provision in a floater is an issuer option that prevents the coupon rate from increasing above a specified maximum rate. Thus, the value of a capped floater is equal to or less than the value of the straight bond. In contrast, the floor provision in a floater is an investor option that prevents the coupon from decreasing below a specified minimum rate. Thus, the value of a floored floater is equal to or higher than the value of the straight bond.
- The characteristics of a convertible bond include the conversion price, which is the applicable share price at which the bondholders can convert their bonds into common shares, and the conversion ratio, which reflects the number of shares of common stock that the bondholders receive from converting their bonds into shares. The conversion price is adjusted in case of corporate actions, such as stock splits, bonus share issuances, and rights and warrants issuances. Convertible bondholders may receive compensation when the issuer pays dividends to its common shareholders, and they may be given the opportunity to either put their bonds or convert their bonds into shares earlier and at more advantageous terms in the case of a change of control.
- A number of investment metrics and ratios help analyze and value convertible bonds. The conversion value indicates the value of the bond if it is converted at the market price of the shares. The minimum value of a convertible bond sets a floor value for the convertible bond at the greater of the conversion value or the straight value. This floor is moving, however, because the straight value is not fixed. The market conversion premium represents the price investors effectively pay for the underlying shares if they buy the convertible bond and then convert it into shares. Scaled by the market price of the shares, it represents the premium payable when buying the convertible bond rather than the underlying common stock.

- Because convertible bonds combine characteristics of bonds, stocks, and options, as well as potentially other features, their valuation and analysis are challenging. Convertible bond investors should consider the factors that affect not only bond prices but also the underlying share price.
- The arbitrage-free framework can be used to value convertible bonds, including callable and putable ones. Each component (straight bond, call option of the stock, and call and/or put option on the bond) can be valued separately.
- The risk–return characteristics of a convertible bond depend on the underlying share price relative to the conversion price. When the underlying share price is well below the conversion price, the convertible bond is "busted" and exhibits mostly bond risk–return characteristics. Thus, it is mainly sensitive to interest rate movements. In contrast, when the underlying share price is well above the conversion price, the convertible bond exhibits mostly stock risk–return characteristics. Thus, its price follows similar movements to the price of the underlying stock. In between these two extremes, the convertible bond trades like a hybrid instrument.

PRACTICE PROBLEMS

The following information relates to Questions 1–10

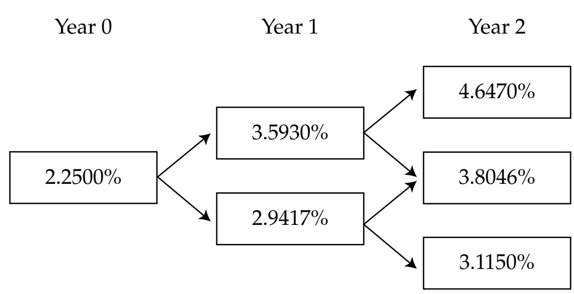
Samuel & Sons is a fixed-income specialty firm that offers advisory services to investment management companies. On 1 October 20X0, Steele Ferguson, a senior analyst at Samuel, is reviewing three fixed-rate bonds issued by a local firm, Pro Star, Inc. The three bonds, whose characteristics are given in Exhibit 1, carry the highest credit rating.

EXHIBIT 1 Fixed-Rate Bonds Issued by Pro Star, Inc.

Bond	Maturity	Coupon	Type of Bond
Bond #1	1 October 20X3	4.40% annual	Option-free
Bond #2	1 October 20X3	4.40% annual	Callable at par on 1 October 20X1 and on 1 October 20X2
Bond #3	1 October 20X3	4.40% annual	Putable at par on 1 October 20X1 and on 1 October 20X2

The one-year, two-year, and three-year par rates are 2.250%, 2.750%, and 3.100%, respectively. Based on an estimated interest rate volatility of 10%, Ferguson constructs the binomial interest rate tree shown in Exhibit 2.

EXHIBIT 2 Binomial Interest Rate Tree



On 19 October 20X0, Ferguson analyzes the convertible bond issued by Pro Star given in Exhibit 3. That day, the option-free value of Pro Star's convertible bond is $1,060 and its stock price $37.50.

EXHIBIT 3 Convertible Bond Issued by Pro Star, Inc.

Issue Date:	6 December 20X0
Maturity date:	6 December 20X4
Coupon rate:	2%
Issue price:	$1,000
Conversion ratio:	31

1. The call feature of Bond #2 is *best* described as:
   A. European style.
   B. American style.
   C. Bermudan style.
2. The bond that would *most likely* protect investors against a significant increase in interest rates is:
   A. Bond #1.
   B. Bond #2.
   C. Bond #3.
3. A fall in interest rates would *most likely* result in:
   A. a decrease in the effective duration of Bond #3.
   B. Bond #3 having more upside potential than Bond #2.
   C. a change in the effective convexity of Bond #3 from positive to negative.
4. The value of Bond #2 is *closest* to:
   A. 102.103% of par.
   B. 103.121% of par.
   C. 103.744% of par.
5. The value of Bond #3 is *closest* to:
   A. 102.103% of par.
   B. 103.688% of par.
   C. 103.744% of par.
6. All else being equal, a rise in interest rates will *most likely* result in the value of the option embedded in Bond #3:
   A. decreasing.
   B. remaining unchanged.
   C. increasing.
7. All else being equal, if Ferguson assumes an interest rate volatility of 15% instead of 10%, the bond that would *most likely* increase in value is:
   A. Bond #1.
   B. Bond #2.
   C. Bond #3.
8. All else being equal, if the shape of the yield curve changes from upward sloping to flattening, the value of the option embedded in Bond #2 will *most likely*:
   A. decrease.
   B. remain unchanged.
   C. increase.

9. The conversion price of the bond in Exhibit 3 is *closest* to:
   A. $26.67.
   B. $32.26.
   C. $34.19.
10. If the market price of Pro Star's common stock falls from its level on 19 October 20X0, the price of the convertible bond will *most likely*:
    A. fall at the same rate as Pro Star's stock price.
    B. fall but at a slightly lower rate than Pro Star's stock price.
    C. be unaffected until Pro Star's stock price reaches the conversion price.

The following information relates to Questions 11–19

Rayes Investment Advisers specializes in fixed-income portfolio management. Meg Rayes, the owner of the firm, would like to add bonds with embedded options to the firm's bond portfolio. Rayes has asked Mingfang Hsu, one of the firm's analysts, to assist her in selecting and analyzing bonds for possible inclusion in the firm's bond portfolio.

Hsu first selects two corporate bonds that are callable at par and have the same characteristics in terms of maturity, credit quality, and call dates. Hsu uses the option adjusted spread (OAS) approach to analyze the bonds, assuming an interest rate volatility of 10%. The results of his analysis are presented in Exhibit 1.

EXHIBIT 1 Summary Results of Hsu's Analysis Using the OAS Approach

Bond	OAS (in bps)
Bond #1	25.5
Bond #2	30.3

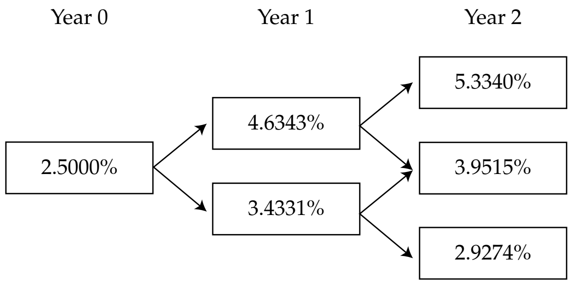
Hsu then selects the four bonds issued by RW, Inc., given in Exhibit 2. These bonds all have a maturity of three years and the same credit rating. Bonds #4 and #5 are identical to Bond #3, an option-free bond, except that they each include an embedded option.

EXHIBIT 2 Bonds Issued by RW, Inc.

Bond	Coupon	Special Provision
Bond #3	4.00% annual	
Bond #4	4.00% annual	Callable at par at the end of years 1 and 2
Bond #5	4.00% annual	Putable at par at the end of years 1 and 2
Bond #6	One-year reference rate annually, set in arrears	

To value and analyze RW's bonds, Hsu uses an estimated interest rate volatility of 15% and constructs the binomial interest rate tree provided in Exhibit 3.

EXHIBIT 3 Binomial Interest Rate Tree Used to Value RW's Bonds



Rayes asks Hsu to determine the sensitivity of Bond #4's price to a 20 bps parallel shift of the benchmark yield curve. The results of Hsu's calculations are shown in Exhibit 4.

EXHIBIT 4 Summary Results of Hsu's Analysis about the Sensitivity of Bond #4's Price to a Parallel Shift of the Benchmark Yield Curve

Magnitude of the Parallel Shift in the Benchmark Yield Curve	+20 bps	−20 bps
Full Price of Bond #4 (% of par)	100.478	101.238

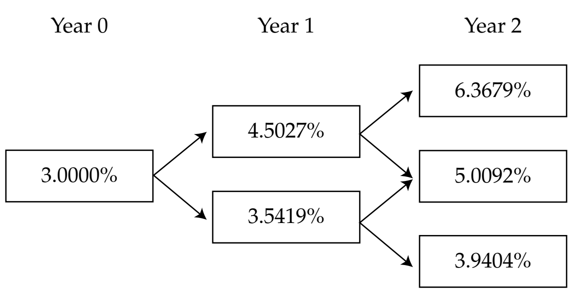
Hsu also selects the two floating-rate bonds issued by Varlep, plc, given in Exhibit 5. These bonds have a maturity of three years and the same credit rating.

EXHIBIT 5 Floating-Rate Bonds Issued by Varlep, plc

Bond	Coupon
Bond #7	One-year reference rate annually, set in arrears, capped at 5.00%
Bond #8	One-year reference rate annually, set in arrears, floored at 3.50%

To value Varlep's bonds, Hsu constructs the binomial interest rate tree provided in Exhibit 6.

EXHIBIT 6 Binomial Interest Rate Tree Used to Value Varlep's Bonds



Last, Hsu selects the two bonds issued by Whorton, Inc., given in Exhibit 7. These bonds are close to their maturity date and are identical, except that Bond #9 includes a conversion option. Whorton's common stock is currently trading at $30 per share.

EXHIBIT 7 Bonds Issued by Whorton, Inc.

Bond	Type of Bond
Bond #9	Convertible bond with a conversion price of $50
Bond #10	Identical to Bond #9 except that it does not include a conversion option

11. Based on Exhibit 1, Rayes would *most likely* conclude that relative to Bond #1, Bond #2 is:
    A. overpriced.
    B. fairly priced.
    C. underpriced.
12. The effective duration of Bond #6 is:
    A. close to 1.
    B. higher than 1 but lower than 3.
    C. higher than 3.
13. In Exhibit 2, the bond whose effective duration might lengthen if interest rates rise is:
    A. Bond #3.
    B. Bond #4.
    C. Bond #5.
14. The effective duration of Bond #4 is *closest* to:
    A. 0.76.
    B. 1.88.
    C. 3.77.
15. The value of Bond #7 is *closest* to:
    A. 99.697% of par.
    B. 99.936% of par.
    C. 101.153% of par.
16. The value of Bond #8 is *closest* to:
    A. 98.116% of par.
    B. 100.000% of par.
    C. 100.485% of par.
17. The value of Bond #9 is equal to the value of Bond #10:
    A. plus the value of a put option on Whorton's common stock.
    B. plus the value of a call option on Whorton's common stock.
    C. minus the value of a call option on Whorton's common stock.
18. The minimum value of Bond #9 is equal to the *greater* of:
    A. the conversion value of Bond #9 and the current value of Bond #10.
    B. the current value of Bond #10 and a call option on Whorton's common stock.
    C. the conversion value of Bond #9 and a call option on Whorton's common stock.
19. The factor that is currently *least likely* to affect the risk–return characteristics of Bond #9 is:
    A. interest rate movements.
    B. Whorton's credit spreads.
    C. Whorton's common stock price movements.

The following information relates to Questions 20–27

John Smith, an investment adviser, meets with Lydia Carter to discuss her pending retirement and potential changes to her investment portfolio. Domestic economic activity has been weakening recently, and Smith's outlook is that equity market values will be lower during the next year. He would like Carter to consider reducing her equity exposure in favor of adding more fixed-income securities to the portfolio.

Government yields have remained low for an extended period, and Smith suggests considering investment-grade corporate bonds to provide additional yield above government debt issues. In light of recent poor employment figures and two consecutive quarters of negative GDP growth, the consensus forecast among economists is that the central bank, at its next meeting this month, will take actions that will lead to lower interest rates.

Smith and Carter review par, spot, and one-year forward rates (Exhibit 1) and four fixed-rate investment-grade bonds issued by Alpha Corporation that are being considered for investment (Exhibit 2).

EXHIBIT 1 Par, Spot, and One-Year Forward Rates (annual coupon payments)

Maturity (Years)	Par Rate (%)	Spot Rate (%)	One-Year Forward (%)
1	1.0000	1.0000	1.0000
2	1.2000	1.2012	1.4028
3	1.2500	1.2515	1.3522

EXHIBIT 2 Selected Fixed-Rate Bonds of Alpha Corporation

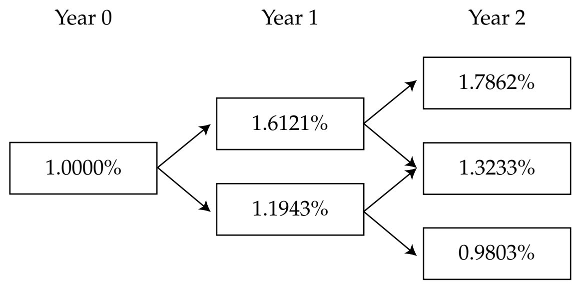
Bond	Annual Coupon	Type of Bond
Bond 1	1.5500%	Straight bond
Bond 2	1.5500%	Convertible bond: currently trading out of the money
Bond 3	1.5500%	Putable bond: putable at par one year and two years from now
Bond 4	1.5500%	Callable bond: callable at par without any lockout periods

Note: All bonds in Exhibit 2 have remaining maturities of exactly three years.

Carter tells Smith that the local news media have been reporting that housing starts, exports, and demand for consumer credit are all relatively strong, even in light of other poor macroeconomic indicators. Smith explains that the divergence in economic data leads him to believe that volatility in interest rates will increase. Smith also states that he recently read a report issued by Brown and Company forecasting that the yield curve could invert within the next six months.

Smith develops a binomial interest rate tree with a 15% interest rate volatility assumption to assess the value of Alpha Corporation's bonds. Exhibit 3 presents the interest rate tree.

EXHIBIT 3 Binomial Interest Rate Tree for Alpha Corporation with 15% Interest Rate Volatility



Carter asks Smith about the possibility of analyzing bonds that have lower credit ratings than the investment-grade Alpha bonds. Smith discusses four other corporate bonds with Carter. Exhibit 4 presents selected data on the four bonds.

EXHIBIT 4 Selected Information on Fixed-Rate Bonds for Beta, Gamma, Delta, and Rho Corporations

Bond	Issuer	Bond Features	Credit Rating
Bond 5	Beta Corporation	Coupon 1.70% Callable in Year 2 OAS of 45 bps	B
Bond 6	Gamma Corporation	Coupon 1.70% Callable in Year 2 OAS of 65 bps	B
Bond 7	Delta Corporation	Coupon 1.70% Callable in Year 2 OAS of 85 bps	B
Bond 8	Rho Corporation	Coupon 1.70% Callable in Year 2 OAS of 105 bps	CCC

Notes: All bonds have remaining maturities of three years. OAS stands for option-adjusted spread.

20. Based on Exhibit 2, and assuming that the forecast for interest rates and Smith's outlook for equity returns are validated, which bond's option is *most likely* to be exercised?
    A. Bond 2
    B. Bond 3
    C. Bond 4
21. Based on Exhibit 2, the current price of Bond 1 is *most likely* greater than the current price of:
    A. Bond 2.
    B. Bond 3.
    C. Bond 4.
22. Assuming the forecast for interest rates is proven accurate, which bond in Exhibit 2 will likely experience the smallest price increase?
    A. Bond 1
    B. Bond 3
    C. Bond 4

23. Based on the information in Exhibit 1 and Exhibit 2, the value of the embedded option in Bond 4 is *closest* to:
    A. nil.
    B. 0.1906.
    C. 0.8789.
24. If Smith's interest rate volatility forecast turns out to be true, which bond in Exhibit 2 is likely to experience the greatest price increase?
    A. Bond 2
    B. Bond 3
    C. Bond 4
25. If the Brown and Company forecast comes true, which of the following is *most likely* to occur? The value of the embedded option in:
    A. Bond 3 decreases.
    B. Bond 4 decreases.
    C. both Bond 3 and Bond 4 increases.
26. Based on Exhibit 2 and Exhibit 3, the market price of Bond 4 is *closest* to:
    A. 100.0000.
    B. 100.5123.
    C. 100.8790.
27. Which of the following conclusions regarding the bonds in Exhibit 4 is correct?
    A. Bond 5 is relatively cheaper than Bond 6.
    B. Bond 7 is relatively cheaper than Bond 6.
    C. Bond 8 is relatively cheaper than Bond 7.

The following information relates to Questions 28–36

Jules Bianchi is a bond analyst for Maneval Investments, Inc. Bianchi gathers data on three corporate bonds, as shown in Exhibit 1.

EXHIBIT 1 Selected Bond Data

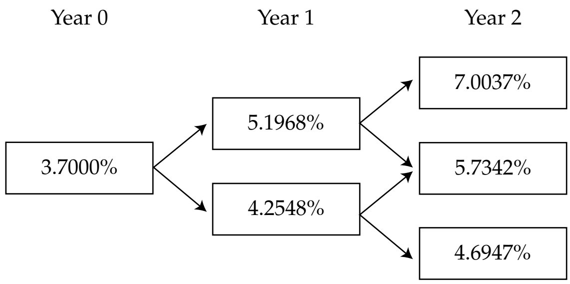
Issuer	Coupon Rate	Price	Bond Description
Ayrault, Inc. (AI)	5.25%	100.200	Callable at par in one year and two years from today
Blum, Inc. (BI)	5.25%	101.300	Option-free
Cresson Enterprises (CE)	5.25%	102.100	Putable at par in one year from today

Note: Each bond has a remaining maturity of three years, annual coupon payments, and a credit rating of BBB.

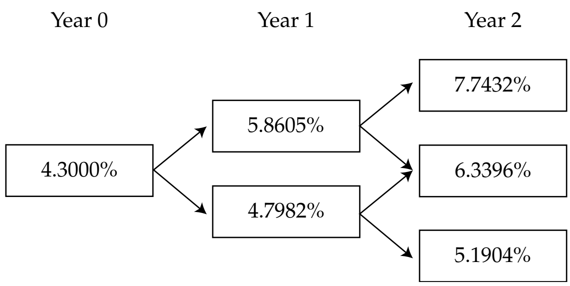
To assess the interest rate risk of the three bonds, Bianchi constructs two binomial interest rate trees based on a 10% interest rate volatility assumption and a current one-year rate of 4%. Panel A of Exhibit 2 provides an interest rate tree assuming the benchmark yield curve shifts down by 30 bps, and Panel B provides an interest rate tree assuming the benchmark yield curve shifts up by 30 bps. Bianchi determines that the AI bond is currently trading at an option-adjusted spread (OAS) of 13.95 bps relative to the benchmark yield curve.

EXHIBIT 2 Binomial Interest Rate Trees

A. Interest Rates Shift Down by 30 bps



B. Interest Rates Shift Up by 30 bps



Armand Gillette, a convertible bond analyst, stops by Bianchi's office to discuss two convertible bonds. One is issued by DeLille Enterprises (DE), and the other is issued by Raffarin Incorporated (RI). Selected data for the two bonds are presented in Exhibits 3 and 4.

EXHIBIT 3 Selected Data for DE Convertible Bond

Issue price	€1,000 at par
Conversion period	13 September 20X5 to 12 September 20X8
Initial conversion price	€10.00 per share
Threshold dividend	€0.50 per share
Change of control conversion price	€8.00 per share
Common stock share price on issue date	€8.70
Share price on 17 September 20X5	€9.10
Convertible bond price on 17 September 20X5	€1,123

EXHIBIT 4 Selected Data for RI Convertible Bond

Straight bond value	€978
Value of embedded issuer call option	€43
Value of embedded investor put option	€26
Value of embedded call option on issuer's stock	€147
Conversion price	€12.50
Current common stock share price	€11.75

Gillette makes the following comments to Bianchi:

- "The DE bond does not contain any call or put options, but the RI bond contains both an embedded call option and put option. I expect that DeLille Enterprises will soon announce a common stock dividend of €0.70 per share."
- "My belief is that, over the next year, Raffarin's share price will appreciate toward the conversion price but not exceed it."

28. Based on Exhibits 1 and 2, the effective duration for the AI bond is *closest* to:
    A. 1.98.
    B. 2.15.
    C. 2.73.
29. If benchmark yields were to fall, which bond in Exhibit 1 would *most likely* experience a decline in effective duration?
    A. AI bond
    B. BI bond
    C. CE bond
30. Based on Exhibit 1, for the BI bond, one-sided:
    A. up-duration will be greater than one-sided down-duration.
    B. down-duration will be greater than one-sided up-duration.
    C. up-duration and one-sided down-duration will be about equal.
31. Based on Exhibit 1, which key rate duration is the largest for the BI bond?
    A. One-year key rate duration
    B. Two-year key rate duration
    C. Three-year key rate duration
32. Which bond in Exhibit 1 *most likely* has the lowest effective convexity?
    A. AI bond
    B. BI bond
    C. CE bond
33. Based on Exhibit 3, if DeLille Enterprises pays the dividend expected by Gillette, the conversion price of the DE bond will:
    A. be adjusted downward.
    B. not be adjusted.
    C. be adjusted upward.
34. Based on Exhibit 3, the market conversion premium per share for the DE bond on 17 September 20X5 is *closest* to:
    A. €0.90.
    B. €2.13.
    C. €2.53.
35. Based on Exhibit 4, the arbitrage-free value of the RI bond is *closest* to:
    A. €814.
    B. €1,056.
    C. €1,108.
36. Based on Exhibit 4 and Gillette's forecast regarding Raffarin's share price, the return on the RI bond over the next year is *most likely* to be:
    A. lower than the return on Raffarin's common shares.
    B. the same as the return on Raffarin's common shares.
    C. higher than the return on Raffarin's common shares.

CHAPTER 10

CREDIT ANALYSIS MODELS

James F. Adams, PhD, CFA
Donald J. Smith, PhD

LEARNING OUTCOMES

The candidate should be able to:

- explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment;
- explain credit scores and credit ratings;
- calculate the expected return on a bond given transition in its credit rating;
- explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses;
- calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters;
- interpret changes in a credit spread;
- explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads;
- compare the credit analysis required for securitized debt to the credit analysis of corporate debt.

1. INTRODUCTION

Credit analysis plays an important role in the broader fixed-income space. Our coverage will go over important concepts, tools, and applications of credit analysis. We first look at modeling credit risk. The inputs to credit risk modeling are the expected exposure to default loss, the loss given default, and the probability of default. We explain these terms and use a numerical example to illustrate the calculation of the credit valuation adjustment for a corporate bond and its credit spread over a government bond yield taken as a proxy for a default-risk-free rate (or default-free rate).

We then discuss credit scoring and credit ratings. Credit scoring is a measure of credit risk used in retail loan markets, and ratings are used in the wholesale bond market. We explain two types of credit analysis models used in practice—structural models and reduced-form models. Both models are highly mathematical and beyond the scope of our coverage. Therefore, we provide only an overview to highlight the key ideas and the similarities and differences between them. We then use the arbitrage-free framework and a binomial interest rate tree to value risky fixed-rate and floating-rate bonds for different assumptions about interest rate volatility. We also build on the credit risk model to interpret changes in credit spreads that arise from changes in the assumed probability of default, the recovery rate, or the exposure to default loss. We also explain the term structure of credit spreads and finally compare the credit analysis required for securitized debt with the credit analysis of corporate bonds.

2. MODELING CREDIT RISK AND THE CREDIT VALUATION ADJUSTMENT

- **explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment**

The difference between the yields to maturity on a corporate bond and a government bond with the same maturity is the most commonly used measure of credit risk. It is called the *credit spread* and is also known in practice as the G-spread. It reveals the compensation to the investor for bearing the default risk of the issuer—the possibility that the issuer fails to make a scheduled payment in full on the due date—and for losses incurred in the event of default.

The terms "default risk" and "credit risk" are sometimes used interchangeably in practice, but we will distinguish between the two in our coverage. Default risk is the narrower term because it addresses the likelihood of an event of default. Credit risk is the broader term because it considers both the default probability and how much is expected to be lost if default occurs. For example, it is possible that the default risk on a collateralized loan is high while the credit risk is low, especially if the value of the collateral is high relative to the amount that is owed.

We assume that the corporate bond and the default-risk-free government bond have the same taxation and liquidity. This is a simplifying assumption, of course. In reality, government bonds typically are more liquid than corporate bonds. Also, differences in liquidity within the universe of corporate bonds are great. Government bonds are available in greater supply than even the most liquid corporates and have demand from a wider set of institutional investors. In addition, government bonds can be used more readily as collateral in repo transactions and for centrally cleared derivatives. Also, there are differences in taxation in some markets. For example, interest income on US corporate bonds is taxable by both the federal and state governments. Government debt, however, is exempt from taxes at the state level. Disregarding tax and liquidity differences allows us to focus on default risk and expected loss as the determining factors for the credit spread.

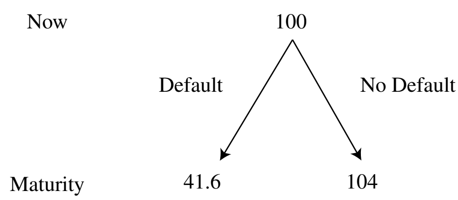
The first factor to consider in modeling credit risk is the **expected exposure** to default loss. This quantity is the projected amount of money the investor could lose if an event of default occurs, before factoring in possible recovery. Although the most common event of default is nonpayment leading to bankruptcy proceedings, the bond prospectus might identify other events of default, such as the failure to meet a different obligation or the violation of a financial covenant.

Consider a one-year, 4% annual payment corporate bond priced at par value. The expected exposure to default loss at the end of the year is simply 104 (per 100 of par value). Later, we will include multiple time periods and volatility in interest rates. That complicates the calculation of expected exposure because we will need to consider the likelihood that the bond price varies as interest rates vary. In this initial example, the exposure is simply the final coupon payment plus the redemption of principal.

The second factor is the assumed **recovery rate**, which is the percentage of the loss recovered from a bond in default. The recovery rate varies by industry, the degree of seniority in the capital structure, the amount of leverage in the capital structure in total, and whether a particular security is secured or otherwise collateralized. We assume a 40% recovery rate for this corporate bond, which is a common baseline assumption in practice. Given the recovery rate assumption, we can determine the assumed **loss given default** (the amount of loss if a default occurs). This is 62.4 per 100 of par value: $104 \times (1 - 0.40) = 62.4$. A related term is *loss severity*; if the recovery rate is 40%, the assumed loss severity is 60%.

Exhibit 1 illustrates the projected cash flows on the corporate bond. If there is no default, the investor receives 104. If default occurs, the investor receives 41.6: $104 - 62.4 = 41.6$. We assume instantaneous recovery, which surely is another simplifying assumption. In practice, lengthy time delays can occur between the event of default and eventual recovery of cash. Notice that we assume that the recovery rate applies to interest as well as principal. One last note is that in the exhibits that we use, calculations may slightly differ on occasion due to rounding at intermediate steps.

EXHIBIT 1 A Simple Credit Risk Example



The third factor is the assumed **probability of default**, which is the probability that a bond issuer will not meet its contractual obligations on schedule. It is important in credit risk modeling to distinguish *risk-neutral* probabilities of default and *actual* (or historical) default probabilities. "Risk-neutral" follows the usage of the term in option pricing. In the risk-neutral option pricing methodology, the expected value for the payoffs is discounted using the risk-free interest rate. The key point is that in getting the expected value of the option, the risk-neutral probabilities associated with the payoffs need to be used. The same idea applies to valuing corporate bonds.

Suppose that a credit rating agency has collected an extensive dataset on the historical default experience for one-year corporate bonds issued by companies having the same business profile as the issuer in this example. It is observed that 99% of the bonds survive and make the full coupon and principal payment at maturity. Just 1% of the bonds default, resulting in an average recovery rate of 40%. Based on these data, the actual default probability for the corporate bond can reasonably be assumed to be 1%.

If the actual probability of default is used to get the expected future value for the corporate bond, the result is 103.376: $(104 \times 0.99) + (41.6 \times 0.01) = 103.376$. Discounting that amount at an assumed risk-free rate of 3% gives a present value of 100.365: $103.376/1.03 = 100.365$. Note that in risk-neutral valuation, the expected value is discounted using the risk-free rate and not the bond's yield to maturity. The key point is that 100.365 overstates the observed value of the bond, which is 100. The issue is to determine the default probability that does produce a value of 100.

Denote the risk-neutral default probability to be P^*. The probability of survival is $1 - P^*$. Given that the corporate bond is priced at 100, $P^* = 1.60\%$. This is found as the solution to P^* in

$$100 = \frac{[104 \times (1 - P^*)] + (41.6 \times P^*)}{1.03}$$

One reason for the difference between actual (or historical) and risk-neutral default probabilities is that actual default probabilities do not include the default risk premium associated with uncertainty over the timing of possible default loss. Another reason is that the observed spread over the yield on a risk-free bond in practice also includes liquidity and tax considerations in addition to credit risk.

To further see the interaction between the credit risk parameters—the expected exposure, the loss given default, and the probability of default—we consider a five-year, zero-coupon corporate bond. Our goal is to determine the fair value for the bond given its credit risk, its yield to maturity, and its spread over a maturity-matching government bond.

Exhibit 2 displays the calculation of the **credit valuation adjustment** (CVA). The CVA is the value of the credit risk in present value terms. In Exhibit 2, LGD stands for the loss given default, POD stands for the probability of default on the given date, POS stands for the probability of survival as of the given date, DF stands for the discount factor, and PV stands for the present value.

EXHIBIT 2 A Five-Year, Zero-Coupon Corporate Bond

Date (1)	Exposure (2)	Recovery (3)	LGD (4)	POD (5)	POS (6)	Expected Loss (7)	DF (8)	PV of Expected Loss (9)
0								
1	88.8487	35.5395	53.3092	1.2500%	98.7500%	0.6664	0.970874	0.6470
2	91.5142	36.6057	54.9085	1.2344%	97.5156%	0.6778	0.942596	0.6389
3	94.2596	37.7038	56.5558	1.2189%	96.2967%	0.6894	0.915142	0.6309
4	97.0874	38.8350	58.2524	1.2037%	95.0930%	0.7012	0.888487	0.6230
5	100.0000	40.0000	60.0000	1.1887%	93.9043%	0.7132	0.862609	0.6152
				6.0957%			CVA =	3.1549

The first step is to get the exposures to default loss. These are shown in Column 2 of Exhibit 2. We assume a flat government bond yield curve at 3.00%. Also, we assume that default occurs only at year-end—on Dates 1, 2, 3, 4, and 5—and that default will not occur on Date 0, the current date. The exposure on Date 5 is 100. For the other dates, we discount using the risk-free rate and the remaining number of years until maturity. For example, exposure at Date 1 is $100/(1.0300)^4 = 88.8487$.

Note that there is no interest rate volatility in this example. In a later section, we will use the arbitrage-free framework to build a binomial interest rate tree for a specified level of volatility. Then, knowing the probability of attaining each node in the tree, we will calculate the *expected exposure* for each date.

Column 3 of Exhibit 2 projects the assumed recovery if default occurs. Here, the recovery rate is a percentage of the exposure. In general, it will be a percentage of the expected exposure, including coupon interest payments, when the model allows for interest rate volatility. We assume for this example that the recovery rate is 40%. The amounts shown in Column 3 are the exposures in Column 2 times 0.40.

Column 4 shows the loss given default. It is the exposure for each date minus the assumed recovery. If the issuer defaults on Date 4, the investor's loss is projected to be 58.2524 (= 97.0874 – 38.8350) per 100 of par value.

The next parameter is the risk-neutral probability of default for each date. In Column 5 of Exhibit 2, we assume that the POD on Date 1 is 1.25%. We use *conditional probabilities of default*, meaning that each year-by-year POD assumes no prior default. These are called hazard rates in statistics. Column 6 reports the probability of survival for each year. The probability of surviving past Date 1 and arriving at Date 2 is 98.75% (= 100% – 1.25%). Therefore, the POD for Date 2 is 1.2344% (= 1.25% × 98.75%), and the POS is 97.5156% (= 98.75% – 1.2344%). The POD for Date 3 is 1.2189% (= 1.25% × 97.5156%), and the POS is 96.2967% (= 97.5156% – 1.2189%). The cumulative probability of default over the five-year lifetime of the corporate bond is 6.0957%, the sum of the PODs in Column 5.

Another method to calculate the POS for each year—a method that is used later in our discussion—is 100% minus the annual default probability raised to the power of the number of years. For example, the probability of the bond surviving until maturity is $(100\% - 1.25\%)^5 = 93.9043\%$. Note that 6.0957% plus 93.9043% equals 100%.

The assumed annual default probability does not need to be the same each year. Later we will show some examples of it changing over the lifetime of the bond.

Column 7 gives the *expected loss* for each date. This is the LGD times the POD. For example, if default occurs on Date 3, the expected loss is 0.6894 per 100 of par value. The exposure is 94.2596. At 40% recovery, the LGD is 56.5558. Assuming no prior default, the POD for that date is 1.2189%. The expected loss of 0.6894 is calculated as 56.5558 times 1.2189%.

Column 8 presents the default-risk-free *discount factors* based on the flat government bond yield curve at 3.00%. The Date 5 discount factor is 0.862609 [$= 1/(1.0300)^5$]. Finally, Column 9 shows the present value of the expected loss for each year. This is the expected loss times the discount factor. The present value of the expected Date 5 loss is 0.6152 per 100 of par value, the expected loss of 0.7132 times 0.862609.

The sum of Column 9 is 3.1549. This amount is known as the credit valuation adjustment. It allows us to calculate the *fair value* of the five-year, zero-coupon corporate bond. If the bond were default free, its price would be 86.2609—that is, the par value of 100 times the Date 5 discount factor. Subtracting the CVA from this amount gives a fair value of 83.1060 (= 86.2609 – 3.1549).

We can now calculate the credit spread on the corporate bond. Given a price of 83.1060, its yield to maturity is 3.77%. The solution for *yield* in this expression is

$$\frac{100}{(1 + \text{Yield})^5} = 83.1060$$

The yield on the five-year, zero-coupon government bond is 3.00%. Therefore, the credit spread is 77 bps: 3.77% – 3.00% = 0.77%. (Note that an approximation for the credit spread commonly used in practice is the annual default probability times 1 minus the recovery rate. In this case, the approximate credit spread is 0.75% [= 1.25% × (1 – 0.40)].) A key point is that the compensation for credit risk received by the investor can be expressed in two ways: (1) as the CVA of 3.1549 in terms of a present value per 100 of par value on Date 0 and (2) as a credit spread of 77 bps in terms of an annual percentage rate for five years.

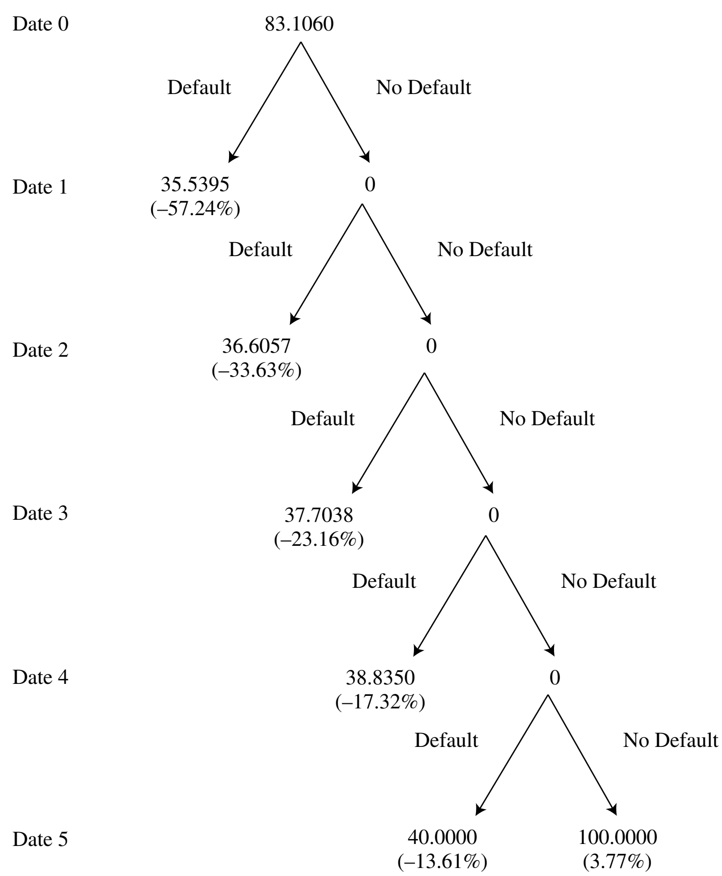
Exhibit 3 provides a display of the projected cash flows and annual rates of return depending on when and if default occurs. On Date 0, the five-year, zero-coupon corporate bond is worth its fair value, 83.1060 per 100 of par value. If on Date 1 the issuer defaults, the investor gets the recoverable amount of 35.5395. The annual rate of return is −57.24%, the solution for the internal rate of return (IRR):

$$83.1060 = \frac{35.5395}{1 + \text{IRR}}$$

$$\text{IRR} = -0.5724$$

If there is no default, the investor receives the coupon payment on that date, which in this case is zero.

EXHIBIT 3 Projected Annual Rates of Return



If the issuer defaults on Date 2, the annual rate of return is −33.63%.

$$83.1060 = \frac{0}{(1 + \text{IRR})^1} + \frac{36.6057}{(1 + \text{IRR})^2}$$

$$\text{IRR} = -0.3363$$

If the default occurs on the maturity date, the annual rate of return "improves" to −13.61%:

$$83.1060 = \frac{0}{(1+\text{IRR})^1} + \frac{0}{(1+\text{IRR})^2} + \frac{0}{(1+\text{IRR})^3} + \frac{0}{(1+\text{IRR})^4} + \frac{40.0000}{(1+\text{IRR})^5}$$

$$\text{IRR} = -0.1361$$

If there is no default, which is most likely because the probability of survival to Date 5 is 93.9043%, the realized rate of return is 3.77%. This reminds us that a yield to maturity on a risky bond is a measure of return to the investor, assuming no default.

The key observation from this example is that the investor faces a wide range of outcomes on the bond depending critically on the *timing* of default. This is a source of the default risk premium that typically is built into the pricing of the bond. Stated differently, the probability of default in credit risk models incorporates the likely time of incidence of default events as well as uncertainty over the timing of the events.

Although this is clearly a simple example of a credit risk model, it does serve to illustrate the interaction between the exposure to default loss for each date, the recovery rate, the loss given default, the probability of default, the expected loss, and the present value of expected loss. It can be made more complex and realistic. Here, the initial probability of default (the hazard rate) used to calculate the conditional PODs and the recovery rate is the same for each year, but these parameters could vary year by year. The government bond yield curve is flat, but it could be upward or downward sloping. Then, the discount factors would need to be calculated sequentially by a process known as "bootstrapping." An example of this process is included later.

In this example, we assume an annual default probability and a recovery rate to get the fair value for the risky corporate bond. This could be reversed. Suppose that we observe that the market price for the five-year, zero-coupon bond is 83.1060 and its credit spread is 77 bps. Then, the same table could be used to get—by trial-and-error search—the annual probability of default that is consistent with the bond price and a recovery rate of 40%. That default probability, which is used to calculate the year-by-year PODs, would be 1.25%. Another possibility is to change the assumed recovery rate. Suppose it is 30% of the exposure. Given the observed bond price and credit spread, the default probability would turn out to be 1.0675%. In that case, the lower recovery rate is offset by the lower probability of default. A higher recovery rate would need to be offset by a higher default probability. In general, for a given price and credit spread, the assumed probability of default and the recovery rate are positively correlated.

EXAMPLE 1 Analysis of Credit Risk (1)

A fixed-income analyst is considering the credit risk over the next year for three corporate bonds currently held in her bond portfolio. Her assessment for the exposure, probability of default, and recovery is summarized in this table:

Corporate Bond	Exposure (per 100 of par value)	Probability of Default	Recovery (per 100 of par value)
A	104	0.75%	40
B	98	0.90%	35
C	92	0.80%	30

Although all three bonds have very similar yields to maturity, the differences in the exposures arise because of differences in their coupon rates.

Based on these assumptions, how would she rank the three bonds, from highest to lowest, in terms of credit risk over the next year?

Solution: She needs to get the loss given default for each bond and multiply that by the probability of default to get the expected loss. The LGD is the exposure minus the assumed recovery.

Corporate Bond	LGD (per 100 of par value)	POD	Expected Loss
A	64	0.75%	0.480
B	63	0.90%	0.567
C	62	0.80%	0.496

Based on the expected losses, Bond B has the highest credit risk and Bond A, the lowest. The ranking is B, C, and A. Note that there is not enough information to recommend a trading strategy because the current prices of the bonds are not given.

EXAMPLE 2 Analysis of Credit Risk (2)

A fixed-income trader at a hedge fund observes a three-year, 5% annual payment corporate bond trading at 104 per 100 of par value. The research team at the hedge fund determines that the risk-neutral annual probability of default used to calculate the conditional POD for each date for the bond, given a recovery rate of 40%, is 1.50%. The government bond yield curve is flat at 2.50%.

Based on these assumptions, does the trader deem the corporate bond to be overvalued or undervalued? By how much? If the trader buys the bond at 104, what are the projected annual rates of return?

Solution: The trader needs to build a table similar to that shown in Exhibit 2; this table is presented in Exhibit 4.

EXHIBIT 4 CVA Calculation for Example 2

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	109.8186	43.9274	65.8911	1.5000%	98.5000%	0.9884	0.975610	0.9643
2	107.4390	42.9756	64.4634	1.4775%	97.0225%	0.9524	0.951814	0.9066
3	105.0000	42.0000	63.0000	1.4553%	95.5672%	0.9169	0.928599	0.8514
				4.4328%			CVA =	2.7222

The exposures are the values for the bond plus the coupon payment for each date assuming a yield to maturity of 2.50%. The exposure is 109.8186 for Date 1 when two years to maturity remain:

$$5 + \frac{5}{(1.0250)^1} + \frac{105}{(1.0250)^2} = 109.8186$$

The assumed recovery for Date 1 is 43.9274 (= 109.8186 × 0.40) for a loss given default of 65.8911 (= 109.8186 – 43.9274). (Note that all calculations are carried out on spreadsheets to preserve precision. The rounded results are reported in the text.) The expected loss is 0.9884 (= 65.8911 × 0.0150). The discount factor for Date 1 is 0.975610 = $1/(1.0250)^1$. The present value of the expected loss is 0.9643 (= 0.9884 × 0.975610).

The credit valuation adjustment for the bond is 2.7222, the sum of the present values of expected loss. If this five-year, 5% bond were default free, its price would be 107.1401.

$$\frac{5}{(1.0250)^1} + \frac{5}{(1.0250)^2} + \frac{105}{(1.0250)^3} = 107.1401$$

Therefore, the fair value of the bond given the assumed credit risk parameters is 104.4178 (= 107.1401 – 2.7222). If this three-year, 5% bond were default free, its price would be 107.1401.

The projected annual rates of return for default on Dates 1, 2, and 3 are –57.76%, –33.27%, and –22.23%, respectively. If there is no default, the rate of return is 3.57%, which is the yield to maturity. Note that these rates of return neglect coupon reinvestment risk because internal rate of return calculations implicitly assume reinvestment at the same rate. The calculations are as follows:

$$104 = \frac{43.9274}{(1 + \text{IRR})^1}$$

$$\text{IRR} = -0.5776$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{42.9756}{(1 + \text{IRR})^2}$$

$$\text{IRR} = -0.3327$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{5}{(1 + \text{IRR})^2} + \frac{42.0000}{(1 + \text{IRR})^3}$$

$$\text{IRR} = -0.2223$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{5}{(1 + \text{IRR})^2} + \frac{105}{(1 + \text{IRR})^3}$$

$$\text{IRR} = 0.0357$$

Environmental, social, and governance (ESG) considerations may also play a role in credit risk assessment. For example, companies responsible for pollution run the risk of fines or other business sanctions, those with poor labor practices risk their reputation and may face customer boycotts or lawsuits, and firms with weak governance are more likely to engage in aggressive or even fraudulent accounting. Estimated probabilities of default and loss given default should incorporate these potential impacts.

Recent years have also seen several types of bond with explicit links to ESG matters. Climate, or green, bonds are typically issued with proceeds earmarked for environmentally beneficial purposes and may come with tax incentives to enhance their attractiveness to investors.

Another category of fixed-income instruments whose special features affect credit risk assessment are catastrophe and pandemic bonds. They resemble an insurance product, rather than a traditional debt instrument. For example, the World Bank issued pandemic bonds in 2017, offering investors high interest payments in return for taking on the risk of losing capital should a pandemic occur, in which case they would pay out aid to poor nations suffering from a serious outbreak of infectious disease. At the time of this writing (July 2020), nearly all the principal from those bonds has been wiped out because caseloads and deaths from COVID-19 have exceeded the bonds' thresholds.

3. CREDIT SCORES AND CREDIT RATINGS

- **explain credit scores and credit ratings**
- **calculate the expected return on a bond given transition in its credit rating**

Credit scores and ratings are used by lenders in deciding to extend credit to a borrower and in determining the terms of the contract. Credit scores are used primarily in the retail lending market for small businesses and individuals. Credit ratings are used in the wholesale market for bonds issued by corporations and government entities, as well as for asset-backed securities (ABS).

Credit scoring methodologies can vary. In some countries, only negative information, such as delinquent payments or outright default, is included. Essentially, everyone has a good credit score until proven otherwise. In other countries, a broader set of information is used to determine the score. A score reflects actual observed factors. In general, credit reporting agencies are national in scope because of differences in legal systems and privacy concerns across countries.

The FICO score, which is the federally registered trademark of the Fair Isaac Corporation, is used in the United States by about 90% of lenders to retail customers. FICO scores are computed using data from consumer credit files collected by three national credit bureaus: Experian, Equifax, and TransUnion. Five primary factors are included in the proprietary algorithm used to get the score:

- 35% for the payment history: This includes the presence or lack of such information as delinquency, bankruptcy, court judgments, repossessions, and foreclosures.
- 30% for the debt burden: This includes credit card debt-to-limit ratios, the number of accounts with positive balances, and the total amount owed.
- 15% for the length of credit history: This includes the average age of accounts on the credit file and the age of the oldest account.
- 10% for the types of credit used: This includes the use of installment payments, consumer finance, and mortgages.
- 10% for recent searches for credit: This includes "hard" credit inquiries when consumers apply for new loans but not "soft" inquiries, such as for employee verification or self-checking one's score.

Fair Isaac Corporation, on its website, notes items that are not included in the FICO credit score: race, color, national origin, sex, marital status, age, salary, occupation, employment history, home address, and child/family support obligations. The company also reports from time to time the distribution across scores, which range from a low of 300 to a perfect score of 850. Exhibit 5 shows the distribution for three particular months: October 2005, before the global financial crisis; April 2009, in the depths of the crisis; and April 2017, well after the crisis. It is evident that the percentage of weak scores increased as economic conditions worsened but has gone down since then. The average FICO score varied from 688 to 687 to 700 during these months.

EXHIBIT 5 Distribution of FICO Scores

FICO Score	October 2005	April 2009	April 2017
300–499	6.6%	7.3%	4.7%
500–549	8.0%	8.7%	6.8%
550–599	9.0%	9.1%	8.5%
600–649	10.2%	9.5%	10.0%
650–699	12.8%	12.0%	13.2%
700–749	16.4%	15.9%	17.1%
750–799	20.1%	19.3%	19.0%
800–850	16.9%	18.2%	20.7%

Source: Fair Isaac Corporation.

EXAMPLE 3 Credit Scoring

Tess Waresmith is a young finance professional who plans to eventually buy a two-family house, live in one unit, and rent the other to help cover the mortgage payments. She is a careful money manager and every year checks her FICO credit score. She is pleased to see that it has improved from 760 last year to 775 this year. Which of these factors can explain the improvement?

A. She is now one year older and has not had any late payments on credit cards during the year.
B. Her bank on its own raised her limit on a credit card from $1,000 to $2,500, but she has maintained the same average monthly balance.
C. She applied for and received a new car loan from her credit union.
D. She refrained from checking her FICO score monthly, which some of her friends do.

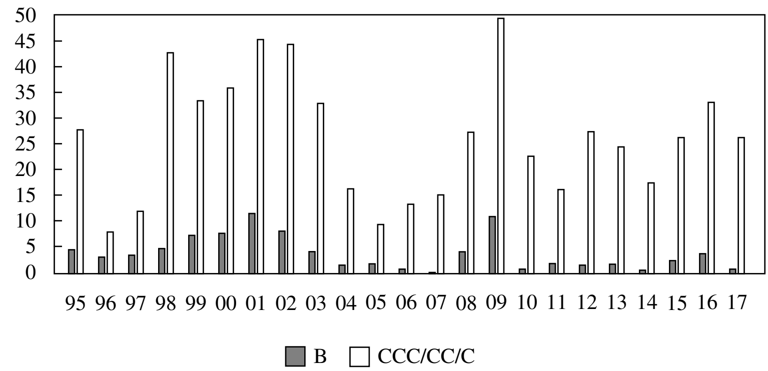
Solution: Factors A, B, and C help explain the improvement. Going down the list:

A. Age itself is not a factor used by Fair Isaac to determine the credit score. However, the average age of the accounts is a factor, as is the age of the oldest account. Therefore, other things being equal, the passage of time tends to improve the score. In general, age and credit score are highly correlated.
B. The credit card debt-to-limit ratio is a component of the debt burden. Having a higher limit for the same average balance reduces the ratio and improves the credit score.
C. Because the car loan is a new type of credit usage and thus does not have any late payments, it has a positive impact on the score.
D. Refraining from self-checking one's credit score has no impact. Self-checking is deemed to be a "soft inquiry" and does not factor into the calibration of the FICO score.

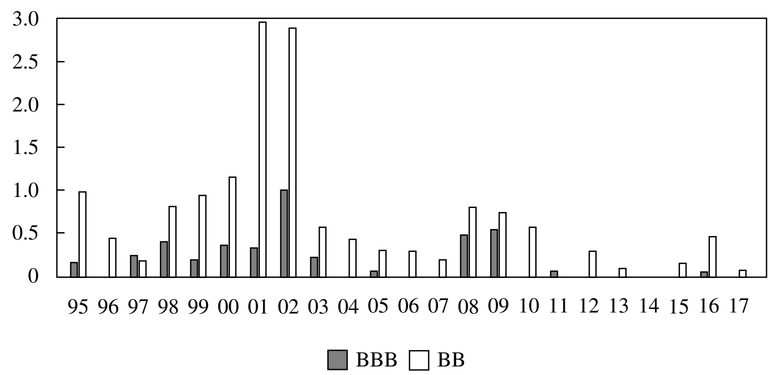
Whereas credit scores are the primary measure of credit risk in retail lending, credit ratings are widely used in corporate and sovereign bond markets. The three major global credit rating agencies are Moody's Investors Service, Standard & Poor's, and Fitch Ratings. Each provides quality ratings for issuers as well as specific issues. Similar to credit scores, these are ordinal ratings focusing on the probability of default. The historical corporate default experience by various ratings for 1995 to 2017 is shown in Exhibit 6.

EXHIBIT 6 Historical Corporate Default Experience by Rating (entries are in %)

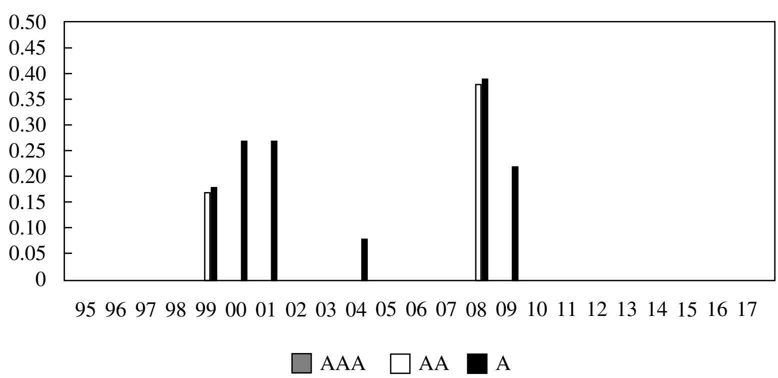
A.



B.



C.



The credit rating agencies consider the expected loss given default by means of *notching*, which is a rating adjustment methodology (covered earlier in the CFA Program curriculum) to reflect the priority of claim for specific debt issues of that issuer and to reflect any subordination. The issuer rating is typically for senior unsecured debt. The rating on subordinated debt is then adjusted, or "notched," by lowering it one or two levels. This inclusion of loss given default in addition to the probability of default explains why they are called "credit ratings" and not just "default ratings."

In addition to the "letter grade," the rating agencies provide an outlook (positive, stable, or negative) for the issuer as well as when the issuer is under "watch." For example, what follows is the history of Standard & Poor's issuer rating for RadioShack Corporation as it moved from BBB– in 1969 to BB+ in 1978, to AAA in 1983, to BB in 2006, and finally to default in 2015:

• 2 May 1969	BBB–
• 13 October 1978	BB+
• 12 December 1980	BB
• 1 April 1981	BBB+
• 7 January 1982	A
• 10 January 1983	AAA
• 28 November 1984	A+/Watch Negative
• 8 August 1991	A/Stable
• 4 January 1993	A/Watch Negative
• 25 February 1993	A–/Stable
• 27 May 1993	A–/Watch Positive
• 17 January 1994	A–/Stable
• 17 October 1996	A–/Negative
• 24 February 1999	A–/Stable
• 13 May 2005	A–/Watch Negative
• 8 August 2005	BBB+/Stable
• 21 April 2006	BBB–/Stable
• 24 July 2006	BBB–/Negative
• 25 October 2006	BB/Negative
• 12 August 2008	BB/Stable
• 21 November 2011	BB–/Stable
• 2 March 2012	B+/Negative
• 30 July 2012	B–/Negative
• 21 November 2012	CCC+/Negative
• 1 August 2013	CCC/Negative
• 20 December 2013	CCC+/Negative
• 16 June 2014	CCC/Negative
• 11 September 2014	CCC–/Negative
• 6 February 2015	D

Source: Standard & Poor's, "2014 Annual Global Corporate Default Study and Rating Transitions," Table 54 (30 April 2015).

The history of RadioShack illustrates that the rating can remain the same for prolonged periods of time. The company was A+ from 1984 to 1991 and A– from 1993 to 2005. The rating agencies report *transition matrixes* based on their historical experience. Exhibit 7 is a representative example. It shows the probabilities of a particular rating transitioning to a different rating over the course of the following year. An A rated issuer has an 87.50% probability of remaining at that level; a 0.05% probability of moving up to AAA (such as RadioShack did in 1983); a 2.50% probability of moving up to AA; an 8.40% probability of moving down to BBB; 0.75% down to BB; 0.60% to B; 0.12% to CCC, CC, or C; and 0.08% to D, where it is in default.

EXHIBIT 7 Representative One-Year Corporate Transition Matrix (entries are in %)

From/To	AAA	AA	A	BBB	BB	B	CCC, CC, C	D
AAA	**90.00**	9.00	0.60	0.15	0.10	0.10	0.05	0.00
AA	1.50	**88.00**	9.50	0.75	0.15	0.05	0.03	0.02
A	0.05	2.50	**87.50**	8.40	0.75	0.60	0.12	0.08
BBB	0.02	0.30	4.80	**85.50**	6.95	1.75	0.45	0.23
BB	0.01	0.06	0.30	7.75	**79.50**	8.75	2.38	1.25
B	0.00	0.05	0.15	1.40	9.15	**76.60**	8.45	4.20
CCC, CC, C	0.00	0.01	0.12	0.87	1.65	18.50	**49.25**	29.60
Credit Spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%	

Exhibit 7 also shows representative credit spreads for a 10-year corporate bond. The credit transition matrix and the credit spreads allow a fixed-income analyst to estimate a one-year rate of return given the possibility of credit rating migration but still no default. Assume that an A rated 10-year corporate bond will have a modified duration of 7.2 at the end of the year given stable yields and spreads. For each possible transition, the analyst can calculate the expected percentage price change as the product of the modified duration and the change in the spread:

From A to AAA:	$-7.2 \times (0.60\% - 1.10\%) = +3.60\%$.
From A to AA:	$-7.2 \times (0.90\% - 1.10\%) = +1.44\%$.
From A to BBB:	$-7.2 \times (1.50\% - 1.10\%) = -2.88\%$.
From A to BB:	$-7.2 \times (3.40\% - 1.10\%) = -16.56\%$.
From A to B:	$-7.2 \times (6.50\% - 1.10\%) = -38.88\%$.
From A to CCC, CC, or C:	$-7.2 \times (9.50\% - 1.10\%) = -60.48\%$.

The probabilities of migration now can be used to calculate the expected percentage change in the bond value over the year. The expected percentage change in bond value for an A rated corporate bond is found by multiplying each expected percentage price change for a possible credit transition by its respective transition probability found in the row associated with the A rating and summing the products:

$$(0.0005 \times 3.60\%) + (0.0250 \times 1.44\%) + (0.8750 \times 0\%) + (0.0840 \times -2.88\%) + (0.0075 \times -16.56\%) + (0.0060 \times -38.88\%) + (0.0012 \times -60.48\%) = -0.6342\%$$

Therefore, the expected return on the bond over the next year is its yield to maturity minus 0.6342%, assuming no default. If the bond was not investment grade, the small probability of a transition to default would need to be taken into consideration.

Credit spread migration typically reduces the expected return for two reasons. First, the probabilities for change are not symmetrically distributed around the current rating. They are skewed toward a downgrade rather than an upgrade. Second, the increase in the credit spread is much larger for downgrades than the decrease in the spread for upgrades.

EXAMPLE 4 The Impact of Credit Migration on Expected Return

Manuel Perello is a wealth manager for several Latin American families who seek to keep a portion of their assets in very high-quality corporate bonds. Mr. Perello explains that the yields to maturity on the bonds should be adjusted for possible *credit spread widening* to measure the expected rate of return over a given time horizon. In his presentation to one of the families, he uses a 10-year, AAA rated corporate bond that would have a modified duration of 7.3 at the end of the year. Using the corporate transition matrix in Exhibit 7, Mr. Perello concludes that the expected return on the bond over the next year can be approximated by the yield to maturity less 32.5 bps to account for a possible credit downgrade even if there is no default. Demonstrate how he arrives at that conclusion.

Solution: First, calculate the expected percentage price change using the modified duration for the bond and the change in the credit spread:

From AAA to AA:	–7.3 × (0.90% – 0.60%) = –2.19%.
From AAA to A:	–7.3 × (1.10% – 0.60%) = –3.65%.
From AAA to BBB:	–7.3 × (1.50% – 0.60%) = –6.57%.
From AAA to BB:	–7.3 × (3.40% – 0.60%) = –20.44%.
From AAA to B:	–7.3 × (6.50% – 0.60%) = –43.07%.
From AAA to CCC, CC, or C:	–7.3 × (9.50% – 0.60%) = –64.97%.

Second, calculate the expected percentage change in bond value over the year using the probabilities associated with the AAA rating row in the corporate transition matrix:

$$(0.9000 \times 0\%) + (0.0900 \times -2.19\%) + (0.0060 \times -3.65\%) + (0.0015 \times -6.57\%) + (0.0010 \times -20.44\%) + (0.0010 \times -43.07\%) + (0.0005 \times -64.97\%) = -0.3249\%$$

4. STRUCTURAL AND REDUCED-FORM CREDIT MODELS

- **explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses**

Credit analysis models fall into two broad categories—structural models and reduced-form models (Fabozzi 2013). Structural models of credit risk date back to the 1970s and the seminal contributions to finance theory by Fischer Black, Myron Scholes, and Robert Merton (Black and Scholes 1973; Merton 1974). Their key insights were that a company defaults on its debt if the value of its assets falls below the amount of its liabilities and that the probability of that event has the features of an option.

Reduced-form varieties emerged in the 1990s (Jarrow and Turnbull 1995; Duffie and Singleton 1999) and avoid a fundamental problem with the structural models. The Black–Scholes–Merton option pricing model explicitly assumes that the assets on which the options are written (i.e., the shares of a company) are actively traded. That assumption is fine for stock options; however, the assets of the company typically do not trade. Reduced-form models get around this problem by not treating default as an endogenous (internal) variable. Instead, the default is an exogenous (external) variable that occurs randomly. Unlike structural models that aim to explain *why* default occurs (i.e., when the asset value falls below the amount of liabilities), reduced-form models aim to explain statistically *when*. This is known as the *default time* and can be modeled using a Poisson stochastic process. The key parameter in this process is the *default intensity*, which is the probability of default over the next time increment. Reduced-form credit risk models are thus also called *intensity-based* and *stochastic default rate* models.

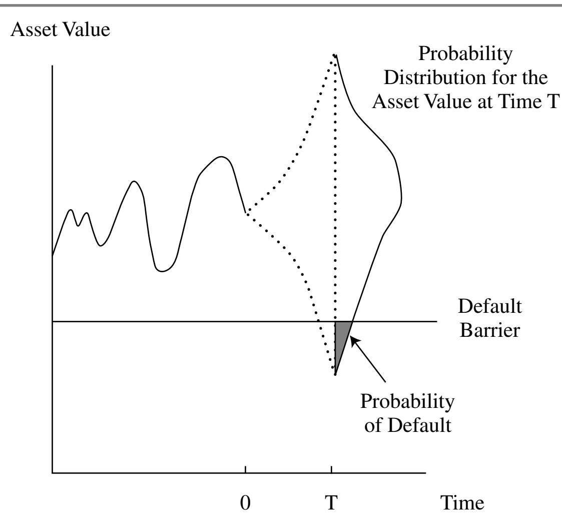
Both types of credit risk model have advantages and disadvantages. Structural models provide insight into the nature of credit risk but can be burdensome to implement. The modeler needs to determine the value of the company, its volatility, and the default barrier that is based on the liabilities of the company. In the model, the company defaults when the value of its assets dips below this default barrier. Although straightforward in theory, it can be difficult in practice because of limitations in available data. Examples of companies hiding debt (Enron Corporation, Tyco International, WorldCom, Parmalat, and Lehman Brothers, to name a few) highlight the challenge to measure the default barrier, especially in times when knowing changes in default probabilities would be most beneficial to investors (Smith 2011).

Reduced-form models have the advantage that the inputs are observable variables, including historical data. The default intensity is estimated using regression analysis on *company-specific* variables (e.g., leverage ratio, net-income-to-assets ratio, and cash-to-assets ratio), and *macroeconomic* variables (e.g., unemployment rate, GDP growth rate, measures of stock market volatility). This flexibility allows the model to directly reflect the business cycle in the credit risk measure.

A disadvantage of reduced-form models is that, unlike structural models, they do not explain the economic reasons for default. Also, reduced-form models assume that default comes as a "surprise" and can occur at any time. In reality, default is rarely a surprise because the issuer usually has been downgraded several times before the final event, as we saw with the RadioShack experience in the previous section.

Exhibit 8 depicts a structural model of default. The vertical axis measures the asset value of the company. It is called a structural model because it depends on the structure of the company's balance sheet—its assets, liabilities, and equity. It also can be called a *company-value* model because the key variable is the asset value of the company. In Exhibit 8, the asset value has been volatile prior to now, time 0, but has remained above the horizontal line that represents the default barrier. If the asset value falls below the barrier, the company defaults on the debt.

EXHIBIT 8 A Structural Model of Default



Source: This exhibit is adapted from Duffie and Singleton (2003, p. 54).

There is a probability distribution for the asset value as of some future date, time T. The probability of default is endogenous to this structural model. It is the portion of the probability distribution that lies below the default barrier. This default probability increases with the variance of the future asset value, with greater time to T, and with greater financial leverage. Less debt in the capital structure lowers the horizontal line and reduces the probability of default. These factors indicate that credit risk is linked to option pricing theory.

An important feature of the structural credit models is that they allow interpretation of debt and equity values in terms of options. Let $A(T)$ be the random asset value as of time T. To simplify, we can assume that the debt liabilities are zero-coupon bonds that mature at time T. These bonds have a face value of K, which represents the default barrier in Exhibit 8. The values for debt and equity at time T are denoted $D(T)$ and $E(T)$ and depend on the relationship between $A(T)$ and K:

$$D(T) + E(T) = A(T) \tag{1}$$

$$E(T) = \max[A(T) - K, 0] \tag{2}$$

$$D(T) = A(T) - \max[A(T) - K, 0] \tag{3}$$

Equation 1 is the balance sheet identity: The market values of debt and equity at time T equal the asset value. Equation 2 indicates that equity is essentially a purchased call option on the assets of the company whereby the strike price is the face value of the debt. It is a long position in a call option because the value of equity goes up when the asset value goes up. Moreover, like options, equity does not take on negative values. Equation 3 shows that in this formulation, the debtholders own the assets of the company and have written the call option held by the shareholders. We can interpret the premium that the debtholders receive for writing the option as the value of having priority of claim in the event that the asset value falls below K. In that case, the value of equity falls to zero and the debtholders own the remaining assets.

Suppose that at time T, $A(T) > K$ so that the call option is in the money to the shareholders. Then, $E(T) = A(T) - K$ and $D(T) = A(T) - [A(T) - K] = K$. Instead, suppose that $A(T) < K$ so that the call option is out of the money and the debt is in default. In this case, $E(T) = 0$ and $D(T) = A(T) - 0 = A(T)$. In both situations, as well as when $A(T) = K$, the balance sheet identity holds. Notice that *limited liability* is an inherent assumption in this model. Equity, like options, does not take on negative values.

EXAMPLE 5 An Equivalent Option Interpretation of Debt and Equity

Carol Feely is a junior credit analyst at one of the major international credit rating agencies. She understands that in the standard structural models, equity is interpreted as a call option on the asset value of the company. However, she is not comfortable with the assumption that it is the debtholders who implicitly own the assets and write a call option on them. She claims that the model should start with the understanding that the shareholders own the net value of the company, which is $A(T) - K$, and that their limited liability is essentially the value of a long position in a put option at a strike price of K. Furthermore, the debtholders own a "risk-free" bond having a value of K at time T and a short position in the put that is held by the shareholders.

Demonstrate that Ms. Feely's "embedded put option" interpretation provides the same values for debt and equity at time T as does the more customary call option structural model.

Solution: A long position in a put option on the asset value at a strike price of K takes the form $\max[K - A(T), 0]$. This put option has intrinsic value to its holder when $K > A(T)$ and is worthless when $K \leq A(T)$. The values for $E(T)$ and $D(T)$ according to Ms. Feely at time T are as follows:

$$E(T) = A(T) - K + \max[K - A(T), 0]$$
$$D(T) = K - \max[K - A(T), 0]$$

If $A(T) > K$ at time T, the put option is out of the money, $E(T) = A(T) - K + 0 = A(T) - K$, and $D(T) = K - 0 = K$. If $A(T) < K$, the put is in the money, $E(T) = A(T) - K + [K - A(T)] = 0$, and $D(T) = K - [K - A(T)] = A(T)$. This interpretation indicates that the value of limited liability to shareholders is the value of the put option that they purchase from the debtholders. Ms. Feely is correct in that the same payoffs as the embedded call option interpretation are obtained.

Although credit risk is inherently linked to option pricing, it is the implementation of structural models that has provided practical value to fixed-income analysis. Many credit rating agencies and consultancies, most notably Moody's KMV Corporation, use option pricing methodologies to estimate such credit risk parameters as the probability of default and the loss given default. Building on the classic Black–Scholes–Merton model and later variants, the model builders use historical data on the company's equity price to estimate volatility, which is a key element in option pricing models.

These advantages and disadvantages indicate that the choice of credit risk model depends on how it is to be used and by whom. Structural models require information best known to the managers of the company (and perhaps their commercial bankers and the credit rating agencies). Therefore, they can be used for internal risk management, for banks' internal credit risk measures, and for publicly available credit ratings. Reduced-form models require only information generally available in financial markets, which suggests that they should be used to value risky debt securities and credit derivatives.

5. VALUING RISKY BONDS IN AN ARBITRAGE-FREE FRAMEWORK

- **calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters**

In this section, we use the arbitrage-free framework to analyze the credit risk of a corporate bond in the context of volatile interest rates (based on Smith 2017). Earlier, we solved for the credit valuation adjustment and the credit spread under the assumptions of no interest rate volatility and a flat government bond yield curve. A binomial interest rate tree for benchmark bond yields allows us to calculate the *expected exposure* to default loss. In addition, we have an upward-sloping yield curve for benchmark bonds. We take the risk-neutral probability of default as given, as if it has been determined using a structural or reduced-form credit model. We also assume a recovery rate if default were to occur that conforms to the seniority of the debt issue and the nature of the issuer's assets.

The first step is to build the binomial interest rate tree under the assumption of no arbitrage. Exhibit 9 displays the data on annual payment benchmark government bonds that are used to build the binomial interest rate tree. This is the *par curve* because each bond is priced at par value. The coupon rates are equal to the yields to maturity because the years to maturity are whole numbers (integers) so that there is no accrued interest. The one-year government bond has a negative yield to reflect the conditions seen in some financial markets. Note that the actual one-year security is likely to be a zero-coupon bond priced at a premium, at 100.2506 per 100 of par value: (100/100.2506) − 1 = −0.0025. However, on a par curve for which all the bonds are priced at 100, it is shown as having a negative coupon rate.

EXHIBIT 9 Par Curve for Annual Payment Benchmark Government Bonds, Spot Rates, Discount Factors, and Forward Rates

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	–0.25%	100	1.002506	–0.2500%	
2	0.75%	100	0.985093	0.7538%	1.7677%
3	1.50%	100	0.955848	1.5166%	3.0596%
4	2.25%	100	0.913225	2.2953%	4.6674%
5	2.75%	100	0.870016	2.8240%	4.9664%

Note: All calculations in this and subsequent exhibits were completed on a spreadsheet; rounded results are reported in the text.

The discount factors and spot rates are bootstrapped using the cash flows on the underlying benchmark bonds in this sequence of equations:

$$100 = (100 - 0.25) \times DF_1$$

$$DF_1 = 1.002506$$

$$100 = (0.75 \times 1.002506) + (100.75 \times DF_2)$$

$$DF_2 = 0.985093$$

$$100 = (1.50 \times 1.002506) + (1.50 \times 0.985093) + (101.50 \times DF_3)$$

$$DF_3 = 0.955848$$

$$100 = (2.25 \times 1.002506) + (2.25 \times 0.985093) + (2.25 \times 0.955848) + (102.25 \times DF_4)$$

$$DF_4 = 0.913225$$

$$100 = (2.75 \times 1.002506) + (2.75 \times 0.985093) + (2.75 \times 0.955848) + (2.75 \times 0.913225) + (102.75 \times DF_5)$$

$$DF_5 = 0.870016$$

The spot (i.e., implied zero-coupon) rates can be calculated from the discount factors; for instance, the two-year spot rate is 0.7538% and the four-year spot rate is 2.2953%:

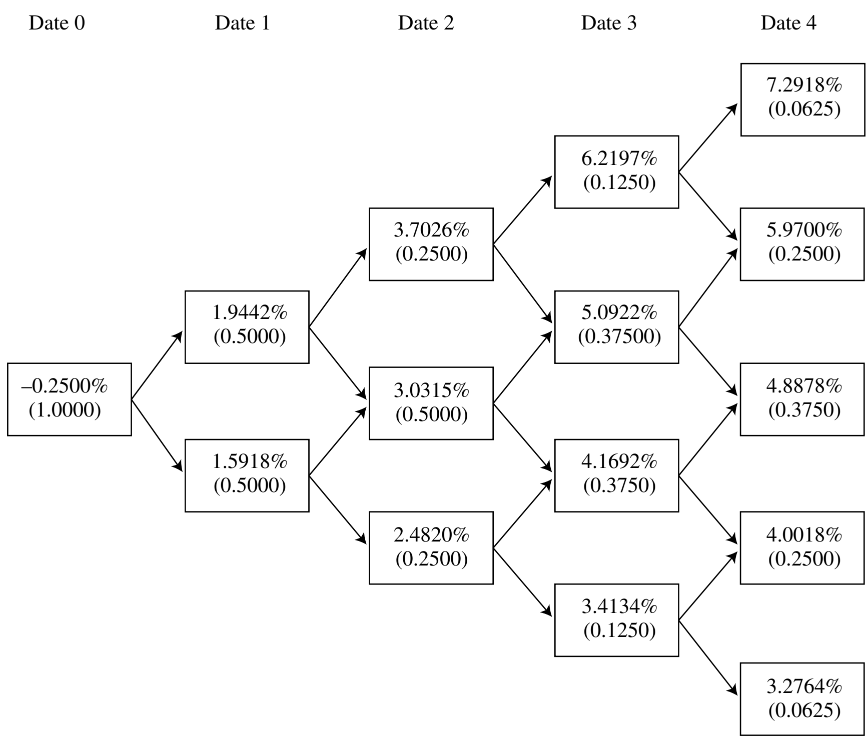
$$\left(\frac{1}{0.985093}\right)^{1/2} - 1 = 0.007538$$

$$\left(\frac{1}{0.913225}\right)^{1/4} - 1 = 0.022953$$

The forward rates are calculated as the ratios of the discount factors. The one-year forward rate two years into the future is 3.0596%: 0.985093/0.955848 – 1 = 0.030596. The one-year forward rate four years into the future is 4.9665%: 0.913225/0.870016 – 1 = 0.049665.

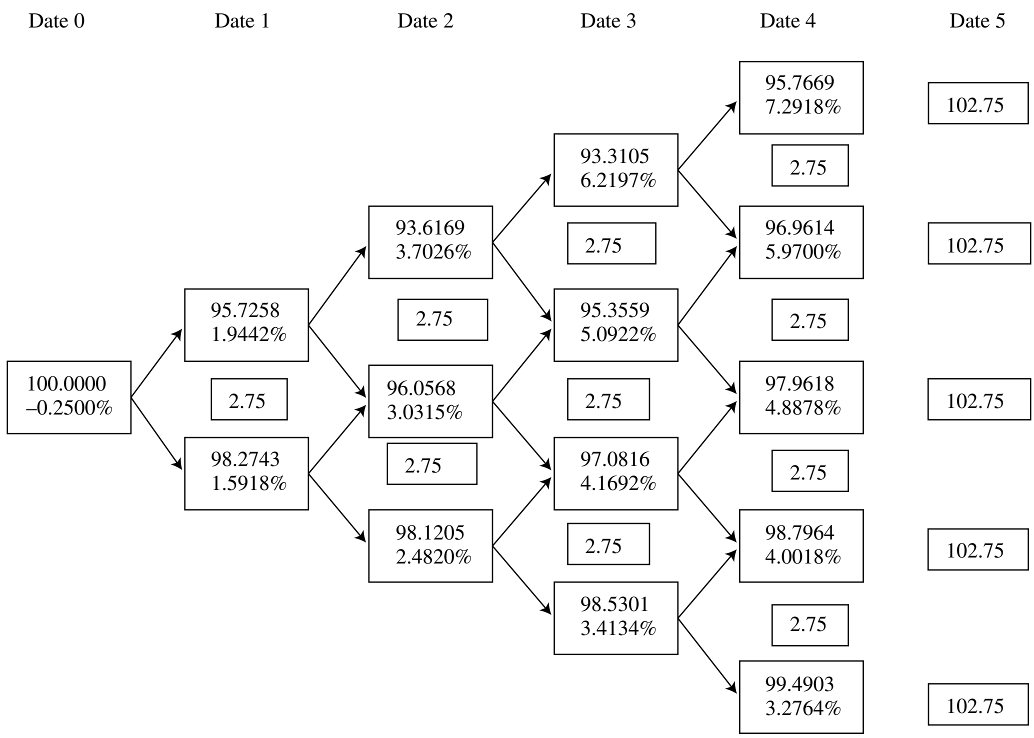
Following the methodology detailed in the "Arbitrage-Free Valuation Framework" topic, we build a binomial interest rate tree for one-year forward rates consistent with the pricing of the benchmark government bonds and an assumption of future interest rate volatility. Here we assume 10% volatility. The resulting binomial interest rate tree is presented in Exhibit 10. Below each rate is the probability of attaining that node in the tree. The current (Date 0) one-year rate of –0.25% will rise to 1.9442% or "fall" to 1.5918% by the end of the year (Date 1) with equal probability. On Date 2, at the end of the second year, the one-year rate will be 3.7026%, 3.0315%, or 2.4820% with probabilities of 0.25, 0.50, and 0.25, respectively. On Date 4, the forward rate will fall within the range of a high of 7.2918% to a low of 3.2764%. For each date, the possible rates are spread out around the forward rates shown in Exhibit 9.

EXHIBIT 10 One-Year Binomial Interest Rate Tree for 10% Volatility



To demonstrate that this is an arbitrage-free binomial interest rate tree, we calculate the Date 0 value of a 2.75% annual payment government bond. We know from Exhibit 9 that this bond is priced at par value. Exhibit 11 shows that the Date 0 value is indeed 100.0000. Notice that the scheduled year-end coupon and principal payments are placed to the right of each forward rate in the tree.

EXHIBIT 11 Valuation of a 2.75% Annual Payment Government Bond



These are the five Date 4 values for the government bond, shown above the interest rate at each node:

$$102.75/1.072918 = 95.7669$$
$$102.75/1.059700 = 96.9614$$
$$102.75/1.048878 = 97.9618$$
$$102.75/1.040018 = 98.7964$$
$$102.75/1.032764 = 99.4903$$

These are the four Date 3 values:

$$\frac{[(0.5 \times 95.7669) + (0.5 \times 96.9614)] + 2.75}{1.062197} = 93.3105$$
$$\frac{[(0.5 \times 96.9614) + (0.5 \times 97.9618)] + 2.75}{1.050922} = 95.3559$$
$$\frac{[(0.5 \times 97.9618) + (0.5 \times 98.7964)] + 2.75}{1.041692} = 97.0816$$
$$\frac{[(0.5 \times 98.7964) + (0.5 \times 99.4903)] + 2.75}{1.034134} = 98.5301$$

Continuing with backward induction, the Date 0 value turns out to be 100.0000, confirming that the binomial interest rate tree has been correctly calibrated.

Now consider a five-year, 3.50% annual payment corporate bond. A fixed-income analyst assigns an annual default probability of 1.25% and a recovery rate of 40% to this bond and assumes 10% volatility in benchmark interest rates. The problem at hand for the analyst is to assess the fair value for the bond under these assumptions. This is done in two steps:

- First, determine the value for the corporate bond assuming no default (VND).
- Second, calculate the credit valuation adjustment.

The fair value of the bond is the VND minus the CVA.

The binomial interest rate tree for the benchmark rates in Exhibit 10 can be used to calculate the VND for the bond. Exhibit 12 shows that the VND is 103.5450 per 100 of par value. This number could also have been obtained more directly by using the benchmark discount factors:

$$(3.50 \times 1.002506) + (3.50 \times 0.985093) + (3.50 \times 0.955848) + (3.50 \times 0.913225) + (103.50 \times 0.870016) = 103.5450$$

The advantage of using the binomial interest rate tree to get the VND is that the same tree is used to calculate the expected exposure to default loss, which is a key element in the credit risk model.

EXHIBIT 12 Value of a 3.50% Annual Payment Corporate Bond Assuming No Default

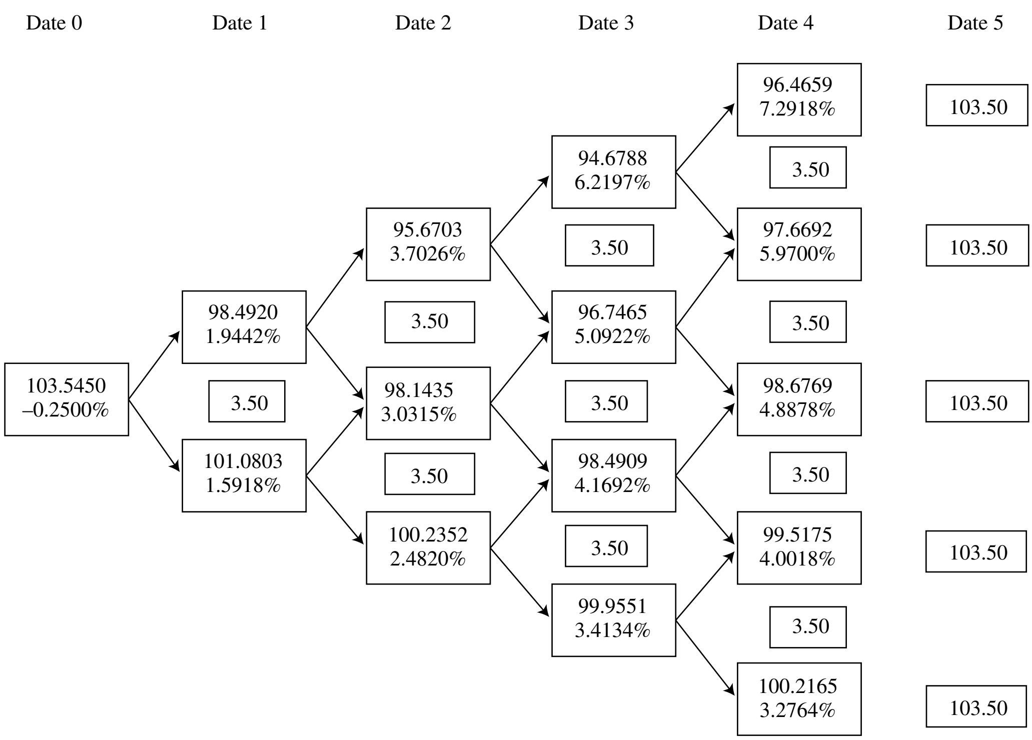


Exhibit 13 shows that the credit valuation adjustment to the value assuming no default is 3.5394 per 100 of par value. The expected exposure for Date 4 is 102.0931, calculated using the bond values at each node, the probability of attaining the node, and the coupon payment:

$$(0.0625 \times 96.4659) + (0.25 \times 97.6692) + (0.375 \times 98.6769) + (0.25 \times 99.5175) + (0.0625 \times 100.2165) + 3.50 = 102.0931$$

(Note again that all calculations are done on a spreadsheet to maintain precision; only the rounded results are reported in the text.) The loss given default for Date 4 is 61.2559 [= 102.0931 × (1 – 0.40)] because the assumed recovery rate is 40% of the exposure. The probability of default at Date 4 is 1.2037%, assuming no prior default. This is based on the probability of survival into the fourth year. It is calculated as

$$1.25\% \times (100\% - 1.25\%)^3 = 1.2037\%$$

The probability of survival after Date 3 is $(100\% - 1.25\%)^3$, and the probability of default on Date 4 is 1.25%. The product of the LGD and the POD is the expected loss. The present value of the expected loss, 0.6734, is the contribution to total CVA for Date 4. The sum of the CVAs for each year is the overall CVA.

EXHIBIT 13 Credit Valuation Adjustment for the 3.50% Annual Payment Corporate Bond

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.2500%	1.002506	0.7766
2	101.5481	60.9289	1.2344%	0.985093	0.7409
3	101.0433	60.6260	1.2189%	0.955848	0.7064
4	102.0931	61.2559	1.2037%	0.913225	0.6734
5	103.5000	62.1000	1.1887%	0.870016	0.6422
			6.0957%	CVA =	3.5394

The fixed-income analyst concludes that the fair value of the corporate bond is 100.0056 per 100 of par value: 103.5450 – 3.5394 = 100.0056. Depending on the current market price for the bond, the analyst might recommend a buy or sell decision.

The yield to maturity (YTM) for the corporate bond given a fair value of 100.0056 is 3.4988%:

$$100.0056 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}$$

$$\text{YTM} = 0.034988$$

The five-year par yield for the government bond in Exhibit 9 is 2.75%. Therefore, the credit spread over the benchmark bond is 0.7488% (= 3.4988% – 2.75%). In practice, the credit spread is typically measured against the actual yield on the comparable-maturity government bond, which might be trading at a premium or a discount.

We can say that the credit risk on this corporate bond is captured by a CVA of 3.5394 per 100 in par value as of Date 0 or as an annual spread of 74.88 bps per year for five years. This conclusion, however, assumes that the observed credit spread is based entirely on credit risk. In fact, there usually are liquidity and tax differences between government and corporate bonds. Those differences are neglected in this analysis to focus on credit risk. Stated differently, the liquidity and tax differences are represented in the credit spread.

EXAMPLE 6 Using Credit Analysis in Decision Making

Lori Boller is a fixed-income money manager specializing in taking long positions on high-yield corporate bonds that she deems to be undervalued. In particular, she looks for bonds for which the credit spread over government securities appears to indicate too high a probability of default or too low a recovery rate if default were to occur. Currently, she is looking at a three-year, 4.00% annual payment bond that is priced at 104 (per 100 of par value). In her opinion, this bond should be priced to reflect an annual default probability of 2.25% given a recovery rate of 40%. Ms. Boller is comfortable with an assumption of 10% volatility in government bond yields over the next few years. Should she consider buying this bond for her portfolio? Use the government par curve in Exhibit 9 and the binomial interest rate tree in Exhibit 10 in the solution.

Solution: Ms. Boller needs to calculate the fair value of the three-year, 4% annual payment corporate bond given her assumptions about the credit risk parameters. The results are shown in Exhibit 14.

EXHIBIT 14 Fair Value of the Three-Year, 4% Annual Payment Corporate Bond

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	64.2541	2.2500%	1.002506	1.4493
2	104.9120	62.9472	2.1994%	0.985093	1.3638
3	104.0000	62.4000	2.1499%	0.955848	1.2823
			6.5993%	CVA =	4.0954

The VND for the bond is 107.3586. The calculations for the bond values in the binomial interest rate tree are as follows:

$$104/1.037026 = 100.2868$$

$$104/1.030315 = 100.9400$$

$$104/1.024820 = 101.4812$$

$$\frac{(0.5 \times 100.2868) + (0.5 \times 100.9400) + 4}{1.019442} = 102.6183$$

$$\frac{(0.5 \times 100.9400) + (0.5 \times 101.4812) + 4}{1.015918} = 103.5621$$

$$\frac{(0.5 \times 102.6183) + (0.5 \times 103.5621) + 4}{0.997500} = 107.3586$$

The CVA for the bond is 4.0954 given the assumption of an annual default probability of 2.25% and a recovery rate of 40% of the expected exposure. The following are calculations for the Date 1 and Date 2 expected exposures:

$$(0.50 \times 102.6183) + (0.50 \times 103.5621) + 4 = 107.0902$$

$$(0.25 \times 100.2868) + (0.50 \times 100.9400) + (0.25 \times 101.4812) + 4 = 104.9120$$

The calculations for the LGD are as follows:

$$107.0902 \times (1 - 0.40) = 64.2541$$
$$104.9120 \times (1 - 0.40) = 62.9472$$
$$104 \times (1 - 0.40) = 62.4000$$

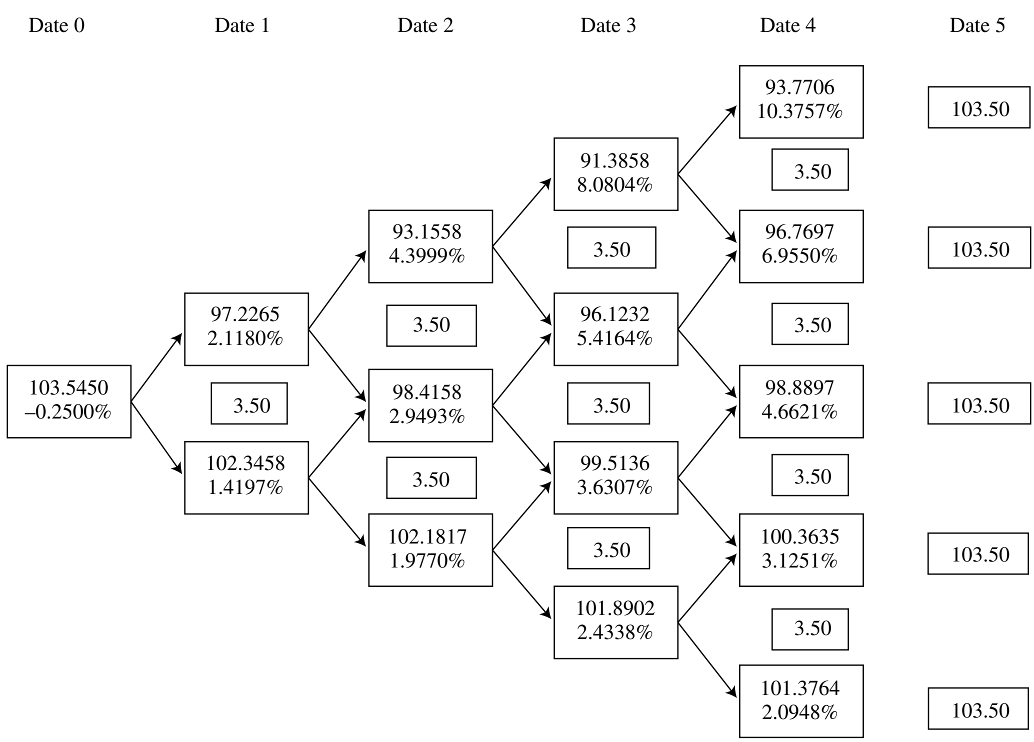
The following are calculations for the POD for Date 2 and Date 3:

$$2.25\% \times (100\% - 2.25\%) = 2.1994\%$$
$$2.25\% \times (100\% - 2.25\%)^2 = 2.1499\%$$

Ms. Boller determines, on the basis of her assumed credit risk parameters, that the fair value for the high-yield corporate bond is 103.2632 (= 107.3586 – 4.0954). Given that the bond is trading at 104, she would likely decline to purchase because in her opinion the bond is overvalued.

A change in the assumed level of interest rate volatility can be shown to have a small impact on the fair value of the corporate bond. Usually the effect of a change in volatility is demonstrated with a bond having an embedded option, such as a callable or putable bond. Here we see an impact of the calculation of CVA on a bond having no embedded options. This is illustrated with Exhibits 15 and 16, which use a no-arbitrage binomial interest rate tree for 20% volatility to value the five-year, 3.50% annual payment corporate bond using the same credit risk parameters as in the previous calculations.

EXHIBIT 15 VND Calculation for the 3.50% Corporate Bond Assuming No Default and 20% Volatility



Notice in Exhibit 15 that with 20% volatility, the range in forward rates for each date is now wider. With 10% volatility, the Date 4 rates go from a low of 3.2764% to a high of 7.2918%. Now, with 20% volatility, the range is from 2.0948% to 10.3757%. The key point is that changing all the bond values still results in a VND of 103.5450. This confirms that the tree has been correctly calibrated and that the assumed level of future interest rate volatility has no impact on the value of a default-risk-free government bond. Changes in the fair value of a corporate bond arising from a change in the assumed rate volatility occur only when there are embedded options and, as demonstrated in Exhibit 16, when there is credit risk.

EXHIBIT 16 CVA Calculation for the 3.50% Corporate Bond Assuming 20% Volatility

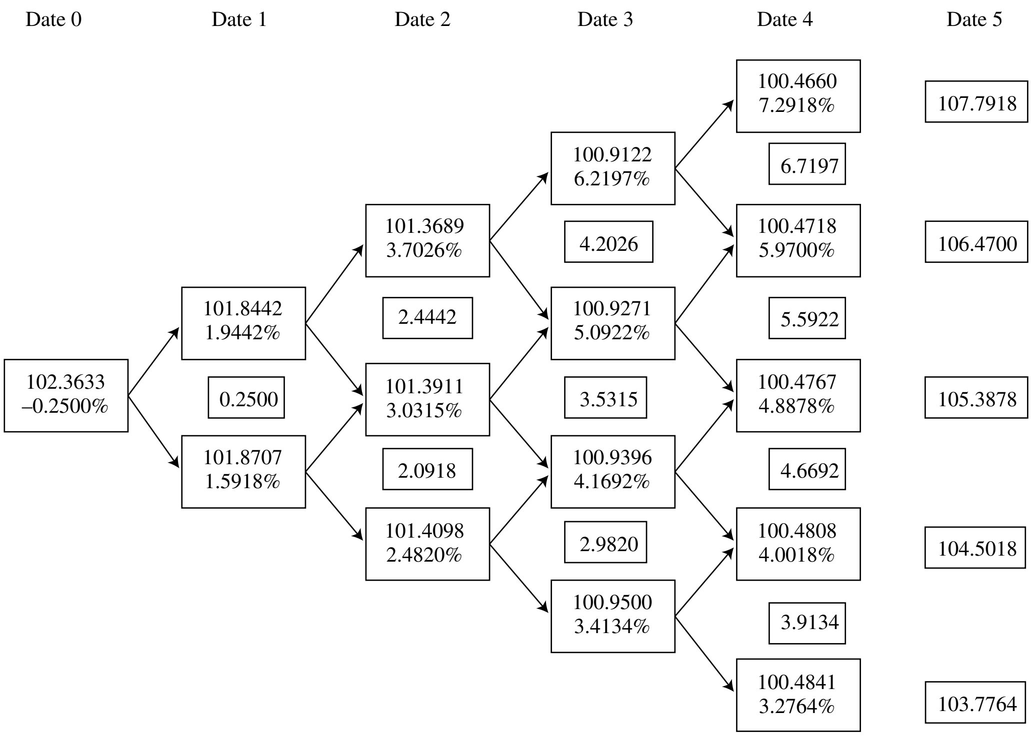
Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.2500%	1.002506	0.7766
2	101.5423	60.9254	1.2344%	0.985093	0.7408
3	101.0233	60.6140	1.2189%	0.955848	0.7062
4	102.0636	61.2382	1.2037%	0.913225	0.6732
5	103.5000	62.1000	1.1887%	0.870016	0.6422
			6.0957%	CVA =	3.5390

Exhibit 16 presents the table to calculate the CVA for 20% volatility. The expected exposures to default loss are slightly lower for Dates 2, 3, and 4 compared with Exhibit 13 for 10% volatility. These small changes feed through the table, reducing the loss given default and the contribution to total CVA for those dates. Overall, the CVA is 3.5390 per 100 of par value. The fair value of the bond is now slightly higher at 100.0060 (= 103.5450 − 3.5390), compared with the value for 10% volatility of 100.0056 (= 103.5450 − 3.5394).

The reason for the small volatility impact on the fair value is the asymmetry in the forward rates produced by the lognormality assumption in the interest rate model. In building the tree, rates are spread out around the implied forward rate for each date—more so the greater the given level of volatility. However, the range is not symmetric about the implied forward rate. For example, the one-year forward rate four years into the future is 4.9665% in Exhibit 9. With 20% volatility, the Date 4 rate at the top of the tree is higher by 5.4092% (= 10.3757% − 4.9665%), while the rate at the bottom of the tree is lower by 2.8717% (= 4.9665% − 2.0948%). The net effect is to reduce the expected exposure to default loss. The top of the tree shows less potential loss because the current value of the bond is lower, which more than offsets the greater exposure to loss at the bottom of the tree.

The arbitrage-free framework can be adapted to value a risky floating-rate note. Consider a five-year "floater" that pays annually the one-year benchmark rate plus 0.50%. This 50 bp addition to the index rate is called the *quoted margin* and typically is fixed over the lifetime of the security. Exhibit 17 demonstrates that the VND for the floater is 102.3633 per 100 of par value, using the binomial interest rate tree for 10% interest rate volatility. Notice that the interest payment is "in arrears," meaning that the rate is set at the beginning of the period and paid at the end of the period. That is why the interest payments set to the right of each rate vary depending on the realized rate in the tree. The interest payment for Date 1 is 0.25 because the Date 0 reference rate is –0.25%: (–0.25% + 0.50%) × 100 = 0.25. The final payment on Date 5 when the floater matures is 105.3878 if the one-year rate is 4.8878% on Date 4: (4.8878% + 0.50%) × 100 + 100 = 105.3878.

EXHIBIT 17 Value of a Floating-Rate Note Paying the Benchmark Rate Plus 0.50% Assuming No Default and 10% Volatility



Notice that the bond values for each date are very similar for the various forward rates. That, of course, is the intent of a floating-rate note. The bond values would all be exactly 100.0000 if the note paid the benchmark rate "flat," meaning a quoted margin of zero. The VND of 102.3633 is obtained via backward induction (i.e., beginning at maturity and working backward in time). The following are the calculations for the bond values for Date 4:

$$107.7918/1.072918 = 100.4660$$
$$106.4700/1.059700 = 100.4718$$
$$105.3878/1.048878 = 100.4767$$
$$104.5018/1.040018 = 100.4808$$
$$103.7764/1.032764 = 100.4841$$

These are the calculations for Date 3:

$$\frac{(0.5 \times 100.4660) + (0.5 \times 100.4718) + 6.7197}{1.062197} = 100.9122$$
$$\frac{(0.50 \times 100.4718) + (0.5 \times 100.4767) + 5.5922}{1.050922} = 100.9271$$
$$\frac{(0.5 \times 100.4767) + (0.5 \times 100.4808) + 4.6692}{1.041692} = 100.9396$$
$$\frac{(0.5 \times 100.4808) + (0.5 \times 100.4841) + 3.9134}{1.034134} = 100.9500$$

These are the calculations for the bond values for Date 2:

$$\frac{(0.5 \times 100.9122) + (0.5 \times 100.9271) + 4.2026}{1.037026} = 101.3689$$

$$\frac{(0.5 \times 100.9271) + (0.5 \times 100.9396) + 3.5315}{1.030315} = 101.3911$$

$$\frac{(0.5 \times 100.9396) + (0.5 \times 100.9500) + 2.9820}{1.024820} = 101.4098$$

These are the calculations for the bond values for Date 1 and Date 0:

$$\frac{(0.5 \times 101.3689) + (0.5 \times 101.3911) + 2.4442}{1.019442} = 101.8442$$

$$\frac{(0.5 \times 101.3911) + (0.5 \times 101.4098) + 2.0918}{1.015918} = 101.8707$$

$$\frac{(0.5 \times 101.8442) + (0.5 \times 101.8707) + 0.2500}{0.997500} = 102.3633$$

Exhibit 18 shows the credit risk table for the floating-rate note. For this example, we assume that for the first three years, the annual default probability is 0.50% and the recovery rate is 20%. The credit risk of the issuer then worsens: For the final two years, the annual probability of default goes up to 0.75% and the recovery rate goes down to 10%. This is an example in which the assumed annual default probability changes over the lifetime of the bond.

EXHIBIT 18 CVA Calculation for the Value of a Floating-Rate Note Paying the Benchmark Rate Plus 0.50%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	102.1074	81.6859	0.5000%	1.002506	0.4095
2	103.6583	82.9266	0.4975%	0.985093	0.4064
3	104.4947	83.5957	0.4950%	0.955848	0.3955
4	105.6535	95.0881	0.7388%	0.913225	0.6416
5	105.4864	94.9377	0.7333%	0.870016	0.6057
			2.9646%	CVA =	2.4586

Note: Credit risk parameter assumptions: for Dates 1–3, annual default probability = 0.50% and recovery rate = 20%; for Dates 4–5, annual default probability = 0.75% and recovery rate = 10%.

The calculation for the expected exposure recognizes that the bond values for each date follow the probabilities of attaining those rates, whereas possible interest payments use the probabilities for the prior date. For example, the expected exposure to default loss for Date 4 is 105.6535:

$$\begin{bmatrix} (0.0625 \times 100.4660) + (0.25 \times 100.4718) + (0.375 \times 100.4767) \\ +(0.25 \times 100.4808) + (0.0625 \times 100.4841) \end{bmatrix} + [(0.125 \times 6.7197)$$

$$+ (0.375 \times 5.5922) + (0.375 \times 4.6692) + (0.125 \times 3.9134)] = 105.6535$$

The first term in brackets is the expected bond value using the Date 4 probabilities for each of the five possible rates. The second term is the expected interest payment using the Date 3 probabilities for each of the four possible rates.

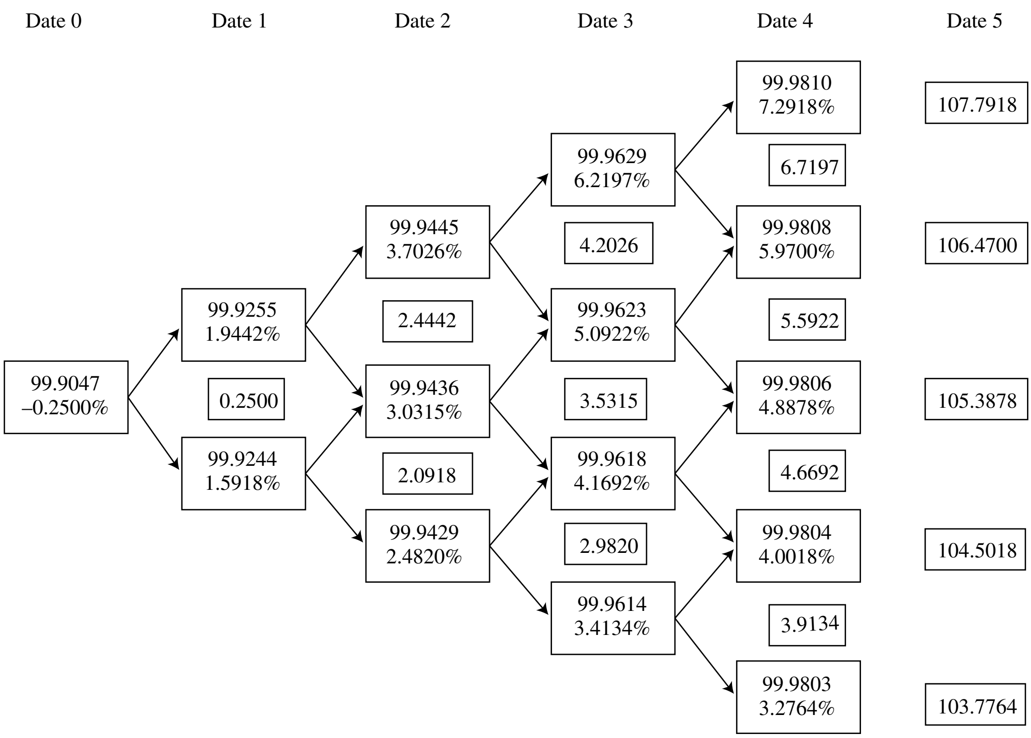
The expected LGD for Date 2 is 82.9266 [= 103.6583 × (1 − 0.20)]; for Date 4, it is 95.0881 [= 105.6535 × (1 − 0.10)]. The PODs in Exhibit 18 reflect the probability of default for each year. For Date 2, the POD is 0.4975%, conditional on no default on Date 1: 0.50% ×

(100% – 0.50%) = 0.4975%. For Date 3, the POD is 0.4950%: 0.50% × (100% – 0.50%)^2 = 0.4950%. The probability of survival into the fourth year is 98.5075%: (100% – 0.50%)^3 = 98.5075%. Therefore, the POD for Date 4 increases to 0.7388% because of the assumed worsening credit risk: 0.75% × 98.5075% = 0.7388%. The probability of survival into the fifth year is 97.7687% (= 98.5075% – 0.7388%). The POD for Date 5 is 0.7333% (= 0.75% × 97.7687%). The cumulative probability of default over the lifetime of the floater is 2.9646%.

Given these assumptions about credit risk, the CVA for the floater is 2.4586. The fair value is 99.9047, the VND of 102.3633 minus the CVA. Because the security is priced below par value, its *discount margin* (DM) must be higher than the quoted margin of 0.50%. The discount margin for a floating-rate note is a yield measure commonly used on floating-rate notes in the same manner that the credit spread is used with fixed-rate bonds.

The arbitrage-free framework can be used to determine the DM for this floater by trial-and-error search (or GoalSeek or Solver in Excel). We add a trial DM to benchmark rates that are used to get the bond values at each node in the tree. Then the trial DM is changed until the Date 0 value matches the fair value of 99.9047. Exhibit 19 shows that the DM for this floater is 0.52046%, slightly above the quoted margin because the security is priced at a small discount below par value.

EXHIBIT 19 The Discount Margin for the Floating-Rate Note Paying the Benchmark Rate Plus 0.50%, Assuming 10% Volatility



These are the calculations for the bond values for Date 2:

$$\frac{(0.5 \times 99.9629) + (0.5 \times 99.9623) + 4.2026}{1 + 0.037026 + 0.0052046} = 99.9445$$

$$\frac{(0.5 \times 99.9623) + (0.5 \times 99.9618) + 3.5315}{1 + 0.030315 + 0.0052046} = 99.9436$$

$$\frac{(0.5 \times 99.9618) + (0.5 \times 99.9614) + 2.9820}{1 + 0.024820 + 0.0052046} = 99.9429$$

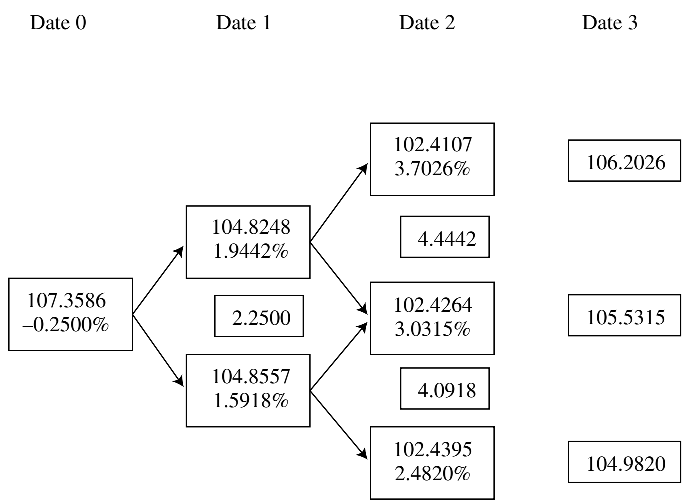
Throughout the binomial interest rate tree, the assumed DM is added to the benchmark rate to factor in credit risk. After a trial-and-error search, a DM of 0.52046% gives the same Date 0 value for the floating-rate note of 99.9047 as is obtained with the VND and CVA models.

EXAMPLE 7 Evaluating a Floating-Rate Note

Omar Yassin is an experienced credit analyst at a fixed-income investment firm. His current assignment is to assess potential purchases of distressed high-yield corporate bonds. One intriguing prospect is a three-year, annual payment floating-rate note paying the one-year benchmark rate plus 2.50%. The floater is rated CCC and is priced at 84 per 100 of par value. Based on various research reports on and prices of the issuer's credit default swaps, Mr. Yassin believes the probability of default in the next year is about 30%. If the issuer goes into bankruptcy at any time, he expects the recovery rate to be at least 50%; it could be as high as 60% because of some valuable real estate holdings. He further believes that if the issuer is able to survive this next year, the default probability for the remaining two years will be only about 10% for each year. Based on these assumptions about the credit risk parameters and an expectation of 10% volatility for interest rates, should Mr. Yassin recommend purchasing the floating-rate note?

Solution: Mr. Yassin calculates the fair value of the three-year, annual payment floating-rate note given his assumptions about the default probabilities and the recovery rate ranging between 50% and 60%. The results are shown in Exhibit 20.

EXHIBIT 20 Fair Value of the Three-Year, Annual Payment Floating-Rate Note Paying the One-Year Rate Plus 2.50%



Assumed 50% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	53.5451	30.0000%	1.002506	16.1038
2	106.6938	53.3469	7.0000%	0.985093	3.6786
3	105.5619	52.7810	6.3000%	0.955848	3.1784
			43.3000%	CVA =	22.9608

Fair value = 107.3586 – 22.9608 = 84.3978.

Assumed 60% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	42.8361	30.0000%	1.002506	12.8830
2	106.6938	42.6775	7.0000%	0.985093	2.9429
3	105.5619	42.2248	6.3000%	0.955848	2.5427
			43.3000%	CVA =	18.3686

Fair value = 107.3586 – 18.3686 = 88.9900.

Each projected interest payment in the tree is the benchmark rate at the beginning of the year plus 2.50% times 100. The rate is –0.25% on Date 0; the "in-arrears" interest payment on Date 1 is 2.2500 [= (–0.25% + 2.50%) × 100]. If the rate is 2.4820% on Date 2, the payment at maturity on Date 3 is 104.9820 [= (2.4820% + 2.50%) × 100 + 100].

The VND for the floater is 107.3586. The calculations for the bond values in the binomial interest rate tree are as follows:

$$106.2026/1.037026 = 102.4107$$

$$105.5315/1.030315 = 102.4264$$

$$104.9820/1.024820 = 102.4395$$

$$\frac{(0.5 \times 102.4107) + (0.5 \times 102.4264) + 4.4442}{1.019442} = 104.8248$$

$$\frac{(0.5 \times 102.4264) + (0.5 \times 102.4395) + 4.0918}{1.015918} = 104.8557$$

$$\frac{(0.5 \times 104.8248) + (0.5 \times 104.8557) + 2.2500}{0.997500} = 107.3586$$

These are the calculations for the expected exposures to default loss:

$$(0.5 \times 104.8248) + (0.5 \times 104.8557) + 2.2500 = 107.0902$$

$$(0.25 \times 102.4107) + (0.5 \times 102.4264) + (0.25 \times 102.4395) + (0.5 \times 4.4442) + (0.5 \times 4.0918) = 106.6938$$

$$(0.25 \times 106.2026) + (0.5 \times 105.5315) + (0.25 \times 104.9820) = 105.5619$$

The assumed default probability for the first year is 30%. The POD for Date 2 is 7.00%, which is the probability of survival into the second year, 70%, times the 10% probability of default. The probability of survival into the third year is 63% (= 70% − 7%); the POD for Date 3 is 6.30% (= 10% × 63%).

The decision to consider purchase of the floating-rate note comes down to the assumption about recovery. Exhibit 20 first shows the results for 50% recovery of the expected exposure. The LGD on Date 2 is 53.3469 [= 106.6938 × (1 − 0.50)]. The overall CVA is 22.9608, giving a fair value of 84.3978 (= 107.3586 − 22.9608). Exhibit 20 next shows the results for 60% recovery. With this assumption, the LGD for Date 2 is just 42.6775 [= 106.6938 × (1 − 0.60)]. Stronger recovery reduces the overall CVA to 18.3686. The fair value for the floater is now 88.9900.

Mr. Yassin should recommend purchasing the distressed floating-rate note. Although there is a significant 43.3% probability of default at some point over the three years, the security appears to be fairly priced at 84 given a recovery rate of 50%. At 60% recovery, it is significantly undervalued.

In addition, there is still a 57.7% (= 100% − 43.3%) chance of no default. Exhibit 21 shows the calculation for the discount margin, which is a measure of the return to the investor assuming no default (like a yield to maturity on a fixed-rate bond). Found by a trial-and-error search, the DM is 8.9148%, considerably higher than the quoted margin because the floater is priced at a deep discount.

EXHIBIT 21 Discount Margin on the Three-Year, Annual Payment Floating-Rate Note Paying the One-Year Rate Plus 2.50%

These are the calculations for the bond values for Date 1 and Date 0:

$$\frac{(0.5 \times 94.3039) + (0.5 \times 94.2698) + 4.4442}{1 + 0.019442 + 0.089148} = 89.0600$$

$$\frac{(0.5 \times 94.2698) + (0.5 \times 94.2415) + 4.0918}{1 + 0.015918 + 0.089148} = 88.9969$$

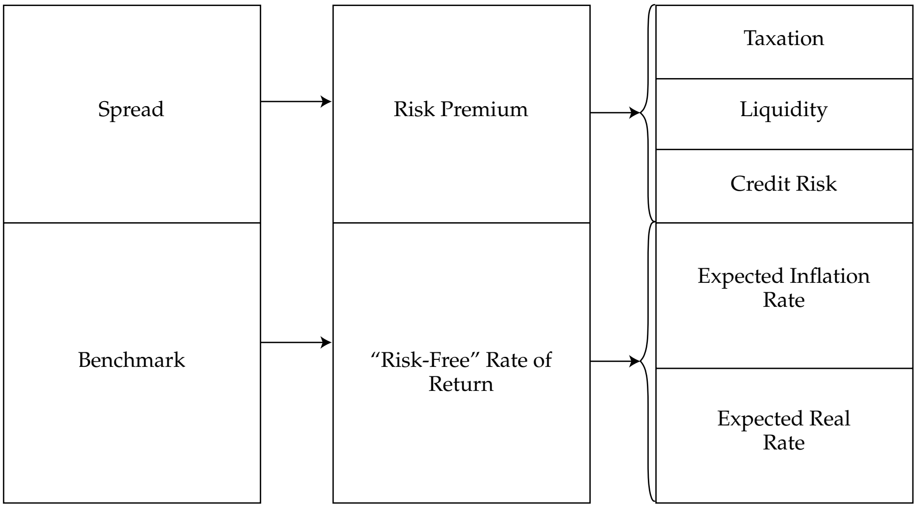
$$\frac{(0.5 \times 89.0600) + (0.5 \times 88.9969) + 2.2500}{1 - 0.0025 + 0.089148} = 84.0000$$

6. INTERPRETING CHANGES IN CREDIT SPREADS

- **interpret changes in a credit spread**

Corporate and benchmark bond yields and the credit spread between them change from day to day. The challenge for a fixed-income analyst is to understand that and be able to explain *why* the yields and spreads change. Exhibit 22 offers a breakdown of the main components of bond yields. Benchmark bond yields, in general, capture the *macroeconomic* factors affecting all debt securities. These are the expected inflation rate and the expected real rate of return. Risk-averse investors in benchmark bonds also might require compensation for uncertainty regarding those variables.

EXHIBIT 22 Components of a Corporate Bond Yield



The spread over the benchmark bond yield captures the *microeconomic* factors that pertain to the corporate issuer and the specific issue itself. The chief microeconomic factor is the expected loss due to default. There also are liquidity and tax differences between the corporate and benchmark bonds. Moreover, it can be difficult to separate these factors. Securities for which it becomes more difficult for analysts to assess a probability of default and a recovery rate typically become less liquid. Similarly, an uncertain tax status on a bond's gains and losses will increase the time and cost to estimate value. That makes the bond less liquid. Another factor in the observed spread between the corporate and benchmark bond yields can be compensation to risk-averse investors for uncertainty regarding credit risk, as well as liquidity and tax factors.

Research groups at major banks and consultancies have been working on models to better include counterparty credit risk, funding costs, and liquidity and taxation effects in the valuations of derivatives. First, a value is obtained using benchmark discount factors, in practice, derived from rates on overnight indexed swaps (OIS). These are interest rate swaps that reference an average daily interest rate. For instance, in the United States this daily rate is the effective federal funds rate. Then this OIS value, which is comparable to the VND in the previous section, is adjusted for the other factors. These valuation adjustments collectively are known as the XVA. The credit valuation adjustment is the most developed and most used in practice. Others include a funding valuation adjustment (FVA), a liquidity valuation adjustment (LVA), and a taxation valuation adjustment (TVA). In principle, the same ideas apply to debt securities in that these XVA comprise the observed spread between corporate and benchmark bond yields. For the purposes of our coverage, we focus only on the credit risk component, the CVA.

We can use the arbitrage-free framework and the credit risk model to examine the connections between the default probability, the recovery rate, and the credit spread. To be sure, this is a simple model to illustrate the much more complex models used in practice. These (which are called *XVA engines*) typically use Monte Carlo simulations for thousands of possible paths for interest rates. Our binomial interest rate tree has only 16 paths for the five years; it's a model of the actual model.

Consider again the five-year, 3.50% annual payment corporate bond examined earlier. In Exhibit 12, the value assuming no default was determined to be 103.5450 per 100 of par value. Now let us use the credit risk model to find the probabilities of default that would be consistent with various credit spreads and a recovery rate of 40%. Suppose, as in Exhibit 7, the credit spread for an AAA rated bond is 0.60%. Using trial-and-error search, we find that an annual probability of default of 1.01% produces a 60 bp credit spread. The credit risk table is presented in Exhibit 23. Notice that the expected exposure to default loss and the loss given default are the same as in Exhibit 13. Only the default probabilities and the contributions to total CVA for each year change.

EXHIBIT 23 CVA Calculation for the 3.50% Corporate Bond Given a Default Probability of 1.01% and a Recovery Rate of 40%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.0100%	1.002506	0.6275
2	101.5481	60.9289	0.9998%	0.985093	0.6001
3	101.0433	60.6260	0.9897%	0.955848	0.5735
4	102.0931	61.2559	0.9797%	0.913225	0.5481
5	103.5000	62.1000	0.9698%	0.870016	0.5240
			4.9490%	CVA =	2.8731

The CVA for the bond is 2.8731 per 100 of par. The fair value is 100.6719 (= 103.5450 – 2.8731). This gives a yield to maturity of 3.35%.

$$100.6719 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}$$

$$\text{YTM} = 0.0335$$

Given that the yield on the five-year benchmark bond is 2.75%, the credit spread is 0.60% (= 3.35% – 2.75%).

We can repeat this exercise for the other credit spreads and ratings shown in Exhibit 7. In each case, trial-and-error search is used to get the initial POD that corresponds to the CVA, the fair value, and the yield to maturity for each assumed spread. The results for the annual and cumulative default probabilities over the five years are shown in Exhibit 24.

EXHIBIT 24 Default Probabilities Consistent with Given Credit Ratings and Spreads and 40% Recovery

Credit Rating	Credit Spread	Annual Default Probability	Cumulative Default Probability
AAA	0.60%	1.01%	4.95%
AA	0.90%	1.49%	7.23%
A	1.10%	1.83%	8.82%
BBB	1.50%	2.48%	11.80%
BB	3.40%	5.64%	25.19%
B	6.50%	10.97%	44.07%
CCC, CC, C	9.50%	16.50%	59.41%

The default probabilities illustrated in Exhibit 24 might seem high, especially given the historical experience presented in Exhibit 6. Since 1995, no AAA rated company has defaulted; still, we model the likelihood to be over 1% for the first year and almost 5% for the next five years. However, as discussed earlier, these are *risk-neutral* probabilities of default and are higher than the actual probabilities because market prices reflect uncertainty over the timing of possible default. Investors are concerned about credit spread widening, especially if they do not intend to hold the bond to maturity. Credit rating migration from year to year, as illustrated in Exhibit 7, is a concern even for a high-quality investment-grade corporate bond. This is captured in the risk-neutral probability of default. Also, we must remember that observed credit spreads reflect more than just credit risk—there also are liquidity and tax differences. That further explains the difference between risk-neutral and actual default probabilities.

The relationship between the assumed recovery rate and the credit spread can be examined in the context of the credit risk model. Suppose that the five-year, 3.50% annual payment corporate bond has an initial probability of default of 1.83%. In Exhibit 24, we see that for a 40% recovery rate, the credit spread is 1.10%. What if the recovery rate is expected to be only 30%? Exhibit 25 shows the credit risk table for that assumption.

EXHIBIT 25 CVA Calculation for the 3.50% Corporate Bond Given a Default Probability of 1.83% and a Recovery Rate of 30%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	72.3003	1.8300%	1.002506	1.3264
2	101.5481	71.0837	1.7965%	0.985093	1.2580
3	101.0433	70.7303	1.7636%	0.955848	1.1923
4	102.0931	71.4652	1.7314%	0.913225	1.1300
5	103.5000	72.4500	1.6997%	0.870016	1.0714
			8.8212%	CVA =	5.9781

The reduction in the recovery rate from 40% to 30% has an impact on LGD and CVA for each year. The overall CVA is 5.9781 per 100 of par value. The fair value for the bond is 97.5670 (= 103.5450 – 5.9781), and the yield to maturity is 4.05%, giving a credit spread of 1.30% (= 4.05% – 2.75%).

$$97.5670 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}$$

$$\text{YTM} = 0.0405$$

This example illustrates how a credit rating agency might use "notching" to combine the expected loss given default and the probability of default in setting the rating for a corporate bond. If the issuer were rated single A, associated with a default probability of 1.83% and a recovery rate of 40% on the company's senior unsecured debt, that debt might have a credit spread of 1.10%, comparable to other A rated companies. This particular bond is subordinated, leading analysts at the rating agency to believe that a lower recovery rate assumption of 30% is applicable. That could justify assigning a lower rating of A– or BBB+ on the subordinated debt, along with its 20 bp higher spread.

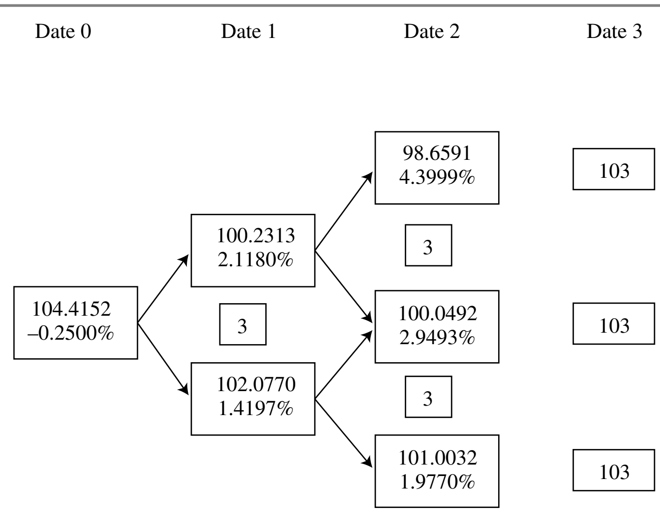
EXAMPLE 8 Evaluating Changes in Credit Risk Parameters

Edward Kapili is a summer intern working on a fixed-income trading desk at a major money-center bank. His supervisor asks him to value a three-year, 3% annual payment corporate bond using a binomial interest rate tree model for 20% volatility and the current par curve for benchmark government bonds. (This is the binomial tree in Exhibit 15.) The assumed annual probability of default is 1.50%, and the recovery rate is 40%.

The supervisor asks Mr. Kapili if the credit spread over the yield on the three-year benchmark bond, which is 1.50% in Exhibit 9, is likely to go up more if the default probability doubles to 3.00% or if the recovery rate halves to 20%. Mr. Kapili's intuition is that doubling the probability of default has a larger impact on the credit spread. Is his intuition correct?

Solution: Mr. Kapili first determines the fair value of the three-year, 3% annual payment bond given the assumptions for the original credit risk parameters. The binomial interest rate tree and credit risk table are presented in Exhibit 26.

EXHIBIT 26 Fair Value of the Three-Year, 3% Annual Payment Corporate Bond Assuming 20% Volatility



Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	62.4925	1.5000%	1.002506	0.9397
2	102.9402	61.7641	1.4775%	0.985093	0.8990
3	103.0000	61.8000	1.4553%	0.955848	0.8597
			4.4328%	CVA =	2.6984

Fair value = 104.4152 – 2.6984 = 101.7168.

The VND for the bond is 104.4152, the CVA is 2.6984, and the fair value is 101.7168 per 100 of par value. The yield to maturity is 2.40%, and the credit spread is 0.90% (= 2.40% – 1.50%).

$$101.7168 = \frac{3}{(1+\text{YTM})^1} + \frac{3}{(1+\text{YTM})^2} + \frac{103}{(1+\text{YTM})^3}$$

$$\text{YTM} = 0.0240$$

Next, Mr. Kapili calculates the fair values under the new credit risk parameters, first for doubling the default probability and second for halving the recovery rate. These tables are shown in Exhibit 27.

EXHIBIT 27 Fair Value Calculations for Doubling the Default Probability and Halving the Recovery Rate

3.00% Default Probability, 40% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	62.4925	3.0000%	1.002506	1.8795
2	102.9402	61.7641	2.9100%	0.985093	1.7705
3	103.0000	61.8000	2.8227%	0.955848	1.6674
			8.7327%	CVA =	5.3174

Fair value = 104.4152 – 5.3174 = 99.0978.

1.50% Default Probability, 20% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	83.3233	1.5000%	1.002506	1.2530
2	102.9402	82.3522	1.4775%	0.985093	1.1986
3	103.0000	82.4000	1.4553%	0.955848	1.1463
			4.4328%	CVA =	3.5978

Fair value = 104.4152 – 3.5978 = 100.8173.

The fair value of the corporate bond falls to 99.0978 when the default probability is raised to 3.00% and the recovery rate stays at 40%. The VND is the same, at 104.4152, and the CVA goes up to 5.3174. The yield to maturity increases to 3.32%, and the credit spread rises to 1.82% (= 3.32% – 1.50%).

$$99.0978 = \frac{3}{(1+\text{YTM})^1} + \frac{3}{(1+\text{YTM})^2} + \frac{103}{(1+\text{YTM})^3}$$

$$\text{YTM} = 0.0332$$

The fair value of the corporate bond falls to 100.8173 when the recovery rate is reduced by half, to 20%, and the default probability is maintained at 1.50%. The VND is again the same, at 104.4152, and the CVA goes up to 3.5978. The yield to maturity increases to 2.71%, and the credit spread rises to 1.21% (= 2.71% – 1.50%).

$$100.8173 = \frac{3}{(1+\text{YTM})^1} + \frac{3}{(1+\text{YTM})^2} + \frac{103}{(1+\text{YTM})^3}$$

$$\text{YTM} = 0.0271$$

Mr. Kapili's intuition is correct: Doubling the default probability has a greater impact on the credit spread than halving the recovery rate.

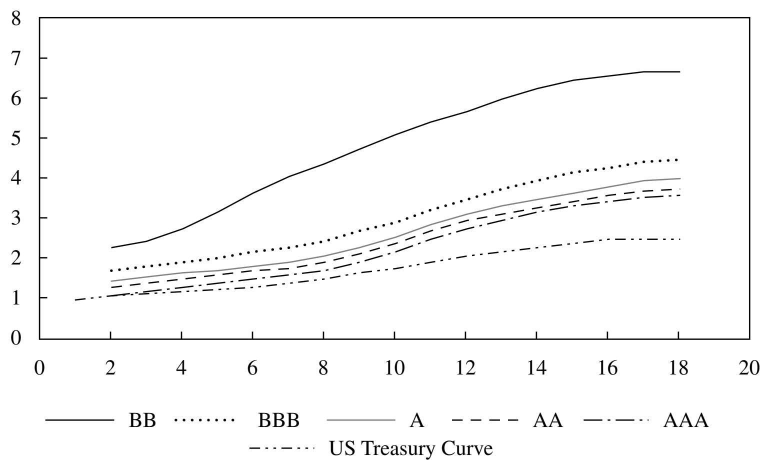
7. THE TERM STRUCTURE OF CREDIT SPREADS

- **explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads**

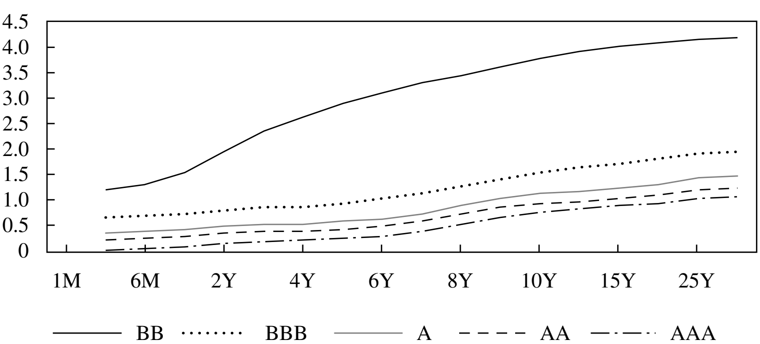
In the same way that the yield curve is composed of the interest rates on a single government issuer's debt across bond maturities, a credit curve shows the spread over a benchmark security for an issuer for outstanding fixed-income securities with shorter to longer maturities. For example, Exhibit 28 shows the relationship between US Treasury yields of a specific maturity and bonds rated AAA, AA, A, BBB, and BB. The total yields of the bonds are shown in Panel A, and spreads over the benchmark Treasury are shown in Panel B.

EXHIBIT 28 Composite Yield Graphs

A. Total Yields



B. Spreads



Source: Bloomberg.

The term structure of credit spreads is a useful gauge for issuers, underwriters, and investors in measuring the risk–return trade-off for a single issuer or a set of issuers across ratings and/or sectors across maturities. Issuers often work with their underwriter to consider the terms of a new issuance or a tender for existing debt based on relative credit spreads across maturities. For example, an investment-grade bond portfolio manager might use the existing credit curve for a particular issuer to determine a bid for a new primary debt issuance as well as to inform trading decisions for secondary debt positions. In some cases, investors, issuers, or underwriters might use the credit spread term structure for a particular rating or corporate sector either to derive prospective pricing for a new issuance or to determine fair value spreads for outstanding securities, which is an extension of matrix pricing. A high-yield debt investor might employ the term structure of credit spreads to gauge the risk/reward trade-offs between debt maturities. Given the impact of monetary and fiscal policies on risky debt markets, policymakers have extended their focus from default-risk-free yield curve dynamics to the term structure of credit spreads.

There are several key drivers of the term structure of credit spreads. First, credit quality is a key factor. For investment-grade securities with the highest credit ratings and extremely low spreads, credit spread migration is only possible in one direction given the implied lower bound of zero on credit spreads. As a result, the credit term structure for the most highly rated securities tends to be either flat or slightly upward sloping. Securities with lower credit quality, however, face greater sensitivity to the credit cycle. The greater likelihood of default associated with high-yield securities generally results in a steeper credit spread curve, both in cases where a weaker economy suggests credit spread widening and when an inverted credit spread curve suggests tighter spreads for longer maturities. As a high-yield bond moves further down the credit spectrum into a more distressed scenario, the contractual cash flows through maturity become less certain—with the value of distressed debt converging to a dollar price equal to the recovery rate as default becomes more certain, regardless of the remaining time to maturity. Such a scenario will result in a steeply inverted credit spread term structure. We now review the determinants of that term structure inversion and other implications of this scenario in more detail.

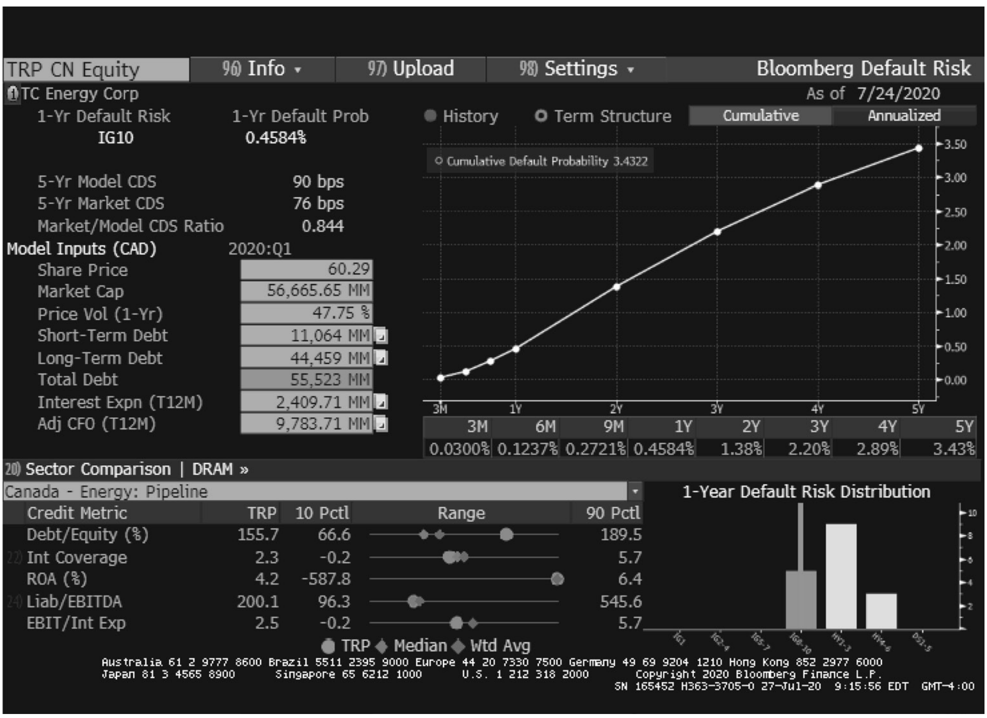
Financial conditions are another critical factor affecting the credit spread term structure. From a macroeconomic perspective, the credit risk of a bond is influenced by expectations for economic growth and inflation. A stronger economic climate is generally associated with higher benchmark yields but lower credit spreads for issuers whose default probability declines during periods of economic growth (cash flows tend to improve and profitability increases under such a scenario). The countercyclical relationship between spreads and benchmark rates is therefore commonly observed across the business cycle.

Market supply and demand dynamics are another critical factor influencing the credit curve term structure. Unlike default-risk-free government securities in developed markets, the relative liquidity of corporate bonds varies widely, with the vast majority of securities not trading on a daily basis. Given that new and most recently issued securities tend to represent the largest proportion of trading volume and are responsible for much of the volatility in credit spreads, the credit curve will be most heavily influenced by the most frequently traded securities. For example, although one might expect the credit curve to steepen for a borrower refinancing near-term maturities with long-term debt, this effect may be partially offset by a tighter bid–offer spread for longer credit maturities. This flattening may also occur within a specific rating or if market participants anticipate significant supply in a particular tenor. Infrequently traded bonds trading with wider bid–offer spreads can also impact the shape of the term structure, so it is important to gauge the size and frequency of trades in bonds across the maturity spectrum to ensure consistency.

Finally, from a microeconomic perspective, company-value model results discussed earlier are another key driver of the credit spread term structure. Under traditional credit analysis, the specific industry or industries within which an issuer operates are considered, as well as key financial ratios, such as cash flow, leverage, and profitability versus sector and ratings peers. This company-specific analysis based on fundamental data has been complemented by more probabilistic, forward-looking structural models for company valuation. These models take stock market valuation, equity volatility, and balance sheet information into account to derive the implied default probability for a company. Holding other factors constant, any micro-economic factor that increases the implied default probability, such as greater equity volatility, will tend to drive a steeper credit spread curve, and the reverse is true with a decline in equity volatility.

Practitioners will frequently employ these tools when analyzing the term structure of credit spreads to determine fair value. For example, the Bloomberg default risk screen (DRSK) shown in Exhibit 29 combines the company-value analysis with fundamental credit ratios for a composite analysis of TransCanada Corporation, a Canadian natural gas transmission and power services company.

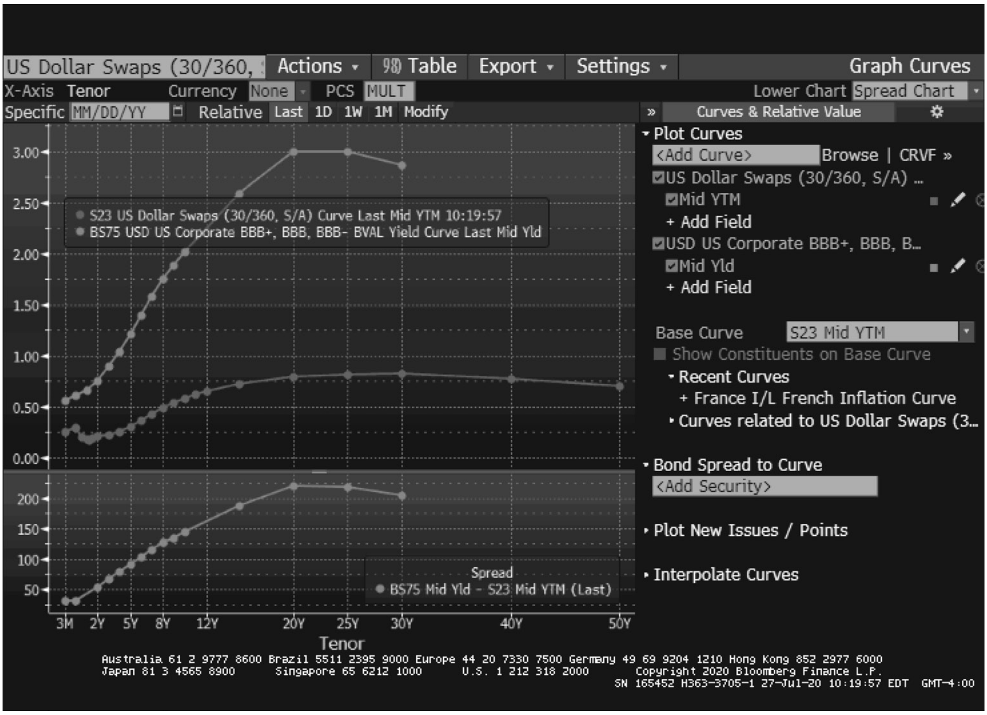
EXHIBIT 29 Default Risk Screen



Source: Bloomberg.

Two further considerations are important when analyzing the term structure of credit spreads. The first concerns the appropriate risk-free or benchmark rates used to determine spreads. A frequently traded government security with the nearest maturity to an outstanding corporate bond generally represents the lowest default risk for developed markets, so this is a logical benchmark choice. However, the duration and maturity of the most liquid or on-the-run government bonds rarely match those of corporate bonds trading in the secondary market, so it is often necessary to interpolate between yields of the two government securities with the closest maturity. Because the interpolation may impact the analysis for less liquid maturities, the benchmark swap curve based on interbank rates is often substituted for the government benchmark because of greater swap market liquidity for off-the-run maturities. For example, Exhibit 30 demonstrates the latter methodology on a Bloomberg screen for a composite of BBB rated US industrial corporate issuers versus the benchmark US dollar swap curve, showing a positive-sloped credit spread term structure across maturities.

EXHIBIT 30 Credit Spreads over Swap Rates



Source: Bloomberg.

The second consideration concerns the all-in spread over the benchmark itself. Term structure analysis should include only bonds with similar credit characteristics, which are typically senior unsecured general obligations of the issuer. Any bonds of the issuer with embedded options, first or second lien provisions, or other unique provisions should be excluded from the analysis. It is also important to note that such securities typically include cross-default provisions so that all securities across the maturity spectrum of a single issuer will be subject to recovery in the event of bankruptcy.

Using the models presented in prior sections, we can demonstrate that the *change* in market expectations of default over time is a key determinant of the shape of the credit curve term structure. This may be shown using a simple extension of the zero-coupon corporate bond example in Exhibit 2 by changing the probability of default. Using a recovery rate of 40% and changing the probability of default from 1.25% to 1.50% raises the credit spread from 77 bps in the original example to 92 bps. These calculations are shown in Exhibit 31.

EXHIBIT 31 Raising the Default Probability of the Five-Year, Zero-Coupon Corporate Bond

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	88.8487	35.5395	53.3092	1.5000%	98.5000%	0.7996	0.970874	0.7763
2	91.5142	36.6057	54.9085	1.4775%	97.0225%	0.8113	0.942596	0.7647
3	94.2596	37.7038	56.5558	1.4553%	95.5672%	0.8231	0.915142	0.7532
4	97.0874	38.8350	58.2524	1.4335%	94.1337%	0.8351	0.888487	0.7419
5	100.0000	40.0000	60.0000	1.4120%	92.7217%	0.8472	0.862609	0.7308
				7.2783%			CVA =	3.7670

Fair value = 86.2609 – 3.7670 = 82.4939.
Yield to maturity = 3.9240%.
Credit spread = 3.9240% – 3.00% = 0.9240%.

Flat credit spread curves imply a relatively stable expectation of default over time, whereas an upward-sloping credit curve implies that investors seek greater compensation for assuming issuer default risk over longer periods. For example, we can illustrate this in terms of a credit spread curve by holding the benchmark rate constant at 3.00% across 3-year, 5-year, and 10-year maturities while increasing the default probability over time. Although one could consider an increase in default probability each year, the following example in Exhibit 32 assumes a 1.00% default probability for Years 1, 2, and 3, a 2.00% probability of default in Years 4 and 5, and a 3.00% default probability in Years 6 through 10, with the recovery rate at a constant 40%. (Note that this is another example of the annual default probability changing over the lifetime of the bonds.) As shown in Exhibit 32, the credit spread rises from 62 bps to 86 bps to 132 bps.

EXHIBIT 32 Increasing the Default Probability for Longer Times to Maturity

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	94.2596	37.7038	56.5558	1.0000%	99.0000%	0.5656	0.970874	0.5491
2	97.0874	38.8350	58.2524	0.9900%	98.0100%	0.5767	0.942596	0.5436
3	100.0000	40.0000	60.0000	0.9801%	97.0299%	0.5881	0.915142	0.5382
				2.9701%			CVA =	1.6308

Fair value = 91.5142 – 1.6308 = 89.8833.
Yield to maturity = 3.6192%.
Credit spread = 3.6192% – 3.00% = 0.6192%.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	88.8487	35.5395	53.3092	1.0000%	99.0000%	0.5331	0.970874	0.5176
2	91.5142	36.6057	54.9085	0.9900%	98.0100%	0.5436	0.942596	0.5124
3	94.2596	37.7038	56.5558	0.9801%	97.0299%	0.5543	0.915142	0.5073

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
4	97.0874	38.8350	58.2524	1.9406%	95.0893%	1.1304	0.888487	1.0044
5	100.0000	40.0000	60.0000	1.9018%	93.1875%	1.1411	0.862609	0.9843
				6.8125%			CVA =	3.5259

Fair value = 86.2609 − 3.5259 = 82.7350.
Yield to maturity = 3.8633%.
Credit spread = 3.8633% − 3.00% = 0.8633%.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	76.647	30.6567	45.9850	1.0000%	99.0000%	0.4599	0.970874	0.4465
2	78.9409	31.5764	47.3646	0.9900%	98.0100%	0.4689	0.942596	0.4420
3	81.3092	32.5237	48.7855	0.9801%	97.0299%	0.4781	0.915142	0.4376
4	83.7484	33.4994	50.2491	1.9406%	95.0893%	0.9751	0.888487	0.8664
5	86.2609	34.5044	51.7565	1.9018%	93.1875%	0.9843	0.862609	0.8491
6	88.8487	35.5395	53.3092	2.7956%	90.3919%	1.4903	0.837484	1.2481
7	91.5142	36.6057	54.9085	2.7118%	87.6801%	1.4890	0.813092	1.2107
8	94.2596	37.7038	56.5558	2.6304%	85.0497%	1.4876	0.789409	1.1744
9	97.0874	38.8350	58.2524	2.5515%	82.4982%	1.4863	0.766417	1.1391
10	100.0000	40.0000	60.0000	2.4749%	80.0233%	1.4850	0.744094	1.1050
				19.9767%			CVA =	8.9187

Fair value = 74.4094 − 8.9187 = 65.4907.
Yield to maturity = 4.3235%.
Credit spread = 4.3235% − 3.00% = 1.3235%.

Positive-sloped credit spread curves are likely when a high-quality issuer with a strong competitive position in a stable industry has low leverage, strong cash flow, and a high profit margin. This type of issuer tends to exhibit very low short-term credit spreads rising with increasing maturity given greater uncertainty due to the macroeconomic environment, potential adverse changes in the competitive landscape, technological change, or other factors that drive a higher implied probability of default over time. Empirical academic studies also tend to support the view that the credit spread term structure is upward-sloping for investment-grade bond portfolios (Bedendo, Cathcart, and El-Jahel 2007).

Alternatively, high-yield issuers in cyclical industries sometimes face a downward-sloping credit term structure because of issuer- or industry-specific reasons. For example, an ownership change resulting from a leveraged buyout or private equity acquisition may often be accompanied by a significant increase in leverage. In such a case, an inverted credit curve may indicate investor expectations that the new owners will create efficiencies in the restructured organization, leading to improved future cash flow and profitability that will benefit debt investors. Another example of an inverted credit term structure might result when issuers in a historically cyclical industry (such as oil and gas exploration or retail) find themselves at the bottom of an economic cycle, with investor expectations of a recovery in the industry tied to improving credit spreads over time.

That said, it is important to distinguish between scenarios where the contractual cash flows of a risky bond are likely to occur and distressed debt scenarios where investors expect to receive only the recovery rate in a likely bankruptcy scenario. Bonds with a very high likelihood of default tend to trade on a price basis that converges toward the recovery rate rather than on a spread to benchmark rates. This scenario leads to credit spread term structures that may be considered more of an "optical" phenomenon rather than a true reflection of the relative risks and rewards of long-term versus short-term bonds from a single issuer, as illustrated in the following discussion.

To demonstrate this using our zero-coupon bond example, let us shift to a scenario where bondholders with 5-year and 10-year bonds outstanding anticipate an imminent default scenario and both bonds trade at a recovery rate of 40%.

Note that if we solve for the fair value and resulting credit spread over the benchmark yield as in the instances where default probability was 1.25%, we end up with the same VNDs for the 5-year and 10-year bonds, respectively. However, when deriving a credit spread value for both securities assuming recovery in a bankruptcy scenario and cross-default provisions across maturities, the credit valuation adjustment representing the sum of expected losses is simply the difference between the VND and the recovery rate.

For the five-year example, we can thus calculate a VND of 86.2609, a CVA of 46.2609, and a fair value with recovery at 40. This results in a yield of 20.1124% and a credit spread over the government bond of 17.1124%. In the 10-year case, the VND may be shown as 74.4094, a CVA of 34.4094, and a fair value at 40. That gives a yield of 9.5958% and a credit spread of 6.5958%. We end up with a steep and inverted "credit spread" curve.

The interpretation of the credit spread term structure is important for investors seeking to capitalize on a market view that differs from that reflected in the credit curve. For example, if a portfolio manager disagrees with the market's expectation of a high near-term default probability that declines over time, she could sell short-term protection in the credit default swap market and buy longer-term protection. In a scenario where the issuer does not default, the investor retains the premium on protection sold and may either retain or choose to sell back the longer-term credit default swap to realize a gain.

8. CREDIT ANALYSIS FOR SECURITIZED DEBT

- **compare the credit analysis required for securitized debt to the credit analysis of corporate debt**

Unlike the general obligation nature of most private or sovereign fixed-income securities, securitized debt allows issuers to finance a specific set of assets or receivables (e.g., mortgages, automobile loans, or credit card receivables) rather than an entire balance sheet. Issuers in securitized debt markets are frequently motivated to undertake financing using these more structured securities given their ability to increase debt capacity and reduce the originator's need to maintain regulatory capital or retain residual risk. The isolation of securitized assets generally decreases the relative financing cost for these assets on a stand-alone basis as compared to a general obligation financing of the debt originator. By freeing up capital, an originator is also able to continue to generate income from further originations. Investors, however, seek to benefit from greater diversification, more stable and predictable underlying cash flows, and a return that is greater than that of securities with similar ratings, which provide a reward for accepting the greater complexity associated with collateralized debt. That said, the credit analysis of such structured finance instruments requires a fundamentally different approach compared with other risky bonds given the underlying collateral, the parties associated with the origination or servicing of the portfolio over the life of the security, and the issuing entity, as well as any structural and credit enhancement features typically present in these transactions.

It is important to distinguish first and foremost among the types of securitized debt issued globally, as well as the various forms. In its summary of structured finance asset types shown in Exhibit 33, the German-based rating agency Scope Ratings AG provides its general approach to credit assessment based not only on the underlying time horizon and collateral but also on asset characteristics referred to as granularity and homogeneity.

EXHIBIT 33 Summary of Asset Types and Characteristics of Core Structured Finance Asset Classes

Deal Type	Underlying Collateral	Risk Horizon	Granularity	Homogeneity	Credit Analysis Approach
Asset-backed CP	Commercial discount credits or credit advances	Short-term	Granular	Homogeneous	Book
Auto ABS	Auto loans or leases	Medium-term	Granular	Homogeneous	Portfolio
CMBS	Commercial mortgages	Typically long-term	Non-granular	Heterogeneous	Loan by loan
Consumer ABS	Consumer loans	Medium-term	Granular	Homogeneous	Portfolio
CRE loans	Commercial real estate loans	Long-term	Non-granular	Heterogeneous	Loan by loan
Credit cards	Credit card balances	Short-term	Granular	Homogeneous	Book
Credit-linked notes/ repackaging	Any financial assets	Typically medium-term	Typically single asset	NA	Pass-through rating/asset by asset
LL CLOs	Leveraged corporate loans	Medium-term	Non-granular	Heterogeneous	Loan by loan
PF CLOs	Project finance debt	Long-term	Non-granular	Heterogeneous	Loan by loan
RMBS	Residential mortgages	Long-term	Granular	Homogeneous	Loan by loan or portfolio
SME ABS	Loans to small- and medium-sized businesses	Typically medium-term	Granular	Mixed	Loan by loan or portfolio
Trade receivables	Commercial credit	Short-term	Typically granular	Homogeneous	Book

Source: Adapted from Scope Ratings AG (2016b, pp. 7–8).

The concept of homogeneity refers to the degree to which underlying debt characteristics within a structured finance instrument are similar across individual obligations. On the one hand, an investor or credit analyst might draw general conclusions about the nature of homogeneous credit card or auto loan obligations given that an individual obligation faces strict eligibility criteria to be included in a specific asset pool. On the other hand, heterogeneous leveraged loan, project finance, or real estate transactions require scrutiny on a loan-by-loan basis given their different characteristics. The granularity of the portfolio refers to the actual number of obligations that make up the overall structured finance instrument. A highly granular portfolio may have hundreds of underlying debtors, suggesting it is appropriate to draw conclusions about creditworthiness based on portfolio summary statistics rather than investigating each borrower. Alternatively, an asset pool with fewer more-discrete or non-granular investments would warrant analysis of each individual obligation.

The combination of asset type and tenor as well as the relative granularity and homogeneity of the underlying obligations drive the approach to credit analysis for a given instrument type. For example, short-term structured finance vehicles with granular, homogeneous assets tend to be evaluated using a statistics-based approach to the existing book of loans. This changes to a portfolio-based approach for medium-term granular and homogeneous obligations because the portfolio is not static but changes over time. For discrete or non-granular heterogeneous portfolios, a loan-by-loan approach to credit analysis is more appropriate. The following example of a credit card securitization will provide further insight into the process.

Exhibit 34 provides a summary from the prospectus of the Synchrony Credit Card Master Note Trust $750,000,000 Series 2016-1 Asset Backed Notes issued in March 2016. As is spelled out in the prospectus, the Synchrony transaction is backed by credit card receivables having the given credit score distribution presented in the exhibit.

EXHIBIT 34 A Structured Debt Example, Composition by FICO Credit Score Range

FICO Credit Score Range	Receivables Outstanding	Percentage of Outstanding
Less than or equal to 599	$995,522,016	6.6%
600 to 659	$2,825,520,245	18.7%
660 to 719	$6,037,695,923	39.9%
720 and above	$5,193,614,599	34.4%
No score	$64,390,707	0.4%
Total	**$15,116,743,490**	**100%**

Source: Synchrony Credit Card Master Note Trust $750,000,000 Series 2016-1 Asset Backed Notes Prospectus (p. 93; available at investors.synchronyfinancial.com).

Investors in this type of ABS will base their probability of default on the mean default probability, recovery rate, and variance of a portfolio of borrowers reflecting the distribution of FICO scores within the pool rather than conducting an analysis of individual borrowers. The prospectus provides a broad set of details beyond the FICO scores of borrowers for further in-depth portfolio analysis, including age of the receivables, average outstanding balances, and delinquency rates.

A heterogeneous portfolio of fewer loans, however, requires a fundamentally different approach. In this instance, each obligation within the asset pool may warrant its own analysis to determine whether an individual commercial property or leveraged company is able to meet its financial obligations under the ABS contract. Here the expected default probability and recovery rate on an asset-by-asset basis is the best gauge of how the investment will perform under various scenarios.

A second critical aspect of the credit exposure associated with ABS relates to the origination and servicing of assets over the life of the transaction. The prospectus and other related documents determine the roles and responsibilities of these related parties over the life of an ABS transaction. Upon inception of the transaction, investors rely on the originator/servicer to establish and enforce loan eligibility criteria, secure and maintain proper documentation and records, and maximize timely repayment and contract enforceability in cases of delinquency. Once the asset pool has been identified, investors are also exposed to operational and counterparty risk over the life of an ABS transaction. That is, they remain exposed to the ability of the servicer to effectively manage and service the portfolio over the life of the transaction. For an auto ABS transaction, this may involve the ability to repossess and sell a vehicle at a price close to the residual value in a timely manner in the event that a borrower is unable to pay, while in a commercial real estate transaction, it may involve identifying and replacing a non-performing tenant. Investors in an asset portfolio whose composition changes over time also face exposure to the replacement of obligors over time. In all such instances, not only is the creditworthiness of the servicer important but also of importance is its track record in meeting these servicing obligations, which are frequently gauged by analyzing the performance of more seasoned transactions handled by the same servicer over the credit cycle.

For example, in the case of the Synchrony Credit Card Master Note Trust transaction, Synchrony Financial acts as servicer of the trust and Synchrony Bank, as sub-servicer, is primarily responsible for receiving and processing collections on the receivables. A potential investor might therefore evaluate not only the performance of other debt backed by credit card receivables but also how outstanding notes serviced by Synchrony have performed over time versus its servicing competitors.

Finally, the structure of a collateralized or secured debt transaction is a critical factor in analyzing this type of investment. These structural aspects include both the nature of the obligor itself, which is often a special purpose entity (SPE) whose sole purpose is to acquire a specified pool of assets and issue ABS to finance the SPE, and any structural enhancements of the transaction, which may include overcollateralization, credit tranching (i.e., tiering the claim priorities of ownership or interest), or other characteristics.

A key question related to the issuer is its relationship to the originator—namely, the degree to which the bankruptcy of the obligor is related to that of the originator. The bankruptcy remoteness is typically determined by whether the transfer of the assets from the originator to the SPE may be deemed a true sale, which otherwise allows for the ability to separate risk between the originator and SPE at a later date.

Second, additional credit enhancements are a key structural element to be evaluated in the context of credit risk. Credit enhancements for ABS take on several forms beyond the bankruptcy remoteness of the SPE. For example, ABS transactions frequently have payout or performance triggers that protect investors in the case of adverse credit events. Certain events related to the servicer or seller—such as failure to make deposits or payments or other adverse events—may trigger early repayment ("amortization") of the security. For consumer transactions such as credit card or automotive ABS, the primary protection against a decline in asset quality for investors is additional return built into the transaction that is greater than the expected or historical loss of the asset pool. This additional return is often called the excess spread. Issuers create subordinated tranches of debt that provide added protection to those rated higher and benefit from a greater excess spread cushion over the life of the financing.

Covered bonds, which originated in Germany in the 18th century but have since been adopted by issuers across Europe, Asia, and Australia, have some similarities with these structured finance investments but also have fundamental differences that warrant special consideration. A covered bond is a senior debt obligation of a financial institution that gives recourse to both the originator/issuer and a predetermined underlying collateral pool. Each country or jurisdiction specifies the eligible collateral types and the specific structures permissible in its covered bond market. Covered bonds most frequently have either commercial or residential mortgages meeting specific criteria or public sector debt as underlying collateral.

The dual recourse to the issuing financial institution and the underlying asset pool has been a hallmark of covered bonds since their inception, but it was also reinforced under the European Union Bank Recovery and Resolution Directive (BRRD; see Scope Ratings AG

2016a). Under the BRRD, covered bonds enjoy unique protection among bank liabilities in the event of restructuring or regulatory intervention. Additionally, the financial institution has the ongoing obligation to maintain sufficient assets in the cover pool to satisfy the claims of covered bondholders at all times, and the obligations of the financial institution with respect to the cover pool are supervised by public or other independent bodies.

Another aspect of covered bonds that needs to be considered in credit analysis is the dynamic nature of the cover pool. In contrast to a static pool of mortgage loans (which expose investors to prepayment risk in the case of US mortgage-backed securities), cover pool sponsors must replace any prepaid or non-performing assets in the cover pool to ensure sufficient cash flows to the maturity of the covered bond.

Analysts should also be aware of various redemption regimes that exist to align the covered bond's cash flows as closely as possible to the original maturity schedule in the event of default of a covered bond's financial sponsor. These include hard-bullet covered bonds; if payments do not occur according to the original schedule, a bond default is triggered and bond payments are accelerated. Another type is soft-bullet covered bonds, which delay the bond default and payment acceleration of bond cash flows until a new final maturity date, which is usually up to a year after the original maturity date. Conditional pass-through covered bonds, in contrast, convert to pass-through securities after the original maturity date if all bond payments have not yet been made.

Credit analysis for covered bonds follows traditional credit analysis in evaluating both the issuer and the cover pool. Given the additional credit enhancements, recovery rates tend to be high and default probabilities low, making covered bonds a relatively safe credit asset. As a result, rating agencies often assign a credit rating to covered bonds that is several notches above that of the issuing financial institution.

SUMMARY

We have covered several important topics in credit analysis. Among the points made are the following:

- Three factors important to modeling credit risk are the expected exposure to default, the recovery rate, and the loss given default.
- These factors permit the calculation of a credit valuation adjustment that is subtracted from the (hypothetical) value of the bond, if it were default risk free, to get the bond's fair value given its credit risk. The credit valuation adjustment is calculated as the sum of the present values of the expected loss for each period in the remaining life of the bond. Expected values are computed using risk-neutral probabilities, and discounting is done at the risk-free rates for the relevant maturities.
- The CVA captures investors' compensation for bearing default risk. The compensation can also be expressed in terms of a credit spread.
- Credit scores and credit ratings are third-party evaluations of creditworthiness used in distinct markets.
- Analysts may use credit ratings and a transition matrix of probabilities to adjust a bond's yield to maturity to reflect the probabilities of credit migration. Credit spread migration typically reduces expected return.

- Credit analysis models fall into two broad categories: structural models and reduced-form models.
- Structural models are based on an option perspective of the positions of the stakeholders of the company. Bondholders are viewed as owning the assets of the company; shareholders have call options on those assets.
- Reduced-form models seek to predict *when* a default may occur, but they do not explain the *why* as structural models do. Reduced-form models, unlike structural models, are based only on observable variables.
- When interest rates are assumed to be volatile, the credit risk of a bond can be estimated in an arbitrage-free valuation framework.
- The discount margin for floating-rate notes is similar to the credit spread for fixed-coupon bonds. The discount margin can also be calculated using an arbitrage-free valuation framework.
- Arbitrage-free valuation can be applied to judge the sensitivity of the credit spread to changes in credit risk parameters.
- The term structure of credit spreads depends on macro and micro factors.
- As it concerns macro factors, the credit spread curve tends to become steeper and to widen in conditions of weak economic activity. Market supply and demand dynamics are important. The most frequently traded securities tend to determine the shape of this curve.
- Issuer- or industry-specific factors, such as the chance of a future leverage-decreasing event, can cause the credit spread curve to flatten or invert.
- When a bond is very likely to default, it often trades close to its recovery value at various maturities; moreover, the credit spread curve is less informative about the relationship between credit risk and maturity.
- For securitized debt, the characteristics of the asset portfolio themselves suggest the best approach for a credit analyst to take when deciding among investments. Important considerations include the relative concentration of assets and their similarity or heterogeneity as it concerns credit risk.

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PRACTICE PROBLEMS

The following information relates to Questions 1–15

Daniela Ibarra is a senior analyst in the fixed-income department of a large wealth management firm. Marten Koning is a junior analyst in the same department, and David Lok is a member of the credit research team.

The firm invests in a variety of bonds. Ibarra is presently analyzing a set of bonds with some similar characteristics, such as four years until maturity and a par value of €1,000. Exhibit 1 includes details of these bonds.

EXHIBIT 1 A Brief Description of the Bonds Being Analyzed

Bond	Description
B1	A zero-coupon, four-year corporate bond with a par value of €1,000. The wealth management firm's research team has estimated that the risk-neutral probability of default for each date for the bond is 1.50%, and the recovery rate is 30%.
B2	A bond similar to B1, except that it has a fixed annual coupon rate of 6% paid annually.
B3	A bond similar to B2 but rated AA.
B4	A bond similar to B2 but the coupon rate is the one-year benchmark rate plus 4%.

Ibarra asks Koning to assist her with analyzing the bonds. She wants him to perform the analysis with the assumptions that there is no interest rate volatility and that the government bond yield curve is flat at 3%.

Ibarra performs the analysis assuming an upward-sloping yield curve and volatile interest rates. Exhibit 2 provides the data on annual payment benchmark government bonds.

She uses these data to construct a binomial interest rate tree based on an assumption of future interest rate volatility of 20%.

EXHIBIT 2 Par Curve for Annual Payment Benchmark Government Bonds

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	–0.25%	€100	1.002506	–0.2500%	
2	0.75%	€100	0.985093	0.7538%	1.7677%
3	1.50%	€100	0.955848	1.5166%	3.0596%
4	2.25%	€100	0.913225	2.2953%	4.6674%

Answer the first five questions (1–5) based on the assumptions made by Marten Koning, the junior analyst. Answer Questions 8–12 based on the assumptions made by Daniela Ibarra, the senior analyst.

Note: All calculations in this problem set are carried out on spreadsheets to preserve precision. The rounded results are reported in the solutions.

1. The market price of Bond B1 is €875. The bond is:
   A. fairly valued.
   B. overvalued.
   C. undervalued.
2. Koning realizes that an increase in the recovery rate would lead to an increase in the bond's fair value, whereas an increase in the probability of default would lead to a decrease in the bond's fair value. He is not sure, however, which effect would be greater. So, he increases both the recovery rate and the probability of default by 25% of their existing estimates and recomputes the bond's fair value. The recomputed fair value is closest to:
   A. €843.14.
   B. €848.00.
   C. €855.91.
3. The fair value of Bond B2 is closest to:
   A. €1,069.34.
   B. €1,111.51.
   C. €1,153.68.
4. The market price of Bond B2 is €1,090. If the bond is purchased at this price and there is a default on Date 3, the rate of return to the bond buyer would be closest to:
   A. −28.38%.
   B. −41.72%.
   C. −69.49%.
5. Bond B3 will have a modified duration of 2.75 at the end of the year. Based on the representative one-year corporate transition matrix in Exhibit 3 and assuming no default, how should the analyst adjust the bond's yield to maturity to assess the expected return on the bond over the next year?

EXHIBIT 3 Representative One-Year Corporate Transition Matrix (entries are in %)

From/To	AAA	AA	A	BBB	BB	B	CCC, CC, C	D
AAA	**90.00**	9.00	0.60	0.15	0.10	0.10	0.05	0.00
AA	1.50	**88.00**	9.50	0.75	0.15	0.05	0.03	0.02
A	0.05	2.50	**87.50**	8.40	0.75	0.60	0.12	0.08
BBB	0.02	0.30	4.80	**85.50**	6.95	1.75	0.45	0.23
BB	0.01	0.06	0.30	7.75	**79.50**	8.75	2.38	1.25
B	0.00	0.05	0.15	1.40	9.15	**76.60**	8.45	4.20
CCC, CC, C	0.00	0.01	0.12	0.87	1.65	18.50	**49.25**	29.60
Credit Spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%	

   A. Add 7.7 bps to YTM.
   B. Subtract 7.7 bps from YTM.
   C. Subtract 9.0 bps from YTM.

6. David Lok has estimated the probability of default of Bond B1 to be 1.50%. He is presenting the approach the research team used to estimate the probability of default. Which of the following statements is Lok likely to make in his presentation if the team used a reduced-form credit model?
   A. Option pricing methodologies were used, with the volatility of the underlying asset estimated based on historical data on the firm's stock price.
   B. Regression analysis was used, with the independent variables including both firm-specific variables, such as the debt ratio and return on assets, and macroeconomic variables, such as the rate of inflation and the unemployment rate.
   C. The default barrier was first estimated, followed by the estimation of the probability of default as the portion of the probability distribution that lies below the default barrier.
7. In the presentation, Lok is asked why the research team chose to use a reduced-form credit model instead of a structural model. Which statement is he likely to make in reply?
   A. Structural models are outdated, having been developed in the 1970s; reduced-form models are more modern, having been developed in the 1990s.
   B. Structural models are overly complex because they require the use of option pricing models, whereas reduced-form models use regression analysis.
   C. Structural models require "inside" information known to company management, whereas reduced-form models can use publicly available data on the firm.
8. As previously mentioned, Ibarra is considering a future interest rate volatility of 20% and an upward-sloping yield curve, as shown in Exhibit 2. Based on her analysis, the fair value of Bond B2 is closest to:
   A. €1,101.24.
   B. €1,141.76.
   C. €1,144.63.
9. Ibarra wants to know the credit spread of Bond B2 over a theoretical comparable-maturity government bond with the same coupon rate as this bond. The foregoing credit spread is closest to:
   A. 108 bps.
   B. 101 bps.
   C. 225 bps.
10. Ibarra is interested in analyzing how a simultaneous decrease in the recovery rate and the probability of default would affect the fair value of Bond B2. She decreases both the recovery rate and the probability of default by 25% of their existing estimates and recomputes the bond's fair value. The recomputed fair value is closest to:
    A. €1,096.59.
    B. €1,108.40.
    C. €1,111.91.
11. The wealth management firm has an existing position in Bond B4. The market price of B4, a floating-rate note, is €1,070. Senior management has asked Ibarra to make a recommendation regarding the existing position. Based on the assumptions used to calculate the estimated fair value only, her recommendation should be to:
    A. add to the existing position.
    B. hold the existing position.
    C. reduce the existing position.
12. The issuer of the floating-rate note, B4, is in the energy industry. Ibarra believes that oil prices are likely to increase significantly in the next year, which will lead to an improvement

in the firm's financial health and a decline in the probability of default from 1.50% in Year 1 to 0.50% in Years 2, 3, and 4. Based on these expectations, which of the following statements is correct?

A. The CVA will decrease to €22.99.
B. The note's fair value will increase to €1,177.26.
C. The value of the FRN, assuming no default, will increase to €1,173.55.

13. The floating-rate note, B4, is currently rated BBB by Standard & Poor's and Fitch Ratings (and Baa by Moody's Investors Service). Based on the research department assumption about the probability of default in Question 10 and her own assumption in Question 11, which action does Ibarra *most likely* expect from the credit rating agencies?
    A. Downgrade from BBB to BB.
    B. Upgrade from BBB to AAA.
    C. Place the issuer on watch with a positive outlook.
14. During the presentation about how the research team estimates the probability of default for a particular bond issuer, Lok is asked for his thoughts on the shape of the term structure of credit spreads. Which statement is he most likely to include in his response?
    A. The term structure of credit spreads typically is flat or slightly upward sloping for high-quality investment-grade bonds. High-yield bonds are more sensitive to the credit cycle, however, and can have a more upwardly sloped term structure of credit spreads than investment-grade bonds or even an inverted curve.
    B. The term structure of credit spreads for corporate bonds is always upward sloping—more so the weaker the credit quality because probabilities of default are positively correlated with the time to maturity.
    C. There is no consistent pattern for the term structure of credit spreads. The shape of the credit term structure depends entirely on industry factors.
15. The final question for Lok is about covered bonds. The person asking says, "I've heard about them but don't know what they are." Which statement is Lok most likely to make to describe a covered bond?
    A. A covered bond is issued in a non-domestic currency. The currency risk is then fully hedged using a currency swap or a package of foreign exchange forward contracts.
    B. A covered bond is issued with an attached credit default swap. It essentially is a "risk-free" government bond.
    C. A covered bond is a senior debt obligation giving recourse to the issuer as well as a predetermined underlying collateral pool, often commercial or residential mortgages.

The following information relates to Questions 16–22

Anna Lebedeva is a fixed-income portfolio manager. Paulina Kowalski, a junior analyst, and Lebedeva meet to review several positions in Lebedeva's portfolio.

Lebedeva begins the meeting by discussing credit rating migration. Kowalski asks Lebedeva about the typical impact of credit rating migration on the expected return on a bond. Lebedeva asks Kowalski to estimate the expected return over the next year on a bond issued by Entre Corp. The BBB rated bond has a yield to maturity of 5.50% and a modified duration of 7.54. Kowalski calculates the expected return on the bond over the next year given the partial credit transition and credit spread data in Exhibit 1. She assumes that market spreads and yields will remain stable over the year.

EXHIBIT 1 One-Year Transition Matrix for BBB Rated Bonds and Credit Spreads

	AAA	AA	A	BBB	BB	B	CCC, CC, C
Probability (%)	0.02	0.30	4.80	85.73	6.95	1.75	0.45
Credit spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%

Lebedeva next asks Kowalski to analyze a three-year bond, issued by VraiRive S.A., using an arbitrage-free framework. The bond's coupon rate is 5%, with interest paid annually and a par value of 100. In her analysis, she makes the following three assumptions:

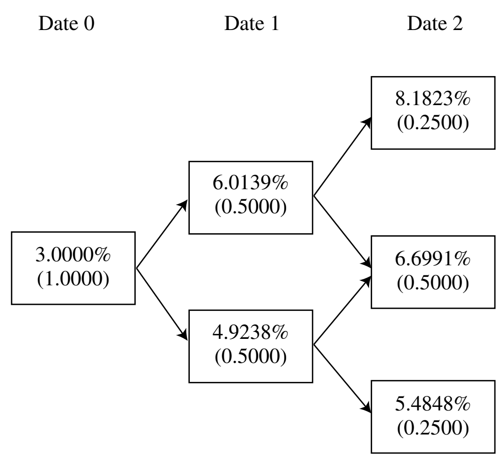
- The annual interest rate volatility is 10%.
- The recovery rate is one-third of the exposure each period.
- The annual probability of default each year is 2.00%.

Selected information on benchmark government bonds for the VraiRive bond is presented in Exhibit 2, and the relevant binomial interest rate tree is presented in Exhibit 3.

EXHIBIT 2 Par Curve Rates for Annual Payment Benchmark Government Bonds

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	3.00%	100	0.970874	3.0000%	3.0000%
2	4.20%	100	0.920560	4.2255%	5.4656%
3	5.00%	100	0.862314	5.0618%	6.7547%

EXHIBIT 3 One-Year Binomial Interest Rate Tree for 10% Volatility (risk-neutral probabilities in parentheses)



Kowalski estimates the value of the VraiRive bond assuming no default (VND) as well as the fair value of the bond. She then estimates the bond's yield to maturity and the bond's credit spread over the benchmark in Exhibit 2. Kowalski asks Lebedeva, "What might cause the bond's credit spread to decrease?"

Lebedeva and Kowalski next discuss the drivers of the term structure of credit spreads. Kowalski tells Lebedeva the following:

Statement 1: The credit term structure for the most highly rated securities tends to be either flat or slightly upward sloping.

Statement 2: The credit term structure for lower-rated securities is often steeper, and credit spreads widen with expectations of strong economic growth.

Next, Kowalski analyzes the outstanding bonds of DLL Corporation, a high-quality issuer with a strong, competitive position. Her focus is to determine the rationale for a positive-sloped credit spread term structure.

Lebedeva ends the meeting by asking Kowalski to recommend a credit analysis approach for a securitized asset-backed security (ABS) held in the portfolio. This non-static asset pool is made up of many medium-term auto loans that are homogeneous, and each loan is small relative to the total value of the pool.

16. The *most appropriate* response to Kowalski's question regarding credit rating migration is that it has:
    A. a negative impact.
    B. no impact.
    C. a positive impact.
17. Based on Exhibit 1, the one-year expected return on the Entre Corp. bond is *closest* to:
    A. 3.73%.
    B. 5.50%.
    C. 7.27%.
18. Based on Kowalski's assumptions and Exhibits 2 and 3, the credit spread on the VraiRive bond is *closest* to:
    A. 0.6949%.
    B. 0.9388%.
    C. 1.4082%.
19. The *most appropriate* response to Kowalski's question relating to the credit spread is:
    A. an increase in the probability of default.
    B. an increase in the loss given default.
    C. a decrease in the risk-neutral probability of default.
20. Which of Kowalski's statements regarding the term structure of credit spreads is correct?
    A. Only Statement 1
    B. Only Statement 2
    C. Both Statement 1 and Statement 2
21. DLL's credit spread term structure is *most* consistent with the firm having:
    A. low leverage.
    B. weak cash flow.
    C. a low profit margin.
22. Given the description of the asset pool of the ABS, Kowalski should recommend a:
    A. loan-by-loan approach.
    B. portfolio-based approach.
    C. statistics-based approach.

The following information relates to Questions 23–30

Lena Liecken is a senior bond analyst at Taurus Investment Management. Kristel Kreming, a junior analyst, works for Liecken in helping conduct fixed-income research for the firm's portfolio managers. Liecken and Kreming meet to discuss several bond positions held in the firm's portfolios.

Bonds I and II both have a maturity of one year, an annual coupon rate of 5%, and a market price equal to par value. The risk-free rate is 3%. Historical default experiences of bonds comparable to Bonds I and II are presented in Exhibit 1.

EXHIBIT 1 Credit Risk Information for Comparable Bonds

Bond	Recovery Rate	Percentage of Bonds That Survive and Make Full Payment
I	40%	98%
II	35%	99%

Bond III is a zero-coupon bond with three years to maturity. Liecken evaluates similar bonds and estimates a recovery rate of 38% and a risk-neutral default probability of 2%, assuming conditional probabilities of default. Kreming creates Exhibit 2 to compute Bond III's credit valuation adjustment. She assumes a flat yield curve at 3%, with exposure, recovery, and loss given default values expressed per 100 of par value.

EXHIBIT 2 Analysis of Bond III

Date	Exposure	Recovery	Loss Given Default	Probability of Default	Probability of Survival	Expected Loss	Present Value of Expected Loss
0							
1	94.2596	35.8186	58.4410	2.0000%	98.0000%	1.1688	1.1348
2	97.0874	36.8932	60.1942	1.9600%	96.0400%	1.1798	1.1121
3	100.0000	38.0000	62.0000	1.9208%	94.1192%	1.1909	1.0898
Sum				5.8808%		3.5395	3.3367

Bond IV is an AA rated bond that matures in five years, has a coupon rate of 6%, and a modified duration of 4.2. Liecken is concerned about whether this bond will be downgraded to an A rating, but she does not expect the bond to default during the next year. Kreming constructs a partial transition matrix, which is presented in Exhibit 3, and suggests using a model to predict the rating change of Bond IV using leverage ratios, return on assets, and macroeconomic variables.

EXHIBIT 3 Partial One-Year Corporate Transition Matrix (entries in %)

From/To	AAA	AA	A
AAA	92.00	6.00	1.00
AA	2.00	89.00	8.00
A	0.05	1.00	85.00
Credit Spread (%)	0.50	1.00	1.75

Kreming calculates the risk-neutral probabilities, compares them with the actual default probabilities of bonds evaluated over the past 10 years, and observes that the actual and risk-neutral probabilities differ. She makes two observations regarding the comparison of these probabilities:

- Observation 1: Actual default probabilities include the default risk premium associated with the uncertainty in the timing of the possible default loss.
- Observation 2: The observed spread over the yield on a risk-free bond in practice includes liquidity and tax considerations, in addition to credit risk.

23. The expected exposure to default loss for Bond I is:
    A. less than the expected exposure for Bond II.
    B. the same as the expected exposure for Bond II.
    C. greater than the expected exposure for Bond II.
24. Based on Exhibit 1, the loss given default for Bond II is:
    A. less than that for Bond I.
    B. the same as that for Bond I.
    C. greater than that for Bond I.
25. Based on Exhibit 1, the expected future value of Bond I at maturity is *closest* to:
    A. 98.80.
    B. 103.74.
    C. 105.00.
26. Based on Exhibit 1, the risk-neutral default probability for Bond I is *closest* to:
    A. 2.000%.
    B. 3.175%.
    C. 4.762%.
27. Based on Exhibit 2, the credit valuation adjustment for Bond III is *closest* to:
    A. 3.3367.
    B. 3.5395.
    C. 5.8808.
28. Based on Exhibit 3, if Bond IV's credit rating changes during the next year to an A rating, its expected price change would be *closest* to:
    A. −8.00%.
    B. −7.35%.
    C. −3.15%.
29. Kreming's suggested model for Bond IV is a:
    A. structural model.
    B. reduced-form model.
    C. term structure model.
30. Which of Kreming's observations regarding actual and risk-neutral default probabilities is correct?
    A. Only Observation 1
    B. Only Observation 2
    C. Both Observation 1 and Observation 2

CHAPTER 11

CREDIT DEFAULT SWAPS

Brian Rose
Don M. Chance, PhD, CFA

LEARNING OUTCOMES

The candidate should be able to:

- describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product;
- describe credit events and settlement protocols with respect to CDS;
- explain the principles underlying and factors that influence the market's pricing of CDS;
- describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve;
- describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments.

1. INTRODUCTION

Derivative instruments in which the underlying is a measure of a borrower's credit quality are widely used and well established in a number of countries. We explore basic definitions of such instruments, explain the main concepts, cover elements of valuation and pricing, and discuss applications.

2. BASIC DEFINITIONS AND CONCEPTS

- **describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product**

A **credit derivative** is a derivative instrument in which the underlying is a measure of a borrower's credit quality. Four types of credit derivatives are (1) total return swaps,

(2) credit spread options, (3) credit-linked notes, and (4) credit default swaps, or CDS. CDS are the most liquid of the four and, as such, are the topic we focus on. In a CDS, one party makes payments to the other and receives in return the promise of compensation if a third party defaults.

In any derivative, the payoff is based on (derived from) the performance of an underlying instrument, rate, or asset that we call the "underlying." For a CDS, the underlying is the credit quality of a borrower. At its most fundamental level, a CDS provides compensation equal to expected recovery when a credit event occurs, but it also changes in value to reflect changes in the market's perception of a borrower's credit quality well in advance of default. The value of a CDS will rise and fall as opinions change about the likelihood and severity of a potential default. The actual event of default might never occur, but a decline in the price of a bond when investors perceive an increase in the likelihood of default is a mark-to-market loss to the bondholder. The most common credit events include bankruptcy, failure to pay, and restructuring. Another type of credit event which may be encountered in sovereign and municipal government bond markets is a moratorium or, more drastically, a repudiation of debt in which the governmental authority declares a moratorium on payments due under the terms of the obligation or challenges the validity of the entire debt obligation. (Other, less common credit events are also defined in the International Swaps and Derivatives Association's Credit Derivatives Definitions, but we will not consider them here.) Credit default swaps are designed to protect creditors against credit events such as these. The industry has expended great effort to provide clear guidance on what credit events are covered by a CDS contract. As with all efforts to write a perfect contract, however, no such device exists and disputes do occasionally arise. We will take a look at these issues later.

In addition to hedging credit risk, investors use CDS to

- leverage their portfolios,
- access maturity exposures not available in the cash market,
- access credit risk while limiting interest rate risk, and
- improve the liquidity of their portfolios given the illiquidity in the corporate bond market.

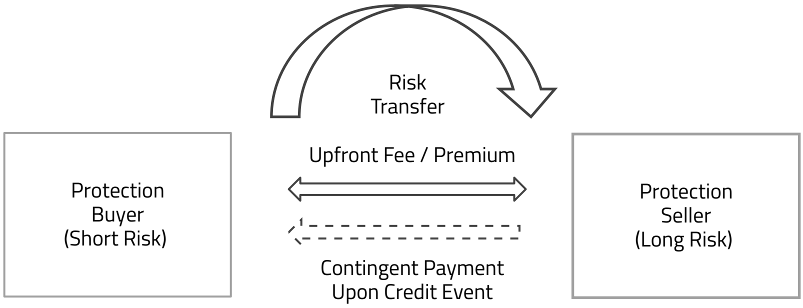
In addition, the CDS market has increased transparency and insight into the actual cost of credit risk. The higher relative liquidity and relative sophistication of CDS investors allow for more accurate price discovery and facilitate trading during liquidity events when the cash market for bonds becomes illiquid. While many of the applications listed above are beyond the scope of this reading, a basic understanding of this important fixed-income tool is necessary for all investment professionals.

Let's now define a **credit default swap**:

> *A credit default swap is a derivative contract between two parties, a credit protection buyer and credit protection seller, in which the buyer makes a series of cash payments to the seller and receives a promise of compensation for credit losses resulting from a credit event in an underlying.*

In a CDS contract there are two counterparties, the **credit protection buyer** and the **credit protection seller**. The buyer agrees to make a series of periodic payments to the seller over the life of the contract (which are determined and fixed at contract initiation) and receives in return a promise that if default occurs, the protection seller will compensate the protection buyer. If default occurs, the periodic payments made by the protection buyer to the protection seller terminate. Exhibit 1 shows the structure of payment flows.

EXHIBIT 1 Payment Structure of a CDS



Credit default swaps are somewhat similar to put options. Put options effectively enable the option holder to sell (put) the underlying security to the option seller if the underlying performs poorly relative to the exercise price. Similarly, in the event of a credit event on the underlying security, the buyer of credit protection receives a payment from the credit protection seller equal to the par or notional value of the security less the expected recovery value. If the credit quality of the underlying deteriorates but there is no outright credit event, the credit protection buyer is compensated only if the contract is unwound. How that compensation occurs and how much protection it provides are some points we will discuss.

A CDS does not eliminate credit risk. The definition of a default in the swap contract may not perfectly align with a traditional default event, so the magnitude of the change in value of the contract may differ from the change in value of the underlying. In addition, the credit protection buyer assumes counterparty risk with respect to the credit protection seller. Although there are no guarantees that the credit protection seller will not default, as was seen with several large financial institutions in the financial crisis that started in 2007, most credit protection sellers are relatively high-quality borrowers. If they were not, they could not be active sellers of credit protection.

The majority of CDS are written on debt issued by corporate borrowers, which will be our focus in this reading. But note that CDS can also be written on the debt of sovereign governments and state and local governments. In addition, CDS can be written on portfolios of loans, mortgages, or debt securities.

2.1. Types of CDS

There are three types of CDS: single-name CDS, index CDS, and tranche CDS. Other CDS-related instruments, such as options on CDS (or CDS swaptions) are beyond the scope of this discussion. A CDS on one specific borrower is called a **single-name CDS**. The borrower is called the **reference entity**, and the contract specifies a **reference obligation**, a particular debt instrument issued by the borrower. Only a small subset of issuers, typically with large outstanding liquid debt, have single-name CDS. The designated instrument is usually a senior unsecured obligation, but the reference obligation is not the only instrument covered by the CDS. Any debt obligation issued by the borrower that is ranked equal to or higher than the reference obligation with respect to the priority of claims is covered. The payoff of the CDS is determined by the **cheapest-to-deliver** obligation, which is the debt instrument that can be purchased and delivered at the lowest cost but has the same seniority as the reference obligation.

EXAMPLE 1 Cheapest-to-Deliver Obligation

Assume that a company with several debt issues trading in the market files for bankruptcy (i.e., a credit event takes place). What is the cheapest-to-deliver obligation for a CDS contract where the reference bond is a five-year senior unsecured bond?

A. A subordinated unsecured bond trading at 20% of par
B. A five-year senior unsecured bond trading at 50% of par
C. A two-year senior unsecured bond trading at 45% of par

Solution: C is correct. The cheapest-to-deliver, or lowest-priced, instrument is the two-year senior unsecured bond trading at 45% of par. Although the bond in A trades at a lower dollar price, it is subordinated and, therefore, does not qualify for coverage under the CDS. Note that even though the CDS holder holds the five-year bonds, he will receive payment on the CDS based on the cheapest-to-deliver obligation, not the specific obligation he holds.

A second type of credit default swap, an **index CDS**, involves a portfolio of single-name CDS. This type of instrument allows participants to take positions on the credit risk of a combination of companies, in much the same way that investors can trade index or exchange-traded funds that are combinations of the equities of companies. The two most commonly traded CDS index products are the North American indexes (CDS) and the European, Asian, and Australian indexes (iTraxx). Correlation of defaults is a strong determinant of a portfolio's behavior. For index CDS, this concept takes the form of a factor called **credit correlation**, and it is a key determinant of the value of an index CDS. Analyzing the effects of those correlations is a highly specialized subject, but be aware that much effort is placed on modeling how defaults by certain companies are connected to defaults by other companies. The more correlated the defaults, the more costly it is to purchase protection for a combination of the companies. In contrast, for a diverse combination of companies whose defaults have low correlations, it will be much less expensive to purchase protection.

A third type of CDS is the **tranche CDS**, which covers a combination of borrowers but only up to pre-specified levels of losses—much in the same manner that asset-backed securities are divided into tranches, each covering particular levels of losses. Coverage of tranche CDS is beyond the scope of this reading.

3. IMPORTANT FEATURES OF CDS MARKETS AND INSTRUMENTS, CREDIT AND SUCCESSION EVENTS, AND SETTLEMENT PROPOSALS

- **describe credit events and settlement protocols with respect to CDS**

As we will describe in more detail later, the CDS market is large, global, and well organized. The unofficial industry governing body is the International Swaps and Derivatives Association (ISDA), which publishes industry-supported conventions that facilitate the functioning of the market. Parties to CDS contracts generally agree that their contracts will conform to ISDA specifications. These terms are specified in a document called the **ISDA Master Agreement**, which the parties to a CDS sign. In Europe, the standard CDS contract is called the Standard Europe Contract, and in the United States and Canada, it is called the Standard North American Contract. Other standardized contracts exist for Asia, Australia, Latin America, and a few other specific countries.

Each CDS contract specifies a **notional amount**, or "notional" for short, which is the amount of protection being purchased. The notional amount can be thought of as the *size* of the contract. It is important to understand that the total notional amount of CDS can exceed the amount of debt outstanding of the reference entity. As we will discuss later, the credit protection buyer does not have to be an actual creditor holding exposure (i.e., owning a loan, bond, or other debt instrument). It can be simply a party that believes that there will be a change in the credit quality of the reference entity.

As with all derivatives, the CDS contract has an expiration or maturity date, and coverage is provided up to that date. The typical maturity range is 1 to 10 years, with 5 years being the most common and actively traded maturity, but the two parties can negotiate any maturity. Maturity dates are typically the 20th day of March, June, September, or December. The March and September maturity dates are the most liquid, as these are when the index CDS contracts roll.

The buyer of a CDS pays a periodic premium to the seller, referred to as the **CDS spread**, which is a return over a market reference rate required to protect against credit risk. It is sometimes referred to as a credit spread. Conceptually, it is the same as the credit spread on a bond, the compensation for bearing credit risk.

An important advancement in the development of CDS has been in establishing standard annual coupon rates on CDS contracts. (Note that the reference bond will make payments that are referred to collectively as the coupon while a CDS on the reference bond will have its own coupon rate.) Formerly, the coupon rate on the CDS was set at the credit spread. If a CDS required a rate of 4% to compensate the protection seller for the assumption of credit risk, the protection buyer made quarterly payments amounting to 4% annually. Now CDS coupon rates are standardized, with the most common coupons being either 1% or 5%. The 1% rate typically is used for a CDS on an investment-grade company or index, and the 5% rate is used for a CDS on a high-yield company or index. Obviously, either standardized rate might not be the appropriate rate to compensate the seller. Clearly, not all investment-grade companies have equivalent credit risk, and not all high-yield companies have equivalent credit risk. In effect, the standard rate may be too high or too low. This discrepancy is accounted for by an **upfront payment**, commonly called the **upfront premium**. The differential between the credit spread and the standard rate is converted to a present value basis. Thus, a credit spread greater than the standard rate would result in a cash payment from the protection buyer to the protection seller. Similarly, a credit spread less than the standard rate would result in a cash payment from the protection seller to the protection buyer.

Regardless of whether either party makes an upfront payment, the reference entity's credit quality could change during the life of the contract, thereby resulting in changes in the value of the CDS. These changes are reflected in the price of the CDS in the market. Consider a high-yield company with a 5% credit spread and a CDS coupon of 5%. Therefore, there is no upfront payment. The protection buyer simply agrees to make payments equal to 5% of the notional over the life of the CDS. Now suppose that at some later date, the reference entity experiences a decrease in its credit quality. The credit protection buyer is thus paying 5% for risk that now merits a rate higher than 5%. The coverage and cost of protection are the same, but the risk being covered is greater. The value of the CDS to the credit protection buyer has, therefore, increased, and if desired, she could unwind the position to capture the gain. The credit protection seller has experienced a loss in value of the instrument because he is receiving 5% to cover a risk that is higher than it was when the contract was initiated. It should be apparent that absent any other exposure to the reference entity, if the credit quality of the reference entity decreases, the credit protection buyer gains and the credit protection seller loses. The market value of the CDS reflects these gains and losses.

The terminology in CDS markets can be confusing. In equity and fixed-income markets, we think of buyers as being long and sellers as being short. In the CDS market, however, that is not always true. In single-name CDS, the *buyer* of credit protection is *short credit exposure* and the *seller* of credit protection is *long credit exposure*. This is consistent with the fact that in the financial world, "shorts" are said to benefit when things go badly. When credit quality deteriorates, the credit protection buyer benefits, and when it improves, the credit protection seller benefits. To make things even more confusing, though, the opposite is true in CDS index positions: The *buyer* of a CDX is *long* credit exposure and the seller of a CDX is *short* credit exposure. To minimize the confusion, we use the terms *credit protection seller* and *credit protection buyer* throughout our discussion.

3.1. Credit and Succession Events

The **credit event** is what defines default by the reference entity—that is, the event that triggers a payment from the credit protection seller to the credit protection buyer. This event must be unambiguous: Did it occur, or did it not? For the market to function well, the answer to this question must be clear.

As previously mentioned, the most common credit events include bankruptcy, failure to pay, and restructuring. **Bankruptcy** is a declaration provided for by a country's laws that typically involves the establishment of a legal procedure that forces creditors to defer their claims. Bankruptcy essentially creates a temporary fence around the company through which the creditors cannot pass. During the bankruptcy process, the defaulting party works with its creditors and the court to attempt to establish a plan for repaying the debt. If that plan fails, there is likely to be a full liquidation of the company, at which time the court determines the payouts to the various creditors. Until liquidation occurs, the company normally continues to operate. Many companies do not liquidate and are able to emerge from bankruptcy. A bankruptcy filing by the reference entity is universally regarded as a credit event in CDS contracts.

Another credit event recognized in standard CDS contracts is **failure to pay**, which occurs when a borrower does not make a scheduled payment of principal or interest on an outstanding obligation after a grace period, without a formal bankruptcy filing. (Failure to pay credit events are defined in the CDS contract. ISDA contracts define failure to pay events uniformly, but the same is not true for bespoke CDS.) The third type of event, **restructuring**, refers to a number of possible events, including reduction or deferral of principal or interest, change in seniority or priority of an obligation, or change in the currency in which principal or interest is scheduled to be paid. To qualify as a credit event, the restructuring must be either involuntary or coercive. An involuntary credit event is one that is forced on the borrower by the creditors. A coercive credit event is one that is forced on the creditors by the borrower. Debt restructuring is not a credit event in the United States; issuers generally restructure under *bankruptcy*, which *is* a credit event. Restructuring is a credit event in other countries where the use of bankruptcy court to reorganize is less common. The Greek debt crisis is a good example of a restructuring that triggered a credit event.

Determination of whether a credit event occurs is done by a 15-member group within the ISDA called the Determinations Committee (DC). Each region of the world has a Determinations Committee, which consists of 10 CDS dealer (sell-side) banks and 5 non-bank (buy-side) end users. To declare a credit event, there must be a supermajority vote of 12 members.

The Determinations Committees also play a role in determining whether a **succession event** occurred. A succession event arises when there is a change in the corporate structure of the reference entity, such as through a merger, a divestiture, a spinoff, or any similar action in which ultimate responsibility for the debt in question becomes unclear. For example, if a company acquires all of the shares of a target company, it ordinarily assumes the target company's debt as well. Many mergers, however, are more complicated and can involve only partial acquisition of shares. Spinoffs and divestitures can also involve some uncertainty about who is responsible for certain debts. When such a question arises, it becomes critical for CDS holders. The question is ordinarily submitted to a Determinations Committee, and its resolution often involves complex legal interpretations of contract provisions and country laws. If a succession event is declared, the CDS contract is modified to reflect the DC's interpretation of whoever it believes becomes the obligor for the original debt. Ultimately, the CDS contract could be split among multiple entities.

3.2. Settlement Protocols

If the DC declares that a credit event has occurred, the two parties to a CDS have the right, but not the obligation, to settle. **Settlement** typically occurs 30 days after declaration of the credit event by the DC. CDS can be settled by **physical settlement** or by **cash settlement**. The former is less common and involves actual delivery of the debt instrument in exchange for a payment by the credit protection seller of the notional amount of the contract. In cash settlement, the credit protection seller pays cash to the credit protection buyer. Determining the amount of that payment is a critical factor because opinions can differ about how much money has actually been lost. The payment should essentially be the loss that the credit protection buyer has incurred, but determining that amount is not straightforward. Default on a debt does not mean that the creditor will lose the entire amount owed. A portion of the loss could be recovered. The percentage of the loss recovered is called the **recovery rate** (RR). (In most models, the recovery rate applies only to the principal.) The complement is called the **loss given default** (LGD), which is essentially an estimate of the expected credit loss. The **payout amount** is determined as the loss given default multiplied by the notional.

$$\text{Loss given default} = 1 - \text{Recovery rate (\%)}$$
$$\text{Payout amount} = \text{LGD} \times \text{Notional}$$

Actual recovery can be a very long process, however, and can occur much later than the payoff date of the CDS. To determine an appropriate LGD, the industry conducts an auction in which major banks and dealers submit bids and offers for the cheapest-to-deliver defaulted debt. This process identifies the market's expectation for the recovery rate and the complementary LGD, and the CDS parties agree to accept the outcome of the auction, even though the actual recovery rate can ultimately be quite different, which is an important point if the CDS protection buyer also holds the underlying debt.

EXAMPLE 2 Settlement Preference

A French company files for bankruptcy, triggering various CDS contracts. It has two series of senior bonds outstanding: Bond A trades at 30% of par, and Bond B trades at 40% of par. Investor X owns €10 million of Bond A and owns €10 million of CDS protection. Investor Y owns €10 million of Bond B and owns €10 million of CDS protection.

1. Determine the recovery rate for both CDS contracts.
2. Explain whether Investor X would prefer to cash settle or physically settle her CDS contract or whether she is indifferent.
3. Explain whether Investor Y would prefer to cash settle or physically settle his CDS contract or whether he is indifferent.

Solution to 1: Bond A is the cheapest-to-deliver obligation, trading at 30% of par, so the recovery rate for both CDS contracts is 30%.

Solution to 2: Investor X has no preference between settlement methods. She can cash settle for €7 million [(1 – 30%) × €10 million] and sell her bond for €3 million, for total proceeds of €10 million. Alternatively, she can physically deliver her entire €10 million face amount of bonds to the counterparty in exchange for €10 million in cash.

Solution to 3: Investor Y would prefer a cash settlement because he owns Bond B, which is worth more than the cheapest-to-deliver obligation. He will receive the same €7 million payout on his CDS contract but can sell Bond B for €4 million, for total proceeds of €11 million. If he were to physically settle his contract, he would receive only €10 million, the face amount of his bond.

3.3. CDS Index Products

So far, we have mostly been focusing on single-name CDS. As noted, there are also index CDS products. A company called Markit has been instrumental in producing CDS indexes. Of course, a CDS index is not in itself a traded instrument any more than a stock index is a traded product. As with the major stock indexes, however, the industry has created traded instruments based on the Markit indexes. These instruments are CDS that generate a payoff based on any default that occurs on any entity covered by the index.

The Markit indexes are classified by region and further classified (or divided) by credit quality. The two most commonly traded regions are North America and Europe. North American indexes are identified by the symbol CDX, and European, Asian, and Australian indexes are identified as iTraxx. Within each geographic category are investment-grade and high-yield indexes. The former are identified as CDX IG and iTraxx Main, each comprising 125 entities. The latter are identified as CDX HY, consisting of 100 entities, and iTraxx Crossover, consisting of up to 75 high-yield entities. Investment-grade index CDS are typically quoted in terms of spreads, whereas high-yield index CDS are quoted in terms of prices. Both types of products use standardized coupons. All CDS indexes are equally weighted. Thus, if there are

125 entities, the settlement on one entity is 1/125 of the notional. (Note that some confusion might arise from quoting certain CDS as prices and some as spreads, but keep in mind that the bond market quotes bonds often as prices and sometimes as yields. For example, a Treasury bond can be described as having a price of 120 or a yield of 2.68%. Both terms, combined with the other characteristics of the bond, imply the same concept.)

Markit updates the components of each index every six months by creating new series while retaining the old series. The latest-created series is called the **on-the-run** series, whereas the older series are called **off-the-run** series. When an investor moves from one series to a new one, the move is called a **roll**. When an entity within an index defaults, that entity is removed from the index and settled as a single-name CDS based on its relative proportion in the index. The index then moves forward with a smaller notional.

Index CDS are typically used to take positions on the credit risk of the sectors covered by the indexes as well as to protect bond portfolios that consist of or are similar to the components of the indexes. (An important reminder: When you *buy* a CDS index position, you are *long the credit exposure*, but when you *buy* a single-name CDS position, you have *bought credit protection*. To avoid confusion, we do not talk about buying and selling CDS herein but focus on the desired exposure, using the terms *buy protection* and *sell protection*.)

Standardization is generally undertaken to increase trading volume, which is somewhat limited in the single-name market with so many highly diverse entities. With CDS indexes on standardized portfolios based on the credit risk of well-identified companies, market participants have responded by trading them in large volumes. Indeed, index CDS are typically more liquid than single-name CDS, with average daily trading volume several times that of single-name CDS.

EXAMPLE 3 Hedging and Exposure Using Index CDS

Assume that an investor sells $500 million of protection using the CDX IG index, which has 125 reference entities. Concerned about the creditworthiness of a few of the components, the investor hedges a portion of the credit risk in each. For Company A, he purchases $3 million of single-name CDS protection, and Company A subsequently defaults.

1. What is the investor's net notional exposure to Company A?
2. What proportion of his exposure to Company A has he hedged?
3. What is the remaining notional on his index CDS trade?

Solution to 1: The investor is long $4 million notional credit exposure ($500 million/125) through the index CDS and is short $3 million notional credit exposure through the single-name CDS. His net notional credit exposure is $1 million.

Solution to 2: He has hedged 75% of his exposure ($3 million out of $4 million).

Solution to 3: His index CDS has $496 million remaining notional credit exposure ($500 million original notional minus the $4 million notional related to Company A, which is no longer in the index).

3.4. Market Characteristics

Credit default swaps trade in the over-the-counter market. To better understand this market, we will first review how credit derivatives and specifically CDS were started.

As financial intermediaries, banks draw funds from savings-surplus sectors, primarily consumers, and channel them to savings-deficit sectors, primarily businesses. Corporate lending is a core element of banking. When a bank makes a corporate loan, it assumes two primary risks. One is that the borrower will not repay principal and interest, and the other is that interest rates will change such that the return the bank is earning on its outstanding loans is less than the rate available on comparable instruments in the marketplace. The former is called **credit risk** or **default risk**, and the latter is called **interest rate risk**. There are many ways to manage interest rate risk. Until around the mid-1990s, credit risk was largely managed using traditional methods—such as analysis of the borrower, its industry, and the macroeconomy—as well as control methods, such as credit limits, monitoring, and collateral. In effect, the only defenses against credit risk were to not make a loan, to lend but require collateral (the value of which is also at risk), or to lend and closely monitor the borrower, hoping that any problems could be foreseen and dealt with before a default occurred.

Around 1995, credit derivatives were created to provide a new and potentially more effective method of managing credit risk. They allow credit risk to be transferred from the lender to another party. In so doing, they facilitate the separation of interest rate risk from credit risk. Banks can then provide their most important service—lending—knowing that the credit risk can be transferred to another party if so desired. This ability to easily transfer credit risk allows banks to greatly expand their loan business. Given that lending is such a large and vital component of any economy, credit derivatives facilitate economic growth and have expanded to cover, and indeed are primarily focused on, the short-, intermediate-, and long-term bond markets. In fact, credit derivatives are more effective in the bond market, in which terms and conditions are far more standard, than in the bank loan market. Of the four types of credit derivatives, credit default swaps have clearly established themselves as the most widely used instrument. Indeed, in today's markets CDS are nearly the only credit derivative used to any great extent. CDS transactions are executed in the over-the-counter market by phone, instant message, or the Bloomberg message service. Trade information is reported to the **Depository Trust and Clearinghouse Corporation**, which is a US-headquartered entity providing post-trade clearing, settlement, and information services for many kinds of securities. Regulations require the central clearing of many CDS contracts, meaning that parties will send their contracts through clearinghouses that collect and distribute payments and impose margin requirements, as well as mark positions to market. Central clearing of CDS has risen dramatically since 2010. Currently, slightly more than half of all CDS are centrally cleared, up from just 10% in 2010.

The CDS market today is considerably smaller than it was prior to the 2008 financial crisis. The Bank for International Settlements reported that as of December 2019, the gross notional amount of CDS was about $7.6 trillion with a market value of $199 billion. (For comparison, the notional amounts for interest rate contracts—forward rate agreements, swaps, options—as of December 2019 was about $449 trillion.) As of December 2007, CDS gross notional was $57.9 trillion, nearly 8 times larger.

More than 90% of all CDS market activity is now derived from trading in five major CDS indexes: iTraxx Europe, iTraxx Europe Crossover, iTraxx Europe Senior Financials, CDX IG, and CDS HY.

4. BASICS OF VALUATION AND PRICING

- **explain the principles underlying and factors that influence the market's pricing of CDS**

Derivatives are typically priced by solving for the cost of a position that fully offsets the underlying exposure and earns the risk-free rate. In the context of CDS, this "price" is the CDS spread or upfront payment for a particular coupon rate under the contract. Although CDS are referred to as "swaps," they in fact resemble options because of the contingent nature of the payment made by the protection seller to the protection buyer if a credit event occurs as established by the ISDA Determinations Committee as outlined above.

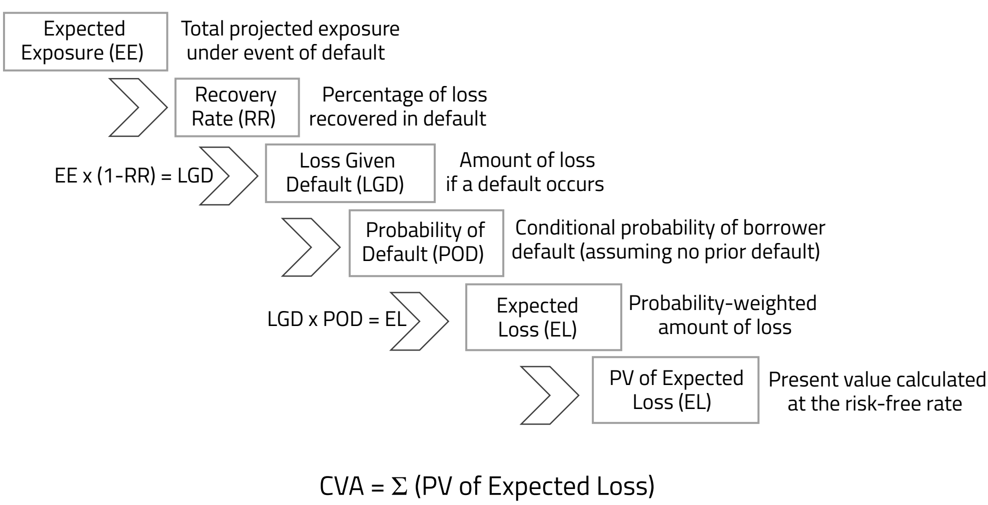
Unlike conventional derivative instruments, the CDS settlement amount under a credit event as declared by the ISDA Determinations Committee is far less clear than for derivatives whose underlying involves actively traded assets, such as equities, interest rates, or currencies. Credit does not "trade" in the traditional sense but, rather, exists implicitly within the bond and loan market. The unique debt structure and composition of each CDS reference entity adds to the complexity of establishing the basis between a CDS contract and a specific outstanding bond or loan.

The details of credit derivative models are beyond the scope of this reading, but it is important for investment industry analysts to have a thorough understanding of the factors that determine CDS pricing.

4.1. Basic Pricing Concepts

In our earlier coverage of credit strategies, we established that the credit valuation adjustment (CVA) may be thought of as the present value of credit risk for a loan, bond, or derivative obligation. In principle, the CVA should, therefore, be a reasonable approximation for the CDS hedge position outlined previously that would leave an investor with a risk-free rate of return. Exhibit 2 summarizes the CVA calculation for a financial exposure.

EXHIBIT 2 Credit Valuation Adjustment



CVA is a function of expected exposure (EE), recovery rate, loss given default, the **probability of default** (POD) to arrive at an expected loss (EL), and a discount factor to arrive at the present value of expected loss.

Considering each of these CVA components in turn, the expected exposure reflects the notional value of the underlying CDS contract. Recall that the recovery rate is the percentage of loss recovered from a bond in default, whereas the loss given default is a function of the loss severity multiplied by the exposure amount.

The probability of default is a key element of CDS pricing that may be illustrated using a simple example. Consider a one-period CDS swap with no upfront payment where we ignore the time value of money and assume that default is possible only at maturity. The fair price of CDS protection for this period for a given borrower may be estimated as

$$\text{CDS spread} \approx (1 - \text{RR}) \times \text{POD}$$

For example, if the probability of default is 2% and the recovery rate is 60%, the estimated CDS spread for the period would be 80 bps for the period. Assuming a $100 notional contract value and a period of a year, the CDS contract fair value would be (the present value of) $0.80.

It is important to note that the POD is a conditional probability over time. That is, assuming a two-period case, the probability of default in Period 2 is contingent on "surviving" to (i.e., not defaulting by) the end of Period 1. Note that we simplify the analysis by assuming discrete times of potential default versus the continuous time assumption common in CDS pricing models.

For example, consider a two-year, 5%, $1,000 loan with one interest payment of $50 due in one year and final interest and principal of $1,050 due in two years. Assume further that we estimate a 2% chance of defaulting on the first payment and a 4% chance of defaulting on the second payment. To calculate the POD over the life of the loan, we first determine the **probability of survival** (POS) for Period 1. The POS is 0.98 (100% minus the 2% POD at T_1) multiplied by 0.96 (100% minus the 4% POD at T_2), approximately 94.08%. Thus, the POD over the life of the loan is 100% − 94.08% = 5.92%.

This conditional probability of default is also known as the **hazard rate**, as described in an earlier reading. The hazard rate is the probability that an event will occur *given that it has not already occurred*.

Now consider another possibility, a 10-year bond with an equivalent hazard rate of 2% each year. Suppose we want to know the probability that the borrower will not default during the entire 10-year period. The probability that a default will occur at some point during the 10 years is one minus the probability of no default in 10 years. The probability of no default in 10 years is $0.98 \times 0.98 \ldots 0.98 = 0.98^{10} = 0.817$. Thus, the probability of default is $1 - 0.817 = 0.183$, or 18.3%. This somewhat simplified example illustrates how a low probability of default in any one period can turn into a surprisingly high probability of default over a longer period of time. Note that we have simplified the analysis by assuming a constant hazard rate, which may not be the case in practice.

EXAMPLE 4 Hazard Rate and Probability of Survival

Assume that a company's hazard rate is a constant 8% per year, or 2% per quarter. An investor sells five-year CDS protection on the company with the premiums paid quarterly over the next five years.

1. What is the probability of survival for the first quarter?
2. What is the conditional probability of survival for the second quarter?
3. What is the probability of survival through the second quarter?

Solution to 1: The probability of survival for the first quarter is 98% (100% minus the 2% hazard rate).

Solution to 2: The conditional probability of survival for the second quarter is also 98%, because the hazard rate is constant at 2%. In other words, *conditional on the company having survived the first quarter*, there is a 2% probability of default in the second quarter.

Solution to 3: The probability of survival through the second quarter is 96.04%. The probability of survival through the first quarter is 98%, and the conditional probability of survival through the second quarter is also 98%. The probability of survival through the second quarter is thus 98% × 98% = 96.04%. Alternatively, 1 − 96.04% = 3.96% is the probability of default sometime during the first two quarters.

Understanding the concept of pricing a CDS is facilitated by recognizing that there are essentially two sides, or legs, of a contract. There is the **protection leg**, which is the contingent payment that the credit protection seller may have to make to the credit protection buyer, and the **premium leg**, which is the series of payments the credit protection buyer promises to make to the credit protection seller. Exhibit 3 provides an illustration of the process.

EXHIBIT 3 Determination of CDS Protection vs. Premium Legs

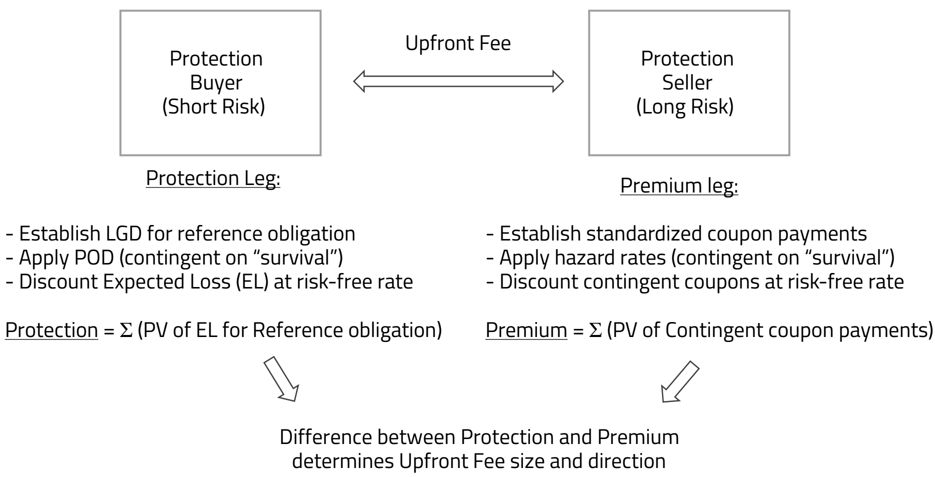


Exhibit 3 shows the upfront payment as the difference in value of the protection and premium legs. The party with a claim on the greater present value must pay the difference at the initiation date of the contract:

$$\text{Upfront payment} = \text{PV (Protection leg)} - \text{PV (Premium leg)}$$

If the result is greater (less) than zero, the protection buyer (seller) pays the protection seller (buyer). Actual CDS pricing and valuation models are more mathematically complex but are based on this conceptual framework.

4.2 The Credit Curve and CDS Pricing Conventions

The credit spread of a debt instrument is the rate in excess of a market reference rate (historically, Libor, although Libor is expected to be replaced by the end of 2021) that investors expect to receive to justify holding the instrument. The reference rate may itself contain some credit risk, as it reflects the rate at which commercial banks lend to one another. The credit spread can be expressed roughly as the probability of default multiplied by the loss given default, with LGD in terms of a percentage. The credit spreads for a range of maturities of a company's debt make up its **credit curve**. The credit curve is somewhat analogous to the term structure of interest rates, which is the set of rates on default-free debt over a range of maturities, but the credit curve applies to non-government borrowers and incorporates credit risk into each rate.

The CDS market for a given borrower is integrated with the credit curve of that borrower. In fact, given the evolution and high degree of efficiency of the CDS market, the credit curve is essentially determined by the CDS rates. The curve is affected by a number of factors, a key one of which is the set of aforementioned hazard rates. A constant hazard rate will tend to flatten the credit curve. Upward-sloping credit curves imply a greater likelihood of default in later years, whereas downward-sloping credit curves imply a greater probability of default in the earlier years. Downward-sloping curves are less common and often a result of severe near-term stress in the financial markets. The credit curve would not be completely flat even if the hazard rates are constant, because of discounting. For example, a company issuing 5- and 10-year zero-coupon bonds could have equally likely probabilities of default and hence equal expected payoffs. The present values of the payoffs are not the same, however, and so the discount rates that equate the present value to the expected payoffs will not be the same.

EXAMPLE 5 Change in Credit Curve

A company's 5-year CDS trades at a credit spread of 300 bps, and its 10-year CDS trades at a credit spread of 500 bps.

1. The company's 5-year spread is unchanged, but the 10-year spread widens by 100 bps. Describe the implication of this change in the credit curve.
2. The company's 10-year spread is unchanged, but the 5-year spread widens by 500 bps. Describe the implication of this change in the credit curve.

Solution to 1: This change implies that although the company is not any riskier in the short term, its longer-term creditworthiness is less attractive. Perhaps the company has adequate liquidity for the time being, but after five years it must begin repaying debt or it will be expected to have cash flow difficulties.

Solution to 2: This change implies that the company's near-term credit risk is now much greater. In fact, the probability of default will decrease if the company can survive for the next five years. Perhaps the company has run into liquidity issues that must be resolved soon, and if not resolved, the company will default.

4.3. CDS Pricing Conventions

With corporate bonds, we typically refer to their values in terms of prices or spreads. The spread is a more informative measure than price. A high-yield bond can be offered with a coupon equal to its yield and, therefore, a price of par value. An investment-grade bond with the same maturity can likewise be offered with a coupon equal to its yield, and therefore, its price is at par. These two bonds would have identical prices at the offering date, and their prices might even be close through much of their lives, but they are quite different bonds. Focusing on their prices would, therefore, provide little information. Their spreads are much more informative. With a market reference rate or the risk-free rate as a benchmark, investors can get a sense for the amount of credit risk implied by their prices, maturities, and coupons. The same is true for CDS. Although CDS have their own prices, their spreads are far more informative.

The reference entity will not necessarily have outstanding debt with credit spreads matching the 1% or 5% standardized coupons conventionally used in CDS contracts. Therefore, the present value of the promised payments from the credit protection buyer to the credit protection seller will most likely be different than the present value of the coupons on the reference entity's debt. The present value difference is the upfront premium paid from one party to the other.

$$\text{Present value of credit spread} = \text{Upfront premium} + \text{Present value of fixed coupon}$$

A good rough approximation used in the industry is that the upfront premium is

$$\text{Upfront premium} = (\text{Credit spread} - \text{Fixed coupon}) \times \text{Duration}$$

The upfront premium must ultimately be converted to a price, which is done by subtracting the percentage premium from 100.

$$\text{Upfront premium \%} = 100 - \text{Price of CDS in currency per 100 par}$$

Note that the duration used here is effective duration, since the cash flows arising from the coupon leg of the CDS are uncertain because they are contingent on the reference entity not defaulting.

EXAMPLE 6 Premiums and Credit Spreads

1. Assume a high-yield company's 10-year credit spread is 600 bps and the duration of the CDS is 8 years. What is the approximate upfront premium required to buy 10-year CDS protection? Assume high-yield companies have 5% coupons on their CDS.
2. Imagine an investor sold five-year protection on an investment-grade company and had to pay a 2% upfront premium to the buyer of protection. Assume the duration of the CDS to be four years. What are the company's credit spreads and the price of the CDS per 100 par?

Solution to 1: To buy 10-year CDS protection, an investor would have to pay a 500 bp coupon plus the present value of the difference between that coupon and the current market spread (600 bps). In this case, the upfront premium would be approximately 100 bps × 8 (duration), or 8% of the notional.

Solution to 2: The value of the upfront premium is equal to the premium (−2%) divided by the duration (4), or −50 bps. The sign of the upfront premium is negative because the seller is paying the premium rather than receiving it. The credit spread is equal to the fixed coupon (100 bps) plus the upfront premium, amortized over the duration of the CDS (−50 bps), or 50 bps. As a reminder, because the company's credit spread is less than the fixed coupon, the protection seller must pay the upfront premium to the protection buyer. The price in currency would be 100 minus the upfront premium, but the latter is negative, so the price is 100 − (−2) = 102.

4.4. Valuation Changes in CDS during Their Lives

As with any traded financial instrument, a CDS has a value that fluctuates during its lifetime. That value is determined in the competitive marketplace. Market participants constantly assess the current credit quality of the reference entity to determine its current value and (implied) credit spread. Clearly, many factors can change over the life of the CDS. By definition, the duration shortens through time. Likewise, the probability of default, the expected loss given default, and the shape of the credit curve will all change as new information is received. The exact valuation procedure of the CDS is precisely the same as it is when the CDS is first issued and simply incorporates the new inputs. The new market value of the CDS reflects gains and losses to the two parties.

Consider the following example of a five-year CDS with a fixed 1% coupon. The credit spread on the reference entity is 2.5%. In promising to pay 1% coupons to receive coverage on a company whose risk justifies 2.5% coupons, the present value of the protection leg exceeds the present value of the payment leg. The difference is the upfront premium, which will be paid by the credit protection buyer to the credit protection seller. During the life of the CDS, assume that the credit quality of the reference entity improves, such that the credit spread is now 2.1%. Now, consider a newly created CDS with the same remaining maturity and 1% coupon. The present value of the payment leg would still be less than the present value of the protection leg, but the difference would be less than it was when the original CDS was created because the risk is now less. Logically, it should be apparent that for the original transaction, the seller has gained and the buyer has lost. The difference between the original upfront premium and the new value is the seller's gain and buyer's loss. A rough approximation of the change in value of the CDS for a given change in spread is as follows:

$$\text{Profit for the buyer of protection} \approx \text{Change in spread in bps} \times \text{Duration} \times \text{Notional}$$

Alternatively, we might be interested in the CDS percentage price change, which is obtained as

$$\text{\% Change in CDS price} = \text{Change in spread in bps} \times \text{Duration}$$

The percentage change in the price of a bond is approximately the change in its yield multiplied by its modified duration. For the CDS, the change in yield is analogous to the change in spread, measured in basis points. The duration of the CDS is analogous to the duration of the bond on which the CDS is written.

EXAMPLE 7 Profit and Loss from Change in Credit Spread

An investor buys $10 million of five-year protection, and the CDS contract has a duration of four years. The company's credit spread was originally 500 bps and widens to 800 bps.

1. Does the investor (credit protection buyer) benefit or lose from the change in credit spread?
2. Estimate the CDS price change and estimated profit to the investor.

Solution to 1: The investor owns protection and is therefore short the credit exposure. As the credit spread widens (the credit quality of the underlying deteriorates), the value of the credit protection she owns increases.

Solution to 2: The percentage price change is estimated as the change in spread (300 bps) multiplied by the duration (4), or 12%. The profit to the investor is 12% times the notional ($10 million), or $1.2 million.

4.5. Monetizing Gains and Losses

As with any financial instrument, changes in the price of a CDS give rise to opportunities to unwind the position and either capture a gain or realize a loss. This process is called **monetizing** a gain or loss. Keep in mind that the protection seller is effectively long the reference entity. He has entered into a contract to insure the debt of the reference entity, for which he receives a series of promised payments and possibly an upfront premium. He clearly benefits if the reference entity's credit quality improves because he continues to receive the same compensation but bears less risk. Using the opposite argument, the credit protection buyer benefits from a deterioration of the reference entity's credit quality. Thus, the credit protection seller is more or less long the company's bonds and the credit protection buyer is more or less short the company's bonds. As the company's credit quality changes through time, the market value of the CDS changes, giving rise to gains and losses for the CDS counterparties. The counterparties can realize those gains and losses by entering into new offsetting contracts, effectively selling their CDS positions to other parties.

Going back to the example in the previous section where the credit quality of the reference entity improved—the credit spread on the reference entity declined from 2.5% to 2.1%. The implied upfront premium on a new CDS that matches the terms of the original CDS with adjusted maturity is now the market value of the original CDS. The premium on the new CDS is smaller than that on the original CDS.

Now, suppose that the protection buyer in the original transaction wants to unwind her position. She would then enter into a new CDS as a protection seller and receive the newly calculated upfront premium. As we noted, this value is less than what he paid originally. Likewise, the protection seller in the original transaction could offset his position by entering into a new CDS as a protection buyer. He would pay an upfront premium that is less than what he originally received. The original protection buyer monetizes a loss, and the seller monetizes a gain. The transaction to unwind the CDS does not need to be done with the same original party, although doing so offers some advantages. Central clearing of CDS transactions facilitates the unwind transaction.

At this point, we have identified two ways of realizing a profit or loss on a CDS. One is to effectively exercise the CDS in response to a default. The other is to unwind the position by entering into a new offsetting CDS in the market. A third, less common method occurs if there is no default. A party can simply hold the position until expiration, at which time the credit protection seller has captured all of the premiums and has not been forced to make any payments, and the seller's obligation for any further payments is terminated. The spread of the CDS will go to zero, in much the same manner as a bond converges toward par as it approaches maturity.

The CDS seller clearly gains, having been paid to bear the risk of default that is becoming increasingly unlikely, and the CDS buyer loses. The buyer loses on the CDS because it paid premiums to receive protection in the event of a default, which did not occur. Although the CDS position itself is a loss, the buyer's overall position is not necessarily a loss. If the buyer is a creditor of the reference entity, the premium "loss" is no different than a homeowner's insurance premium payment on his house; he wouldn't consider that payment a loss simply because his house did not burn down.

5. APPLICATIONS OF CDS

- **describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve**

Credit default swaps, as demonstrated, facilitate the transfer of credit risk. As simple as that concept seems, there are many different circumstances under which CDS are used. In this section, we consider some applications of this instrument.

Any derivative instrument has two general uses. One is to exploit an expected movement in the underlying. The derivative typically requires less capital and is usually an easier instrument in which to create a short economic exposure as compared with the underlying. The derivatives market can also be more efficient, meaning that it can react to information more rapidly and have more liquidity than the market for the underlying. Thus, information or an expectation of movement in the underlying can often be exploited much more efficiently with the derivative than with the underlying directly.

The other trading opportunity facilitated by derivatives is in valuation differences between the derivative and the underlying. If the derivative is mispriced relative to the underlying, one can take the appropriate position in the derivative and an offsetting position in the underlying. If the valuation assessment is correct and other investors come to the same conclusion, the values of the derivative and underlying will converge, and the investor will earn a return that is essentially free of risk because the risk of the underlying has been hedged away by holding offsetting positions in the derivative and the underlying. Whether this happens as planned depends on both the efficiency of the market and the quality of the valuation model. Differences can also exist between the derivative and other derivatives on the same underlying.

These two general types of uses are also the major applications of CDS. We will refer to them as managing credit exposures, meaning the taking on or shedding of credit risk in light of changing expectations and/or valuation disparities. With valuation disparities, the focus is on differences in the pricing of credit risk in the CDS market relative to that of the underlying bonds.

5.1. Managing Credit Exposures

The most basic application of a CDS is to increase or decrease credit exposure. The most obvious such application is for a lender to buy protection to reduce its credit exposure to a borrower. For the seller of protection, the trade adds credit exposure. A lender's justification for using a CDS seems obvious. The lender may have assumed too much credit risk but does not want to sell the bond or loan because there can be significant transaction costs, because later it may want the bond or loan back, or because the market for the bond or loan is relatively illiquid. If the risk is temporary, it is almost always easier to temporarily reduce risk by using a CDS. Beyond financial institutions, any organization exposed to credit risk is potentially a candidate for using CDS.

The justification for selling credit protection is somewhat less obvious. The seller can be a CDS dealer, whose objective is to profit from making markets in CDS. A dealer typically attempts to manage its exposure by either diversifying its credit risks or hedging the risk by entering into a transaction with yet another party, such as by shorting the debt or equity of the reference entity, often accompanied by investment of the funds in a repurchase agreement, or repo. If the dealer manages the risk effectively, the risk assumed in selling the CDS is essentially offset when the payment for assuming the risk exceeds the cost of removing the risk. Achieving this outcome successfully requires sophisticated credit risk modeling.

Although dealers make up a large percentage of protection sellers, not all sellers are dealers. Consider that any bondholder is a buyer of credit and interest rate risk. If the bondholder wants only credit risk, it can obtain it by selling protection, which would require far less capital and incur potentially lower overall transaction costs than buying the bond. Moreover, the CDS can be more liquid than the bond, so the position can be unwound much more easily.

As noted, it is apparent why a party making a loan might want credit protection. Consider, however, that a party with no exposure to the reference entity might also purchase credit protection. Such a position is called a **naked credit default swap**, and it has resulted in some controversy in regulatory and political circles. In buying protection without owning the underlying, the investor is taking a position that the entity's credit quality will deteriorate, whereas the seller of protection without owning the underlying is taking the position that the entity's credit quality will improve or that the CDS was overpriced.

Some regulators and politicians believe it is inappropriate for a party with no exposure to a borrower to speculate that the borrower's financial condition will deteriorate. This controversy accelerated during the financial crisis of 2008-2009 because many investors bought protection without owning the underlying and benefited from the crisis.

The counterargument, however, is that elsewhere in the financial markets, such bets are made all of the time in the form of long puts, short futures, and short sales of stocks and bonds. These instruments are generally accepted as a means of protecting oneself against poor performance in the financial markets. Credit protection is also a means of protecting oneself against poor performance. In addition, proponents of naked CDS argue that they bring liquidity to the credit market, potentially providing more stability, not less. Nonetheless, naked CDS trading is banned in Europe for sovereign debt, although it is generally permitted otherwise.

CDS trading strategies, with or without naked exposure, can take several forms. An investor can choose to be long or short the credit exposure, as we have previously discussed. Alternatively, the party can be a credit protection seller on one reference entity and a credit protection buyer on a different entity. This is called a **long/short credit trade**. This transaction is a bet that the credit position of one entity will improve relative to that of another. The two entities might be related in some way or might produce substitute goods. For example, one might take a position that because of competition and changes in the luxury car industry, the credit quality of Daimler will improve and that of BMW will weaken, so selling protection on Daimler and buying protection on BMW would be appropriate. Similarly, an investor may undertake a long/short trade based on other factors, such as environmental, social, and governance (ESG) considerations. For instance, an investor may be concerned about a company's poor ESG-related practices and policies relative to another company. In this case, the investor could buy protection using the CDS of a company with weak ESG practices and policies and sell protection using the CDS of a company with strong ESG practices and policies. Example 8 provides a case study of ESG considerations in a long/short ESG trade.

EXAMPLE 8 Long/Short Trade with ESG Considerations

Overview

An analyst is evaluating two US apparel companies: Atelier and Trapp. Atelier is a large company that focuses on high-end apparel brands. It is profitable despite a high cost structure. Trapp is smaller and less profitable than Atelier. Trapp focuses on less expensive brands and strives to keep costs low. Both companies purchase their merchandise from suppliers all over the world. The analyst recognizes that apparel companies must maintain adequate oversight over their suppliers to control the risks of reputational damage and inventory disruptions. Supplier issues are particularly relevant for Atelier and Trapp following a recent fire that occurred at the factory of Global Textiles, a major supplier to both companies. The fire resulted in multiple casualties and unfavorable news headlines.

The analyst notices a significant difference in the way Atelier and Trapp approach ESG considerations. After the fire at its supplier, Atelier signed an "Accord on Fire and Building Safety," which is a legally binding agreement between global apparel manufacturers, retailers, and trade unions in the country where the fire occurred. After signing the accord, Atelier made a concerted effort to fix and enhance machinery in factories of its suppliers. Its objective was to improve workplace safety—notably, to reduce lost employee time due to factory incidents and the rate of factory accidents and fatalities.

Investors view Atelier's corporate governance system favorably because management interests and stakeholder interests are strongly aligned. Atelier's board of directors includes a high percentage of independent directors and is notably diverse. In contrast, Trapp's founder is the majority owner of the company and serves as CEO and chairman of the board of directors. Furthermore, Trapp's board is composed mainly of individuals who have minimal industry expertise. As a consequence, Trapp's board was unprepared to adequately respond to the Global Textiles fire. Given the lack of independence and expertise of Trapp's board, investors consider Trapp's corporate governance system to be poor. Because of its emphasis on low costs and reflecting its less experienced board, Trapp chose not to sign the accord.

Implications for CDS

Single-name CDS on both Atelier and Trapp are actively traded in the market, although Trapp's CDS is less liquid. Before the Global Textiles fire, five-year CDS for Trapp traded at a spread of 250 bps, compared to a spread of 150 bps for the five-year CDS for Atelier. The difference in spreads reflects Trapp's lower trading liquidity, perceived lower creditworthiness (primarily reflecting its smaller size and lower profitability), and hence higher default risk relative to Atelier.

After the Global Textiles fire, spreads on the CDS for all companies in the apparel sector widened considerably. Credit spreads for the five-year CDS on Atelier widened by 60 bps (to 210 bps), and credit spreads for the five-year CDS on Trapp widened by 75 bps (to 325 bps). The analyst believes that over the longer term, the implications of the fire at Global Textiles will be even more adverse for Trapp relative to Atelier. The analyst's view largely reflects Trapp's higher ESG-related risks, especially the perceived weaker safety in its factories and its weaker corporate governance system. In particular, the analyst believes that spreads of Trapp's CDS will remain wider than their pre-fire level of 250 bps, but Atelier's CDS spreads will return to their pre-fire level of 150 bps.

Describe how the analyst can use CDS to exploit the potential opportunity.

Solution
The analyst can try to exploit the potential opportunity by buying protection (shorting the credit) on Trapp using five-year CDS and selling protection (going long the credit) on Atelier using five-year CDS. This trade would reflect both the anticipated continuing adverse spreads for Trapp relative to the pre-fire level and the return of spreads for Atelier to their lower pre-fire levels. For example, assume Atelier's five-year CDS spread returns to 150 bps from 210 bps, but Trapp's five-year CDS spread narrows to just 300 bps from 325 bps. The difference in spreads between the two companies' CDS would have widened from 115 bps (325 bps – 210 bps) right after the factory fire occurred to 150 bps (300 bps – 150 bps). This 35 bp difference in spread would represent profit (excluding trading costs) to the analyst from the long/short trade.

Similar to a long/short trade involving individual entities (companies), an investor might also create a long/short trade using CDS indexes. For example, if the investor anticipates a weakening economy, she could buy protection using a high-yield CDS index and sell protection using an investment-grade CDS index. As high-yield spreads widen relative to investment-grade spreads, the trade would realize a profit. As another example, a trader expecting a strengthening in the Asian economy relative to the European economy could buy protection using a European CDS index and sell protection using an Asian CDS index. As Asia spreads narrow relative to European spreads, the trade would realize a profit.

Another type of long/short trade, called a **curve trade**, involves buying single-name or index protection at one maturity and selling protection on the same reference entity at a different maturity. Consider two CDS maturities, which we will call the short-term and the long-term to keep things simple. We will assume the more common situation of an upward-sloping credit curve, meaning that long-term CDS rates (and credit spreads) are higher than short-term rates. If the curve changes shape, it becomes either steeper or flatter. A steeper (flatter) curve means that long-term credit risk increases (decreases) relative to short-term credit risk. An investor who believes that long-term credit risk will increase relative to short-term credit risk (credit curve steepening) can buy protection by buying a long-term single-name CDS or selling a long-term CDS index and sell protection by selling a short-term single-name CDS or buying a short-term CDS index. In the short run, a curve-steepening trade is bullish. It implies that the short-term outlook for the reference entity is better than the long-term outlook. In the short run, a curve-flattening trade is bearish. It implies that the short-run outlook for the reference entity looks worse than the long-run outlook and reflects the expectation of near-term problems for the reference entity.

EXAMPLE 9 Curve Trading

An investor owns some intermediate-term bonds issued by a company and has become concerned about the risk of a near-term default, although he is not very concerned about a default in the long term. The company's two-year CDS currently trades at 350 bps, and the four-year CDS is at 600 bps.

1. Describe a potential curve trade that the investor could use to hedge the default risk.
2. Explain why an investor may prefer to use a curve trade as a hedge against the company's default risk rather than simply buying protection on the reference entity.

Solution to 1: The investor anticipates a flattening credit curve for the reference company, with spreads rising at the shorter end of the curve. Thus, he would buy credit protection on the two year (buy the two-year single-name CDS) while selling credit protection further out on the curve (sell the four-year single-name CDS).

Solution to 2: The long/short trade reduces the cost of buying near-term credit protection, with the cost of the credit protection offset by the premium received from selling protection further out on the curve. This works only as long as the investor's expectations about the relative risk of near- and longer-term default hold true.

Of course, there can be changes to the credit curve that take the form of simple shifts in the general level of the curve, whereby all spreads go up or down by roughly equal amounts. As with long-duration bonds relative to short-duration bonds, the values of longer-term CDS will move more than those of shorter-term CDS. As an example, a trader who believes that all spreads will go up will want to be a buyer of credit protection but will realize that longer-term CDS will move more than short-term CDS. Thus, she might want to buy protection at the longer part of the curve and hedge by selling protection at the shorter part of the curve. She will balance the sizes of the positions so that the volatility of the position she believes will gain in value will be more than that of the other position. If more risk is desired, she might choose to trade only the more volatile leg.

6. VALUATION DIFFERENCES AND BASIS TRADING

- **describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments**

Different investors will have different assessments of the price of credit risk. Such differences of opinion will lead to valuation disparities. Clearly, there can be only one appropriate price at which credit risk can be eliminated, but that price is not easy to determine. The party that has the best estimate of the appropriate price of credit risk can capitalize on its knowledge or ability at the expense of another party. Any such comparative advantage can be captured by trading the CDS against either the reference entity's debt or equity or derivatives on its debt or equity, but such trading is critically dependent on the accuracy of models that isolate the credit risk component of the return. The details of those models are left to CDS specialists, but it is important for candidates to understand the basic ideas.

The yield on the bond issued by the reference entity to a CDS contains a factor that reflects the credit risk. In principle, the amount of yield attributable to credit risk on the bond should be the same as the credit spread on a CDS. It is, after all, the compensation paid to the party assuming the credit risk, regardless of whether that risk is borne by a bondholder or a CDS seller. But there may be a difference in the credit risk compensation in the bond market and CDS market. This differential pricing can arise from mere differences of opinions, differences in models used by participants in the two markets, differences in liquidity in the two markets, and supply and demand conditions in the repo market, which is a primary source of financing for bond purchases. A difference in the credit spreads in these two markets is the foundation of a strategy known as a **basis trade**.

The general idea behind most basis trades is that any such mispricing is likely to be temporary and the spreads should return to equivalence when the market recognizes the disparity.

For example, suppose the bond market implies a 5% credit risk premium whereas the CDS market implies a 4% credit risk premium. The trader does not know which is correct but believes these two rates will eventually converge. From the perspective of the CDS, its risk premium is too low relative to the bond credit risk premium. From the perspective of the bond, its risk premium is too high relative to the CDS market, which means its price is too low. So, the CDS market could be pricing in too little credit risk, and/or the bond market could be pricing in too much credit risk. Either market could be correct; it does not matter. The investor would buy the bond at a price that appears to overestimate its credit risk and, at the same time, buy credit protection at what appears to be an unjustifiably low premium, simultaneously hedging interest rate risk exposure with a duration strategy or interest rate derivatives. The risk is balanced because the default potential on the bond is protected by the CDS. If convergence occurs, the trade would capture the 1% differential in the two markets.

To determine the profit potential of such a trade, it is necessary to decompose the bond yield into the risk-free rate plus the funding spread plus the credit spread. The risk-free rate plus the funding spread is essentially the market reference rate. The credit spread is then the excess of the yield over the market reference rate and can be compared with the credit spread in the CDS market. If the spread is higher in the bond market than in the CDS market, it is said to be a negative basis. If the spread is higher in the CDS market than in the bond market, it is said to be a positive basis. Note that in practice, the above decomposition can be complicated by the existence of embedded options, such as with callable and convertible bonds or when the bond is not selling near par. Those factors would need to be accounted for in the calculations.

EXAMPLE 10 Bonds vs. Credit Default Swaps

An investor wants to be long the credit risk of a given company. The company's bond currently yields 6% and matures in five years. A comparable five-year CDS contract has a credit spread of 3.25%. The investor can borrow at Libor, which is currently 2.5%.

1. Calculate the bond's credit spread.
2. Identify a basis trade that would exploit the current situation.

Solution to 1: The bond's credit spread is equal to the yield (6%) minus the market reference rate (2.5%). Therefore, the bond's credit spread is currently 3.5%.

Solution to 2: The bond and CDS markets imply different credit spreads. Credit risk is cheap in the CDS market (3.25%) relative to the bond market (3.5%). The investor should buy protection in the CDS market at 3.25% and go long the bond, with its 3.5% credit spread, netting 25 bps.

Another type of trade using CDS can occur within the instruments issued by a single entity. Credit risk is an element of virtually every unsecured debt instrument or the capital leases issued by a company. Each of these instruments is priced to reflect the appropriate credit risk. Investors can use the CDS market to first determine whether any of these instruments is incorrectly priced relative to the CDS and then buy the cheaper one and sell the more expensive one. Again, there is the assumption that the market will adjust. This type of trading is much more complex, however, because priority of claims means that not all of the instruments pay off equally if default occurs.

EXAMPLE 11 Using CDS to Trade on a Leveraged Buyout

An investor believes that a company will undergo a leveraged buyout (LBO) transaction, whereby it will issue large amounts of debt and use the proceeds to repurchase all of the publicly traded equity, leaving the company owned by management and a few insiders.

1. Why might the CDS spread change?
2. What equity-versus-credit trade might an investor execute in anticipation of such a corporate action?

Solution to 1: Taking on the additional debt will almost surely increase the probability of default, thereby increasing the CDS spread.

Solution to 2: The investor might consider buying the stock and buying credit protection. Both legs will profit if the LBO occurs because the stock price will rise as the company repurchases all outstanding equity and the CDS price will rise as its spread widens to reflect the increased probability of default.

CDS indexes also create an opportunity for a type of arbitrage trade. If the cost of the index is not equivalent to the aggregate cost of the index components, an investor might go long the cheaper instrument and short the more expensive instrument. There is the implicit assumption that convergence will occur. If it does, the investor gains the benefit while basically having neutralized the risk. Transaction costs in this type of arbitrage trade can be quite significant and nullify the profit potential for all but the largest investors.

SUMMARY

- A credit default swap (CDS) is a contract between two parties in which one party purchases protection from another party against losses from the default of a borrower for a defined period of time.
- A CDS is written on the debt of a third party, called the reference entity, whose relevant debt is called the reference obligation, typically a senior unsecured bond.
- A CDS written on a particular reference obligation normally provides coverage for all obligations of the reference entity that have equal or higher seniority.
- The two parties to the CDS are the credit protection buyer, who is said to be short the reference entity's credit, and the credit protection seller, who is said to be long the reference entity's credit.
- The CDS pays off upon occurrence of a credit event, which includes bankruptcy, failure to pay, and, in some countries, involuntary restructuring.
- Settlement of a CDS can occur through a cash payment from the credit protection seller to the credit protection buyer as determined by the cheapest-to-deliver obligation of the reference entity or by physical delivery of the reference obligation from the protection buyer to the protection seller in exchange for the CDS notional.
- A cash settlement payoff is determined by an auction of the reference entity's debt, which gives the market's assessment of the likely recovery rate. The credit protection buyer must accept the outcome of the auction even though the ultimate recovery rate could differ.

- CDS can be constructed on a single entity or as indexes containing multiple entities. Bespoke CDS or baskets of CDS are also common.
- The fixed payments made from CDS buyer to CDS seller are customarily set at a fixed annual rate of 1% for investment-grade debt or 5% for high-yield debt.
- Valuation of a CDS is determined by estimating the present value of the payment leg, which is the series of payments made from the protection buyer to the protection seller, and the present value of the protection leg, which is the payment from the protection seller to the protection buyer in event of default. If the present value of the payment leg is greater than the present value of the protection leg, the protection buyer pays an upfront premium to the seller. If the present value of the protection leg is greater than the present value of the payment leg, the seller pays an upfront premium to the buyer.
- An important determinant of the value of the expected payments is the hazard rate, the probability of default given that default has not already occurred.
- CDS prices are often quoted in terms of credit spreads, the implied number of basis points that the credit protection seller receives from the credit protection buyer to justify providing the protection.
- Credit spreads are often expressed in terms of a credit curve, which expresses the relationship between the credit spreads on bonds of different maturities for the same borrower.
- CDS change in value over their lives as the credit quality of the reference entity changes, which leads to gains and losses for the counterparties, even though default may not have occurred or may never occur. CDS spreads approach zero as the CDS approaches maturity.
- Either party can monetize an accumulated gain or loss by entering into an offsetting position that matches the terms of the original CDS.
- CDS are used to increase or decrease credit exposures or to capitalize on different assessments of the cost of credit among different instruments tied to the reference entity, such as debt, equity, and derivatives of debt and equity.

PRACTICE PROBLEMS

The following information relates to Questions 1–6

UNAB Corporation

On 1 January 20X2, Deem Advisors purchased a $10 million six-year senior unsecured bond issued by UNAB Corporation. Six months later (1 July 20X2), concerned about the portfolio's credit exposure to UNAB, Doris Morrison, the chief investment officer at Deem Advisors, buys $10 million protection on UNAB with a standardized coupon rate of 5%. The reference obligation of the CDS is the UNAB bond owned by Deem Advisors. UNAB adheres to the ISDA CDS protocols.

On 1 January 20X3, Morrison asks Bill Watt, a derivatives analyst, to assess the current credit quality of UNAB bonds and the value of Deem Advisors' CDS on UNAB debt. Watt gathers the following information on UNAB's debt issues currently trading in the market:

Bond 1: A two-year senior unsecured bond trading at 40% of par
Bond 2: A five-year senior unsecured bond trading at 50% of par
Bond 3: A five-year subordinated unsecured bond trading at 20% of par

With respect to the credit quality of UNAB, Watt makes the following statement:

> "There is severe near-term stress in the financial markets, and UNAB's credit curve clearly reflects the difficult environment."

On 1 July 20X3, UNAB fails to make a scheduled interest payment on the outstanding subordinated unsecured obligation after a grace period; however, the company does not file for bankruptcy. Morrison asks Watt to determine if UNAB experienced a credit event and, if so, to recommend a settlement preference.

Kand Corporation

Morrison is considering purchasing protection on Kand Corporation debt to hedge the portfolio's position in Kand. She instructs Watt to determine if an upfront payment would be required and, if so, the amount of the premium. Watt presents the information for the CDS in Exhibit 1.

EXHIBIT 1 Summary Data for 10-year CDS on Kand Corporation

Credit spread	700 bps
Duration	7 years
Coupon rate	5%

Morrison purchases 10-year protection on Kand Corporation debt. Two months later the credit spread for Kand Corporation has increased by 200 bps. Morrison asks Watt to close out the firm's CDS position on Kand Corporation by entering into a new, offsetting contract.

Tollunt Corporation

Deem Advisors' chief credit analyst recently reported that Tollunt Corporation's five-year bond is currently yielding 7% and a comparable CDS contract has a credit spread of 4.25%. Since the current market reference rate is 2.5%, Watt has recommended executing a basis trade to take advantage of the pricing of Tollunt's bonds and CDS. The basis trade would consist of purchasing both the bond and the CDS contract.

1. If UNAB experienced a credit event on 1 July, Watt should recommend that Deem Advisors:
   A. prefer a cash settlement.
   B. prefer a physical settlement.
   C. be indifferent between a cash or a physical settlement.
2. According to Watt's statement, the shape of UNAB's credit curve is *most likely*:
   A. flat.
   B. upward-sloping.
   C. downward-sloping.
3. Should Watt conclude that UNAB experienced a credit event?
   A. Yes
   B. No, because UNAB did not file for bankruptcy
   C. No, because the failure to pay occurred on a subordinated unsecured bond
4. Based on Exhibit 1, the upfront premium as a percent of the notional for the CDS protection on Kand Corporation would be *closest* to:
   A. 2.0%.
   B. 9.8%.
   C. 14.0%.

5. If Deem Advisors enters into a new offsetting contract two months after purchasing protection on Kand Corporation, this action will most likely result in:
   A. a loss on the CDS position.
   B. a profit on the CDS position.
   C. neither a loss nor a profit on the CDS position.
6. If convergence occurs in the bond and CDS markets for Tollunt Corporation, a basis trade will capture a profit *closest* to:
   A. 0.25%.
   B. 1.75%.
   C. 2.75%.

The following information relates to Questions 7–14

John Smith, a fixed-income portfolio manager at a €10 billion sovereign wealth fund (the Fund), meets with Sofia Chan, a derivatives strategist with Shire Gate Securities (SGS), to discuss investment opportunities for the Fund. Chan notes that SGS adheres to ISDA (International Swaps and Derivatives Association) protocols for credit default swap (CDS) transactions and that any contract must conform to ISDA specifications. Before the Fund can engage in trading CDS products with SGS, the Fund must satisfy compliance requirements.

Smith explains to Chan that fixed-income derivatives strategies are being contemplated for both hedging and trading purposes. Given the size and diversified nature of the Fund, Smith asks Chan to recommend a type of CDS that would allow the Fund to simultaneously fully hedge multiple fixed-income exposures.

Smith and Chan discuss opportunities to add trading profits to the Fund. Smith asks Chan to determine the probability of default associated with a five-year investment-grade bond issued by Orion Industrial. Selected data on the Orion Industrial bond are presented in Exhibit 1.

EXHIBIT 1 Selected Data on Orion Industrial Five-Year Bond

Year	Hazard Rate
1	0.22%
2	0.35%
3	0.50%
4	0.65%
5	0.80%

Chan explains that a single-name CDS can also be used to add profit to the Fund over time. Chan describes a hypothetical trade in which the Fund sells £6 million of five-year CDS protection on Orion, where the CDS contract has a duration of 3.9 years. Chan assumes that the Fund closes the position six months later, after Orion's credit spread narrowed from 150 bps to 100 bps.

Chan discusses the mechanics of a long/short trade. In order to structure a number of potential trades, Chan and Smith exchange their respective views on individual companies and global economies. Chan and Smith agree on the following outlooks.

Outlook 1: The European economy will weaken.
Outlook 2: The US economy will strengthen relative to that of Canada.
Outlook 3: The credit quality of electric car manufacturers will improve relative to that of traditional car manufacturers.

Chan believes US macroeconomic data are improving and that the general economy will strengthen in the short term. Chan suggests that a curve trade could be used by the Fund to capitalize on her short-term view of a steepening of the US credit curve.

Another short-term trading opportunity that Smith and Chan discuss involves the merger and acquisition market. SGS believes that Delta Corporation may make an unsolicited bid at a premium to the market price for all of the publicly traded shares of Zega, Inc. Zega's market capitalization and capital structure are comparable to Delta's; both firms are highly levered. It is anticipated that Delta will issue new equity along with 5- and 10-year senior unsecured debt to fund the acquisition, which will significantly increase its debt ratio.

7. To satisfy the compliance requirements referenced by Chan, the Fund is *most likely* required to:
   A. set a notional amount.
   B. post an upfront payment.
   C. sign an ISDA master agreement.
8. Which type of CDS should Chan recommend to Smith?
   A. CDS index
   B. Tranche CDS
   C. Single-name CDS
9. Based on Exhibit 1, the probability of Orion defaulting on the bond during the first three years is *closest* to:
   A. 1.07%.
   B. 2.50%.
   C. 3.85%.
10. To close the position on the hypothetical Orion trade, the Fund:
    A. sells protection at a higher premium than it paid at the start of the trade.
    B. buys protection at a lower premium than it received at the start of the trade.
    C. buys protection at a higher premium than it received at the start of the trade.
11. The hypothetical Orion trade generated an approximate:
    A. loss of £117,000.
    B. gain of £117,000.
    C. gain of £234,000.
12. Based on the three economic outlook statements, a profitable long/short trade would be to:
    A. sell protection using a Canadian CDX IG and buy protection using a US CDX IG.
    B. buy protection using an iTraxx Crossover and sell protection using an iTraxx Main.
    C. buy protection using an electric car CDS and sell protection using a traditional car CDS.
13. The curve trade that would *best* capitalize on Chan's view of the US credit curve is to:
    A. buy protection using a 20-year CDX and buy protection using a 2-year CDX.
    B. buy protection using a 20-year CDX and sell protection using a 2-year CDX.
    C. sell protection using a 20-year CDX and buy protection using a 2-year CDX.
14. A profitable equity-versus-credit trade involving Delta and Zega is to:
    A. short Zega shares and buy protection on Delta using the 10-year CDS.
    B. go long Zega shares and buy protection on Delta using 5-year CDS.
    C. go long Delta shares and buy protection on Delta using 5-year CDS.

PART III

FIXED INCOME PORTFOLIO MANAGEMENT

CHAPTER 12

OVERVIEW OF FIXED-INCOME PORTFOLIO MANAGEMENT

Bernd Hanke, PhD, CFA
Brian J. Henderson, PhD, CFA

LEARNING OUTCOMES

The candidate should be able to:

- discuss roles of fixed-income securities in portfolios and how fixed-income mandates may be classified;
- describe fixed-income portfolio measures of risk and return as well as correlation characteristics;
- describe bond market liquidity, including the differences among market sub-sectors, and discuss the effect of liquidity on fixed-income portfolio management;
- describe and interpret a model for fixed-income returns;
- discuss the use of leverage, alternative methods for leveraging, and risks that leverage creates in fixed-income portfolios;
- discuss differences in managing fixed-income portfolios for taxable and tax-exempt investors.

1. INTRODUCTION

Investors often seek regular income from their investments as well as a predetermined date when their capital will be returned. Fixed-income investments offer both.

Fixed-income instruments include a broad range of publicly traded securities (such as commercial paper, notes, and bonds traded through exchanges as well as OTC) and non-publicly traded instruments (such as loans and private placements). Individual loans or fixed-income obligations may be bundled into a pool of assets supporting such instruments as asset-backed securities and covered bonds. Fixed-income portfolio managers combine these diverse instruments across issuers, maturities, and jurisdictions to meet the various needs of investors. We discuss the different roles of fixed-income securities in portfolios and explain the two main types of fixed-income mandates—liability-based mandates and total return mandates—as well as bond market liquidity. We also provide an overview of portfolio measures, instruments, and vehicles used in fixed-income portfolio management and introduce a model of how a bond position's total expected return can be decomposed.

2. ROLES OF FIXED-INCOME SECURITIES IN PORTFOLIOS

- **discuss roles of fixed-income securities in portfolios and how fixed-income mandates may be classified**

Fixed-income securities serve important roles in investment portfolios, including diversification, regular cash flows, and possible inflation hedging. We will briefly review the roles in turn.

2.1. Diversification Benefits

Fixed-income investments can provide diversification benefits when combined with other asset classes in a portfolio. Recall that a major reason portfolios can effectively reduce risk is that combining securities whose returns are not perfectly correlated (i.e., a correlation coefficient of less than +1.0) provides risk diversification. Lower correlations are associated with higher diversification benefits and lower risk. The challenge in diversifying risk is to find assets with correlations much lower than +1.0.

Correlations of fixed-income and equity securities vary, but adding fixed-income exposure to portfolios that include equity securities is usually an effective way to obtain diversification benefits. Fixed-income investments may also provide risk reduction because of their low correlations with other asset classes, such as real estate and commodities. Exhibit 1 shows the correlation between the S&P 500 Index and various fixed-income categories based on total returns (monthly) over a 20-year period ending in December 2019.

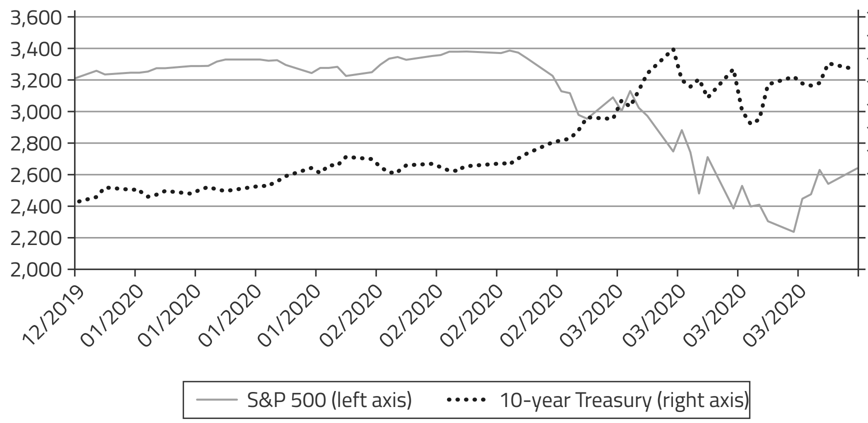
EXHIBIT 1 Total Return Correlations between US Fixed Income and Equities

	Fixed-Income Indexes						
	US Aggregate	10Y US Treasury	US Corporate Bonds	Global Aggregate	US TIPS	US High Yield	Emerging Market (USD)
S&P 500	−0.09	−0.30	0.20	0.15	0.02	0.63	0.51

Note: Bloomberg Barclays Indices are shown.
Source: Bloomberg.

Exhibit 2 shows the divergent performance of US equities and bonds from the end of 2019 to the end of March 2020. For example, bonds outperformed equities amid the fears over the global COVID-19 pandemic in Q1 2020.

EXHIBIT 2 Returns of S&P 500 vs. 10-Year Treasuries, 12 December 2019–31 March 2020



Note: Daily data; constant-maturity 10-year Treasuries used.

Within the fixed-income asset class, the correlation between fixed-income indexes will be driven largely by the interest rate component (i.e., duration) and by geography. Rate changes can explain a significant amount of movement in fixed-income securities prices. The credit component or credit spread will likely result in diversification given differences in sectors, credit quality, and geography. For example, investment-grade securities may exhibit less correlation with below-investment-grade securities and with emerging market securities and equities. The rate component of the return can be isolated by calculating correlations using excess returns (this is more meaningful when evaluating returns across fixed-income sectors). Exhibit 3 shows correlations on an excess return basis between various fixed-income indexes.

EXHIBIT 3 Excess Return Correlations of Barclays Bloomberg Indexes over a 20-Year Period

	US Aggregate	US Corporate	Global Aggregate	US High Yield	Emerging Market (USD)
US Aggregate	1.00				
US Corporate	0.93	1.00			
Global Aggregate	0.88	0.86	1.00		
US High Yield	0.86	0.84	0.76	1.00	
Emerging Market (USD)	0.79	0.76	0.74	0.80	1.00

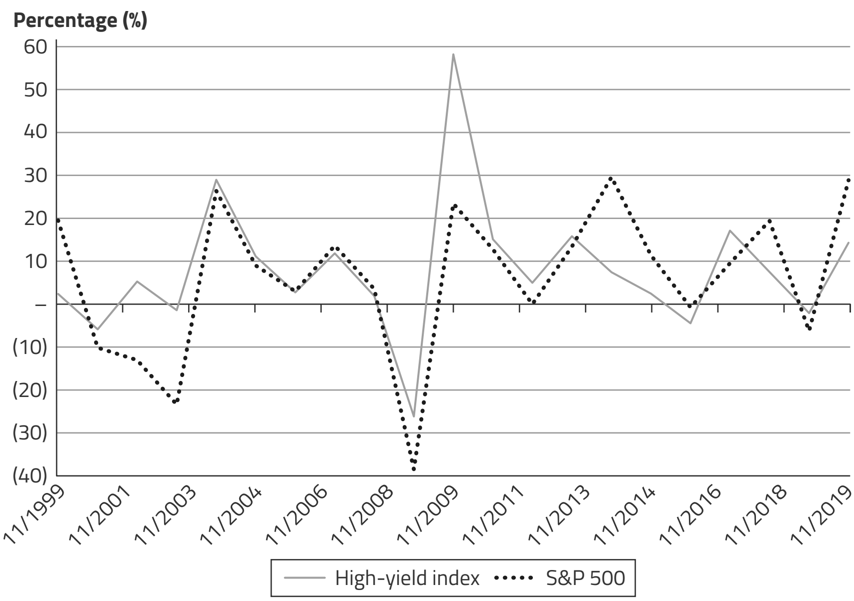
Notes: Bloomberg Barclays Indices shown. Based on monthly data over 20 years ending December 2019.
Source: Bloomberg.

Importantly, correlations are not constant over time. During a long historical period, the average correlation of returns between two asset classes may be low, but in any particular period, the correlation can differ from the average correlation. During periods of market stress, investors may exhibit a "flight to quality" by buying safer assets, such as government bonds (increasing their prices), and selling riskier assets, such as equity securities and high-yield bonds (lowering their prices). These actions may decrease the correlation between government bonds and equity securities, as well as between government bonds and high-yield bonds. At the same time, the correlation between riskier assets, such as equity securities and high-yield bonds, may increase.

Note that similar to correlations, volatility (standard deviation) of asset class returns may also vary over time. If interest rate volatility increases, bonds, particularly those with long maturities, can exhibit higher near-term volatility relative to the average volatility over a long historical period. The standard deviation of returns for lower-credit-quality (high-yield) bonds can rise significantly during times of financial stress, because as credit quality declines and the probability of default increases, investors often view these bonds as being more similar to equities.

Exhibit 4 shows the annual returns of the S&P 500 versus the Bloomberg Barclays US Corporate High Yield Index over a 20-year period ending in December 2019. It illustrates how the fixed-income sector and equities can behave in a similar way. Recall that both asset classes are strongly linked to the issuer's business performance and fundamentals. Over the 20-year period, the average return was 7.96% and 6.26% for the high-yield index and the S&P 500, respectively, and the standard deviation was 15.54% and 17.02%, respectively. The correlation was 0.69.

EXHIBIT 4 Relationship between S&P 500 and High-Yield Returns



2.2. Benefits of Regular Cash Flows

Fixed-income investments typically produce regular cash flows for a portfolio. Regular cash flows allow investors—both individual and institutional—to meet known future obligations, such as tuition payments, pension obligations, and payouts on life insurance policies. In these cases, future liabilities can be estimated with some reasonable certainty. Fixed-income securities are often acquired and "dedicated" to funding those future liabilities. In dedicated portfolios, fixed-income securities are selected with cash flows matching the timing and magnitude of projected future liabilities.

It is important to note that reliance on regular cash flows assumes that no credit event (such as an issuer missing a scheduled interest or principal payment) or other market event (such as a decrease in interest rates that causes an increase in prepayments of mortgages underlying mortgage-backed securities) will occur. These events may cause actual cash flows of fixed-income securities to differ from expected cash flows. If any credit or market event occurs or is forecasted to occur, a portfolio manager may need to adjust the portfolio.

2.3. Inflation-Hedging Potential

Some fixed-income securities can provide a hedge for inflation. Bonds with floating-rate coupons can protect interest income from inflation because the market reference rate should adjust for inflation over time. The principal payment at maturity is unadjusted for inflation. Inflation-linked bonds provide investors with valuable inflation-hedging benefits by paying a return that is directly linked to an index of consumer prices and adjusting the principal for inflation. The return on inflation-linked bonds, therefore, includes a real return plus an additional component that is tied directly to the inflation rate. All else equal, inflation-linked bonds typically exhibit lower return volatility than conventional bonds and equities do because the volatility of the returns on inflation-linked bonds depends on the volatility of *real*, rather than *nominal*, interest rates. The volatility of real interest rates is typically lower than the volatility of nominal interest rates that drive the returns of conventional bonds and equities.

Many governments in developed countries and some in developing countries have issued inflation-linked bonds, as have financial and non-financial corporate issuers. For investors with long investment horizons, especially institutions facing long-term liabilities (for example, defined benefit pension plans and life insurance companies), inflation-linked bonds are particularly useful.

Adding inflation-indexed bonds to diversified portfolios of bonds and equities typically results in superior risk-adjusted real portfolio returns. This improvement occurs because inflation-linked bonds can effectively represent a separate asset class, since they offer returns that differ from those of other asset classes and add to market completeness. Introducing inflation-linked bonds to an asset allocation strategy can result in a superior mean–variance-efficient frontier.

EXAMPLE 1 Adding Fixed-Income Securities to a Portfolio

Mary is anxious about the level of risk in her portfolio because of a recent period of increased equity market volatility. Most of her wealth is invested in a diversified global equity portfolio.

She contacts two wealth management firms (Firm A and Firm B) for advice. In her conversations with each adviser, she expresses her desire to reduce her portfolio's risk and to have a portfolio that generates a cash flow stream with consistent purchasing power over her 15-year investment horizon.

The correlation coefficient of Mary's diversified global equity portfolio with a diversified fixed-coupon bond portfolio is –0.10 and with a diversified inflation-linked bond portfolio is 0.10. The correlation coefficient between a diversified fixed-coupon bond portfolio and a diversified inflation-linked bond portfolio is 0.65.

The adviser from Firm A suggests diversifying half of her investment assets into nominal fixed-coupon bonds. The adviser from Firm B also suggests diversification but recommends that Mary invest 25% of her investment assets in fixed-coupon bonds and 25% in inflation-linked bonds.

Evaluate the advice given to Mary by each adviser on the basis of her stated desires regarding portfolio risk reduction and cash flow stream. Recommend which advice Mary should follow, making sure to discuss the following concepts in your answer:

a. Diversification benefits
b. Cash flow benefits
c. Inflation-hedging benefits

Solution:

Advice from Firm A: Diversifying into fixed-coupon bonds would offer substantial diversification benefits in lowering overall portfolio volatility (risk) given the negative correlation of –0.10. The portfolio's volatility, measured by standard deviation, would be lower than the weighted sum of standard deviations of the diversified global equity portfolio and the diversified fixed-coupon bond portfolio. The portfolio will generate regular cash flows because it includes fixed-coupon bonds. This advice, however, does not address Mary's desire to have the cash flows maintain purchasing power over time and thus serve as an inflation hedge.

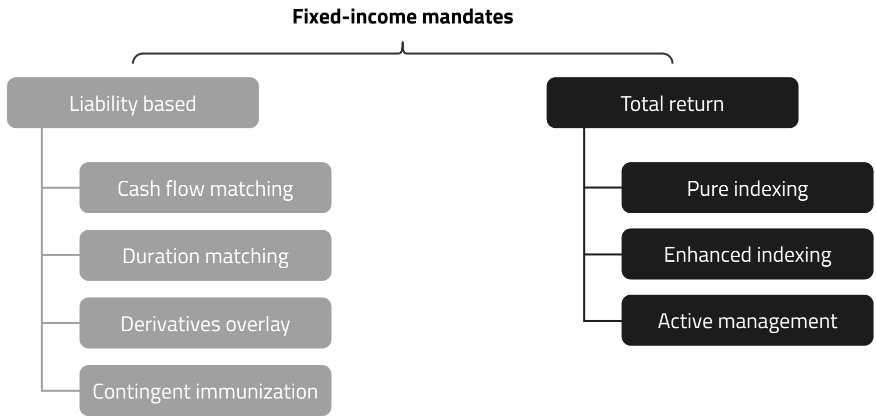
Advice from Firm B: Diversifying into both fixed-coupon bonds and inflation-linked bonds offers additional diversification benefits beyond that offered by fixed-coupon bonds only. The correlation between diversified global equities and inflation-linked bonds is only 0.10. The correlation between nominal fixed-coupon bonds and inflation-linked bonds is 0.65, which is also less than 1.0. The portfolio will generate regular cash flows because of the inclusion of fixed-coupon and inflation-linked bonds. Adding the inflation-linked bonds helps at least partially address Mary's desire for consistent purchasing power over her investment horizon.

Which Advice to Choose: On the basis of her stated desires and the analysis given, Mary should follow the advice provided by Firm B.

3. CLASSIFYING FIXED-INCOME MANDATES

The previous section covered the roles of fixed-income securities in portfolios and the benefits these securities provide. When investment mandates include an allocation to fixed income, investors need to decide how to add fixed-income securities to portfolios. Fixed-income mandates can be broadly classified into liability-based mandates and total return mandates. Exhibit 5 provides a broad overview of the different types of mandates, splitting the universe into two broad categories—liability-based mandates and total return mandates.

EXHIBIT 5 Fixed-Income Mandates



3.1. Liability-Based Mandates

Liability-based mandates are investments that take an investor's future obligations into consideration. Liability-based mandates are managed to match or cover expected liability payments (future cash outflows) with future projected cash inflows. As such, they are also referred to as asset/liability management (ALM) or mandates that use liability-driven investments (LDIs). These types of mandates are structured in a way to ensure that a liability or a stream of liabilities (e.g., a company's pension liabilities or those projected by insurance companies) can be covered and that any risk of shortfalls or deficient cash inflows is minimized. **Cash flow matching** is an immunization approach that attempts to ensure that all future liability payouts are matched precisely by cash flows from bonds or fixed-income derivatives. **Duration matching** is an immunization approach that is based on the duration of assets and liabilities. Ideally, the liabilities being matched (the liability portfolio) and the portfolio of assets (the bond portfolio) should be affected similarly by a change in interest rates. The mandates may use futures contracts (such as in a derivatives overlay) and, as in the case of **contingent immunization**—a hybrid approach that combines immunization with an active management approach when assets exceed the present value of liabilities—may allow for active bond portfolio management. Such liability-based mandates, which will be covered in detail later, are important because of their extensive use by such entities as pension plans and insurance companies.

3.2. Total Return Mandates

Total return mandates are generally managed to either track or outperform a market-weighted fixed-income benchmark, such as the Bloomberg Barclays Global Aggregate Bond Index. They are used by many types of investors, including individuals, foundations, endowments, sovereign wealth funds, and defined contribution retirement plans. Liability-based and total return mandates exhibit common features, such as the goal to achieve the highest risk-adjusted returns (or perhaps highest yields to maturity) given a set of constraints. The two types of mandates, however, have fundamentally different objectives. A common total return approach is **pure indexing**. It attempts to replicate a bond index as closely as possible and is sometimes referred to as "full replication." Under this approach, the targeted **active return** (portfolio return minus benchmark return, also known as "tracking difference") and **active risk** (annualized standard deviation of active returns, also known as the benchmark **tracking risk** or **tracking error**) are both zero. In practice, even if the active risk is zero, the realized portfolio return will almost always be lower than the corresponding index return because of trading costs and management fees. We will explain the limitations of this approach later, in our coverage of index-based strategies.

An **enhanced indexing approach** maintains a close link to the benchmark but seeks to generate some outperformance relative to the benchmark. As with the pure indexing approach, in practice, enhanced indexing allows small deviations in portfolio holdings from the benchmark index but tracks the benchmark's primary risk factor exposures very closely (particularly duration). Unlike the pure indexing approach, however, minor risk factor mismatches (e.g., sector or quality bets) are used in enhanced indexing.

Active management allows larger risk factor mismatches relative to a benchmark index. These mismatches may cause significant return differences between the active portfolio and the underlying benchmark. Most notably, portfolio managers may take views on portfolio duration that differ markedly from the duration of the underlying benchmark. To take advantage of potential opportunities in changing market environments, active managers may incur significant portfolio turnover—often considerably higher than the underlying benchmark's turnover. Active portfolio managers normally charge higher management fees than pure or enhanced indexing managers charge.

Exhibit 6 summarizes the key features of the total return approaches.

EXHIBIT 6 Total Return Approaches: Key Features

	Pure Indexing	Enhanced Indexing	Active Management
Objective	Match benchmark return and risk as closely as possible	Modest outperformance (generally 20–30 bps) of benchmark while active risk is kept low (typically around 50 bps or lower)	Higher outperformance (generally around 50 bps or more) of benchmark and higher active risk levels
Portfolio weights	Ideally the same as benchmark or only slight mismatches	Small deviations from underlying benchmark	Significant deviations from underlying benchmark
Target risk factor profile	Aims to match risk factors exactly	Most primary risk factors are closely matched (in particular, duration)	Large risk factor deviations from benchmark (in particular, duration; note that some active strategies do not take large risk factor deviations and focus on high idiosyncratic risk)
Turnover	Similar to underlying benchmark	Slightly higher than underlying benchmark	Considerably higher than underlying benchmark

3.3. Fixed-Income Mandates with ESG Considerations

Some fixed-income mandates include a requirement that environmental, social, and governance (ESG) factors be considered during the investment process. When considering these factors, an analyst or portfolio manager may look for evidence of whether the portfolio contains companies whose operations are favorable or unfavorable in the context of ESG and whether such companies' actions and resource management practices reflect a sustainable business model. For example, the analyst or portfolio manager may consider whether a company's activities involved significant environmental damage, instances of unfair labor practices, or lapses in corporate governance integrity. For companies that do not fare favorably in an ESG analysis, investors may assume that these companies are more likely to encounter future ESG-related incidents that could cause serious reputational and financial damage to the company. Such incidents could impair a company's credit quality and result in a decline in both the price of the company's bonds and the performance of a portfolio containing those bonds.

EXAMPLE 2 The Characteristics of Different Total Return Approaches

A consultant for a large corporate pension plan is looking at three funds (Funds X, Y, and Z) as part of the pension plan's global fixed-income allocation. All three funds use the Bloomberg Barclays Global Aggregate Bond Index as a benchmark. Exhibit 7 provides characteristics of each fund and the index. Identify the approach (pure indexing, enhanced indexing, or active management) that is *most likely* used by each fund and support your choices by referencing the information in Exhibit 7.

EXHIBIT 7 Characteristics of Funds X, Y, and Z and the Bloomberg Barclays Global Aggregate Bond Index

Risk and Return Characteristics	Fund X	Fund Y	Fund Z	Bloomberg Barclays Global Aggregate Bond Index
Average maturity (years)	8.61	8.35	9.45	8.34
Modified duration (years)	6.37	6.35	7.37	6.34
Average yield to maturity (%)	1.49	1.42	1.55	1.43
Convexity	0.65	0.60	0.72	0.60
Quality				
AAA	41.10	41.20	40.11	41.24
AA	15.32	15.13	14.15	15.05
A	28.01	28.51	29.32	28.78
BBB	14.53	14.51	15.23	14.55
BB	0.59	0.55	1.02	0.35
Not rated	0.45	0.10	0.17	0.05
Maturity Exposure				
0–3 years	21.43	21.67	19.20	21.80
3–5 years	23.01	24.17	22.21	24.23
5–10 years	32.23	31.55	35.21	31.67
10+ years	23.33	22.61	23.38	22.30

Risk and Return Characteristics	Fund X	Fund Y	Fund Z	Bloomberg Barclays Global Aggregate Bond Index
Country Exposure				
United States	42.55	39.44	35.11	39.56
Japan	11.43	18.33	13.33	18.36
France	7.10	6.11	6.01	6.08
United Kingdom	3.44	5.87	4.33	5.99
Germany	6.70	5.23	4.50	5.30
Italy	4.80	4.01	4.43	4.07
Canada	4.44	3.12	5.32	3.15
Other	19.54	17.89	26.97	17.49

Notes: Quality, maturity exposure, and country exposure are shown as a percentage of the total for each fund and the index. Weights do not always sum to 100 because of rounding. Historical data used as of February 2016.
Source: Barclays Research.

Solution: Fund X most likely uses an enhanced indexing approach. Fund X's modified duration and convexity are very close to those of the benchmark but still differ slightly. The average maturity of Fund X is slightly longer than that of the benchmark, whereas Fund X's average yield to maturity is slightly higher than that of the benchmark. Fund X also has deviations in quality, maturity exposure, and country exposures from the benchmark, providing further evidence of an enhanced indexing approach. Some of these deviations are meaningful; for example, Fund X has a relatively strong underweighting in Japan.

Fund Y most likely uses a pure indexing approach because it provides the closest match to the Bloomberg Barclays Global Aggregate Bond Index. The risk and return characteristics are almost identical for Fund Y and the benchmark. Furthermore, quality, maturity exposure, and country exposure deviations from the benchmark are very minor.

Fund Z most likely uses an active management approach because risk and return characteristics, quality, maturity exposure, and country exposure differ markedly from the index. The difference can be seen most notably with the mismatch in modified duration (7.37 for Fund Z versus 6.34 for the benchmark). Other differences between Fund Z and the index exist, but a sizable duration mismatch provides the strongest evidence of an active management approach.

4. FIXED-INCOME PORTFOLIO MEASURES

- **describe fixed-income portfolio measures of risk and return as well as correlation characteristics**

We first provide a brief review of fixed-income risk and return measures introduced in earlier lessons (Exhibit 8).

EXHIBIT 8 Bond Risk and Return Measures

Macaulay duration (MacDur)	Macaulay duration is a weighted average of the time to receipt of the bond's promised payments, where the weights are the shares of the full price that correspond to each of the bond's promised future payments.
Modified duration (ModDur)	The Macaulay duration statistic divided by one plus the yield per period, which estimates the percentage price change (including accrued interest) for a bond given a change in its yield to maturity.
Effective duration (EffDur)	The sensitivity of the bond's price to a change in a benchmark yield curve (i.e., using a parallel shift in the benchmark yield curve [ΔCurve]). Effective duration is essential to the measurement of the interest rate risk of a complex bond where future cash flows are uncertain.
Key rate duration (KeyRatDur, also called *partial duration*)	A measure of a bond's sensitivity to a change in the benchmark yield curve at a specific maturity point or segment. Key rate durations help identify "shaping risk" for a bond or a portfolio—that is, its sensitivity to changes in the shape of the benchmark yield curve (e.g., the yield curve becoming steeper or flatter or showing more or less curvature).
Empirical duration	A measure of interest rate sensitivity that is determined from market data—that is, run a regression of bond price returns on changes in a benchmark interest rate (for example, the price returns of a 10-year euro-denominated corporate bond could be regressed on changes in the 10-year German bund or the 10-year Euribor swap rate).
Money duration	A measure of the price change in units of the currency in which the bond is denominated. Money duration can be stated per 100 of par value or in terms of the bond's actual position size in the portfolio. Commonly called "dollar duration" in the United States.
Price value of a basis point (PVBP)	An estimate of the change in a bond's price given a 1 bp change in yield to maturity. PVBP "scales" money duration so that it can be interpreted as money gained or lost for each basis point change in the reference interest rate. Also referred to in North America as the "dollar value of an 0.01" (pronounced *oh-one*) and abbreviated as DV01. It is calibrated to a bond's par value of 100; for example, a DV01 of \$0.08 is equivalent to 8 cents per 100 points. (The terms PVBP and DV01 are used interchangeably; we will generally use PVBP, but DV01 has the same meaning.) A related statistic to PVBP, sometimes called "basis point value" (or BPV), is the money duration times 0.0001 (1 bp).
Convexity	A second-order effect that describes a bond's price behavior for larger yield movements. It captures the extent to which the yield/price relationship deviates from a linear relationship. If a bond has positive convexity, the expected return of the bond will be higher than the return of an identical-duration, lower-convexity bond if interest rates change. This price behavior is valuable to investors, and therefore, a bond with higher convexity might be expected to have a lower yield to maturity than a similar-duration bond with less convexity. Nominal convexity calculations assume that the cash flows do not change when yields to maturity change.
Effective convexity (EffCon)	A curve convexity statistic that measures the secondary effect of a change in a benchmark yield curve. A pricing model is used to determine the new prices when the benchmark curve is shifted upward (PV+) and downward (PV−) by the same amount (ΔCurve), holding other factors constant.

Exhibit 8 provides a reminder of convexity and why it is valuable. It is likely to be even more valuable when interest rate volatility is expected to increase. This dynamic tends to drive changes in the shape of the yield curve: As convexity becomes more valuable, investors will bid up prices on the longer-maturity bonds (which have more convexity), and the long end of the curve may decline or even invert (or invert further), increasing the curvature of the yield curve. A helpful heuristic for understanding convexity is that for zero-coupon (option free) bonds, the following are true:

- Macaulay durations increase linearly with maturity: A 30-year zero-coupon bond has three times the duration of a 10-year zero-coupon bond. Convexity is approximately proportional to duration squared; therefore, a 30-year zero-coupon bond has about nine times the convexity of a 10-year zero-coupon bond.
- Coupon-paying bonds have more convexity than zero-coupon bonds of the same duration: A 30-year coupon-paying bond with a duration of approximately 18 years has more convexity than an 18-year zero-coupon bond. The more widely dispersed a bond's cash flows are around the duration point, the more convexity it will exhibit. For this reason, a zero-coupon bond has the lowest convexity of all bonds of a given duration.

SCALING CONVENTIONS

Convexity statistics must always be interpreted carefully because there is no convention for how they should be presented. When calculating the impact of convexity in approximating returns, the proper accounting for the scaling of convexity is important. Note that some data vendors report the convexity statistic divided by 100, whereas other applications may use the "raw" number.

4.1. Portfolio Measures of Risk and Return

Building on the measures of risk and return that apply to individual fixed-income securities, we now provide an overview of measures of risk and return applicable to portfolios of fixed-income securities. We will then illustrate their use in fixed income in a portfolio management scenario and refer to them in the subsequent coverage of liability-driven investing and total return strategies.

Bond portfolio duration is the sensitivity of a portfolio of bonds to small changes in interest rates. Recall that it can be calculated as the weighted average of time to receipt of the aggregate cash flows or, more commonly, as the weighted average of the individual bond durations of the portfolio.

Modified duration of a bond portfolio indicates the percentage change in the market value given a change in yield to maturity. If the modified duration of a portfolio is 15, then for a 100 bp increase or decrease in yield to maturity, the market value of the portfolio is expected to decrease or increase by about 15%. Modified duration of a portfolio comprising *j* fixed-income securities can be estimated as

$$\text{AvgModDur} = \sum_{j=1}^{J} \text{ModDur}_j \left(\frac{\text{MV}_j}{\text{MV}} \right) \tag{1}$$

where MV stands for market value of the portfolio and MV_j is the market value of a specific bond.

Convexity of a bond portfolio can be a valuable tool when positioning a portfolio. Importantly, it is a second-order effect; it operates behind duration in importance and can largely be ignored for small yield changes. When convexity is added with the use of derivatives, however, it can be extremely important to returns. This effect will be demonstrated later. Negative convexity may also be an important factor in a bond's or a portfolio's returns. For bonds with short option positions embedded in their structures (such as mortgage-backed securities or callable bonds) or portfolios with short option positions, the convexity effect may be large. For a portfolio comprising *j* fixed-income securities, it can be estimated as

$$\text{AvgConvexity} = \sum_{j=1}^{J} \text{Convexity}_j \left(\frac{\text{MV}_j}{\text{MV}} \right) \tag{2}$$

Adding convexity to a portfolio is not costless. Portfolios with higher convexity are most often characterized by lower yields to maturity. Investors will be willing to pay for increased convexity when they expect yields to change by more than enough to cover the amount given up in yield to maturity. Convexity is more valuable when yields to maturity are more volatile. A portfolio's convexity can be altered by shifting the maturity/duration distribution of bonds in the portfolio, by adding individual bonds with the desired convexity properties, or by using derivatives.

Effective duration and convexity of a portfolio are the relevant summary statistics when future cash flows of bonds in a portfolio are contingent on interest rate changes.

$$\text{Effectiveduration(EffDur)} = \frac{(\text{PV}_-) - (\text{PV}_+)}{2(\Delta\text{Curve})(\text{PV}_0)} \tag{3}$$

$$\text{Effectiveconvexity(EffCon)} = \frac{(\text{PV}_-) + (\text{PV}_+) - 2(\text{PV}_0)}{(\Delta\text{Curve})^2(\text{PV}_0)} \tag{4}$$

Spread duration is a useful measure for determining a portfolio's sensitivity to changes in credit spreads. Duration indicates the percentage price effect of an interest rate change on a bond, and spread duration measures the effect of a change in yield spread on a bond's price. Spread duration provides the approximate percentage increase (decrease) in bond price expected for a 1% decrease (increase) in credit spread.

Duration times spread (DTS) is a modification of the spread duration definition to incorporate the empirical observation that spread changes across the credit spectrum tend to occur on a *proportional percentage* basis rather than being based on *absolute* basis point changes. This measure, reviewed in detail in a later lesson, weights the spread duration by a factor equal to the current credit spread, increasing the magnitude of expected price changes for a given change in spread.

Portfolio dispersion captures the variance of the times to receipt of cash flows with respect to the duration. It is used in measuring interest rate immunization for liabilities. Whereas Macaulay duration is the weighted *average* of the times to receipt of cash flows, dispersion is the weighted *variance*. It measures the extent to which the payments are spread out around the duration. Convexity is affected by the dispersion of cash flows. Higher cash flow dispersion leads to an increase in convexity.

4.2. Correlations between Fixed-Income Sectors

Correlation characteristics refer to the interplay between benchmark rates, spreads, and such factors as currencies. Correlations between fixed-income sectors within a market are likely to be higher than those across markets given country-specific factors, such as central bank policy, economic growth, and inflation. In developed economies, investment-grade securities with a low probability of default are highly correlated with interest rate changes in the sovereign yield curve. Below-investment-grade securities are affected more by changes in spread than by changes in general interest rates and often exhibit stronger correlations with equity markets. Recall that correlations between interest rates and spreads can often be negative. As the economy worsens, interest rates fall and spreads widen, and the reverse occurs when the economy improves. Correlations for global government bonds will be partly driven by changes in interest rates but also by changes in local currency exchange rates.

4.3. Use of Measures of Risk and Return in Portfolio Management

We now provide an overview of how portfolio measures may be used by fund managers to reflect their views.

4.3.1. Portfolio Duration in Total Return Mandates

Total return mandates that are actively managed often use a top-down approach to establish the large risk factors in a portfolio combined with a bottom-up approach of individual security selection. The analytics discussed earlier can be used to measure and manage the macroeconomic risk factors in the portfolio. Portfolio managers develop or use a forecast of the direction of the economy and an assessment of the current business, political, and regulatory environment to develop themes that can be reflected in the portfolio. On the basis of expectations for changes in interest rates and the shape of the yield curve, portfolio managers can adjust the duration of a portfolio to reflect their view. For example, if the portfolio manager expects interest rates to rise and the yield curve to steepen, she would reduce the exposure of the portfolio to longer-dated bonds relative to the benchmark, which would reduce portfolio duration. If her view materialized as expected, all else equal, the fund would outperform the benchmark, resulting in active excess returns.

4.3.2. Managing Credit Exposure Using Spread Duration

Portfolio managers often use the spread duration measures introduced earlier to gauge the portfolio's sensitivity to changes in credit spreads. A portfolio manager expecting credit spreads to narrow may wish to increase the spread duration in an actively managed portfolio. The manager may face constraints, such as a target duration, rating-based restrictions, or limits to derivatives use, as part of the investment mandate. A second way to increase the portfolio credit exposure is to reduce the average credit rating of the portfolio; for example, reduce A rated names and increase BBB rated credits. In this case, the duration times spread measure may be a more appropriate measure of portfolio value changes. These active portfolio management tools are addressed in more detail in a later lesson on credit strategies.

The single bond risk and return measures discussed previously at an aggregate level will determine the large risk factors for the portfolio. The portfolio manager will select securities as part of the portfolio construction process to achieve a targeted level of tracking error or active risk relative to a benchmark. The contribution to duration, convexity, spread duration, and DTS of a single bond to the portfolio is weighted by the market value of the position relative to the total market value of the portfolio. The portfolio manager will select a diversified universe of holdings to construct the portfolio in the manner he believes will optimize expected return and risk.

4.3.3. Relative Value Concept

Relative value is a key concept in the active management of fixed-income portfolios that describes the selection of the most attractive individual securities to populate the portfolio with, using ranking and comparing. Portfolio managers analyze and rank securities on the basis of such considerations as valuation, issuer fundamentals, and market technical conditions (supply and demand). This analysis is carried out across sectors, issuers, and individual securities to select securities with the most attractive risk and return profiles. The portfolio manager will establish a time horizon over which the relative value analysis is applied. The single bond characteristics can be used to express an active position relative to the benchmark. For example, each bond has a distinct key rate duration (KeyRateDur) profile. If the portfolio manager wants to establish a bullet or barbell position as part of the active risk decision, bonds with a specific KeyRateDur profile will be selected. Similarly, the portfolio manager can select securities that in aggregate have more/less DTS than the benchmark if she is bullish/bearish on corporate bond spreads. The selection of the most attractive individual securities to populate the portfolio will apply relative value analysis to compare and rank securities. In the context of the efficient frontier, those securities that offer the most expected return for a given level of risk would offer the best relative value.

The positioning of the portfolio reflects the portfolio manager's total return expectations for the market and relative returns versus the benchmark, given his views with regard to both the direction of interest rates and credit spread changes. Diversification considerations ensure that idiosyncratic risks are within acceptable risk parameters.

EXAMPLE 3

1. Which of the following best describes a measure of sensitivity to changes in yields to maturity for a portfolio of bonds with cash flows contingent on interest rate changes?
   A. Portfolio dispersion
   B. Modified duration
   C. Effective duration
2. Which of the following is a true statement about portfolio dispersion?
   A. It can be described as the variance of time to the receipt of cash flows.
   B. The higher the dispersion, the lower the convexity of the portfolio.
   C. It determines the portfolio's sensitivity to changes in credit spreads.

Solutions:

1. C is correct. Effective duration is particularly relevant in scenarios where the cash flows from the bonds held in a portfolio are contingent on changes in interest rates.
2. A is correct. Dispersion measures the variance of the time to receive cash flows from the fixed-income securities held.

5. BOND MARKET LIQUIDITY

- **describe bond market liquidity, including the differences among market sub-sectors, and discuss the effect of liquidity on fixed-income portfolio management**

A liquid security is one that may be transacted quickly with little effect on the security's price. Fixed-income securities vary greatly in their liquidity.

Compared with equities, fixed-income markets are generally less liquid. The global fixed-income universe contains many individual bonds with varying features. Many issuers have multiple bonds outstanding with their own unique maturity dates, coupon rates, early redemption features, and other specific features.

An important structural feature affecting liquidity is that fixed-income markets are typically over-the-counter dealer markets. Search costs (the costs of finding a willing counterparty) exist in bond markets because investors may have to locate desired bonds. In addition, when either buying or selling, investors may have to obtain quotes from various dealers to obtain the most advantageous pricing. With limited, although improving, sources for transaction prices and quotes, bond markets are ordinarily less transparent than equity markets. Liquidity, search costs, and price transparency are closely related to the type of issuer and its credit quality. An investor is likely to find that bonds of a highly creditworthy government issuer are more liquid, have greater price transparency, and have lower search costs than bonds of, for example, a corporate issuer with lower credit quality.

Bond liquidity is typically highest immediately after issuance. For example, an on-the-run bond issue (the most recently issued bonds) of a highly creditworthy sovereign entity is typically more liquid than a bond with similar features—including maturity—that was issued previously (an off-the-run bond). On-the-run bonds also trade at narrow bid–ask spreads. This difference in liquidity is typically present even if the off-the-run bond was issued only one or two months earlier. One reason for this phenomenon is that soon after bonds are issued, dealers normally have a supply of the bonds in inventory, but as time goes by and bonds are traded, many are purchased by buy-and-hold investors. Once in the possession of such investors, those bonds are no longer available for trading.

Recall that liquidity typically affects bond yields to maturity. Bond investors require higher yields for investing in illiquid securities relative to otherwise identical securities that are more liquid. The higher yield to maturity compensates investors for the costs they may encounter if they try to sell illiquid bonds prior to maturity. These costs include the opportunity costs associated with the delays in finding trading counterparties, as well as the bid–ask spread (which is a direct loss of wealth). The incremental yield to maturity investors require for holding illiquid bonds instead of liquid bonds is referred to as a *liquidity premium*. The magnitude of the liquidity premium normally varies depending on such factors as the issuer, the issue size, and time to maturity. For example, when a 10-year US Treasury bond shifts from on-the-run to off-the-run status, it typically trades at a yield to maturity several basis points above that of the new on-the-run bond.

5.1. Liquidity among Bond Market Sub-Sectors

Bond market liquidity varies across sub-sectors. These sub-sectors can be categorized by such key features as issuer type, credit quality, issue size, and maturity. The global bond market includes sovereign government bonds, non-sovereign government bonds, government-related bonds, corporate bonds, and securitized bonds (such as asset-backed securities and commercial mortgage-backed securities). Sovereign government bonds are typically more liquid than corporate and non-sovereign government bonds. Their superior liquidity relates to their large issuance size, use as benchmark bonds, acceptance as collateral in the repo market, and well-recognized issuers. Sovereign government bonds of countries with high credit quality and large issuance are typically more liquid than bonds of lower-credit-quality countries.

Corporate bonds are issued by many different companies and represent a wide spectrum of credit quality. For corporate bonds with low credit quality, it can be difficult to find a counterparty dealer with the securities in inventory or willing to take them into inventory. Bonds of infrequent issuers are often less liquid than the bonds of issuers with many outstanding issues because market participants are less familiar with companies that seldom issue debt. In addition, smaller issues are generally less liquid than larger issues because small bond issues are typically excluded from major bond indexes with minimum issue size requirements.

5.2. The Effects of Liquidity on Fixed-Income Portfolio Management

Liquidity concerns influence fixed-income portfolio management in multiple ways, including pricing, portfolio construction, and consideration of alternatives to bonds (such as derivatives).

5.2.1. Pricing

Sources for pricing of recent bond transactions—notably corporate bonds—are not always readily available. Note that price transparency is improving in some bond markets. In the United States, the Financial Industry Regulatory Authority's Trade Reporting and Compliance Engine (TRACE) and the Municipal Securities Rulemaking Board's Electronic Municipal Market Access (EMMA) are electronic systems that help increase transparency in corporate and municipal bond markets, and similar initiatives play a similar role elsewhere for corporate bonds traded on market exchanges, increasing pricing transparency. In most bond markets, however, the lack of transparency in corporate bond trading presents a challenge.

Because many bonds do not trade or trade infrequently, using recent transaction prices to represent current value is not practical. Reliance on last traded prices, which may be out of date and may not incorporate current market conditions, could result in costly trading decisions. The determinants of corporate bond value, including interest rates, credit spreads, and liquidity premiums, change frequently. One solution to the pricing problem is to use matrix pricing that makes use of observable liquid benchmark yields of similar maturity and duration as well as benchmark spreads of bonds with comparable times to maturity, credit quality, and sector or security type to estimate the current market yield and price.

5.2.2. Portfolio Construction

Investors' liquidity preferences directly influence portfolio construction. In constructing a portfolio, investors must consider the important trade-off between yield to maturity and liquidity. As mentioned previously, illiquid bonds typically have higher yields to maturity; a buy-and-hold investor seeking higher returns will often prefer less liquid bonds with higher yields to maturity. In contrast, investors who prefer greater liquidity will likely sacrifice returns and choose more liquid bonds with lower yields to maturity. Some investors may restrict their portfolio holdings to bonds within a certain maturity range. This restriction reduces the need to sell bonds to generate needed cash inflows. In such cases, the investors that anticipate their liquidity needs may give up the higher yield to maturity typically available to longer-term bonds. In addition to avoiding longer-term bonds, investors with liquidity concerns may also avoid small issues and private placements of corporate bonds.

A challenge in bond portfolio construction relates to the dealer market. Bond dealers often carry an inventory of bonds because buy and sell orders do not arrive simultaneously. A dealer is not certain how long bonds will remain in its inventory. Less liquid bonds are likely to remain in inventory longer than liquid bonds. A dealer provides bid–ask quotes (prices at which it will buy and sell) on bonds of its choice. Some illiquid bonds will not have quotes, particularly bid quotes, from any dealer. A number of different factors determine the bid–ask spread. Riskier bonds often have higher bid–ask spreads because of dealers' aversion to hold those bonds in inventory. Because bond dealers must finance their inventories, the dealers incur costs in both obtaining funding and holding those bonds. Dealers seek to cover their costs and make a profit through the bid–ask spread, and therefore, the spread will be higher for illiquid bonds that are likely to remain in inventory longer.

A bond's bid–ask spread is also a function of the bond's complexity and how easily market participants can analyze the issuer's creditworthiness. Bid–ask spreads in government bonds are generally lower than spreads in corporate bonds or structured financial instruments, such as asset-backed securities. Conventional (plain vanilla) corporate bonds normally have lower spreads than corporate bonds with non-standard or complex features, such as embedded options. Bonds of large, high-credit-quality corporations that have many outstanding bond issues are the most liquid among corporate bonds, and thus they have relatively low bid–ask spreads compared with smaller, less creditworthy companies.

Illiquidity directly increases bid–ask spreads of bonds, which increases the cost of trading. Higher transaction costs reduce the benefits of active portfolio decisions and may decrease portfolio managers' willingness to adjust their portfolios to take advantage of opportunities that present themselves. As an example to quantify trading costs, if a corporate bond with a 15-year duration is being quoted by dealers with a 10 bp bid–ask spread, the cost impact to the portfolio is approximately 1.50% ($0.0010 \times 15 \times 100 = 1.50\%$). The portfolio manager would buy the bond at $100, and when the portfolio is priced (typically at bid or the midpoint between the bid and the ask), the bond would have a value of $98.50, reducing total portfolio return. This is the price that would be realized if the bond were sold, holding other factors constant. To mitigate trading costs, investors can participate in the primary or new issue market where bonds are typically issued at a discount to the price at which a similar issue trades in the secondary market.

KNOWLEDGE CHECK

Rank the following instruments from the usually most liquid to the least liquid:

- Low-credit-quality corporate bond
- Recently issued on-the-run sovereign bond issued by a high-credit-quality government
- High-credit-quality corporate bond
- Sovereign bond issued a year ago by a high-credit-quality government

Solution:

- Recently issued on-the-run sovereign bond issued by a high-credit-quality government
- Sovereign bond issued a year ago by a high-credit-quality government
- High-credit-quality corporate bond
- Low-credit-quality corporate bond

5.2.3. Alternatives to Direct Investment in Bonds

Because transacting in fixed-income securities may present challenges resulting from low liquidity in many segments of the fixed-income market, fund managers may use alternative methods to establish bond market exposures. The methods we outline are applicable across different fixed-income mandates. We will take a more in-depth look at the ones particularly relevant to passive and liability-driven mandates later as part of our coverage dedicated to such mandates. Next, we provide an overview of the most common methods—specifically, mutual funds, exchange-traded funds (ETFs), exchange-traded derivatives, and OTC derivatives. In considering direct versus indirect investments, the asset manager must weigh the ongoing fees associated with such instruments as mutual funds and ETFs against the bid–offer cost of direct investment in the underlying securities.

ETFs and mutual funds. These products provide an alternative to transacting in individual bonds. They are more liquid than the underlying securities. Mutual funds are pooled investment vehicles whose shares or units represent a proportional share in the ownership of the assets in an underlying portfolio. In the case of open-end mutual funds, new shares may be redeemed or issued at the fund's net asset value (NAV) established at the end of each trading day based on the fund's valuation of all existing assets minus liabilities, divided by the total number of shares outstanding. Bond mutual fund investors enjoy the advantage of being able to redeem holdings at the fund's NAV rather than needing to sell illiquid positions. The benefit from economies of scale is usually the overriding factor for smaller investors in their choice of a bond mutual fund over direct investment. Because bonds often trade at a minimum lot size of USD1 million or higher per bond, successful replication of a broad index or construction of a diversified actively managed portfolio could easily require hundreds of millions of dollars in investments. Therefore, the greater diversification across fixed-income markets achievable by a larger fund may be well worth the additional cost in terms of an upfront load in some instances and an annual management fee.

Although investors benefit from increased diversification, the fund must outline its stated investment objectives and periodic fees, but actual security holdings are available only on a retroactive basis. Unlike the underlying securities, bond mutual funds have no maturity date; the fund manager continuously purchases and sells bonds to track index performance, and monthly interest payments fluctuate on the basis of fund holdings.

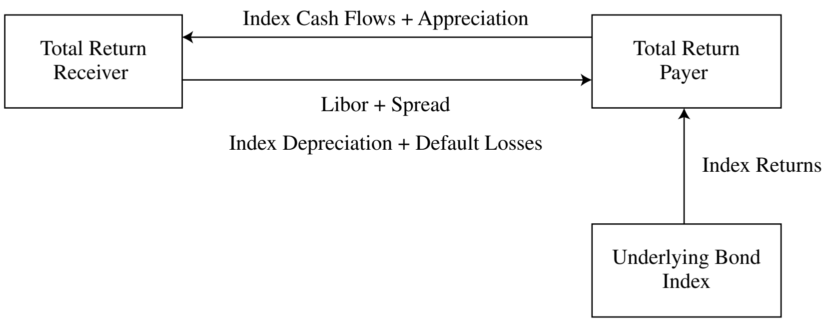
Exchange-traded funds share some mutual fund characteristics but have more tradability features. Investors benefit from greater bond ETF liquidity versus mutual funds given their availability to be purchased or sold throughout the trading day.

Exchange traded derivatives. Futures and options on futures provide exposure to underlying bonds. Being exchange traded, they involve financial instruments with standardized terms, documentation, and pricing traded on an organized exchange. Exchange-traded products also include interest rate products and options for interest rate–related ETFs.

OTC derivatives. Interest rate swaps are the most widely used OTC derivative worldwide and entail customized arrangements between two counterparties that reference an underlying market price or index. Some interest rate swaps are liquid, with multiple swap dealers posting competitive two-way quotes. In addition to interest rate swaps, fixed-income portfolio managers use inflation swaps, total return swaps, and credit swaps to alter their portfolio exposure. Because they trade over the counter, swaps may be tailored to an investor's specific needs.

A total return swap (TRS), a common over-the-counter portfolio derivative strategy, combines elements of interest rate swaps and credit derivatives. Similar to an interest rate swap, a total return swap involves the periodic exchange of cash flows between two parties for the life of the contract. Unlike an interest rate swap, in which counterparties exchange a stream of fixed cash flows versus a floating-rate benchmark such as the MRR (the market reference rate) to transform fixed assets or liabilities to a variable exposure, a TRS has a periodic exchange based on a reference obligation that is an underlying equity, commodity, or bond index. Exhibit 9 outlines the most basic TRS structure. The **total return receiver** receives both the cash flows from the underlying index and any appreciation in the index over the period in exchange for paying the MRR plus a predetermined spread. The **total return payer** is responsible for paying the reference obligation cash flows and return to the receiver but will also be compensated by the receiver for any depreciation in the index or default losses incurred by the portfolio.

EXHIBIT 9 Total Return Swap Mechanics



The TRS transaction is an over-the-counter derivative contract based on an ISDA (International Swaps and Derivatives Association) master agreement. This contract specifies a notional amount, periodic cash flows, and final maturity, as well as the credit and other legal provisions related to the transaction. The historical attractiveness of using TRS stemmed from the efficient risk transfer on the reference obligation from one counterparty to another on a confidential basis without requiring the full cash outlay associated with the mutual fund or ETF purchase. In fact, another way to think of the TRS is as a synthetic secured financing transaction in which the investor (the total return receiver) benefits from more-advantageous funding terms faced by a dealer (typically the total return payer) offering to facilitate the transaction.

The potential for both a smaller initial cash outlay and lower swap bid–offer costs compared with the transaction costs of direct purchase or use of a mutual fund or ETF are the most compelling reasons to consider a TRS to add fixed-income exposure.

That said, several considerations may offset these benefits in a number of instances:

- The investor does not legally own the underlying assets but, rather, has a combined synthetic long position in both the market and the credit risk of the index that is contingent on the performance of the total return payer. The total return receiver must both perform the necessary credit due diligence on its counterparty and face the rollover risk at maturity of having the ability to renew the contract with reasonable pricing and business terms in the future.
- Structural changes to the market and greater regulatory oversight, particularly capital rules affecting dealers, have raised the cost and increased the operational burden of these transactions because of the need to collateralize mark-to-market positions frequently and within shorter timeframe.

6. A MODEL FOR FIXED-INCOME RETURNS

- **describe and interpret a model for fixed-income returns**

Investors often have views on future changes in the yield curve and structure or restructure their portfolios accordingly. Investment strategies should be evaluated in terms of expected returns rather than just yields to maturity. A bond's yield to maturity provides an incomplete measure of its expected return. Instead, expected fixed-income returns consist of a number of different components in addition to yield to maturity. Examining these components leads to a better understanding of the driving forces behind expected returns—on individual bonds and fixed-income portfolios. The focus is on *expected* as opposed to *realized* returns, which may be decomposed in a similar manner.

6.1. Decomposing Expected Returns

Decomposing expected fixed-income returns allows an investor to differentiate among several important return components. At the most general level, expected returns, denoted as E(*R*), can be decomposed (approximately) in the following manner:

E(*R*) ≈ Coupon income

+/− Rolldown return

+/− E(ΔPrice due to investor's view of benchmark yield)

+/− E(ΔPrice due to investor's view of yield spreads)

+/− E(ΔPrice due to investor's view of currency value changes),

where E(. . .) represents effects on expected returns based on expectations of the item in parentheses and Δ represents "change." The decomposition holds only approximately and ignores taxes (note that some of the material on decomposing expected returns has been adapted from Hanke and Seals [2010]).

6.1.1. Coupon Income

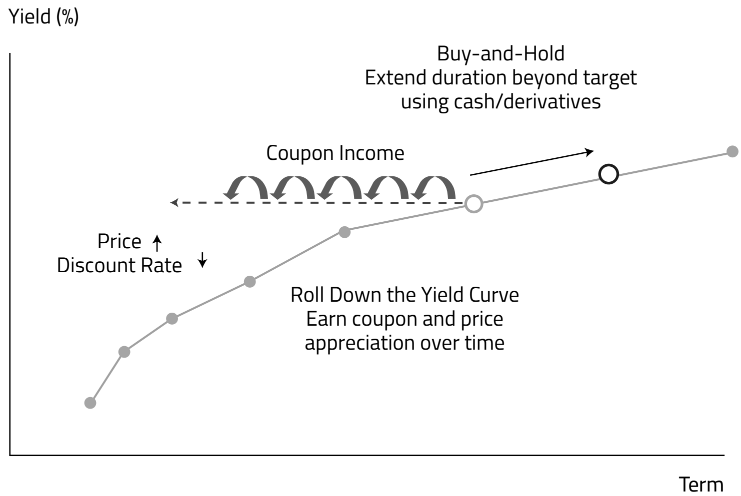
Coupon income is the income that an investor receives from coupon payments relative to the bond's price and interest on reinvestment income. Assuming there is no reinvestment income, coupon income equals a bond's annual current yield.

Coupon income (or Current yield) = Annual coupon payment/Current bond price

6.1.2. Rolldown Return

The rolldown return, sometimes referred to as "rolldown and carry return," results from the bond "rolling down" the yield curve as the time to maturity decreases (see Exhibit 10), assuming zero interest rate volatility. Bond prices change as time passes even if the market discount rate remains the same. As time passes, a bond's price typically moves closer to par. This price movement is illustrated by the constant-yield price trajectory, which shows the "pull to par" effect on the price of a bond trading at a premium or a discount to par value. If the issuer does not default, the price of a bond approaches par value as its time to maturity approaches zero.

EXHIBIT 10 Rolling down the Yield Curve Effect



The rolldown return equals the bond's percentage price change assuming an unchanged yield curve over the strategy horizon. Bonds trading at a premium to their par value will experience capital losses during their remaining life, and bonds trading at a discount relative to their par value will experience capital gains during their remaining life.

To compute the rolldown return, the bond has to be revalued at the end of the strategy horizon assuming an unchanged yield curve. Then the rolldown return is as follows:

$$\text{Rolldown return} = \frac{\left(\text{Bond price}_{\textit{End-of-horizon period}} - \text{Bond price}_{\textit{Beginning-of-horizon period}}\right)}{\text{Bond price}_{\textit{Beginning-of-horizon period}}} \quad (5)$$

The sum of the coupon income and the rolldown return may be referred to as the bond's *rolling yield*.

6.1.3. Views of Benchmark Yields

The expected change in price based on investor's views of benchmark yields to maturity and the term structure of yield volatility reflects an investor's expectation of changes in yields to maturity and yield volatility over the investment horizon. This expected change is zero if the investor expects yield curves and yield volatility to remain unchanged. Assuming the investor does expect a change in the yield curve, this expected return component is computed as follows:

$$\begin{aligned}&\text{E(Change in price based on investor's views of yields and yield volatility)}\\&= (-\text{ModDur} \times \Delta\text{Yield}) + [\tfrac{1}{2} \times \text{Convexity} \times (\Delta\text{Yield})^2]\end{aligned} \quad (6)$$

where ModDur is the modified duration of a bond, ΔYield is the expected change in yield to maturity, and Convexity reflects the second-order effect of the price–yield relationship. Note that for bonds with embedded options, the duration and convexity measures used should be effective duration and effective convexity. Also, in contrast to fixed-coupon bonds, floating-rate notes have a modified duration that is largely due to spread changes, as described in detail later.

6.1.4. Views of Yield Spreads

The expected change in price based on an investor's view of yield spreads reflects an investor's expectation of changes in market credit spreads over the investment horizon. When economic or credit conditions are improving, spreads are typically said to tighten, thereby reducing the required yield to maturity on the bond. Deteriorating conditions would conversely result in higher required yields to maturity. This component of expected return reflects general market conditions rather than any spread changes due to issuer-specific (or idiosyncratic) risk and is computed as follows:

$$\text{E}(\Delta\text{Price based on investor's views of yield spreads}) \\ = (-\text{ModDur} \times \Delta\text{Yield}) + [\tfrac{1}{2} \times \text{Convexity} \times (\Delta\text{Yield})^2] \quad (7)$$

Yield spreads can also fluctuate on the basis of idiosyncratic risk. Credit migration refers to credit quality changes that may result in an issuer downgrade or upgrade. This can result in either lower spreads for higher ratings or higher spreads for lower ratings affecting the expected return on bonds. Higher-quality credits tend to have low probabilities of default but can experience changes in bond prices due to an anticipated or actual migration. The price impact is calculated in the same way as noted previously for market changes in yield to maturity. Note that investors face price declines on non-defaulted bonds if spreads widen. Yearly default rates can vary significantly and are more severe for speculative-grade (high-yield) issues.

6.1.5. Views of Currency Value Changes

If an investor holds bonds denominated in a currency other than her home currency, she also needs to factor in any expected fluctuations in the currency exchange rate or expected currency gains or losses over the investment horizon. The magnitude and direction of the change in the exchange rate can be based on a variety of factors, including the manager's own view, results from surveys, or a quantitative model output. It can also be based on the exchange rate that can be locked in over the investment horizon using currency forwards.

Return measured in functional currency terms (domestic currency returns of foreign currency assets) can be shown as $R_{DC} = (1 + R_{FC})(1 + R_{FX}) - 1$ for a single asset or

$$R_{DC} = \sum_{i=1}^{n} \omega_i(1 + R_{FC,i})(1 + R_{FX,i}) - 1 \quad (8)$$

for a portfolio, where R_{DC} and R_{FC} are the domestic and foreign currency returns expressed as a percentage, R_{FX} is the percentage change of the domestic currency versus the foreign currency, and ω_i is the respective portfolio weight of each foreign currency asset (in domestic currency terms), with the sum of ω_i equal to 1. In the context of the return decomposition framework, R_{DC} simply combines the third factor, E(ΔPrice due to investor's view of benchmark yield), and the fifth factor, +/− E(ΔFunctional currency value), in the expected fixed-income return model.

EXAMPLE 4 Decomposing Expected Returns

Ann Smith works for a US investment firm in its London office. She manages the firm's British pound–denominated corporate bond portfolio. Her department head in New York City has asked Smith to make a presentation on next year's total expected return of her portfolio in US dollars and the components of this return. Exhibit 11 shows information on the portfolio and Smith's expectations for the next year. Expected return (for the bond portfolio) and its components are on an annualized basis, and any potential coupons are assumed to be paid annually. Calculate the total expected return of Smith's bond portfolio, assuming no reinvestment income.

EXHIBIT 11 Portfolio Characteristics and Expectations

Notional principal of portfolio (in millions)	£100
Average bond coupon payment (per £100 par value)	£2.75
Coupon frequency	Annual
Investment horizon	1 year
Current average bond price	£97.12
Expected average bond price in one year (assuming an unchanged yield curve)	£97.27
Average bond convexity in one year	18
Average bond modified duration in one year	3.70
Expected average benchmark yield-to-maturity change	0.26%
Expected change in spread (spread expected to narrow in this scenario)	–0.10%
Expected currency losses (£ depreciation versus US$)	0.50%

Solution: The portfolio's coupon income is 2.83%. The portfolio has an average coupon of £2.75 on a £100 notional principal and currently trades at £97.11. The coupon income over a one-year horizon is 2.83% = £2.75/£97.11.

In one year's time, assuming an unchanged yield curve and zero interest rate volatility, the rolldown return is 0.17% = (£97.27 – £97.11)/£97.11.

The rolling yield, which is the sum of the coupon income and the rolldown return, is 3.00% = 2.83% + 0.17%.

The expected change in price based on Smith's views of benchmark yields to maturity is –0.96%, calculated as follows: The bond portfolio has a modified duration of 3.70 and a convexity statistic of 18. Smith expects an average benchmark yield-to-maturity change of 0.26%. Smith expects to incur a decrease in prices and a reduction in return based on her rate view. The expected change in price based on Smith's views of yields to maturity and yield spreads is thus $-0.0096 = (-3.70 \times 0.0026) + [\frac{1}{2} \times 18 \times (0.0026)^2]$. So the expected reduction in return based on Smith's rate view is 0.96%.

Smith expects an impact from the 0.1% change (narrowing in this scenario) in spread in her well-diversified investment-grade bond portfolio. The impact on the expected return is, therefore, $0.37\% = [-3.70 \times (-0.0010)] + [1/2 \times 18 \times (-0.0010)^2]$.

Smith expects the British pound, the foreign currency in which her bond position is denominated, to depreciate by an annualized 50 bps (or 0.5%) over the investment horizon against the US dollar, the home country currency. The expected currency loss to the portfolio is thus 0.50%.

The total expected return on Smith's bond position is 1.91%, as summarized in Exhibit 12.

EXHIBIT 12 Return Component Calculations

Return Component	Formula	Calculation
Coupon income	Annual coupon payment/Current bond price	£2.75/£97.11 = 2.83%
+ Rolldown return	$\frac{\left(\text{Bond price}_{\text{End-of-horizon period}} - \text{Bond price}_{\text{Beginning-of-horizon period}}\right)}{\text{Bond price}_{\text{Beginning-of-horizon period}}}$	(£97.27 – £97.11)/£97.11 = 0.17%
= Rolling yield	Coupon income + Rolldown return	2.83% + 0.17% = 3.00%
+/– E(ΔPrice* based on Smith's benchmark yield view)	$(-\text{ModDur} \times \Delta\text{Yield}) + [½ \times \text{Convexity} \times (\Delta\text{Yield})^2]$	$(-3.70 \times 0.0026) + [½ \times 18 \times (0.0026)^2] = -0.96\%$
+/– E(ΔPrice due to investor's view of credit spreads)	$(-\text{ModDur} \times \Delta\text{Spread}) + [½ \times \text{Convexity} \times (\Delta\text{Spread})^2]$	$(-3.70 \times -0.0010) + [1/2 \times 18 \times (-0.0010)^2] = 0.37$
+/– E(Currency gains or losses)	Given	–0.50%
= Total expected return		**1.91%**

*Note that the change in price in the context of this example refers to the change in portfolio value.

6.2. Estimation of the Inputs

In the model for fixed-income returns discussed earlier, some of the individual expected return components can be more easily estimated than others. The easiest component to estimate is the coupon income. The return model's most uncertain individual components are the investor's views of changes in benchmark yields and yield spreads and expected currency movements. These components are normally based on purely qualitative (subjective) criteria, a quantitative model (including surveys), or a mixture of the two. Although a quantitative approach may seem more objective, there are a number of quantitative models that can be used, each with different methodologies associated with the underlying calculations.

6.3. Limitations of the Expected Return Decomposition

The return decomposition just described is an approximation; only duration and convexity are used to summarize the price–yield relationship. In addition, the model implicitly assumes that all intermediate cash flows of the bond are reinvested at the yield to maturity, which results in different coupon reinvestment rates for different bonds.

The model also ignores other factors, such as local richness/cheapness effects and potential financing advantages. Local richness/cheapness effects are deviations of individual maturity segments from the fitted yield curve, which was obtained using a curve estimation technique. Yield curve estimation techniques produce relatively smooth curves, and there are likely slight deviations from the curve in practice. There may be financing advantages to certain maturity segments in the repo market. The repo market provides a form of short-term borrowing for dealers in government securities who sell government bonds to other market participants overnight and buy them back, typically on the following day. In most cases, local richness/cheapness effects and financing advantages tend to be relatively small and are thus not included in the expected return decomposition model.

EXAMPLE 5 Components of Expected Return

Kevin Tucker manages a global bond portfolio. At a recent investment committee meeting, Tucker discussed his portfolio's domestic (very high-credit-quality) government bond allocation with another committee member. The other committee member argued that if the yield curve is expected to remain unchanged, the only determinants of a domestic government bond's expected return are its coupon payment and its price.

Explain why the other committee member is incorrect, including a description of the additional expected return components that need to be included.

Solution: A bond's coupon payment and its price allow only its coupon income to be computed. Coupon income is an incomplete measure of a bond's expected return. For domestic government bonds, in addition to coupon income, the rolldown return needs to be considered. The rolldown return results from the fact that bonds are pulled to par as the time to maturity decreases, even if the yield curve is expected to remain unchanged over the investment horizon. Currency gains and losses would also need to be considered in a global portfolio. Because the portfolio consists of government bonds with very high credit quality, the view on yield spreads is less relevant for Tucker's analysis. For government and corporate bonds with lower credit quality, however, yield spreads would also need to be considered as additional return components.

7. LEVERAGE

- **discuss the use of leverage, alternative methods for leveraging, and risks that leverage creates in fixed-income portfolios**

Leverage is the use of borrowed capital to increase the magnitude of portfolio positions, and it is an important tool for fixed-income portfolio managers. By using leverage, fixed-income portfolio managers may be able to increase portfolio returns relative to what they can achieve in unleveraged portfolios.

Managers often have mandates that place limits on the types of securities they may hold. Simultaneously, managers may have return objectives that are difficult to achieve, especially during low–interest rate environments. Through the use of leverage, a manager can increase his investment exposure and may be able to increase the returns to fixed-income asset classes that typically have low returns. The increased return potential, however, comes at the cost of increased risk: If losses occur, these would be higher than in unleveraged positions.

7.1. Using Leverage

Leverage increases portfolio returns if the securities in the portfolio have returns higher than the cost of borrowing. In an unleveraged portfolio, the return on the portfolio (r_p) equals the return on invested funds (r_I). When the manager uses leverage, however, the invested funds exceed the portfolio's equity by the amount that is borrowed.

The leveraged portfolio return, r_p, can be expressed as the total investment gains per unit of invested capital:

$$r_P = \frac{\text{Portfolio return}}{\text{Portfolio equity}} = \frac{r_I \times (V_E + V_B) - (V_B \times r_B)}{V_E} \tag{9}$$

where

V_E = Value of the portfolio's equity
V_B = Borrowed funds
r_B = Borrowing rate (cost of borrowing)
r_I = Return on the invested funds (investment returns)
r_p = Return on the levered portfolio

The numerator represents the total return on the portfolio assets, $r_I \times (V_E + V_B)$, minus the cost of borrowing, $V_B \times r_B$, divided by the portfolio's equity.

The leveraged portfolio return can be decomposed further to better identify the effect of leverage on returns:

$$\begin{aligned} r_P &= \frac{r_I \times (V_E + V_B) - (V_B \times r_B)}{V_E} \\ &= \frac{(r_I \times V_E) + [V_B \times (r_I - r_B)]}{V_E} \\ &= r_I + \frac{V_B}{V_E}(r_I - r_B) \end{aligned}$$

This expression decomposes the leveraged portfolio return into the return on invested funds and a portion that accounts for the effect of leverage. If $r_I > r_B$, then the second term is positive because the rate of return on invested funds exceeds the borrowing rate; in this case, leverage increases the portfolio's return. If $r_I < r_B$, then the second term is negative because the rate of return on invested funds is less than the borrowing rate; in this case, the use of leverage decreases the portfolio's return. The degree to which the leverage increases or decreases portfolio returns is proportional to the use of leverage (amount borrowed), V_B/V_E, and the amount by which investment return differs from the cost of borrowing, $r_I - r_B$.

7.2. Methods for Leveraging Fixed-Income Portfolios

Fixed-income portfolio managers have a variety of tools available to create leveraged portfolio exposures—notably, the use of financial derivatives and borrowing via collateralized money markets. Derivatives and borrowing are explicit forms of leverage. Other forms of leverage, such as the use of structured financial instruments, are more implicit. We provide a description of the most common ones.

7.2.1. Futures Contracts

Futures contracts embed significant leverage because they permit the counterparties to gain exposure to a large quantity of the underlying asset without having to actually transact in the underlying. Futures contracts can be obtained for a modest investment that comes in the form of a margin deposit. A futures contract's notional value equals the current value of the underlying asset multiplied by the multiplier, or the quantity of the underlying asset controlled by the contract.

The futures leverage is the ratio of the futures exposure (in excess of the margin deposit) normalized by the amount of margin required to control the notional amount. We can calculate the futures leverage using the following equation:

$$\text{Leverage}_{Futures} = \frac{\text{Notional value} - \text{Margin}}{\text{Margin}} \tag{10}$$

7.2.2. Swap Agreements

An interest rate swap can be viewed as a portfolio of bonds. In an interest rate swap, the fixed-rate payer is effectively short a fixed-rate bond and long a floating-rate bond. When interest rates increase, the value of the swap to the fixed-rate payer increases because the present value of the fixed-rate liability decreases and the floating-rate payments received increase. The fixed-rate receiver in the interest rate swap agreement effectively has a long position in a fixed-rate bond and a short position in a floating-rate bond. If interest rates decline, the value of the swap to the fixed-rate receiver increases because the present value of the fixed-rate asset increases and the floating-rate payments made decrease.

Because interest rate swaps are economically equivalent to a long–short bond portfolio, they provide leveraged exposure to bonds; the only capital required to enter into swap agreements is collateral required by the counterparties. Collateral for interest rate swap agreements has historically occurred between the two (or more) counterparties in the transaction. Increasingly, collateral for interest rate and other swaps occurs through central clearinghouses.

7.2.3. Repurchase Agreements

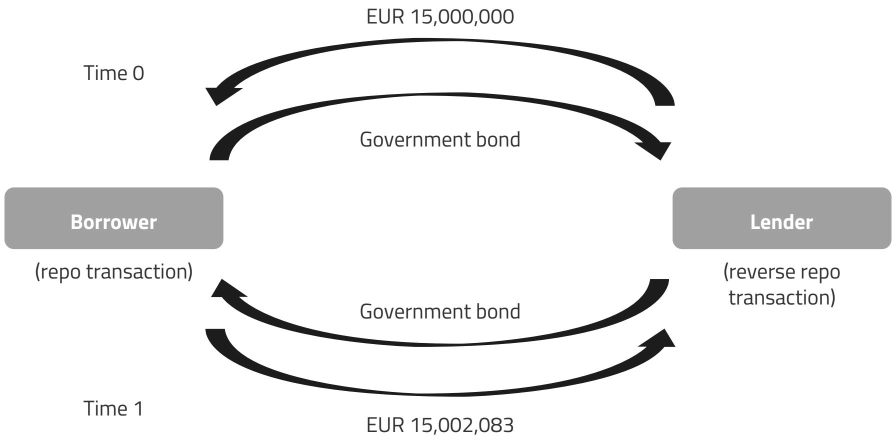
Repurchase agreements (repos) are an important source of short-term financing for fixed-income security dealers and other financial institutions, as evidenced by the trillions of dollars of repo transactions that take place annually. In a repurchase agreement, a security owner agrees to sell a security for a specific cash amount while simultaneously agreeing to repurchase the security at a specified future date (typically one day later) and price. Repos are thus effectively collateralized loans. When discussing a repo, the transaction normally refers to the borrower's standpoint; from the standpoint of the lender (such as a money market fund), these agreements are referred to as **reverse repos**. Exhibit 13 illustrates the transaction.

The interest rate on a repurchase agreement, called the **repo rate**, is the difference between the security's selling price and its repurchase price. For example, consider a dealer wishing to finance a EUR15 million bond position with a repurchase agreement. The dealer enters into an overnight repo at a repo rate of 5%. We can compute the price at which she agrees to repurchase this bond after one day as the EUR15 million value today plus one day of interest. The interest amount is computed as follows:

$$\text{Dollar interest} = \text{Principal amount} \times \text{Repo rate} \times (\text{Term of repo in days}/360)$$

Continuing with the example, the dollar interest is EUR2,083.33 = EUR15 million × 5% × (1/360). Thus, the dealer will repurchase the bond the next day for EUR15,002,083.33.

EXHIBIT 13 Repo and Reverse Repo



The term, or length, of a repurchase agreement is measured in days. Overnight repos are common, although they are often rolled over to create longer-term funding. A repo agreement may be cash driven or security driven. Cash-driven transactions feature one party that owns bonds and wants to borrow cash, as in the foregoing example. Cash-driven transactions usually feature "general collateral"—securities commonly accepted by investors and dealers, such as Treasury bonds. In a security-driven transaction, the lender typically seeks a particular security. The motives may be for hedging, arbitrage, or speculation.

Credit risk is a concern in a repo agreement, in particular for the counterparty that lends capital. Protection against a default by the borrower is provided by the underlying collateral bonds. Additional credit protection comes from the "haircut," the amount by which the collateral's value exceeds the repo principal amount. For example, haircuts for high-quality government bonds typically range from 1% to 3% and are higher for other types of bonds. The size of the haircut serves to not only protect the lender against a potential default by the borrower but also to limit the borrower's net leverage capacity. Generally, the size of the haircut increases as the price volatility of the underlying collateral increases.

Repos are categorized as bilateral repos or tri-party repos, depending on the way they are settled. Bilateral repos are conducted directly between two institutions, and settlement is typically conducted as "delivery versus payment," meaning that the exchanges of cash and collateral occur simultaneously through a central custodian (for example, the Depository Trust Company in the United States). Bilateral repos are usually used for security-driven transactions. Tri-party repo transactions involve a third party that provides settlement and collateral management services. Most cash-motivated repo transactions against general collateral are conducted as tri-party repo transactions.

7.2.4. Security Lending

Security lending is another form of collateralized lending and is closely linked to the repo market. The primary motive of security lending transactions is to facilitate short sales, which involve the sale of securities the seller does not own. A short seller must borrow the securities he has sold short in order to deliver them upon trade settlement. Another motive for security lending transactions is financing, or collateralized borrowing. In a financing-motivated security loan, a bond owner lends the bond to another investor in exchange for cash.

Security lending transactions are collateralized by cash or high-credit-quality bonds. In the United States, most transactions feature cash collateral, although in many other countries, highly rated bonds are used as collateral. Typically, security lenders require collateral valued in excess of the value of the borrowed securities when bonds are used as collateral. For example, if high-quality government bonds are used as collateral, the lender may require bonds valued at 102% of the value of the borrowed securities. The extra 2% functions in the same way as the haircut in the repo market, providing extra protection against borrower default. The collateral required will increase if lower-quality bonds are used as collateral.

In security lending transactions with cash collateral, the security borrower typically pays the security lender, typically a long-only investment fund, a fee equal to a percentage of the value of the securities loaned. For securities that are readily available for lending, that fee is small. The security lender earns an additional return by reinvesting the cash collateral. In cases where the security loan is initiated for financing purposes, the lending fee is typically negative, indicating that the security lender pays the security borrower a fee in exchange for its use of the cash.

When bonds are posted as collateral, the income earned on the collateral usually exceeds the security lending rate; the security lender (who is in possession of the bonds as collateral) usually repays the security borrower a portion of the interest earned on the bond collateral. The term **rebate rate** refers to the portion of the collateral earnings rate that is repaid to the security borrower by the security lender. This relationship can be expressed as follows:

$$\text{Rebate rate} = \text{Collateral earnings rate} - \text{Security lending rate}$$

When securities are difficult to borrow, typically because there is high demand to short those securities, the rebate rate may be negative, which means the fee for borrowing the securities is greater than the return earned on the collateral. In this case, the security borrower pays a fee to the security lender in addition to forgoing the interest earned on the collateral.

There are important differences between repurchase agreements and security lending transactions. Unlike repurchase agreements, security lending transactions are typically open-ended. The security lender may recall the securities at any time, forcing the borrower to deliver the bonds by buying them back or borrowing from another lender. Similarly, the borrower may deliver the borrowed securities back to the lender at any time, forcing the lender, or its agent, to return the collateral (cash or bonds) and search for another borrower.

7.3. Risks of Leverage

Leverage alters the risk–return properties of an investment portfolio. A heavily leveraged portfolio may incur significant losses even when portfolio assets suffer only moderate valuation declines.

Leverage can lead to forced liquidations. If the value of the portfolio decreases, the portfolio's equity relative to borrowing levels is reduced and the portfolio's leverage increases. Portfolio assets may be sold in order to pay off borrowing and reduce leverage. If portfolio assets are not liquidated, then the overall leverage increases, corresponding to higher levels of risk. Decreases in portfolio value can lead to forced liquidations even if market conditions are unfavorable for selling—for example, during crisis periods. The term "fire sale" refers to forced liquidations at prices that are below fair value as a result of the seller's need for immediate liquidation. Reducing leverage, declining asset values, and forced sales have the potential to create spiraling effects that can result in severe declines in values and reduction in market liquidity.

Additionally, reassessments of counterparty risk typically occur during extreme market conditions, such as during the 2008–09 financial crisis. During periods of financial crisis, counterparties to short-term financing arrangements, such as credit lines, repurchase agreements, and security lending agreements, may withdraw their financing. These withdrawals undermine the ability of leveraged market participants to maintain their investment exposures. Thus, leveraged investors may be forced to reduce their investment exposure at exactly the worst time—that is, when prices are depressed.

EXAMPLE 6 Using Leverage in a Fixed-Income Portfolio

Arturo manages a mutual fund that is benchmarked to the Global Aggregate Bond Index. He currently has a bullish view of the global economy and believes corporate bond spreads are attractive. He is bearish on US Treasury interest rates given his economic growth forecast and expects rates to increase. The fund's US corporate bond holdings have a duration of seven years. He believes the best opportunities are in emerging market securities, and in particular, he is bullish on Brazilian rates, expecting them to decrease. The fund has experienced strong inflows recently and is fully invested. Arturo is evaluating tools to potentially increase the fund's total return by creating leveraged fixed-income exposures.

Given Arturo's plan to leverage exposures in his fund, discuss how he would achieve his objectives and identify the strategy risks.

Solution: The mutual fund is fully invested and, therefore, Arturo needs to use leverage to potentially increase his returns. His bearish view on US Treasury interest rates would require that he reduce the fund's seven-year duration contributed by the US corporate bond holdings. He can sell the number of futures contracts on US Treasuries whose notional value and associated duration would offset the duration of the corporate bonds to his new target duration. Doing so would allow him to retain exposure (spread duration) to the corporate bonds whose spreads may contract as the economy grows while shedding the interest rate exposure, since he believes rates will rise, adversely affecting bond prices.

Arturo's bullish view on Brazilian rates can be expressed by entering into a receive fixed-rate, pay floating-rate swap on Brazilian rates. The fund will effectively have the equivalent of a fixed-rate bond that will appreciate in price if his view materializes and Brazilian interest rates fall.

Both the short US Treasury futures and long Brazilian interest rate swap positions are leveraged since the only capital used is the collateral required by the counterparties. The risk to the leveraged strategy is that if Arturo's view on either position turns out to be incorrect, losses are magnified. This may also require positions to be closed and assets sold to cover the losses, which may occur at an inopportune time if the markets have sold off.

8. FIXED-INCOME PORTFOLIO TAXATION

- **discuss differences in managing fixed-income portfolios for taxable and tax-exempt investors**

A tax-exempt investor's objective is to achieve the highest possible risk-adjusted returns net of fees and transaction costs. A prudent taxable investor needs to also consider the effects of taxes on both expected and realized net investment returns.

The investment management industry has traditionally made investment decisions based on pretax returns as though investors are tax exempt (such as pension funds in many countries; see Rogers [2006]). The majority of the world's investable assets, however, are owned by taxable investors, who are concerned with after-tax, rather than pretax, returns.

Taxes may differ among investor types, among countries, and on the basis of income source, such as interest or capital gains. In many countries, pension funds are exempt from taxes but corporations generally have to pay tax on their investments. Many countries make some allowance for tax-sheltered investments that individuals can use (up to certain limits). These types of tax shelters generally offer either an exemption from tax on investment income or a deferral of taxes until an investor draws money from the shelter (usually after retirement). Such shelters allow returns to accrue on a pretax basis until retirement, which can provide substantial benefits. In a fixed-income context for taxable investors, coupon payments (interest income) are typically taxed at the investor's normal income tax rate. Capital gains, however, may be taxed at a lower effective rate than an investor's normal income tax rate. In some countries, income from special types of fixed-income securities, such as bonds issued by a sovereign government, a non-sovereign government, or various government agencies, may be taxed at a lower effective rate or even not taxed.

Specific tax rules vary among jurisdictions. Any discussion of the effect of taxes on investor returns—and, therefore, on how portfolios should optimally be managed for taxable investors—is especially challenging if it needs to apply on a global level. Although accounting standards have become more harmonized globally, any kind of tax harmonization among countries is not likely to occur anytime soon. An investor should consider how taxes affect investment income in the country where the income is earned and how the investment income is treated when it is repatriated to the investor's home country. Treaties between countries may affect tax treatment of investment income. Taxes are complicated and can make investment decisions difficult. Portfolio managers who manage assets for taxable individual investors, as opposed to tax-exempt investors, need to consider a number of issues.

8.1. Principles of Fixed-Income Taxation

Although tax codes differ among jurisdictions, there are certain principles that most tax codes have in common with regard to taxation of fixed-income investments:

- The two primary sources of investment income that affect taxes for fixed-income securities are coupon payments (interest income) and capital gains or losses.
- In general, tax is payable only on capital gains and interest income that have actually been received. In some countries, an exception to this rule applies to zero-coupon bonds. Imputed interest that is taxed throughout a zero-coupon bond's life may be calculated. This method of taxation ensures that tax is paid over the bond's life and that the return on a zero-coupon bond is not taxed entirely as a capital gain.
- Capital gains are frequently taxed at a lower effective tax rate than interest income.
- Capital losses generally cannot be used to reduce sources of income other than capital gains. Capital losses reduce capital gains in the tax year in which they occur. If capital losses exceed

capital gains in the year, they can often be "carried forward" and applied to gains in future years; in some countries, losses may also be "carried back" to reduce capital gains taxes paid in prior years. Limits on the number of years that capital losses can be carried forward or back typically exist.
- In some countries, short-term capital gains are taxed at a different (usually higher) rate than long-term capital gains.

An investor or portfolio manager generally has no control over the timing of when coupon income is received and the related income tax must be paid. However, he or she can generally decide the timing of the sale of investments and, therefore, has some control over the timing of realized capital gains and losses. This control can be valuable for a taxable investor because it may be optimal to delay realizing gains and related tax payments and to realize losses as early as possible. This type of tax-driven strategic behavior is referred to as tax-loss harvesting.

Key points for managing taxable fixed-income portfolios include the following:

- Selectively offset capital gains and losses for tax purposes.
- If short-term capital gains tax rates are higher than long-term capital gains tax rates, then be judicious when realizing short-term gains.
- Realize losses taking into account tax consequences. They may be used to offset current or future capital gains for tax purposes.
- Control turnover in the fund. In general, the lower the turnover, the longer capital gains tax payments can be deferred.
- Consider the trade-off between capital gains and income for tax purposes.

8.2. Investment Vehicles and Taxes

The choice of investment vehicle often affects how investments are taxed at the final investor level. In a pooled investment vehicle (sometimes referred to as a *collective investment scheme*), such as a mutual fund, interest income is generally taxed at the final investor level when it occurs—regardless of whether the fund reinvests interest income or pays it out to investors. In other words, the fund is considered to have distributed interest income for tax purposes in the year it is received even if it does not actually pay it out to investors. Taxation of capital gains arising from the individual investments within a fund is often treated differently in different countries.

Some countries, such as the United States, use what is known as *pass-through treatment* of capital gains in mutual funds. Realized net capital gains in the underlying securities of a fund are treated as if distributed to investors in the year that they arise, and investors need to include the gains on their tax returns. Other countries, such as the United Kingdom, do not use pass-through treatment. Realized capital gains arising within a fund increase the net asset value of the fund shares that investors hold. Investors pay taxes on the net capital gain when they sell their fund shares. This tax treatment leads to a deferral in capital gains tax payments. A UK portfolio manager's decisions on when to realize capital gains or losses do not affect the timing of tax payments on capital gains by investors.

In a separately managed account, an investor typically pays tax on realized gains in the underlying securities at the time they occur. The investor holds the securities directly rather than through shares in a fund. For separately managed accounts, the portfolio manager needs to consider tax consequences for the investor when making investment decisions.

Tax-loss harvesting, which we defined earlier as deferring the realization of gains and realizing capital losses early, allows investors to accumulate gains on a pretax basis. The deferral of taxes increases the present value of investments for the investor.

EXAMPLE 7 Managing Taxable and Tax-Exempt Portfolios

A bond portfolio manager needs to raise €10,000,000 in cash to cover outflows in the portfolio she manages. To satisfy her cash demands, she considers one of two corporate bond positions for potential liquidation: Position A and Position B. For tax purposes, capital gains receive pass-through treatment; realized net capital gains in the underlying securities of a fund are treated as if distributed to investors in the year that they arise. Assume that the capital gains tax rate is 28% and the income tax rate for interest is 45%. Exhibit 14 provides relevant data for the two bond positions.

EXHIBIT 14 Selected Data for Two Bonds

	Position A		Position B
Current market value	€10,000,000		€10,000,000
Capital gain/loss	€1,000,000		–€1,000,000
Coupon rate	5.00%		5.00%
Remaining maturity	10 years		10 years
Income tax rate		45%	
Capital gains tax rate		28%	

The portfolio manager considers Position A to be slightly overvalued and Position B to be slightly undervalued. Assume that the two bond positions are identical with regard to all other relevant characteristics. How should the portfolio manager optimally liquidate bond positions if she manages the portfolio for:

1. tax-exempt investors?
2. taxable investors?

Solution to 1: The taxation of capital gains and capital losses has minimal consequences for tax-exempt investors. Consistent with the portfolio manager's investment views, the portfolio manager would likely liquidate Position A, which she considers slightly overvalued, rather than liquidating Position B, which she considers slightly undervalued.

Solution to 2: All else equal, portfolio managers for taxable investors should have an incentive to defer capital gains taxes and realize capital losses early (tax-loss harvesting) so that losses can be used to offset current or future capital gains. Despite the slight undervaluation of the position, the portfolio manager might want to liquidate Position B because of its embedded capital loss, which will result in a lower realized net capital gain being distributed to investors. This decision is based on the assumption that there are no other capital losses in the portfolio that can be used to offset other capital gains. Despite the slight overvaluation of Position A, its liquidation would be less desirable for a taxable investor because of the required capital gains tax.

SUMMARY

- Fixed-income investments provide diversification benefits in a portfolio context. These benefits arise from the generally low correlations of fixed-income investments with other major asset classes, such as equities.
- Floating-rate and inflation-linked bonds can be used to hedge inflation risk.
- Fixed-income investments have regular cash flows, which is beneficial for the purposes of funding future liabilities.
- For liability-based fixed-income mandates, portfolio construction follows two main approaches—cash flow matching and duration matching—to match fixed-income assets with future liabilities.
- Total return mandates are generally structured to either track or outperform a benchmark.
- Total return mandates can be classified into various approaches according to their target active return and active risk levels. Approaches range from pure indexing to enhanced indexing to active management.
- Bond Portfolio Duration is the sensitivity of a portfolio of bonds to small changes in interest rates. It can be calculated as the weighted average of time to receipt of the aggregate cash flows or, more commonly, as the weighted average of the individual bond durations that comprise the portfolio.
- Modified Duration of a Bond Portfolio indicates the percentage change in the market value given a change in yield-to-maturity. Modified duration of a portfolio comprising *j* fixed income securities can be estimated as

$$\text{AvgModDur} = \sum_{j=1}^{J} \text{ModDur}_j \left(\frac{\text{MV}_j}{\text{MV}} \right)$$

where MV stands for market value of the portfolio and MV_j is the market value of a specific bond.

- Convexity of a bond portfolio is a second-order effect; it operates behind duration in importance and can largely be ignored for small yield changes. When convexity is added with the use of derivatives, however, it can be extremely important to returns.
- Effective duration and convexity of a portfolio are the relevant summary statistics when future cash flows of bonds in a portfolio are contingent on interest rate changes.
- Spread duration is a useful measure for determining a portfolio's sensitivity to changes in credit spreads. It provides the approximate percentage increase (decrease) in bond price expected for a 1% decrease (increase) in credit spread.
- Duration times spread is a modification of the spread duration definition to incorporate the empirical observation that spread changes across the credit spectrum tend to occur on a *proportional percentage* basis rather than being based on *absolute* basis point changes.
- Portfolio dispersion captures the variance of the times to receipt of cash flows around the duration. It is used in measuring interest rate immunization for liabilities.
- Duration management is the primary tool used by fixed-income portfolio managers.
- Convexity supplements duration as a measure of a bond's price sensitivity for larger movements in interest rates. Adjusting convexity can be an important portfolio management tool.
- For two portfolios with the same duration, the portfolio with higher convexity has higher sensitivity to large declines in yields to maturity and lower sensitivity to large increases in yields to maturity.
- Interest rate derivatives can be used effectively to increase or decrease duration and convexity in a bond portfolio.

- Liquidity is an important consideration in fixed-income portfolio management. Bonds are generally less liquid than equities, and liquidity varies greatly across sectors.
- Liquidity affects pricing in fixed-income markets because many bonds either do not trade or trade infrequently.
- Liquidity affects portfolio construction because there is a trade-off between liquidity and yield to maturity. Less liquid bonds have higher yields to maturity, all else being equal, and may be more desirable for buy-and-hold investors. Investors anticipating liquidity needs may forgo higher yields to maturity for more liquid bonds.
- Investors can obtain exposure to the bond market using mutual funds and ETFs that track a bond index. Shares in mutual funds are redeemable at the net asset value with a one-day time lag. ETF shares have the advantage of trading on an exchange.
- A total return swap, an over-the-counter derivative, allows an institutional investor to transform an asset or liability from one asset category to another—for instance, from variable-rate cash flows referencing the market reference rate to the total return on a particular bond index.
- A total return swap can have some advantages over a direct investment in a bond mutual fund or ETF. As a derivative, it requires less initial cash outlay than direct investment in the bond portfolio for similar performance but carries counterparty risk.
- As a customized over-the-counter product, a TRS can offer exposure to assets that are difficult to access directly, such as some high-yield and commercial loan investments.
- When evaluating fixed-income investment strategies, it is important to consider expected returns and to understand the various components of expected returns.
- Decomposing expected fixed-income returns allows investors to understand the different sources of returns given expected changes in bond market conditions.
- A model for expected fixed-income returns can decompose them into the following components: coupon income, rolldown return, expected change in price based on investor's views of yields to maturity and yield spreads, and expected currency gains or losses.
- Leverage is the use of borrowed capital to increase the magnitude of portfolio positions. By using leverage, fixed-income portfolio managers may be able to increase portfolio returns relative to what they can achieve in unleveraged portfolios. The potential for increased returns, however, comes with increased risk.
- Methods for leveraging fixed-income portfolios include the use of futures contracts, swap agreements, repurchase agreements, structured financial instruments, and security lending.
- Taxes can complicate investment decisions in fixed-income portfolio management. Complications result from the differences in taxation among investor types, countries, and income sources.
- The two primary sources of investment income that affect taxes for fixed-income securities are coupon payments (interest income) and capital gains or losses. Tax is usually payable only on capital gains and interest income that have actually been received.
- Capital gains are frequently taxed at a lower effective tax rate than interest income. If capital losses exceed capital gains in the year, they can often be "carried forward" and applied to gains in future years.

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PRACTICE PROBLEMS

The following information relates to Questions 1–6

Cécile is a junior analyst for an international wealth management firm. Her supervisor, Margit, asks Cécile to evaluate three fixed-income funds as part of the firm's global fixed-income offerings. Selected financial data for the funds Aschel, Permot, and Rosaiso are presented in Exhibit 1. In Cécile's initial review, she assumes that there is no reinvestment income and that the yield curve remains unchanged.

EXHIBIT 1 Selected Data on Fixed-Income Funds

	Aschel	Permot	Rosaiso
Current average bond price	$117.00	$91.50	$94.60
Expected average bond price in one year (end of Year 1)	$114.00	$96.00	$97.00
Average modified duration	7.07	7.38	6.99
Average annual coupon payment	$3.63	$6.07	$6.36
Present value of portfolio's assets (millions)	$136.33	$68.50	$74.38
Bond type*			
Fixed-coupon bonds	95%	38%	62%
Floating-coupon bonds	2%	34%	17%
Inflation-linked bonds	3%	28%	21%
Quality*			
AAA	65%	15%	20%
BBB	35%	65%	50%
B	0%	20%	20%
Not rated	0%	0%	10%
Value of portfolio's equity (millions)	$94.33		
Value of borrowed funds (millions)	$42.00		
Borrowing rate	2.80%		
Return on invested funds	6.20%		

*Bond type and quality are shown as a percentage of total for each fund.

After further review of the composition of each of the funds, Cécile makes the following notes:

Note 1: Aschel is the only fund of the three that uses leverage.
Note 2: Rosaiso is the only fund of the three that holds a significant number of bonds with embedded options.

Margit asks Cécile to analyze liability-based mandates for a meeting with Villash Foundation. Villash Foundation is a tax-exempt client. Prior to the meeting, Cécile identifies what she considers to be two key features of a liability-based mandate.

Feature 1 It can minimize the risk of deficient cash inflows for a company.

Feature 2 It matches expected liability payments with future projected cash inflows.

Two years later, Margit learns that Villash Foundation needs $5 million in cash to meet liabilities. She asks Cécile to analyze two bonds for possible liquidation. Selected data on the two bonds are presented in Exhibit 2.

EXHIBIT 2 Selected Data for Bonds 1 and 2

	Bond 1	Bond 2
Current market value	$5,000,000	$5,000,000
Capital gain/loss	400,000	–400,000
Coupon rate	2.05%	2.05%
Remaining maturity	8 years	8 years
Investment view	Overvalued	Undervalued
Income tax rate	39%	
Capital gains tax rate	30%	

1. Based on Exhibit 1, which fund provides the highest level of protection against inflation for coupon payments?
   A. Aschel
   B. Permot
   C. Rosaiso
2. Based on Exhibit 1, the rolling yield of Aschel over a one-year investment horizon is *closest* to:
   A. –2.56%.
   B. 0.54%.
   C. 5.66%.
3. The leveraged portfolio return for Aschel is *closest* to:
   A. 7.25%.
   B. 7.71%.
   C. 8.96%.
4. Based on Note 2, Rosaiso is the only fund for which the expected change in price based on the investor's views of yields to maturity and yield spreads should be calculated using:
   A. convexity.
   B. modified duration.
   C. effective duration.
5. Is Cécile correct with respect to key features of liability-based mandates?
   A. Yes
   B. No, only Feature 1 is correct.
   C. No, only Feature 2 is correct.
6. Based on Exhibit 2, the optimal strategy to meet Villash Foundation's cash needs is the sale of:
   A. 100% of Bond 1.
   B. 100% of Bond 2.
   C. 50% of Bond 1 and 50% of Bond 2.

The following information relates to Questions 7–12

Celia is chief investment officer for the Topanga Investors Fund, which invests in equities and fixed income. The clients in the fund are all taxable investors. The fixed-income allocation includes a domestic (US) bond portfolio and an externally managed global bond portfolio.

The domestic bond portfolio has a total return mandate, which specifies a long-term return objective of 25 basis points (bps) over the benchmark index. Relative to the benchmark, small deviations in sector weightings are permitted, such risk factors as duration must closely match, and tracking error is expected to be less than 50 bps per year.

The objectives for the domestic bond portfolio include the ability to fund future liabilities, protect interest income from short-term inflation, and minimize the correlation with the fund's equity portfolio. The correlation between the fund's domestic bond portfolio and equity portfolio is currently 0.14. Celia plans to reduce the fund's equity allocation and increase the allocation to the domestic bond portfolio. She reviews two possible investment strategies.

Strategy 1 Purchase AAA rated fixed-coupon corporate bonds with a modified duration of two years and a correlation coefficient with the equity portfolio of −0.15.

Strategy 2 Purchase US government agency floating-coupon bonds with a modified duration of one month and a correlation coefficient with the equity portfolio of −0.10.

Celia realizes that the fund's return may decrease if the equity allocation of the fund is reduced. Celia decides to liquidate $20 million of US Treasuries that are currently owned and to invest the proceeds in the US corporate bond sector. To fulfill this strategy, Celia asks Dan, a newly hired analyst for the fund, to recommend specific Treasuries to sell and corporate bonds to purchase.

Dan recommends Treasuries from the existing portfolio that he believes are overvalued and will generate capital gains. Celia asks Dan why he chose only overvalued bonds with capital gains and did not include any bonds with capital losses. Dan responds with two statements.

Statement 1: Taxable investors should prioritize selling overvalued bonds and always sell them before selling bonds that are viewed as fairly valued or undervalued.

Statement 2: Taxable investors should never intentionally realize capital losses.

Regarding the purchase of corporate bonds, Dan collects relevant data, which are presented in Exhibit 1.

EXHIBIT 1 Selected Data on Three US Corporate Bonds

Bond Characteristics	Bond 1	Bond 2	Bond 3
Credit quality	AA	AA	A
Issue size ($ millions)	100	75	75
Maturity (years)	5	7	7
Total issuance outstanding ($ millions)	1,000	1,500	1,000
Months since issuance	New issue	3	6

Celia and Dan review the total expected 12-month return (assuming no reinvestment income) for the global bond portfolio. Selected financial data are presented in Exhibit 2.

EXHIBIT 2 Selected Data on Global Bond Portfolio

Notional principal of portfolio (in millions)	€200
Average bond coupon payment (per €100 par value)	€2.25
Coupon frequency	Annual
Investment horizon	1 year
Current average bond price	€98.45
Expected average bond price in one year (assuming an unchanged yield curve)	€98.62
Average bond convexity	22
Average bond modified duration	5.19
Expected average benchmark yield-to-maturity change	0.15%
Expected change in credit spread (widening)	0.13%
Expected currency gains (€ appreciation vs. $)	0.65%

Celia contemplates adding a new manager to the global bond portfolio. She reviews three proposals and determines that each manager uses the same index as its benchmark but pursues a different total return approach, as presented in Exhibit 3.

EXHIBIT 3 New Manager Proposals: Fixed-Income Portfolio Characteristics

Sector Weights (%)	Manager A	Manager B	Manager C	Index
Government	53.5	52.5	47.8	54.1
Agency/quasi-agency	16.2	16.4	13.4	16.0
Corporate	20.0	22.2	25.1	19.8
MBS	10.3	8.9	13.7	10.1
Risk and Return Characteristics	**Manager A**	**Manager B**	**Manager C**	**Index**
Average maturity (years)	7.63	7.84	8.55	7.56
Modified duration (years)	5.23	5.25	6.16	5.22
Average yield to maturity (%)	1.98	2.08	2.12	1.99
Turnover (%)	207	220	290	205

7. Which approach to its total return mandate is the fund's domestic bond portfolio *most likely* to use?
   A. Pure indexing
   B. Enhanced indexing
   C. Active management
8. Strategy 2 is *most likely* preferred to Strategy 1 for meeting the objective of:
   A. protecting against inflation.
   B. funding future liabilities.
   C. minimizing the correlation of the fund's domestic bond portfolio and equity portfolio.
9. Are Dan's statements to Celia that support Dan's choice of bonds to sell correct?
   A. Only Statement 1 is correct.
   B. Only Statement 2 is correct.
   C. Neither Statement 1 nor Statement 2 is correct.

10. Based on Exhibit 1, which bond *most likely* has the highest liquidity premium?
    A. Bond 1
    B. Bond 2
    C. Bond 3
11. Based on Exhibit 2, the total expected return of the fund's global bond portfolio is *closest* to:
    A. 0.90%.
    B. 1.66%.
    C. 3.76%.
12. Based on Exhibit 3, which manager is *most likely* to have an active management total return mandate?
    A. Manager A
    B. Manager B
    C. Manager C

CHAPTER 13

LIABILITY-DRIVEN AND INDEX-BASED STRATEGIES

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Donald J. Smith, PhD

LEARNING OUTCOMES

The candidate should be able to:

- describe liability-driven investing;
- evaluate strategies for managing a single liability;
- compare strategies for a single liability and for multiple liabilities, including alternative means of implementation;
- describe construction, benefits, limitations, and risk–return characteristics of a laddered bond portfolio;
- evaluate liability-based strategies under various interest rate scenarios and select a strategy to achieve a portfolio's objectives;
- explain risks associated with managing a portfolio against a liability structure;
- discuss bond indexes and the challenges of managing a fixed-income portfolio to mimic the characteristics of a bond index;
- compare alternative methods for establishing bond market exposure passively;
- discuss criteria for selecting a benchmark and justify the selection of a benchmark.

1. INTRODUCTION

Fixed-income instruments make up nearly three-quarters of all global financial assets available to investors. It is thus not surprising that bonds are a critical component of most investment portfolios. In our coverage of structured and passive total return fixed-income investment strategies, we explain that "passive" does not simply mean "buy and hold." The primary strategies discussed—immunization and indexation—can entail frequent rebalancing of the bond portfolio. We also note that "passive" stands in contrast to "active" fixed-income strategies that are based on the asset manager's particular view on interest rate and credit market conditions.

We explain liability-driven investing by demonstrating how to best structure a fixed-income portfolio when considering both the asset and liability sides of the investor's balance sheet. It is first important to have a thorough understanding of both the timing and relative certainty of future financial obligations. Because it is rare to find a bond investment whose characteristics perfectly match one's obligations, we introduce the idea of structuring a bond portfolio to match the future cash flows of one or more liabilities that have bond-like characteristics. Asset–liability management (ALM) strategies are based on the concept that investors incorporate both rate-sensitive assets and liabilities into the portfolio decision-making process. When the liabilities are given and assets are managed, liability-driven investing (LDI), a common type of ALM strategy, may be used to ensure adequate funding for an insurance portfolio, a pension plan, or an individual's budget after retirement. The techniques and risks associated with LDI are introduced using a single liability and then are expanded to cover both cash flow and duration-matching techniques and multiple liabilities. This strategy, known as immunization, may be viewed simply as a special case of interest rate hedging.

We then turn our attention to index-based investment strategies, through which investors gain a broader exposure to fixed-income markets, rather than tailoring investments to match a specific liability profile. We explain the advantages of index-based investing, such as diversification, but we also note that the depth and breadth of bond markets make both creating and tracking an index more challenging than in the equity markets. We also explore a variety of alternatives in matching a bond index, from full replication to enhanced indexing using primary risk factors. Finally, we explain that it is critical to select a benchmark that is most relevant to a specific investor based on factors such as the targeted duration profile and risk appetite.

2. LIABILITY-DRIVEN INVESTING

- **describe liability-driven investing**

Let us start with the example of a 45-year-old investor who plans to retire at age 65 and who would like to secure a stable stream of income thereafter. It is quite probable that he currently has a diversified portfolio that includes bonds, equities, and possibly other asset classes. Our focus here is on the fixed-income portion of his overall portfolio. We will assume that the investor builds the bond portfolio (immediately) and will add to it each year. Upon retirement, he plans to sell the bonds and buy an annuity that will pay a fixed benefit for his remaining lifetime. This investor's initial 20-year time horizon is critical to identifying and measuring the impact on retirement income arising from future interest rate volatility, and it forms the initial frame of reference for understanding and dealing with interest rate risk.

More generally, the frame of reference is in the form of a balance sheet of rate-sensitive assets and liabilities. In the example of the 45-year-old investor, the asset is the growing bond portfolio and the liability is the present value of the annuity that the investor requires to satisfy the fixed lifetime benefit.

2.1. Liability-Driven Investing vs. Asset-Driven Liabilities

Liability-driven investing (LDI) and asset-driven liabilities (ADL) are special cases of ALM. The key difference is that with ADL, the assets are given and the liabilities are structured to manage interest rate risk; whereas with LDI, which is much more common, the liabilities are given and the assets are managed. As an example of LDI, a life insurance company acquires a liability portfolio based on the insurance policies underwritten by its sales force. Another example involves the future employee benefits promised by a defined benefit pension plan, which create a portfolio of rate-sensitive liabilities. In each circumstance, the liabilities are defined and result from routine business and financial management decisions. The present value of those liabilities depends on current interest rates (as well as other factors). A life insurance or pension manager will use the estimated interest rate sensitivity of plan liabilities as a starting point when making investment portfolio decisions. This process often requires building a model for the liabilities.

With ADL, the asset side of the balance sheet results from a company's underlying businesses, and the debt manager seeks a liability structure to reduce interest rate risk. One example might be a leasing company with short-term contracts that chooses to finance itself with short-term debt. The company is aiming to match the maturities of its assets and liabilities to minimize risk. Alternatively, a manufacturing company might identify that its operating revenues are highly correlated with the business cycle. Monetary policy is typically managed so there is positive correlation between interest rates and the business cycle. Central banks lower policy rates when the economy is weak and raise them when it is strong. Therefore, this company has a natural preference for variable-rate liabilities so that operating revenue and interest expense rise and fall together.

2.2. Types of Liabilities

An LDI strategy starts with analyzing the size and timing of the entity's liabilities. Exhibit 1 shows a classification scheme for this analysis.

EXHIBIT 1 Classification of Liabilities

Liability Type	Amount of Cash Outlay	Timing of Cash Outlay	Examples
Type I	Known	Known	Traditional fixed-income bond with no embedded options
Type II	Known	Uncertain	Callable and putable bonds Term life insurance policy (timing of death unknown)
Type III	Uncertain	Known	Floating rate note – interest payments depend on future interest rates Inflation-protected securities – amounts of interest and principal payments tied to inflation
Type IV	Uncertain	Uncertain	Property and casualty insurance (weather events difficult to predict)

Type I: MacDur, ModDur, money duration, and the PVBP can be used to measure the interest rate sensitivity

Types II–IV: Effective duration needed to estimate interest rate sensitivity. Calculated using a model for:
- Uncertain amount and/or timing of the cash flows
- Initial assumption about the yield curve

Note that effective duration is needed with Types II, III, and IV liabilities, based on initial assumptions about the yield curve. Then, the yield curve is shifted up and down to obtain new estimates for the present value of the liabilities. We demonstrate this process later for the sponsor of a defined benefit pension plan, which is another example of an entity with Type IV liabilities.

EXAMPLE 1

Modern Mortgage, a savings bank, decides to establish an ALCO (asset–liability committee) to improve risk management and coordination of its loan and deposit rate-setting processes. Modern's primary assets are long-term, fixed-rate, monthly payment, fully amortizing residential mortgage loans. The mortgage loans are prime quality and have loan-to-value ratios that average 80%. The loans are pre-payable at par value by the homeowners at no fee. Modern also holds a portfolio of non-callable, fixed-income government bonds (considered free of default risk) of varying maturities to manage its liquidity needs. The primary liabilities are demand and time deposits that are fully guaranteed by a government deposit insurance fund. The demand deposits are redeemable by check or debit card. The time deposits have fixed rates and maturities ranging from 90 days to three years and are redeemable before maturity at a small fee. The banking-sector regulator in the country in which Modern operates has introduced a new capital requirement for savings banks. In accordance with the requirement, contingent convertible long-term bonds are issued by the savings bank and sold to institutional investors. The key feature is that if defaults on the mortgage loans reach a certain level or the savings bank's capital ratio drops below a certain level, as determined by the regulator, the bonds convert to equity at a specified price per share.

As a first step, the ALCO needs to identify the types of assets and liabilities that comprise its balance sheet using the classification scheme in Exhibit 1. Type I has certain amounts and dates for its cash flows; Type II has known amounts but uncertain dates; Type III has specified dates but unknown amounts; and Type IV has uncertain amounts and dates.

Specify and explain the classification scheme for the following:

1. Residential mortgage loans
2. Government bonds
3. Demand and time deposits
4. Contingent convertible bonds

Solution to 1: Residential mortgage loans are Type IV assets to the savings bank. The timing of interest and principal cash flows is uncertain because of the prepayment option held by the homeowner. This type of call option is complex. Homeowners might elect to prepay for many reasons, including sale of the property as well as the opportunity to refinance if interest rates come down. Therefore, a prepayment model is needed to project the timing of future cash flows. Default risk also affects the projected amount of the cash flow for each date. Even if the *average* loan-to-value ratio is 80%, indicating high-quality mortgages, some loans could have higher ratios and be more subject to default, especially if home prices decline.

Solution to 2: Fixed-rate government bonds are Type I assets because the coupon and principal payment dates and amounts are determined at issuance.

Solution to 3: Demand and time deposits are Type II liabilities from the savings bank's perspective. The deposit amounts are known, but the depositor can redeem the deposits prior to maturity, creating uncertainty about timing.

Solution to 4: The contingent convertible bonds are Type IV liabilities. The presence of the conversion option makes both the amount and timing of cash flows uncertain.

3. INTEREST RATE IMMUNIZATION: MANAGING THE INTEREST RATE RISK OF A SINGLE LIABILITY

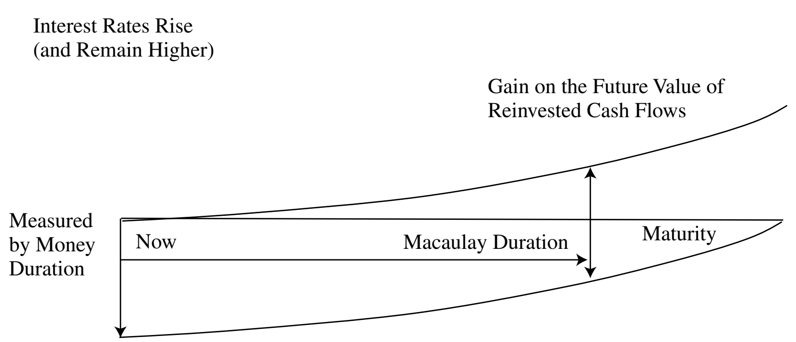
- **evaluate strategies for managing a single liability**

Liability-driven investing in most circumstances is used to manage the interest rate risk on multiple liabilities. In this section, we focus on only a single liability to demonstrate the techniques and risks of the classic investment strategy known as interest rate **immunization**. Immunization is the process of structuring and managing a fixed-income bond portfolio to minimize the variance in the realized rate of return over a known time horizon. This variance arises from the volatility of future interest rates. Default risk is neglected at this point because the portfolio bonds are assumed to have default probabilities that approach zero.

The most obvious way to immunize the interest rate risk on a single liability is to buy a zero-coupon bond that matures on the obligation's due date. The bond's face value matches the liability amount. There is no cash flow reinvestment risk because there are no coupon payments to reinvest, and there is no price risk because the bond is held to maturity. Any interest rate volatility over the bond's lifetime is irrelevant in terms of the asset's ability to pay off the liability. The problem is that in many financial markets, zero-coupon bonds are not available. Nevertheless, the perfect immunization provided by a zero-coupon bond sets a standard to measure the performance of immunizing strategies using coupon-bearing bonds.

Exhibit 2 and Exhibit 3 illustrate the connection between immunization and the duration of a traditional coupon-bearing fixed-income bond.

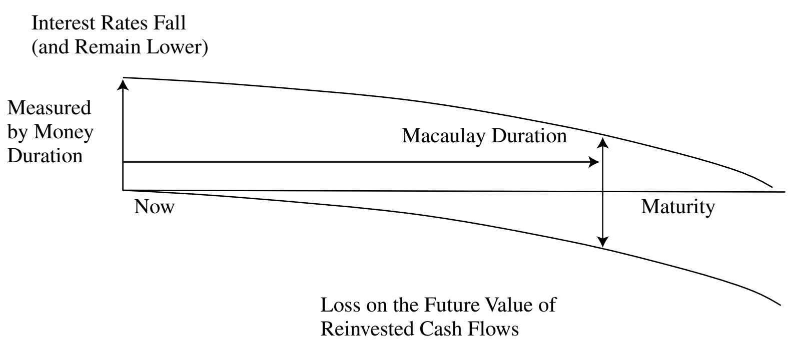
EXHIBIT 2 Immunization with a Single Bond: Rate Rise Scenario



Assume that the bond is currently priced at par value. Then, an instantaneous, one-time, upward (parallel) shift occurs in the yield curve. The bond's value falls. That drop in value is estimated by the money duration of the bond. Recall that the money duration is the bond's modified duration statistic multiplied by the price. Subsequently, the bond price will be "pulled to par" as the maturity date nears (assuming no default, of course). But another factor is at work. Assuming interest rates remain higher, the future value of reinvested coupon payments goes up. It is shown by the rising line as more and more payments are received and reinvested at the higher interest rates.

The key detail to note in Exhibit 2 is that at some point in time, the two effects—the price effect and the coupon reinvestment effect—cancel each other out. The remarkable result is that this point in time turns out to be the bond's Macaulay duration (for a zero-coupon bond, its Macaulay duration is its maturity). Therefore, an investor having an investment horizon equal to the bond's Macaulay duration is effectively protected, or immunized, from interest rate risk in that price, and coupon reinvestment effects offset for either higher or lower rates. Exhibit 3 shows the same effect for an immediate downward shift in interest rates.

EXHIBIT 3 Immunization: Interest Rate Fall Scenario



3.1. A Numerical Example of Immunization

We now show that the strategy of matching the Macaulay duration to the investment horizon works for a bond portfolio as well as for an individual security. Suppose that some entity has a single liability of EUR 250 million due 15 February 2027. Further assume that the current date is 15 February 2021, so the investment horizon is six years. The asset manager for the entity seeks to build a three-bond portfolio to earn a rate of return sufficient to pay off the obligation.

3.1.1. Portfolio Features

Exhibit 4 reports the prices, yields, risk statistics (Macaulay duration and convexity), and par values for the chosen portfolio. The portfolio's current market value is EUR 200,052,250 (= EUR 47,117,500 + EUR 97,056,750 + EUR 55,878,000). The semi-annual coupon payments on the bonds occur on 15 February and 15 August of each year (note that we have chosen to use bonds that pay coupons semi-annually, which is not always the case). The price is per 100 of par value, and the yield to maturity is on a street-convention semi-annual bond basis (meaning an annual percentage rate having a periodicity of two). Both the Macaulay duration and the convexity are annualized. (Note that in practice, some bond data vendors report the convexity statistic divided by 100.)

EXHIBIT 4 The Bond Portfolio to Immunize the Single Liability

	2.5-Year Bond	7-Year Bond	10-Year Bond
Coupon rate	1.50%	3.25%	5.00%
Maturity date	15 August 2023	15 February 2028	15 February 2031
Price	100.25	99.75	100.50
Yield to maturity	1.3979%	3.2903%	4.9360%
Par value	47,000,000	97,300,000	55,600,000
Market value	47,117,500	97,056,750	55,878,000
Macaulay duration	2.463	6.316	7.995
Convexity	7.253	44.257	73.747
Allocation	23.55%	48.52%	27.93%

Exhibit 5 shows the cash flows and calculations used to obtain the relevant portfolio statistics. The third column aggregates the coupon and principal payments received for each date from the three bonds.

EXHIBIT 5 Portfolio Statistics

Time	Date	Cash Flow	PV of Cash Flow	Weight	Time × Weight	Dispersion	Convexity
0	15-Feb-21	–200,052,250					
1	15-Aug-21	3,323,625	3,262,282	0.0163	0.0163	1.9735	0.0326
2	15-Feb-22	3,323,625	3,202,071	0.0160	0.0320	1.6009	0.0960
3	15-Aug-22	3,323,625	3,142,971	0.0157	0.0471	1.2728	0.1885
4	15-Feb-23	3,323,625	3,084,962	0.0154	0.0617	0.9871	0.3084
5	15-Aug-23	50,323,625	45,847,871	0.2292	1.1459	11.2324	6.8754
6	15-Feb-24	2,971,125	2,656,915	0.0133	0.0797	0.4782	0.5578
7	15-Aug-24	2,971,125	2,607,877	0.0130	0.0913	0.3260	0.7300
8	15-Feb-25	2,971,125	2,559,744	0.0128	0.1024	0.2048	0.9213
9	15-Aug-25	2,971,125	2,512,500	0.0126	0.1130	0.1131	1.1303
10	15-Feb-26	2,971,125	2,466,127	0.0123	0.1233	0.0493	1.3560
11	15-Aug-26	2,971,125	2,420,610	0.0121	0.1331	0.0121	1.5972
12	15-Feb-27	2,971,125	2,375,934	0.0119	0.1425	0.0000	1.8527
13	15-Aug-27	2,971,125	2,332,082	0.0117	0.1515	0.0116	2.1216
14	15-Feb-28	100,271,125	77,251,729	0.3862	5.4062	1.5434	81.0931
15	15-Aug-28	1,390,000	1,051,130	0.0053	0.0788	0.0473	1.2610
16	15-Feb-29	1,390,000	1,031,730	0.0052	0.0825	0.0825	1.4028
17	15-Aug-29	1,390,000	1,012,688	0.0051	0.0861	0.1265	1.5490
18	15-Feb-30	1,390,000	993,997	0.0050	0.0894	0.1788	1.6993
19	15-Aug-30	1,390,000	975,651	0.0049	0.0927	0.2389	1.8533
20	15-Feb-31	56,990,000	39,263,380	0.1963	3.9253	12.5585	82.4316
			200,052,250	1.0000	12.0008	33.0378	189.0580

For instance, EUR 3,323,625 is the sum of the coupon payments for the first four dates:

(1.50% × 0.5 × EUR 47,000,000) + (3.25% × 0.5 × EUR 97,300,000) + (5.00% × 0.5 × EUR 55,600,000) = EUR 352,500 + EUR 1,581,125 + EUR 1,390,000 = EUR 3,323,625

On 15 August 2023, the principal of EUR 47,000,000 is redeemed so that the total cash flow is EUR 50,323,625. The next eight cash flows represent the coupon payments on the second and third bonds, and so forth.

The internal rate of return on the cash flows in column 3 for the 20 semi-annual periods, including the portfolio's initial market value on 15 February 2021, is 1.8804%. Annualized on a semi-annual bond basis, the portfolio's cash flow yield is 3.7608% (= 2 × 1.8804%). This yield is significantly higher than the market value-weighted average of the individual bond yields-to-maturity presented in Exhibit 4, which equals 3.3043%.

(1.3979% × 0.2355) + (3.2903% × 0.4852) + (4.9360% × 0.2793) = 3.3043%

This difference arises because of the steepness in the yield curve. The key point is that the goal of the immunization strategy is to achieve a rate of return close to 3.76%, not 3.30%.

The fourth column in Exhibit 5 shows the present values for each of the aggregate cash flows, calculated using the internal rate of return per period (1.8804%) as the discount rate. For example, the combined payment of EUR100,271,125 due on 15 February 2028 has a present value of EUR77,251,729. [*Note: Calculations are carried out on a spreadsheet that preserves precision. For readability and to avoid clutter, the exhibits and text report rounded results. For example, the following calculation gives 77,251,498 with the numbers shown on the left hand-side, but it gives 77,251,729, the amount shown on the right hand-side, when the precise semi-annual cash flow yield, 1.0188037819%, is used.*]

$$\frac{100{,}271{,}125}{(1.018804)^{14}} = 77{,}251{,}729$$

The sum of the present values in column 4 of Exhibit 5 is EUR200,052,250, the current market value for the bond portfolio.

3.1.2. Portfolio Duration

The sixth column of Exhibit 5 is used to obtain the portfolio's Macaulay duration. This duration statistic is the weighted average of the times to the receipt of cash flow, whereby the share of total market value for each date is the weight. Column 5 shows the weights, which are the PV of each cash flow divided by the total PV of EUR200,052,250. The times to receipt of cash flow (the times from column 1) are multiplied by the weights and then summed. For example, the contribution to total portfolio duration for the second cash flow on 15 February 2022 is 0.0320 (= 2 × 0.0160). The sum of column 6 is 12.0008. That is the Macaulay duration for the portfolio in terms of semi-annual periods. Annualized, it is 6.0004 (= 12.0008/2). It is now clear why the asset manager for the entity chose this portfolio: The portfolio Macaulay duration matches the investment horizon of six years.

In practice, it is common to estimate the portfolio duration using the market value-weighted average of the individual durations for each bond. Exhibit 4 shows those individual durations and the allocation percentages for each bond. The average Macaulay duration is (2.463 × 0.2355) + (6.316 × 0.4852) + (7.995 × 0.2793) = 5.8776.

The difference, as with the cash flow yield and the market value-weighted average yield, arises because the yield curve is not flat. When the yield curve is upwardly sloped, average duration (5.8776) is less than the portfolio duration (6.0004). This difference in duration statistics is important because using the average duration in building the immunizing portfolio instead of the portfolio duration would introduce model risk to the strategy, as we will see later.

3.1.3. Portfolio Dispersion

The sum of the seventh column in Exhibit 5 is the portfolio dispersion statistic. Recall that whereas Macaulay duration is the weighted *average* of the times to receipt of cash flow, dispersion is the weighted *variance*. It measures the extent to which the payments are spread out around the duration. For example, the contribution to total portfolio dispersion for the fifth cash flow on 15 August 2023 is 11.2324: $(5 - 12.0008)^2 \times 0.2292 = 11.2324$.

This portfolio's dispersion is 33.0378 in terms of semi-annual periods. Annualized, it is 8.2594 (= 33.0378/4). The Macaulay duration statistic is annualized by dividing by the periodicity of the bonds (two payments per year); dispersion (and convexity, which follows) is annualized by dividing by the periodicity squared (i.e., $2^2 = 4$ for semi-annual payment bonds).

3.1.4. Portfolio Convexity

The portfolio convexity is calculated with the eighth column. It is the sum of the times to the receipt of cash flow, multiplied by those times plus one, multiplied by the shares of market value for each date (weight), and all divided by one plus the cash flow yield squared. For example, the contribution to the sum for the 14th payment on 15 February 2028 is 81.0931 (= 14 × 15 × 0.3862). The sum of the column is 189.0580. The convexity in semi-annual periods is 182.1437:

$$\frac{189.0580}{(1.018804)^2} = 182.1437$$

The annualized convexity for the portfolio is 45.5359 (= 182.1437/4). This result is slightly higher than the market value-weighted average of the individual convexity statistics (for each bond) reported in Exhibit 4:

$$(7.253 \times 0.2355) + (44.257 \times 0.4852) + (73.747 \times 0.2793) = 43.7786$$

As with the average yield and duration, this difference results from the slope of the yield curve. The convexity statistic can be used to improve the estimate for the change in portfolio market value following a change in interest rates than is provided by duration alone. That is, convexity is the second-order effect, whereas duration is the first-order effect.

There is an interesting connection among the portfolio convexity, Macaulay duration, dispersion, and cash flow yield in immunized portfolio convexity, also known as the "portfolio convexity statistic":

$$\text{Immunized Portfolio Convexity} = \frac{\text{MacDur}^2 + \text{MacDur} + \text{Dispersion}}{(1 + \text{Cash flow yield})^2} \qquad (1)$$

In terms of semi-annual periods, the Macaulay duration for this portfolio is 12.0008, the dispersion is 33.0378, and the cash flow yield is 1.8804%.

$$\text{Immunized Portfolio Convexity} = \frac{12.0008^2 + 12.0008 + 33.0378}{(1.018804)^2} = 182.1437$$

The portfolio dispersion and convexity statistics are used to assess the structural risk to the interest rate immunization strategy. Structural risk arises from the potential for shifts and twists to the yield curve. This risk is discussed later.

3.1.5. Investment Horizon and Immunization

We now demonstrate how matching the Macaulay duration for the portfolio to the investment horizon leads to interest rate immunization. The first three columns of Exhibit 6 are identical to the ones in Exhibit 5.

The fourth column shows the values of the cash flows as of the horizon date of 15 February 2027, assuming that the cash flow yield remains unchanged at 3.7608%. For instance, the future value of the EUR3,323,625 in coupon payments received on 15 August 2021 is EUR4,079,520:

$$3{,}323{,}625 \times \left(1 + \frac{0.037608}{2}\right)^{11} = 4{,}079{,}520$$

The value of the last cash flow for EUR56,990,000 on 15 February 2031 is EUR49,099,099 as of the horizon date of 15 February 2027:

$$\frac{56{,}990{,}000}{\left(1 + \frac{0.037608}{2}\right)^{8}} = 49{,}099{,}099$$

We assume that all of the payments received before the horizon date are reinvested at the cash flow yield. All of the payments received after the horizon date are sold at their discounted values. The sum of the fourth column in Exhibit 6 is EUR250,167,000, which is more than enough to pay off the EUR250 million liability. The six-year holding period rate of return (ROR), also called the horizon yield, is 3.7608%. It is based on the original market value and the total return and is the solution for ROR:

$$200{,}052{,}250 = \frac{250{,}167{,}000}{\left(1 + \frac{\text{ROR}}{2}\right)^{12}},\ \text{ROR} = 0.037608$$

The holding period rate of return equals the cash flow yield for the portfolio. This equivalence is the multi-bond version of the well-known result for a single bond: The realized rate of return matches the yield to maturity only if coupon payments are reinvested at that same yield and if the bond is held to maturity or sold at a point on the constant-yield price trajectory.

EXHIBIT 6 Interest Rate Immunization

Time	Date	Cash Flow	Total Return at 3.7608%	Total Return at 2.7608%	Total Return at 4.7608%
0	15-Feb-21	−200,052,250			
1	15-Aug-21	3,323,625	4,079,520	3,864,613	4,305,237
2	15-Feb-22	3,323,625	4,004,225	3,811,992	4,205,138
3	15-Aug-22	3,323,625	3,930,319	3,760,088	4,107,366
4	15-Feb-23	3,323,625	3,857,777	3,708,891	4,011,868
5	15-Aug-23	50,323,625	57,333,230	55,392,367	59,332,093
6	15-Feb-24	2,971,125	3,322,498	3,225,856	3,421,542
7	15-Aug-24	2,971,125	3,261,175	3,181,932	3,341,989

Time	Date	Cash Flow	Total Return at 3.7608%	Total Return at 2.7608%	Total Return at 4.7608%
8	15-Feb-25	2,971,125	3,200,984	3,138,607	3,264,286
9	15-Aug-25	2,971,125	3,141,904	3,095,871	3,188,390
10	15-Feb-26	2,971,125	3,083,914	3,053,718	3,114,258
11	15-Aug-26	2,971,125	3,026,994	3,012,138	3,041,850
12	15-Feb-27	2,971,125	2,971,125	2,971,125	2,971,125
13	15-Aug-27	2,971,125	2,916,287	2,930,670	2,902,045
14	15-Feb-28	100,271,125	96,603,888	97,559,123	95,662,614
15	15-Aug-28	1,390,000	1,314,446	1,333,991	1,295,282
16	15-Feb-29	1,390,000	1,290,186	1,315,827	1,265,166
17	15-Aug-29	1,390,000	1,266,373	1,297,911	1,235,750
18	15-Feb-30	1,390,000	1,242,999	1,280,238	1,207,018
19	15-Aug-30	1,390,000	1,220,058	1,262,806	1,178,955
20	15-Feb-31	56,990,000	49,099,099	51,070,094	47,213,270
			250,167,000	250,267,858	250,265,241

3.1.6. A Drop in the Cash Flow Yield Scenario

The fifth column in Exhibit 6 repeats the calculations for the assumption of an instantaneous, one-time, 100 bp drop in the cash flow yield on 15 February 2021. The future values of all cash flows received are now lower because they are reinvested at 2.7608% instead of 3.7608%. For example, the payment of EUR50,323,625 on 15 August 2023, which contains the principal redemption on the 2.5-year bond, grows to only EUR55,392,367:

$$50{,}323{,}625 \times \left(1 + \frac{0.027608}{2}\right)^7 = 55{,}392{,}367$$

The value of the last cash flow is now higher because it is discounted at the lower cash flow yield:

$$\frac{56{,}990{,}000}{\left(1 + \frac{0.027608}{2}\right)^8} = 51{,}070{,}094$$

The important result is that the total return as of the horizon date is EUR250,267,858, demonstrating that the cash flow reinvestment effect is balanced by the price effect, as illustrated for a single bond in Exhibit 2. The holding-period rate of return is 3.7676%:

$$200{,}052{,}250 = \frac{250{,}267{,}858}{\left(1 + \frac{\text{ROR}}{2}\right)^{12}},\ \text{ROR} = 0.037676$$

3.1.7. An Increase in the Cash Flow Yield Scenario

To complete the example, the sixth column in Exhibit 6 reports the results for an instantaneous, one-time, 100 bp jump in the cash flow yield, up to 4.7608% from 3.7608%. In this case, the future values of the reinvested cash flows are higher and the discounted values of cash flows due after the horizon date are lower. Nevertheless, the total return of EUR250,265,241 for the six-year investment horizon is enough to pay off the liability. The horizon yield is 3.7674%:

$$200{,}052{,}250 = \frac{250{,}265{,}241}{\left(1 + \frac{\text{ROR}}{2}\right)^{12}}, \text{ ROR} = 0.037674$$

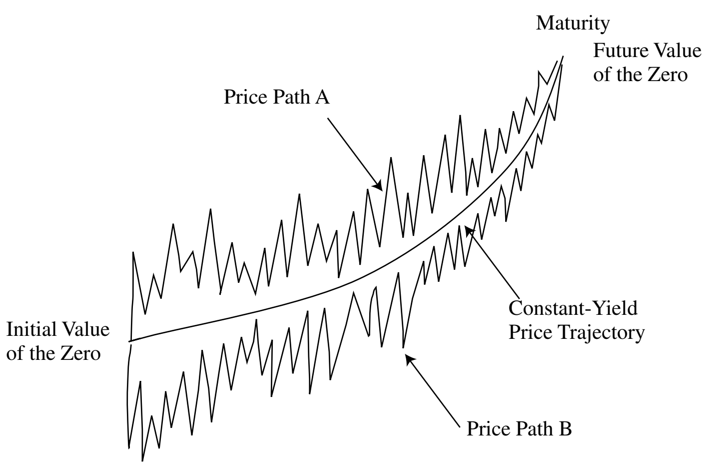
This numerical exercise demonstrates interest rate immunization using a portfolio of fixed-income bonds. The total returns and holding period rates of return are virtually the same—in fact, slightly higher because of convexity—whether the cash flow yield goes up or down.

3.1.8. Immunization and Rebalancing

Exhibit 5 is somewhat misleading, however, because it suggests that immunization is a buy-and-hold passive investment strategy. It suggests that the entity will (a) hold on the horizon date of 15 February 2027 the same positions in what then will be one-year, 3.25% and four-year, 5% bonds and (b) sell the bonds on that date. This suggestion is misleading because the portfolio must be frequently rebalanced to stay on its target duration. As time passes, the portfolio's Macaulay duration changes but not in line with the change in the remainder of the investment horizon. For example, after five years, the investment horizon as of 15 February 2026 is just one remaining year. The portfolio Macaulay duration at that time needs to be 1.000. The asset manager will have had to execute some trades by then, substantially reducing the holdings in what is then the five-year, 5% bond.

Exhibit 7 offers another way to illustrate interest rate immunization. An immunization strategy is essentially "zero replication." We know that the perfect bond to lock in the six-year holding period rate of return is a six-year zero-coupon bond having a face value that matches the EUR250 million liability. The idea is to originally structure and then manage over time a portfolio of coupon-bearing bonds that replicates the period-to-period performance of the zero-coupon bond. Therefore, immunization is essentially just an interest rate hedging strategy. As the yield on the zero-coupon bond rises and falls, there will be unrealized losses and gains. In Exhibit 7, this is illustrated by the zero-coupon bond's value moving below and above the constant-yield price trajectory. Two paths for the zero-coupon yield are presented: Path A for generally lower rates (and higher values) and Path B for higher rates (and lower values). Regardless, the market value of the zero-coupon bond will be "pulled to par" as maturity nears.

EXHIBIT 7 Interest Rate Immunization as Zero Replication



Immunizing with coupon-bearing bonds entails continuously matching the portfolio Macaulay duration with the Macaulay duration of the zero-coupon bond over time and as the yield curve shifts, even though the zero-coupon bond could be hypothetical and not exist in reality. Also, in order to fully match the liability, the bond portfolio's initial market value has to match or exceed the present value of the zero-coupon bond. The Macaulay duration of that, perhaps hypothetical, zero-coupon bond always matches the investment horizon. Immunization will be achieved if any ensuing change in the cash flow yield on the bond portfolio is equal to the change in the yield to maturity on the zero-coupon bond. That equivalence will ensure that the change in the bond portfolio's market value is close to the change in the market value of the zero-coupon bond. Therefore, at the end of the six-year investment horizon, the bond portfolio's market value should meet or exceed the face value of the zero-coupon bond, regardless of the path for interest rates over the six years.

3.1.9. Immunization and Shifts in the Yield Curve

The key assumption to achieve immunization is the statement that "any ensuing change in the cash flow yield on the bond portfolio is equal to the change in the yield to maturity on the zero-coupon bond." A *sufficient*, but not *necessary*, condition for that statement is a parallel (or shape-preserving) shift to the yield curve whereby all yields change by the same amount. *Sufficient* means that if the yield curve shift is parallel, the change in the bond portfolio's cash flow yield will equal the change in yield to maturity of the zero-coupon bond, which is enough to ensure immunization. To achieve immunization, however, it is not *necessary* that the yield curve shifts in a parallel manner. That is, in some cases, the immunization property can prevail even with non-parallel yield curve movements, such as an upward and steepening shift (sometimes called a "bear steepener"), an upward and flattening shift (a "bear flattener"), a downward and steepening shift (a "bull steepener"), or a downward and flattening shift (a "bull flattener").

Exhibits 8 and 9 demonstrate this observation. Exhibit 8 presents three different upward yield curve shifts. The first is a parallel shift of 102.08 bps for each of the three bond yields. The second is a steepening shift of 72.19 bps for the 2.5-year bond, 94.96 bps for the 7-year bond, and 120.82 bps for the 10-year bond. The third is a flattening shift, whereby the yields on the three bonds increase by 145.81 bps, 109.48 bps, and 79.59 bps, respectively. The key point is that each of these yield curve shifts results in the same 100 bp increase in the cash flow yield from 3.7608% to 4.7608%. Moreover, each shift in the yield curve produces virtually the same reduction in the portfolio's market value.

EXHIBIT 8 Some Upward Yield Curve Shifts That Achieve Interest Rate Immunization

	Change in 2.5-Year Yield	Change in 7-Year Yield	Change in 10-Year Yield	Change in Cash Flow Yield	Change in Market Value
Upward and parallel	+102.08 bps	+102.08 bps	+102.08 bps	+100 bps	−11,340,537
Upward and steepening	+72.19 bps	+94.96 bps	+120.82 bps	+100 bps	−11,340,195
Upward and flattening	+145.81 bps	+109.48 bps	+79.59 bps	+100 bps	−11,340,183

Exhibit 9 shows the results for three downward shifts in the yield curve. The first is a parallel shift of 102.06 bps. The second and third are downward and steepening (−129.00 bps, −104.52 bps, and −92.00 bps for the 2.5-year, 7-year, and 10-year bonds) and downward and flattening (−55.76 bps, −86.32 bps, and −134.08 bps). Each shift results in the same 100 bp decrease in the cash flow yield from 3.7608% to 2.7608% and virtually the same increase in the market value of the portfolio.

EXHIBIT 9 Some Downward Yield Curve Shifts That Achieve Interest Rate Immunization

	Change in 2.5-Year Yield	Change in 7-Year Yield	Change in 10-Year Yield	Change in Cash Flow Yield	Change in Market Value
Downward and parallel	−102.06 bps	−102.06 bps	−102.06 bps	−100 bps	12,251,212
Downward and steepening	−129.00 bps	−104.52 bps	−92.00 bps	−100 bps	12,251,333
Downward and flattening	−55.76 bps	−86.32 bps	−134.08 bps	−100 bps	12,251,484

Notice that the interest rate immunization property shown in Exhibit 6 rests only on the change in the cash flow yield going up or down by 100 bps. It is not necessary to assume that the change in the value of the immunizing portfolio arises only from a parallel shift in the yield curve. In the same manner, the immunization property illustrated in Exhibit 7 requires only that the change in the value of the immunizing portfolio, one that has a Macaulay duration matching the investment horizon, is close to the change in the value of the zero-coupon bond that provides perfect immunization. Exhibits 8 and 9 demonstrate that some non-parallel as well as parallel shifts can satisfy those conditions. Of course, there are many other non-parallel shifts for which those conditions are not met.

In general, the interest rate risk to an immunization strategy is that the change in the cash flow yield on the portfolio is not the same as on the ideal zero-coupon bond. This difference can occur with twists to the shape of the yield curve, in addition to some non-parallel shifts.

Exhibits 10 and 11 portray two such twists. To exaggerate the risk, assume that the immunizing portfolio has a “barbell” structure in that it is composed of half short-term bonds and half long-term bonds. The portfolio Macaulay duration for the barbell is six years. The zero-coupon bond that provides perfect immunization has a maturity (and Macaulay duration) also of six years.

Exhibit 10 shows a steepening twist to the yield curve. The twist is assumed to occur at the six-year point to indicate that the value of the zero-coupon bond does not change. Short-term yields go down and long-term yields go up by approximately the same amount. The value of the barbell portfolio goes down because the losses on the long-term positions exceed the gains on the short-term holdings as a result of the difference in duration between the holdings and the equivalence in the assumed changes in yield. Therefore, this portfolio does not track the value of the zero-coupon bond for such a scenario.

EXHIBIT 10 Immunization Risk and Steepening Twist

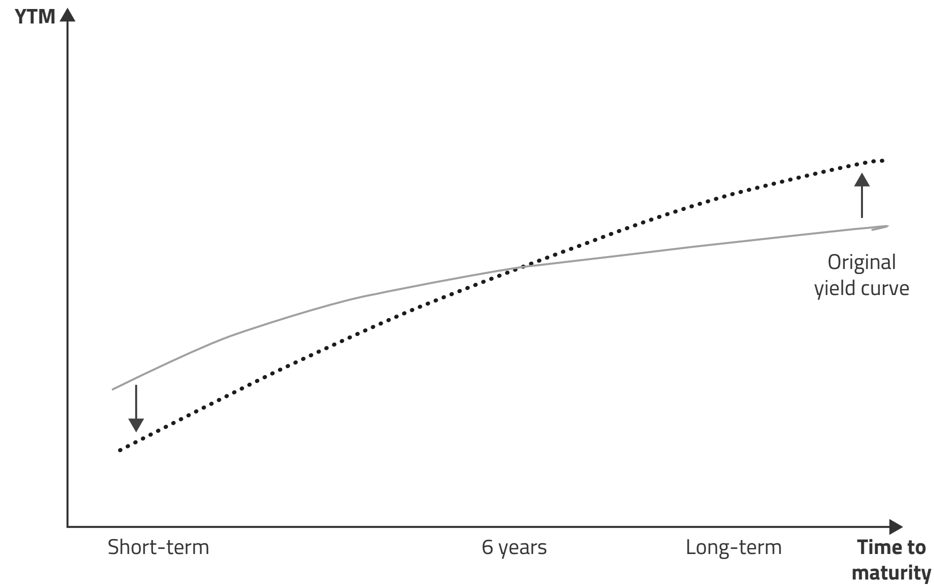
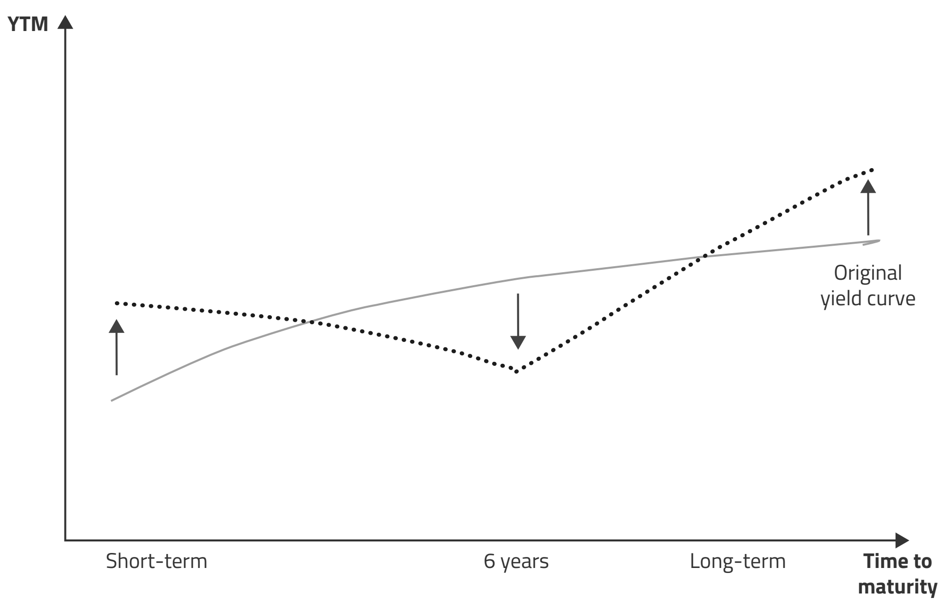


Exhibit 11 illustrates a dramatic twist in the shape of the yield curve. Short-term and long-term yields go up while the six-year yields go down. This type of twist is a butterfly movement, in this case a "positive butterfly." (In a "negative butterfly" twist, short-term and long-term yields go down and intermediate-term yields go up.) The immunizing portfolio decreases in value as its yields go up and the zero-coupon bond goes up in value. Again, for this particular scenario, the portfolio does not track the change in the value of the bond that provides perfect immunization. Fortunately for those entities that pursue interest rate immunization, these types of twists are rare. Most yield curve shifts are generally parallel, with some steepening and flattening, especially for maturities beyond a few years.

EXHIBIT 11 Immunization Risk and a Butterfly Yield Curve Movement



3.1.10. Structural Risk in Immunization Strategy

Exhibits 10 and 11 also illustrate how to reduce **structural risk** to an immunizing strategy. Structural risk arises from portfolio design, particularly the choice of the portfolio allocations. The risk is that yield curve twists and non-parallel shifts lead to changes in the cash flow yield that do not match the yield to maturity of the zero-coupon bond that provides for perfect immunization. Structural risk is reduced by minimizing the dispersion of the bond positions, going from a barbell design to more of a bullet portfolio that concentrates the component bonds' durations around the investment horizon. At the limit, a zero-coupon bond that matches the date of the single obligation has, by design, no structural risk.

Equation 1 (immunized portfolio convexity) indicates that minimizing portfolio dispersion is the same as minimizing the portfolio convexity for a given Macaulay duration and cash flow yield. An advantage to using convexity to measure the extent of structural risk is that the portfolio statistic can be approximated by the market value-weighted average of the individual bonds' convexities. A problem with estimating portfolio dispersion using the weighted average of dispersion statistics for individual bonds is that it can be misleading. Consider a portfolio of all zero-coupon bonds of varying maturities. Each individual bond has zero dispersion (because it has only one payment), so the market value-weighted average is also zero. Clearly, the portfolio overall can have significant (non-zero) dispersion.

In summary, the characteristics of a bond portfolio structured to immunize a single liability are that it:

- has an initial market value that equals or exceeds the present value of the liability;
- has a portfolio Macaulay duration that matches the liability's due date;
- minimizes the portfolio convexity statistic.

This portfolio must be regularly rebalanced over the horizon to maintain the target duration, because the portfolio Macaulay duration changes as time passes and as yields change. The portfolio manager needs to weigh the trade-off between incurring transaction costs from rebalancing and allowing some duration gap. This and other risks to immunization—for instance, those arising from the use of interest rate derivatives to match the duration of assets to the investment horizon—are covered later.

EXAMPLE 2

An institutional client asks a fixed-income investment adviser to recommend a portfolio to immunize a single 10-year liability. It is understood that the chosen portfolio will need to be rebalanced over time to maintain its target duration. The adviser proposes two portfolios of coupon-bearing government bonds because zero-coupon bonds are not available. The portfolios have the same market value. The institutional client's objective is to minimize the variance in the realized rate of return over the 10-year horizon. The two portfolios have the following risk and return statistics:

	Portfolio A	Portfolio B
Cash flow yield	7.64%	7.65%
Macaulay duration	9.98	10.01
Convexity	107.88	129.43

These statistics are based on aggregating the interest and principal cash flows for the bonds that constitute the portfolios; they are not market value-weighted averages of the yields, durations, and convexities of the individual bonds. The cash flow yield is stated on a semi-annual bond basis, meaning an annual percentage rate having a periodicity of two; the Macaulay durations and convexities are annualized.

Indicate the portfolio that the investment adviser should recommend, and explain the reasoning.

Solution: The adviser should recommend Portfolio A. First, notice that the cash flow yields of both portfolios are virtually the same and that both portfolios have Macaulay durations very close to 10, the horizon for the liability. It would be wrong and misleading to recommend Portfolio B because it has a "higher yield" and a "duration closer to the investment horizon of 10 years." In practical terms, a difference of 1 bp in yield is not likely to be significant, nor is the difference of 0.03 in annual duration.

Given the fact that the portfolio yields and durations are essentially the same, the choice depends on the difference in convexity. The difference between 129.43 and 107.88, however, is meaningful. In general, convexity is a desirable property of fixed-income bonds. All else being equal (meaning the same yield and duration), a more convex bond gains more if the yield goes down and loses less if the yield goes up than a less convex bond.

The client's objective, however, is to minimize the variance in the realized rate of return over the 10-year horizon. That objective indicates a conservative immunization strategy achieved by building the duration-matching portfolio and minimizing the portfolio convexity. Such an approach minimizes the dispersion of cash flows around the Macaulay duration and makes the portfolio closer to the zero-coupon bond that would provide perfect immunization; see Equation 1.

The structural risk to the immunization strategy is the potential for non-parallel shifts and twists to the yield curve, which lead to changes in the cash flow yield that do not track the change in the yield on the zero-coupon bond. This risk is minimized by selecting the portfolio with the lower convexity (and dispersion of cash flows).

Note that default risk is neglected in this discussion because the portfolio consists of government bonds that presumably have default probabilities approaching zero.

4. INTEREST RATE IMMUNIZATION: MANAGING THE INTEREST RATE RISK OF MULTIPLE LIABILITIES

- **compare strategies for a single liability and for multiple liabilities, including alternative means of implementation**
- **describe construction, benefits, limitations, and risk–return characteristics of a laddered bond portfolio**

The principle of interest rate immunization applies to multiple liabilities in addition to a single liability. For now, we continue to assume that these are Type I cash flows in that the scheduled amounts and payment dates are known to the asset manager. In particular, we assume that the same three bonds from Exhibits 4 and 5, which were assets in the single-liability immunization, are now themselves liabilities to be immunized. This assumption allows us to use the same portfolio statistics as in the previous section. The entity in the examples that follow seeks to immunize the cash flows in column 3 (the cash flow column) of Exhibit 5 from Dates 1 through 20, and so it needs to build a portfolio of assets that will allow it to pay those cash flows. The present value of the (now) corporate debt liabilities is EUR200,052,250. The cash flow yield is 3.76%; the Macaulay duration is 6.00; and the convexity is 45.54. We use the portfolio statistics rather than the market value-weighted averages because they better summarize Type I liabilities.

In this section, we discuss several approaches to manage these liabilities:

- *Cash flow matching*, which entails building a dedicated portfolio of zero-coupon or fixed-income bonds to ensure that there are sufficient cash inflows to pay the scheduled cash outflows (a related concept, the so-called "laddered portfolio," also falls into the cash flow matching category of approaches);
- *Duration matching*, which extends the ideas of the previous section to a portfolio of debt liabilities;
- *Derivatives overlay*, in particular using futures contracts on government bonds in the immunization strategy; and
- *Contingent immunization*, which allows for active bond portfolio management as long as the surplus is above a designated threshold.

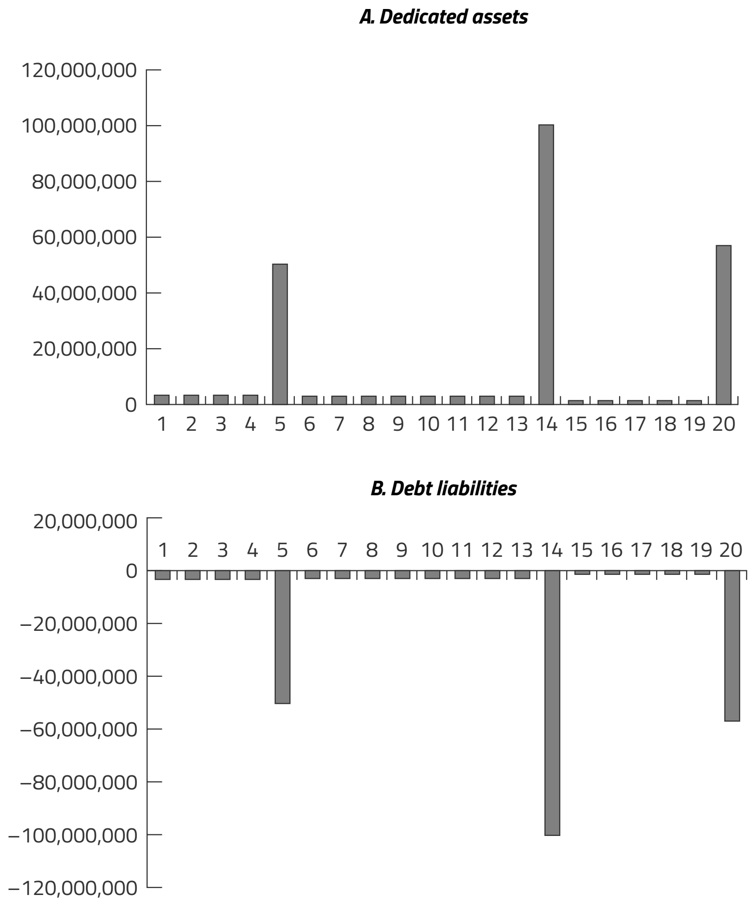
4.1. Cash Flow Matching

A classic strategy to eliminate the interest rate risk arising from multiple liabilities is to build a dedicated asset portfolio of high-quality fixed-income bonds that, as closely as possible, matches the amount and timing of the scheduled cash outflows. "Dedicated" means that the bonds are placed in a held-to-maturity portfolio. A natural question is, if the entity has enough cash to build the dedicated bond portfolio, why not just use that cash to buy back and retire the liabilities? The answer is that the buyback strategy is difficult and costly to implement if the bonds are widely held by buy-and-hold institutional and retail investors. Most corporate bonds are rather illiquid, so buying them back on the open market is likely to drive up the purchase price. Cash flow matching can be a better use of the available cash assets.

A corporate finance motivation for cash flow matching is to improve the company's credit rating. The entity has sufficient cash assets to retire the debt liabilities, and dedicating the bonds effectively accomplishes that objective. Under some circumstances, a corporation might even be able to remove both the dedicated asset portfolio and the debt liabilities from its balance sheet through the process of **accounting defeasance**. Also called in-substance defeasance, accounting defeasance is a way of extinguishing a debt obligation by setting aside sufficient high-quality securities, such as US Treasury notes, to repay the liability.

Panel A in Exhibit 12 illustrates the dedicated cash flow matching asset portfolio. These assets could be zero-coupon bonds or traditional fixed-income securities. Panel B represents the amount and timing of the debt liabilities. The amounts come from the third column in Exhibit 5 and are the sum of the coupon and principal payments on the three debt securities.

EXHIBIT 12 Cash Flow Matching



A concern when implementing this strategy is the *cash-in-advance constraint*. That means securities are not sold to meet obligations; instead, sufficient funds must be available on or before each liability payment date to meet the obligation. The design of traditional bonds—a fixed coupon rate and principal redemption at maturity—is a problem if the liability stream, unlike in Exhibit 12, is a level payment annuity. That scenario could lead to large cash holdings between payment dates and, therefore, cash flow reinvestment risk, especially if yields on high-quality, short-term investments are low (or worse, negative).

EXAMPLE 3

Alfred Simonsson is assistant treasurer at a Swedish lumber company. The company has sold a large tract of land and now has sufficient cash holdings to retire some of its debt liabilities. The company's accounting department assures Alfred that its external auditors will approve of a defeasement strategy if Swedish government bonds are purchased to match the interest and principal payments on the liabilities. Following is the schedule of payments due on the debt as of June Year 1 that the company plans to defease:

June Year 2	SEK 3,710,000
June Year 3	SEK 6,620,000
June Year 4	SEK 4,410,000
June Year 5	SEK 5,250,000

The following Swedish government bonds are available. Interest on the bonds is paid annually in May of each year.

Coupon Rate	Maturity Date
2.75%	May Year 2
3.50%	May Year 3
4.75%	May Year 4
5.50%	May Year 5

How much in par value for each government bond will Alfred need to buy to defease the debt liabilities, assuming that the minimum denomination in each security is SEK 10,000?

Solution: The cash flow matching portfolio is built by starting with the last liability of SEK 5,250,000 in June Year 5. If there were no minimum denomination, that liability could be funded with the 5.50% bonds due May Year 5 having a par value of SEK 4,976,303 (= SEK 5,250,000/1.0550). To deal with the constraint, however, Alfred buys SEK 4,980,000 in par value. That bond pays SEK 5,253,900 (= SEK 4,980,000 × 1.0550) at maturity. This holding also pays SEK 273,900 (= SEK 4,980,000 × 0.0550) in coupon interest in May Years 2, 3, and 4.

Then move to the June Year 4 obligation, which is SEK 4,136,100 after subtracting the SEK 273,900 received on the 5.50% bond: SEK 4,410,000 − SEK 273,900 = SEK 4,136,100. Alfred buys SEK 3,950,000 in par value of the 4.75% bond due May Year 4. That bond pays SEK 4,137,625 (= SEK 3,950,000 × 1.0475) at maturity and SEK 187,625 in interest in May Year 2 and Year 3.

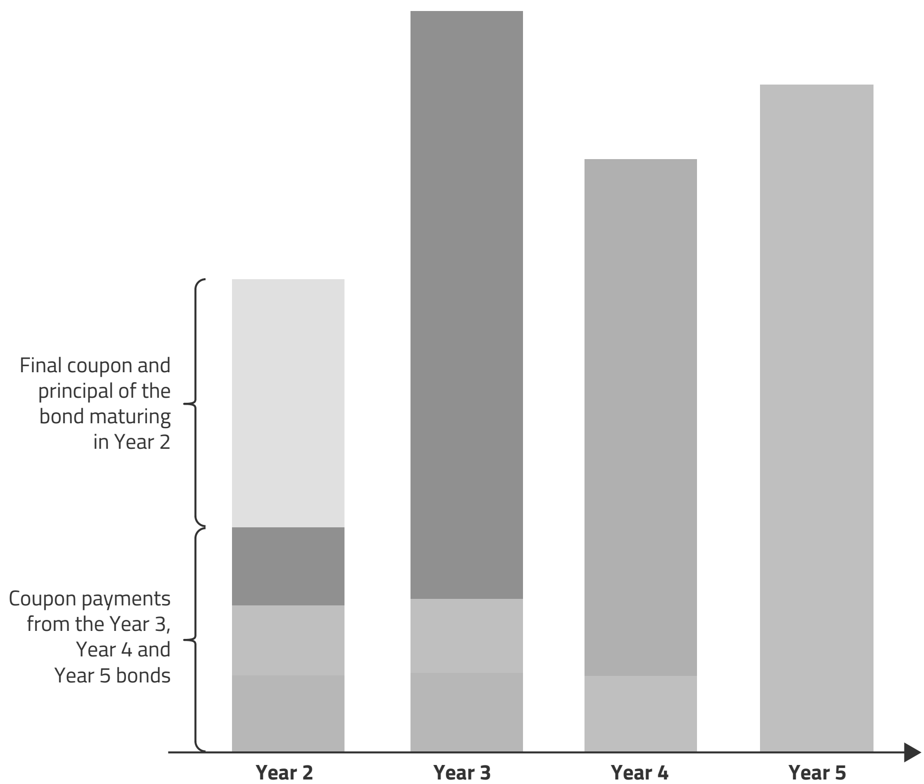
The net obligation in June Year 3 is SEK 6,158,475 (= SEK 6,620,000 − SEK 273,900 − SEK 187,625) after subtracting the interest received on the longer-maturity bonds. The company can buy SEK 5,950,000 in par value of the 3.50% bond due May Year 3. At maturity, this bond pays SEK 6,158,250 (= SEK 5,950,000 × 1.0350). The small shortfall of SEK 225 (= SEK 6,158,475 − SEK 6,158,250) can be made up because the funds received in May are reinvested until June. This bond also pays SEK 208,250 in interest in May Year 2.

Finally, Alfred needs to buy SEK 2,960,000 in par value of the 2.75% bond due May Year 2. This bond pays SEK 3,041,400 (= SEK 2,960,000 × 1.0275) in May Year 2. The final coupon and principal, plus the interest on the 5.50%, 4.75%, and 3.50% bonds, total SEK 3,711,175 (= SEK 3,041,400 + SEK 273,900 + SEK 187,625 + SEK 208,250). That amount is used to pay off the June Year 2 obligation of SEK 3,710,000. Note that the excess could be kept in a bank account to cover the Year 3 shortfall.

In sum, Alfred buys the following portfolio:

Bond	Par Value
2.75% due May Year 2	SEK 2,960,000
3.50% due May Year 3	SEK 5,950,000
4.75% due May Year 4	SEK 3,950,000
5.50% due May year 5	SEK 4,980,000

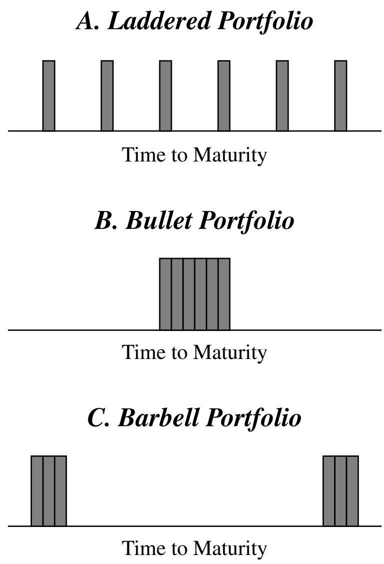
The following chart illustrates the cash flow matching bond portfolio: Each bar represents the par amount of a bond maturing in that year plus coupon payments from bonds maturing in later years.



4.2. Laddered Portfolios

A popular fixed-income investment strategy in the wealth management industry is to build a "laddered" portfolio for clients. Exhibit 13 illustrates this approach, along with two other maturity-based strategies—a "bullet" portfolio and a "barbell" portfolio. The laddered portfolio spreads the bonds' maturities and par values more or less evenly along the yield curve. The bullet portfolio concentrates the bonds at a particular point on the yield curve, whereas the barbell portfolio places the bonds at the short-term and long-term ends of the curve. In principle, each can have the same portfolio duration statistic and approximately the same change in value for a parallel shift in the yield curve. A non-parallel shift or a twist in the curve, however, leads to very different outcomes for the bullet and barbell structures. An obvious advantage to the laddered portfolio is protection from shifts and twists—the cash flows are essentially "diversified" across the time spectrum.

EXHIBIT 13 Laddered, Bullet, and Barbell Fixed-Income Portfolios



4.2.1. Benefits of Using Laddered Portfolios

This "diversification" over time provides the investor a balanced position between the two sources of interest rate risk—cash flow reinvestment and market price volatility. Bonds mature each year and are reinvested at the longer-term end of the ladder, typically at higher rates than short-term securities. Over time, the laddered portfolio likely includes bonds that were purchased at high interest rates as well as low interest rates. Investors familiar with "dollar cost averaging" will see the similarity. In addition, reinvesting funds as bonds mature maintains the duration of the overall portfolio.

Another attractive feature to the laddered portfolio apparent in Exhibit 13 is in convexity. Convexity, technically, is the second-order effect on the value of an asset or liability given a change in the yield to maturity. Importantly, it is affected by the dispersion of cash flows, as indicated in Equation 1, repeated here:

$$\text{Immunized Portfolio Convexity} = \frac{\text{MacDur}^2 + \text{MacDur} + \text{Dispersion}}{(1 + \text{Cash flow yield})^2}.$$

If the three portfolios have the same duration (and cash flow yield), then the barbell clearly has the highest convexity and the bullet the lowest. The laddered portfolio will tend to have relatively high convexity because its cash flows by design are spread over the time line. Compared with the barbell, the laddered portfolio has much less cash flow reinvestment risk.

In practice, perhaps the most desirable aspect of the laddered portfolio is in liquidity management. This aspect is particularly relevant if the bonds are not actively traded, as is the case for many corporate securities. As time passes, there is always a bond that is close to redemption. Its duration will be low so that its price is fairly stable even in a time of interest rate volatility. If the client needs immediate cash, the soon-to-mature bond makes for high-quality collateral on a personal loan or, for an institution, a repo contract. As the bonds mature, the final coupon and principal can be deployed for consumption or reinvested in a long-term bond at the back of the ladder.

4.2.2. Using ETFs to Build Laddered Portfolios

Another way for a wealth manager to build a laddered portfolio for a client is to use fixed-maturity corporate bond exchange-traded funds (ETFs). These ETFs have a designated year of maturity and credit risk profile—for instance, 2024 investment-grade corporate bonds. The passively managed, low-cost ETF seeks to replicate the performance of an index of, for instance, 50 held-to-maturity investment-grade corporate bonds that mature in 2021. As discussed in previous sections, the ETF manager can use a stratified sampling approach to track the index.

Suppose that in 2021, the wealth manager buys for the client roughly equal positions in the 2022 through 2029 fixed-maturity corporate bond ETFs. These purchases create a laddered portfolio that should provide the same benefits as holding the bonds directly—price stability in the soonest-to-mature ETF and greater convexity than holding more of a bullet portfolio. Moreover, the ETFs should be more liquid than positions in the actual bonds.

But laddered portfolios are not without limitations. For many investors, the decision to build a laddered bond portfolio should be weighed against buying shares in a fixed-income mutual fund, especially if the portfolio consists of a limited number of corporate bonds. Clearly, the mutual fund provides greater diversification of default risk. Moreover, actual bonds can entail a much higher cost of acquisition. If the entire investment needs to be liquidated, the mutual fund shares can be redeemed more quickly than the bonds can be sold, and likely at a better price.

EXAMPLE 4

Mr. Zheng is a Shanghai-based wealth adviser. A major client of his, the Wang family, holds most of its assets in residential property and equity investments and relies on regular cash flows from those holdings. Zheng recommends that the Wang family also have a laddered portfolio of Chinese government bonds. He suggests the following portfolio, priced for settlement on 1 January 2021:

Coupon Rate	Payment Frequency	Maturity	Flat Price	Yield (s.a.)	Par Value (CNY)	Market Value (CNY)
3.22%	Annual	26-Mar-22	101.7493	1.758%	10 Million	10,422,826
3.14%	Annual	8-Sept-24	102.1336	2.508%	10 Million	10,312,292
3.05%	Annual	22-Oct-26	101.4045	2.764%	10 Million	10,199,779
2.99%	Semi-annual	15-Oct-29	101.4454	2.803%	10 Million	10,208,611
					40 Million	41,143,508

The yields to maturity on the first three bonds have been converted from a periodicity of one to two in order to report them on a consistent semi-annual bond basis, as indicated by "(s.a.)." The total market value of the portfolio is CNY 41,143,508. The cash flow yield for the portfolio is 2.661%, whereas the market value-weighted average yield is 2.455%.

Most important for his presentation to the senior members of the Wang family is the schedule for the 30 cash flows:

1	26-Mar-21	322,000	16	8-Sep-24	10,314,000
2	15-Apr-21	149,500	17	15-Oct-24	149,500
3	8-Sep-21	314,000	18	22-Oct-24	305,000
4	15-Oct-21	149,500	19	15-Apr-25	149,500
5	22-Oct-21	305,000	20	15-Oct-25	149,500
6	26-Mar-22	10,322,000	21	22-Oct-25	305,000
7	15-Apr-22	149,500	22	15-Apr-26	149,500
8	8-Sep-22	314,000	23	15-Oct-26	149,500
9	15-Oct-22	149,500	24	22-Oct-26	10,305,000
10	22-Oct-22	305,000	25	15-Apr-27	149,500
11	15-Apr-23	149,500	26	15-Oct-27	149,500
12	8-Sep-23	314,000	27	15-Apr-28	149,500
13	15-Oct-23	149,500	28	15-Oct-28	149,500
14	22-Oct-23	305,000	29	15-Apr-29	149,500
15	15-Apr-24	149,500	30	15-Oct-29	10,149,500

Indicate the main points that Zheng should emphasize in this presentation about the laddered portfolio to senior members of the Wang family.

Solution: Zheng should emphasize three features of the portfolio:

1. **High credit quality.** Given that the family already has substantial holdings in residential property and equity, which are subject to price volatility and risk, investments in government bonds provide the Wang family with holdings in a very low-risk asset class.
2. **Liquidity.** The schedule of payments shows that coupon payments are received each year. These funds can be used for any cash need, including household expenses. The large principal payments can be reinvested in longer-term government bonds at the back of the ladder.
3. **Yield curve diversification.** The bond investments are spread out along four segments of the government bond yield curve. If they were concentrated at a single point, the portfolio would have the risk of higher yields at that point. By spreading out the maturities in the ladder formation, the portfolio has the benefit of diversification.

4.3. Duration Matching

- **evaluate liability-based strategies under various interest rate scenarios and select a strategy to achieve a portfolio's objectives**

Duration matching to immunize multiple liabilities is based on similar principles to those covered earlier in relation to a single liability. A portfolio of fixed-income bonds is structured and managed to track the performance of the zero-coupon bonds that would perfectly lock in the rates of return needed to pay off the corporate debt liabilities identified in Exhibit 5. Recall that in the case of a single liability, the immunization strategy is to match the portfolio Macaulay duration with the investment horizon. Also, the initial investment needs to match (or exceed) the present value of the liability. These two conditions can be combined to prescribe that the money duration of the immunizing portfolio matches the money duration of the debt liabilities. Money duration, or "dollar duration," is the portfolio modified duration multiplied by the market value (recall that modified duration is the portfolio Macaulay duration divided by one plus the cash flow yield per period). With multiple liabilities, matching money durations is useful because the market values and cash flow yields of the assets and liabilities are not necessarily equal.

The money duration for the debt liabilities is EUR 1,178,237,935:

$$\left[\frac{\text{Portfolio MacDur}}{\left(1+\frac{\text{Annualized CF Yield}}{2}\right)}\right]\times \text{PV of Debt liabilities}$$

$$=\left[\frac{6.0004}{\left(1+\frac{0.037608}{2}\right)}\right]\times 200{,}052{,}250 = 1{,}178{,}237{,}935$$

The term in brackets is the annualized modified duration for the bond portfolio. To keep the numbers manageable, we use the basis point value (BPV) measure for money duration. This measure is the money duration multiplied by 1 bp. The BPV is EUR 117,824 (= EUR 1,178,237,935 × 0.0001). For each 1 bp change in the cash flow yield, the market value changes by approximately EUR 117,824. It is an approximation because convexity is not included. A closely related risk measure is the present value of a basis point (PVBP), also called the PV01 (present value of an "01," meaning 1 bp) and, in North America, the DV01 (dollar value of an "01").

Exhibit 14 shows the three bonds purchased by the asset manager on 15 February 2021. The total cash outlay on that date is EUR 202,224,094 (= EUR 41,772,719 + EUR 99,750,000 + EUR 60,701,375 = the market values of the three bonds). Exhibit 15 presents the table used to calculate the cash flow yield and the risk statistics. The annualized cash flow yield is 3.5822%. It is the internal rate of return on the cash flows in the second column of Exhibit 15, multiplied by two. The annualized Macaulay duration for the portfolio is 5.9308 (= 11.8615/2), and the modified duration is 5.8264 (= 5.9308/[1 + 0.035822/2]). The annualized dispersion and convexity statistics are 12.3048 (= 49.2194/4) and 48.6846 (= $\{201.7767/[1 + 0.035822/2]^2\}/4$), respectively. Notice that the first few cash flows for the assets in Exhibit 15 are less than the liability payments in Exhibit 5. That disparity indicates that some of the bonds held in the asset portfolio will need to be sold to meet the obligations.

EXHIBIT 14 The Bond Portfolio to Immunize the Multiple Liabilities

	1.5-Year Bond	6-Year Bond	11.5-Year Bond
Coupon rate	1.00%	2.875%	4.50%
Maturity date	15 August 2022	15 February 2027	15 August 2032
Price	99.875	99.75	100.25
Yield to maturity	1.0842%	2.9207%	4.4720%
Par value	41,825,000	100,000,000	60,550,000
Market value	41,772,719	99,750,000	60,701,375
Macaulay duration	1.493	5.553	9.105
Convexity	2.950	34.149	96.056
Allocation	20.657%	49.326%	30.017%

EXHIBIT 15 Portfolio Statistics

Time	Date	Cash Flow	PV of Cash Flow	Weight	Time × Weight	Dispersion	Convexity
0	15-Feb-21	−202,224,094					
1	15-Aug-21	3,009,000	2,956,054	0.0146	0.0146	1.7245	0.0292
2	15-Feb-22	3,009,000	2,904,040	0.0144	0.0287	1.3966	0.0862
3	15-Aug-22	44,834,000	42,508,728	0.2102	0.6306	16.5068	2.5225
4	15-Feb-23	2,799,875	2,607,951	0.0129	0.0516	0.7970	0.2579
5	15-Aug-23	2,799,875	2,562,062	0.0127	0.0633	0.5965	0.3801
6	15-Feb-24	2,799,875	2,516,981	0.0124	0.0747	0.4276	0.5228
7	15-Aug-24	2,799,875	2,472,692	0.0122	0.0856	0.2890	0.6847
8	15-Feb-25	2,799,875	2,429,183	0.0120	0.0961	0.1791	0.8649
9	15-Aug-25	2,799,875	2,386,440	0.0118	0.1062	0.0966	1.0621
10	15-Feb-26	2,799,875	2,344,449	0.0116	0.1159	0.0402	1.2753
11	15-Aug-26	2,799,875	2,303,196	0.0114	0.1253	0.0085	1.5034
12	15-Feb-27	102,799,875	83,075,901	0.4108	4.9297	0.0079	64.0865
13	15-Aug-27	1,362,375	1,081,607	0.0053	0.0695	0.0069	0.9734
14	15-Feb-28	1,362,375	1,062,575	0.0053	0.0736	0.0240	1.1034
15	15-Aug-28	1,362,375	1,043,878	0.0052	0.0774	0.0508	1.2389
16	15-Feb-29	1,362,375	1,025,510	0.0051	0.0811	0.0869	1.3794
17	15-Aug29	1,362,375	1,007,465	0.0050	0.0847	0.1315	1.5245
18	15-Feb-30	1,362,375	989,738	0.0049	0.0881	0.1844	1.6738
19	15-Aug-30	1,362,375	972,323	0.0048	0.0914	0.2450	1.8271
20	15-Feb-31	1,362,375	955,214	0.0047	0.0945	0.3129	1.9839
21	15-Aug-31	1,362,375	938,406	0.0046	0.0974	0.3875	2.1439
22	15-Feb-32	1,362,375	921,894	0.0046	0.1003	0.4686	2.3067
23	15-Aug-32	61,912,375	41,157,805	0.2035	4.6811	25.2505	112.3462
			202,224,094	1.0000	11.8615	49.2194	201.7767

The market value of the immunizing fixed-income bonds is EUR 202,224,094. That amount is higher than the value of the liabilities, which is EUR 200,052,250. The reason for the difference in market values as of 15 February 2021 is the difference in the cash flow yields. The high-quality assets needed to immunize the corporate liabilities have a cash flow yield of 3.5822%, which is lower than the cash flow yield of 3.7608% on the debt obligations. The assets grow at a lower rate and, therefore, need to start at a higher level. If we discount the debt liabilities scheduled in the third column of Exhibit 5 at 3.5822%, the present value is EUR 202,170,671, indicating that initially, the immunizing portfolio is slightly overfunded. Importantly, the asset portfolio BPV is EUR 117,824 (= 202,224,094 × 5.8264 × 0.0001), matching the BPV for the debt liabilities.

There is another meaningful difference in the structure of the asset and liability portfolios. Although the money durations are the same, the dispersion and convexity statistics for the assets are greater than for the liabilities—12.30 compared with 8.26 for dispersion, and 48.68 compared with 45.54 for convexity. This difference is required to achieve immunization for multiple liabilities. (Mathematically, in the optimization problem, to minimize the difference in the change in the values of assets and liabilities, the first derivative leads to matching money duration [or BPV] and the second derivative to having higher dispersion.) Intuitively, this condition follows from the general result that, for equal durations, a more convex portfolio generally outperforms a less convex portfolio (higher gains if yields fall, lower losses if yields rise). But, as in the case of immunizing a single liability, the dispersion of the assets should be as low as possible subject to being greater than or equal to the dispersion of the liabilities to mitigate the effect of non-parallel shifts in the yield curve. Note that from Equation 1, higher dispersion implies higher convexity when the Macaulay durations and cash flow yields are equal.

4.3.1. Duration Matching—Parallel Shift Example

Some numerical examples are useful to illustrate that immunization of multiple liabilities is essentially an interest rate risk hedging strategy. The idea is that changes in the market value of the asset portfolio closely match changes in the debt liabilities whether interest rates rise or fall. Exhibits 16 through 19 demonstrate this dynamic.

First, we allow the yield curve to shift upward in a parallel manner. The yields on the bonds in Exhibit 14 go up instantaneously by 25 bps on 15 February 2021, immediately after the asset portfolio is purchased. That increase results in a drop in market value of EUR 2,842,408. The yields on the debt liabilities in Exhibit 5 also go up by 25 bps, dropping the market value by EUR 2,858,681. The difference is EUR 16,273, a small amount given that the size of portfolios exceeds EUR 200 million. This scenario implicitly assumes no change in the corporate entity's credit risk.

EXHIBIT 16 Immunizing Multiple Liabilities: Upward Parallel Shift

	Immunizing Assets	Debt Liabilities	Difference
ΔMarket value	−2,842,408	−2,858,681	16,273
ΔCash flow yield	0.2437%	0.2449%	−0.0012%
ΔPortfolio BPV	−2,370	−2,207	−163

Next, we shift the yield curve downward by 25 bps. Both the asset and liability portfolios gain market value by almost the same amount. The difference is only EUR 12,504.

EXHIBIT 17 Immunizing Multiple Liabilities: Downward Parallel Shift

Downward Parallel Shift	Immunizing Assets	Debt Liabilities	Difference
ΔMarket value	2,900,910	2,913,414	−12,504
ΔCash flow yield	−0.2437%	−0.2449%	0.0012%
ΔPortfolio BPV	2,429	2,256	173

The driving factor behind the success of the strategy given these upward and downward shifts is that the portfolio durations are matched and changes in the cash flow yields are very close: 24.37 bps for the assets and 24.49 bps for the liabilities. In Exhibit 17, the asset portfolio rises slightly less than the liabilities when the yield curve shifts down in a parallel manner by 25 bps. Hence, the loss is EUR 12,504 despite the greater convexity of the assets. That disparity is explained by the slightly higher decrease in the cash flow yield on the liabilities. As explained in the previous section, a parallel shift is a sufficient but not necessary condition for immunization. Although not shown in the exhibits, an upward non-parallel shift of 15.9 bps in the 1.5-year bond, 23.6 bps in the 6-year bond, and 27.5 bps in the 11.5-year bond leads to virtually the same change in market value (EUR 2,842,308) as the 25 bp parallel shift. Those particular changes are chosen because they result in the same change in the cash flow yield of 24.37 bps.

4.3.2. Duration Matching—Yield Curve Twist Scenario

The structural risk to the immunization strategy is apparent in Exhibit 18. This scenario is the steepening twist in which short-term yields on high-quality bonds go down while long-term yields go up. The 1.5-year yield is assumed to drop by 25 bps. The 6-year yield remains the same, and the 11.5-year yield goes up by 25 bps. These changes lead to a loss of EUR 1,178,071 in the asset portfolio as the cash flow yield increases by 10.04 bps. The maturities of the debt liabilities differ from those of the assets. For simplicity, we assume that those yields change in proportion to the differences in maturity around the six-year pivot point for the twist. The 2.5-year yield drops by 19.44 bps (= 25 bps × 3.5/4.5), the 7-year yield goes up by 4.55 bps (= 25 bps × 1/5.5), and the 10-year goes up by 18.18 bps (= 25 bps × 4/5.5). The market value of the liabilities drops by only EUR 835,156 because the cash flow yield increases by only 7.11 bps. The value of the assets goes down by more than the liabilities—the difference is EUR 342,915. The steepening twist to the shape of the yield curve is the source of the loss.

EXHIBIT 18 Immunizing Multiple Liabilities: Steepening Twist

	Immunizing Assets	Debt Liabilities	Difference
ΔMarket value	−1,178,071	−835,156	−342,915
ΔCash flow yield	0.1004%	0.0711%	0.0293%
ΔPortfolio BPV	−984	−645	−339

The results of the fourth scenario show that a flattening twist can lead to a comparable gain if long-term high-quality yields fall while short-term yields rise (Exhibit 19). We make the same assumptions about proportionate changes in the yields. In this case, the cash flow yield of the assets goes down more and the market value rises higher than the debt liabilities. Clearly, an entity that pursues immunization of multiple liabilities hopes that steepening twists are balanced out by flattening twists and that most yield curve shifts are more or less parallel.

EXHIBIT 19 Immunizing Multiple Liabilities: Flattening Twist

	Immunizing Assets	Debt Liabilities	Difference
ΔMarket value	1,215,285	850,957	364,328
ΔCash flow yield	−0.1027%	−0.0720%	−0.0307%
ΔPortfolio BPV	1,016	658	358

The above illustrations (in Exhibits 16–19) also report the changes in the portfolio BPVs for the assets and liabilities. Before the yield curve shifts and twists, the BPVs are matched at EUR 117,824. Afterward, there is a small money duration mismatch. In theory, the asset manager needs to rebalance the portfolio immediately. In practice, the manager likely waits until the mismatch is large enough to justify the transaction costs in selling some bonds and buying others. Another method to rebalance the portfolio is to use interest rate derivatives.

EXAMPLE 5

A Japanese corporation recently sold one of its lines of business and would like to use the cash to retire the debt liabilities that financed those assets. Summary statistics for the multiple debt liabilities, which range in maturity from three to seven years, are market value, JPY 110.4 billion; portfolio modified duration, 5.84; portfolio convexity, 46.08; and BPV, JPY 64.47 million.

An investment bank working with the corporation offers three alternatives to accomplish the objective:

1. **Bond tender offer.** The corporation would buy back the debt liabilities on the open market, paying a premium above the market price. The corporation currently has a single-A rating and hopes for an upgrade once its balance sheet is improved by retiring the debt. The investment bank anticipates that the tender offer would have to be at a price commensurate with a triple-A rating to entice the bondholders to sell. The bonds are widely held by domestic and international institutional investors.
2. **Cash flow matching.** The corporation buys a portfolio of government bonds that matches, as closely as possible, the coupon interest and principal redemptions on the debt liabilities. The investment bank is highly confident that the corporation's external auditors will agree to accounting defeasement because the purchased bonds are government securities. That agreement will allow the corporation to remove both the defeasing asset portfolio and the liabilities from the balance sheet.
3. **Duration matching.** The corporation buys a portfolio of high-quality corporate bonds that matches the duration of the debt liabilities. Interest rate derivatives contracts will be used to keep the duration on its target as time passes and yields change. The investment bank thinks it is very unlikely that the external auditors will allow this strategy to qualify for accounting defeasement. The corporation can explain to investors and the rating agencies in the management section of its annual report, however, that it is aiming to "effectively defease" the debt. To carry out this strategy, the investment bank suggests three different portfolios of investment-grade corporate bonds that range in maturity from 2 years to 10 years. Each portfolio has a

market value of about JPY 115 billion, which is considered sufficient to pay off the liabilities.

	Portfolio A	Portfolio B	Portfolio C
Modified duration	5.60	5.61	5.85
Convexity	42.89	50.11	46.09
BPV (in millions)	JPY 64.50	JPY 64.51	JPY 67.28

After some deliberation and discussion with the investment bankers and external auditors, the corporation's CFO chooses Strategy 3, duration matching.

1. Indicate the likely trade-offs that led the corporate CFO to choose the duration-matching strategy over the tender offer and cash flow matching.
2. Indicate the portfolio that the corporation should choose to carry out the duration-matching strategy.

Solution to 1: The likely trade-offs are between removing the debt liabilities from the balance sheet, either by directly buying the bonds from investors or by accounting defeasement via cash flow matching, and the cost of the strategy. The tender offer entails buying the bonds at a triple-A price, which would likely be considerably higher than at a single-A price. Cash flow matching entails buying even more expensive government bonds. The duration-matching strategy can be implemented at a lower cost because the asset portfolio consists of less expensive investment-grade bonds. The CFO has chosen the lowest-cost strategy, even though the debt liabilities will remain on the balance sheet.

Solution to 2: The corporation should recommend Portfolio B. Portfolio C closely matches the modified duration (as well as the convexity) of the liabilities. Duration matching when the market values of the assets and liabilities differ, however, entails matching the money durations, in particular the BPVs. The choice then comes down to Portfolios A and B. Although both have BPVs close to the liabilities, it is incorrect to choose A based on its BPV being "closer."

The important difference between Portfolios A and B lies in the convexities. To immunize multiple liabilities, the convexity (and dispersion of cash flows) of the assets needs to be greater than the liabilities. Therefore, Portfolio A does not meet that condition.

Recall that in Example 2, the correct immunizing portfolio is the one with the lower convexity, which minimizes the structural risk to the strategy. But, that bond portfolio still has a convexity greater than the zero-coupon bond that would provide perfect immunization. This greater convexity of the immunizing portfolio is because the dispersion of the zero-coupon bond is zero and the durations are the same. As seen in Equation 1, that dispersion implies a lower convexity statistic.

4.4. Derivatives Overlay

Interest rate derivatives can be a cost-effective method to rebalance the immunizing portfolio to keep it on its target duration as the yield curve shifts and twists and as time passes. Suppose that in the duration-matching example shown earlier, there is a much larger instantaneous upward shift in the yield curve on 15 February 2021. In particular, all yields shift up by 100 bps. Because yields and duration are inversely related, the portfolio duration statistics go down, as does the market value. The BPV of the immunizing asset portfolio decreases from EUR 117,824 to EUR 108,679, a drop of EUR 9,145. The BPV for the debt liabilities goes down to EUR 109,278, a drop of EUR 8,546. There is now a money duration gap of –EUR 599 (= EUR 108,679 – EUR 109,278). The asset manager could sell some of the 1%, 1.5-year bonds and buy some more of the 4.50%, 11-year bonds to close the money duration gap. A more efficient and lower-cost rebalancing strategy, however, is likely to buy, or go long, a few interest rate futures contracts to rebalance the portfolio.

To address the question of the required number of contracts to close, or reduce, a duration gap, we change the example from euros to US dollars. Doing so allows us to illustrate the calculations for the required number of futures contracts using the actively traded 10-year US Treasury note futures contract offered at the CME Group. The present value of corporate debt liabilities shown in Exhibits 4 and 5 now is assumed to be USD 200,052,250. Risk and return statistics are invariant to currency denomination, so the portfolio Macaulay duration is still 6.0004 and the BPV is USD 117,824.

In the previous example for duration matching of multiple liabilities, the asset manager purchased three bonds with maturities of 1, 6, and 11 years. In this next scenario, we assume that the asset manager buys a portfolio of high-quality, short-term bonds. This portfolio has a market value of USD 222,750,000, Macaulay duration of 0.8532, and cash flow yield of 1.9804%. Discounting the debt liabilities in the third column of Exhibit 5 at 1.9804% gives a present value of USD 222,552,788. This value indicates that the immunizing portfolio is overfunded on 15 February 2021. The BPV for the asset portfolio is USD 18,819:

$$\left[\frac{0.8532}{\left(1+\frac{0.019804}{2}\right)}\right] \times 222{,}750{,}000 \times 0.0001 = 18{,}819$$

The asset manager might elect to hold a portfolio of short-term bonds rather than intermediate-term and long-term securities for a number of reasons, including greater liquidity, perception of finer pricing in the short-term market, or that the entity faces liquidity constraints and needs to hold these short-term bonds to meet regulatory requirements. A derivatives overlay strategy is then used to close the duration gap while keeping the underlying portfolio unchanged. In general, a derivatives overlay transforms some aspect of the underlying portfolio—the currency could be changed with foreign exchange derivatives or the credit risk profile with credit default swap contracts. Here, interest rate derivatives are used to change the interest rate risk profile, increasing the portfolio BPV from USD 18,819 to USD 117,824.

Details of interest rate futures contracts are covered elsewhere. Here we note some specific features of the 10-year US Treasury note contract traded at the CME Group relevant for this example. Each contract is for USD 100,000 in par value and has delivery dates in March, June, September, and December.

Conversion factors that are used to make the qualifying T-notes roughly equivalent for delivery by the contract seller, or short position, are based on an arbitrary yield to maturity of 6.00%. If the eligible T-note has a coupon rate below (above) 6.00%, the conversion factor is less (more) than 1.0000. The invoice price paid by the buyer of the contract, the long position, at the expiration of the contract is the futures price multiplied by the conversion factor, plus accrued interest. The logic of this design is that if the contract seller chooses to deliver a qualifying T-note having a lower (higher) coupon rate than 6.00%, the buyer pays a lower (higher) price.

The key point is that, although the eligible T-notes are roughly equivalent, one will be identified as the cheapest-to-deliver (CTD) security. Importantly, the duration of the 10-year T-note futures contract is assumed to be the duration of the CTD T-note. A factor in determining the CTD T-note is that the conversion factors for each qualifying security are based on the arbitrary assumption of a 6.00% yield to maturity. In practice, when yields are below 6.00% the CTD security typically is the qualifying T-note having the lowest duration. Therefore, the 10-year T-note futures contract essentially has been acting as a 6.5-year contract. (That explains the motivation for introducing the Ultra 10-year contract—to provide a hedging instrument more closely tied to the 10-year T-note traded in the cash market.)

To illustrate the importance of using the risk statistics for the CTD T-note, Exhibit 20 reports two hypothetical qualifying securities for the March 2021 10-year futures contract. One is designated the 6.5-year T-note. It has a coupon rate of 2.75% and matures on 15 November 2027. As of 15 February 2021, it is assumed to be priced to yield 3.8088%. Its BPV per USD 100,000 in par value is USD 56.8727, and its conversion factor is 0.8226. The other is the on-the-run 10-year T-note. Its coupon rate is 4.00%, and it matures on 15 February 2031. Its BPV is USD 81.6607, and its conversion factor is 0.8516.

EXHIBIT 20 Two Qualifying T-Notes for the March 2021 10-Year T-Note Futures Contract as of 15 February 2021 (hypothetical example)

	6.5-Year T-Note	10-Year T-Note
Coupon rate	2.75%	4.00%
Maturity date	15 November 2027	15 February 2031
Full price per 100,000 in par value	USD 94,449	USD 99,900
Yield to maturity	3.8088%	4.0122%
Modified duration	6.0215	8.1742
BPV per 100,000 in par value	56.8727	81.6607
Conversion factor	0.8226	0.8516

The calculation of the required number of futures contract, denoted N_f, comes from this relationship:

$$\text{Asset portfolio BPV} + (N_f \times \text{Futures BPV}) = \text{Liability portfolio BPV} \tag{2}$$

Inherent in this expression is the important idea that although futures contracts have a market value of zero as a result of daily mark-to-market valuation and settlement, they can add to or subtract from the asset portfolio BPV. This equation can be rearranged to isolate N_f:

$$N_f = \frac{\text{Liability portfolio BPV} - \text{Asset portfolio BPV}}{\text{Futures BPV}} \tag{3}$$

If N_f is a positive number, the asset manager buys, or goes long, the required number of futures contracts. Doing so raises the money duration of the assets to match that of the liabilities. If N_f is a negative number, the asset manager sells, or goes short, futures contracts to reduce the money duration. In our problem, the asset portfolio BPV is USD 18,819 and the liability portfolio BPV is USD 117,824. Therefore, N_f is a large positive number and depends on the BPV for the futures contract. The exact formulation for the Futures BPV is complicated, however, and goes beyond the scope of our coverage. It involves such details as the number of days until the expiration of the contract, the interest rate for that period, and the accrued interest on the deliverable bond. To simplify, we use an approximation formula that is common in practice:

$$\text{Futures BPV} \approx \frac{\text{BPV}_{\text{CTD}}}{\text{CF}_{\text{CTD}}} \tag{4}$$

where CF_{CTD} is the conversion factor for the CTD security.

If the CTD security is the 6.5-year T-note shown in Exhibit 20, the Futures BPV is estimated to be USD 69.1377 (= 56.8727/0.8226). Then, the required number of contracts is approximately 1,432:

$$\frac{117{,}824 - 18{,}819}{69.1377} = 1{,}432$$

But, if the CTD security is the 10-year T-note, the Futures BPV is USD 95.8909 (= 81.6607/0.8516). To close the money duration gap, the required number of contracts is only 1,032:

$$\frac{117{,}824 - 18{,}819}{95.8909} = 1{,}032$$

Clearly, the asset manager must know the CTD T-note to use in the derivatives overlay strategy. The difference of 400 futures contracts is significant.

The asset manager has established a synthetic "barbell" strategy: having positions in the short-term and longer-term segments of the yield curve. The term "synthetic" means "created with derivatives." The underlying asset portfolio is concentrated in the short-term market. The derivatives portfolio is either at the 6.5-year or 10-year segment of the yield curve. CME Group also has actively traded two-year and five-year Treasury futures contracts. Therefore, the asset manager could choose to spread out the futures contracts across other segments of the yield curve. That diversification reduces the structural risk to the immunization strategy arising from non-parallel shifts and twists to the curve.

EXAMPLE 6

A Frankfurt-based asset manager uses the Long Bund contract traded at the Intercontinental Exchange (ICE) futures exchange to manage the gaps that arise from "duration drift" in a portfolio of German government bonds that are used to immunize a portfolio of corporate debt liabilities. This futures contract has a notional principal of EUR 100,000 and is based on a 6% coupon rate. The German government bonds that are eligible for delivery have maturities between 8.5 years and 10.5 years.

Currently, the corporate debt liabilities have a market value of EUR 330,224,185, a modified duration of 7.23, and a BPV of EUR 238,752. The asset portfolio has a market value of EUR 332,216,004, a modified duration of 7.42, and a BPV of EUR 246,504. The duration drift has arisen because of a widening spread between corporate and government bond yields as interest rates in general have come down. The lower yields on government bonds have increased the modified durations relative to corporates.

Based on the deliverable bond, the asset manager estimates that the BPV for each futures contract is EUR 65.11.

1. Does the asset manager go long (buy) or go short (sell) the futures contract?
2. How many contracts does the manager buy or sell to close the duration gap?

Solution to 1: The asset manager needs to go short (or sell) Long Bund futures contracts. The money duration of the assets, as measured by the BPV, is greater than the money duration of debt liabilities. This relationship is true of the modified duration statistics as well, but the money duration is a better measure of the gap because the market values differ.

Solution to 2: Use Equation 3 to get the requisite number of futures contracts to sell.

$$N_f = \frac{\text{Liability portfolio BPV} - \text{Asset portfolio BPV}}{\text{Futures BPV}}$$

where Liability portfolio BPV = 238,752, Asset portfolio BPV = 246,504, and Futures BPV = 65.11.

$$N_f = \frac{238{,}752 - 246{,}504}{65.11} = -119.06$$

The minus sign indicates the need to go short (or sell) 119 contracts to close the duration gap.

4.5. Contingent Immunization

We have seen that the initial market value for the immunizing asset portfolio can vary according to the strategy chosen by the asset manager. Earlier, in the duration-matching example, the initial market value of the asset portfolio was EUR 202,224,094, while the liabilities were EUR 200,052,250. The derivatives overlay example is to hold a portfolio of short-term bonds having a market value of USD 222,750,000 and 1,432 10-year futures contracts (assuming that the CTD eligible security is the 6.5-year T-note) to immunize the liability of USD 200,052,250.

The difference between the market values of the assets and liabilities is the **surplus**. The initial surplus in the duration-matching example is EUR 2,171,844 (= EUR 202,224,094 – EUR 200,052,250); the surplus in the derivatives overlay example is USD 22,697,750 (= EUR 222,750,000 –EUR 200,052,250). The presence of a significant surplus allows the asset manager to consider a hybrid passive–active strategy known as **contingent immunization**. The idea behind contingent immunization is that the asset manager can pursue active investment strategies, as if operating under a total return mandate, as long as the surplus is above a designated threshold. If the actively managed assets perform poorly, however, and the surplus evaporates, the mandate reverts to the purely passive strategy of building a duration-matching portfolio and then managing it to remain on duration target.

In principle, when the surplus is above a sufficient threshold, the manager may increase portfolio risk in any asset category, including equity, fixed income, and alternative investments. The manager could also buy out-of-the-money commodity options contracts or credit default swaps. The objective is to attain portfolio gains in order to reduce the cost of retiring the debt obligations without falling below the minimum funding threshold. Obviously, liquidity is an important criterion in selecting the investments because the positions would need to be unwound if losses cause the surplus to near the threshold.

A natural setting for contingent immunization is in the fixed-income derivatives overlay strategy. Instead of buying, or going long, 1,432 10-year T-note futures contracts, the asset manager could intentionally over-hedge or under-hedge, depending on the held view on rate volatility at the 6.5-year segment of the Treasury yield curve. That segment matters because the 10-year T-note futures contract price responds to changes in the yield of the CTD security. The asset manager could buy more (less) than 1,432 contracts if she expects the 6.5-year Treasury yield to go down (up) and the futures price to go up (down).

Suppose that on 15 February 2021, the price of the March 10-year T-note futures contract is quoted to be 121-03. The price is 121 and 3/32 percent of USD 100,000, which is the contract size. Therefore, the delivery price in March would be USD 121,093.75 multiplied by the conversion factor, plus the accrued interest. What matters to the asset manager is the change in the settlement futures price from day to day. For each futures contract, the gain or loss is USD 31.25 for each 1/32nd change in the futures price, calculated as 1/32 percent of USD 100,000.

Now suppose that the asset manager anticipates an upward shift in the yield curve. Such a shift would cause bond prices to drop in both the Treasury cash and futures markets. Suppose that the quoted March futures price drops from 121-03 to 119-22. That is a 45/32nd change in the price and causes a loss of USD 1,406.25 (= 45 × USD 31.25) per contract. If the asset manager holds 1,432 long contracts, the loss that day is USD 2,013,750 (= USD 1,406.25 × 1,432). But if the asset manager is allowed to under-hedge, he could have dramatically reduced the number of long futures contracts and maybe even gone short in anticipation of the upward shift. The presence of the surplus allows the manager the opportunity to take a view on interest rates and save some of the cost of the strategy to retire the debt liabilities. The objective is to be over-hedged when yields are expected to fall and under-hedged when they are expected to rise.

EXAMPLE 7

An asset manager is asked to build and manage a portfolio of fixed-income bonds to retire multiple corporate debt liabilities. The debt liabilities have a market value of GBP 50,652,108, a modified duration of 7.15, and a BPV of GBP 36,216.

The asset manager buys a portfolio of British government bonds having a market value of GBP 64,271,055, a modified duration of 3.75, and a BPV of GBP 24,102. The initial surplus of GBP 13,618,947 and the negative duration gap of GBP 12,114 are intentional. The surplus allows the manager to pursue a contingent immunization strategy to retire the debt at, hopefully, a lower cost than a more conservative duration-matching approach. The duration gap requires the manager to buy, or go long, interest rate futures contracts to close the gap. The manager can choose to over-hedge or under-hedge, however, depending on market circumstances.

The futures contract that the manager buys is based on 10-year gilts having a par value of GBP 100,000. It is estimated to have a BPV of GBP 98.2533 per contract. Currently, the asset manager has purchased, or gone long, 160 contracts.

Which statement *best* describes the asset manager's hedging strategy and the held view on future 10-year gilt interest rates? The asset manager is:

A. over-hedging because the rate view is that 10-year yields will be rising.
B. over-hedging because the rate view is that 10-year yields will be falling.

C. under-hedging because the rate view is that 10-year yields will be rising.
D. under-hedging because the rate view is that 10-year yields will be falling.

Solution: B is correct. The asset manager is over-hedging because the rate view is that 10-year yields will be falling. First calculate the number of contracts (N_f) needed to fully hedge (or immunize) the debt liabilities. The general relationship is Equation 2: Asset portfolio BPV + (N_f × Futures BPV) = Liability portfolio BPV.

Asset portfolio BPV is GBP 24,102; Futures BPV is 98.2533; and Liability portfolio BPV is 36,216.

$$24{,}102 + (N_f \times 98.2533) = 36{,}216$$

$$N_f = 123.3$$

The asset manager is over-hedging because a position in 160 long futures contracts is more than what is needed to close the duration gap. Long, or purchased, positions in interest rate futures contracts gain when futures prices rise and rates go down. The anticipated gains from the strategic decision to over-hedge in this case further increase the surplus and reduce the cost of retiring the debt liabilities.

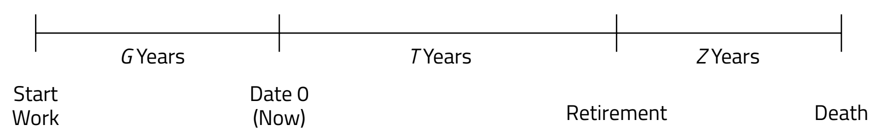
5. LIABILITY-DRIVEN INVESTING: AN EXAMPLE OF A DEFINED BENEFIT PENSION PLAN

Earlier we introduced four types of liabilities: Types I, II, III, and IV. Defined benefit (DB) pension plan obligations are a good example of Type IV liabilities for which both the aggregate amounts and dates are uncertain. An LDI strategy for this entity starts with a model for these liabilities. We first explain the model assumptions and then calculate future liabilities.

5.1. Model Assumptions

We reveal some of the assumptions that go into this complex financial modeling problem by assuming the work history and retirement profile for a representative employee covered by the pension plan. We assume that this employee has worked for *G* years, a sufficient length of time to ensure that the retirement benefits are vested. The employee is expected to work for another *T* years and then to retire and live for *Z* years. Exhibit 21 illustrates this time line.

EXHIBIT 21 Timeline Assumptions for the Representative Employee



In this final pay DB example, the retired employee receives a fixed lifetime annuity based on her wage at the time of retirement, denoted W_T. Some pension plans index the annual retirement benefit to inflation. Our example assumes an annuity fixed in nominal terms, calculated as the final wage, W_T, multiplied by a multiplier, m, multiplied by the total number of years worked, $G + T$.

There are two general measures of the retirement obligations as of Time 0—the accumulated benefit obligation (ABO) and the projected benefit obligation (PBO). The ABO calculates the liability based on the G years worked and the current annual wage, denoted W_0, even though the annuity paid in retirement is based on W_T (final wage) and $G + T$ years. The use of the current annual wage and the number of years worked is because the ABO represents the *legal liability* today of the plan sponsor if the plan were to be closed or converted to another type of plan, such as a defined contribution (DC) plan.The ABO is the present value of the projected annuity, discounted at an annual rate r on high-quality corporate bonds (government regulators and the accounting authorities allow high-quality corporate bonds to be used to discount the future liabilities), which for simplicity we assume applies for all periods (a flat yield curve).

$$\text{ABO} = \frac{1}{(1+r)^T} \times \left[\frac{m \times G \times W_0}{1+r} + \frac{m \times G \times W_0}{(1+r)^2} + \cdots + \frac{m \times G \times W_0}{(1+r)^Z}\right]$$

The term in brackets is the value of the Z-year annuity as of year T, and that sum is discounted back over T years to Time 0.

The PBO liability measure uses the projected wage for year T instead of the current wage in the Z-year annuity.

$$\text{PBO} = \frac{1}{(1+r)^T} \times \left[\frac{m \times G \times W_T}{1+r} + \frac{m \times G \times W_T}{(1+r)^2} + \cdots + \frac{m \times G \times W_T}{(1+r)^Z}\right]$$

Although the ABO is the legal obligation to the plan sponsor, the PBO is the liability reported in financial statements and used to assess the plan's funding status. The plan is over-funded (under-funded) if the current fair value of assets is more (less) than the present value of the promised retirement benefits.

The next step is to consider how wages evolve between dates 0 and T. We denote w to be the average annual wage growth rate for the employee's remaining work life of T years. Therefore, the relationship between W_0 and W_T is $W_T = W_0 \times (1 + w)^T$.

After some algebraic manipulation and substitution, the two liability measures can be written more compactly as follows:

$$\text{ABO} = \frac{m \times G \times W_0}{(1+r)^T} \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^Z}\right] \text{ and}$$

$$\text{PBO} = \frac{m \times G \times W_0 \times (1+w)^T}{(1+r)^T} \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^Z}\right]$$

Note that the PBO always will be larger than the ABO by the factor of $(1 + w)^T$, assuming positive wage growth in nominal terms.

We see in this simple model several of the important assumptions that go into using an LDI strategy to manage these Type IV liabilities. The assumed post-retirement lifetime (Z years) is critical. A higher value for Z increases both the ABO and PBO measures of liability. The pension plan faces *longevity risk*, which is the risk that employees live longer in their retirement years than assumed in the models. Some plans have become under-funded and have had to increase assets because regulators required that they recognize longer life expectancies. Another important assumption is the time until retirement (T years). In the ABO measure, increases in T reduce the liability. That result also holds for the PBO as long as wage growth (w) is lower than the discount rate (r). Assuming w is less than r is reasonable if it can be assumed that employees over time generally are compensated for price inflation and some part of real economic growth, as well as for seniority and productivity improvements, but overall the labor income growth rate does not quite keep pace with the nominal return on high-quality financial assets.

5.2. Model Inputs

We now use a numerical example to show how the effective durations of ABO and PBO liability measures are calculated. Assume that $m = 0.02$, $G = 25$, $T = 10$, $Z = 17$, W_0 = USD 50,000, and $r = 0.05$. We also assume that the wage growth rate w is an arbitrarily chosen constant fraction of the yield on high-quality corporate bonds r—in particular, that $w = 0.9 \times r$ so that $w = 0.045$ (= 0.9×0.05). Based on these assumptions, the ABO and PBO for the representative employee are USD 173,032 and USD 268,714, respectively.

$$\text{ABO} = \frac{m \times G \times W_0}{(1+r)^T} \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^Z}\right]$$

$$= \frac{0.02 \times 25 \times 50{,}000}{(1.05)^{10}} \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{17}}\right] = 173{,}032$$

$$\text{PBO} = \frac{m \times G \times W_0 \times (1+w)^T}{(1+r)^T} \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^Z}\right]$$

$$= \frac{0.02 \times 25 \times 50{,}000 \times (1.045)^{10}}{(1.05)^{10}} \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{17}}\right] = 268{,}714$$

If the plan covers 10,000 similar employees, the total liability is approximately USD 1.730 billion ABO and USD 2.687 billion PBO. Assuming that the pension plan has assets with a market value of USD 2.700 billion, the plan currently is overfunded by both measures of liability.

5.3. Calculating Durations

Recall that in general, the effective durations for assets or liabilities are obtained by raising and lowering the assumed yield curve in the valuation model and recalculating the present values.

$$\text{Effective duration} = \frac{(\text{PV}_-) - (\text{PV}_+)}{2 \times \Delta\text{Curve} \times (\text{PV}_0)} \quad (5)$$

PV_0 is the initial value, PV_- is the new value after the yield curve is lowered by ΔCurve, and PV_+ is the value after the yield curve is raised. In this simple model with a flat yield curve, we raise r from 0.05 to 0.06 (and w from 0.045 to 0.054) and lower r from 0.05 to 0.04 (and w from 0.045 to 0.036); therefore, ΔCurve = 0.01.

Given our assumptions, ABO_0 is USD 173,032. Redoing the calculations for the higher and lower values for r and w gives USD 146,261 for ABO_+ and USD 205,467 for ABO_-. The ABO effective duration is 17.1.

$$\text{ABO duration} = \frac{(\text{PV}_-) - (\text{PV}_+)}{2 \times \Delta\text{Curve} \times (\text{PV}_0)} = \frac{205{,}467 - 146{,}261}{2 \times 0.01 \times 173{,}032} = 17.1$$

Repeating the calculations for the PBO liability measure gives USD 247,477 for PBO_+ and USD 292,644 for PBO_-. Given that PBO_0 is 268,714, the PBO duration is 8.4.

$$\text{PBO} = \frac{292{,}644 - 247{,}477}{2 \times 0.01 \times 268{,}714} = \frac{(PV_-) - (PV_+)}{2 \times \Delta\text{Curve} \times (PV_0)} = 8.4$$

These calculations indicate the challenge facing the fund manager. There is a significant difference between having liabilities of USD 1.730 billion and an effective duration of 17.1, as measured by the ABO, and liabilities of USD 2.687 billion and an effective duration of 8.4, as measured by the PBO. The ABO BPV is USD 2,958,300 (= USD 1.730 billion × 17.1 × 0.0001), and the PBO BPV is USD 2,257,080 (= USD 2.687 billion × 8.4 × 0.0001). The plan sponsor must decide which liability measure to use for risk management and asset allocation. For example, if the corporation anticipates that it might be a target for an acquisition and that the acquirer likely would want to convert the retirement plan from defined benefit to defined contribution, the ABO measure matters more than the PBO.

We assume that the corporate sponsor sees itself as an ongoing independent institution that preserves the pension plan's current design. Therefore, PBO is the appropriate measure for pension plan liabilities. The plan is fully funded in that the market value of assets, assumed to be USD 2.700 billion, exceeds the PBO of USD 2.687 billion, giving a surplus of only USD 13 million. That surplus disappears quickly if yields on high-quality corporate bonds that are used to discount the projected benefits drop by about 5 bps to 6 bps. Note that the surplus divided by the PBO BPV is 5.76 (= 13,000,000/2,257,080). Interest rate risk is a major concern to the plan sponsor because changes in the funding status flow through the income statement, thereby affecting reported earnings per share.

Lower yields also raise the market value of assets depending on how those assets are allocated. We assume that the current asset allocation is 50% equity, 40% fixed income, and 10% alternatives. The fixed-income portfolio is managed to track an index of well-diversified corporate bonds—such indexes are covered later. Relevant at this point is that the chosen bond index reports a modified duration of 5.5.

The problem is to assign a duration for the equity and alternative investments. To be conservative, we assume that there is no stable and predictable relationship between valuations on those asset classes and market interest rates. Therefore, equity duration and alternatives duration are assumed to be zero. Assuming zero duration does not imply that equity and alternatives have no interest rate risk. Effective duration estimates the percentage change in value arising from a change in nominal interest rates. The effect on equity and alternatives depends on *why* the nominal rate changes, especially if that rate change is not widely anticipated in the market. Higher or lower interest rates can arise from a change in expected inflation, a change in monetary policy, or a change in macroeconomic conditions. Only fixed-income securities have a well-defined connection between market values and the yield curve. Nevertheless, assumptions are a source of model risk, as discussed in the next section.

Given these assumptions, we conclude that the asset BPV is USD 594,000 = USD 2.700 billion × [(0.50 × 0) + (0.40 × 5.5) + (0.10 × 0)] × 0.0001. The term in brackets is the estimated effective duration for the asset portfolio, calculated using the shares of market value as the weights. Clearly, the pension plan is running a significant duration gap—the asset BPV of USD 594,000 is much lower than the liability BPV of USD 2,257,080, using the PBO measure. If all yields go down by 10 bps, the market value of assets goes up by approximately USD 5.940 million and the present value of liabilities goes up by USD 22.571 million. The pension plan would have a deficit and be deemed under-funded.

5.4. Addressing the Duration Gap

The pension fund manager can choose to reduce, or even eliminate, the duration gap using derivatives. We consider several scenarios, starting with futures. We then consider the use of swaps and options to enter an interest rate swap.

5.4.1. Using Futures to Reduce the Duration Gap

For example, suppose the Ultra 10-year Treasury futures contract at the Chicago Mercantile Exchange (CME) has a BPV of USD 95.8909 because the on-the-run T-note is the CTD security. Using Equation 3, the pension plan would need to buy, or go long, 17,343 contracts to fully hedge the interest rate risk created by the duration gap:

$$N_f = \frac{\text{Liability portfolio BPV} - \text{Asset portfolio BPV}}{\text{Futures BPV}}$$

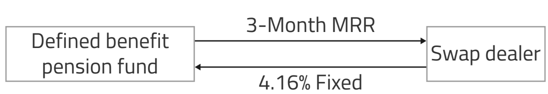
$$= \frac{2{,}257{,}080 - 594{,}000}{95.8909} = 17{,}343$$

One concern with hedging with futures is the need for daily oversight of the positions. That need arises because futures contracts are marked to market and settled at the end of each trading day into the margin account. Suppose that the fund did buy 17,343 futures contracts and 10-year Treasury yields go up by 5 bps. Given that the Futures BPV is USD 95.8909 per contract, the *realized* loss that day is more than USD 8.315 million: USD $95.8909 \times 5 \times 17{,}343 = 8{,}315{,}179$. That amount is offset by the *unrealized* reduction in the present value of liabilities. Such a large position in futures contracts would lead to significant daily cash inflows and outflows. For that reason, such hedging problems as the one facing the pension fund often are addressed with over-the-counter interest rate swaps rather than exchange-traded futures contracts.

5.4.2. Using Interest Rate Swaps to Reduce Duration Gap

Suppose that the pension fund manager can enter a 30-year, receive-fixed, interest rate swap against the three-month Market Reference Rate (MRR). The fixed rate on the swap is 4.16%. Assume its effective duration is +16.73, and its BPV is +0.1673 per USD 100 of notional principal. Exhibit 22 illustrates this swap.

EXHIBIT 22 Interest Rate Swap



The risk statistics for an interest rate swap can be obtained from interpreting the contract as a combination of bonds. From the pension fund's perspective, the swap is viewed as buying a 30-year, 4.16% fixed-rate bond from the swap dealer and financing that purchase by issuing a 30-year floating-rate note (FRN) that pays the three-month MRR.

Swaps are typically quoted as a fixed rate against the MRR flat, meaning no spread. The spread over the MRR is put into the fixed rate. For instance, a swap of 4.00% against the MRR flat is the same as a swap of 4.25% against MRR + 0.25%. The swap's money duration is taken to be the (high) duration of the fixed-rate bond minus the (low) duration of the FRN. That explains why a receive-fixed swap has positive duration. From the swap dealer's perspective, the contract is viewed as purchasing a (low duration) FRN that is financed by issuing a (high duration) fixed-rate bond. Hence, the swap has negative duration to the dealer.

The notional principal (NP) on the interest rate swap needed to close the duration gap to zero can be calculated with this expression:

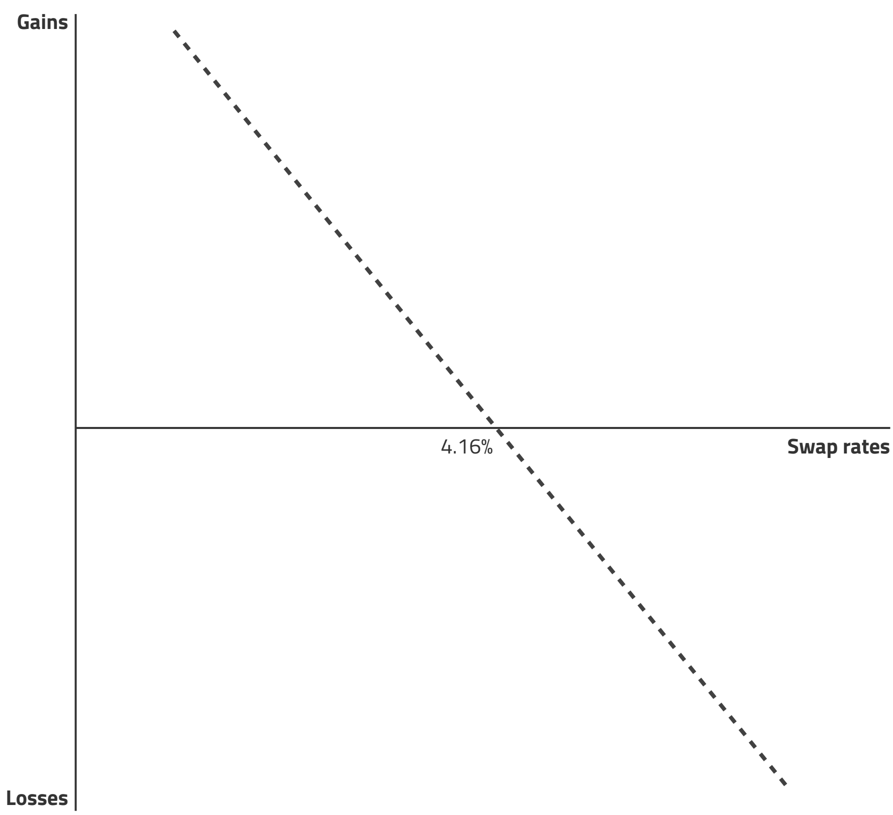
$$\text{Asset BPV} + \left[\text{NP} \times \frac{\text{Swap BPV}}{100}\right] = \text{Liability BPV} \qquad (6)$$

This is similar to Equation 2 for futures contracts. Given that the Asset BPV is USD 594,000 and the Liability BPV is USD 2,257,080 using the PBO measure, the required notional principal for the receive-fixed swap having a BPV of 0.1673 is about USD 994 million.

$$594{,}000 + \left[\text{NP} \times \frac{0.1673}{100}\right] = 2{,}257{,}080,\ \text{NP} = 994{,}070{,}532$$

Exhibit 23 shows the simplified payoff from entering into the receive-fixed swap.

EXHIBIT 23 Receive-Fixed Swap Payoff



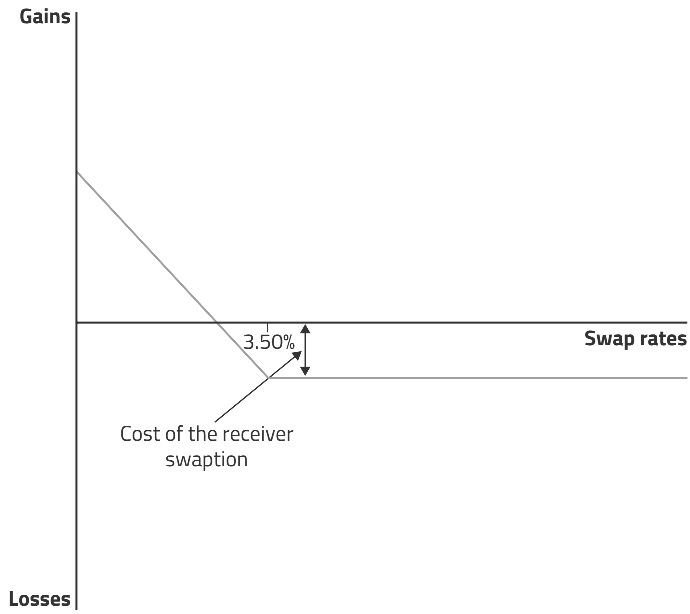
We use the term "hedging ratio" (or "interest rate hedging ratio," since the focus is on reducing interest rate risk) to indicate the extent of interest rate risk management. A hedging ratio of 0% indicates no hedging at all. The pension plan retains the significant negative duration gap and the risk of lower corporate bond yields if it does not hedge. A hedging ratio of 100% indicates an attempt to fully balance, or to immunize, the assets and liabilities. In this case, the plan manager enters the receive-fixed swap for a notional principal of USD 994 million. In practice, partial hedges are common—the manager's task is to select the hedging ratio between 0% and 100%. The initial use of derivatives entails moving up a substantial learning curve. It is important that all stakeholders to the retirement plan understand the hedging strategy. These stakeholders include the plan sponsor, the regulatory authorities, the auditors, the employees covered by the plan, and perhaps even the employees' union representatives. Interest rate swaps typically have a value of zero at initiation. If swap rates rise, the value of the receive-fixed swap becomes negative and stakeholders will need an explanation of those losses. If the contract is collateralized, the pension fund will have to post cash or marketable securities with the swap dealer. We discuss collateralization further in the next section. The key point is that in all likelihood, the prudent course of action for the plan manager is to use a partial hedge rather than attempt to reduce the duration gap to zero.

One possibility is that the plan sponsor allows the manager some flexibility (called "strategic hedging") in selecting the hedging ratio. For example, the mandate could be to stay within a range of 25% to 75%. When the manager anticipates lower market rates and gains on receive-fixed interest rate swaps, the manager prefers to be at the top of an allowable range. On the other hand, if market (swap) rates are expected to go up, the manager could reduce the hedging ratio to the lower end of the range. The performance of the strategic hedging decisions can be measured against a strategy of maintaining a preset hedging ratio, for instance, 50%. That strategy means entering the receive-fixed swap for a notional principal of USD 497 million, which is about half of the notional principal needed to attempt to immunize the plan from interest rate risk.

5.4.3. Using Options to Reduce Duration Gap

Another consideration for the plan manager is whether to use an option-based derivatives overlay strategy. Instead of entering a 30-year, receive-fixed interest rate swap against the three-month MRR, the pension fund could purchase an option to enter a similar receive-fixed swap. This contract is called a receiver swaption. The cost is a known amount paid up front. Suppose that the strike rate on the swaption is 3.50%. Given that the current 30-year swap fixed rate is assumed to be 4.16%, this receiver swaption is out of the money. The swap rate would have to fall by 66 bps (= 4.16% − 3.50%) for the swap contract to have intrinsic value. Suppose that the swaption premium is 100 bps, an amount based on the assumed level of interest rate volatility and the time to expiration (the next date that liabilities are measured and reported). Given a notional principal of USD 497 million, the pension plan pays USD 4.97 million (= USD 497 million × 0.0100) up front to buy the swaption. (This example neglects that the 3.50% swap has a somewhat higher effective duration and BPV than the 4.16% swap.) Exhibit 24 shows the payoff profile of the receiver swaption.

EXHIBIT 24 Received Swaption Payoff Profile

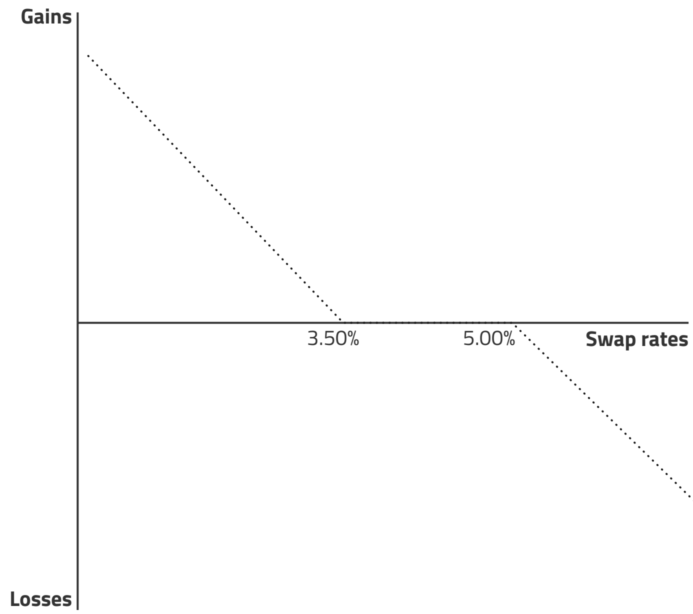


When the expiration date arrives, the plan exercises the swaption if 30-year swap rates are below 3.50%. The plan could "take delivery" of the swap and receive what has become an above-market fixed rate for payment of the three-month MRR. Or, the plan could close out the swap with the counterparty to capture the present value of the annuity based on the difference between the contractual fixed rate of 3.50% and the fixed rate in the swap market, multiplied by the notional principal. This gain partially offsets the loss incurred on the higher value for the pension plan liabilities. If 30-year swap rates are equal to or above 3.50% at expiration, the plan lets the swaption expire.

5.4.4. Using a Swaption Collar

Another derivatives overlay is a swaption collar. The plan buys the same receiver swaption, but instead of paying the premium of USD 4.97 million in cash, the plan writes a payer swaption. Suppose that a strike rate of 5.00% on the payer swaption generates an upfront premium of 100 bps. Therefore, the combination is a "zero-cost" collar, at least in terms of the initial expense. If 30-year swap rates are below 3.50% at expiration, the purchased receiver swaption is in the money and the option is exercised. If the swap rate is between 3.50% and 5.00%, both swaptions are out of the money. But if the swap rate exceeds 5.00%, the payer swaption is in the money to the counterparty. As the writer of the contract, the pension plan is obligated to receive a fixed rate of only 5.00% when the going market rate is higher. The plan could continue with the swap but, in practice, would more likely seek to close it out by making a payment to the counterparty for the fair value of the contract. Note that potential losses on the receive-fixed swap and swaption collar are *time-deferred* and *rate-contingent* and therefore are uncertain. Exhibit 25 illustrates the payoff profile of the swaption collar.

EXHIBIT 25 Swaption Collar

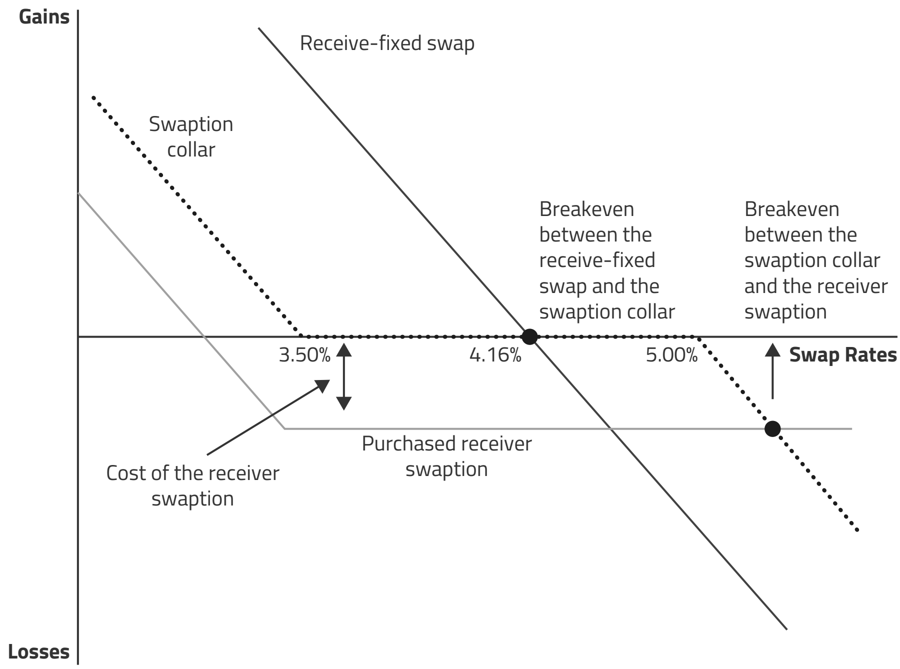


5.4.5. Selecting a Suitable Hedging Strategy

Hedging decisions involve a number of factors, including accounting and tax treatment for the derivatives used in the overlay strategy. An important consideration is the various stakeholders' sensitivity to losses on the derivatives. Obviously, the plan manager is a "hero" if yields suddenly go down and if any of the three strategies—enter the receive-fixed swap, buy the receiver swaption, or enter the swaption collar—are undertaken. Note that swap rates do not need to go below 3.50% for the receiver swaption to generate an immediate gain. Its market value would go up if market rates fall (an increase in the value of the option), and it could be sold for more than the purchase price. The problem for the manager, however, occurs if yields suddenly and unexpectedly go up, leading to a significant loss on the hedge. Will being hedged be deemed a managerial mistake by some of the stakeholders?

A factor in the choice of derivatives overlay is the plan manager's view on future interest rates, particularly on high-quality corporate bond yields at the time of the next reporting for liabilities. An irony to interest rate risk management is that the view on rates is part of decision making even when uncertainty about future rates is the motive for hedging. Exhibit 26 brings together the payoffs on the three derivatives and the breakeven rates that facilitate the choice of contract.

EXHIBIT 26 Payoffs on Received-Fixed Swap, Receiver Swaption, and Swaption Collar



Consider first the receive-fixed swap payoff. We assume it has a notional principal of USD 497 million (a 50% hedging ratio). There are gains (losses) if rates on otherwise comparable 30-year swaps are below (above) 4.16%. In reality, the payoff line is not linear as shown in the exhibit. Suppose the swap rate moves down to 4.10%. The gain is the present value of the 30-year annuity of USD 149,100 (= [0.0416 – 0.0410] × 0.5 × USD 497,000,000) per period, assuming semi-annual payments. Assuming that 4.10% is the correct rate for discounting, the gain is about USD 5.12 million:

$$\frac{149{,}100}{\left(1+\frac{0.0410}{2}\right)^1}+\frac{149{,}100}{\left(1+\frac{0.0410}{2}\right)^2}+\cdots+\frac{149{,}100}{\left(1+\frac{0.0410}{2}\right)^{60}}=5{,}120{,}670$$

If the swap rate moves up to 4.22%, the annuity is still USD 149,100. But the loss is about USD 5.05 million using 4.22% to discount the cash flows.

$$\frac{149{,}100}{\left(1+\frac{0.0422}{2}\right)^1}+\frac{149{,}100}{\left(1+\frac{0.0422}{2}\right)^2}+\cdots+\frac{149{,}100}{\left(1+\frac{0.0422}{2}\right)^{60}}=5{,}047{,}526$$

The payoffs, including the initial cost, for the purchased 3.50% receiver swaption are shown as the thin line in Exhibit 26. The premium paid at purchase is USD 4.97 million, assuming that the quoted price is 100 bps and the notional principal is USD 497 million. The dotted line shows the payoffs on the swaption collar. It is composed of the long position in the 3.50% receiver swaption and the short position in the 5.00% payer swaption. There is a gain if the swap rate is below 3.50% and a loss if the rate is above 5.00%.

Decision making is facilitated by breakeven numbers. It is easier to ask "do we expect the rate to be above or below a certain number" than to state a well-articulated probability distribution for the future rate. Exhibit 26 shows two breakeven rates. If the plan manager expects the swap rate to be at or below 4.16%, the receive-fixed swap is preferred. Its gains are higher than the other two derivatives overlays. If the manager expects the swap rate to be above 4.16%, however, the swaption collar is attractive because the swap would be incurring a loss. At some point above 5.00%, the purchased receiver swaption is better because it limits the loss. That breakeven rate can be found by trial-and-error search. The task is to find the swap rate that generates a loss that is more than the USD 4.97 million purchase price for the receiver swaption.

Suppose the swap rate goes up to 5.07% on the date that the liabilities are measured and reported. The fair value of the written 5.00% payer swaption starts with the 30-year annuity of USD 173,950 [= (0.0507 – 0.0500) × 0.5 × USD 497,000,000]. The loss of about USD 5.33 million is the present value of that annuity, discounted at 5.07%.

$$\frac{173{,}950}{\left(1+\frac{0.0507}{2}\right)^{1}}+\frac{173{,}950}{\left(1+\frac{0.0507}{2}\right)^{2}}+\cdots+\frac{173{,}950}{\left(1+\frac{0.0507}{2}\right)^{60}}=5{,}333{,}951$$

Therefore, if the plan manager expects the swap rate to be above 5.07%, the purchased receiver swaption is preferred.

In summary, many decisions go into the LDI strategy for defined benefit pension plans. Given the assumptions that lie behind the calculations of the asset BPV and the liability BPV, including the important choice between the ABO and PBO measure of liabilities, the plan manager faces a significant duration gap. The hedging ratio—the percentage of the duration gap to close—is a key decision that might depend on the held view on future interest rates—in particular, on high-quality corporate bond yields that are used to measure the liabilities. Then, given the determined hedging ratio, the choice of derivatives overlay is made. That decision once again depends on many factors, including the view on future rates.

EXAMPLE 8

A corporation is concerned about the defined benefit pension plan that it sponsors for its unionized employees. Because of recent declines in corporate bond yields and weak performance in its equity investments, the plan finds itself to be only about 80% funded based on the PBO measure. That fact is raising concerns with its employees as well as with the rating agencies. Currently, the present value of the corporation's retirement obligations is estimated by the plan's actuarial advisers to be about USD 1.321 billion using the PBO measure of liabilities. The corporation has no plans to close the defined benefit plan but is concerned about having to report the funding status in its financial statements. The market value of its asset portfolio is USD 1.032 billion—the plan is underfunded by USD 289 million.

The pension fund's asset allocation is rather aggressive: 70% equity, 10% alternative assets, and 20% fixed income. The fund manager hopes that a recovering equity market will reverse the deficit and ultimately return the plan to a fully funded position. Still, the manager is concerned about tightening corporate spreads as the economy improves. That scenario could lead to lower discount rates that are used to calculate the present value of the liabilities and offset any gains in the stock market. The actuarial advisers to the plan estimate that the effective duration of the liabilities is 9.2, so the BPV is USD 1.215 million. The corporate sponsor requires that the manager assume an effective duration of zero on equity and alternative assets. The fixed-income portfolio consists mostly of long-term bonds, including significant holdings of zero-coupon government securities. Its effective duration is estimated to be 25.6. Taken together, the asset BPV is USD 528,384. The negative money duration gap is substantial.

The pension plan has hired a qualified professional asset manager (QPAM) to offer advice on derivatives overlay strategies and to execute the contracts with a commercial bank. The QPAM suggests that the pension plan consider the use of interest rate derivatives to partially close the duration gap between its assets and liabilities.

The QPAM has identified three interest rate derivatives strategies that can be executed with the commercial bank. The first is a 30-year, 3.80% receive-fixed swap referencing the three-month MRR. The swap's effective duration is +17.51, and its BPV is 0.1751 per USD 100 of notional principal. The second is a receiver swaption having a strike rate of 3.60%. The plan pays a premium of 145 bps up front to buy the right to enter a 30-year swap as the fixed-rate receiver. The expiration date is set to match the date when the pension plan next reports its funding status. The third is a swaption collar, the combination of buying the 3.60% receiver swaption and writing a 4.25% payer swaption. The premiums on the two swaptions offset, so this is a "zero-cost" collar.

After some discussions with the rates desk at the commercial bank and a conversation with the bank's strategy group, the plan manager instructs the QPAM to select the 3.80% receive-fixed interest rate swap. Moreover, the manager chooses a hedging ratio of 75%.

1. Calculate the notional principal on the interest rate swap to achieve the 75% hedging ratio.
2. Indicate the plan manager's likely view on future 30-year swap fixed rates given the decision to choose the swap rather than the purchased receiver swaption or the swaption collar.

Solution to 1: First calculate the notional principal needed to close the duration gap between assets and liabilities to zero using Equation 6.

$$\text{Asset BPV} + \left(\text{NP} \times \frac{\text{Swap BPV}}{100}\right) = \text{Liability BPV}$$

Asset BPV is USD 528,384; Swap BPV is 0.1751 per 100 of notional principal; and Liability BPV is USD 1.215 million.

$$528{,}384 + \left(\text{NP} \times \frac{0.1751}{100}\right) = 1{,}215{,}000; \text{NP} = 392{,}127{,}927$$

A 100% hedging ratio requires a receive-fixed interest rate swap having a notional principal of about USD 392 million. For a hedging ratio of 75%, the notional principal needs to be about USD 294 million (= 392 × 0.75).

Solution to 2: The plan manager's likely view is that the 30-year swap rate will be less than 3.80%. Then the gains on the receive-fixed interest rate swap exceed those on the swaption collar (i.e., not profitable until the swap rate falls below 3.60%) and on the purchased receiver swaption (i.e., not profitable until the swap rate falls sufficiently below 3.60% to recover the premium paid), as illustrated in Exhibit 26. Note that if the 30-year swap rate exceeds 3.80%, then the receive-fixed interest rate swap will begin losing immediately. Losses on the swaption collar will not begin until the rate rises above 4.25%, while losses on the purchased receiver swaption (at any swap rate above 3.60%) are limited to the premium paid.

Notice that this rate view is also consistent with the concern about lower corporate bond yields and the relatively high hedging ratio.

6. RISKS IN LIABILITY-DRIVEN INVESTING

- **explain risks associated with managing a portfolio against a liability structure**

We have mentioned in previous sections some of the risks to LDI strategies for single and multiple liabilities. In this section, we review those risks and introduce some new ones. The essential relationship for full interest rate hedging is summarized in this expression:

$$\text{Asset BPV} \times \Delta\text{Asset yields} + \text{Hedge BPV} \times \Delta\text{Hedge yields}$$
$$\approx \text{Liability BPV} \times \Delta\text{Liability yields} \quad (7)$$

ΔAsset yields, ΔHedge yields, and ΔLiability yields are measured in basis points. This equation describes an immunization strategy (a hedging ratio of 100%) whereby the intent is to match the changes in market value on each side of the balance sheet when yields change. Doing so entails matching the money duration of assets and liabilities. We know, however, that entities also choose to partially hedge interest rate risk by selecting a hedging ratio less than 100%. In any case, Equation 7 serves to indicate the source of the risks to LDI. The "approximately equals" sign (≈) in the equation results from ignoring higher-order terms, such as convexity.

6.1. Model Risk in Liability-Driven Investing

We encounter model risk in financial modeling whenever assumptions are made about future events and approximations are used to measure key parameters. The risk is that those assumptions turn out to be wrong and the approximations are inaccurate. For example, in our earlier defined benefit pension plan example, we assumed that the effective durations for investments in equity and alternative assets are zero. That assumption introduces the risk that Asset BPV is mis-measured if, in fact, those market values change as the yield curve shifts. The modeling problem is that the effect on those asset classes is not predictable or stable because it depends on the reason for the change in nominal interest rates. Unlike fixed-income bonds, an increase in expected inflation can have a very different effect on equity and alternative asset valuations than an increase in the real rate.

Measurement error for Asset BPV can even arise in the classic immunization strategy for Type I cash flows, which have set amounts and dates. In practice, it is common to approximate the asset portfolio duration using the weighted average of the individual durations for the component bonds. A better approach to achieve immunization, however, uses the cash flow yield to discount the future coupon and principal payments. This error is minimized when the underlying yield curve is flat or when future cash flows are concentrated in the flattest segment of the curve.

A similar problem arises in measuring Hedge BPV. When we illustrated the use of derivatives overlays to immunize, we used a common approximation for the Futures BPV. Equation 4 estimates it to be the BPV for the qualifying CTD security divided by its conversion factor. A more developed calculation involving short-term rates and accrued interest, however, could change the number of contracts needed to hedge the interest rate risk. Although the error introduced by using an approximation might not be large, it still can be a source of underperformance in the hedging strategy.

Model risk in obtaining a measure of Liability BPV is evident in the earlier defined benefit pension plan example. Measuring a defined benefit pension plan's liability is clearly a difficult financial modeling problem. Even the simple models for the two liability measures (the ABO and PBO) necessarily require many assumptions about the future, including the dates when employees retire and their wage levels at those times. The difficulty in projecting life spans of retirees covered by the pension plan leads to longevity risk. The risk is the sponsor has not provided sufficient assets to make the longer-than-expected payout stream. More, and harder-to-make, assumptions are needed to deal with Type IV liabilities and lead to greater uncertainty regarding the models' outputs.

Implicit in Equation 7 is the assumption that all yields change by the same number of basis points—that is, ΔAsset Yields, ΔHedge Yields, and ΔLiability Yields are equal. That is a strong assumption—and a source of risk—if the particular fixed-income assets, the derivatives, and the liabilities are positioned at varying points along the benchmark bond yield curve and at varying spreads to that curve. In Section 3 on immunizing the interest rate risk on a single liability by structuring and managing a portfolio of fixed-income bonds, we point out that a parallel yield curve shift is a sufficient, but not necessary, condition to achieve the desired outcome. Non-parallel shifts as well as twists to the yield curve can result in changes to the cash flow yield on the immunizing portfolio that do not match the change in the yield on the zero-coupon bond that provides perfect immunization. Minimizing dispersion of the cash flows in the asset portfolio mitigates this risk.

Generally, the framework for thinking about interest rate risk rests on changes in the benchmark bond yield curve, which usually is the yield curve for government bonds. In practice, however, ΔAsset Yields and ΔLiability Yields often refer to various classes of corporate bonds. In the pension fund example, the fund holds a portfolio of fixed-income bonds that tracks a well-diversified index of corporate bonds that may include non-investment-grade securities. The present value of retirement benefits, however, depends on yields on high-quality corporate bonds. Therefore, a risk is that the respective spreads on the broad index and the high-quality sector do not move in unison with a shift in the government bond yield curve. A similar spread risk is present in the earlier example of immunizing multiple Type I liabilities. The difference is that the assets in that example are of higher quality than the liabilities.

6.2. Spread Risk in Liability-Driven Investing

Spread risk also is apparent in the derivatives overlay LDI strategies. We illustrated how futures contracts can be used to hedge the interest rate risk of the multiple liabilities, either passively or contingently. In particular, the futures contracts are on 10-year US Treasury notes, whereas the liabilities are corporate obligations. Movements in the corporate–Treasury yield spread introduce risk to the hedging strategy. Usually, yields on high-quality corporate bonds are less volatile than on more-liquid Treasuries. Government bonds are used in a wide variety of hedging as well as speculative trading strategies by institutional investors. Also, inflows of international funds typically are placed in government bonds, at least until they are allocated to other asset classes. Those factors lead to greater volatility in Treasury yields than comparable-maturity corporate bonds.

Another source of spread risk is the use of interest rate swap overlays. We showed how receive-fixed swaps, purchased receiver swaptions, and swaption collars can reduce the duration gap between pension plan assets and liabilities. In that example, ΔHedge Yields refers to fixed rates on interest rate swaps referencing the three-month MRR. The spread risk is between high-quality corporate bond yields and swap rates. Typically, there is less volatility in the corporate/swap spread than in the corporate/Treasury spread because both the MRR and corporate bond yields contain credit risk vis-à-vis Treasuries. Therefore, one of the usual advantages to hedging corporate bond risk with interest rate swaps is that those derivatives pose less spread risk than Treasury futures contracts.

6.3. Counterparty Credit Risk

Counterparty credit risk is a concern if the interest rate swap overlays are uncollateralized, as was common before the 2008–2009 global financial crisis. Suppose that the interest rate swap portrayed in Exhibit 22 does not have a collateral agreement, or Credit Support Annex (CSA), to the standard International Swaps and Derivatives Association (ISDA) contract. The credit risk facing the pension plan is that the swap dealer defaults at a time when the replacement swap fixed rate is below 4.16%. In the same manner, the credit risk facing the dealer is that the pension plan defaults at the time when the market rate on a comparable swap is above 4.16%. Therefore, credit risk entails the joint probability of default by the counterparty and movement in market rates that results in the swap being valued as an asset.

Since the 2008–2009 global financial crisis, over-the-counter derivatives increasingly include a CSA to the ISDA contract to mitigate counterparty credit risk. Collateral provisions vary. A typical CSA calls for a zero threshold, meaning that only the counterparty for which the swap has negative market value posts collateral, which usually is cash but can be highly marketable securities. The CSA can be one way (only the "weaker" counterparty needs to post collateral when the swap has negative market value from its perspective) or two way (either counterparty is obligated to post collateral when the swap has negative market value). The threshold could be positive, meaning that the swap has to have a certain negative value before collateral needs to be exchanged. Another possibility is that one or both counterparties are required to post a certain amount of collateral, called an independent amount, even if the swap has zero or positive value. This provision makes the CSA similar to the use of margin accounts with exchange-traded futures contracts.

Collateralization on derivatives used in an LDI strategy introduces a new risk factor—the risk that available collateral becomes exhausted. That risk is particularly important for the pension plan example, in which the plan would need to enter a sizable derivatives overlay to even use a 50% hedging ratio, let alone to fully hedge the interest rate risk. That is because the duration gap between assets and liabilities is often large, especially for plans having a significant equity allocation. Therefore, the probability of exhausting collateral is a factor in determining the hedging ratio and the permissible range in the ratio if strategic hedging is allowed.

The same concern about cash management and collateral availability arises with the use of exchange-traded futures contracts. These contracts entail daily mark-to-market valuation and settlement into a margin account. This process requires daily oversight because cash moves into or out of the margin account at the close of each trading day. In contrast, the CSA on a collateralized swap agreement typically allows the party a few days to post additional cash or marketable securities. Also, there usually is a *minimum transfer amount* to mitigate the transaction costs for small inconsequential payments.

6.4. Asset Liquidity Risk

Asset liquidity becomes a risk factor in strategies that combine active investing to the otherwise passive fixed-income portfolios. This risk is particularly important with contingent immunization. In the presence of a surplus above a sufficient threshold, the manager may increase portfolio risk by using active management. But if losses reduce the surplus to some minimum amount, the positions need to be adjusted to revert to a passive duration-matching fixed-income portfolio of high-quality bonds. Distressed assets that become hard to value, such as tranches of subprime mortgage-backed securities, also become illiquid during financial crises.

In summary, an LDI manager has a fundamental choice between managing interest rate risk with asset allocation and with derivatives overlays. As with all financial management decisions, the choice depends on a thorough evaluation of risk and return trade-offs. In some circumstances, derivatives might be deemed too expensive or risky, particularly with regard to available collateral and cash holdings. Then the manager might choose to increase holdings of long-term, high-quality bonds that have high duration statistics. The growth of government zero-coupon bonds, such as US Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities), facilitates that asset reallocation process.

EXAMPLE 9

A derivatives consultant, a former head of interest rate swaps trading at a major London bank, is asked by a Spanish corporation to devise an overlay strategy to "effectively defease" a large debt liability. That means that there are dedicated assets to retire the debt even if both assets and the liability remain on the balance sheet. The corporation currently has enough euro-denominated cash assets to retire the bonds, but its bank advises that acquiring the securities via a tender offer at this time will be prohibitively expensive.

The 10-year fixed-rate bonds are callable at par value in three years. This is a one-time call option. If the issuer does not exercise the option, the bonds are then non-callable for the remaining time to maturity. The corporation's CFO anticipates higher benchmark interest rates in the coming years. Therefore, the strategy of investing the available funds for three years and then calling the debt is questionable because the embedded call option might be "out of the money" when the call date arrives. Moreover, it is likely that the cost to buy the bonds on the open market at that time will still be prohibitive.

The corporation has considered a cash flow matching approach by buying a corporate bond having the same credit rating and a call structure (call date and call price) close to the corporation's own debt liability. However, the bank working with the CFO has been unable to identify an acceptable bond. Instead, the bank suggests that the corporation buy a 10-year non-callable, fixed-rate corporate bond and use a swaption to mimic the characteristics of the embedded call option. The idea is to transform the callable bond (the liability) into a non-callable security synthetically using the swaption. Then the newly purchased non-callable bond "effectively" defeases the transformed "non-callable" debt liability.

To confirm the bank's recommendation for the derivatives overlay, the CFO turns to the derivatives consultant, asking if the corporation should (1) buy a payer swaption, (2) buy a receiver swaption, (3) write a payer swaption, or (4) write a receiver swaption. The time frames for the swaptions correspond to the embedded call option.

They are "3y7y" contracts, an option to enter a seven-year interest rate swap in three years. The CFO also asks the consultant about the risks to the recommended swaption position.

1. Indicate the swaption position that the derivatives consultant should recommend to the corporation.
2. Indicate the risks in using the derivatives overlay.

Solution to 1: The derivatives consultant should recommend that the corporation choose the fourth option and write a receiver swaption—that is, an option that gives the swaption buyer the right to enter into a swap to receive fixed and pay floating. When the corporation issued the callable bond, it effectively bought the call option, giving the corporation the flexibility to refinance at a lower cost of borrowed funds if benchmark rates and/or the corporation's credit spread narrows. Writing the receiver swaption "sells" that call option, and the corporation captures the value of the embedded call option by means of the premium received. Suppose that market rates in three years are higher than the strike rate on the swaption and the yield on the debt security. Then both options—the embedded call option in the bond liability, as well as the swaption—expire out of the money. The asset and liability both have seven years until maturity and are non-callable. Suppose instead that market rates fall and bond prices go up. Both options are now in the money. The corporation sells the seven-year bonds (the assets) and uses the proceeds to call the debt liabilities at par value. The gain on that transaction offsets the loss on closing out the swaption with the counterparty.

Solution to 2: Potential risks to using swaptions include (1) credit risk if the swaption is not collateralized, (2) "collateral exhaustion risk" if it is collateralized, and (3) spread risk between swap fixed rates and the corporation's cost of funds. First, suppose the receiver swaption is not collateralized. In general, the credit risk on an option is unilateral, meaning that the buyer bears the credit risk of the writer. That unilateral risk assumes the premium is paid in full upon entering the contract; in other words, the buyer has met their entire obligation. Therefore, the corporation as the swaption writer would have no additional credit exposure to the buyer. Second, assume that the swaption is collateralized. As the writer of the option, the corporation would need to regularly post cash collateral or marketable securities with either the counterparty or a third-party clearinghouse. The risk is that the corporation exhausts its available cash or holdings of marketable securities and cannot maintain the hedge. Spread risk arises because the value of the embedded call option in three years depends on the corporation's cost of funds at that time, including its credit risk. The value of the swaption depends only on seven-year swap fixed rates at that time. In particular, the risk is that the corporate/swap spread widens when benchmark rates are low and both options can be exercised. If the corporate spread over the benchmark rate goes up, the gain in the embedded call option is reduced. If the swap spread over the same benchmark rate goes down, the loss on the swaption increases. Fortunately, corporate and swap spreads over benchmark rates are usually positively correlated, but still the risk of an unexpected change in the spread should be identified.

7. BOND INDEXES AND THE CHALLENGES OF MATCHING A FIXED-INCOME PORTFOLIO TO AN INDEX

- **discuss bond indexes and the challenges of managing a fixed-income portfolio to mimic the characteristics of a bond index**

Though the need to offset liabilities through immunization requires a specific bond portfolio, many investors seek a broader exposure to the fixed-income universe. These investors may be attracted to the risk versus return characteristics available in bond markets, or they may seek to allocate a portion of their investable assets to fixed income as part of a well-diversified multi-asset portfolio. In either case, an investment strategy based on a bond market index offers an investor the ability to gain broad exposure to the fixed-income universe. Index-based investments generally offer investors the possibility of greater diversification and lower fees as well as avoiding the downside risk from seeking positive excess returns over time from active management.

An investor seeking to offset a specific liability through immunization gauges the success of his strategy based on how closely the chosen bonds offset the future liability or liabilities under different interest rate scenarios. In contrast, an investor seeking to match the returns of a bond market index will gauge an investment strategy's success in terms of how closely the chosen market portfolio mirrors the return of the underlying bond market index. Deviation of returns on the selected portfolio from bond market index returns are referred to as **tracking risk** or **tracking error**. Investors use several methods to match an underlying market index (Volpert 2012). The first of these is **pure indexing**, in which the investor aims to replicate an existing market index by purchasing all of the constituent securities in the index to minimize tracking risk. The purchase of all securities within an index is known as the **full replication approach**. In **enhanced indexing strategy**, the investor purchases fewer securities than the full set of index constituents but matches primary risk factors (discussed later) reflected in the index. This strategy aims to replicate the index performance under different market scenarios more efficiently than the full replication of a pure indexing approach. **Active management** involves taking positions in primary risk factors that deviate from those of the index in order to generate excess return.

Casual financial market observers usually refer to an equity market index to gauge overall financial market sentiment. Examples often consist of a small set of underlying securities, such as the Dow Jones Industrial Average of 30 US stocks, the CAC 40 traded on Euronext in Paris, or the 50 constituent companies in the Hang Seng Index, which represent more than half the market capitalization of the Hong Kong stock market. When bond markets are mentioned at all, the price and yield of the most recently issued benchmark government bond is typically referenced rather than a bond market index. This contrast reflects the unwieldy nature of bond markets for both the average investor and financial professionals alike.

Although rarely highlighted in the financial press, investments based on bond market indexes form a very substantial proportion of financial assets held by investors. Fixed-income markets have unique characteristics that make them difficult to track, and investors therefore face significant challenges in replicating a bond market index. These challenges include:

- the size and breadth of bond markets,
- the wide array of fixed-income security characteristics,
- unique issuance and trading patterns of bonds versus other securities, and
- the effect of these patterns on index composition and construction, pricing, and valuation.

We will tackle each of these issues and their implications for fixed-income investors.

7.1. Size and Breadth of the Fixed-Income Universe

Fixed-income markets are much larger and broader than equity markets, and the number of fixed-income securities outstanding is vastly larger as reflected in broad market indexes. For instance, the MSCI World Index, capturing equities in 23 developed market countries and 85% of the available market capitalization in each market, consists of about 1,600 securities, whereas the Bloomberg Barclays Global Aggregate Index, covering global investment-grade debt from 24 local currency markets, consists of more than 16,000 securities. Those fixed-income issuers represent a much wider range of borrowers than the relatively narrow universe of companies issuing equity securities. For example, the oldest and most widely recognized US bond market index, the Bloomberg Barclays US Aggregate Index (one of four regional aggregate benchmarks that constitute the Bloomberg Barclays Global Aggregate Index), includes US Treasuries, government agency securities, corporate bonds, mortgage-backed securities, asset-backed securities, and commercial mortgage-backed securities. Although the large number of index constituents provides a means of risk diversification, in practice it is neither feasible nor cost-effective for investors to pursue a full replication approach with a broad fixed-income market index.

7.2. Array of Characteristics

Different maturities, ratings, call/put features, and varying levels of security and subordination give rise to a much wider array of public and private bonds available to investors. Exhibit 27 illustrates the number of publicly traded fixed-income and equity securities outstanding for a select group of major global issuers.

EXHIBIT 27 Debt and Equity Securities Outstanding for Select Issuers

Issuer	Fixed-Income Securities	Common Equity Securities	Preferred Equity Securities
Royal Dutch Shell PLC	57	1	0
BHP Billiton Limited	22	1	0
Johnson & Johnson	37	1	0
Ford Motor Company	104	2	2

Source: Bloomberg as of 14 October 2020. Bonds with more than $50 million outstanding included.

As of October 2020, Royal Dutch Shell had 57 bonds outstanding across four currencies, some of which were both fixed and floating rate, with a range of maturities from under a year to bonds maturing in 2052. The existence of many debt securities for a particular issuer suggests that many near substitutes may exist for an investor seeking to pursue an enhanced index strategy. That said, the relative liquidity and performance characteristics of those bonds may differ greatly depending on how recently the bond was issued and how close its coupon is to the yield currently required to price the bond at par.

7.3. Unique Issuance and Trading Patterns

Unlike equity securities, which trade primarily over an exchange, fixed-income markets are largely over-the-counter markets that rely on broker/dealers as principals to trade in these securities using a quote-based execution process rather than the order-based trading systems common in equity markets. The rising cost of maintaining risk-weighted assets on dealer balance sheets as a result of Basel III capital requirements has had an adverse effect on fixed-income trading and liquidity. Broker/dealers have reduced bond inventories because of higher capital costs. With lower trading inventories, dealers have both a limited appetite to facilitate trading at narrow bid–offer spreads and are less willing to support larger "block" trades, preferring execution in smaller trade sizes. Finally, a significant decline in proprietary trading among dealers has had a greater pricing effect on less liquid or "off-the-run" bonds. Although many see these structural changes in fixed-income trading acting as a catalyst for more electronic trading, this trend will likely be most significant for the most liquid fixed-income securities in developed markets, with a more gradual effect on less frequently traded fixed-income securities worldwide. Fixed-income trading in many markets is difficult to track. In some markets, regulators developed systems that facilitate mandatory reporting of over-the-counter transactions in eligible fixed-income securities, such as the US Trade Reporting and Compliance Engine (TRACE) system. All broker/dealers that are Financial Industry Regulatory Authority (FINRA) member firms must report corporate bond transactions within 15 minutes of occurrence. It is important to note the distinct nature of fixed-income trading versus equities. The vast majority of fixed-income securities either do not trade at all or trade only a few times during the year. Only a small fraction trade every business day, according to MarketAxess, a leading electronic trading provider. It is also important to note that the average trade size in dollar terms in the US investment-grade bond market is roughly 70 times the size of the average stock trade.

The illiquid nature of most fixed-income instruments gives rise to pricing and valuation challenges for asset managers. For fixed-income instruments that are not actively traded and therefore do not have an observable price, it is common to use an estimation process known as **matrix pricing** or **evaluated pricing**. Matrix pricing makes use of observable liquid benchmark yields, such as Treasuries of similar maturity and duration, as well as the benchmark spreads of bonds with comparable times to maturity, credit quality, and sector or security type in order to estimate the current market yield and price. In practice, asset managers will typically outsource this function to a global custodian or external vendor. This estimation analysis is another potential source of variation between index performance and portfolio returns.

The complexity of trading and valuing individual fixed-income securities further underscores the challenges associated with managing an index-based bond portfolio. Fixed-income indexes change frequently as a result of both new debt issuance and the maturity of outstanding bonds. Bond index eligibility is also affected by changes in ratings and bond callability. As a result, rebalancing of bond market indexes usually occurs monthly rather than semi-annually or annually as it does for equity indexes. Fixed-income investors pursuing a pure indexing strategy therefore must also incur greater transaction costs associated with maintaining a bond portfolio consistent with the index.

7.4. Primary Risk Factors

Given the significant hurdles involved in bond index matching, asset managers typically seek to target the primary risk factors present in a fixed-income index through a diversified portfolio. Volpert (2012) summarized these primary indexing risk factors as follows:

- **Portfolio modified adjusted duration.** Effective duration, or the sensitivity of a bond's price to a change in a benchmark yield curve, is an important primary factor as a first approximation of an index's exposure to interest rate changes. It is important to factor in option-adjusted duration so that the analysis reflects securities with embedded call risk. Larger rate moves should incorporate the second-order convexity adjustment to increase accuracy.

- **Key rate duration.** Although effective duration may be a sufficient measure for small rate changes and parallel yield curve shifts, the **key rate duration** takes into account rate changes in a specific maturity along the yield curve while holding the remaining rates constant. This measure of duration gauges the index's sensitivity to non-parallel yield curve shifts. By effectively matching the key rate durations between the portfolio and the underlying index, a manager can significantly reduce the portfolio's exposure to changes in the yield curve.
- **Percent in sector and quality.** Index yield is most effectively matched by targeting the same percentage weights across fixed-income sectors and credit quality, assuming that maturity parameters have also been met.
- **Sector and quality spread duration contribution.** The portfolio manager can minimize deviations from the benchmark by matching the amounts of index duration associated with the respective issuer sectors and quality categories. The former refers to the issuer type and/or industry segment of the bond issuer. In the case of the latter, the risk that a bond's price will change in response to an idiosyncratic rate move rather than an overall market yield change is known as spread risk. For non-government fixed-income securities, we separate the yield to maturity into a benchmark yield (typically the most recently issued or on-the-run government bond with the closest time to maturity) and a spread reflecting the difference between the benchmark yield and the security-specific yield. **Spread duration** refers to the change in a non-Treasury security's price given a widening or narrowing of the spread compared with the benchmark. Matching the relative quality between the portfolio and the fixed-income index will minimize this risk.
- **Sector/coupon/maturity call weights.** Asset managers face a number of challenges in matching price/yield sensitivity beyond the use of effective duration. Although convexity is a useful second-order condition that should be used to improve this approximation, the negative convexity of callable bonds may distort the call exposure of an index and lead to costly rebalancing when rates shift. As a result, managers should seek to match the sector, coupon, and maturity weights of callable bonds by sector. Doing so is particularly important in the mortgage sector because of the refinancing of high-coupon securities with lower-coupon bonds.
- **Issuer exposure.** Concentration of issuers within a portfolio exposes the asset manager to issuer-specific event risk. The manager should therefore seek to match the portfolio duration effect from holdings in each issuer.

Another method used to address a portfolio's sensitivity to rate changes along the yield curve is referred to as the **present value of distribution of cash flows methodology**. This approach seeks to approximate and match the yield curve risk of an index over discrete time periods referred to as cash flow vertices, and it involves several steps, as follows:

1. The manager divides the cash flows for each non-callable security in the index into discrete semi-annual periods, aggregates them, and then adds the cash flows for callable securities in the index based on the probability of call for each given period.
2. The present value of aggregated cash flows for each semi-annual period is computed, with the total present value of all such aggregated cash flows equal to the index's present value. The percentage of the present value of each cash flow vertex is calculated.
3. The time period is then multiplied by the vertex's proportionate share of the index. (The first cash flow at 6 months is equal to 1; the second cash flow at 12 months is equal to 2; the third cash flow at 18 months is equal to 3, etc.) Because each cash flow represents an effective zero-coupon payment in the corresponding period, the time period reflects the

duration of the cash flow. For example, if the third vertex represents 3% of all cash flows, the third period's contribution to duration might be 1.5 years × 3.0%, or 0.045.

4. Finally, each period's contribution to duration is added to arrive at a total representing the bond index's duration. The portfolio being managed will be largely protected from deviations from the benchmark associated with yield curve changes by matching the percentage of the portfolio's present value that comes due at specific points in time with that of the index.

The goal of matching these primary indexing risk factors is to minimize tracking error, the standard deviation of a portfolio's active return for a given period, whereby active return is defined as follows:

$$\text{Active return} = \text{Portfolio return} - \text{Benchmark index return}$$

If we assume that returns are normally distributed around the mean, then from a statistical perspective, 68% of those returns will lie within one standard deviation of the mean. Therefore, if a fund's tracking error is 50 bps, then for approximately two-thirds of the time period observations, we would expect the fund's return to be less than 50 bps above or below the index's return.

EXAMPLE 10

Cindy Cheng, a Hong Kong-based portfolio manager, has established the All Asia Dragon Fund, a fixed-income fund designed to outperform the Markit iBoxx Asian Local Bond Index (ALBI). The ALBI tracks the total return performance of liquid bonds denominated in local currencies in the following markets: Chinese mainland, Hong Kong SAR, India, Indonesia, Korea, Malaysia, the Philippines, Singapore, Taiwan region, and Thailand. The index includes both government and non-government bond issues, with constituent selection criteria by government as well as weights designed to balance the desire for liquidity and stability ("Markit iBoxx ALBI Index Guide," January 2016, Markit Ltd).

Individual bond weightings in the index are based on market capitalization, and market weights, reviewed annually, are designed to reflect the investability of developing Asian local currency bonds available to international investors. These weights are driven by local market size and market capitalization, secondary bond market liquidity, accessibility to foreign investors, and development of infrastructure that supports fixed-income investment and trading, such as credit ratings, yield curves, and derivatives products.

Given the large number of bonds in the index, Cheng uses a representative sample of the bonds to construct the fund. She chooses bonds so that the fund's duration, market weights, and sector/quality percentage weights closely match the ALBI. Given the complexity of managing bond investments in these local markets, Cheng is targeting a 1.25% tracking error for the fund.

1. Interpret Cheng's tracking error target for the All Asia Dragon Fund.
2. One of Cheng's largest institutional investors has encouraged her to reduce tracking error. Suggest steps Cheng could take to minimize this risk in the fund.

Solution to 1: The target tracking error of 1.25% means that assuming normally distributed returns, in 68% or two-thirds of time periods, the All Asia Dragon Fund should have a return that is within 1.25% of the ALBI.

Solution to 2: Cheng could further reduce tracking error beyond her choice of duration, market, and sector/quality weightings to mirror the index by using the present value of distribution of cash flows methodology outlined earlier. By doing so, she can better align the contribution to portfolio duration that comes from each market, sector, and issuer type based on credit quality.

Cheng should consider matching the amount of index duration that comes from each sector, as well as matching the amount of index duration that comes from various quality categories across government and non-government bonds, to minimize tracking error.

Finally, Cheng should evaluate the portfolio duration coming from each issuer to minimize event risk. Again, this evaluation should occur on a duration basis rather than as a percentage of market value to quantify the exposure more accurately versus the benchmark ALBI.

8. ALTERNATIVE METHODS FOR ESTABLISHING PASSIVE BOND MARKET EXPOSURE

- **compare alternative methods for establishing bond market exposure passively**

Why is passive bond market exposure attractive for investors? A **passive investment** in the fixed-income market may be defined as one that seeks to mimic the prevailing characteristics of the overall investments available in terms of credit quality, type of borrower, maturity, and duration rather than express a specific market view. This approach is consistent with the efficient markets hypothesis in that the portfolio manager seeks to simply replicate broader fixed-income market performance rather than outperform the market. Stated differently, establishing passive bond market exposure does not require the in-depth economic, market, or security analysis necessary to achieve an above-market return, nor does it require the high trading frequency of active management, which should lead to lower costs for managing and servicing a portfolio. Finally, the stated goal of matching the performance of a broad-based bond index is consistent with the highest degree of portfolio diversification.

Several methods exist for establishing a passive bond market exposure. In what follows, we will explore both full index replication as well as an enhanced indexing strategy and compare the risks, costs, and relative liquidity of these strategies when applied to the bond market.

8.1. Full Replication

Bond market index replication is the most straightforward strategy that a manager can use to mimic index performance. Use of full replication reflects the belief or expectation that i) an active manager cannot consistently outperform the index on a risk-adjusted basis; ii) the investor cannot identify a skilled manager in advance; or iii) the investor is not prepared to go through periods of underperformance. Initial index replication does not require manager analysis but rather involves sourcing a wide range of securities in exact proportion to the index, many of which may be thinly traded. The manager's ongoing task under full replication is to purchase or sell bonds when there are changes to the index in addition to managing inflows and outflows for a specific fund. For example, the manager may have to sell when a security no longer meets the index criteria, such as when a security either matures or is downgraded. For the Bloomberg Barclays US Aggregate Bond Index, a fixed-income security becomes ineligible when it either has a maturity of less than one year or is downgraded below an average minimum investment-grade rating. On the other hand, managers must purchase newly issued securities that meet index criteria to maintain full replication, which, depending on the index, may occur quite frequently. Rolling bond maturities, as well as frequent new issuance eligible for inclusion in the index, drive a monthly rebalancing for most fixed-income indexes. The number of purchases and sales required to maintain an exact proportional allocation would be very significant for most bond indexes. As a result, although the large number of index constituents may well provide the best means of risk diversification, in practice it is neither feasible nor cost-effective for investors to pursue full replication for broad-based fixed-income indexes. It is impractical for all but the most narrow indexes. Investors that wish to have exposure to an index would in practice rely on one of many ETFs that exist in this space.

8.2. Enhanced Indexing

Many limitations of the full replication approach are addressed by an enhanced indexing strategy. This approach's goal is to mirror the most important index characteristics and still closely track index performance over time while purchasing fewer securities. This general approach is referred to as a **stratified sampling** or **cell approach** to indexing. First, each cell or significant index portfolio characteristic is identified and mapped to the current index. Second, the fixed-income portfolio manager identifies a subset of bonds or bond-linked exposures, such as derivatives, with characteristics that correspond to the index. Finally, the positions in each cell are adjusted over time given changes to the underlying index versus existing portfolio positions. For example, say a fixed-income index contains 1,000 fixed-income securities, 10% of which are AAA rated. The portfolio manager might choose five to ten AAA rated securities within a cell in order to mimic the performance of the AAA rated bonds within the index.

Enhanced indexing is also of critical importance to investors who consider environmental, social, or other factors when selecting a fixed-income portfolio. These comprise several categories, including ESG or socially responsible investing (SRI), which refer to the explicit inclusion of ethical, environmental, or social criteria when selecting a portfolio. Additional categories include sustainability, which includes companies addressing their ESG risks, and green bonds, which fund projects with direct environmental benefits. There are two main components to incorporating ESG factors in an index. First, a business involvement screen excludes issuers involved in business lines or such activities as alcohol, gambling, tobacco, adult entertainment, nuclear power, and firearms. Second, an ESG rating, provided by one of numerous third-party companies, is applied. MSCI, one such company, provides ratings on an "AAA to CCC" scale using a rules-based methodology according to the companies' exposure to ESG risks and how well they are managed relative to peers. When building a sustainable index, for example, Bloomberg will apply such rules as those previously described and then further filter the constituents for business involvement and an MSCI minimum rating requirement of BBB. Given the proliferation of ESG data providers, there are differences across methodologies and ratings. Consider also that the pillar of ESG that may be most important to an investor may not have the same emphasis in the rating criteria used by the index provider.

8.2.1. Enhancement Strategies

Volpert (2012) outlines a number of enhancement strategies available to portfolio managers seeking to reduce the component of tracking error associated with the expenses and transaction costs of portfolio management as follows:

- **Lower cost enhancements.** The most obvious enhancement is in the area of cost reduction, whether this involves minimizing fund expenses or introducing a more competitive trading process to reduce the bid–offer cost of trading.
- **Issue selection enhancements.** The use of bond valuation models to identify specific issues that are undervalued or "cheap" to their implied value provides another opportunity to enhance return.
- **Yield curve enhancements.** The use of analytical models to gauge and calculate relative value across the term structure of interest rates allows managers to develop strategies to both overweight maturities that are considered undervalued and underweight those that appear to be richly priced.
- **Sector/quality enhancements.** This strategy involves overweighting specific bond and credit sectors across the business cycle to enhance returns. Other sectors are underweighted as a result. This approach may tilt exposure toward corporates given a greater yield spread per unit of duration exposure or shorter maturities, or it may over- or underweight specific sectors or qualities based on analysis of the business cycle.

For example, a manager may increase her allocation to Treasuries over corporates when significant spread widening is anticipated, or reverse this allocation if spread narrowing is deemed more likely.

- **Call exposure enhancements.** Because effective duration is a sufficient risk measure only for relatively small rate changes, anticipated larger yield changes may affect bond performance significantly, especially when a bond shifts from trading to maturity to trading to an earlier call date. Large expected yield changes increase the value of call protection, and any significant differences from index exposure should incorporate potentially large tracking risk implications, as well as the implicit market view that this difference implies. For example, an anticipated drop in yields might cause a callable bond to shift from being priced on a yield-to-maturity basis to a yield-to-call basis. Callable fixed-income securities (priced on a yield-to-call basis) trading above par tend to be less price sensitive for a given effective duration than those priced on a yield-to-maturity basis, suggesting a manager should use metrics other than effective duration in this case when changing exposure.

The stratified sampling approach provides an asset manager the ability to optimize portfolio performance across these characteristics with fewer securities than would be required through full index replication. By matching portfolio performance as closely as possible, investment managers also seek to minimize tracking error, limit the need to purchase or sell thinly traded securities, and/or frequently rebalance the portfolio as would be required when precisely matching the index.

EXAMPLE 11

Adelaide Super, a superannuation fund, offers a range of fixed interest (or fixed-income) investment choices to its members. Superannuation funds are Australian government-supported arrangements for Australian workers to save for retirement, which combine a government-mandated minimum percentage of wages contributed by employers with a voluntary employee contribution that offers tax benefits. Superannuation plans are similar to defined contribution plans common in the United States, Europe, and Asia.

Three of the bond fund choices Adelaide Super offers are as follows:

- **Dundee Australian Fixed-Income Fund.** The investment objective is to outperform the Bloomberg AusBond Composite Index in the medium to long term. The index includes investment-grade fixed-interest bonds with a minimum of one month to maturity issued in the Australian debt market under Australian law, including the government, semi-government, credit, and supranational/sovereign sectors. The index includes AUD-denominated bonds only. The investment strategy is to match index duration but add value through fundamental and model-driven return strategies.
- **Newcastleton Australian Bond Fund.** The fund aims to outperform the Bloomberg AusBond Composite Index over any three-year rolling period, before fees, expenses, and taxes, and uses multiple strategies, such as duration, curve positioning, and credit and sector rotation rather than one strategy, allowing the fund to take advantage of opportunities across fixed-income markets under all market conditions.
- **Paisley Fixed-Interest Fund.** The fund aims to provide investment returns after fees in excess of the fund's benchmark, which is the Bloomberg AusBond Bank Bill Index and the Bloomberg AusBond Composite Index (equally weighted) by investing in a diversified portfolio of Australian income-producing assets. Paisley seeks to minimize transaction costs via a buy-and-hold strategy, as opposed to active management. The AusBond Bank Bill Index is based on the bank bill market, which is the short-term market (90 days or less) in which Australian banks borrow from and lend to one another via bank bills.

Rank the three fixed-income funds in order of risk profile, and suggest a typical employee for whom this might be a suitable investment.

Solution: The Paisley Fixed-Interest Fund represents the lowest risk of the three fund choices, given both its choice of underlying bond index (half of which is in short-term securities) and lack of active management strategies. The Paisley Fund could be a suitable choice for an investor near retirement who is seeking income with a minimum risk profile.

The Dundee Fund represents a medium risk profile given the choice of the composite benchmark and suggests an enhanced approach to indexing. This fund may be the best choice for a middle-aged worker seeking to add a fixed-income component with moderate risk to his portfolio.

The Newcastleton Fund has the highest risk of the three choices and is an example of an actively managed fund that has a mandate to take positions in primary risk factors, such as duration and credit, that deviate from those of the index in order to generate excess return. This fund could be an appropriate choice for a younger worker who is seeking exposure to fixed income but willing to accommodate higher risk.

8.3. Alternatives to Investing Directly in Fixed-Income Securities

Recall that a number of alternatives to direct investing into bonds are available to investment managers. We have shown earlier that index-based exposure can be obtained through the following traded products, such as ETFs that offer greater liquidity than the underlying securities or other alternatives, such as mutual funds (i.e., pooled investment vehicles whose shares or units represent a proportional share in the ownership of the assets in an underlying portfolio). Investors benefit from greater bond ETF liquidity versus mutual funds given their availability to be purchased or sold throughout the trading day. Recall that authorized ETF participants—who enter into an agreement with the distributor of the fund, purchasing shares from or selling ETF shares to the fund creation units—would be encouraged to engage in arbitrage to profit from any significant divergence between the market price of the underlying fixed-income securities portfolio and an ETF's net asset value (NAV). That said, the fact that many fixed-income securities are either thinly traded or not traded at all might allow such a divergence to persist to a much greater degree for a bond ETF than might be the case in the equity market.

Another alternative to direct investing in fixed-income securities are index-based total return swaps, common over-the-counter instruments. Recall that similar to an interest rate swap, a **total return swap** involves the periodic exchange of cash flows between two parties for the life of the contract. Unlike an interest rate swap—in which counterparties exchange a stream of fixed cash flows versus a floating-rate benchmark, such as the MRR, to transform fixed assets or liabilities to a variable exposure—a total return swap (TRS) has a periodic exchange based on a reference obligation that is an underlying equity, commodity, or bond index. The total return receiver receives both the cash flows from the underlying index as well as any appreciation in the index over the period in exchange for paying the MRR plus a predetermined spread. The total return payer is responsible for paying the reference obligation cash flows and return to the receiver but will also be compensated by the receiver for any depreciation in the index or default losses incurred on the portfolio.

A TRS can have some advantages over a direct investment in a bond mutual fund or ETF. As a derivative, it requires less initial cash outlay than direct investment in the bond portfolio for similar performance. A TRS also carries counterparty credit risk, however. As a customized over-the-counter product, a TRS can offer exposure to assets that are difficult to access directly, such as some high-yield and commercial loan investments.

9. BENCHMARK SELECTION

- **discuss criteria for selecting a benchmark and justify the selection of a benchmark**

The choice of a benchmark is perhaps an investment manager's most important decision beyond the passive versus active decision or the form that the investment takes, as described earlier. Benchmark selection is one of the final steps in the broader asset allocation process.

The asset allocation process starts with a clear delineation of the portfolio manager's investment goals and objectives. Examples of such goals might include the protection of funds (especially against inflation), broad market replication, predictable returns within acceptable risk parameters, or maximum absolute returns through opportunistic means. The manager must agree on an investment policy with the asset owners, beneficiaries, and other constituents outlining return objectives, risk tolerance, and constraints to narrow choices available in the broader capital markets to meet these objectives. Recall that a strategic asset allocation targeting specific weightings for each permissible asset class is the result of this process, while a tactical asset allocation range often provides the investment manager some short-term flexibility to deviate from these weightings in response to anticipated market changes.

Bonds figure prominently in most asset allocations given that they represent the largest fraction of global capital markets, capture a wide range of issuers, and, as borrowed funds, represent claims that should involve lower risk than common equity. Choosing a fixed-income benchmark is unique, however, in that the investor usually has some degree of fixed-income exposure embedded within its asset/liability portfolio, as outlined in the foregoing immunization and liability-driven investing examples. The investment manager must therefore consider these implicit or explicit duration preferences when choosing a fixed-income benchmark.

Benchmark selection must factor in the broad range of issuers and characteristics available in the fixed-income markets. In general, the use of an index as a widely accepted benchmark requires clear, transparent rules for security inclusion and weighting, investability, daily valuation and availability of past returns, and turnover. Unlike in equity indexes, fixed-income market dynamics can drive deviation from a stable benchmark sought by investors for a number of reasons:

- The finite maturity of bonds in a static portfolio implies that duration will drift downward over time.
- Market dynamics and issuer preferences tend to dictate both issuer composition for broad-based indexes as well as maturity selection for narrower indexes. For example, as shown in Exhibit 28, the composition of the Bloomberg Barclays US Aggregate Bond Index changed significantly during the years prior to and after the 2008 global financial crisis, with a large increase in securitized debt pre-crisis and a significant rise in government debt thereafter:

EXHIBIT 28 Bloomberg Barclays US Aggregate Bond Index Sector Allocation, Selected Years

Year	Government	Corporate	Securitized
1993	53.0%	17.0%	30.0%
1998	46.0%	22.0%	32.0%
2000	38.0%	24.0%	39.0%
2005	40.2%	19.5%	40.2%
2008	38.6%	17.7%	43.7%
2010	45.8%	18.8%	35.5%
2015	44.8%	24.2%	31.0%
2020	43.4%	27.3%	29.3%

Sources: Lehman Brothers; Barclays.

Separately, a corporate debt index investor might find her benchmark choice no longer desirable if issuers refinance maturing bonds for longer maturities and extend overall debt duration.

The dynamics of fixed-income markets require investors to more actively understand and define their underlying duration preferences as well as a desired risk and return profile within their fixed-income allocation when conducting benchmark selection. Expressed differently, the desired duration profile may be considered the portfolio "beta," with the targeted duration equal to an investor's preferred duration exposure. Once these parameters are clear, investors may wish to combine several well-defined sub-benchmark categories into an overall benchmark. Examples of sub-benchmark categories might include Treasuries (or domestic sovereign bonds), US agencies or other asset-backed securities, corporate bonds, high-yield bonds, bank loans, developed markets global debt, or emerging markets debt.

For fixed-income investors seeking to reduce the cost of active management, an alternative known as **smart beta** has emerged. Smart beta involves the use of simple, transparent, rules-based strategies as a basis for investment decisions. The starting point for smart beta investors is an analysis of the well-established, static strategies that tend to drive excess portfolio returns. In theory, asset managers who are able to isolate and pursue such strategies can capture a significant proportion of these excess returns without the significantly higher fees associated with active management. Although the use of smart beta strategies is more established among equity managers, fixed-income managers are increasing their use of these techniques as well (see Staal, Corsi, Shores, and Woida 2015).

EXAMPLE 12

Given the significant rise in regional bond issuance following the 2008 global financial crisis, Next Europe Asset Management Limited aims to grow its assets under management by attracting a variety of new local Eurozone investors to the broader set of alternatives available in the current fixed-income market. Several of the indexes that Next Europe offers as a basis for investment are as follows:

- **S&P Eurozone Sovereign Bond Index.** This index consists of fixed-rate, sovereign debt publicly issued by Eurozone national governments for their domestic markets with various maturities including 1 to 3 years, 3 to 5 years, 5 to 7 years, 7 to 10 years, and 10+ years. For example, the 1- to 3-year index had a weighted average maturity of 1.91 years and a modified duration of 1.87 as of 31 July 2020 (www.spglobal.com).
- **Bloomberg EUR Investment Grade European Corporate Bond Index (BERC).** The BERC index consists of local, EUR-based corporate debt issuance in Eurozone countries and had an effective duration of 5.28 as of September 2020.
- **Bloomberg EUR High Yield Corporate Bond Index (BEUH).** This index consists of sub-investment-grade, EUR-denominated bonds issued by Eurozone-based corporations. It had an effective duration of 3.68 as of September 2020 (www.bloombergindexes.com).
- **FTSE Pfandbrief Index.** The Pfandbrief, which represents the largest segment of the German private debt market, is a bond issued by German mortgage banks, collateralized by long-term assets, such as real estate or public sector loans. These securities are also referred to as covered bonds and are being used as a model for similar issuance in other European countries.

The FTSE Pfandbrief indexes include jumbo Pfandbriefs from German issuers as well as those of comparable structure and quality from other Eurozone countries. The sub-indexes offer a range of maturities, including 1 to 3 years, 3 to 5 years, 5 to 7 years, 7 to 10 years, and 10+ years (www.ftse.com/products/indices).

Which of the above indexes would be suitable for the following investor portfolios?

1. A highly risk-averse investor who is sensitive to fluctuations in portfolio value.
2. A new German private university that has established an endowment with a very long-term investment horizon.
3. A Danish life insurer relying on the fixed-income portfolio managed by Next Europe to meet both short-term claims as well as offset long-term obligations.

Solution to 1: Given this investor's high degree of risk aversion, an index with short or intermediate duration with limited credit risk would be most appropriate to limit market value risk. Of the alternatives listed, the S&P Eurozone Sovereign Bond 1–3 Years Index or the FTSE 1–3 Years Pfandbrief Index (given the high credit quality of covered bonds) would be appropriate choices.

Solution to 2: This investor's very long investment horizon suggests that the BERC is an appropriate index, because it has the longest duration of the indexes given. In addition, the long-term S&P Eurozone Sovereign Bond or FTSE Pfandbrief indexes (10+ years) could be appropriate choices as well. Next Europe should consider the trade-off between duration and risk in its discussion with the endowment.

Solution to 3: The Danish life insurer faces two types of future obligation, namely a short-term outlay for expected claims and a long-term horizon for future obligations. For the short-term exposure, stability of market value is a primary consideration, and the insurer would seek an index with low market risk. Of the above alternatives, the 1–3 Years S&P Sovereign Bond or the FTSE Pfandbrief 1–3 Years would be the best choices. The longer-term alternatives in the Solution to 2 would be most appropriate for the long-term future obligations.

SUMMARY

- Structured fixed-income investing requires a frame of reference, such as a balance sheet, to structure the bond portfolio. This frame of reference can be as simple as the time to retirement for an individual or as complex as a balance sheet of rate-sensitive assets and liabilities for a company.
- Assets and liabilities can be categorized by the degree of certainty surrounding the amount and timing of cash flows. Type I assets and liabilities, such as traditional fixed-rate bonds with no embedded options, have known amounts and payment dates. For Type I assets and liabilities, such yield duration statistics as Macaulay, modified, and money duration apply.
- Type II, III, and IV assets and liabilities have uncertain amounts and/or uncertain timing of payment. For Type II, III, and IV assets and liabilities, curve duration statistics, such as effective duration, are needed. A model is used to obtain the estimated values when the yield curve shifts up and down by the same amount.
- Immunization is the process of structuring and managing a fixed-income portfolio to minimize the variance in the realized rate of return over a known investment horizon.
- In the case of a single liability, immunization is achieved by matching the Macaulay duration of the bond portfolio to the horizon date. As time passes and bond yields change, the duration of the bonds changes and the portfolio needs to be rebalanced. This rebalancing can be accomplished by buying and selling bonds or using interest rate derivatives, such as futures contracts and interest rate swaps.
- An immunization strategy aims to lock in the cash flow yield on the portfolio, which is the internal rate of return on the cash flows. It is not the weighted average of the yields to maturity on the bonds that constitute the portfolio.

- The risk to immunization is that as the yield curve shifts and twists, the cash flow yield on the bond portfolio does not match the change in the yield on the zero-coupon bond that would provide for perfect immunization.
- A sufficient, but not necessary, condition for immunization is a parallel (or shape-preserving) shift whereby all yields change by the same amount in the same direction. If the change in the cash flow yield is the same as that on the zero-coupon bond being replicated, immunization can be achieved even with a non-parallel shift to the yield curve.
- Structural risk to immunization arises from some non-parallel shifts and twists to the yield curve. This risk is reduced by minimizing the dispersion of cash flows in the portfolio, which can be accomplished by minimizing the convexity statistic for the portfolio. Concentrating the cash flows around the horizon date makes the immunizing portfolio closely track the zero-coupon bond that provides for perfect immunization.
- For multiple liabilities, one method of immunization is cash flow matching. A portfolio of high-quality zero-coupon or fixed-income bonds is purchased to match as closely as possible the amount and timing of the liabilities.
- A motive for cash flow matching can be accounting defeasance, whereby both the assets and liabilities are removed from the balance sheet.
- A laddered bond portfolio is a common investment strategy in the wealth management industry. The laddered portfolio offers "diversification" over the yield curve compared with "bullet" or "barbell" portfolios. This structure is especially attractive in stable, upwardly sloped yield curve environments as maturing short-term debt is replaced with higher-yielding long-term debt at the back of the ladder.
- A laddered portfolio offers an increase in convexity because the cash flows have greater dispersions than a more concentrated (bullet) portfolio.
- A laddered portfolio provides liquidity in that it always contains a soon-to-mature bond that could provide high-quality, low-duration collateral on a repo contract if needed.
- Immunization of multiple liabilities can be achieved by structuring and managing a portfolio of fixed-income bonds. Because the market values of the assets and liabilities differ, the strategy is to match the money durations. The money duration is the modified duration multiplied by the market value. The basis point value is a measure of money duration calculated by multiplying the money duration by 0.0001.
- The conditions to immunize multiple liabilities are that (1) the market value of assets is greater than or equal to the market value of the liabilities, (2) the asset basis point value (BPV) equals the liability BPV, and (3) the dispersion of cash flows and the convexity of assets are greater than those of the liabilities.
- A derivatives overlay—for example, interest rate futures contracts—can be used to immunize single or multiple liabilities.
- The number of futures contracts needed to immunize is the liability BPV minus the asset BPV, divided by the futures BPV. If the result is a positive number, the entity buys, or goes long, futures contracts. If the result is a negative number, the entity sells, or goes short, futures contracts. The futures BPV can be approximated by the BPV for the cheapest-to-deliver security divided by the conversion factor for the cheapest-to-deliver security.
- Contingent immunization adds active management of the surplus, which is the difference between the asset and liability market values, with the intent to reduce the overall cost of retiring the liabilities. In principle, any asset classes can be used for the active investment. The entity can choose to over-hedge or under-hedge the number of futures contracts needed for passive immunization.

- Liability-driven investing (LDI) often is used for complex rate-sensitive liabilities, such as those for a defined benefit pension plan. The retirement benefits for covered employees depend on many variables, such as years of employment, age at retirement, wage level at retirement, and expected lifetime. There are different measures for the liabilities: for instance, the accumulated benefit obligation (ABO) that is based on current wages and the projected benefit obligation (PBO) that is based on expected future wages. For each liability measure (ABO or PBO), a model is used to extract the effective duration and BPV.
- Interest rate swap overlays can be used to reduce the duration gap as measured by the asset and liability BPVs. There often is a large gap because pension funds hold sizable asset positions in equities that have low or zero effective durations and their liability durations are high.
- The hedging ratio is the percentage of the duration gap that is closed with the derivatives. A hedging ratio of zero implies no hedging. A hedging ratio of 100% implies immunization—that is, complete removal of interest rate risk.
- Strategic hedging is the active management of the hedging ratio. Because asset BPVs are less than liability BPVs in typical pension funds, the derivatives overlay requires the use of receive-fixed interest rate swaps. Because receive-fixed swaps gain value as current swap market rates fall, the fund manager could choose to raise the hedging ratio when lower rates are anticipated. If rates are expected to go up, the manager could strategically reduce the hedging ratio.
- An alternative to the receive-fixed interest rate swap is a purchased receiver swaption. This swaption confers to the buyer the right to enter the swap as the fixed-rate receiver. Because of its negative duration gap (asset BPV is less than liability BPV), the typical pension plan suffers when interest rates fall and could become underfunded. The gain on the receiver swaption as rates decline offsets the losses on the balance sheet.
- Another alternative is a swaption collar, the combination of buying the receiver swaption and writing a payer swaption. The premium received on the payer swaption that is written offsets the premium needed to buy the receiver swaption.
- The choice among hedging with the receive-fixed swap, the purchased receiver swaption, and the swaption collar depends in part on the pension fund manager's view on future interest rates. If rates are expected to be low, the receive-fixed swap typically is the preferred derivative. If rates are expected to go up, the swaption collar can become attractive. And if rates are projected to reach a certain threshold that depends on the option costs and the strike rates, the purchased receiver swaption can become the favored choice.
- Model risks arise in LDI strategies because of the many assumptions in the models and approximations used to measure key parameters. For example, the liability BPV for the defined benefit pension plan depends on the choice of measure (ABO or PBO) and the assumptions that go into the model regarding future events (e.g., wage levels, time of retirement, and time of death).
- Spread risk in LDI strategies arises because it is common to assume equal changes in asset, liability, and hedging instrument yields when calculating the number of futures contracts, or the notional principal on an interest rate swap, to attain a particular hedging ratio. The assets and liabilities are often on corporate securities, however, and their spreads to benchmark yields can vary over time.
- Investing in a fund that tracks a bond market index offers the benefits of both diversification and low administrative costs. Tracking risk arises when the fund manager chooses to buy only a subset of the index, a strategy called enhanced indexing, because fully replicating the index can be impractical as a result of the large number of bonds in the fixed-income universe.

- Corporate bonds are often illiquid. Matrix pricing uses available data on comparable securities to estimate the fair value of the illiquid bonds.
- The primary risk factors encountered by an investor tracking a bond index include decisions regarding duration (option-adjusted duration for callable bonds, convexity for possible large yield shifts, and key rate durations for non-parallel shifts) and portfolio weights (assigned by sector, credit quality, maturity, coupon rate, and issuer).
- Index replication is one method to establish a passive exposure to the bond market. The manager buys or sells bonds only when there are changes to the index. Full replication can be expensive, however, as well as infeasible for broad-based fixed-income indexes that include many illiquid bonds.
- Several enhancement strategies can reduce the costs to track a bond index: lowering trading costs, using models to identify undervalued bonds and to gauge relative value at varying points along the yield curve, over/under weighting specific credit sectors over the business cycle, and evaluating specific call features to identify value given large yield changes.
- Investors can obtain passive exposure to the bond market using ETFs or mutual funds. Exchange-traded fund (ETF) shares have the advantage of trading on an exchange throughout the day.
- A total return swap, an over-the-counter derivative, allows an institutional investor to transform an asset or liability from one asset category to another—for instance, from variable-rate cash flows referencing the MRR to the total return on a particular bond index.
- A total return swap (TRS) can have some advantages over a direct investment in a bond mutual fund or ETF. As a derivative, it requires less initial cash outlay than direct investment in the bond portfolio for similar performance. A TRS also carries counterparty credit risk, however. As a customized over-the-counter product, a TRS can offer exposure to assets that are difficult to access directly, such as some high-yield and commercial loan investments.
- Selecting a particular bond index is a major decision for a fixed-income investment manager. Selection is guided by the specified goals and objectives for the investment. The decision should recognize several features of bond indexes: (1) Given that bonds have finite maturities, the duration of the index drifts down over time; (2) the composition of the index changes over time with the business cycle and maturity preferences of issuers.

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PRACTICE PROBLEMS

The following information relates to Questions 1–8

Serena is a risk management specialist with Liability Protection Advisors. Trey, CFO of Kiest Manufacturing, enlists Serena's help with three projects. The first project is to defease some of

Kiest's existing fixed-rate bonds that are maturing in each of the next three years. The bonds have no call or put provisions and pay interest annually. Exhibit 1 presents the payment schedule for the bonds.

EXHIBIT 1 Kiest Manufacturing Bond Payment Schedule (as of beginning of Year 1)

Maturity Date	Payment Amount
End of Year 1	$9,572,000
End of Year 2	$8,392,000
End of Year 3	$8,200,000

The second project for Serena is to help Trey immunize a $20 million portfolio of liabilities. The liabilities range from 3.00 years to 8.50 years with a Macaulay duration of 5.34 years, cash flow yield of 3.25%, portfolio convexity of 33.05, and basis point value (BPV) of $10,505. Serena suggested employing a duration-matching strategy using one of the three AAA rated bond portfolios presented in Exhibit 2.

EXHIBIT 2 Possible AAA Rated Duration-Matching Portfolios

	Portfolio A	Portfolio B	Portfolio C
Bonds (term, coupon)	4.5 years, 2.63% 7.0 years, 3.50%	3.0 years, 2.00% 6.0 years, 3.25% 8.5 years, 3.88%	1.5 years, 1.25% 11.5 years, 4.38%
Macaulay duration	5.35	5.34	5.36
Cash flow yield	3.16%	3.33%	3.88%
Convexity	31.98	34.51	50.21
BPV	$10,524	$10,506	$10,516

Serena explains to Trey that the underlying duration-matching strategy is based on the following three assumptions.

1. Yield curve shifts in the future will be parallel.
2. Bond types and quality will closely match those of the liabilities.
3. The portfolio will be rebalanced by buying or selling bonds rather than using derivatives.

The third project for Serena is to make a significant direct investment in broadly diversified global bonds for Kiest's pension plan. Kiest has a young workforce, and thus, the plan has a long-term investment horizon. Trey needs Serena's help to select a benchmark index that is appropriate for Kiest's young workforce. Serena discusses three benchmark candidates, presented in Exhibit 3.

EXHIBIT 3 Global Bond Index Benchmark Candidates

Index Name	Effective Duration	Index Characteristics
Global Aggregate	7.73	Market cap weighted; Treasuries, corporates, agency, securitized debt
Global Aggregate GDP Weighted	7.71	Same as Global Aggregate, except GDP weighted
Global High Yield	4.18	GDP weighted; sovereign, agency, corporate debt

With the benchmark selected, Trey provides guidelines to Serena directing her to (1) use the most cost-effective method to track the benchmark and (2) provide low tracking error.

After providing Trey with advice on direct investment, Serena offered him additional information on alternative indirect investment strategies using (1) bond mutual funds, (2) exchange-traded funds (ETFs), and (3) total return swaps. Trey expresses interest in using bond mutual funds rather than the other strategies for the following reasons.

- **Reason 1:** Total return swaps have much higher transaction costs and initial cash outlay than bond mutual funds.
- **Reason 2:** Unlike bond mutual funds, bond ETFs can trade at discounts to their underlying indexes, and those discounts can persist.
- **Reason 3:** Bond mutual funds can be traded throughout the day at the net asset value of the underlying bonds.

1. Based on Exhibit 1, Kiest's liabilities would be classified as:
   A. Type I.
   B. Type II.
   C. Type III.
2. Based on Exhibit 2, the portfolio with the greatest structural risk is:
   A. Portfolio A.
   B. Portfolio B.
   C. Portfolio C.
3. Which portfolio in Exhibit 2 fails to meet the requirements to achieve immunization for multiple liabilities?
   A. Portfolio A
   B. Portfolio B
   C. Portfolio C
4. Based on Exhibit 2, relative to Portfolio C, Portfolio B:
   A. has higher cash flow reinvestment risk.
   B. is a more desirable portfolio for liquidity management.
   C. provides less protection from yield curve shifts and twists.
5. Serena's three assumptions regarding the duration-matching strategy indicate the presence of:
   A. model risk.
   B. spread risk.
   C. counterparty credit risk.
6. The global bond benchmark in Exhibit 3 that is least appropriate for Kiest to use is the:
   A. Global Aggregate Index.
   B. Global High Yield Index.
   C. Global Aggregate GDP Weighted Index.
7. To meet both of Trey's guidelines for the pension's bond fund investment, Serena should recommend:
   A. pure indexing.
   B. enhanced indexing.
   C. active management.
8. Which of Trey's reasons for choosing bond mutual funds as an investment vehicle is correct?
   A. Reason 1
   B. Reason 2
   C. Reason 3

The following information relates to questions 9–16

SD&R Capital (SD&R), a global asset management company, specializes in fixed-income investments. Molly, chief investment officer, is meeting with a prospective client, Leah of DePuy Financial Company (DFC).

Leah informs Molly that DFC's previous fixed-income manager focused on the interest rate sensitivities of assets and liabilities when making asset allocation decisions. Molly explains that, in contrast, SD&R's investment process first analyzes the size and timing of client liabilities, and then it builds an asset portfolio based on the interest rate sensitivity of those liabilities.

Molly notes that SD&R generally uses actively managed portfolios designed to earn a return in excess of the benchmark portfolio. For clients interested in passive exposure to fixed-income instruments, SD&R offers two additional approaches.

- **Approach 1:** Seeks to fully replicate a small range of benchmarks consisting of government bonds.
- **Approach 2:** Follows an enhanced indexing process for a subset of the bonds included in the Bloomberg Barclays US Aggregate Bond Index. Approach 2 may also be customized to reflect client preferences.

To illustrate SD&R's immunization approach for controlling portfolio interest rate risk, Molly discusses a hypothetical portfolio composed of two non-callable, investment-grade bonds. The portfolio has a weighted average yield-to-maturity of 9.55%, a weighted average coupon rate of 10.25%, and a cash flow yield of 9.85%.

Leah informs Molly that DFC has a single $500 million liability due in nine years, and she wants SD&R to construct a bond portfolio that earns a rate of return sufficient to pay off the obligation. Leah expresses concern about the risks associated with an immunization strategy for this obligation. In response, Molly makes the following statements about liability-driven investing:

- **Statement 1:** Although the amount and date of SD&R's liability is known with certainty, measurement errors associated with key parameters relative to interest rate changes may adversely affect the bond portfolios.
- **Statement 2:** A cash flow matching strategy will mitigate the risk from non-parallel shifts in the yield curve.

Molly provides the four US dollar–denominated bond portfolios in Exhibit 1 for consideration. Molly explains that the portfolios consist of non-callable, investment-grade corporate and government bonds of various maturities because zero-coupon bonds are unavailable.

EXHIBIT 1 Proposed Bond Portfolios to Immunize SD&R Single Liability

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Cash flow yield	7.48%	7.50%	7.53%	7.51%
Average time to maturity	11.2 years	9.8 years	9.0 years	10.1 years
Macaulay duration	9.8	8.9	8.0	9.1
Market value-weighted duration	9.1	8.5	7.8	8.6
Convexity	154.11	131.75	130.00	109.32

The discussion turns to benchmark selection. DFC's previous fixed-income manager used a custom benchmark with the following characteristics:

- **Characteristic 1:** The benchmark portfolio invests only in investment-grade bonds of US corporations with a minimum issuance size of $250 million.
- **Characteristic 2:** Valuation occurs on a weekly basis, because many of the bonds in the index are valued weekly.
- **Characteristic 3:** Historical prices and portfolio turnover are available for review.

Molly explains that, in order to evaluate the asset allocation process, fixed-income portfolios should have an appropriate benchmark. Leah asks for benchmark advice regarding DFC's portfolio of short-term and intermediate-term bonds, all denominated in US dollars. Molly presents three possible benchmarks in Exhibit 2.

EXHIBIT 2 Proposed Benchmark Portfolios

Benchmark	Index	Composition	Duration
1	Bloomberg Barclays US Bond Index	80% US government bonds 20% US corporate bonds	8.7
2 Index Blend	50% Bloomberg Barclays US Corporate Bond Index	100% US corporate bonds	7.5
	50% Bloomberg Barclays Short-Term Treasury Index	100% short-term US government debt	0.5
3	Bloomberg Barclays Global Aggregate Bond Index	60% EUR-denominated corporate bonds 40% US-denominated corporate debt	12.3

9. The investment process followed by DFC's previous fixed-income manager is *best* described as:
   A. asset-driven liabilities.
   B. liability-driven investing.
   C. asset–liability management.
10. Relative to Approach 1 of gaining passive exposure, an advantage of Approach 2 is that it:
    A. minimizes tracking error.
    B. requires less risk analysis.
    C. is more appropriate for socially responsible investors.
11. The two-bond hypothetical portfolio's immunization goal is to lock in a rate of return equal to:
    A. 9.55%.
    B. 9.85%.
    C. 10.25%.
12. Which of Molly's statements about liability-driven investing is (are) correct?
    A. Statement 1 only.
    B. Statement 2 only.
    C. Both Statement 1 and Statement 2.
13. Based on Exhibit 1, which of the portfolios will *best* immunize SD&R's single liability?
    A. Portfolio 1
    B. Portfolio 2
    C. Portfolio 3

14. Which of the portfolios in Exhibit 1 *best* minimizes the structural risk to a single-liability immunization strategy?
    A. Portfolio 1
    B. Portfolio 3
    C. Portfolio 4
15. Which of the custom benchmark's characteristics violates the requirements for an appropriate benchmark portfolio?
    A. Characteristic 1
    B. Characteristic 2
    C. Characteristic 3
16. Based on DFC's bond holdings and Exhibit 2, Molly should recommend:
    A. Benchmark 1.
    B. Benchmark 2.
    C. Benchmark 3.

The following information relates to Questions 17–22

Doug, the newly hired chief financial officer for the City of Radford, asks the deputy financial manager, Hui, to prepare an analysis of the current investment portfolio and the city's current and future obligations. The city has multiple liabilities of different amounts and maturities relating to the pension fund, infrastructure repairs, and various other obligations.

Hui observes that the current fixed-income portfolio is structured to match the duration of each liability. Previously, this structure caused the city to access a line of credit for temporary mismatches resulting from changes in the term structure of interest rates.

Doug asks Hui for different strategies to manage the interest rate risk of the city's fixed-income investment portfolio against one-time shifts in the yield curve. Hui considers two different strategies:

- Strategy 1: Immunization of the single liabilities using zero-coupon bonds held to maturity.
- Strategy 2: Immunization of the single liabilities using coupon-bearing bonds while continuously matching duration.

The city also manages a separate, smaller bond portfolio for the Radford School District. During the next five years, the school district has obligations for school expansions and renovations. The funds needed for those obligations are invested in the Bloomberg Barclays US Aggregate Index. Doug asks Hui which portfolio management strategy would be most efficient in mimicking this index.

A Radford School Board member has stated that she prefers a bond portfolio structure that provides diversification over time, as well as liquidity. In addressing the board member's inquiry, Hui examines a bullet portfolio, a barbell portfolio, and a laddered portfolio.

17. A disadvantage of Strategy 1 is that:
    A. price risk still exists.
    B. interest rate volatility introduces risk to effective matching.
    C. there may not be enough bonds available to match all liabilities.
18. Which duration measure should be matched when implementing Strategy 2?
    A. Key rate
    B. Modified
    C. Macaulay

19. An upward shift in the yield curve on Strategy 2 will *most likely* result in the:
    A. price effect canceling the coupon reinvestment effect.
    B. price effect being greater than the coupon reinvestment effect.
    C. coupon reinvestment effect being greater than the price effect.
20. The effects of a non-parallel shift in the yield curve on Strategy 2 can be reduced by:
    A. minimizing the convexity of the bond portfolio.
    B. maximizing the cash flow yield of the bond portfolio.
    C. minimizing the difference between liability duration and bond-portfolio duration.
21. Hui's response to Doug's question about the most efficient portfolio management strategy should be:
    A. full replication.
    B. active management.
    C. an enhanced indexing strategy.
22. Which portfolio structure should Hui recommend that would satisfy the school board member's preference?
    A. Bullet portfolio
    B. Barbell portfolio
    C. Laddered portfolio

The following information relates to Questions 23–25

Chaopraya is an investment advisor for high-net-worth individuals. One of her clients, Schuylkill, plans to fund her grandson's college education and considers two options:

- Option 1: Contribute a lump sum of $300,000 in 10 years.
- Option 2: Contribute four level annual payments of $76,500 starting in 10 years.

The grandson will start college in 10 years. Schuylkill seeks to immunize the contribution today.

For Option 1, Chaopraya calculates the present value of the $300,000 as $234,535. To immunize the future single outflow, Chaopraya considers three bond portfolios given that no zero-coupon government bonds are available. The three portfolios consist of non-callable, fixed-rate, coupon-bearing government bonds considered free of default risk. Chaopraya prepares a comparative analysis of the three portfolios, presented in Exhibit 1.

EXHIBIT 1 Results of Comparative Analysis of Potential Portfolios

	Portfolio A	Portfolio B	Portfolio C
Market value	$235,727	$233,428	$235,306
Cash flow yield	2.504%	2.506%	2.502%
Macaulay duration	9.998	10.002	9.503
Convexity	119.055	121.498	108.091

Chaopraya evaluates the three bond portfolios and selects one to recommend to Schuylkill.

23. Recommend the portfolio in Exhibit 1 that would *best* achieve the immunization. Justify your response.

Template for Question 23

Recommend the portfolio in Exhibit 1 that would *best* achieve the immunization. (circle one)	**Justify** your response.
Portfolio A	
Portfolio B	
Portfolio C	

Schuylkill and Chaopraya now discuss Option 2.

Chaopraya estimates the present value of the four future cash flows as $230,372, with a money duration of $2,609,700 and convexity of 135.142. She considers three possible portfolios to immunize the future payments, as presented in Exhibit 2.

EXHIBIT 2 Data for Bond Portfolios to Immunize Four Annual Contributions

	Portfolio 1	Portfolio 2	Portfolio 3
Market value	$245,178	$248,230	$251,337
Cash flow yield	2.521%	2.520%	2.516%
Money duration	2,609,981	2,609,442	2,609,707
Convexity	147.640	139.851	132.865

24. Determine the *most appropriate* immunization portfolio in Exhibit 2. Justify your decision.

Template for Question 24

Determine the *most appropriate* immunization portfolio in Exhibit 2. (circle one)	**Justify** your decision.
Portfolio 1	
Portfolio 2	
Portfolio 3	

After selecting a portfolio to immunize Schuylkill's multiple future outflows, Chaopraya prepares a report on how this immunization strategy would respond to various interest rate scenarios. The scenario analysis is presented in Exhibit 3.

EXHIBIT 3 Projected Portfolio Response to Interest Rate Scenarios

	Immunizing Portfolio	Outflow Portfolio	Difference
Upward parallel shift			
Δ Market value	−6,410	−6,427	18
Δ Cash flow yield	0.250%	0.250%	0.000%
Δ Portfolio BPV	−9	−8	−1
Downward parallel shift			
Δ Market value	6,626	6,622	4
Δ Cash flow yield	−0.250%	−0.250%	0.000%
Δ Portfolio BPV	9	8	1

	Immunizing Portfolio	Outflow Portfolio	Difference
Steepening twist			
Δ Market value	−1,912	347	−2,259
Δ Cash flow yield	0.074%	−0.013%	0.087%
Δ Portfolio BPV	−3	0	−3
Flattening twist			
Δ Market value	1,966	−343	2,309
Δ Cash flow yield	−0.075%	0.013%	−0.088%
Δ Portfolio BPV	3	0	3

25. Discuss the effectiveness of Chaopraya's immunization strategy in terms of duration gaps.

CHAPTER 14

YIELD CURVE STRATEGIES

Robert W. Kopprasch, PhD, CFA
Steven V. Mann, PhD

LEARNING OUTCOMES

The candidate should be able to:

- describe the factors affecting fixed-income portfolio returns due to a change in benchmark yields;
- formulate a portfolio positioning strategy given forward interest rates and an interest rate view that coincides with the market view;
- formulate a portfolio positioning strategy given forward interest rates and an interest rate view that diverges from the market view in terms of rate level, slope, and shape;
- formulate a portfolio positioning strategy based upon expected changes in interest rate volatility;
- evaluate a portfolio's sensitivity using key rate durations of the portfolio and its benchmark;
- discuss yield curve strategies across currencies;
- evaluate the expected return and risks of a yield curve strategy.

1. INTRODUCTION

The size and breadth of global fixed-income markets, as well as the term structure of interest rates within and across countries, lead investors to consider numerous factors when creating and managing a bond portfolio. While fixed-income index replication and bond portfolios that consider both an investor's assets and liabilities were addressed earlier in the curriculum, we now turn our attention to active bond portfolio management. In contrast to a passive index strategy, active fixed-income management involves taking positions in primary risk factors that deviate from those of an index in order to generate excess return. Financial analysts who can successfully apply fixed-income concepts and tools to evaluate yield curve changes and position a portfolio based upon an interest rate view find this to be a valuable skill throughout their careers.

Prioritizing fixed-income risk factors is a key first step. In what follows, we focus on the yield curve, which represents the term structure of interest rates for government or benchmark securities, with the assumption that all promised principal and interest payments take place. Fixed-income securities, which trade at a spread above the benchmark to compensate investors for credit and liquidity risk, will be addressed later in the curriculum. The starting point for active portfolio managers is the current term structure of benchmark interest rates and an interest rate view established using macroeconomic variables introduced earlier. In what follows, we demonstrate how managers may position a fixed-income portfolio to capitalize on expectations regarding the level, slope, or shape (curvature) of yield curves using both long and short cash positions, derivatives, and leverage.

2. KEY YIELD CURVE AND FIXED-INCOME CONCEPTS FOR ACTIVE MANAGERS

- **describe the factors affecting fixed-income portfolio returns due to a change in benchmark yields**

The factors comprising an investor's expected fixed-income portfolio returns introduced earlier in the curriculum are summarized in Equation 1:

$$\begin{aligned} E(R) \approx\ & \text{Coupon income} \\ & +/- \text{ Rolldown return} \\ & +/- E\,(\Delta \text{ Price due to investor's view of benchmark yields}) \\ & +/- E\,(\Delta \text{ Price due to investor's view of yield spreads}) \\ & +/- E\,(\Delta \text{ Price due to investor's view of currency value changes}) \end{aligned} \tag{1}$$

Sections 2 and 3 will focus on actively managing the first three components of Equation 1, and Section 4 will include changes in currency. Credit strategies driving yield spreads will be discussed in a later lesson. As active management hinges on an investor's ability to identify actionable trades with specific securities, our review of yield curve and fixed-income concepts focuses on these practical considerations.

2.1. Yield Curve Dynamics

When someone refers to "the yield curve," this implies that one yield curve for a given issuer applies to all investors. In fact, a yield curve is a stylized representation of the yields-to-maturity available to investors at various maturities for a specific issuer or group of issuers. Yield curve models make certain assumptions that may vary by investor or by the intended use of the curve, raising such issues as the following:

- Asynchronous observations of various maturities on the curve
- Maturity gaps that require interpolation and/or smoothing
- Observations that seem inconsistent with neighboring values
- Use of on-the-run bonds only versus all marketable bonds (i.e., including off-the-run bonds)
- Differences in accounting, regulatory, or tax treatment of certain bonds that may make them look like outliers

As an example, a yield curve of the most recently issued, or on-the-run, securities may differ significantly from one that includes off-the-run securities. Off-the-run bonds are typically less liquid than on-the-run bonds, and hence they have a lower price (higher yield-to-maturity). Inclusion of off-the-run bonds will tend to "pull" the yield curve higher.

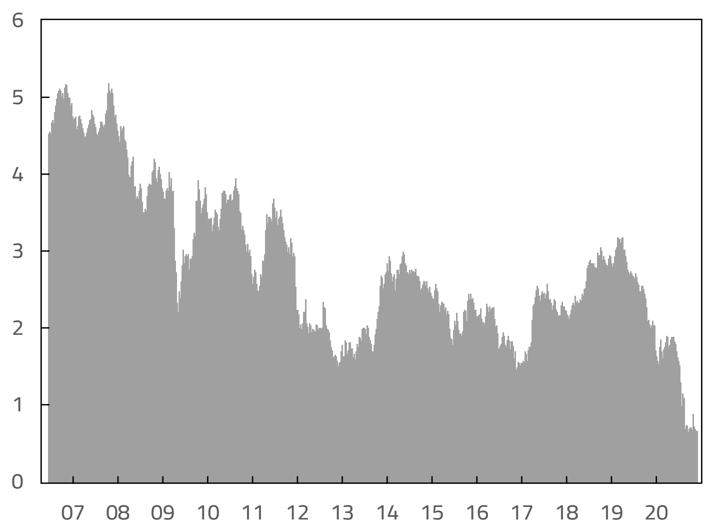
This illustrates two key points about yield curves. First, although we often take reported yield curves as a "given," they often do not consist of traded securities and must be derived from available bond yields-to-maturity using some type of model. This is particularly true for constant maturity yields, shown in some of the following exhibits. A constant maturity yield estimates, for example, what a hypothetical 5-year yield-to-maturity would be if a bond were available with exactly five years to maturity. While some derivatives reference the daily constant maturity yield, the current on-the-run 5-year Treasury issued before today has a maturity of less than five years. Estimating a constant maturity 5-year yield typically requires interpolating the yields-to-maturity on actively traded bonds with maturities *near* five years. Different models and assumptions can produce different yield curves. The difference between models becomes more pronounced as yields-to-maturity are converted to spot and forward rates (as spot and forward rate curves amplify yield curve steepness and curvature).

Second, a tradeoff exists between yield-to-maturity and liquidity. Active management strategies must assess this tradeoff when selecting bonds for the portfolio, especially if frequent trading is anticipated. While off-the-run bonds may earn a higher return if held to maturity, buying and selling them will likely involve increased trading costs (especially in a market crisis).

Primary yield curve risk factors are often categorized by three types: a change in (1) level (a parallel "shift" in the yield curve); (2) slope (a flattening or steepening "twist" of the yield curve); and (3) shape or curvature (or "butterfly movement"). Earlier in the curriculum, principal components analysis was used to decompose yield curve changes into these three separate factors. Level, slope, and curvature movements over time accounted for approximately 82%, 12%, and 4%, respectively, of US Treasury yield curve changes. Although based upon a specific historical period, the consistency of these results over time and across global markets underscores the importance of these factors in realizing excess portfolio returns under an active yield curve strategy.

The following exhibits provide historical context for the three yield curve factors using constant maturity US Treasury yields. Exhibit 1 shows US 10-year constant maturity yield levels.

EXHIBIT 1 10-Year US Treasury Yield, 2007–2020 (%)



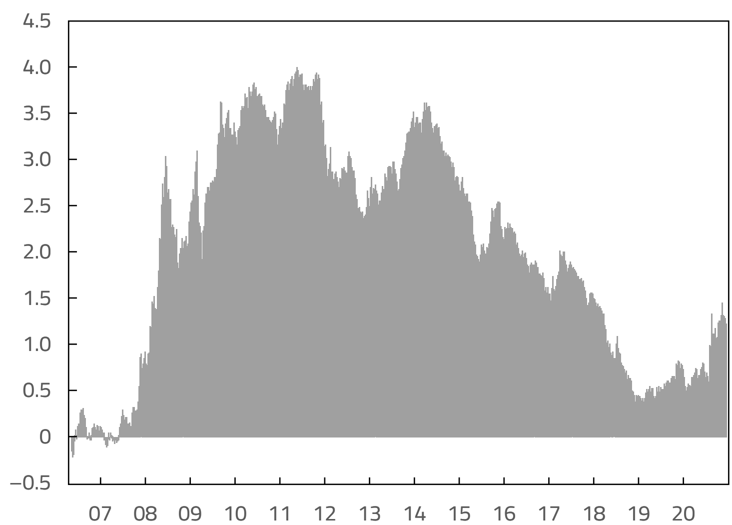
Source: US Federal Reserve.

During the period shown in Exhibit 1, 10-year US Treasury yields-to-maturity demonstrated significant volatility, falling to new lows in 2020 amid a flight to quality during the COVID-19 pandemic. Slower growth and accommodative monetary policy in the form of quantitative easing among global central banks since the 2008 global financial crisis years has driven government yields to zero and below. In 2020, negative yields were common on many Japanese, German, and Swiss government bonds, among others.

A change in yield *level* (or parallel shift) occurs when all yields-to-maturity represented on the curve change by the same number of basis points. Under this assumption, a portfolio manager might use a first-order duration statistic to approximate the impact of an expected yield curve change on portfolio value. This implies that yield curve changes occur only in parallel shifts, which is unreliable in cases where the yield curve's slope and curvature also change. Larger yield curve changes necessitate the inclusion of second-order effects in order to better measure changes in portfolio value.

Yield curve *slope* is often defined as the difference in basis points between the yield-to-maturity on a long-maturity bond and the yield-to-maturity on a shorter-maturity bond. For example, as of July 2020, the slope as measured by the 2s30s spread, or the difference between the 30-year Treasury bond (30s) and the 2-year Treasury note (2s) yields-to-maturity (1.43% and 0.16%, respectively), was 127 bps. Exhibit 2 shows the 2s30s spread for US Treasury constant maturity yields. As this spread increases, or widens, the yield curve is said to steepen, while a decrease, or narrowing, is referred to as a flattening of the yield curve. In most instances, the spread is positive and the yield curve is upward-sloping. If the spread turns negative, as was the case just prior to the 2008 global financial crisis, the yield curve is described as "inverted."

EXHIBIT 2 2s30s US Yield Spread, 2007–2020 (%)



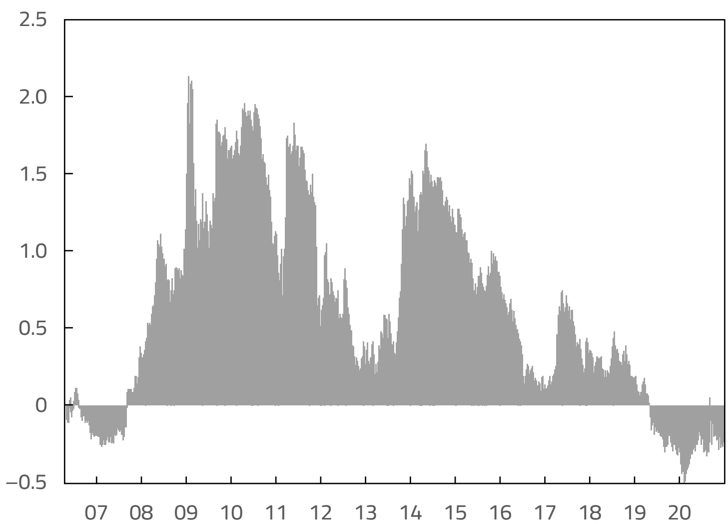
Source: US Federal Reserve.

Yield curve shape or *curvature* is the relationship between yields-to-maturity at the short end of the curve, at a midpoint along the curve (often referred to as the "belly" of the curve), and at the long end of the curve. A common measure of yield curve curvature is the **butterfly spread**:

$$\text{Butterfly spread} = -(\text{Short-term yield}) + (2 \times \text{Medium-term yield}) - \text{Long-term yield} \quad (2)$$

The butterfly spread takes on larger positive values when the yield curve has more curvature. Exhibit 3 displays this measure of curvature for the US Treasury constant maturity yield curve using 2-year, 10-year, and 30-year tenors. Curvature indicates a difference between medium-term yields and a linear interpolation between short-term and long-term yields-to-maturity. A positive butterfly spread indicates a "humped" or concave shape to the midpoint of the curve, while a "saucer" or convex shape indicates the spread is negative. The butterfly spread changes when intermediate-term yield-to-maturity changes are of a different magnitude than those on the wings (the short- and long-end of the curve). Note that as in the case of yield curve slope, the butterfly spread was generally positive until 2020, except for the period just prior to the 2008 global financial crisis.

EXHIBIT 3 US Butterfly Spread (2s/10s/30s), 2007–2020 (%)

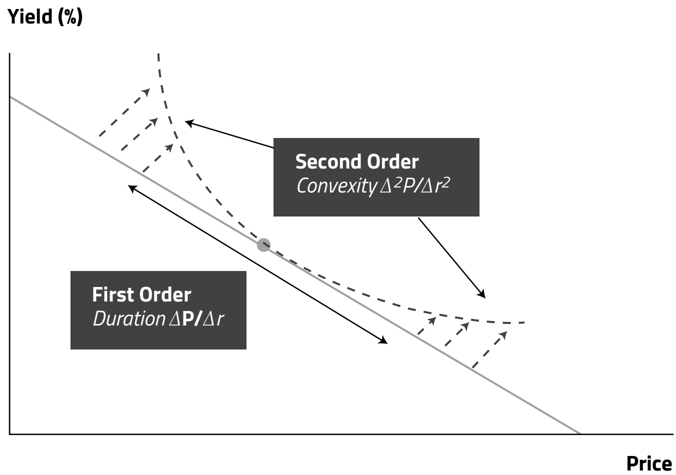


Source: US Federal Reserve.

2.2. Duration and Convexity

As active managers position their portfolios to capitalize on expected changes in the level, slope, and curvature of the benchmark yield curve, the anticipated change in portfolio value due to yield-to-maturity changes are captured by the third term in Equation 1—namely, the expected change in price due to investor's view of benchmark yields. The price/yield relationship for fixed-income bonds was established earlier in the curriculum as the combination of two factors: a negative, linear first-order factor (*duration*) and a usually positive, non-linear second-order factor (*convexity*), as shown in Exhibit 4.

EXHIBIT 4 Price–Yield Relationship for a Fixed-Income Bond



The third term in Equation 1, *E* (Δ Price due to investor's view of benchmark yield), combines the duration and convexity effects in Equation 3 of the percentage change in the full price ($\%\Delta PV^{Full}$) for a single bond as introduced earlier:

$$\%\Delta PV^{Full} \approx -(\text{ModDur} \times \Delta\text{Yield}) + [½ \times \text{Convexity} \times (\Delta\text{Yield})^2] \quad (3)$$

Fixed-income portfolio managers often approximate changes in a bond portfolio's present value (PV) by substituting market value (MV)-weighted averages for modified duration and convexity into Equation 3.

$$\text{AvgModDur} = \sum_{j=1}^{J} \text{ModDur}_j \times \left(\frac{\text{MV}_j}{\text{MV}}\right) \quad (4)$$

$$\text{AvgConvexity} = \sum_{j=1}^{J} \text{Convexity}_j \times \left(\frac{\text{MV}_j}{\text{MV}}\right) \quad (5)$$

Active managers focus on the *incremental* effect on these summary statistics for a portfolio by adding or selling bonds in the portfolio or by buying and selling fixed-income derivatives. Duration is a first-order effect that attempts to capture a linear relationship between bond prices and yield-to-maturity. Convexity is a second-order effect that describes a bond's price behavior for larger movements in yield-to-maturity. This additional term is a positive amount on a traditional (option-free) fixed-rate bond for either a yield increase or decrease, causing the yield/price relationship to deviate from a linear relationship. Because duration is a first-order effect, it follows that duration management—accounting for changes in yield curve level—will usually be a more important consideration for portfolio performance than convexity management. This is consistent with our previous discussion of the relative importance of the yield curve level, slope, and curvature. As we shall see later in this lesson, convexity management is more closely associated with yield curve slope and shape changes.

All else equal, positive convexity is a valuable feature in bonds. If a bond has higher positive convexity than an otherwise identical bond, then the bond price increases more if interest rates decrease (and decreases less if interest rates increase) than the duration estimate would suggest. Said another way, the expected price change of a bond with positive convexity for a given rate change will be higher than the price change of an identical-duration, lower-convexity bond. This price behavior is valuable to investors; therefore, a bond with higher convexity might be expected to have a lower yield-to-maturity than a similar-duration bond with less convexity. All else equal, bonds with longer durations have higher convexity than bonds with shorter durations. Also, as noted earlier in the curriculum, convexity is affected by the *dispersion* of cash flows—that is, the *variance* of the times to receipt of cash flow. Higher cash flow dispersion leads to an increase in convexity. This is in contrast to Macaulay duration, which measures the weighted *average* of the times to cash flow receipt. Note that throughout this lesson, we will use "raw" versus scaled (or "raw" divided by 100) convexity figures often seen on trading platforms. We can see the convexity effect by comparing two bond portfolios:

EXAMPLE 1 US Treasury Securities Portfolio

Tenor	Coupon	Price	ModDur	Convexity
2y	0.250%	\$100	1.994	5.0
5y	0.875%	\$100	4.880	26.5
10y	2.000%	\$100	9.023	90.8

Consider two \$50 million portfolios: Portfolio A is fully invested in the 5-year Treasury bond, and Portfolio B is an investment split between the 2-year (58.94%) and the 10-year (41.06%) bonds. The Portfolio B weights were chosen to (approximately) match the 5-year bond duration of 4.88. How will the value of these portfolios change if all three Treasury yields-to-maturity immediately rise or fall by 50 bps?

Using Equation 3, we can derive the percentage value change for Portfolios A and B as well as the dollar value of each \$50 million investment:

Portfolio	+ 50 bps % Δ Price	+ 50 bps Δ Price	– 50 bps % Δ Price	– 50 bps Δ Price
A	–2.407%	(\$1,203,438)	2.473%	\$1,236,563
B	–2.390%	(\$1,194,883)	2.490%	\$1,245,170

For example, for the case of a 50 bp increase in rates:

Portfolio A:

$$-2.407\% = (-4.880 \times 0.005) + [0.5 \times 26.5 \times (0.005^2)]$$

Portfolio B:

$$-2.390\% = 0.5894 \times (-1.994 \times 0.005) + [(0.5 \times 5 \times (0.005^2)] + 0.4106 \times (-9.023 \times 0.005) + [(0.5 \times 90.8 \times (0.005^2)]$$

Note that Portfolio B gains *more* (\$8,607) than Portfolio A when rates fall 50 bps and loses *less* (\$8,555) than Portfolio A when rates rise by 50 bps.

The first portfolio concentrated in a single intermediate maturity is often referred to as a **bullet** portfolio. The second portfolio, with similar duration but combining short- and long-term maturities, is a **barbell** portfolio. Although the bullet and barbell have the same duration, the barbell's higher convexity (40.229 versus 26.5 for the bullet) results in a larger gain as yields-to-maturity rise and a smaller loss when yields-to-maturity fall. Convexity is therefore valuable when interest rate volatility is expected to rise. This dynamic tends to cause investors to bid up prices on more convex, longer-maturity bonds, which drives changes in yield curve shape. As a result, the long end of the curve may decline or even invert (or invert further), increasing the curvature of the yield curve.

EXAMPLE 2 Portfolio Convexity

Portfolio convexity is a second-order effect that causes the value of a portfolio to respond to a change in yields-to-maturity in a non-linear manner. Which of the following best describes the effect of positive portfolio convexity for a given change in yield-to-maturity?

a. Convexity causes a greater increase in price for a decline in yields-to-maturity and a greater decrease in price when yields-to-maturity rise.
b. Convexity causes a smaller increase in price for a decline in yields-to-maturity and a greater decrease in price when yields-to-maturity rise.
c. Convexity causes a greater increase in price for a decline in yields-to-maturity and a smaller decrease in price when yields-to-maturity rise.

The correct answer is c. Note that the convexity component of Equation 3 involves squaring the change in yield-to-maturity, or [$\frac{1}{2} \times \text{Convexity} \times (\Delta\text{Yield})^2$], making the term positive as long as portfolio convexity is positive. This adds to the overall portfolio gain when yields-to-maturity decline and reduces the portfolio loss when yields-to-maturity rise.

3. YIELD CURVE STRATEGIES

- **formulate a portfolio positioning strategy given forward interest rates and an interest rate view that coincides with the market view**
- **formulate a portfolio positioning strategy given forward interest rates and an interest rate view that diverges from the market view in terms of rate level, slope, and shape**
- **formulate a portfolio positioning strategy based upon expected changes in interest rate volatility**
- **evaluate a portfolio's sensitivity using key rate durations of the portfolio and its benchmark**

Earlier in the curriculum, we established that yield curves are usually upward-sloping, with diminishing marginal yield-to-maturity increases at longer tenors—that is, flatter at longer maturities. As nominal yields-to-maturity incorporate an expected inflation premium, positively sloped yield curves are consistent with market expectations of rising or stable future inflation and relatively strong economic growth. Investor expectations of higher yields-to-maturity for assuming the increased interest rate risk of long-term bonds also contribute to this positive slope. Active managers often begin with growth and inflation forecasts, which they then translate into expected yield curve level, slope, and/or curvature changes. If their forecasts coincide with today's yield curve, managers will choose active strategies that are consistent with a static or stable yield curve. If their forecasts differ from what today's yield curve implies about these future yield curve characteristics, managers will position the portfolio to generate excess return based upon this divergent view, within the constraints of their investment mandate, using the cash and derivatives strategies we discuss next.

3.1. Static Yield Curve

A portfolio manager may believe that bonds are fairly priced and that the existing yield curve will remain unchanged over an investment horizon.

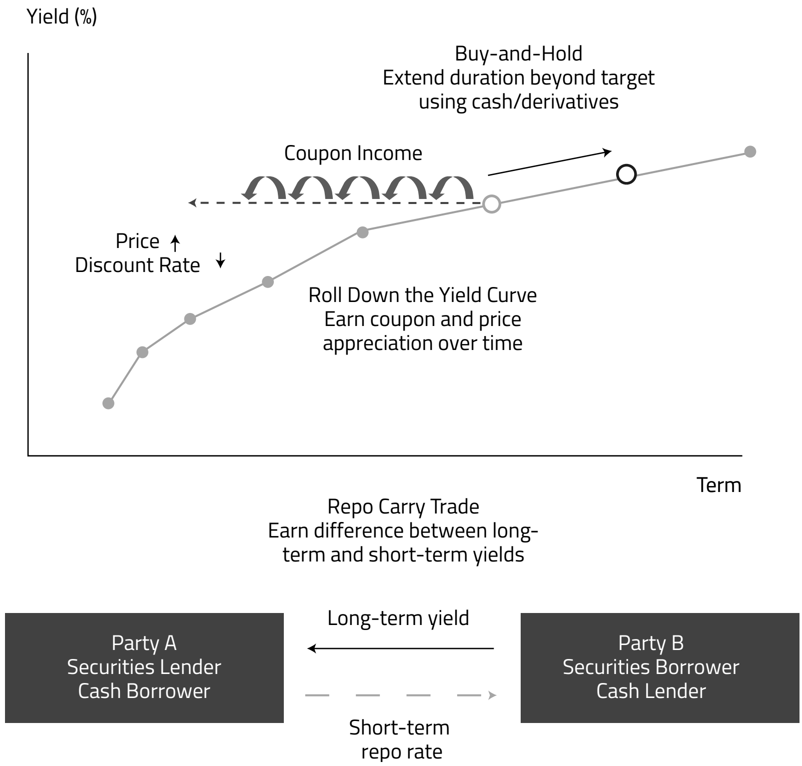
The two basic ways in which a manager may actively position a bond portfolio versus a benchmark index to generate excess return from a static or stable yield curve is to increase risk by adding either duration or leverage to the portfolio. If the yield curve is upward-sloping, longer duration exposure will result in a higher yield-to-maturity over time, while the "repo carry" trade (the difference between a higher-yielding instrument purchased and a lower-yielding (financing) instrument) will also generate excess returns.

Starting with cash-based instruments, "buy-and-hold" is an obvious strategy if the yield curve is upward-sloping. In an active context, this involves buying bonds with duration above the benchmark without active trading during a subsequent period. If the relationship between long- and short-term yields-to-maturity remains stable over this period, the manager is rewarded with higher return from the incremental duration. "Rolling down" the yield curve, a concept introduced previously, differs slightly from the "buy-and-hold" approach in terms of the investment time horizon and expected accumulation. The rolldown return component of Equation 1 (sometimes referred to as "carry-rolldown") incorporates not only coupon income (adjusted over time for any price difference from par) but also additional return from the passage of time and the investor's ability to sell the shorter-maturity bond in the future at a higher price (lower yield-to-maturity due to the upward-sloping yield curve) at the end of the investment horizon. If the yield curve is upward-sloping, buying bonds with a maturity *beyond* the investment horizon offers a total return (higher coupon plus price appreciation) greater than the purchase of a bond with maturity *matching* the investment horizon if the curve remains static. Finally, a common strategy known as a repurchase agreement or repo trade may be used in an expected stable rate environment to add leverage risk to the portfolio. The repo market involves buying a long-term security and financing it at a short-term rate below the long-term yield-to-maturity—that is, earning a positive "repo carry." At the end of the trade, the bond is sold and the repo is unwound. These cash-based strategies are summarized in Exhibits 5 and 6.

EXHIBIT 5 Cash-Based Static Yield Curve Strategies

Strategy	Description	Income	Objective
Buy-and-hold	Constant without active trading	Coupon income	Add duration beyond target given static yield curve view
Rolling down the yield curve	Constant, with Δ Price as maturity shortens	Coupon income +/− Rolldown return	Add duration and increased return if future shorter-term yields are below current yield-to-maturity
Repo carry trade	Finance bond purchase in repo market	(Coupon income +/− Rolldown return)—Financing cost	Generate repo carry return if coupon plus rolldown exceeds financing cost

EXHIBIT 6 Carry, Rolldown, and Buy-and-Hold Strategies under a Static Yield Curve



Excess return under these strategies depends upon stable rate levels and yield curve shape. Note that a more nuanced "buy-and-hold" strategy under this scenario could also involve less liquid and higher-yielding government bonds (such as off-the-run bonds). The lack of portfolio turnover may make the strategy seem passive, but in fact it may be quite aggressive as it introduces liquidity risk, a topic addressed in detail later in the curriculum. The ability to benefit from price appreciation by selling a shorter-dated bond at a premium when rolling down (or riding) the yield curve hinges on a reasonably static and upward-sloping yield curve. Not only will the repo carry be maintained under this yield curve scenario, but it also will generate excess return due to the reduced cash outlay versus a term bond purchase.

Active managers whose investment mandate extends to the use of synthetic means to increase risk by adding duration or leverage to the portfolio might consider using the derivatives-based strategies in Exhibit 7 to increase duration exposure beyond a benchmark target. Although the long futures example is similar to rolling down the yield curve, it relies solely on price appreciation rather than bond coupon income. The receive-fixed swap, on the other hand, is similar to the cash-based repo carry trade, but the investor receives the fixed swap rate and pays a market reference rate (MRR), which is often referred to as "swap carry."

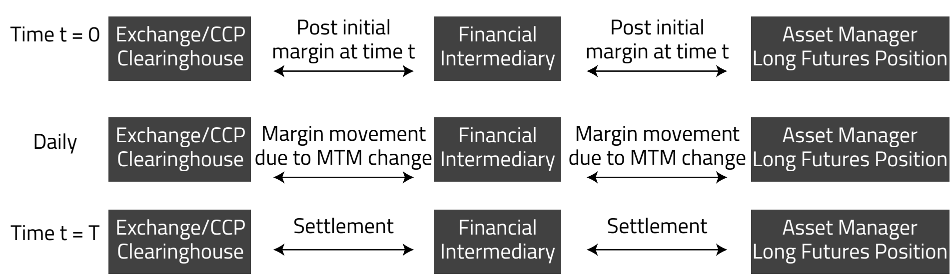
EXHIBIT 7 Derivatives-Based Static Yield Curve Strategies

Strategy	Description	Targeted Return	Goal
Long futures position	Purchase contract for future bond delivery	(Δ Price / Δ Bond yield) − Margin cost	Synthetically increase duration (up-front margin and daily mark-to-market valuation)
Receive-fixed swap	Fixed-rate receiver on an interest rate swap	(Swap rate − MRR) + (Δ Swap mark-to-market / Δ Swap yield)	Synthetically increase portfolio duration (up-front / mark-to-market collateral) + / − Swap carry

As mentioned previously in the curriculum, global exchanges offer a wide range of derivatives contracts across swap, bond, and short-term market reference rates for different settlement dates, and over the counter (OTC) contracts may be uniquely tailored to end user needs. Our treatment here is limited to futures and swaps and will extend to options in a later section.

Although margining was historically limited to exchange-traded derivatives, the advent of derivatives central counterparty (CCP) clearing mandated by regulatory authorities following the 2008 global financial crisis to mitigate counterparty risk has given rise to similar cash flow implications for OTC derivatives. Active managers using both exchange-traded and OTC derivatives must therefore maintain sufficient cash or eligible collateral to fulfill margin or collateral requirements. They must also factor any resulting foregone portfolio return into their overall performance. That said, since the initial cash outlay for a derivative is limited to initial margin or collateral as opposed to the full price for a cash bond purchase, derivatives have a high degree of implicit leverage. That is, a small move in price/yield can have a very large effect on a derivative's mark-to-market value (MTM) relative to the margin posted. Exhibit 8 shows these cash flow mechanics. This outsized price effect makes derivatives effective instruments for fixed-income portfolio management.

EXHIBIT 8 Derivatives Cash Flow Impact for a Fixed-Income Portfolio



For example, bond futures involve a contract to take delivery of a bond on a specific future date. Changes in the futures contract value mirror those of the underlying bond's price over time, allowing an investor to create an exposure profile similar to a long bond position by purchasing this contract with a fraction of the outlay of a cash bond purchase. While futures contracts are covered in detail elsewhere in the curriculum, for our purposes here it is important to establish the basis point value (BPV) of a futures contract. Most government bond futures are traded and settled using the least costly or cheapest-to-deliver (CTD) bond among those eligible for future delivery. For example, the CME Group's Ultra 10-Year US Treasury Note Futures contract specifies delivery of an original 10-year issue Treasury security with not less than 9 years, five months and not more than 10 years to maturity with an assumed 6% yield-to-maturity and contract size of $100,000. The "duration" of the bond futures contract is assumed to match that of the CTD security. In order to determine the futures BPV, we use the following approximation introduced previously:

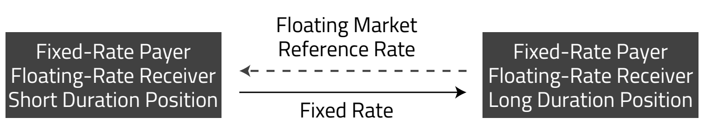
$$\text{Futures BPV} \approx \text{BPV}_{\text{CTD}} / \text{CF}_{\text{CTD}} \tag{6}$$

where CF_{CTD} is the conversion factor for the CTD security. For government bond futures with a fixed basket of underlying bonds, such as Australian Treasury bond futures, the futures BPV simply equals the BPV of an underlying basket of bonds.

The manager in Example 1 can replicate the 10-year Treasury exposure using futures by matching the BPV of the cash bond. As explained elsewhere, the BPV of the $20.53 million (or 41.06% × $50 million) 10-year Treasury position equals the modified duration (9.023) multiplied by the full price (also known as the money duration) times one basis point, or $18,524. If the CTD security under the Ultra 10-Year Futures contract is a Treasury bond also priced at par but with 9.5 years remaining to maturity, modified duration of 8.84, and a conversion factor of 0.684, then each $100,000 futures contract has a BPV of $129.24 ($88.40/0.684). The manager must therefore buy approximately 143 futures contracts ($18,524/$129.24) to replicate the exposure. Note that as shown in Exhibit 8, this will involve an outlay of initial margin and margin movement due to MTM changes rather than investment of full principal.

An interest rate swap involves the net exchange of fixed-for-floating payments, where the fixed rate (swap rate) is derived from short-term market reference rates for a given tenor. As shown in Exhibit 9, the swap contract may be seen as a combination of bonds, namely a fixed-rate bond versus a floating-rate bond of the same maturity.

EXHIBIT 9 Swaps as a Duration Management Tool



Note the similarities between the "carry" trade in Exhibit 5 and the receive-fixed interest rate swap position on the right in Exhibit 9. The fixed-rate receiver is "long" a fixed-rate term bond and "short" a floating-rate bond, giving rise to an exposure profile that mimics a "long" cash bond position by increasing duration. A swap's BPV may be estimated using Equation 7.

$$\text{Swap BPV} = \text{ModDur}_{\text{Swap}} \times \text{Swap Notional}/10{,}000 \tag{7}$$

The difference between the receive-fixed swap and long fixed-rate bond positions is best understood via an example.

EXAMPLE 3 Calculating Bond versus Swap Returns

Say a UK-based manager seeks to extend duration beyond an index by adding 10-year exposure. The manager considers either buying and holding a 10-year, 2.25% semi-annual coupon UK government bond priced at £93.947, with a corresponding yield-to-maturity of 2.9535%, or entering a new 10-year, GBP receive-fixed interest rate swap at 2.8535% versus the six-month GBP MRR currently set at 0.5925%. We compare the results of both strategies over a six-month time horizon for a £100 million par value during which both the bond yield-to-maturity and swap rates fall 50 bps. We ignore day count details in the calculation.

Position	Income	Price Appreciation/MTM	Gain in 6 Months
10y UK bond	£1,125,000	£4,337,779	£5,462,778
10y GBP swap	£1,130,500	£4,234,260	£5,364,760

The relevant return components from Equation 1 are income, namely coupon income for the bond versus "carry" for the swap, and E (Δ Price due to investor's view of benchmark yield) in the form of price appreciation for the bond versus an MTM gain for the swap:

10-Year UK Government Bond:

Coupon income = £1,125,000, or (2.25%/2) × £100 million.

Price appreciation = £4,337,779. Using Excel, this is the difference between the 10-year, or [PV (0.029535/2, 20, 1.125, 100)], and the 9.5-year bond at the lower yield-to-maturity, or [PV (0.024535/2, 19, 1.125, 100)] × £1 million.

We can separate bond price appreciation into two components:

Rolldown return: The difference between the 10-year and 9.5-year PV with *no* change in yield-to-maturity of £262,363, or [PV (0.029535/2, 20, 1.125, 100)] – [PV (0.029535/2, 19, 1.125, 100)] × £1 million].

E **(Δ Price due to investor's view of benchmark yield):** The difference in price for a 50 bp shift of the 9.5-year bond of £4,075,415, or [PV (0.024535, 19, 1.125, 100)] – [PV (0.029535, 19, 1.125, 100)] × £1 million.

10-Year GBP Swap:

Swap carry = £1,130,500, or [(2.8535% – 0.5925%)/2] × £100,000,000.

Swap MTM gain = £4,234,260. The swap MTM gain equals the difference between the fixed leg and floating leg, which is currently at par. The fixed leg equals the 9.5-year swap value given a 50 bp shift in the fixed swap rate, which is £104,234,260, or

[PV(0.023535/2, 19, 2.8535/2, 100)] × £1 million, and the floating leg is priced at par and therefore equal to £100,000,000.

We can use Equation 7 to derive an approximate swap MTM change of £4,159,000 by multiplying swap BPV (8.318 × £100 million) by 50 bps. As in the case of a bond future, the cash outlay for the swap is limited to required collateral or margin for the transaction as opposed to the bond's full cash price. Note that for the purposes of this example, we have ignored any interest on the difference between the bond investment and the cash outlay for the swap.

While these strategies are designed to gain from a static or stable interest rate term structure, we now turn to portfolio positioning in a changing yield curve environment.

EXAMPLE 4 Static Yield Curve Strategies under Curve Inversion

An investment manager who pursues the cash-based yield curve strategies described in Exhibit 5 faces an inverted yield curve (with a decline in long-term yields-to-maturity and a sharp increase in short-term yields-to-maturity) instead. Which of the following is the *least* likely portfolio outcome under this scenario?

a. The manager realizes a loss on a "buy-and-hold" position that extends duration beyond that of the index.
b. The manager faces negative carry when financing a bond purchase in the repo market.
c. The manager is able to reinvest coupon income from a yield curve rolldown strategy at a higher short-term yield-to-maturity.

Solution: The correct answer is a. The fall in long-term yields-to-maturity will lead to price *appreciation* under the "buy-and-hold" strategy. The difference between long-term and short-term yields-to-maturity in b will fall, leading to negative carry if short-term yields-to-maturity rise sharply. As for c, higher short-term yields-to-maturity will enable the manager to reinvest bond coupon payments at a higher rate.

3.2. Dynamic Yield Curve

Exhibits 1 through 3 show that yield curves are dynamic over time, with significant changes in the level, slope, and curvature of rates across maturities. Unless otherwise specified, the sole focus here is on instantaneous yield-to-maturity changes affecting E(Δ Price due to investor's view of benchmark yields), the third component of Equation 1.

3.2.1. Divergent Rate Level View

The principal components analysis cited earlier underscores that rate level changes are the key driver of changes in single bond or bond portfolio values. The first term in Equation 3 shows that bond value changes result from yield-to-maturity changes multiplied by a duration statistic. For active fixed-income managers with a divergent rate level view, positioning the portfolio to increase profit as yield levels fall or minimizing losses as yield levels rise is of primary importance. To be clear, a divergent rate level view implies an expectation of a *parallel* shift in the yield curve, as shown in Exhibit 10.

EXHIBIT 10 Yield Level Changes

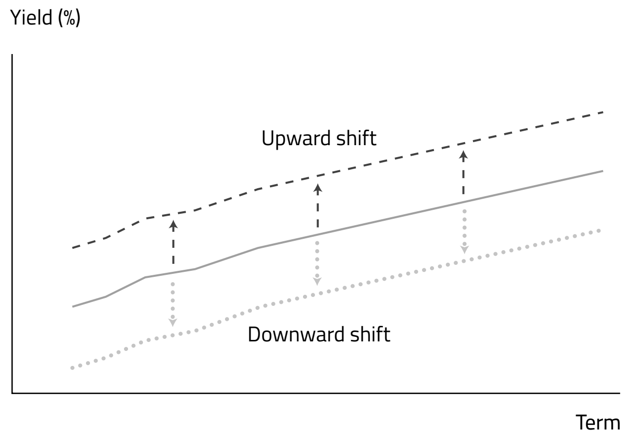


Exhibit 1 shows a general decline in bond yield levels, referred to as a bull market, since 2007. This trend began in late 1981 when the 10-year US Treasury yield-to-maturity peaked at nearly 16%, a consequence of contractionary US Federal Reserve monetary policy in which the short-term federal funds rate was raised to 20% to combat double-digit inflation. Extending duration beyond a target index over this period was a winning active strategy, despite occasional periods of yield increases. Exhibit 11 summarizes the major strategies an active manager might pursue if she expects lower yield levels and downside risks.

EXHIBIT 11 Major Yield Curve Strategies to Increase Portfolio Duration

Strategy	Description	Expected Excess Return	Downside Risks
Cash bond purchase ("bullet")	Extend duration with longer-dated bonds	Price appreciation as yield-to-maturity declines	Higher yield levels
Receive-fixed swap	Fixed-rate receiver on an interest rate swap	Swap MTM gain plus "carry" (fixed minus floating rate)	Higher swap yield levels and/or higher floating rates
Long futures position	Purchase contract for forward bond delivery	Futures MTM gain – Margin cost	Higher bond yields and/or higher margin cost

Assume the "index" portfolio equally weights the 2-, 5-, and 10-year Treasuries priced at par from Example 1, while a higher duration "active" portfolio is weighted 25% for 2- and

5-year Treasuries, respectively, and 50% in 10-year Treasuries. Average portfolio statistics are summarized here:

Portfolio	Coupon	Modified Duration	Convexity
Index	1.042%	5.299	40.8
Active (25/25/50)	1.281%	6.230	53.3

We can see from this table that the active portfolio has a blended coupon nearly 24 bps above that of the index.

We now turn to the impact of a parallel yield curve shift on the index versus active portfolios. Assuming an instantaneous 30 bp downward shift in yields-to-maturity, the index portfolio value would rise by approximately 1.608%, or $(-5.299 \times -0.003) + 0.5 \times (40.8) \times (-0.003^2)$, versus an estimated 1.893% increase for the actively managed portfolio, a positive difference of nearly $285,000 for a $100 million portfolio.

EXAMPLE 5 Portfolio Impact of Higher Yield-to-Maturity Levels

Consider a $50 million Treasury portfolio equally weighted between 2-, 5-, and 10-year Treasuries using parameters from the prior example as the index, and an active portfolio with 20% each in 2- and 5-year Treasuries and the remaining 60% invested in 10-year Treasuries. Which of the following is closest to the active versus index portfolio value change due to a 40 bp rise in yields-to-maturity?

a. Active portfolio declines by $181,197 more than the index portfolio
b. Active portfolio declines by $289,915 more than the index portfolio
c. Index portfolio declines by $289,915 more than the active portfolio

Solution: The correct answer is b. First, we must establish average portfolio statistics for the 20/20/60 portfolio using a weighted average of duration (6.79 versus 5.299 for the index) and convexity (60.8 versus 40.8 for the index). Second, using these portfolio statistics, we must calculate $\%\Delta PV^{Full}$, as shown in Equation 3, for both the index and active portfolios, which are −2.087% for the index and −2.667% for the active portfolio, respectively. Finally, we multiply the difference of −0.58% by the $50 million notional to get −$289,915.

Receive-fixed swaps or long futures positions may be used in place of a cash bond strategy to take an active view on rates. Note that most fixed-income managers will tend to favor option-free over callable bonds if taking a divergent rate level view due to the greater liquidity of option-free bonds. An exception to this arises when investors formulate portfolio positioning strategies based upon expected changes in interest rate volatility, as we will discuss in detail later in this lesson.

As 2020 began, some analysts expected government yields-to-maturity to eventually rise following over a decade of quantitative easing after the 2008 global financial crisis. However, yields instead reached new lows during 2020 when the COVID-19 pandemic caused a sharp economic slowdown, prompting additional monetary and fiscal policy stimulus. If analysts expected a strong economic rebound to increase yield levels, they might seek to lessen the adverse impact of higher rate levels by reducing duration. Exhibit 12 outlines major strategies to achieve this goal.

EXHIBIT 12 Major Yield Curve Strategies to Reduce Portfolio Duration

Strategy	Description	Expected Excess Return	Downside Risks
Cash bond sale ("bullet")	Reduce duration with short sale/switch to shorter-dated bonds	Smaller price decline as yield-to-maturity increases	Lower yield levels
Pay-fixed (interest rate swap)	Fixed-rate payer on an interest rate swap	Swap MTM gain plus "swap carry" (MRR – Fixed swap rate)	Swap MTM loss amid lower swap yield levels and/or lower floating rates
Short futures position	Sell contract for forward bond delivery	Futures MTM gain – Margin cost	Futures MTM loss amid lower bond yields and/or higher margin cost

Returning to our "index" portfolio of equally weighted 2-, 5-, and 10-year Treasuries, we now consider an active portfolio positioned to reduce downside exposure to higher yields-to-maturity versus the index. In order to limit changes to the bond portfolio, the manager chooses a swap strategy instead.

EXAMPLE 6 Five-Year Pay-Fixed Swap Overlay

In this example, the manager enters into a pay-fixed swap overlay with a notional principal equal to one-half of the size of the total bond portfolio. We will focus solely on first-order effects of yield changes on price (ignoring coupon income and swap carry) to determine the active and index portfolio impact. As the pay-fixed swap is a "short" duration position, it is a negative contribution to portfolio duration and therefore subtracted from rather than added to the portfolio. Recall the $100 million "index" portfolio has a modified duration of 5.299, or (1.994 + 4.88 + 9.023)/3. If the manager enters a $50 million notional 5-year pay-fixed swap with an assumed modified duration of 4.32, the portfolio's modified duration falls to 3.139, or [(5.299 × 100) – (4.32 × 50)]/100. Stated differently, the bond portfolio BPV falls from $52,990 to $31,390 with the swap. For a 25 bp yield increase, this $21,600 reduction in active portfolio BPV reduces the adverse impact of higher rates by approximately $540,000 versus the "index" portfolio.

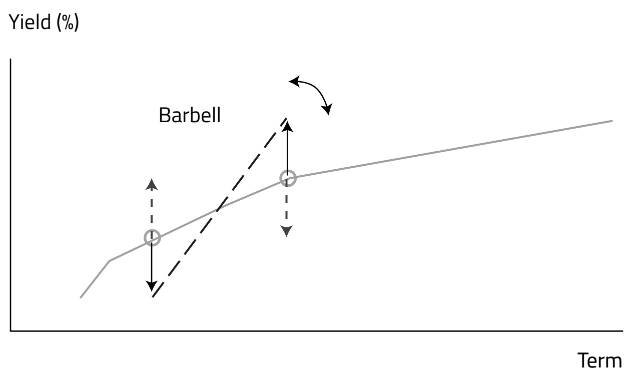
One point worth noting related to short duration positions is that with the exception of distressed debt situations addressed later in the curriculum, the uncertain cost and availability of individual bonds to borrow and sell short leads many active managers to favor the use of derivatives over short sales to establish a short bond position. Derivatives also facilitate duration changes without interfering with other active bond strategies within a portfolio.

Portfolio managers frequently use average duration and yield level changes to estimate bond portfolio performance in broad terms. However, these approximations are only reasonable if we assume a parallel yield curve shift. As Exhibits 2 and 3 show, non-parallel changes, or shifts in the slope and/or shape of the yield curve, occur frequently and require closer examination of individual positions and rate changes across maturities.

3.2.2. Divergent Yield Curve Slope View

Exhibit 2 established that while a positively sloped yield curve prevails under most economic scenarios, this difference between long-term and short-term yields-to-maturity can vary significantly over time. Changes in monetary policy, as well as expectations for growth and inflation, affect yields differently across the term structure, resulting in an increase (steepening) or decrease (flattening) in this spread. Although the **barbell** strategy combining extreme maturities is often referred to in a long-only context as in Example 1, here we take a more generalized approach in which the short-term and long-term security positions within the barbell trade may move in opposite directions—that is, combining a "short" and a "long" position. This type of barbell is an effective tool employed by managers to position a bond portfolio for yield curve steepening or flattening changes, as shown in Exhibit 13.

EXHIBIT 13 Barbell Strategy for a Yield Curve Slope Change



A manager could certainly use a bullet to increase or decrease exposure to a specific maturity in anticipation of a price change that changes yield curve slope, but a *combination* of positions in both short and long maturities with greater cash flow dispersion is particularly well-suited to position for yield curve slope changes or twists. Managers combine long or short positions in either maturity segment to take advantage of expected yield curve slope changes—which may be duration neutral, net long, or short duration depending upon *how* the curve is expected to steepen or flatten in the future. Also, in some instances, the investment policy statement may allow managers to use bonds, swaps, and/or futures to achieve this objective. Finally, while not all strategies shown are cash neutral, here we focus solely on portfolio value changes due to yield changes, ignoring any associated funding or other costs that might arise as a result.

Yield curve steepener strategies seek to gain from an increase in yield curve slope, or a greater difference between long-term and short-term yields-to-maturity. This may be achieved by combining a "long" shorter-dated bond position with a "short" longer-dated bond position.

For example, assume an active manager seeks to benefit from yield curve steepening with a net zero duration by purchasing the 2-year Treasury and selling the 10-year Treasury securities from our earlier example, both of which are priced at par.

Tenor	Coupon	Position ($ MM)	Modified Duration	Convexity
Long 2y	0.25%	163.8	1.994	5.0
Short 10y	2.00%	–36.2	9.023	90.8

Note that here and throughout the lesson, negative portfolio positions reflect a "short" position. We can approximate the impact of *parallel* yield curve changes using portfolio duration and convexity. Portfolio duration is approximately zero, or [1.994 × 163.8/(163.8 – 36.2)] + [9.023 × –36.2/(163.8 – 36.2)], and portfolio convexity equals –19.34, or [5.0 × 163.8/(163.8 – 36.2)] + [9.023 × –36.2/(163.8 – 36.2). A 25 bp increase in *both* 2-year and 10-year Treasury yields-to-maturity therefore has no duration effect on the portfolio, although negative convexity leads to a 0.006%, or $7,712 decline in portfolio value, or $127,600,000 $\times 0.5 \times -19.34 \times 0.0025^2$.

However, changes in the *difference* between short- and long-term yields-to-maturity are not captured by portfolio duration or convexity but rather require assessment of individual positions. For example, if yield curve *slope* increases from 175 bps to 225 bps due to a 25 bp *decline* in 2-year yields-to-maturity and a 25 bp *rise* in 10-year yields-to-maturity, the portfolio increases in value by $1,625,412 as follows:

2y: $\$819{,}102 = \$163{,}800{,}000 \times (-1.994 \times -0.0025 + 0.5 \times 5.0 \times -0.0025^2)$

10y: $\$806{,}310 = -\$36{,}200{,}000 \times (-9.023 \times 0.0025 + 0.5 \times 90.8 \times 0.0025^2)$

EXAMPLE 7 Barbell Performance under a Flattening Yield Curve

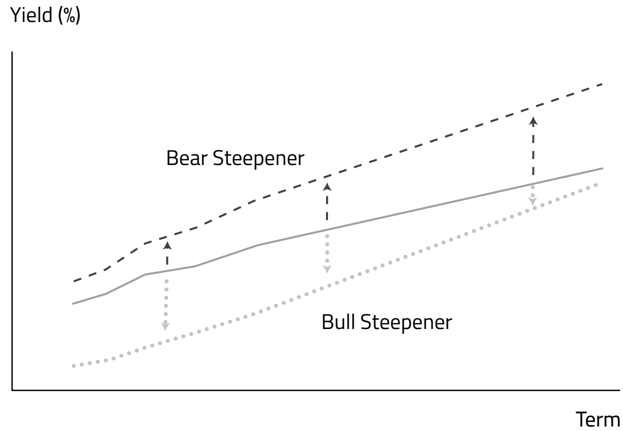
Consider a Treasury portfolio consisting of a $124.6 million long 2-year zero-coupon Treasury with an annualized 2% yield-to-maturity and a short $25.41 million 10-year zero-coupon bond with a 4% yield-to-maturity. Calculate the net portfolio duration and solve for the first-order change in portfolio value based upon modified duration assuming a 25 bp rise in 2-year yield-to-maturity and a 30 bp decline in 10-year yield-to-maturity.

First, recall from earlier in the curriculum that Macaulay duration (MacDur) is equal to maturity for zero-coupon bonds and modified duration (ModDur) is equal to MacDur/1+*r*, where *r* is the yield per period. We can therefore solve for the modified duration of the 2-year zero as 1.96 (= 2/1.02) and the 10-year zero as 9.62 (= 10/1.04), so net portfolio duration equals zero, or (124.6/150 × 1.96) + (25.4/150 × –9.62).

We may show that the 2-year Treasury BPV is close to $24,430 (= 1.96 × 124,600,000/10,000) and the 10-year Treasury position BPV is also approximately $24,430 (= 9.61 × 25,410,000/10,000), but it is a short position. Therefore a 25 bp *increase* in 2-year yield-to-maturity *decreases* portfolio value by $610,750 (25 bps × $24,430), while a 30 bp *decrease* in the 10-year yield-to-maturity also *decreases* portfolio value (due to the short position) by an additional $732,900 (= 30 bps × $24,430), for a total approximate portfolio *loss* of $1,343,650.

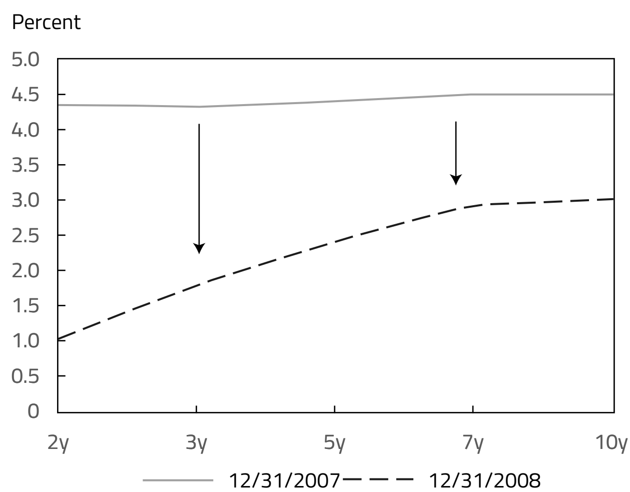
The portfolio manager is indifferent as to whether the portfolio gain from a greater slope arises due to a greater change in value from short-term or long-term yield movements as the duration is matched between the two positions. Two variations of a steeper yield curve adapted from Smith (2014) are shown in Exhibit 14.

EXHIBIT 14 Yield Curve Slope Changes—Steepening



In an earlier lesson on establishing a rate view, we highlighted a **bull steepening** scenario under which short-term yields-to-maturity fall by more than long-term yields-to-maturity if the monetary authority cuts benchmark rates to stimulate economic activity during a recession. Exhibit 15 shows the bull steepening that occurred in the UK gilt yield curve amid the 2008 global financial crisis. After reaching a cycle peak of 5.75% in July 2007, the Bank of England cut its monetary policy base rate six times, down to 2.00% in early December 2008, due to weakening economic conditions and financial market stress.

EXHIBIT 15 UK Government Yields, 2007 versus 2008 (Year End)



Source: Bloomberg.

On the other hand, a **bear steepening** occurs when long-term yields-to-maturity rise more than short-term yields-to-maturity. This could result from a jump in long-term rates amid higher growth and inflation expectations while short-term rates remain unchanged. In this case, an analyst might expect the next central bank policy change to be a monetary tightening to curb inflation.

Bull or bear steepening expectations will change the strategy an active fixed-income manager might pursue, as seen in Exhibit 16.

EXHIBIT 16 Yield Curve Steepener Strategies

Strategy	Description	Expected Excess Return	Downside Risks
Duration neutral	Net zero duration	Portfolio gain from yield curve slope increase	Yield curve flattening
Bear steepener	Net negative ("short") duration	Portfolio gain from slope increase and/or rising yields	Yield curve flattening and/or lower yields
Bull steepener	Net positive ("long") duration	Portfolio gain from slope increase and/or lower yields	Yield curve flattening and/or higher yields

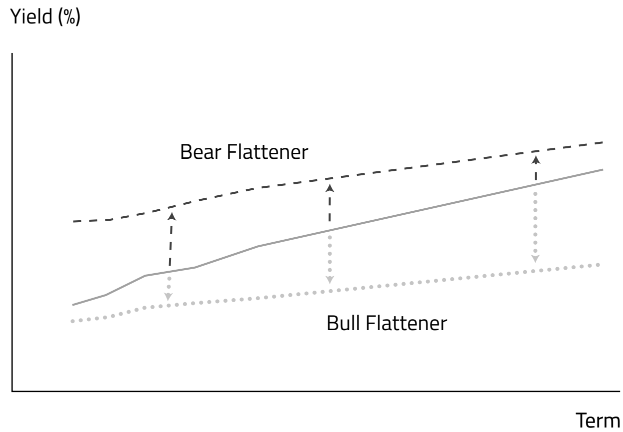
For example, assume an active manager expects the next yield curve change to be a bull steepening and establishes the following portfolio using the same 2-year and 10-year Treasury securities as in our prior examples.

Tenor	Coupon	Position ($ MM)	Modified Duration	Convexity
Long 2y	0.25%	213.8	1.994	5.0
Short 10y	2.00%	–36.2	9.023	90.8

In contrast to the earlier duration-matched steepener, the bull steepener increases the 2-year long Treasury position by $50 million, introducing a net long duration position to capitalize on an anticipated greater decline in short-term yields-to-maturity. We can see this by solving for portfolio duration of 0.5613, or $[1.994 \times 213.8/(213.8 - 36.2)] + [9.023 \times -36.2/(213.8 - 36.2)]$, which is equivalent to a portfolio BPV of approximately $9,969, or $0.5613 \times [(\$213{,}800{,}000 - \$36{,}200{,}000)/10{,}000]$. We may use this portfolio BPV to estimate the approximate portfolio gain if the 2-year yield-to-maturity falls by 25 bps more than the 10-year yield-to-maturity rises, which is equal to $249,225 (= 25 bps × $9,969).

Yield curve flattening involves an anticipated narrowing of the difference between long-term and short-term yields-to-maturity, two basic variations of which are shown in Exhibit 17 and are adapted from Smith (2014).

EXHIBIT 17 Yield Curve Slope Changes—Flattening



A flatter yield curve may follow monetary policy actions due to changing growth and inflation expectations. For example, a **bear flattening** scenario might follow the bear steepening move seen in Exhibit 15 if policymakers respond to rising inflation expectations and higher long-term rates by raising short-term policy rates. It was established earlier in the curriculum that investors sell higher risk assets and buy default risk-free government bonds in a flight to quality during highly uncertain markets, a situation which often contributes to **bull flattening** as long-term rates fall more than short-term rates. Flattener strategies may use a barbell strategy, which reverses the exposure profile of a steepener—namely, a "short" short-term bond position and a "long" long-term bond position. The bull and bear variations of this strategy are summarized in Exhibit 18.

EXHIBIT 18 Yield Curve Flattener Strategies

Strategy	Description	Expected Excess Return	Downside Risks
Duration neutral	Net zero duration position	Portfolio gain from yield curve slope decrease	Yield curve steepening
Bear flattener	Net negative duration position	Portfolio gain from slope decrease and/or rising yields	Yield curve steepening and/or lower yields
Bull flattener	Net positive duration position	Portfolio gain from slope decrease and/or lower yields	Yield curve steepening and/or higher yields

Say, for example, a French investor expects the government yield curve to flatten over the next six months following years of quantitative easing by the European Central Bank through 2019. Her lack of a view as to whether this will occur amid lower or higher rates causes her to choose a duration neutral flattener using available French government (OAT) zero-coupon securities. She decides to enter the following trade at the beginning of 2020:

Tenor	Yield	Price	Notional (€ MM)	Modified Duration	Position BPV	Convexity
Short 2y	–0.65%	€101.313	–83.24	2.013	(€16,975)	6.1
Long 10y	0.04%	€99.601	17.05	9.996	€16,977	110

Note that as the Excel PRICE function returns a #NUM! error value for bonds with negative yields-to-maturity, we calculate the 2-year OAT zero-coupon bond price of 101.313 using $100/(1 - 0.0065)^2$. The initial portfolio BPV close to zero tells us that parallel yield curve shifts will have little effect on portfolio value, while the short 2-year and long 10-year trades position the manager to profit from a decline in the current 69 bp spread between 2- and 10-year OAT yields-to-maturity. After six months, the portfolio looks as follows:

Tenor	Yield	Price	Notional (€ MM)	Modified Duration	Convexity
Short 1.5y	–0.63%	€100.95	–83.24	1.51	3.8
Long 9.5y	–0.20%	€101.92	17.05	9.52	100.2

At the end of six months (June 2020), the sharp decline in economic growth and inflation expectations due to the COVID-19 pandemic caused the OAT yield curve to flatten as the 10-year yield-to-maturity fell. The six-month barbell return of €695,332 is comprised of rolldown return and yield changes, calculated as follows:

Rolldown Return Zero-coupon bonds usually accrete in value as time passes if rates remain constant and the yield-to-maturity is positive. However, under negative yields-to-maturity, amortization of the bond's premium will typically result in a rolldown *loss*. In our example, the investor is short the original 2-year zero and therefore realizes a rolldown *gain* on the short position. Rolldown return on the barbell may be shown to be approximately €277,924, as follows:

"Short" 2-year: $-€83.24\text{ MM} \times [1/(1 + -0.65\%)^2] - [1/(1 + -0.65\%)^{1.5}]$

"Long" 10-year: $€17.05\text{ MM} \times [1/(1 + 0.04\%)^{10}] - [1/(1 + 0.04\%)^{9.5}]$

Δ *Price Due to Benchmark Yield Changes* The yield difference falls from 69 bps to 43 bps, mostly due to a 24 bp decline in the 10-year yield-to-maturity. Note that the Excel DURATION and MDURATION functions also return a #NUM! error for negative yields-to-maturity. We may use either price changes, as shown next, or the modified duration and convexity statistics as of the end of the investment horizon, just shown, to calculate a return of €417,408 using Equation 3.

"Short" 2-year: $-€83.24\text{ MM} \times [1/(1 + -0.65\%)^{1.5}] - [1/(1 + -0.63\%)^{1.5}]$

"Long" 10-year: $€17.05\text{ MM} \times [1/(1 + 0.04\%)^{9.5}] - 1/(1 + -0.20\%)^{9.5}]$

As we have considered duration-neutral, long, and short duration strategies to position the portfolio for expected yield curve slope changes, average duration is clearly no longer a sufficient summary statistic. A barbell strategy has greater cash flow dispersion and is therefore more convex than a bullet strategy, implying that its value will decrease by less than a bullet if yields-to-maturity rise and increase by more than a bullet if yields-to-maturity fall. We therefore must consider portfolio convexity in addition to duration when weighing yield curve slope strategies under different scenarios.

3.2.3. Divergent Yield Curve Shape View

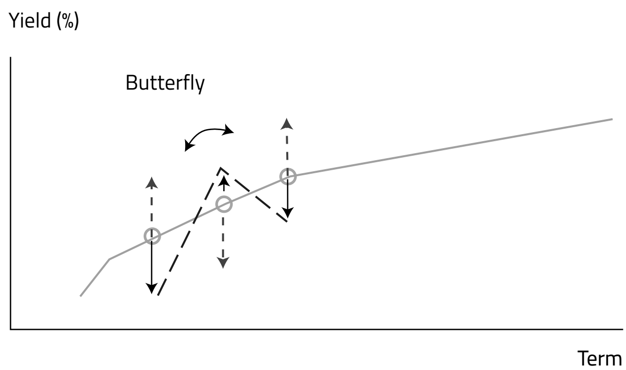
As described in Section 2.1, yield curve shape or curvature describes the relationship between short-, medium-, and long-term yields-to-maturity across the term structure. Recall from

Equation 2 that we quantify the butterfly spread by subtracting both short- and long-term rates from twice the intermediate yield-to-maturity. Since the difference between short- and medium-term rates is typically greater than that between medium- and long-term rates, the butterfly spread is usually positive, as seen earlier in Exhibit 3.

What factors drive yield curve curvature changes as distinct from rate level or curve slope changes? The segmented markets hypothesis introduced previously offers one explanation: Different market participants face either regulatory or economic asset/liability management constraints that drive the supply and demand for fixed-income instruments within different segments of the term structure. For example, a potential factor driving the apparent butterfly spread volatility in Exhibit 3 is the active central bank purchases of Treasury securities at specific maturities under its quantitative easing policy.

The most common yield curve curvature strategy combines a long bullet with a short barbell portfolio (or vice versa) in what is referred to as a **butterfly strategy** to capitalize on expected yield curve shape changes. The short-term and long-term bond positions of the barbell form the "wings," while the intermediate-term bullet bond position forms the "body" of the butterfly, as illustrated in Exhibit 19. Note that unlike the steepener and flattener cases, the investor is either "long" or "short" *both* a short-term and long-term bond and enters into an intermediate-term bullet trade in the opposite direction.

EXHIBIT 19 Butterfly Strategy



For example, consider a situation in which an active manager expects the butterfly spread to rise due to lower 2- and 10-year yields-to-maturity and a higher 5-year Treasury yield-to-maturity. Using the same portfolio statistics as in prior examples with bonds priced at par, consider the following combined *short* (5-year) bullet and *long* (2-year and 10-year) barbell strategy.

Tenor	Coupon	Position ($ MM)	Modified Duration	Position BPV	Convexity
Long 2y	0.25%	110	1.994	$21,934	5.0
Short 5y	0.875%	–248.3	4.88	($121,170)	26.5
Long 10y	2.00%	110	9.023	$99,253	90.8

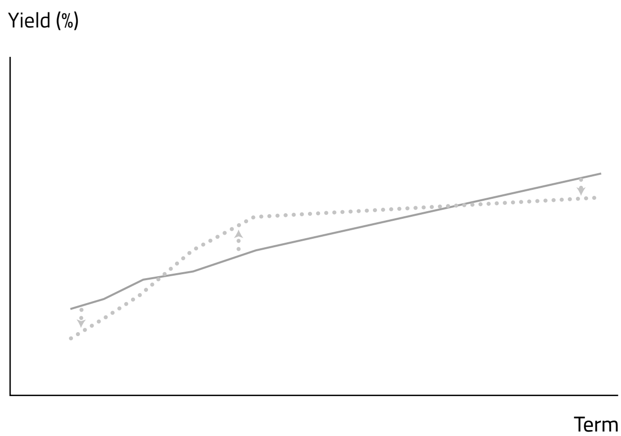
While the sum of portfolio positions (−$28.3 MM) shows that the investor has a net "short" bond position, we can verify the strategy is duration neutral by either adding up the position BPVs or calculating the portfolio duration, or [1.994 × (110/−28.3)] + [4.88 × (−248.3/−28.3)] + [9.023 × (110/−28.3)] to confirm that both are approximately zero. The portfolio convexity may be shown as −139.9, or [5.0 × (110/−28.3)] + [26.5 × (−248.3/−28.3)] + [90.8 × (110/−28.3)].

How does this portfolio perform if 2- and 10-year Treasury yields-to-maturity fall by 25 bps each and the 5-year yield-to-maturity rises by 50 bps? A duration-based estimate multiplying each position BPV by the respective yield change gives us an approximation of $9,089,075, or (+25 bps × $21,934(+ −(50 bps × −$121,188) + (+25 bps × $99,253). A more precise answer of $9,038,877 incorporating convexity for each position may be derived using Equation 3. You might ask why the precise portfolio value change is below our approximation. The answer lies in the relative *magnitude* of yield changes across the curve. Since the 5-year yield-to-maturity is assumed to increase by 50 bps rather than 25 bps, the convexity impact of the short bullet position outweighs that of the long barbell. Although the portfolio is nearly immune to parallel yield curve changes with a BPV close to zero, the portfolio gain in our example coincides with an increase in the butterfly spread from −50 bps to +100 bps.

This example shows that an active manager's specific view on *how* yield curve shape will change will dictate the details of the combined bullet and barbell strategy. Exhibit 20, adapted from Smith (2014), shows both the **negative butterfly** view just shown as well as a **positive butterfly**, which indicates a *decrease* in the butterfly spread due to an expected rise in short- and long-term yields-to-maturity combined with a lower medium-term yield-to-maturity. Note that a positive butterfly view indicates a decrease in butterfly spread due to a bond's inverse price–yield relationship.

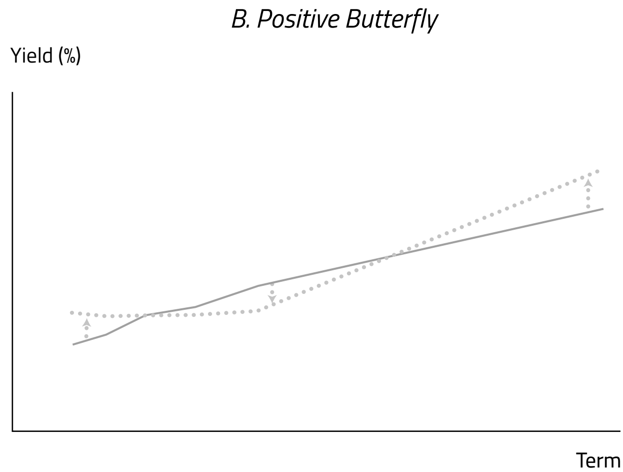
EXHIBIT 20 Yield Curve Curvature Changes

A. Negative Butterfly



(*continued*)

EXHIBIT 20 (Continued)



Note that as in the case of yield curve slope strategies, the *combination* of a short bullet and long barbell increases portfolio convexity due to higher cash flow dispersion, making this a more meaningful portfolio risk measure for this strategy than average duration (which remains neutral in the Exhibit 20 example). Exhibit 21 summarizes the two butterfly strategies.

EXHIBIT 21 Yield Curve Curvature Strategies

Expected Scenario	Investor's Expectation	Active Position
Negative butterfly	Lower short- and long-term yields, Higher medium-term yields	Short bullet, Long barbell (long positions in short- and long-term bonds)
Positive butterfly	Higher short- and long-term yields, Lower medium-term yields	Long bullet, Short barbell (short positions in short- and long-term bonds)

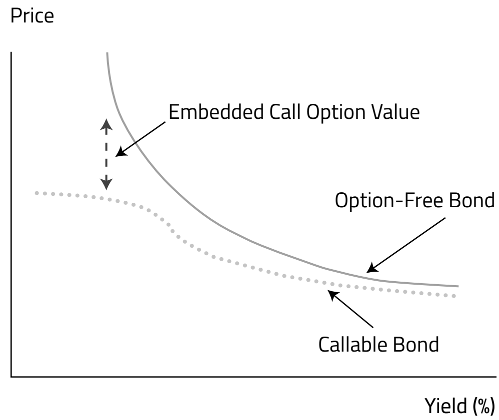
3.2.4. Yield Curve Volatility Strategies

While the prior sections focused on strategies using option-free bonds and swaps and futures as opposed to bonds with embedded options and stand-alone option strategies, we now explicitly address the role of volatility in active fixed-income management.

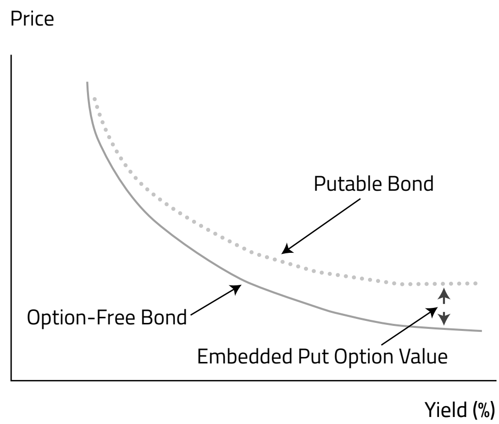
Option-only strategies play a more modest role in overall yield curve management. In markets such as in the United States where a significant portion of outstanding fixed-income bonds, such as asset-backed securities, have embedded options, investors use cash bond positions with embedded options more frequently than stand-alone options to manage volatility. For example, as of 2019 approximately 30% of the Bloomberg Barclays US Aggregate Bond Index was comprised of securitized debt, which mostly includes bonds with embedded options. As outlined earlier, the purchase of a bond call (put) option offers an investor the right, but not the obligation, to buy (sell) an underlying bond at a pre-determined strike price. An active manager's choice between purchasing or selling bonds with embedded call or put options versus an option-free bond with otherwise similar characteristics hinges upon expected changes in the option value and whether the investor is "short" volatility (i.e., has sold the right to call a bond at a fixed price to the issuer), as in the case of callable bonds, or "long" volatility (i.e., owns the right to sell the bond at a fixed price to the issuer), as for putable bonds. Exhibit 22 shows how callable and putable bond prices change versus option-free bonds as yields-to-maturity change.

EXHIBIT 22 Callable and Putable versus Option-Free Bonds

A. Callable Bond



B. Putable Bond



EXAMPLE 8 Option-Free Bonds versus Callable/Putable Bonds

An investment manager is considering an incremental position in a callable, putable, or option-free bond with otherwise comparable characteristics. If she expects a downward parallel shift in the yield curve, it would be most profitable to be:

a. long a callable bond.
b. short a putable bond.
c. long an option-free bond.

Solution: "C" is correct. The value of a bond with an embedded option is equal to the sum of the value of an option-free bond plus the value to the embedded option. The bond investor can be either long or short the embedded option, depending on the type of bond. With a callable bond, the embedded call option is owned by the issuer of the bond, who can exercise this option if yields-to-maturity decrease (the bond investor is short the call option). With a putable bond, the embedded put option is owned by the bond investor, who can exercise the option if yields-to-maturity increase. For a decrease in yields-to-maturity—as given in the question—the value of the embedded call option increases and the value of the embedded put option decreases. This means that a long position in a callable bond ("A") would underperform compared to a long position in an option-free bond. A short position in a putable bond ("B") would underperform a long position in an option-free bond primarily because yields-to-maturity were declining, although the declining value of the embedded put option would mitigate some of the loss (the seller of the putable bond has "sold" the embedded put).

As mentioned earlier in the curriculum, effective duration and convexity are the relevant summary statistics when future bond cash flows are contingent upon interest rate changes.

$$\text{Effective Duration(EffDur)} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\,\text{Curve})(PV_0)} \tag{8}$$

$$\text{Effective Convexity(EffCon)} = \frac{(PV_{-}) + (PV_{+}) - 2(PV_0)}{(\Delta\,\text{Curve})^2 \times (PV_0)} \tag{9}$$

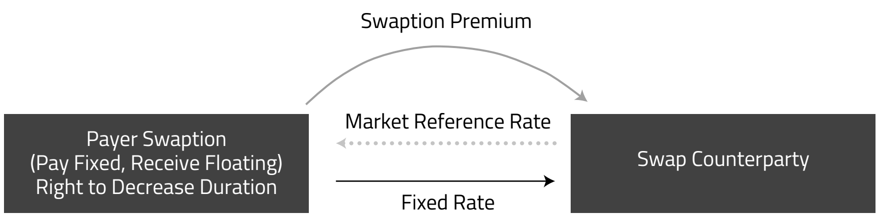
In Equations 8 and 9, PV− and PV+ are the portfolio value changes from a decrease and increase in yield-to-maturity, respectively, PV_0 is the original portfolio value, and ΔCurve is the change in the benchmark yield-to-maturity.

Although cash-based yield curve volatility strategies are limited to the availability of liquid callable or putable bonds, several stand-alone derivatives strategies involve the right, but not the obligation, to change portfolio duration and convexity based upon an interest rate-sensitive payoff profile.

Interest rate put and call options are generally based upon a bond's price, not yield-to-maturity. Therefore, the purchase of a bond call option provides an investor the right, but not the obligation, to acquire an underlying bond at a pre-determined strike price. This purchased call option adds convexity to the portfolio and will be exercised if the bond price appreciates beyond the strike price (i.e., generally at a lower yield-to-maturity). On the other hand, a purchased bond put option benefits the owner if prices fall (i.e., yields-to-maturity rise) beyond the strike prior to expiration. Sale of a bond put (call) option limits an investor's return to the up-front premium received in exchange for assuming the potential cost of exercise if bond prices fall below (rise above) the pre-determined strike. Note that the option seller must post margin based on exchange or counterparty requirements until expiration.

An interest rate **swaption** involves the right to enter into an interest rate swap at a specific strike price in the future. This instrument grants the contingent right to increase or decrease portfolio duration. For example, Exhibit 23 shows a purchased payer swaption, which a manager might purchase to benefit from higher rates using an option-based strategy.

EXHIBIT 23 Purchased Payer Swaption



Options on bond futures contracts are liquid exchange-traded instruments frequently used by fixed-income market participants to buy or sell the right to enter into a futures position. Long option, swaption, and bond futures option strategies are summarized in Exhibit 24.

EXHIBIT 24 Long Option, Swaption, and Bond Futures Option Strategies

Strategy	Description	Targeted Return	Portfolio Impact
Long bond call option	Purchase right to take forward bond delivery	Max (Bond price at lower yield – Strike price, 0) – Call premium	Increase portfolio duration and convexity (up-front premium)
Long bond put option	Purchase right to deliver bond in the future	Max (Strike price – Bond price at higher yield, 0) – Put premium	Decrease portfolio duration and convexity (up-front premium)
Long payer swaption	Own the right to pay-fixed on an interest rate swap at a strike rate	Max (Strike rate – Swap rate, 0) – Swaption premium	Decrease in portfolio duration and convexity (up-front premium)
Long receiver swaption	Own the right to receive-fixed on an interest rate swap at a strike rate	Max (Swap rate – Strike rate, 0) – Swaption premium	Increase in portfolio duration and convexity (up-front premium)
Long call option on bond future	Own the right to take forward bond delivery at a strike price	Max (Bond futures price at lower yield – Strike price, 0) – Call premium	Increase in portfolio duration and convexity (up-front premium)
Long put option on bond future	Own the right to deliver bond in the future at a strike price	Max (Strike price – Bond futures price at higher yield, 0) – Put premium	Decrease in portfolio duration and convexity (up-front premium)

EXAMPLE 9 Choice of Option Strategy

A parallel upward shift in the yield curve is expected. Which of the following would be the best option strategy?

a. Long a receiver swaption
b. Short a payer swaption
c. Long a put option on a bond futures contract

Solution: C is correct. With an expected upward shift in the yield curve, the portfolio manager would want to reduce portfolio duration in anticipation of lower bond prices. A put option increases in value as the yield curve shifts upward, while the price of the underlying bond declines below the strike. A is incorrect because a receiver swaption is an option to receive-fixed in an interest rate swap. With fixed-rate bond prices expected to fall as rates rise, the portfolio manager would not want to exercise an option to receive a fixed strike rate, which is similar to owning a fixed-rate bond. B is incorrect because a payer swaption is an option to pay-fixed/receive-floating in an interest rate swap. A *long*, not a short, position in a payer swaption would benefit from higher rates.

In an expected stable or static yield curve environment, an active manager may aim to "sell" volatility in the form of either owning callable bonds (which is an implicit "sale" of an option) or selling stand-alone options in order to earn premium income, if this is within the investment mandate. The active portfolio decision here depends upon the manager's view as to whether future realized volatility will be greater or less than the implied volatility, as reflected by the price of a stand-alone option or a bond with embedded options. The manager will benefit if rates remain relatively constant and the bond is not called and/or the options sold expire worthless. Alternatively, if yield curve volatility is expected to increase, a manager may prefer to be long volatility in order to capitalize on large changes in level, yield curve slope, and/or shape using option-based contracts.

EXAMPLE 10 Option-Free versus Callable and Putable Bonds Amid Higher Yield Levels

Given a parallel shift upwards in the yield curve, what is the most likely ordering in terms of expected decline in value—from least to most—for otherwise comparable bonds? Assume that the embedded options are deep out-of-the-money.

a. Callable bond, option-free bond, putable bond
b. Putable bond, callable bond, option-free bond
c. Putable bond, option-free bond, callable bond

Solution: Answer: B is correct. The value of a bond with an embedded option may be considered as the value of an option-free bond plus the value of the embedded option. While the upward shift in the yield curve will cause the option-free component of each bond to depreciate in value, this change in yields-to-maturity will also affect the value of embedded options.

For a putable bond, the bond investor has the option to "put" the bond back to the issuer if yields-to-maturity rise. The more rates rise, the more valuable this embedded option becomes. This increasing option value will partially offset the decline in value of the putable bond relative to the option-free bond. This can be seen in the lower panel of Exhibit 22: The dotted line for the putable bond has a flatter slope than the solid line for the option-free bond; its price will decrease more slowly as yields-to-maturity increase.

For a callable bond, the bond issuer has an option to "call" the bond if yields-to-maturity decline; the more rates rise, the lower the call option value. Since the bond investor is short the embedded option and the value of the embedded option has fallen, this will partially offset the decline in the value of the callable bond relative to the option-free bond. The top panel of Exhibit 22 shows that the dotted line for the callable bond has a flatter slope than the solid line for the option-free bond.

As rates continue to increase, the embedded option for the putable bond rises in value more quickly at the margin as it shifts toward becoming an in-the-money option. In contrast, the deep out-of-the-money embedded call option moves further out-of-the-money as rates increase and the marginal impact of further rate increases declines.

3.3. Key Rate Duration for a Portfolio

So far, we have evaluated changes in yield curve level, slope, and curvature using one, two, and three specific maturity points across the term structure of interest rates, respectively. The concept of **key rate duration** (or partial duration) introduced previously measures portfolio sensitivity over a set of maturities along the yield curve, with the sum of key rate durations being identical to the effective duration:

$$\text{KeyRateDur}_k = -\frac{1}{\text{PV}} \times \frac{\Delta \text{PV}}{\Delta r_k} \tag{10}$$

$$\sum_{k=1}^{n} \text{KeyRateDur}_k = \text{EffDur} \tag{11}$$

where r_k represents the kth key rate. In contrast to effective duration, key rate durations help identify "shaping risk" for a bond—that is, a bond's sensitivity to changes in the shape of the benchmark yield curve. By breaking down a portfolio into its individual duration components by maturity, an active manager can pinpoint and quantify key exposures along the curve, as illustrated in the following simplified zero-coupon bond example.

Compare a passive zero-coupon US Treasury bond portfolio versus an actively managed portfolio:

"Index" Zero-Coupon Portfolio

Tenor	Coupon	Annualized Yield	Price (per $100)	Position ($ MM)	ModDur	KeyRateDur
2y	0.00%	1%	98.03	98.03	1.980	0.738
5y	0.00%	2%	90.57	90.57	4.902	1.688
10y	0.00%	3%	74.40	74.40	9.709	2.747

Assume the "index" portfolio is simply weighted by the price of the respective 2-, 5-, and 10-year bonds for a total portfolio value of $263 million, or $1 million × (98.03 + 90.57 + 74.4). We can calculate the portfolio modified duration as 5.173, or [1.98 × (98.03/263)] + [4.902 × (90.57/263)] + [9.709 × (74.40/263)]. Or, we could calculate each key rate duration by maturity, as in the far right column. For example, the 2-year key rate duration (KeyRate-Dur_2) equals 0.738, or 1.98 × (98.03/263). Note that these three key rate duration values also sum to the portfolio value of 5.173.

"Active" Zero-Coupon Portfolio

Tenor	Coupon	Annualized Yield	Price (per $100)	Position ($ MM)	ModDur	KeyRateDur
2y	0.00%	1%	98.03	51.40	1.980	0.387
5y	0.00%	2%	90.57	–46.00	4.902	–0.857
10y	0.00%	3%	74.40	257.60	9.709	9.509

As in the case of the "index" portfolio, the "active" zero-coupon portfolio has a value of $263 million, or [$1 million × (51.4 – 46 + 257.6)], but the portfolio duration is greater at 9.039, or [1.98 × (51.4/263)] + [4.902 × (–46/263)] + [9.709 × (257.6/263)]. Note that the short 5-year active position has a negative key rate duration of –0.857, or 4.902 × (–46/257.6).

By now, you may have noticed that our active manager is positioned for the combination of a negative butterfly and a bull flattening at the long end of the yield curve. However, a comparison of the active versus index portfolio duration summary statistic does not tell the entire story. Instead, we can compare the key rate or partial durations for specific maturities across the index and active portfolios to better understand exposure differences:

Tenor	Active	Index	Difference
2y	0.39	0.74	–0.35
5y	–0.86	1.69	–2.55
10y	9.51	2.75	6.76
Portfolio	9.04	5.17	–3.87

The key rate duration differences in this chart provide more detailed information regarding the exposure differences across maturities. For example, the negative differences for 2-year and 5-year maturities (–0.35 and –2.55, respectively) indicate that the active portfolio has lower exposure to short-term rates than the index portfolio. The large positive difference in the 10-year tenor shows that the active portfolio has far greater exposure to 10-year yield-to-maturity changes. This simple zero-coupon bond example may be extended to portfolios consisting of fixed-coupon bonds, swaps, and other rate-sensitive instruments that may be included in a fixed-income portfolio, as seen in the following example.

EXAMPLE 11 Key Rate Duration

A fixed-income manager is presented with the following key rate duration summary of his actively managed bond portfolio versus an equally weighted index portfolio across 5-, 10-, and 30-year maturities:

Tenor	Active	Index	Difference
5y	–1.188	1.633	–2.821
10y	2.909	3.200	–0.291
30y	11	8.067	2.933
Portfolio	12.72	12.9	–0.179

Assume the active manager has invested in the index bond portfolio and used only derivatives to create the active portfolio. Which of the following most likely represents the manager's synthetic positions?

a. Receive-fixed 5-year swap, short 10-year futures, and pay-fixed 30-year swap
b. Pay-fixed 5-year swap, short 10-year futures, and receive-fixed 30-year swap
c. Short 5-year futures, long 10-year futures, and receive-fixed 30-year swap

Solution: Answer: B is correct. The key rate duration summary shows the investor to be net short 5- and 10-year key rate duration and long 30-year key rate duration versus the index. A combines synthetic long, short, and short positions in the 5-, 10-, and 30-year maturities, respectively. C combines short, long, and long positions across the curve. The combination of a pay-fixed (short duration) 5-year swap, a short 10-year futures position, and a receive-fixed (long duration) 30-year swap is, therefore, the best answer.

4. ACTIVE FIXED-INCOME MANAGEMENT ACROSS CURRENCIES

- **discuss yield curve strategies across currencies**

The benefits of investing across borders to maximize return and diversify exposure is a consistent theme among portfolio managers. While both the tools as well as the strategic considerations of active versus passive currency risk management within an investment portfolio are addressed elsewhere, here we will primarily focus on extending our analysis of yield curve strategies from a single yield curve to multiple yield curves across currencies.

An earlier currency lesson noted that investors measure return in functional currency terms—that is, considering domestic currency returns on foreign currency assets, as shown in Equations 12 and 13.

$$\text{Single asset: } R_{DC} = (1 + R_{FC})\ (1 + R_{FX}) - 1 \tag{12}$$

$$\text{Portfolio: } R_{DC} = \sum_{i=1}^{n} \omega_i (1 + R_{FC,i})(1 + R_{FX,i}) - 1 \tag{13}$$

R_{DC} and R_{FC} are the domestic and foreign currency returns expressed as a percentage, R_{FX} is the percentage change of the domestic versus foreign currency, while ω_i is the respective portfolio weight of each foreign currency asset (in domestic currency terms) with the sum of ω_i equal to 1. In the context of Equation 1, R_{DC} simply combines the third factor, +/−*E* (Δ Price due to investor's view of benchmark yield), and the fifth factor, +/−*E* (Δ Price due to investor's view of currency value changes), factors in the expected fixed-income return model.

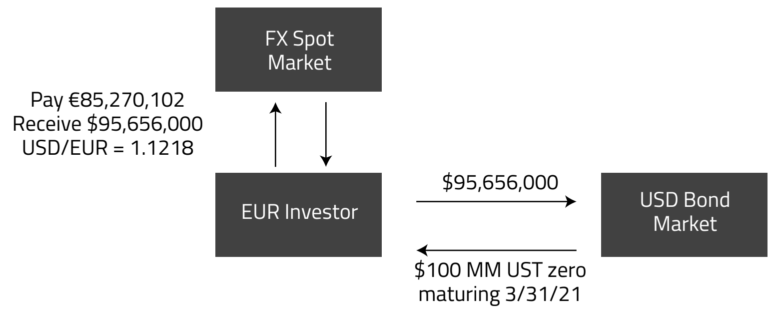
In a previous term structure lesson, we highlighted several macroeconomic factors that influence the bond term premium and required returns, such as inflation, economic growth, and monetary policy. Differences in these factors across countries are frequently reflected in the relative term structure of interest rates as well as in exchange rates.

For example, after a decade of economic expansion following the 2008 global financial crisis, the US Federal Reserve's earlier reversal of quantitative easing versus the European Central

Bank through 2019 led to significantly higher short-term government yields-to-maturity in the United States versus Europe.

Against this historical backdrop, assume a German fixed-income manager decides to buy short-term US Treasuries to take advantage of higher USD yields-to-maturity. At the end of March 2019, a USD Treasury zero-coupon bond maturing on 31 March 2021 had a price at 95.656, with an approximate yield-to-maturity of 2.25%. Based upon the then-current USD/EUR spot rate of 1.1218 (that is, $1.1218 = €1), the manager pays €85,270,102 (= $95,656,000/1.1218) for a $100 million face value Treasury security, as seen in Exhibit 25.

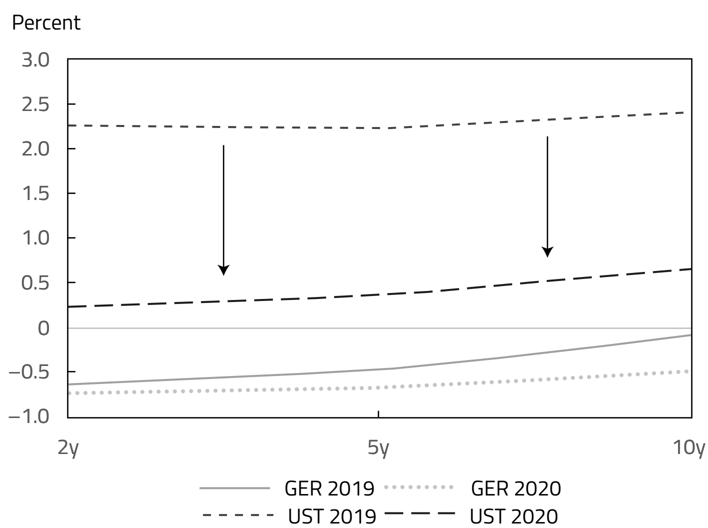
EXHIBIT 25 USD/EUR Spot Trade and US Treasury Zero Purchase



As in the single currency yield curve case, the investor will benefit from bond price appreciation if the US Treasury yield-to-maturity falls during the holding period. In addition, since her domestic returns are measured in EUR, she will also benefit if the USD she receives upon sale of the bond or at maturity buy more EUR per USD in the future—that is, if USD/EUR decreases (i.e., USD *appreciates* versus EUR).

In fact, the flight to quality induced by the COVID-19 pandemic in early 2020 led to a sharp decline in US Treasury yields-to-maturity. Exhibit 26 shows how the relationship between US and German government rates changed between March 2019 and March 2020.

EXHIBIT 26 US vs. German Government Yield Curves, 2019 and 2020



Source: Bloomberg.

As a result, one year after purchase (31 March 2020), the US Treasury zero-coupon bond maturing 31 March 2021 traded at a price of 100.028 and the USD/EUR spot was 1.1031.

Now we calculate the German investor's 1-year domestic currency return from holding the $100 million par value US Treasury zero-coupon bond.

Equation 12 separates this return into two key components:

$\boldsymbol{R}_{\mathbf{FC}}$: 4.57%, = ($100,028,000/$95,656,000) − 1, as the investor receives $100,028,000 upon sale of the US Treasury bond purchased a year earlier at $95,656,000.

$\boldsymbol{R}_{\mathbf{FX}}$: 1.70%, = (1.1218/1.1031 − 1), as the investor converted €85,270,102 into USD to purchase the bond at 1.1218 and then converted USD proceeds back to EUR at 1.1031. The EUR depreciated (i.e., lower USD/EUR spot rate) over the 1-year period.

$\boldsymbol{R}_{\mathbf{DC}}$ may be shown to be 6.34%, solved either using Equation 12 or directly for the 1-year return on investment in EUR terms, = (€90,678,996/€ 85,270,102) −1.

In contrast to the *unhedged* 1-year example, let us now assume that the German manager fully hedges the foreign currency risk associated with the US Treasury bond purchase and holds it instead for two years, at which time she receives the bond's face value of $100,000,000. Specifically, the manager enters a 2-year FX forward agreement at the time of bond purchase to sell the future $100,000,000 payment upon bond maturity and buy EUR at the then current 2-year USD/EUR forward rate of 1.1870, locking in a certain €84,245,998, = $100,000,000/1.1870, in two years' time.

If fully hedged, the expected annualized return, R_{DC}, in EUR terms on the 2-year US Treasury zero-coupon bond hedged EUR investment over two years is equal to −0.60%, = $(€84{,}245{,}998/€85{,}270{,}102)^{0.5} - 1$, which matches the 2-year annualized German government zero-coupon bond yield-to-maturity upon inception. This may also be calculated using Equation 12, with R_{FC} = 2.25% and R_{FX} = −2.785%, or $(1.1218/1.1870)^{0.5} - 1$.

The fully hedged investment example is a reminder from earlier lessons that **covered interest rate parity** establishes a fundamental no-arbitrage relationship between spot and forward rates for individual cash flows in T periods, as shown in Equation 14.

$$F\left(\frac{\text{DC}}{\text{FC}}, T\right) = S_0(\text{DC/FC})\frac{(1 + r_{\text{DC}})^T}{(1 + r_{\text{FC}})^T} \tag{14}$$

F denotes the forward rate; S is the spot rate; and r_{DC} and r_{FC} reflect the respective domestic and foreign currency risk-free rates. If an investor uses a forward contract to fully hedge foreign currency cash flows, she should expect to earn the domestic risk-free rate, as seen in our example. Recall also that this implies in general that a higher-yielding currency will trade at a forward discount, while a lower-yielding currency will trade at a premium. This is consistent with USD/EUR spot versus forward exchange rates (1.1218 spot versus the 1.187 2-year forward rate) as well as the relationship between USD rates and EUR rates in 2019, as shown in Exhibit 26.

In contrast, **uncovered interest rate parity** suggests that over time, the returns on unhedged foreign currency exposure will be the same as on a domestic currency investment. Although forward FX rates should in theory be an unbiased predictor of future spot FX rates if uncovered interest rate parity holds, in practice investors sometimes seek to exploit a persistent divergence from interest rate parity conditions (known as the **forward rate bias**) by investing in higher-yielding currencies, which is in some cases enhanced by borrowing in lower-yielding currencies.

This demonstrates that active fixed-income strategies across currencies must factor in views on currency appreciation versus depreciation as well as yield curve changes across countries.

Our investor's USD versus EUR interest rate view in the previous example combined with an implicit view that USD/EUR would remain relatively stable led to the highest return in the unhedged case with a 1-year investment horizon. This stands in contrast to the relationship between USD/EUR spot and 2-year forward rates at the inception of the trade on 31 March 2019, when implied (annualized) EUR appreciation was 3.73%, = $(1.187/1.1031)^{0.5} - 1$.

The European fixed-income manager in our example might use leverage instead of cash by borrowing in euros when buying the 2-year US Treasury zero. This is an extension of the single currency repo carry trade shown in Exhibit 5, in which an investor borrows short-term in one currency and invests in another higher-yielding currency. This **carry trade across currencies** is a potential source of additional income subject to short-term availability if the positive interest rate differential persists for the life of the transaction. Given the preponderance of fixed-rate coupon versus zero-coupon bonds, our analysis turns next to these securities. As in the case of the fully hedged German investor in US Treasuries, we first establish the necessary building blocks to replicate a risk-free domestic currency return when investing in a foreign currency fixed-income coupon bond. We then consider how an active investor might deviate from this exposure profile to generate excess return.

Consider the example of a Japan-based investor who buys a fixed-rate USD coupon bond. In order to fully hedge JPY domestic currency cash flows for the foreign currency bond, as in the case of the earlier German investor, the investor must first sell Japanese yen (JPY) and purchase USD at the current spot rate to purchase the bond. At the end of each semi-annual interest period, the investor receives a USD coupon, which must be converted at the future JPY/USD spot rate (that is, the number of JPY required to buy one USD). At maturity, the investor receives the final semi-annual coupon and principal, which must be converted to JPY using the future JPY/USD spot rate to receive the final payment in domestic currency.

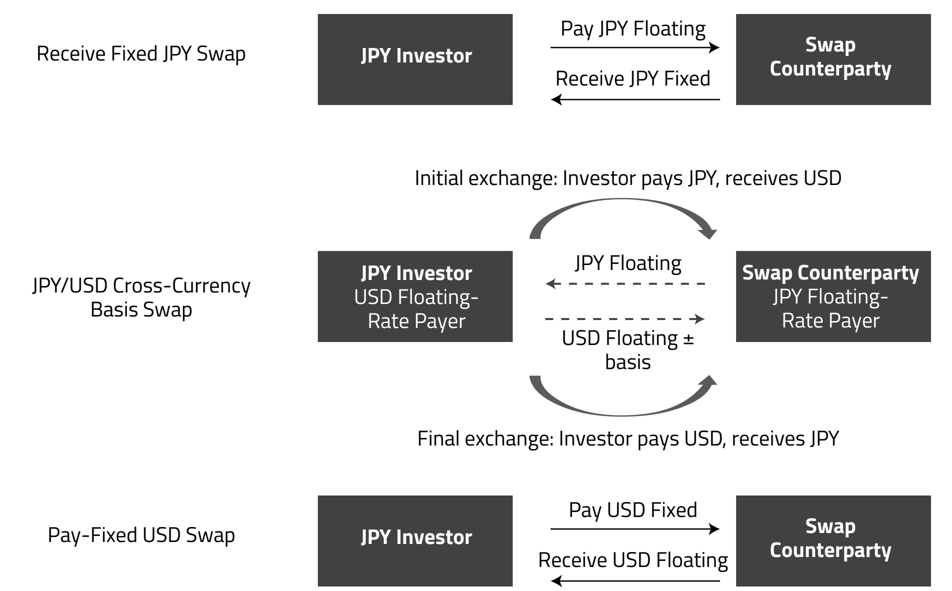
The fixed-rate foreign currency bond exposes the Japanese investor to a series of FX forward exposures that may be hedged upon purchase with a cross-currency swap, as seen in Exhibit 27 with the example of a par 10-year US Treasury bond with a 0.625% coupon issued in May 2020.

EXHIBIT 27 Fixed-Fixed Cross-Currency Swap Diagram and Details

Trade Details	JPY/USD Fixed-Fixed Cross-Currency Swap
Start date	15 May 2020
Maturity date	15 May 2030
Fixed USD payer	JPY Investor
Fixed JPY payer	Swap counterparty
Initial exchange	JPY investor pays JPY10.706 billion and receives USD100 million as of 15 May 2020
Fixed USD rate	0.625% Semiannual, Act/Act
Fixed JPY rate	–0.726% Semiannual, Act/365
Final exchange	JPY investor pays USD100 million and receives JPY10.706 billion as of 15 May 2030

Note that the fixed-fixed cross-currency swap components, shown in Exhibit 28, are a combination of three distinct hedging transactions: a receive-fixed JPY interest rate swap, a USD-JPY **cross-currency basis swap** involving the exchange of floating JPY for floating USD payments, and a pay-fixed USD interest rate swap.

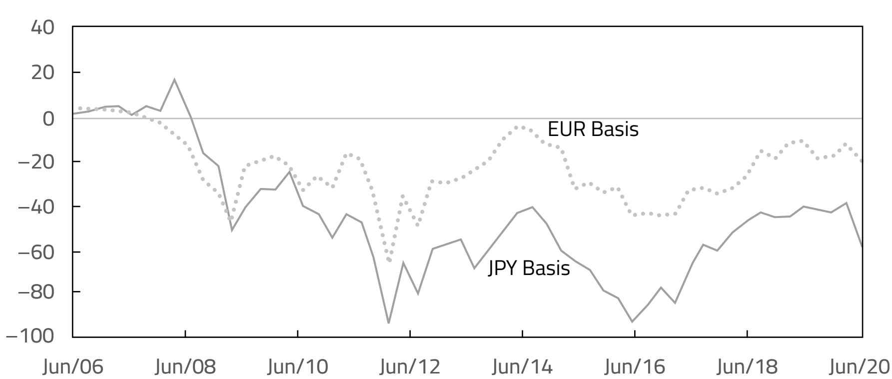
EXHIBIT 28 Fixed-Fixed Cross-Currency Swap Components



CROSS-CURRENCY BASIS AND COVERED INTEREST RATE PARITY

The “basis” or spread, as shown in the cross-currency basis swap, is the difference between the USD interest rate and the synthetic USD interest rate derived from swapping JPY into US dollars. A positive (negative) currency basis means that the direct USD interest rate is higher (lower) than the synthetic USD interest rate. While covered interest rate parity suggests that cross-currency basis should be close to zero, Exhibit 29 shows that the JPY and EUR cross-currency basis was persistently negative following the 2008 global financial crisis.

EXHIBIT 29 Five-Year JPY and EUR Cross-Currency Basis, 2006–2020



Cross-currency basis is widely seen as a barometer for global financial conditions. For example, greater credit and liquidity risk within the EU financial sector and the European Central Bank's aggressive quantitative easing have been cited as causes of the wider USD/EUR cross-currency basis.

Du, Tepper, and Verdelhan (2018) investigate the persistent no-arbitrage violation of covered interest rate parity implied by wider cross-currency basis observed across G-10 countries and offer several explanations. First, higher financial intermediation costs since the 2008 global financial crisis, such as higher bank regulatory capital requirements, prevent market participants from taking advantage of basis arbitrage opportunities. Second, covered interest rate parity violations suggest international imbalances in the form of high demand for investments in high interest rate currencies and a large supply of savings in low interest rate currencies. These deviations are magnified by divergent monetary policies across jurisdictions.

The building blocks of the fixed-fixed cross-currency swap shown in Exhibit 28 offer an active fixed-income investor a simplified framework within which one can take interest rate or currency positions to deviate from a risk-free domestic currency return. For example, by foregoing the pay USD fixed swap, the JPY investor takes a USD rate view by earning the USD fixed coupon and paying USD floating while fully hedging the currency exposure via the cross-currency basis swap. Similar principles apply as in the single currency case—namely, to go long (or overweight) assets expected to appreciate and go short (or underweight) assets expected to decline in value or appreciate less. The overweight and underweight bond positions may now be denominated in different currencies, with the active strategy often using an underweight position in one currency to fund an overweight position in another. The resulting yield curve strategy faces three potential risks: (1) yield curve movements—level, slope, or curvature—in the overweight currency; (2) yield curve changes in the underweight currency; and (3) exchange rate changes.

Consider the following unhedged example of a higher- versus lower-yielding currency.

EXAMPLE 12 MXN Carry Trade

Consider the case of a portfolio manager examining a cross-currency carry trade between US dollar (USD) and Mexican peso (MXN) money market rates. The manager is contemplating borrowing in USD for one year and investing in 90-day Mexican treasury bills, rolling them over at maturity for the next 12 months. Assume that today's 1-year USD interest rate is 1.85%, the 90-day MXN interest rate is currently 7.70% (annualized), and the MXN/USD spot exchange rate is 19.15 (that is, it takes 19.15 MXN to buy one USD).

If the manager expected that Mexican money market rates and the MXN/USD exchange rate would remain stable, the expected profit from this carry trade is:

$$(1 + 0.0770/4)^4 - (1 + 0.0185) \approx 6.08\%$$

However, money market and exchange rates are rarely stable; this trade is exposed to changes in both the 90-day MXN interest rate and the MXN/USD spot exchange rate.

(The 1-year fixed-rate USD loan eliminates exposure to USD rate changes.) Assume that 90-day MXN interest rates and exchange rates change as follows over the 12-month period.

Rate / Time	Today	90 Days	180 Days	270 Days	360 Days
90-day MXN rate	7.70%	7.85%	8.15%	8.20%	N/A
MXN/USD spot rate	19.15	18.05	19.05	18.80	19.65

Note that 90-day MXN yields-to-maturity rose and that MXN depreciated slightly versus USD over the 360-day period. If the manager had rolled over this trade for the full 12 months, the realized return would have been:

$$\left(1+\frac{0.0770}{4}\right)\left(1+\frac{0.0785}{4}\right)\left(1+\frac{0.0815}{4}\right)\left(1+\frac{0.082}{4}\right)\times\frac{19.15}{19.65}$$
$$-(1+0.0185)\approx 3.61\%$$

While the cross-currency carry trade was ultimately profitable, it was exposed to risks over the horizon; moreover, despite the rise in 90-day MXN yields-to-maturity, a late-period MXN depreciation undercut the profitability of the trade. This underscores the fact that carry trades are unhedged and are most successful in stable (low volatility) markets: Unforeseen market volatility can quickly erase even the most attractive cross-currency carry opportunities. For example, in the first quarter of 2020 at the start of the COVID-19 pandemic, MXN depreciated against the USD by approximately 25% in just over a month.

While an endless number of unhedged strategies seeking to capitalize on a level, slope, or curvature view across currencies exist, Exhibit 30 summarizes several of these major strategies.

EXHIBIT 30 Active Cross-Currency Strategies

Strategy	Purchase	Sell / Borrow	Expected Unhedged Return
Receive-fixed/ pay-fixed	High-yielding fixed-income asset	Lower-yield fixed-rate loan	Carry (higher yield minus lower yield) assuming uncovered interest parity does not hold
Receive-fixed/ pay-floating	High-yielding fixed-rate asset	Short-term, lower yield floating-rate loan rolled over until maturity	Carry (higher yield minus lower yield) plus long- versus short-term rate differential for lower-yielding currency
Receive-floating/ pay-fixed	High-yield floating-rate asset	Lower-yield fixed-rate loan	Carry (higher floating yield minus lower fixed yield)
Receive-floating/ pay-floating	High-yield floating-rate asset	Short-term, lower yield floating-rate loan rolled over until maturity	Carry (higher floating yield minus lower floating yield)

EXAMPLE 13 Bear Flattening Impact

A fixed-income manager is considering a foreign currency fixed-income investment in a relatively high-yielding market, where she expects bear flattening to occur in the near future and her lower-yielding domestic yield curve to remain stable and upward-sloping. Under this scenario, which of the following strategies will generate the largest carry benefit if her interest rate view is realized?

a. Receive-fixed in foreign currency, pay-fixed in domestic currency
b. Receive-fixed in foreign currency, pay-floating in domestic currency
c. Receive-floating in foreign currency, pay-floating in domestic currency

Solution: The correct answer is C. If the higher-yielding foreign currency experiences a bear flattening in the yield curve as the manager expects, then foreign currency short-term yields-to-maturity will increase by more than long-term yields-to-maturity; thus she will want receive-floating in foreign currency. Given the upward-sloping domestic yield curve, we would expect the carry difference between receiving foreign currency floating rates and paying domestic currency floating rates to be the highest.

5. A FRAMEWORK FOR EVALUATING YIELD CURVE STRATEGIES

- **evaluate the expected return and risks of a yield curve strategy**

The factors affecting the expected return of a fixed-income portfolio were summarized in Equation 1. The key underlying assumption in this calculation is that the inputs rely on the fixed-income manager's expectations under an active strategy. As we have seen earlier, unexpected changes to the level, slope, and shape of the yield curve as well as currency changes can impact a portfolio's value in a number of ways—as quantified by the use of portfolio duration and convexity statistics in Equation 3 for a single currency and in Equation 13 for a multicurrency portfolio.

Practitioners frequently evaluate fixed-income portfolio risk using **scenario analysis**, which involves changing multiple assumptions at once to assess the overall impact of unexpected market changes on a portfolio's value. Managers may use historical rate and currency changes or conduct specific stress tests using this analysis. For example, a leveraged investor might evaluate how much rates or currencies must move before she faces a collateral or margin call or is forced to unwind a position. Fixed-income portfolio models offer practitioners a variety of historical or user-defined scenarios. The following scenario analysis example shows how this may be done for the US Treasury portfolio seen earlier.

EXAMPLE 14 Scenario Analysis—US Treasury Securities Portfolio

Tenor	Coupon	Price	Modified Duration	Convexity
2y	0.25%	100	1.994	5.0
5y	0.875%	100	4.88	26.5
10y	2.00%	100	9.023	90.8

In Example 1, we compared two $50 million portfolios. Portfolio A is fully invested in the 5-year Treasury bond, while Portfolio B is split between 2-year (58.94%) and 10-year (41.06%) bonds to match a 5-year bond duration of 4.88. Rather than the earlier parallel yield curve shift, we now analyze two yield curve slope scenarios—namely, an immediate bear steepening and bull flattening of the US Treasury yield curve. The bear steepening scenario involves a 50 bp and 100 bp *rise* in 5- and 10-year yields-to-maturity, respectively, while the bull flattening is assumed to result from a 50 bp fall in 5-year rates and a 100 bp fall in 10-year rates. Using Equation 3, our scenario analysis looks as follows:

Scenario	Portfolio A % Δ Price	Portfolio A Δ Price	Portfolio B % Δ Price	Portfolio B Δ Price
Bear steepening	–2.407%	($1,203,437)	–3.518%	($1,759,216)
Bull flattening	2.473%	$1,236,563	3.891%	$1,945,628

We may conclude from our analysis that although Portfolios A and B have similar duration and therefore perform similarly if the yield curve experiences a parallel shift (except for the convexity difference) seen in Example 1, they perform very differently under various yield curve slope scenarios.

The fixed-income portfolio risk and return impact of rolldown return versus carry, changes in the level, slope, and shape of a single currency yield curve, and an extension to multiple currencies (where spot and forward FX rates are related to relative interest rates) are best illustrated with a pair of examples.

EXAMPLE 15 AUD Bullet versus Barbell

A US-based portfolio manager plans to invest in Australian zero-coupon bonds denominated in Australian dollars (AUD). He projects that over the next 12 months, the Australian zero-coupon yield curve will experience a downward parallel shift of 60 bps and that AUD will appreciate 0.25% against USD. The manager is weighing bullet and barbell strategies using the following data:

Statistic	Bullet	Barbell
Investment horizon (years)	1.0	1.0
Average bond price in portfolio (today)	98.00	98.00
Average portfolio bond price (in 1 year/stable yield curve)	99.75	100.00
Expected portfolio effective duration (in 1 year)	3.95	3.95
Expected portfolio convexity (in 1 year)	19.50	34.00
Expected change in AUD zero-coupon yields	–0.60%	–0.60%
Expected change in AUD versus USD	+0.25%	+0.25%

Solve for the expected return over the 1-year investment horizon for each portfolio using the step-by-step estimation approach in Equation 1.

Rolldown Return

The sum of coupon income (in %) and the price effect on bonds from "rolling down the yield curve." Since both portfolios contain only zero-coupon bonds, there is no coupon income and we calculate the rolldown return using $(PV_1 - PV_0) / PV_0$, where PV_0 is today's bond price and PV_1 is the bond price in one year, assuming no shift in the yield curve.

1. **Bullet:** 1.7857% = (99.75 − 98.00) / 98.00
2. **Barbell:** 2.0408% = (100.000 − 98.00) / 98.00

E **(Δ Price Due to Investor's View of Benchmark Yield)**

The effect of the interest rate view on expected portfolio return may be estimated using Equation 3, using effective duration and convexity in one year's time to evaluate the expected 60 bp downward parallel yield curve shift:

1. **Bullet:** 2.4051% = $(-3.95 \times -0.0060) + [1/2 \times 19.5 \times (-0.0060)^2]$
2. **Barbell:** 2.4312% = $(-3.95 \times -0.0060) + [1/2 \times 34.0 \times (-0.0060)^2]$

$E(R) \approx$ % Rolldown return + E (% Δ Price due to investor's view of benchmark yield) + E (% Δ Price due to investor's view of currency value changes)

In addition to rolldown return and expected price changes due to changes in yield-to-maturity, the expected 0.25% appreciation of AUD versus USD must be incorporated in order to arrive at the USD investor's domestic currency return. Using Equation 12, R_{FC} equals the sum of rolldown return and changes in price due to yield-to-maturity changes, while R_{FX} is 0.25%. Expected returns are as follows:

$$E(R_1) = 4.4513\%, \text{ or } [(1 + 0.017857 + 0.024051) \times (1.0025)] - 1$$
$$E(R_2) = 4.7332\%, \text{ or } [(1 + 0.020408 + 0.024312) \times (1.0025)] - 1$$

Overall, the barbell outperforms the bullet by approximately 28 bps. Rolldown return contributes most of this outperformance. Rolldown return contributed approximately 25.5 bps of outperformance (i.e., 2.0408% − 1.7857%) for the barbell, and the greater convexity of the barbell portfolio contributed just over 2.6 bps of outperformance (i.e., 2.4312% − 2.4051%). Currency exposure had the same impact on both strategies. The strong rolldown contribution is likely driven by the stronger price appreciation (under the stable yield curve assumption) of longer-maturity zeros in the barbell portfolio relative to the price appreciation of the intermediate zeros in the bullet portfolio as the bonds ride the curve over the 1-year horizon to a shorter maturity.

EXAMPLE 16 US Treasury Bullet versus Barbell

Assume a 1-year investment horizon for a portfolio manager considering US Treasury market strategies. The manager is considering two strategies to capitalize on an expected rise in US Treasury security zero-coupon yield levels of 50 bps in the next 12 months:

1. A bullet portfolio fully invested in 5-year zero-coupon notes currently priced at 94.5392.
2. A barbell portfolio: 62.97% is invested in 2-year zero-coupon notes priced at 98.7816, and 37.03% is invested in 10-year zero-coupon bonds priced at 83.7906.

Further assumptions for evaluating these portfolios are shown here:

Statistic	Bullet	Barbell
Investment horizon (years)	1.0	1.0
Average bond price in portfolio (today)	94.5392	92.6437
Average portfolio bond price (in 1 year/stable yield curve)	96.0503	94.3525
Expected portfolio effective duration (in 1 year)	3.98	3.98
Expected portfolio convexity (in 1 year)	17.82	32.57
Expected change in US Treasury zero-coupon yields	0.50%	0.50%

Solve for the expected return over the 1-year investment horizon for each portfolio using the step-by-step estimation approach in Equation 1.

Rolldown Return

The sum of coupon income (in %) and the price effect on bonds from "rolling down the yield curve." Since both portfolios contain only zero-coupon bonds, there is no coupon income and we calculate the rolldown return using $(PV_1 - PV_0) / PV_0$, where PV_0 is today's bond price and PV_1 is the bond price in one year, assuming no shift in the yield curve.

Bullet: $(96.0503 - 94.5392) \div 94.5392 = 1.5984\%$
Barbell: $(94.3525 - 92.6437) \div 92.6437 = 1.8445\%$

E **(Δ Price Due to Investor's View of Benchmark Yield)**

The effect of the interest rate view on expected portfolio return may be estimated with Equation 3, using effective duration and convexity in one year's time to evaluate the expected 50 bp upward parallel yield curve shift:

Bullet: $-1.9677\% = (-3.98 \times 0.0050) + [1/2 \times 17.82 \times (0.0050)^2]$
Barbell: $-1.9493\% = (-3.98 \times 0.0050) + [1/2 \times 32.57 \times (0.0050)^2]$

Expected total return in percentage terms for each portfolio is equal to:

$E(R)$ = % Rolldown return + E (% Δ Price due to investor's view of benchmark yield)

The total expected return over the 1-year investment horizon for the bullet portfolio is therefore –0.3693%, or 1.5984% – 1.9677%, and the expected return for the barbell portfolio is –0.1048%, or 1.8445% – 1.9493%.

If the manager's expected market scenario materializes, the barbell portfolio outperforms the bullet portfolio by 26 bps. The higher barbell convexity contributed just under 2 bps of outperformance, whereas the rolldown return contributed nearly 25 bps. Stronger price appreciation (under the stable yield curve assumption) resulted from a greater rolldown effect from the 10-year zeros in the barbell versus the 5-year zeros over one year.

SUMMARY

This chapter addresses active fixed-income yield curve management using cash- and derivative-based strategies to generate returns, which exceed those of a benchmark index due to yield curve changes. The following are the main points in the chapter:

- A par yield curve is a stylized representation of yields-to-maturity available to investors at various maturities, which often does not consist of traded securities but must be extracted from available bond yields using a model.
- Primary yield curve risk factors may be categorized by changes in level (or a parallel "shift"), slope (a flatter or steeper yield curve), and shape or curvature.
- Yield curve slope measures the difference between the yield-to-maturity on a long-maturity bond and the yield-to-maturity on a shorter-maturity bond. Curvature is the relationship between short-, intermediate-, and long-term yields-to-maturity.
- Fixed-income portfolio managers can approximate actual and anticipated bond portfolio value changes using portfolio duration and convexity measures. Duration measures the linear relationship between bond prices and yield-to-maturity. Convexity is a second-order effect describing a bond's price behavior for larger rate movements and is affected by cash flow dispersion.
- A barbell portfolio combining short- and long-term bond positions will have greater convexity than a bullet portfolio concentrated in a single maturity for a given duration.
- Active managers seeking excess return in an expected static yield curve environment that is upward-sloping can use a buy-and-hold strategy to increase duration, roll down the yield curve, or use leverage via a carry trade in cash markets. Receive-fixed swaps and long futures positions replicate this exposure in the derivatives market.
- Derivatives offer the opportunity to synthetically change exposure with a far smaller initial cash outlay than cash strategies but require managers to maintain sufficient cash or eligible securities to fulfill margin or collateral requirements.
- Active fixed-income managers with a divergent rate level view increase duration exposure above a target if yields-to-maturity are expected to decline and reduce duration if expecting higher yields-to-maturity to minimize losses.
- Yield curve steepeners seek to gain from a greater spread between short- and long-term yields-to-maturity by combining a "long" short-dated bond position with a "short" long-dated bond position, while a flattener involves sale of short-term bonds and purchase of long-term bonds.

- Steepener and flattener strategies may be net duration neutral or net long or short duration depending upon a manager's view of how the yield curve slope will change—that is, the relative contribution of short- and long-term yield-to-maturity changes to the expected yield curve slope change.
- The butterfly strategy combining a long bullet with a short barbell portfolio (or vice versa) is commonly used to capitalize on expected yield curve shape changes.
- Active managers capitalize on a view as to whether future realized interest rate volatility will be greater or less than implied volatility by purchasing or selling bonds with embedded options or by using stand-alone interest rate options.
- Stand-alone interest rate put and call options are generally based upon a bond's price, not yield-to-maturity.
- Interest rate swaptions and options on bond futures are among the common tools used by active managers to alter portfolio duration and convexity subject to yield-to-maturity changes. An interest rate swaption involves the right to enter into an interest rate swap at a specific strike price in the future, while an option on a bond future involves the right, not the obligation, to buy or sell a futures contract.
- Key rate durations can be used in active fixed-income management to identify a bond portfolio's sensitivity to changes in the shape of the benchmark yield curve, allowing an active manager to quantify exposures along the curve.
- Fixed-income managers engaged in active yield curve strategies across currencies measure excess return from active management in functional currency terms—that is, considering domestic currency returns on foreign currency assets within a portfolio.
- Interest rate parity establishes the fundamental relationship between spot and forward exchange rates, with a higher-yielding currency trading at a forward discount and a lower-yielding currency trading at a premium.
- Covered interest rate parity involves the use of a forward contract to lock in domestic currency proceeds, while uncovered interest rate parity suggests that over time, the returns on unhedged foreign currency exposure will be the same as on a domestic currency investment.
- Active investors use the carry trade across currencies to take advantage of divergence from interest rate parity by borrowing in a lower-yield currency and investing in a higher-yield currency.
- A cross-currency swap enables investors to fully hedge the domestic currency value of cash flows associated with foreign currency bonds.
- Active managers deviate from fully hedged foreign currency bond cash flows by entering overweight and underweight bond positions denominated in different currencies, often using an underweight position in one currency to fund an overweight position in another.
- Investors evaluate the expected return on an active fixed-income portfolio strategy by combining coupon income and rolldown return with expected portfolio changes based on benchmark yield-to-maturity, credit, and currency value changes over the investment horizon.
- Unexpected market changes or risks to portfolio value are frequently evaluated using scenario analysis.

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PRACTICE PROBLEMS

The following information relates to Questions 1–8

A Sydney-based fixed-income portfolio manager is considering the following Commonwealth of Australia government bonds traded on the ASX (Australian Stock Exchange):

Tenor	Coupon	Yield	Price	Modified Duration	Convexity
2y	5.75%	0.28%	110.90	1.922	4.9
4.5y	3.25%	0.55%	111.98	4.241	22.1
9y	2.50%	1.10%	111.97	8.175	85.2

The manager is considering portfolio strategies based upon various interest rate scenarios over the next 12 months. She is considering three long-only government bond portfolio alternatives, as follows:

Bullet: Invest solely in 4.5-year government bonds
Barbell: Invest equally in 2-year and 9-year government bonds
Equal weights: Invest equally in 2-year, 4.5-year, and 9-year bonds

1. The portfolio alternative with the *highest* modified duration is the:
   A. bullet portfolio.
   B. barbell portfolio.
   C. equally weighted portfolio.
2. The manager estimates that accelerated economic growth in Australia will increase the *level* of government yields-to-maturity by 50 bps. Under this scenario, which of the three portfolios experiences the *smallest* decline in market value?
   A. Bullet portfolio
   B. Barbell portfolio
   C. Equally weighted portfolio
3. Assume the manager is able to extend her mandate by adding derivatives strategies to the three portfolio alternatives. The best way to position her portfolio to benefit from a *bear flattening* scenario is to combine a:
   A. 2-year receive-fixed Australian dollar (AUD) swap with the *same* money duration as the bullet portfolio.
   B. 2-year pay-fixed AUD swap with *twice* the money duration as the 2-year government bond in the barbell portfolio.
   C. 9-year receive-fixed AUD swap with *twice* the money duration as the 9-year government bond position in the equally weighted portfolio.
4. In her market research, the manager learns that ASX 3-year and 10-year Treasury bond futures are the most liquid products for investors trading and hedging medium- to long-term Australian dollar (AUD) interest rates. Although neither contract matches the exact characteristics of the cash bonds of her choice, which of the following additions to a barbell portfolio *best* positions her to gain under a *bull flattening* scenario?
   A. Purchase a 3-year Treasury bond future matching the money duration of the short-term (2-year) position.
   B. Sell a 3-year Treasury bond future matching the money duration of the short-term bond position.

   C. Purchase a 10-year Treasury bond future matching the money duration of the long-term bond position.
5. An economic slowdown is expected to result in a 25 bp decline in Australian yield *levels*. Which portfolio alternative will experience the largest gain under this scenario?
   A. Bullet portfolio
   B. Barbell portfolio
   C. Equally weighted portfolio
6. The portfolio alternative with the *least* exposure to convexity is the:
   A. bullet portfolio.
   B. barbell portfolio.
   C. equally weighted portfolio.
7. The current butterfly spread for the Australian government yield curve based upon the manager's portfolio choices is:
   A. 83 bps.
   B. 28 bps.
   C. −28 bps.
8. If the manager has a positive butterfly view on Australian government yields-to-maturity, the *best* portfolio position strategy to pursue is to:
   A. purchase the bullet portfolio and sell the barbell portfolio.
   B. sell the bullet portfolio and sell the barbell portfolio.
   C. purchase the equally weighted portfolio and sell the barbell portfolio.

---

9. An analyst manages an active fixed-income fund that is benchmarked to the Bloomberg Barclays US Treasury Index. This index of US government bonds currently has a modified portfolio duration of 7.25 and an average maturity of 8.5 years. The yield curve is upward-sloping and expected to remain unchanged. Which of the following is the *least* attractive portfolio positioning strategy in a static curve environment?
   A. Purchasing a 10-year zero-coupon bond with a yield of 2% and a price of 82.035
   B. Entering a pay-fixed, 30-year USD interest rate swap
   C. Purchasing a 20-year Treasury and financing it in the repo market
10. A Dutch investor considering a 5-year EUR government bond purchase expects yields-to-maturity to decline by 25 bps in the next six months. Which of the following statements about the rolldown return is *correct*?
   A. The rolldown return equals the difference between the price of the 5-year bond and that of a 4.5-year bond at the lower yield-to-maturity.
   B. The rolldown return consists of the 5-year bond's basis point value multiplied by the expected 25 bp yield-to-maturity change over the next six months.
   C. The rolldown return will be negative if the 5-year bond has a zero coupon and is trading at a premium.
11. An investment manager is considering decreasing portfolio duration versus a benchmark index given her expectations of an upward parallel shift in the yield curve. If she has a choice between a callable, putable, or option-free bond with otherwise comparable characteristics, the most profitable position would be to:
   A. own the callable bond.
   B. own the putable bond.
   C. own the option-free bond.

12. An active fixed-income manager holds a portfolio of commercial and residential mortgage-backed securities that tracks the Bloomberg Barclays US Mortgage-Backed Securities Index. Which of the following choices is the most relevant portfolio statistic for evaluating the first-order change in his portfolio's value for a given change in benchmark yield?
    A. Effective duration
    B. Macaulay duration
    C. Modified duration
13. An active fund trader seeks to capitalize on an expected steepening of the current upward-sloping yield curve using option-based fixed-income instruments. Which of the following portfolio positioning strategies *best* positions her to gain if her interest rate view is realized?
    A. Sell a 30-year receiver swaption and a 2-year bond put option.
    B. Purchase a 30-year receiver swaption and a 2-year bond put option.
    C. Purchase a 30-year payer swaption and a 2-year bond call option.

The following information relates to Questions 14–17

A financial analyst at an in-house asset manager fund has created the following spreadsheet of key rate durations to compare her active position to that of a benchmark index so she can compare the rate sensitivities across maturities.

Tenor	$KeyRateDur^{Active}$	$KeyRateDur^{Index}$	Difference
2y	−0.532	0.738	−1.270
5y	0.324	1.688	−1.364
10y	5.181	2.747	2.434
30y	1.142	2.162	−1.020
Portfolio	6.115	7.335	−1.220

14. Which of the following statements is true if yield *levels* increase by 50 bps?
    A. The active portfolio will outperform the index portfolio by approximately 61 bps.
    B. The index portfolio will outperform the active portfolio by approximately 61 bps.
    C. The index portfolio will outperform the active portfolio by approximately 21 bps.
15. Which of the following statements best characterizes how the active portfolio is positioned for yield curve changes *relative* to the index portfolio?
    A. The active portfolio is positioned to benefit from a bear steepening of the yield curve versus the benchmark portfolio.
    B. The active portfolio is positioned to benefit from a positive butterfly movement in the shape of the yield curve versus the index.
    C. The active portfolio is positioned to benefit from yield curve flattening versus the index.
16. Which of the following derivatives strategies would *best* offset the yield curve exposure difference between the active and index portfolios?
    A. Add a pay-fixed 10-year swap and long 2-year, 5-year, and 30-year bond futures positions to the active portfolio.
    B. Add a receive-fixed 30-year swap, a pay-fixed 10-year swap, and short positions in 2-year and 5-year bond futures to the active portfolio.
    C. Add a pay-fixed 10-year swap, a short 30-year bond futures, and long 2-year and 5-year bond futures positions to the active portfolio.

17. Which of the following statements best describes the forward rate bias?
    A. Investors tend to favor fixed-income investments in currencies that trade at a premium on a forward basis.
    B. Investors tend to hedge fixed-income investments in higher-yielding currencies given the potential for lower returns due to currency depreciation.
    C. Investors tend to favor unhedged fixed-income investments in higher-yielding currencies that are sometimes enhanced by borrowing in lower-yielding currencies.

The following information relates to Questions 18–20

A US-based fixed-income portfolio manager is examining unhedged investments in Thai baht (THB) zero-coupon government bonds issued in Thailand and is considering two investment strategies:

1. **Buy-and-hold:** Purchase a 1-year, THB zero-coupon bond with a current yield-to-maturity of 1.00%.
2. **Roll down the THB yield curve:** Purchase a 2-year zero-coupon note with a current yield-to-maturity of 2.00% and sell it in a year.

THB proceeds under each strategy will be converted into USD at the end of the 1-year investment horizon. The manager expects a stable THB yield curve and that THB will appreciate by 1.5% relative to USD. The following information is used to analyze these two investment strategies:

Statistic	Buy and Hold	Yield Curve Rolldown
Investment horizon (years)	1.0	1.0
Bond maturity at purchase (years)	1.0	2.0
Yield-to-maturity (today)	1.00%	2.00%
Average portfolio bond price (today)	99.0090	96.1169
Expected average portfolio bond price (in 1 year)	100.00	99.0090
Expected currency gains (in 1 year)	1.5%	1.5%

18. The *rolldown returns* over the 1-year investment horizon for the Buy-and-Hold and Yield Curve Rolldown portfolios are closest to:
    A. 1.00% for the Buy-and-Hold portfolio and 3.01% for the Yield Curve Rolldown portfolio, respectively.
    B. 0.991% for the Buy-and-Hold portfolio and 3.01% for the Yield Curve Rolldown portfolio, respectively.
    C. 0.991% for the Buy-and-Hold portfolio and 2.09% for the Yield Curve Rolldown portfolio, respectively.

19. The *total expected return* over the 1-year investment horizon for the Buy-and-Hold and Yield Curve Rolldown portfolios are closest to:
    A. 2.515% for the Buy-and-Hold portfolio and 4.555% for the Yield Curve rolldown portfolio, respectively.
    B. 2.42% for the Buy-and-Hold portfolio and 4.51% for the Yield Curve Rolldown portfolio, respectively.
    C. 2.491% for the Buy-and-Hold portfolio and 3.59% for the Yield Curve Rolldown portfolio, respectively.
20. Which of the following statements best describes how the expected total return results would *change* if THB yields were to rise significantly over the investment horizon?
    A. Both the Buy-and-Hold and Yield Curve Rolldown expected portfolio returns would *increase* due to higher THB yields.
    B. Both the Buy-and-Hold and Yield Curve Rolldown expected portfolio returns would *decrease* due to higher THB yields.
    C. The Buy-and-Hold expected portfolio returns would be *unchanged* and the Yield Curve Rolldown expected portfolio returns would *decrease* due to the rise in yields.

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21. An active investor enters a duration-neutral yield curve flattening trade that combines 2-year and 10-year Treasury positions. Under which of the following yield curve scenarios would you expect the investor to realize the *greatest* portfolio gain?
    A. Bear flattening
    B. Bull flattening
    C. Yield curve inversion

CHAPTER 15

FIXED-INCOME ACTIVE MANAGEMENT: CREDIT STRATEGIES

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LEARNING OUTCOMES

The candidate should be able to:

- describe risk considerations for spread-based fixed-income portfolios;
- discuss the advantages and disadvantages of credit spread measures for spread-based fixed-income portfolios, and explain why option-adjusted spread is considered the most appropriate measure;
- discuss bottom-up approaches to credit strategies;
- discuss top-down approaches to credit strategies;
- discuss liquidity risk in credit markets and how liquidity risk can be managed in a credit portfolio;
- describe how to assess and manage tail risk in credit portfolios;
- discuss the use of credit default swap strategies in active fixed-income portfolio management;
- discuss various portfolio positioning strategies that managers can use to implement a specific credit spread view;
- discuss considerations in constructing and managing portfolios across international credit markets;
- describe the use of structured financial instruments as an alternative to corporate bonds in credit portfolios;
- describe key inputs, outputs, and considerations in using analytical tools to manage fixed-income portfolios.

1. INTRODUCTION

Most fixed-income instruments trade at a nominal yield to maturity (YTM) that lies above that for an equivalent government or benchmark bond of similar maturity. This **yield spread** or difference compensates investors for the risk that they might not receive interest and principal cash flows as expected, whether as a result of a financially distressed corporate borrower, a sovereign issuer unable (or unwilling) to meet scheduled payments, or a deterioration in credit quality in an underlying pool of assets of a structured instrument such as an asset-backed security. A portion of the yield spread reflects the bid–offer cost of buying or selling a particular bond versus a government security, a liquidity premium that varies based on market conditions. Active managers of spread-based fixed-income portfolios take positions in credit and other risk factors that vary from those of an index to generate excess return versus passive index replication. Financial analysts who build on their foundational knowledge by mastering these more advanced fixed-income concepts and tools will broaden their career opportunities in the investment industry.

We begin by reviewing expected fixed-income portfolio return components with a particular focus on credit spreads. These spreads are not directly observable but rather derived from market information. Similar to benchmark yield curves, credit-spread curves are often defined by spread level and slope, and usually grouped by credit rating to gauge relative risk as well as to anticipate and act on expected changes in these relationships over the business cycle. We outline credit spread measures for fixed- and floating-rate bonds and quantify the effect of spread changes on portfolio value. Building blocks for active credit management beyond individual bonds include exchange-traded funds (ETFs), structured financial instruments, and derivative products such as credit default swaps (CDS). These tools are used to describe bottom-up and top-down active credit management approaches as well as how managers position spread-based fixed-income portfolios to capitalize on a market view.

2. KEY CREDIT AND SPREAD CONCEPTS FOR ACTIVE MANAGEMENT

- **describe risk considerations for spread-based fixed-income portfolios**

Managers seeking to maximize fixed-income portfolio returns will usually buy securities with a higher YTM (and lower equivalent price) than a comparable default risk-free government bond. The excess return targeted by active managers of spread-based fixed-income portfolios is captured in the fourth term of the now familiar fixed-income return equation:

$$\begin{aligned} E(R) \approx\ & \text{Coupon income} \\ & +/- \text{ Rolldown return} \\ & +/- E\ (\Delta \text{ Price due to investor's view of benchmark yields}) \\ & +/- E\ (\Delta \text{ Price due to investor's view of yield spreads}) \\ & +/- E\ (\Delta \text{ Price due to investor's view of currency value changes}) \end{aligned} \tag{1}$$

Similar to the benchmark yield curve addressed earlier in the curriculum, yield spreads for a specific bond issuance over a comparable government bond cannot be directly observed but are rather derived or estimated from market information. This yield spread is a risk premium that primarily compensates investors for assuming credit and liquidity risks.

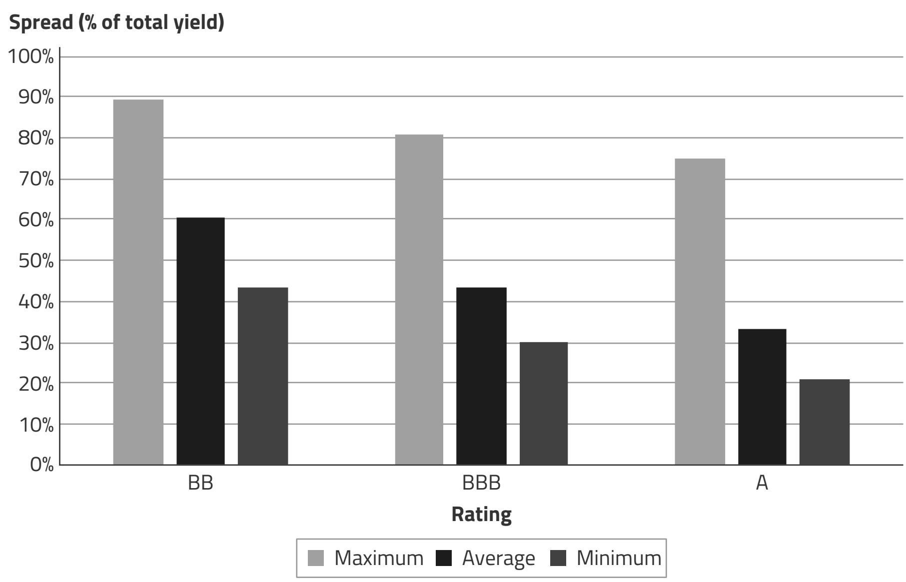
While credit risk for a specific borrower depends on both the likelihood of default and the loss severity in a default scenario, credit risk for a specific bond *issuance* also depends on the period over which payments are promised, the relative seniority of the debt claim, and the sources of repayment, such as the value of underlying collateral, among other factors.

Liquidity risk refers to an investor's ability to readily buy or sell a specific security. The YTM difference (or bid–ask spread) between the purchase and sale price of a bond depends on market conditions and on the specific supply-and-demand dynamics of each fixed-income security. As active fixed-income portfolio managers identify and pursue specific credit strategies, they must also consider trading costs when calculating expected excess returns.

2.1. Credit Risk Considerations

Yield spreads over default risk-free government bonds mostly compensate investors for the potential of not receiving promised cash flows (issuer default) and for the loss severity if a default occurs. Spreads range widely across ratings categories and time periods. For example, Exhibit 1 shows yield spreads as a percentage of total YTM for A-, BBB-, and BB rated US corporate issuers from mid-2009 to mid-2020.

EXHIBIT 1 Yield Spreads as a Percentage of YTM, 2009–2020



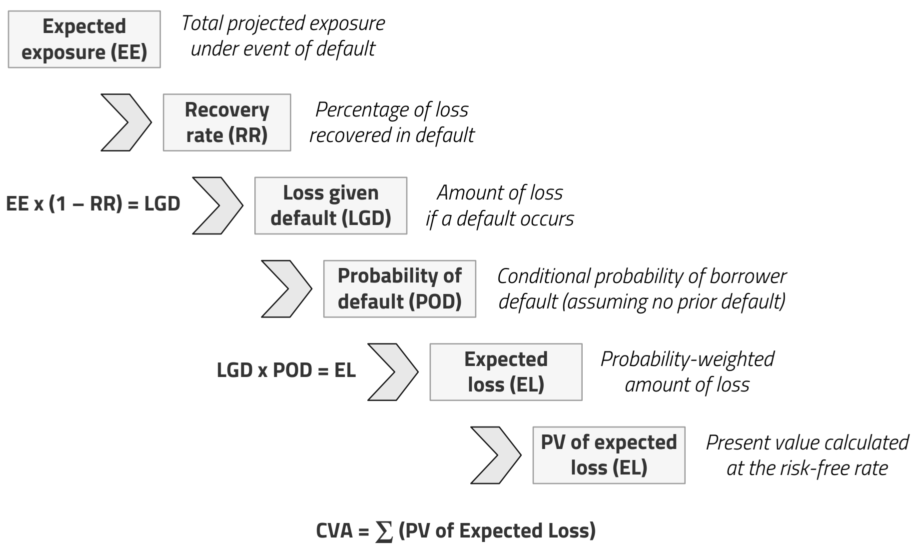
Source: Bloomberg.

On average, 60% of total YTM was attributable to yield spread for BB rated issuers versus 33% for A rated issuers over the period. This percentage was at a minimum for all rating categories in 2010 as the US economy recovered from the 2008–09 financial crisis and reached its peak in early 2020 during the economic slowdown due to the COVID-19 pandemic. The higher average proportion of all-in yield attributable to credit risk warrants a greater focus on this factor among high-yield investors over the credit cycle.

2.1.1. Default Probabilities and Recovery Rates

The **credit valuation adjustment (CVA)** framework shown earlier in the curriculum and in Exhibit 2 comprises the present value of credit risk for a loan, bond, or derivative obligation.

EXHIBIT 2 Credit Valuation Adjustment

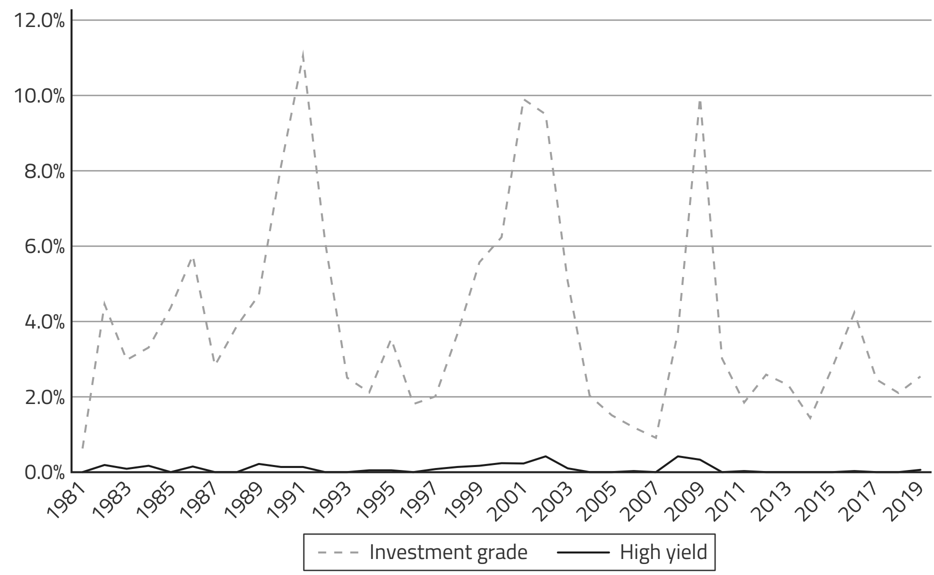


The CVA framework provides a useful means to evaluate the two key components of credit risk. These include (1) **default risk** (also called **probability of default** [POD]), or the likelihood that a borrower defaults or fails to meet its obligation to make full and timely payments of principal and interest according to the terms of the debt security; and (2) **loss severity** (also called loss given default [LGD]), which is the amount of loss if a default occurs. POD is usually expressed as a percentage in annual terms. LGD is most often expressed as a percentage of par value. Recall that the one-period credit spread estimate from an earlier lesson on CDS where we ignored the time value of money was simply the product of LGD and POD (Spread ≈ LGD × POD). This implies that a simple one-period POD can be approximated by dividing credit spread by LGD (POD ≈ Spread/LGD). While this estimate works well for bonds trading close to par, distressed bonds tend to trade on a price rather than a spread basis, which approaches the recovery rate (1 – LGD) as default becomes likely.

The historical POD and the LGD rate is much lower for investment-grade bonds than for high-yield bonds. A **credit loss rate** represents the *realized* percentage of par value lost to default for a group of bonds, or the bonds' default rate multiplied by the loss severity. According to Moody's Investors Service, the highest annual credit loss rate for US investment-grade corporate bonds from 1983 to 2019 was 0.41%, with an average of just 0.05%. For high-yield bonds, the average credit loss rate over the same period was

2.53%, and in several years, usually around economic recessions, losses exceeded 5%. Exhibit 3 shows global annual corporate default rates from S&P Global Ratings for a similar period.

EXHIBIT 3 Annual Global Corporate Bond Default Rates (%)



Source: S&P Global Ratings.

Exhibit 3 makes clear that the likelihood of default rises significantly as the economy slows, reaching peaks during the 1990–91, 2001, and 2008 recessions. The percentage of par value lost in a default scenario depends on a bond's (or loan's) relative position in the capital structure and whether it is secured or unsecured, as shown in Exhibit 4.

EXHIBIT 4 Average Volume Weighted US Corporate Debt Recovery Rates, 1983–2019

First lien bank loan	64%
Second lien bank loan	29%
Senior unsecured bank loan	44%
First lien bond	55%
Second lien bond	45%
Senior unsecured bond	35%
Senior subordinated bond	27%
Subordinated bond	28%
Junior subordinated bond	14%

Source: Moody's Investors Service.

EXAMPLE 1 Estimating Credit Spreads Using POD and LGD

A bank analyst observes a first lien bank loan maturing in two years with a spread of 100 bps from an issuer considering a new second lien bank loan. Using average historical volume weighted corporate debt recovery rates (RR) as a guide, what is the estimated credit spread for the new second lien bank loan?

Answer:

1. Using the POD approximation (POD ≈ Spread/LGD and LGD = (1 − RR)), the analyst uses the current first lien bank loan credit spread and expected first lien bank loan recovery rate to estimate the issuer's POD to be 2.778% (=1.00%/(1 – 0.64)).
2. Using the issuer POD from Answer 1 and the expected second lien bank loan recovery rate of 29%, the bank analyst solves for the expected second lien spread using (POD × LGD) to get 197 bps (=2.778% × (1 – 0.29)).

2.1.2. Default versus Credit Migration

Although actual defaults are relatively rare among higher-rated bond issuers, changes in the *relative* assessment of creditworthiness occurs more frequently. **Credit migration**, or the likelihood of a change in a bond's public credit rating, usually has a negative effect on bond prices. This effect occurs because the chance of downgrade exceeds that of an upgrade, and the yield spread increase at lower credit ratings is far greater than the spread decrease in the event of a credit upgrade.

The POD versus credit migration varies significantly across the credit spectrum. For example, Exhibit 5 shows the two-year average rate of global corporate default and one-notch downgrade.

EXHIBIT 5 Two-Year Average Global Corporate Default/Downgrade,1981–2019

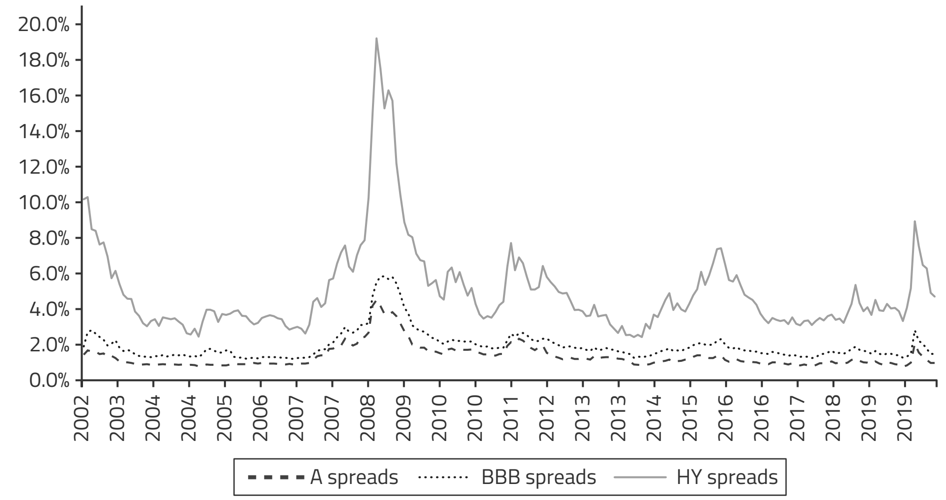
Statistic/Rating	AAA	AA	A	BBB	BB	B	CCC
Default Probability (%)	0.03	0.06	0.14	0.45	1.96	7.83	36.49
One Notch Downgrade (%)	16.22	13.79	8.81	5.66	9.82	5.22	

Source: S&P Global Ratings.

Investors typically categorize credit risk using public debt ratings, distinguishing between investment-grade and high-yield market segments. Investment-grade bonds generally have higher credit ratings, lower default risk, and higher recovery in the event of default and offer lower all-in yields to maturity. High-yield bonds usually have higher yields to maturity as a result of lower (sub-investment or speculative grade) credit ratings, higher default risk, and lower recovery in the event of default. In an earlier yield curve strategies lesson, changes in the level, slope, and shape of the government bond term structure across maturities were established as primary risk factors. The level and slope of credit spread curves are often categorized by public credit rating to distinguish relative market changes across the credit spectrum.

For example, the relative historical yield spread *level* across public rating categories for US corporate borrowers is shown in Exhibit 6.

EXHIBIT 6 A, BBB, and High-Yield US 10-year Corporate Spread Levels (%)



Source: Bloomberg.

Lower-rated bonds face a greater impact from adverse market events, as evidenced by the widening gap between BBB rated and high-yield bonds during the 2008 financial crisis and the COVID-19 pandemic in 2020.

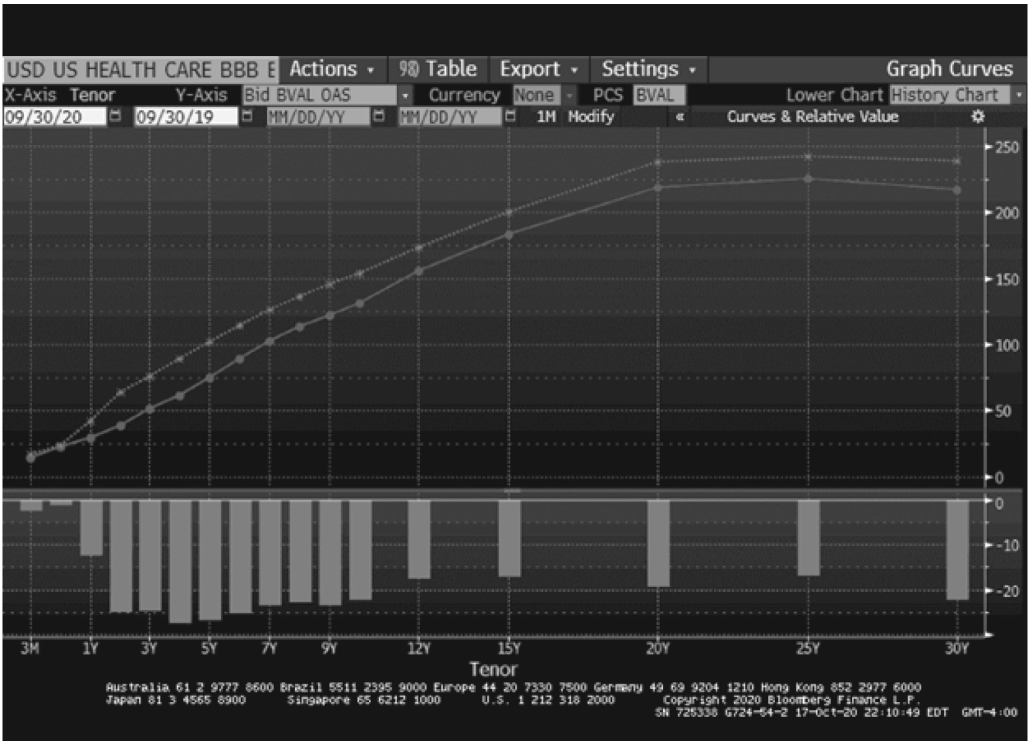
2.1.3. Credit Spread Curves

Active managers often position spread-based portfolios to capitalize on expected credit spread curve changes in a way similar to the benchmark yield curve strategies seen in an earlier lesson. While frequent issuers with many bonds outstanding across maturities have their own issuer-specific credit curve, credit spread curves are usually categorized by rating, issuer type, and/or corporate sector. These curves are derived from the difference between all-in yields to maturity for bonds within each respective category and a government benchmark bond or swap yield curve, with adjustments for specific credit spread measures covered in detail later. For example, Exhibit 7 shows the decline in option-adjusted spreads for US BBB rated health care companies over a one-year period from the end of Q3 2019 to 2020, with the bar graph at the bottom showing the decrease for each maturity.

Primary credit risk factors for a specific issuer include the level and slope of the issuer's credit spread curve. For instance, ignoring liquidity differences across maturities, an upward-sloping credit spread curve suggests a relatively low near-term default probability that rises over time as the likelihood of downgrade and/or default increases. A flatter credit spread curve in contrast indicates that downgrade/default probabilities are equally likely in the near- and long-term.

Credit spread curve changes are broadly driven by the **credit cycle**, the expansion and contraction of credit over the business cycle, which translates into asset price changes based on default and recovery expectations across maturities and rating categories. Exhibit 8 outlines

EXHIBIT 7 BBB Rated US Corporate Health Care Spreads, 2019–2020



Source: Bloomberg.

key credit cycle characteristics and the general effect on credit spread curve levels and slope for high- and low-rated issuers.

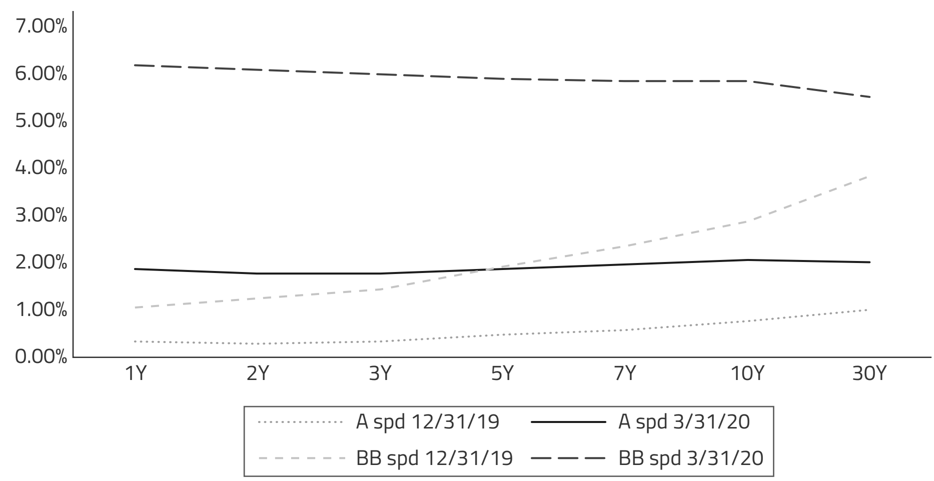
EXHIBIT 8 General Credit Cycle Characteristics

	Early Expansion (Recovery)	Late Expansion	Peak	Contraction (Recession)
Economic Activity	Stable	Accelerating	Decelerating	Declining
Corporate Profitability	Rising	Peak	Stable	Falling
Corporate Leverage	Falling	Stable	Rising	Peak
Corporate Defaults	Peak	Falling	Stable	Rising
Credit Spread Level	Stable	Falling	Rising	Peak
Credit Spread Slope	Stable for high grade, inverted for low ratings	Steeper for both higher and lower ratings	Steeper for both higher and lower ratings	Flatter for high grade, inverted for low ratings

Exhibits 3 and 6 demonstrate the significant variability in annual credit loss rates and credit spread changes, respectively, across the ratings spectrum. Lower-rated issuers tend to experience greater slope and level changes over the credit cycle, including more frequent inversion of the credit curve, given their larger rise in annual credit losses during economic downturns. Higher-rated issuers, in contrast, face smaller credit spread changes and usually exhibit upward-sloping credit curves and fewer credit losses during periods of economic contraction. Credit spread differences *between* major ratings categories tend to narrow during periods of strong economic growth and widen when growth is expected to slow.

For example, consider the widening of BB versus single-A US corporate spreads during Q1 2020 shown in Exhibit 9. The difference between two-year BB spreads and A spreads for the same tenor more than tripled over this three-month period.

EXHIBIT 9 BB versus A Credit Spread Curve, 2019 versus 2020



Source: Bloomberg.

EXAMPLE 2 Credit Cycle and Credit Spread Curve Changes

Which of the following *best* describes the expected shape of the credit spread curve in an economic downturn?

A. Investment-grade and high-yield issuers usually experience similar credit spread curve steepening because of declining corporate profitability.
B. High-yield issuers usually experience more spread curve steepening than investment-grade issuers because higher leverage leads to a greater decline in profitability.
C. High-yield issuers often experience more pronounced flattening or credit spread curve inversion in an economic downturn because the probability of downgrade or default is higher in the near term than the long term.

Solution: The correct answer is C. While investment-grade and high-yield issuers both experience declining profitability in an economic downturn, as in answers A and B, this usually leads to a flatter credit spread curve for investment-grade issuers and often to credit spread curve inversion for high-yield issuers, given a rise in near-term downgrades and defaults.

Actual price movements of lower-rated bonds can be quite different from what analytical models based on benchmark rates and credit spreads would predict under issuer-specific and market stress scenarios. For example, issuer financial distress will cause a bond's price to diverge from what a model using benchmark rates would suggest. As an issuer nears default, the price of its bond approaches the estimated recovery rate, regardless of the current benchmark YTM, because investors no longer expect to receive risky future coupon payments. Under a "flight to quality" market stress scenario, investors sell high-risk, low-rated bonds, which fall in price, and purchase government bonds, which experience price appreciation. This observed negative correlation between high-yield credit spreads and government benchmark yields to maturity often leads fixed-income practitioners to use statistical models and historical bond market data to estimate **empirical duration** rather than rely on analytical duration estimates based on duration and convexity. This market stress scenario is addressed in the following example.

EXAMPLE 3 Empirical versus Analytical Duration

A high-yield bond fund manager is considering adding a US$50 million face value, five-year, 6.75% semiannual coupon bond with a YTM of 5.40% to an active portfolio. The manager uses regression analysis to estimate the bond's empirical duration to be 2.95. Calculate the bond's analytical duration, and estimate the difference in the expected versus actual market value change for this position, given a 50 bp decline in benchmark yields to maturity using these two measures.

Solution:

1. Solve for the bond's analytical duration by using the Excel MDURATION function (MDURATION(settlement, maturity, coupon, yield, frequency, basis)) using a settlement date of 1 January 2022, maturity of 1 January 2027, a 6.75% coupon,

5.40% YTM, semiannual frequency and basis of 0 (30/360 day count) to get 4.234. Note the analytical duration is greater than the observed empirical duration of 2.95.

2. The bond position value can be calculated using the Excel PRICE function (PRICE (settlement, maturity, coupon, yield, frequency, basis)) to solve for a price of 105.847 per 100 face value, or a price of US$52,923,500 for a US$50 million face value.
3. The difference in percentage market value change can be estimated using the 0.50% yield change multiplied by modified duration (–ModDur × ΔYield) for the two estimates. If the benchmark YTM declines by 50 bps, then

   Analytical duration estimate: 2.117% = (–4.234 × –0.5%)
   Empirical duration estimate: 1.475% = (–2.95 × –0.5%)
   The analytical duration calculation overestimates the price gain versus the empirical duration estimate.

4. The difference is 0.642% (2.117% – 1.475%), or an expected US$339,769 (=0.642% × $52,923,500) value difference between the two measures.

While the concept of empirical duration emphasizes the *direction* of high-yield credit spread changes versus benchmark rates, as suggested earlier, the *magnitude* of credit spread changes is greater for lower- versus higher-rate bonds. As we will see later in the lesson, this empirical observation leads to the use of credit spread measure changes based on *percentage* as opposed to absolute credit spread changes for lower-rated issuers.

2.2. Credit Spread Measures

- **discuss the advantages and disadvantages of credit spread measures for spread-based fixed-income portfolios, and explain why option-adjusted spread is considered the most appropriate measure**

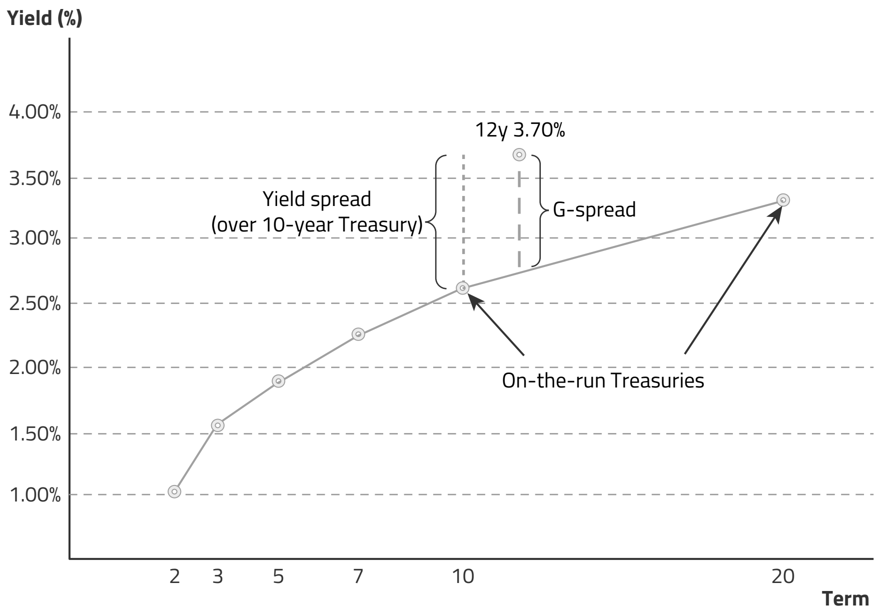
2.2.1. Fixed-Rate Bond Credit Spread Measures

The estimation of yield spreads from market information gives rise to several measures of the difference between a fixed-rate bond's YTM and a benchmark rate. Recall that the YTM is an internal rate of return calculation of all bond cash flows that assumes any earlier payments are reinvested at the same rate and the bond is held to maturity. Spread comparisons are accurate when comparing bonds with identical maturities but different coupons. Because bond maturities vary in practice, a mismatch arises that creates measurement bias if the yield curve is sloped. As a bond rolls down the curve, the benchmark security can also change over time. Finally, yield-based measures do not accurately gauge the return of carry-based strategies often used by active managers (for example, long a risky bond, short a default risk-free position in the repo market).

The yield spread (or benchmark spread) defined earlier as the simple difference between a bond's YTM and the YTM of an on-the-run government bond of similar maturity is easy to calculate and interpret for option-free bonds, and it is particularly useful for infrequently traded bonds. The yield spread also facilitates the approximation of bond price changes for a given benchmark YTM change, assuming a constant yield spread. That said, this simple government bond–based measure has both curve slope and maturity mismatch biases and lacks consistency over time because government benchmarks change as a bond nears maturity.

The **G-spread** uses constant maturity Treasury yields to maturity as the benchmark. Exhibit 10 shows the difference between yield spread and G-spread measures using the example of a bond with 12 years remaining to maturity. While the yield spread for this bond would likely be quoted over a 10-year government benchmark rate, the G-spread involves an interpolation between 10-year and 20-year government yields to maturity.

EXHIBIT 10 Yield Spread versus G-Spread



EXAMPLE 4 Yield Spread versus G-Spread

A portfolio manager considers the following annual coupon bonds:

Issuer	Term	Coupon	Yield	ModDur
Bank	8y	2.75%	2.68%	7.10
Government	7y	1.5%	1.39%	6.61
Government	10y	1.625%	1.66%	9.16

1. Calculate the yield spread and G-spread for the bank bond.
2. An increase in expected inflation causes the government yield curve to steepen, with a 20-point rise in the 10-year government bond YTM and no change in the 7-year government YTM. If the respective bank bond yield spread measures remain unchanged, calculate the expected bank bond percentage price change in each case, and explain which is a more accurate representation of the market change in this case.

Solution to 1: Yield spread for the bank bond is 1.290%, or the simple difference between the 2.68% bank bond YTM and the 1.39% YTM of the nearest on-the-run government bond.

The G-spread is the difference between the bank bond YTM and a linear interpolation of the YTMs of the 7-year government bond (r_{7yr}) and the 10-year government bond (r_{10yr}). Calculate the approximate 8-year government rate as follows:

1. Solve for the weights of the 7-year and the 10-year bond in the interpolation calculation.

$$\text{7-year bond weight} = w_7 = 66.7\% \; (= (10 - 8)/(10 - 7))$$
$$\text{10-year bond weight} = w_{10} = 33.3\% \; (\text{or } (1 - w_7))$$
$$\text{Note that } (w_7 \times 7) + (w_{10} \times 10 = 8).$$

2. The 8-year government rate is a weighted average of the 7-year bond rate and the 10-year bond rate using the weights in Step 1.

$$r_{8yr} = w_7 \times r_{7yr} + w_{10} \times r_{10yr}$$
$$= (66.7\% \times 1.39\%) + (33.3\% \times 1.66\%) = 1.48\%$$

3. The G-spread, or the difference between the bank bond YTM and the 8-year government rate, equals 1.20% (= 2.68% – 1.48%).

Solution to 2: For the yield spread measure, neither the 1.20% spread nor the 7-year government rate of 1.39% has changed, so an analyst considering only these two factors would expect the bank bond price to remain unchanged.

However, for the G-spread measure, the 15 bp increase in the 10-year government YTM causes the 8-year interpolated government YTM to change.

1. The 7-year and the 10-year bond weights for the interpolation are the same as for Question 1, $w_7 = 66.7\%$ and $w_{10} = 33.3\%$.
2. The new 8-year government rate is a weighted average of the 7-year bond rate and the 10-year bond rate using the weights in Step 1.

$$r_{8yr} = w_7 \times r_{7yr} + w_{10} \times r_{10yr}$$
$$= (66.7\% \times 1.39\%) + (33.3\% \times 1.81\%) = 1.53\%$$

3. The bank bond YTM has risen by 0.05% to 2.73% (= 1.53% + 1.20%).
4. The bank bond price change can be estimated by multiplying the yield change by modified duration (–ModDur × ΔYield) as in earlier lessons. This change can be calculated as –0.355% (= –7.1 × 0.05%).

   Note that we can confirm this using the Excel PV function (= –PV (rate, nper, pmt, FV, type)) where "rate" is the interest rate per period (0.0268), "nper" is the number of periods (8), "pmt" is the periodic coupon (2.75), "FV" is future value (100), and "type" corresponds to payments made at the end of each period (0).

   Initial bank bond price: 100.50 (= –PV (0.0268, 8, 2.75, 100, 0)
   New bank bond price: 100.14 (= –PV (0.0273, 8, 2.75, 100, 0)
   Price change: –0.354% (= (100.14 – 100.50)/100.50)

   The G-spread calculation provides a more accurate representation of the estimated bank bond price change in this case because it incorporates the term structure of interest rates.

The **I-spread (interpolated spread)** uses interest rate swaps as the benchmark. Recall that swap rates are derived using short-term lending or market reference rates (MRRs) rather than default-risk-free rates, and unlike government bonds, they are quoted across all maturities. Short-term MRR were historically survey-based Libor rates and are transitioning to transaction-based, secured overnight funding rates. The spread over an MRR-based benchmark can be interpreted as a *relative* rather than absolute credit risk measure for a given bond issuer. An issuer might use the I-spread to determine the relative cost of fixed-rate versus floating-rate borrowing alternatives, while an investor can use the MRR spread to compare pricing more readily across issuers and maturities. Swap benchmarks have the added benefit of directly measuring all-in bond YTMs with an instrument that can be used both as a duration hedge and to measure carry return more accurately for a leveraged position. While the I-spread addresses the maturity mismatch of bonds and benchmarks as raised earlier, it incorporates yield levels using a point on the curve to estimate a risky bond's yield spread rather than the term structure of interest rates and is limited to option-free bonds as a credit risk measure.

Asset swaps convert a bond's periodic fixed coupon to MRR plus (or minus) a spread. If the bond is priced close to par, this spread approximately equals the bond's credit risk over the MRR. Exhibit 11 shows the mechanics of an asset swap.

EXHIBIT 11 Asset Swap Mechanics

The **asset swap spread (ASW)** is the difference between the bond's fixed coupon rate and the fixed rate on an interest rate swap versus MRR, which matches the coupon dates for the remaining life of the bond. If we assume an investor purchases a bond at par, the asset swap transforms the fixed-rate coupon to an equivalent spread over MRR for the life of the bond. Note that under a bond default scenario, the asset manager would still face the mark-to-market settlement of the swap.

EXAMPLE 5 ASW versus I-Spread

Consider the information from the bank and government annual coupon bonds from the prior example:

Issuer	Term	Coupon	Yield	ModDur
Bank	8y	2.75%	2.68%	7.10
Government	7y	1.5%	1.39%	6.61
Government	10y	1.625%	1.66%	9.16

Assuming that 7- and 10-year swap spreads over the respective government benchmark yields to maturity are 15 bps and 20 bps, calculate the ASW and the I-spread for the bank bond, and interpret the difference between the two.

Solution:

1. Solve for the weights of the 7-year and the 10-year bond in the interpolation calculation.

$$\text{7-year bond weight} = w_7 = 66.7\% \ (= (10 - 8)/(10 - 7)).$$
$$\text{10-year bond weight} = w_{10} = 33.3\% \ (\text{or } (1 - w_7)).$$
$$\text{Note that } (w_7 \times 7) + (w_{10} \times 10) = 8.$$

2. The interpolated 8-year swap rate is a weighted average of the 7-year swap rate (1.54% = 1.39% + 0.15%) and the 10-year swap rate (1.86% = 1.66% + 0.20%).

$$r_{Swap8yr} = w_7 \times r_{Swap7yr} + w_{10} \times r_{Swap10yr}$$
$$(66.7\% \times 1.54\%) + (33.3\% \times 1.86\%) = 1.647\%$$

3. The ASW equals the difference between the bank bond *coupon* of 2.75% and the 8-year swap rate of 1.647%, or 110.3 bps.
4. The I-spread is the difference between the bank bond's current YTM of 2.68% and the 8-year swap rate of 1.647%, or 103.3 bps.

The ASW is an estimate of the spread over MRR versus the bond's original coupon rate to maturity, while the I-spread is an estimate of the spread over MRR for a new par bond from the bank issuer, with the difference largely reflecting the premium or discount of the outstanding bond price.

While both the G-spread and I-spread use the same discount rate for each cash flow, a more precise approach incorporating the term structure of interest rates is to derive a constant spread over a government (or interest rate swap) spot curve instead. This spread is known as the **zero-volatility spread (Z-spread)** of a bond over the benchmark rate. The Z-spread formula shown in Equation 2 was introduced in an earlier reading.

$$PV = \frac{PMT}{(1 + z_1 + Z)^1} + \frac{PMT}{(1 + z_2 + Z)^2} + \ldots + \frac{PMT + FV}{(1 + z_N + Z)^N} \quad (2)$$

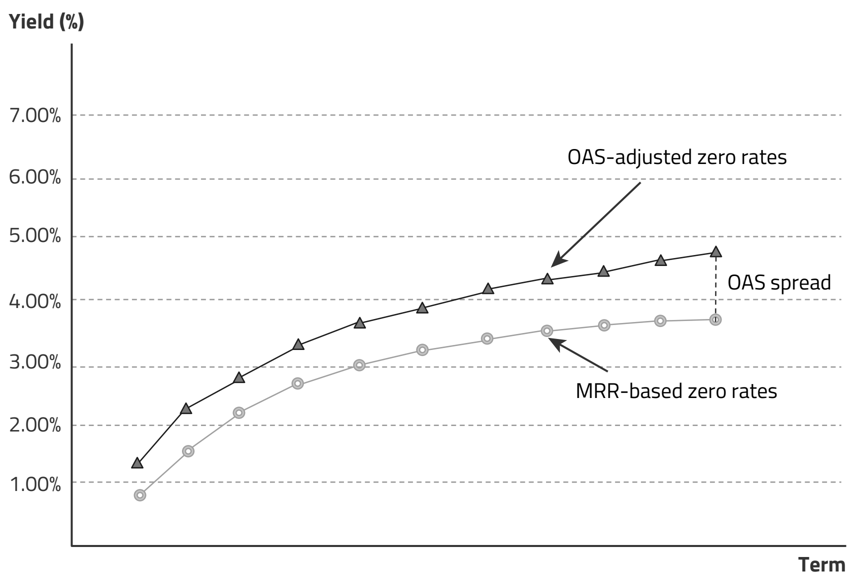
Here the bond price (PV) is a function of coupon (PMT) and principal (FV) payments in the numerator with respective benchmark spot rates $z_1 \ldots z_N$ derived from the swap or government yield curve and a constant Z-spread per period (Z) in the denominator discounted as of a coupon date. While more accurate than either the G-spread or I-spread, this is a more complex calculation that is conducted by practitioners using either a spreadsheet or other analytical model.

Credit default swap (CDS) basis refers to the difference between the Z-spread on a specific bond and the CDS spread of the same (or interpolated) maturity for the same issuer.

Recall from earlier in the curriculum that a CDS is a derivative contract in which a protection buyer makes a series of premium (or CDS spread) payments to a protection seller in exchange for compensation for credit losses (or the difference between par and the recovery rate) under a credit event. Negative basis arises if the yield spread is above the CDS spread, and positive basis indicates a yield spread in excess of the CDS market. Although spreads for a single issuer across bond and CDS markets should be closely aligned in principle, in practice, CDS basis arises because of such factors as bond price differences from par, accrued interest, and varying contract terms, among other items. As in the case of asset swaps, CDS basis is a pricing measure, but unlike ASW, a CDS contract is terminated and settled following a credit event with no residual interest rate swap mark-to-market exposure. Similar to the I-spread using swaps or the asset swap just mentioned, CDS basis is a useful credit measure for investors actively trading or hedging credit risk using CDS, as addressed in detail later.

The **option-adjusted spread (OAS)** is a generalization of the Z-spread calculation that incorporates bond option pricing based on assumed interest rate volatility. Earlier readings established the use of the term structure of zero rates combined with a volatility assumption to derive forward interest rates used to value bonds with embedded options. The OAS is the constant yield spread over the zero curve, which makes the arbitrage-free value of such a bond equal to its market price as shown in Exhibit 12. Note that the Z-spread for an option-free bond is simply its OAS, assuming zero volatility.

EXHIBIT 12 OAS



The OAS approach is the most appropriate yield spread measure for active fixed-income portfolio managers because it provides a consistent basis for comparing credit risk yield spreads for option-free, callable, putable, and structured fixed-income instruments. OAS calculations typically rely on fixed-income analytical models that incorporate the current term structure of interest rates, interest rate volatility, and term structure model factors introduced earlier in addition to the specific option-based features of a particular bond. Although OAS provides the best means to facilitate yield spread comparisons across different fixed-income securities, the main drawback of the OAS is that it is highly dependent on volatility and other model assumptions. For example, returns on structured financial instruments are highly dependent on prepayment versus extension risk, as outlined in an earlier lesson. While some analytical models calculate OAS using a standard or constant prepayment speed assumption, values based on historical or empirical analysis might provide very different and more accurate results. Also, the theoretical nature of the OAS calculation implies that bonds with embedded options are unlikely to realize the spread implied by the bond's OAS. Despite these shortcomings, OAS is the most widely accepted credit spread measure for comparing bonds with and without optionality across a fixed-income bond portfolio.

EXAMPLE 6 Portfolio OAS

A European portfolio manager is presented with the following information on a portfolio of two bonds. Calculate the OAS of the portfolio.

Issuer	Face Value	Price*	Accrued Interest*	OAS (bps p.a.)
Company A	€ 100,000,000	95	1.5	125
Company B	€ 200,000,000	97	2.0	150

*Per 100 of par value.

Solution:

1. Solve for bond and portfolio values:
   A. Company A bond: €96,500,000 = €100,000,000 × (0.95 + 0.015)
   B. Company B bond: €198,000,000 = €200,000,000 × (0.97 + 0.02)
   C. Portfolio: €294,500,000 = €96,500,000 + €198,000,000
2. Solve for portfolio weights w_A and w_B:
   A. Company A (w_A): 32.8% = €96,500,000/€294,500,000
   B. Company B (w_B): 67.2% = €198,000,000/€294,500,000
3. Solve for portfolio OAS using $(OAS_A \times w_A) + (OAS_B \times w_B)$.

$$\text{OAS} = 142 \text{ bps p.a.} = (0.328 \times 125 \text{ bps}) + (0.672 \times 150 \text{ bps})$$

Exhibit 13 summarizes these fixed-rate bond credit spread measures adapted from O'Kane and Sen (2005).

EXHIBIT 13 Key Fixed-Rate Bond Credit Spread Measures

Spread	Description	Advantages	Disadvantages
Yield spread	Difference between bond YTM and government benchmark of similar tenor	Simple to calculate and observe	Maturity mismatch, curve slope bias, and inconsistent over time
G-spread (Government spread)	Spread over interpolated government bond	Transparent and maturity matching default risk-free bond	Subject to changes in government bond demand
I-spread (Interpolated spread)	Yield spread over swap rate of same tenor	Spread versus market based (MRR) measure often used as hedge or for carry trade	Point estimate of term structure and limited to option-free bonds
ASW (Asset swap)	Spread over MRR of fixed bond coupon	Traded spread to convert current bond coupon to MRR plus a spread	Tradable spread rather than spread measure corresponding to cashflows and limited to option-free bonds
Z-spread (Zero volatility spread)	Yield spread over a government (or swap) spot curve	Accurately captures term structure of government or swap zero rates	More complex calculation limited to option-free bonds
CDS Basis	Yield spread versus CDS spread of same tenor	Interpolated CDS spread versus Z-spread	Traded spread rather than spread measure corresponding to cashflows and limited to option-free bonds
OAS	Yield spread using Z-spread including bond option volatility	Provides generalized comparison for valuing risky option-free bonds with bonds with embedded options	Complex calculation based on volatility and prepayment assumptions; bonds with embedded options are unlikely to earn OAS over time

EXAMPLE 7 Comparison of Fixed-Rate Bond Credit Spread Measures

1. An active manager observes a yield spread for an outstanding corporate bond that is above the G-spread for that same bond. Which of the following is the most likely explanation for the difference?
   A. The government benchmark bond used to calculate the yield spread has a *shorter* maturity than the corporate bond, and the benchmark yield curve is *upward* sloping.
   B. The government benchmark bond used to calculate the yield spread has a *shorter* maturity than the corporate bond, and the benchmark curve is *downward* sloping.
   C. The government benchmark bond used to calculate the yield spread has a *longer* maturity than the corporate bond, and the benchmark yield curve is *downward* sloping.

2. An active manager is weighing the purchase of two callable bonds with similar credit risks and the same final maturity. Which of the two bonds is more likely to be called on the next call date?
   A. The bond with the lower ASW
   B. The bond with the lower Z-spread
   C. The bond with the lower OAS

Solution to 1: The correct answer is A. For a given all-in YTM, the lower the (on-the-run or interpolated) benchmark rate, the higher the relevant spread measure over the benchmark. Therefore, the higher yield spread versus G-spread most likely arises from the government benchmark having a shorter maturity than the bond and an upward sloping government yield curve. As for B and C, the yield spread would be lower than the G-spread for a downward sloping yield curve.

Solution to 2: The correct answer is C. The OAS measure is best suited to compare the impact of embedded options on similar bonds because it incorporates a volatility assumption to account for the value of bond options. Answer A indicates the spread over MRR for an outstanding bond swapped versus the original coupon rate, while the Z-spread in B assumes zero volatility and therefore does not capture the value of bond options.

2.2.2. Floating-Rate Note Credit Spread Measures

In contrast to fixed-rate bonds, floating-rate notes (FRNs) pay a periodic interest coupon comprising a variable MRR plus a (usually) constant yield spread. While fixed- and floating-rate bonds both decline in price if credit risk rises, interest rate risk on these bond types differs, and the associated FRN credit spread measures warrant our attention.

An earlier reading provided a simplified framework for valuing a floating-rate bond on a payment date, shown in Equation 3:

$$PV = \frac{\left(\frac{(\text{MRR} + \text{QM}) \times \text{FV}}{m}\right)}{\left(1 + \frac{(\text{MRR} + \text{DM})}{m}\right)^1} + \frac{\left(\frac{(\text{MRR} + \text{QM}) \times \text{FV}}{m}\right)}{\left(1 + \frac{(\text{MRR} + \text{DM})}{m}\right)^2} + \cdots + \frac{\left(\frac{(\text{MRR} + \text{QM}) \times \text{FV}}{m}\right) + \text{FV}}{\left(1 + \frac{(\text{MRR} + \text{DM})}{m}\right)^N} \quad (3)$$

Each interest payment is MRR plus the **quoted margin** (QM) times par (FV) and divided by m, the number of periods per year. Rather than a fixed YTM as for fixed-rate bonds, the periodic discount rate per period is MRR plus the **discount margin** (DM) divided by the periodicity (m), or (MRR + DM)/m. Note that for the purposes of Equation 3, MRR is based on current MRR and therefore implies a flat forward curve. The QM is the yield spread over the MRR established upon issuance to compensate investors for assuming the credit risk of the issuer. While some FRN bond indentures include an increase or decrease in the QM if public ratings or other criteria change, given that this spread is usually fixed through maturity, the QM does not reflect credit risk changes over time.

The discount (or required) margin is the yield spread versus the MRR such that the FRN is priced at par on a rate reset date. For example, assume an FRN issued at par value pays three-month MRR plus 1.50%. The QM is 150 bps. If the issuer's credit risk remains unchanged, the DM also equals 150 bps. On each quarterly reset date, the floater will be priced at par value. Between coupon dates, the flat price will be at a premium or discount to par value if MRR falls or rises. If on a reset date, the DM falls to 125 bps because of an issuer upgrade, the FRN will be priced at a premium above par value. The amount of the premium is the present value of the premium future cash flows. The annuity difference of 25 bps per period is calculated for the remaining life of the bond. Exhibit 14 summarizes the relationship between the QM versus DM and an FRN's price on any reset date.

EXHIBIT 14 FRN Discount, Premium, and Par Pricing

FRN price	Description	QM versus DM
Par	FRN trades at a price (PV) equal to its future value (FV)	QM = DM
Discount	FRN trades at PV < FV	QM < DM
Premium	FRN trades at PV > FV	QM > DM

EXAMPLE 8 Discount Margin

A London-based investor owns a five-year £100 million FRN that pays three-month MRR + 1.75% on a quarterly basis. The current MRR of 0.50% is assumed to remain constant over time. If the issuer's credit risk deteriorates and the DM rises to 2.25%, explain whether the FRN is trading at a discount or premium, and calculate the price difference from par.

Solution: The FRN is trading at a discount because the QM is below the DM. We can solve for the price difference using the following steps.

1. Solve for the quarterly interest payment (= (MRR + QM) × FV/m) in the numerator and the discount rate (= (MRR + DM)/m) in the denominator of Equation 3 with QM = 1.75%, DM = 2.25%, MRR = 0.50%, and m = 4.
   A. Quarterly interest payment: £562,500 (= (0.50% + 1.75%) × £100,000,000/4)
   B. Discount rate: 0.6875% (= (0.50% + 2.25%)/4)

2. Solve for the new price using results from 1A and 1B with N = 20.

$$£97{,}671{,}718 = \frac{£562{,}500}{(1 + 0.6875\%)} + \frac{£562{,}500}{(1 + 0.6875\%)^2} + \frac{£562{,}500}{(1 + 0.6875\%)^3} + \ldots + \frac{£100{,}562{,}500}{(1 + 0.6875\%)^{20}}$$

3. The price difference is £2,328,282 (= £100,000,000 – £ 97,671,718).

The **zero-discount margin (Z-DM)** incorporates forward MRR into the yield spread calculation for FRNs. As in the case of the zero-volatility spread for fixed-rate bonds shown earlier, the Z-DM is the fixed periodic adjustment applied to the FRN pricing model to solve for the observed market price. As Equation 4 shows, this calculation incorporates the respective benchmark spot rates z_i derived from the swap or government yield curve for the Z-spread into the FRN pricing model shown earlier.

$$\text{PV} = \frac{\left(\frac{(\text{MRR} + \text{QM}) \times \text{FV}}{m}\right)}{\left(1 + \frac{(\text{MRR} + Z - \text{DM})}{m}\right)^1} + \frac{\left(\frac{(z_2 + \text{QM}) \times \text{FV}}{m}\right)}{\left(1 + \frac{(z_2 + Z - \text{DM})}{m}\right)^2} + \cdots + \frac{\left(\frac{(z_N + \text{QM}) \times \text{FV}}{m}\right) + \text{FV}}{\left(1 + \frac{(z_N + Z - \text{DM})}{m}\right)^N} \quad (4)$$

As in the case of the Z-spread for fixed-rate bonds, the Z-DM will change based on changes in the MRR forward curve. For example, in an upward-sloping yield curve, the Z-DM will be below the DM. Also, the Z-DM assumes an unchanged QM and that the FRN will remain outstanding until maturity. Exhibit 15 summarizes FRN credit spreads as adapted from O'Kane and Sen (2005).

EXHIBIT 15 Key FRN Credit Spread Measures

Spread	Description	Advantages	Disadvantages
QM	Yield spread over MRR of original FRN	Represents periodic spread related FRN cash flow	Does not capture changes in credit risk over time
DM	Yield spread over MRR to price FRN at par	Establishes spread difference from QM with constant MRR	Assumes a flat MRR zero curve
Z-DM	Yield spread over MRR curve	Incorporates forward MRR rates in yield spread measure	More complex calculation and yield spread does not match FRN cash flows

EXAMPLE 9 Floating-Rate Credit Spread Measure

An Australian investor holds a three-year FRN with a coupon of three-month MRR + 1.25%. Given an expected strong economic recovery, she anticipates a rise in Australian MRR over the next three years and an improvement in the FRN issuer's creditworthiness. Which of the following credit spread measures does she expect to be the *lowest* as a result?

A. QM
B. DM
C. Z-DM

Solution: The correct answer is C. The QM will be above the DM if issuer creditworthiness improves. As MRRs rise over the next three years, the upward-sloping curve will cause the Z-DM to remain below the DM.

2.2.3. Portfolio Return Impact of Yield Spreads

We now turn from credit spread measures to their impact on expected portfolio return. The first and third variables in Equation 1, namely roll-down return and E (Δ Price due to investor's view of yield spreads), are directly relevant for active managers targeting excess return above a benchmark portfolio using credit strategies.

In the first instance, recall from earlier lessons that investors "rolling down" the yield curve accumulate coupon income and additional return from fixed-rate bond price appreciation over an investment horizon if benchmark rates are positive and the yield curve slopes upward. For fixed-rate bonds priced at a spread over the benchmark, roll-down return from coupon income is higher by the bond's original credit spread. The roll-down return due to price appreciation will also be higher than for an otherwise identical government security because the higher-yielding instrument will generate greater carry over time. Note that this higher return comes with greater risk and assumes all promised payments take place and the bond remains outstanding—that is, no default or prepayment occurs, and the bond is not called.

EXAMPLE 10 Corporate versus Government Bond Roll Down

A London-based investor wants to estimate roll-down return attributable to a fixed-rate, option-free corporate bond versus UK gilts over the next six months assuming a static, upward-sloping government yield curve and a constant credit spread. The corporate bond has exactly 10 years remaining to maturity, a semiannual coupon of 3.25%, and a YTM of 2.75%, while the closest maturity UK gilt is a 1.75% coupon currently yielding 1.80%, with 9.5 years remaining to maturity.

1. Calculate the annualized roll-down return to the UK corporate bond versus the government bond over the next six months.
2. Describe how the relative roll-down return would change if the investor were to use an interpolated government benchmark rather than the actual 9.5-year gilt.

Solution to 1: Solve for the annualized difference in roll-down return by calculating the change in price plus the coupon income for both the corporate bond and the government bond.

1. Calculate the corporate bond roll-down return per £100 face value. For price changes, use the Excel PV function (= –PV(rate, nper, pmt, FV, type)) where "rate" is the interest rate per period (0.0275/2), "nper" is the number of periods (20), "pmt" is the periodic coupon (3.25/2), "FV" is future value (100), and "type" corresponds to payments made at the end of each period (0).
   A. Initial price is 104.346 (= –PV (0.0275/2, 20, 3.25/2, 100, 0)).
   B. Price in six months is 104.155 (= –PV (0.0275/2, 19, 3.25/2, 100, 0)). Price *depreciation* is 0.18% (= (104.155 – 104.346)/104.346).
   C. Six-month coupon income is 1.625 (= 3.25/2), or equal to 1.557% (= 1.625/104.346), which combined (without rounding) with –0.18% from B results in a 1.375% six-month return (2.75% annualized).

2. Calculate the UK gilt price change and coupon income.
   A. Initial price is 99.565 (= –PV (0.018/2, 19, 1.75/2, 100, 0)).
   B. Price in six months is 99.586 (= –PV (0.018/2, 18, 1.75/2, 100, 0)). Price *appreciation* is 0.021% (= (99.586 – 99.565)/99.565).
   C. Six-month coupon income is 0.875 (=1.75/2), or equal to 0.879% (0.875/99.565), which combined with +0.021% equals 0.9% for six months (1.80% annualized).

The annualized roll-down return difference is the 2.75% corporate bond realized return less the 1.80% UK gilt realized return, or 0.95%.

Solution to 2: The interpolated benchmark involves the use of the most liquid, on-the-run government bonds to derive a hypothetical 10-year UK gilt YTM. Because the UK gilt yield curve is upward sloping in this example, we can conclude that the relative roll-down return using an interpolated benchmark would be lower than the 0.95% difference in Question 1.

Active credit managers often view the E (Δ Price due to investor's view of yield spreads) term in Equation 1 on a stand-alone basis because they manage benchmark rate risks separately from credit. Equation 5 is similar to equations from earlier lessons quantifying the change in bond price for a given YTM change, but it is limited here to yield spread changes, or $\%\Delta PV^{Spread}$ (= ΔPV/ΔSpread).

$$\%\Delta\,PV^{Spread} \approx -(\text{EffSpreadDur} \times \Delta\text{Spread}) + (\tfrac{1}{2} \times \text{EffSpreadCon} \times (\Delta\text{Spread})^2) \quad (5)$$

where effective spread duration (EffSpreadDur) and effective spread convexity (EffSpreadCon) reflect spread rather than curve changes, and ΔSpread is typically defined as the change in OAS.

$$\text{EffSpreadDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Spread})(PV_0)} \quad (6)$$

$$\text{EffSpreadCon} = \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta\text{Spread})^2 \times (PV_0)} \quad (7)$$

The first term of Equation 5 is sometimes simply referred to as **spread duration**, or, alternatively, as **OAS duration** when OAS is the underlying spread. Active managers approximate bond portfolio value changes due to spread changes by substituting market value–weighted averages for the duration and convexity measures in Equation 5. As noted earlier, spread changes for lower-rated bonds tend to be consistent on a proportional percentage rather than absolute basis; therefore, adjusting spread duration to capture this **Duration Times Spread (DTS)** effect is important, as in Equation 8.

$$\text{DTS} \approx (\text{EffSpreadDur} \times \text{Spread}) \quad (8)$$

A portfolio's DTS is the market value–weighted average of DTS of its individual bonds, and spread changes of a portfolio are measured on a percentage (ΔSpread/Spread) basis rather than in absolute basis point terms, as in the following example.

EXAMPLE 11 DTS Example

A financial analyst compares a portfolio evenly split between two technology company bonds trading at par to an index with an average OAS of 125 bps.

Issuer	OAS	EffSpreadDur
A Rated Bond	100 bps	3.0
BB Rated Bond	300 bps	4.0

Calculate the portfolio DTS, and estimate how the technology bond portfolio will perform if index OAS widens by 10 bps.

Solution: Portfolio DTS is the market value–weighted average of DTS based on Equation 8, or $\sum_{i=1}^{n} w_i(\text{EffSpreadDur}_i \times \text{Spread}_i)$.

1. Portfolio DTS in this two-asset example is $w_A(\text{EffSpreadDur}_A \times \text{Spread}_A) + w_{BB}(\text{EffSpreadDur}_{BB} \times \text{Spread}_{BB})$ with equal weights ($w_A = w_{BB} = 0.50$). Solve for portfolio DTS of 750 (= (0.5 × 100 bps × 3.0) + (0.5 × 300 bps × 4.0)).
2. Index spread widening of 10 bps is equivalent to 8% (10 bps/125 bps spread) on a ΔSpread/Spread basis. We can therefore calculate the estimated basis point change in the technology bond portfolio by multiplying the portfolio DTS of 750 by the 8% expected percentage spread change to get an expected 60 bps p.a. widening for the technology bond portfolio.

As active credit managers consider *incremental* effects of credit-based portfolio decisions, they often use spread duration–based statistics to gauge the first-order impact of spread movements. For example, Equation 9 approximates the annualized **excess spread** return for a spread-based bond:

$$\text{ExcessSpread} \approx \text{Spread}_0 - (\text{EffSpreadDur} \times \Delta\text{Spread}) \quad (9)$$

Spread_0 is the initial yield spread, which changes to (Spread_0/Periods Per Year) for holding periods of less than a year. Note that this calculation assumes no defaults for the period in question. While relatively rare, as an event of default grows more likely, expected future bond cash flows are impaired, and a bond's value instead approaches the present value of expected recovery. The annualized expected excess return shown in Equation 10 incorporates both default probability and loss severity:

$$\text{E [ExcessSpread]} \approx \text{Spread}_0 - (\text{EffSpreadDur} \times \Delta\text{Spread}) - (\text{POD} \times \text{LGD}) \quad (10)$$

Equation 10 captures a key goal of active credit management, which is to maximize expected spread return in excess of the portfolio credit loss or realized percentage of par value lost to defaults over time.

EXAMPLE 12 Excess Spread and Expected Excess Spread

A corporate bond has an effective spread duration of five years and a credit spread of 2.75% (275 bps).

1. What is the approximate excess return if the bond is held for six months and the credit spread narrows 50 bps to 2.25%? Assume the spread duration remains at five years and that the bond does not experience default losses.
2. What is the instantaneous (holding period of zero) excess return if the spread rises to 3.25%?
3. Assume the bond has a 1% annualized expected POD and expected loss severity of 60% in the event of default. What is the expected excess return if the bond is held for six months and the credit spread is expected to fall to 2.25%?

Solution to 1: Using Equation 9 (Spread_0 – (EffSpreadDur × ΔSpread)), the excess return on the bond is 3.875% = (2.75% × 0.5) – [(2.25% – 2.75%) × 5].

Solution to 2: Using Equation 9, the instantaneous excess return on the bond is approximately –2.5% = (2.75% × 0) – [(3.25% – 2.75%) × 5].

Solution to 3: Using Equation 10 (Spread_0 – (EffSpreadDur × ΔSpread) – (POD × LGD)), the expected excess return on the bond is approximately 3.575% = (2.75% × 0.5) – [(2.25% – 2.75%) × 5] – (0.5 × 1% × 60%).

Finally, we must address the difference in duration as an interest rate sensitivity measure for FRNs versus fixed-rate bonds. The periodic reset of MRRs in both the FRN numerator and denominator leads to a *rate* duration of near zero for floaters trading at par on a reset date (prior to MRR reset). As we saw in an earlier DM example, changes in *spread* (DM or Z-DM) are the key driver of price changes for a given FRN yield change. The respective FRN rate and spread duration measures are shown in Equations 11 and 12 and demonstrated in the following example.

$$\text{EffRateDur}_{\text{FRN}} = \frac{(\text{PV}_{-}) - (\text{PV}_{+})}{2 \times (\Delta\text{MRR})(\text{PV}_0)} \tag{11}$$

$$\text{EffSpreadDur}_{\text{FRN}} = \frac{(\text{PV}_{-}) - (\text{PV}_{+})}{2 \times (\Delta\text{DM})(\text{PV}_0)} \tag{12}$$

We return to the example of a five-year £100 million FRN at three-month MRR + 1.75%, with a DM of 2.25% and a 0.50% MRR priced at £97,671,718. We can derive the FRN's effective rate duration by first calculating PV_- and PV_+ using a spreadsheet by shifting MRR down and up by 0.05% as follows:

$$PV_0 = £97{,}671{,}718 = \frac{£562{,}500}{(1 + 0.6875\%)} + \frac{£562{,}500}{(1 + 0.6875\%)^2} + \cdots + \frac{£100{,}562{,}500}{(1 + 0.6875\%)^{20}}$$

$$PV_- = £97{,}668{,}746 = \frac{£550{,}000}{(1 + 0.6750\%)} + \frac{£550{,}000}{(1 + 0.6750\%)^2} + \cdots + \frac{£100{,}550{,}000}{(1 + 0.6750\%)^{20}}$$

$$PV_+ = £97{,}674{,}685 = \frac{£575{,}000}{(1 + 0.7000\%)} + \frac{£575{,}000}{(1 + 0.7000\%)^2} + \cdots + \frac{£100{,}575{,}000}{(1 + 0.7000\%)^{20}}$$

Solving for $\text{EffRateDur}_{\text{FRN}}$, we arrive at a rate duration of −0.061, which is slightly negative because the floater trades at a discount. The spread duration statistic $\text{EffSpreadDur}_{\text{FRN}}$ is calculated in a similar manner by shifting DM down and up by 0.05%, with PV− and PV+ equal to £97,972,684 and £97,515,401 and $\text{EffSpreadDur}_{\text{FRN}}$ equal to 4.682.

3. CREDIT STRATEGIES

3.1. Bottom-Up Credit Strategies

- **discuss bottom-up approaches to credit strategies**

As active fixed-income managers consider the selection process for spread-based bond portfolio investments, they must assess different ways in which to maximize excess spread across the fixed-income issuer types, industries, and instruments within their prescribed investment mandate. A fundamental choice these investors face is whether to engage in an individual security selection process or bottom-up approach; a macro- or market-based, top-down approach in pursuing this objective; or a combination of both.

Fundamental credit analysis covered earlier in the curriculum considers the basis on which a specific issuer can satisfy its interest and principal payments through bond maturity. Analysts often assess unsecured corporate bonds using factors such as profitability and leverage to identify the sources and variability of cash flows available to an issuer to service debt. These measures are usually chosen and compared relative to an industry and/or the jurisdiction in which the issuer operates. In the case of a sovereign borrower, the relevant metric is the economic activity within a government's jurisdiction and the government's ability and willingness to levy taxes and generate sufficient revenue to meet its obligations. Alternatively, for a special purpose entity issuer with bonds backed by mortgage-based or other securitized cash flows, a credit measure of both the residential borrowers and underlying collateral value as well as internal credit enhancements are among the primary factors considered in the assessment.

While individual bonds across all these issuer types are usually rated by at least two of the major credit rating agencies, active managers typically conduct their own credit assessment of individual borrowers rather than relying on ratings, which are frequently used to define a mandate (e.g., investment grade versus high yield), categorize, or benchmark investments of similar credit quality.

3.1.1. Defining the Credit Universe

A bottom-up approach typically begins with a manager defining the universe of eligible bonds within a mandate and then grouping the universe into categories that allow consistent relative value analysis across comparable borrowers. For example, a corporate bond portfolio manager is likely to divide eligible bonds into industry sectors, such as media and telecommunications and industrials, as well as into subsectors and/or firms located in different jurisdictions. Media and telecommunications subsectors include firms in the cable and satellite industries, internet media, and telecommunications carriers. Within each sector or subsector based on either industry classification methodologies or a customized approach, she can use relative value analysis to determine the bonds that are attractively valued.

EXAMPLE 13 Dividing the Credit Universe

An investor is conducting a relative value analysis on global bond issuers in the health care sector. He is trying to decide whether the global health care sector is a sufficiently narrow sector for his analysis. Through his research, he has determined the following:

- Biotech and pharmaceutical companies are active globally across Europe, Asia, and the Americas.
- Health care facilities are typically local in nature and tend to sell into only one of these three regions.
- Medical equipment and devices is a more cyclical business, and many of these firms are part of multi-industry companies in which health care accounts for a smaller fraction of overall company sales.

Describe considerations that the investor can use in determining how to best divide the health care sector into comparable companies.

Solution: An investor typically seeks to isolate a sector that contains a set of companies for which he expects company-level risks, rather than industry or macro risks, to be the dominant factors. Based on the investor's analysis, biotech and pharmaceutical companies differ meaningfully from health care facilities and medical equipment manufacturers. Health care facilities have a narrow regional focus in contrast to the global focus in biotech and pharma.

The investor might therefore want to divide the global health care sector into global biotech and global pharmaceuticals. Hospitals and other health care facilities warrant separate treatment given their narrow geographic focus and different industry drivers. He might want to consider a different approach to medical device companies given their multi-industry profiles.

3.1.2. Bottom-Up Credit Analysis

Once the credit universe has been divided into sectors and prospective bonds identified, the investor evaluates each issuer's implied credit risk comparing company-specific financial information to spread-related compensation for assuming default, credit migration, and liquidity risks for comparative purposes.

Beyond the prospects within a company's industry, its competitive position within that industry, and operating history, financial ratios are a valuable tool to compare creditworthiness across firms. Earlier lessons stressed the value of key ratios, including profitability and cash flow, leverage, and debt coverage, which are summarized in Exhibit 16.

EXHIBIT 16 Key Financial Ratios for Bottom-Up Credit Analysis

Ratio	Description	Advantages	Disadvantages
EBITDA/ Total Assets	**Profitability** Cash flow as a percentage of assets	Combines operating income with non-cash expense	Ignores capital expenditures and working capital changes
Debt/ Capital	**Leverage** Fraction of company's capital financed with debt	Direct measure of relative reliance on debt financing	More relevant for investment-grade than high-yield issuers
EBITDA/ Interest Expense	**Coverage** Cash flow available to service debt	Measures relative issuer ability to meet debt payments	Volatile measure for firms with high cash flow variability

While offering a relatively consistent basis for comparison across firms and over time, reliance on financial ratios based on publicly available accounting data alone is of limited value because of comparability issues across firms and industries as well as the historical nature of financial statements. Alternative measures combine several relevant financial ratios with market-based measures to establish a forward-looking approach to creditworthiness.

A previous lesson established that statistical credit analysis models to measure individual issuer creditworthiness can be categorized as either **reduced form credit models** or **structural credit models**. Reduced form models solve for **default intensity**, or the POD over a specific time period, using observable company-specific variables such as financial ratios and recovery assumptions as well as macroeconomic variables, including economic growth and market volatility measures. Structural credit models use market-based variables to estimate the market value of an issuer's assets and the volatility of asset value. The likelihood of default is defined as the probability of the asset value falling below that of liabilities.

An early example of the reduced form approach is the **Z-score** established by Altman (1968), which combined liquidity (working capital/total assets), profitability (retained earnings/total assets), asset efficiency (EBIT/ total assets), market versus book value of equity, and asset turnover (sales/total assets) factors weighted by coefficients to form a composite score. Each composite, or Z-score, was used to classify manufacturing firms into those expected to remain solvent and those anticipated to go bankrupt. Similar to credit scoring models, this multiple discriminant analysis reduces the dimensionality of the input variables to a single cutoff Z-score that represents the default threshold, as shown in the following example.

EXAMPLE 14 Z-Score Comparison of Two Firms

A United Kingdom–based financial analyst considers a Z-score model in evaluating two publicly traded non-manufacturing companies as follows:

$$\text{Z-Score Model} = 1.2 \times A + 1.4 \times B + 3.3 \times C + 0.6 \times D + 0.999 \times E,$$

where

A is Working Capital/Total Assets

B is Retained Earnings/Total Assets

C is EBIT/Total Assets

D is Market Value of Equity/Total Liabilities

E is Sales/Total Assets

Firms with a Z-score greater than 3.0 are considered financially sound, those scoring between 3.0 and 1.8 are at greater risk of financial distress, and those with a Z-score below 1.8 are likely to face insolvency.

1. Calculate the Z-score for Firm 1 and Firm 2. Which has a higher likelihood of financial distress based on this measure?

Financial Data (GBP thousands)/Firm	Firm 1	Firm 2
Total Sales	23,110	15,270
EBIT	6,910	2,350
Current Assets	7,560	4,990
Total Assets	36,360	23,998
Current Liabilities	5,400	3,564
Total Liabilities	9,970	10,050
Retained Earnings	20,890	13,787
Market Value of Equity	29,000	18,270

2. Evaluate the most likely reasons for the difference in creditworthiness between the two firms based on the Z-score model factors.

Solution to 1: First, calculate the respective ratios for both firms as follows, noting that working capital is equal to current assets minus current liabilities:

Z-Score Factors	Firm 1	Firm 2
Working Capital/Assets	0.059	0.059
Retained Earnings/Assets	0.575	0.575
EBIT/Total Assets	0.190	0.098
Market Value of Equity/Total Liabilities	2.909	1.818
Sales/Total Assets	0.636	0.636

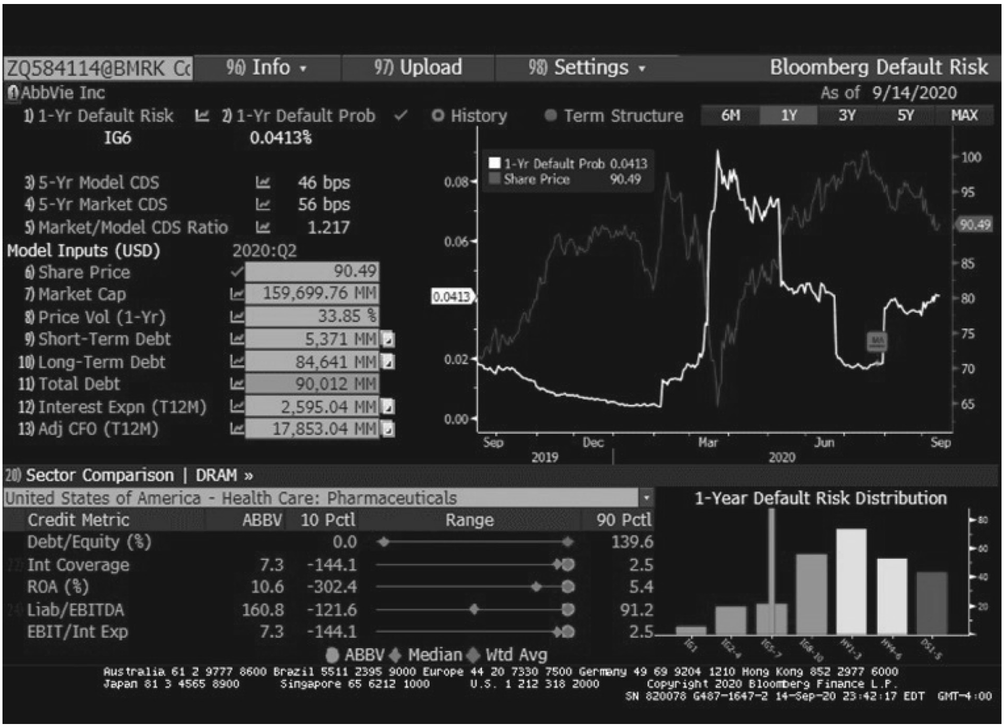
Solving for the respective Z-scores, we find that Firm 1 has a Z-score of 3.883, while Firm 2 has a Z-score of 2.925. Firm 2 therefore has a greater likelihood of financial distress.

Solution to 2: Comparing the respective Z-score ratios of Firm 1 and Firm 2, we find that Firm 2 has a far lower asset efficiency (EBIT/Total Assets of 9.8% versus 19% for Firm 1) and a lower relative equity market value (Market Value of Equity/Total Liabilities of 1.818 versus 2.909 for Firm 1) than Firm 1, while all other ratios are comparable.

Structural credit models used in practice include Moody's Analytics Expected Default Frequency (EDF) and Bloomberg's Default Risk (DRSK) models, both of which provide daily POD estimates for a broad range of issuers over a selected period. The EDF model estimates a forward-looking POD defined as the point at which the market value of assets falls below a firm's obligations. The model uses asset volatility to determine the likelihood of reaching the default point and is calibrated for different industries, regions, and observed credit market dynamics.

Bloomberg's DRSK model estimate for AbbVie Inc., as shown in Exhibit 17, includes a market-based asset value measure derived from equity market capitalization and equity volatility as well as a default threshold measured using the book value of liabilities. These and other DRSK model inputs in the left column of the screen can be defined by users and compared within and across industry sectors. In addition to the one-year POD estimate of 0.0413%, DRSK calculates a "model" CDS spread (upper left corner), which can be compared to the actual market CDS spread.

EXHIBIT 17 Bloomberg DRSK Model Estimate for AbbVie Inc.



Source: Bloomberg.

Both the EDF and DRSK approaches are sometimes referred to as "distance to default" models because a probability distribution is used to determine how far an issuer's current market value of assets is from the default threshold for a given period.

EXAMPLE 15 "Distance to Default" Models

An active manager is weighing an investment in the bonds of two issuers in the same industry with identical PODs using a structural credit model. Which of the following changes to the model inputs for one of the issuers would lead the analyst to expect an increase in the POD for that issuer?

A. An increase in the issuer's coverage ratio
B. An increase in the volatility of the issuer's stock price
C. A decrease in the issuer's leverage ratio

Solution: The correct answer is B. Higher equity volatility increases the likelihood that the market value of the issuer's assets will fall below the default threshold. A higher coverage ratio in A implies higher cash flow as a percentage of assets, increasing the issuer's ability to service its debt obligations. The decrease in the issuer's leverage ratio in C represents a decline in the amount of debt versus equity, reducing the issuer's likelihood of financial distress.

3.1.3. Bottom-Up Relative Value Analysis

Given two issuers with similar credit risk, the investor will typically choose bonds of the issuer with the higher yield spread, given the greater potential for excess returns. For issuers with different credit-related risk, the investor must decide whether the additional spread is sufficient compensation for the incremental exposure. The excess expected return calculation in Equation 10 captures the relationship between yield spreads and the components of credit risk, as seen in the following example.

EXAMPLE 16 Comparing Investments Using Expected Excess Return

A portfolio manager considers two industrial bonds for a one-year investment:

Issuer	Rating	EffSpreadDur	YTM	Z-Spread
A Rated Industrial	A2	5.0	4.0%	100 bps
B Rated Industrial	B2	7.0	6.5%	350 bps

The manager observes a historical annual default probability of 0.27% for A2 rated issuers and 3.19% for B2 rated issuers and assumes a 40% recovery rate for both bonds.

1. Compute the estimated excess return for each bond assuming no change in spreads, and interpret whether the B rated bond spread provides sufficient compensation for the incremental risk.
2. Which bond is more attractive if spreads are expected to widen by 10%?

Solution to 1: As per Equation 10,

$$E\ [\text{ExcessSpread}] \approx \text{Spread}_0 - (\text{EffSpreadDur} \times \Delta\text{Spread}) - (\text{POD} \times \text{LGD}).$$

A rated expected excess return is 0.84% = 1% – (5 × 0) – (0.27% × 60%). B rated expected excess return is 1.59% = 3.5% – (7 × 0) – (3.19% × 60%). The B rated bond appears to provide sufficient compensation for the added risk.

Solution to 2: Recalculate Equation 10 with ΔSpread of 10 bps for the A rated bond and 35 bps for the B rated bond.

A rated excess return is 0.34% = 1% – (5 × 0.1%) – (0.27% × 60%).
B rated excess return is –0.86% = 3.5% – (7 × 0.35%) – (3.19% × 60%).

The A rated bond is more attractive under this scenario.

In practice, bonds from different issuers usually also have various maturity, embedded call or put provisions, liquidity, and other characteristics, so these additional features should be taken into account during the security selection process. For example, structural differences such as callability or priority within the capital structure must be factored in because they affect valuation. Also, bonds recently issued in larger tranches by frequent issuers will tend to have narrower bid–offer spreads and greater daily transaction volume, allowing investors to buy or sell the bond at a lower cost. This feature is likely to be of greater importance to investors who expect short-term spread narrowing and/or have a relatively short investment time horizon. Note that relative liquidity tends to decline over time, particularly if the same issuer returns to the bond market and offers a price concession for new debt. If, on the other hand, an investor has a longer investment horizon with the flexibility to hold a bond to maturity, he might be able to increase excess return via a greater liquidity premium. Finally, other factors driving potential yield spread differences to be considered include split ratings or negative ratings outlooks, potential merger and acquisition activity, and other positive or negative company events not adequately reflected in the analysis.

When deciding among frequent issuers with several bond issues outstanding, investors might consider using credit spread curves for these issuers across maturities to gauge relative value.

EXAMPLE 17 Using Spread Curves in Relative Value Analysis

A United States–based issuer has the following option-free bonds outstanding:

Outstanding Debt	Term	Coupon	Price	YTM
2-year issue	2	4.25%	106.7	0.864%
5-year issue	5	3.25%	106	1.984%
15-year issue	15	2.75%	91	3.528%

Current on-the-run US Treasury YTMs are as follows:

Tenor	Coupon	Price
2y	0.250%	100
5y	0.875%	100
10y	2.000%	100
20y	2.250%	100

An investor considers the purchase of a new 10-year issue from the company and expects the new bond to include a 10 bp new issue premium. What is the fair value spread for the new issue based on outstanding debt?

1. First, solve for the credit spreads for outstanding bonds as the difference in the YTM from an actual or interpolated government bond:

   5-year spread: 110.9 bps (= 1.984% – 0.875%)
   15-year spread: Solve for 10- and 20-year bond interpolation weights.
   10-year weight: w_{10} = 0.50% (= (20 – 10)/(15 – 10))
   20-year weight: w_{20} = 0.50% (= (1 – w_{10}))
   15-year interpolated bond: 2.125% = (2.00% × 0.5) + (2.25% × 0.5)
   15-year spread: 140.3 bps (= 3.528% – 2.125%)

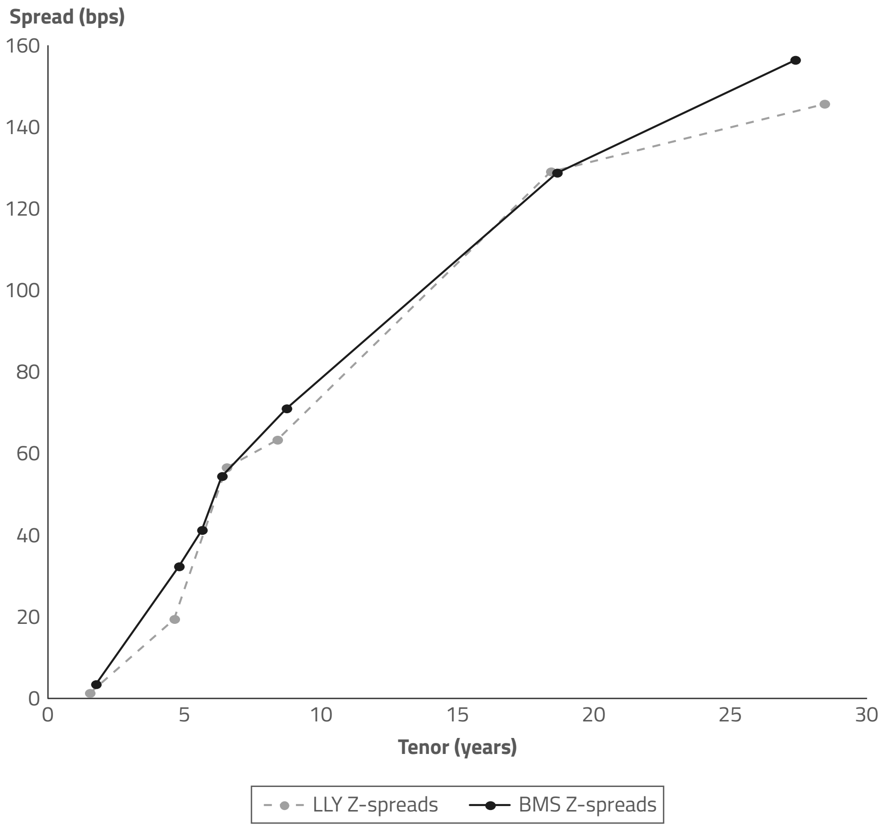
2. Derive the implied 10-year new issue spread by interpolating the 5- and 15-year credit spreads using the same interpolation weights as for Treasuries and adding the 10 bp new issue premium.

   10-year spread: 135.6 bps = 0.1% + (1.109% × 0.5 + 1.403% × 0.5)

Many issuers have several bond issues, each of which typically has a different maturity and duration. To reflect the various maturities, a spread curve can be developed for each issuer and can be useful in conducting relative value analysis. A spread curve is the fitted curve of credit spreads for similar bonds of an issuer plotted against the maturity of those bonds.

Exhibit 18 plots the Z-spread versus maturities for select outstanding bonds of two A2/A+ rated health care companies, Eli Lilly (LLY) and Bristol-Myers Squibb (BMS), which have similar probabilities of default.

EXHIBIT 18 Spread Curves for Eli Lilly and Bristol-Myers Squibb



Source: Bloomberg.

These spread curves are closely aligned except in roughly five-year and nearly 30-year maturities, where the BMS spreads are approximately 10 bps wider than those of LLY. If the bonds have similar features and liquidity, then a manager might conclude that the market perceives BMS credit risk to be slightly higher than that of LLY. However, if the manager believes that BMS is the stronger credit, several actions are possible depending on portfolio objectives and constraints. For example, if the investment mandate is to outperform a benchmark using long-only positions, the manager might overweight BMS bonds and underweight LLY bonds relative to the benchmark. If the objective is to generate positive absolute returns, underweighting or avoiding LLY bonds is less appropriate because such actions are meaningful only in the context of a benchmark. If permitted, the manager could also consider a long–short CDS strategy outlined later.

Once a manager has identified specific issuers and bond maturities to actively over- or underweight versus a benchmark, the next important step is to quantify and track these active investments in the context of the primary indexing risk factors identified in an earlier lesson in the active portfolio construction process. For example, if an investor chooses to overweight specific health care industry issuers versus the respective sector and spread duration contributions of the benchmark index, the difference in portfolio weights between the active and index positions establishes a basis upon which excess return can be measured going forward.

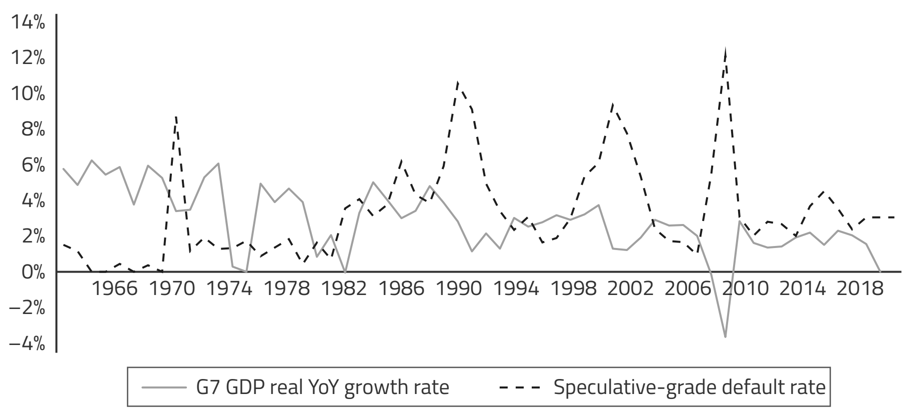
3.2. Top-Down Credit Strategies

- **discuss top-down approaches to credit strategies**

A top-down approach to credit strategy focuses on a broader set of factors affecting the bond universe in contrast to the more detailed and issuer-specific bottom-up approach. Macro factors critical to credit investors include economic growth, real rates and inflation, changes in expected market volatility and risk appetite, recent credit spread changes, industry trends, geopolitical risk, and currency movements. Assessment of these factors guides investors in selecting credit market sectors with attractive relative value characteristics, with an increased bond allocation to more attractive sectors and an underweight (or possibly short bond positions in) less favorable sectors. Top-down investors frequently use broader sector distinctions than under a bottom-up approach. For example, a top-down investor expecting credit spreads to narrow might favor the relative value opportunity of high-yield bonds over investment-grade bonds.

GDP growth is critical to the credit cycle, as seen in Exhibit 19, which shows global speculative-grade default rates versus the real GDP growth rate among G7 countries from 1962 to 2019. Sharp declines in GDP growth are often associated with rising default rates.

EXHIBIT 19 Global Speculative-Grade Default Rate and Real GDP Growth Rate for G7 countries, 1962–2019



Sources: Moody's Investors Service, OECD (IHS Markit).

A portfolio manager or analyst might decide to factor this relationship into the investment decision-making process; for example, an above-consensus real GDP growth forecast might lead to an increased high-yield allocation if future defaults are expected to remain below market expectations.

3.2.1. Assessing Credit Quality in a Top-Down Approach

Active top-down and bottom-up credit managers frequently use public ratings to categorize and rank the credit quality of bonds within a portfolio. As investors compare investments across credit ratings, the fact that default risk rises more rapidly as ratings decline is important to consider. The use of weighted factors, such as those established by Moody's based on the likelihood of credit loss over a specific period versus ordinal factors across the credit spectrum, enables managers to capture this effect more accurately, as demonstrated in Exhibit 20.

EXHIBIT 20 Weighted versus Ordinal Credit Rating Categories

Moody's	S&P	Fitch	Ordinal	Weighted
Aaa	AAA	AAA	1	1
Aa1	AA+	AA+	2	10
Aa2	AA	AA	3	20
Aa3	AA–	AA–	4	40
A1	A+	A+	5	70
A2	A	A	6	120
A3	A–	A–	7	180
Baa1	BBB+	BBB+	8	260
Baa2	BBB	BBB	9	360
Baa3	BBB–	BBB–	10	610
Ba1	BB+	BB+	11	940
Ba2	BB	BB	12	1,350
Ba3	BB–	BB–	13	1,766
B1	B+	B+	14	2,220
B2	B	B	15	2,720
B3	B–	B–	16	3,490
Caa1	CCC+	CCC+	17	4,770
Caa2	CCC	CCC	18	6,500
Caa3	CCC–	CCC–	19	10,000
Ca	CC	CC	20	

Source: Moody's Investors Service.

The impact of weighted ratings is best demonstrated using an example. For instance, assume a manager is assessing credit quality for a portfolio in which half of the bonds are rated A1/A+ and the other half are rated Ba3/BB-. Using an ordinal scale, the average portfolio credit quality score is 9 (= 50% × 5 + 50% × 13), which corresponds to an average rating of Baa2/BBB in Exhibit 20. However, using the weighted scale at the far right, the portfolio's average credit quality score is 918 (= 50% × 70 + 50% × 1,766), or closer to Ba1/BB+, two levels (notches) below the average rating derived using an ordinal scale.

Earlier readings underscored the risks of relying on public credit ratings, in particular that ratings tend to lag the market's pricing of credit risk critical to an active investor. In addition, one should note that S&P's and Moody's ratings capture different types of risks, with S&P ratings focused on the POD, while Moody's focuses on expected losses, which could influence historical comparisons. The credit rating time horizon is also critical because ratings agencies issue both short-term and long-term ratings for specific issuers, which might warrant additional attention. For these reasons, active managers often prefer to use credit spread measures such as OAS to measure average portfolio credit quality. To calculate a portfolio's average OAS, each bond's individual OAS is weighted by its market value. A manager might also group bonds by OAS categories, which are sometimes mapped to public ratings for comparative purposes.

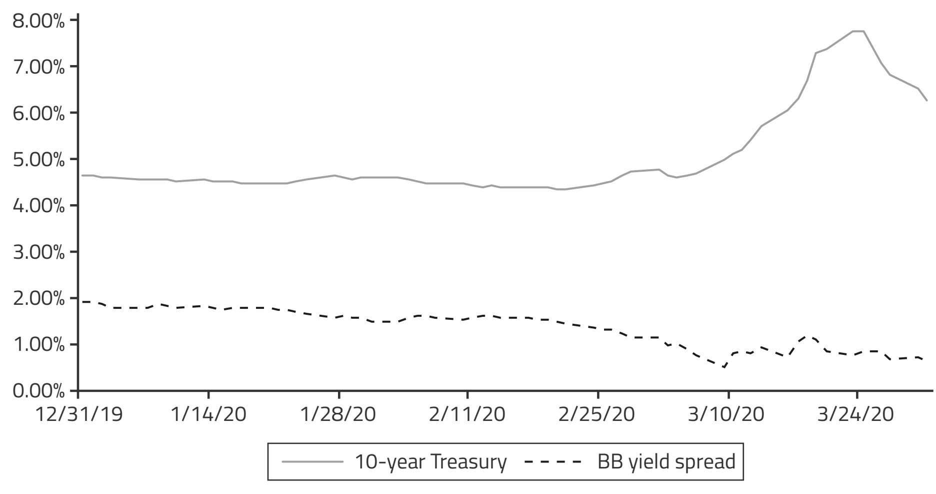
The use of spread-based rather than rating-based measures also facilitates the measurement of changes in portfolio value due to spread changes. As shown earlier, Equation 5 provides a framework to quantify portfolio value changes due to yield spread movements:

$$\%\Delta PV^{Spread} = -(\text{EffSpreadDur} \times \Delta\text{Spread}) + (½ \times \text{EffSpreadCon} \times (\Delta\text{Spread})^2)$$

Smaller yield spread changes are often estimated using the first term in Equation 5. This analytical duration approach provides a reasonable approximation of the price–yield spread relationship for investment-grade bonds with low credit spreads. However, for bonds with greater default risk further down the credit spectrum, changes to both the EffSpreadDur and the ΔSpread terms might be required to accurately reflect empirical observations of how credit risk changes affect overall portfolio value.

In isolating portfolio value changes due to yield spread changes using EffSpreadDur, Equation 5 implicitly assumes that government bond YTMs and credit spreads are uncorrelated, independent variables. However, empirical duration estimates using statistical models often diverge from analytical duration calculations over time and in different interest rate environments. For instance, under a "flight to quality" scenario, the macroeconomic factors driving government bond YTMs *lower* will cause high-yield bond credit spreads to *rise* as the result of an expectation of a greater likelihood and higher severity of financial distress, as shown in Exhibit 21 during the COVID-19 pandemic in early 2020.

EXHIBIT 21 US Treasury Yields versus US Corporate BB Spreads, 2020



Source: Bloomberg.

As for ΔSpread, recall the empirical observation that bonds trading at wider spreads usually experience larger spread changes, which are proportional to the DTS measure in Equation 8.

These greater changes in bond spread have an impact similar to that of the weighted Moody's credit rating categories in Exhibit 18.

EXAMPLE 18 Top-Down Excess Returns

An investor has formed expectations across four bond rating categories and intends to overweight the category with the highest expected excess return over the next 12 months. Evaluate which rating group is the most attractive based on the information in the following table and assuming no change in spread duration:

Rating Category	Current OAS	Expected ΔOAS	Expected Loss (POD × LGD)	EffSpreadDur
A	1.05%	–0.25%	0.06%	5.5
Baa	1.35%	–0.35%	0.30%	6.0
Ba	2.45%	–0.50%	0.60%	4.5
B	3.50%	–0.75%	3.00%	4.0

Solution: The following table summarizes expected excess returns E [ExcessSpread] ≈ Spread_0 – (EffSpreadDur × ΔSpread) – (POD × LGD) for each of the four rating categories. For example, expected excess return for rating category A is 2.37% (= 1.05% – (5.5 × –0.25%) – 0.06%).

Rating Category	Current OAS	Expected ΔOAS	Expected Loss (POD × LGD)	EffSpreadDur	E(Excess Return)
A	1.05%	–0.25%	0.06%	5.5	2.37%
Baa	1.35%	–0.35%	0.30%	6.0	3.15%
Ba	2.45%	–0.50%	0.60%	4.5	4.10%
B	3.50%	–0.75%	3.00%	4.0	3.50%

Given that the Ba category has the highest expected excess return, it is the most attractive rating category to overweight in the portfolio.

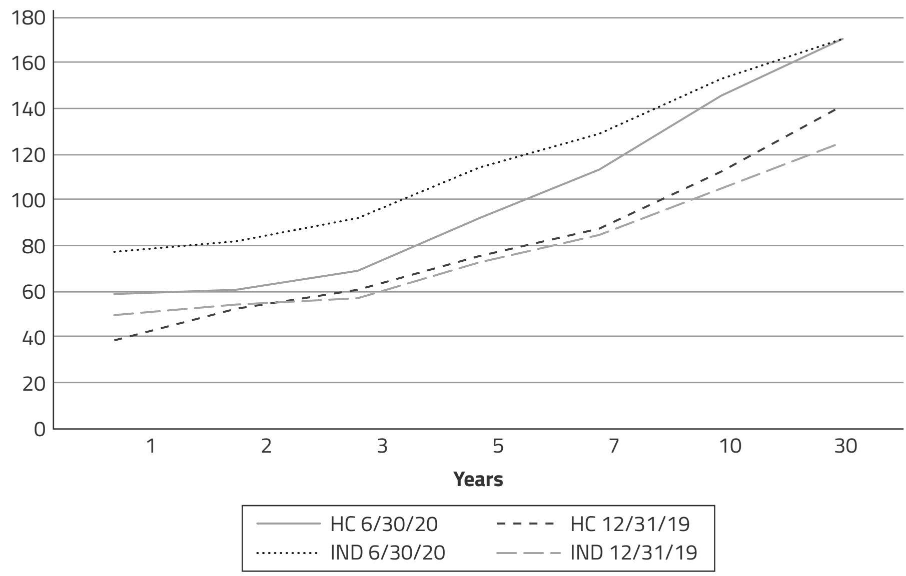
3.2.2. Sector Allocation in a Top-Down Approach

Industry sector allocations (or weightings) are an important part of a top-down approach to credit strategy. To determine which sector(s) to over- or underweight, an active portfolio manager usually begins with an interest rate and overall market view established using macroeconomic variables introduced earlier. This view is a key step in determining whether specific sectors of the economy are likely to over- or underperform over the manager's investment time horizon.

Quantitative methods such as regression analysis are often used in making industry allocation decisions. For example, the average spread of bonds within an individual industry sector and rating category might be compared with the average spread of the bonds with the same rating but excluding the chosen industry sector. Alternatively, a portfolio manager might also use financial ratios in comparing sector spreads and sector leverage. Generally speaking, higher leverage should imply higher credit risk and thus wider spreads. A portfolio manager could therefore compare sectors on a spread-versus-leverage basis to identify relative value opportunities.

Sector- and rating-specific spread curves are a useful tool in guiding decision making for top-down sector allocations. A comparison of curves combined with an investor's view could lead to credit portfolio positioning based on a view that a specific credit spread curve will flatten or steepen, or that two spread curves will converge or diverge. For example, Exhibit 22 shows the divergence in industrial versus health care spreads for BBB rated US issuers over the first half of 2020 during the COVID-19 pandemic. The flatter industrial credit spread curve reflects that sector's relatively weak credit outlook versus health care over the period.

EXHIBIT 22 US BBB Industrial versus Health Care Spreads (bps p.a.)



Source: Bloomberg.

3.3. Factor-Based Credit Strategies

While the top-down approach to fixed-income portfolio construction outlined in the previous section grouped investment choices by sector and public ratings, active credit investors are increasingly turning to strategies based on style factors.

3.3.1. Key Factors Affecting Credit Spreads

Factor investing has long been applied in equity markets as noted in earlier lessons, but the application of systematic risk factors such as size, value, and momentum in fixed-income markets is relatively new. For example, Israel, Palhares, and Richardson (2018) established a framework for evaluating excess corporate bond returns based on a number of characteristics, evaluating their significance in explaining fixed-income returns. The authors found strong evidence of positive risk-adjusted returns to measures of carry, defensive, momentum, and value. Exhibit 23 lists these four factors, their rationale, and the measures used in their analysis.

EXHIBIT 23 Selected Fixed-Income Factors

Factor	Rationale	Measures Used
Carry	Expected return measure if POD or aggregate risk premium is unchanged	OAS
Defensive	Empirical research suggests safer low-risk assets deliver higher risk-adjusted returns	Market-based leverage, gross profitability, and low duration
Momentum	Bonds with higher recent returns outperform those with lower recent returns	Trailing six-month excess bond and equity returns
Value	Low market value versus fundamental value indicates greater than expected return	Bond spread less default probability measure, which includes rating, duration, and excess return volatility

The returns represented diversification with respect to common market risk sources such as equity or credit risk premia and are similar in characteristic to those factors shown to be significant in equity markets, with some adjustments. Investigation of the source of returns suggested neither traditional risk exposures nor mispricing provided a comprehensive explanation for the excess returns.

3.3.2. Environmental, Social, and Governance Factors

The growing relevance of environmental, social, and governance (ESG) factors in active portfolio management is evidenced by growing adoption of the Principles for Responsible Investment. This independent body established in partnership with the United Nations to promote ESG factors in investing has more than 3,000 signatories worldwide with more than $100 trillion in assets under management.

Active credit investors usually incorporate ESG factors into portfolio strategies in one of three basic ways:

- The use of screens to either exclude specific industries with less favorable ESG characteristics, such as firearms, tobacco, or coal, or to rule out specific companies or sovereign issuers with ESG-specific ratings below a threshold
- Use of ESG ratings to target issuers within a given sector or rating category with relatively favorable ESG characteristics while matching a specific index risk and return
- Targeting fixed-income investments that directly fund ESG-specific initiatives

ESG-specific ratings for private and public issuers are a key element in the portfolio selection process. The wide range of quantitative and qualitative criteria used to measure ESG attributes and differences in methodology and weighting leads to greater dispersion in ESG versus credit ratings. That said, ESG and credit ratings tend to be positively correlated for two reasons. First, issuers with more financial resources are better able to meet more stringent ESG standards, while those with a greater likelihood of financial distress often face governance or other adverse risks. Second, major rating agencies now explicitly incorporate ESG risks into the traditional credit rating process. In 2019, Moody's cited ESG risks as a material factor in one-third of its credit rating actions among private sector issuers.

Green bonds are fixed-income instruments that directly fund ESG-related initiatives such as those related to environmental or climate benefits. This rapidly growing segment of the fixed-income market includes corporate, financial institution, and public issuers where bond proceeds are directed to projects that reduce air pollution, recycle post-consumer waste products, underwrite environmental remediation projects, and invest in alternative construction materials for environmentally sustainable buildings. Issuers frequently agree to voluntary guidelines such as the International Capital Market Association's Green Bond Principles (2018) to ensure that these securities meet investor ESG requirements. Although green bonds usually rank pari passu (or at the same level) with the issuer's outstanding senior unsecured bonds and therefore reflect similar pricing, the favorable ESG characteristics often result in greater investor demand than for standard debt issues. For example, in October 2020, the European Union issued €17 billion in new 10-year and 20-year debt in its first-ever offering of social bonds to finance its COVID-19 pandemic-related job support program. At nearly 14 times the issuance size, the €233 billion in investor orders for the new bonds represented the largest demand ever for a primary bond issuance.

4. LIQUIDITY AND TAIL RISK

- **discuss liquidity risk in credit markets and how liquidity risk can be managed in a credit portfolio**
- **describe how to assess and manage tail risk in credit portfolios**

4.1. Liquidity Risk

The feasibility and cost of buying and selling fixed-income instruments are important considerations for active investors. Trading volumes and bid–offer costs vary widely across fixed-income markets and regions. For instance, sovereign bonds in large developed markets are highly liquid, usually offering institutional bid–offer spreads in secondary markets for on-the-run securities of less than one basis point during trading hours. Smaller, off-the-run corporate bonds or structured notes, on the other hand, might command bid–offer spreads of 10 bps or more and take days to execute, given that many outstanding bonds do not trade at all on a given trading day.

Consider, for example, the US corporate bond market, wherein a single major issuer might have dozens of outstanding debt tranches of varying tenor, currency, or other feature, each separately traded and identifiable via a specific CUSIP or ISIN (International Securities Identification Number). As mentioned earlier in the curriculum, individual bond issuance and trading has historically taken place in over-the-counter (OTC) markets as opposed to on an exchange. OTC market liquidity rests with individual dealers, their specific portfolio and depth of inventory, and appetite to supply liquidity at a cost. Corporate bonds are traditionally traded on a request-for-quote basis, in which investors reach out to multiple dealers to request a fixed price quote for a specific trade size. The use of electronic trading platforms for bond trading has grown because higher regulatory capital requirements reduced bond inventories among dealers after the 2008–09 global financial crisis. While electronic trading platforms comprised less than one-third of individual corporate bond trading volume as of 2020, trading in bond portfolios and bond ETFs, addressed later in this lesson, has grown in importance.

Transaction cost estimates in bond markets vary significantly from those in equity markets because of market structure differences. Price discovery for infrequently traded individual bonds often begins with **matrix pricing (or evaluated pricing)** techniques introduced earlier in the curriculum using bonds from similar issuers and actively traded government benchmarks to establish a bond's fair value. For bonds quoted actively on a request-for-quote system by individual dealers, the effective spread transaction cost statistic introduced in an earlier lesson and shown in Equation 13 provides an estimate of trading cost.

$$\text{Trade size} \times \begin{cases} \text{Trade price} - (\text{Bid} + \text{Ask}/2)\text{for buy orders} \\ (\text{Bid} + \text{Ask}/2) - \text{Trade price for sell orders} \end{cases} \tag{13}$$

However, the effective spread is an inadequate gauge of trading costs for positions that are traded in smaller orders over time and/or whose execution affects market spreads. A separate, ex-post liquidity gauge specific to the US corporate bond market is the TRACE (Trade Reporting and Compliance Engine) reporting system introduced in 2002 to track real-time price and volume reporting for bond transactions. Portfolio managers will often review recent TRACE trading activity to gauge the estimated cost of trading a bond position.

Active portfolio managers take several steps in managing the liquidity risk of bond portfolios, given the significant market risk involved in trading less liquid positions. First, active managers will usually favor on-the-run government bonds or most recently issued corporate or other bonds for short-term tactical portfolio positioning, while reserving relatively illiquid positions for buy-and-hold strategies or strategic positioning to minimize expected return erosion due to trading costs. Second, active managers might consider liquid alternatives to individual bond trades to close portfolio gaps where active management adds little value, or to react quickly to rapidly changing markets. These alternatives include CDS outlined later and bond ETFs.

Fixed-income ETFs are liquid, exchange-traded bond portfolios that create and redeem shares using an OTC primary market that exists between a set of institutional investors (or **authorized participants**) and the ETF sponsor. These ETF shares trade in the secondary market on an exchange, overcoming the liquidity constraints of individual OTC-traded bonds. Bond ETFs have enjoyed significant growth and are available across the credit spectrum as well as for different maturities and in different markets. Although the underlying cash flow exposures are similar, ETFs usually neither mature nor experience duration drift (with the exception of target maturity ETFs) as do individual bonds. As ETF sponsors target a specific index or profile, ETFs offer relatively constant portfolio duration and pay variable monthly interest based on the overall portfolio. Active credit managers use ETFs to quickly and efficiently overweight or underweight exposures in rapidly changing markets and to take on strategic exposure in segments of the market where individual or bottom-up bond selection is less of a focus.

When relatively illiquid bond positions are purchased or sold over longer periods, portfolio managers might consider hedging strategies such as asset swaps to mitigate the benchmark risk of a portfolio position as outlined in the following example.

EXAMPLE 19 Using Asset Swaps to Manage Liquidity Risk

Recall the earlier example of a United States–based issuer with the following option-free bonds outstanding:

Outstanding Debt	Term	Coupon	Price	YTM
2-year issue	2	4.25%	106.7	0.864%
5-year issue	5	3.25%	106.0	1.984%
15-year issue	15	2.75%	91.0	3.528%

Assume the investor instead holds a US$50 million face value position in the outstanding 15-year bond. Historical TRACE data suggest an average $5 million daily trading volume in the 15-year bond. Which of the following statements *best* describes how the issuer might use an asset swap to manage the benchmark interest rate risk associated with liquidating this bond position?

A. The investor should enter into an asset swap where he receives fixed and pays floating, unwinding the swap position once the bond position is sold.
B. The investor should enter into an asset swap where he pays fixed and receives floating, unwinding the swap position once the bond position is sold.
C. The investor should enter into an asset swap where he pays fixed and receives floating, unwinding the swap position over time in proportion to the amount of the bond sold.

Solution: The correct answer is C. Because the investor's bond position represents a long position (i.e., long both spread duration and benchmark duration), the best hedge would be a short-duration (or pay-fixed swap) position rather than A. As for B, the hedge unwind occurs once the bond position is sold rather than over time, which exposes the investor to benchmark interest rate risk for the portion of the bond sold. The proportional swap unwind in C ensures that the offsetting swap position matches the benchmark interest rate risk of the bond.

4.2. Tail Risk

Extreme adverse outcomes that exceed those to be expected from a normally distributed portfolio are often referred to as tail events. In the context of active fixed-income management, the measurement and management of tail risk involves stress testing a portfolio's value based on the key fixed-income returns factors in Equation 1. In an earlier lesson on measuring and managing market risk, **value at risk (VaR)** was introduced as a measure of the minimum portfolio loss expected to occur over a given time period at a specific confidence level. For example, a 5% daily VaR of €8.7 million implies that a portfolio manager should expect a daily portfolio loss of *at least* €8.7 million on 5% of all trading days. Assuming normally distributed portfolio returns, the 5% confidence level translates to an outcome at least 1.65 standard deviations below the mean, while a 1% confidence interval lies at least 2.33 standard deviations below the mean. Risk managers often use expected returns, volatilities, and correlations to estimate parametric VaR in addition to either historical simulation or Monte Carlo methods. The following example shows a simple parametric VaR calculation for a bond position.

EXAMPLE 20 Fixed-Rate Bond VaR

Consider the earlier case of an investor holding $50 million face value of a 15-year bond with a coupon of 2.75%, a current YTM of 3.528%, and a price of 91 per 100 of face value. What is the VaR for the full bond price at a 99% confidence interval for one month (assuming 21 trading days in the month) if daily yield volatility is 1.75% and we assume a normal distribution?

Solution: First, we solve for the expected change in YTM based on a 99% confidence interval for the bond and a 1.75% yield volatility over 21 trading days, which equals 18.7 bps = (1.75% × 2.33 standard deviations × $\sqrt{21}$).We can quantify the bond's market value change using either a duration approximation or the actual price change as follows. We can use the Excel MDURATION function to solve for the bond's duration as 12.025. We can therefore approximate the change in bond value using the familiar (−ModDur × ΔYield) expression as \$1,023,147 = (\$50 million × 0.91 × (−12.025 × .00187)). We can also use the Excel PRICE function to directly calculate the new price of 88.982 and multiply the price change of 2.018 by the face value to get \$1,009,000.

The simplicity and transparency of VaR can be misleading if it is used as a tool for quantifying tail risk for several reasons. First, VaR tends to underestimate the frequency and severity of extreme adverse events. It also fails to capture the downside correlation and liquidity risks associated with market stress scenarios. Finally, although VaR addresses *minimum* loss for a specific confidence level, it fails to quantify the *average* or expected loss under an extreme adverse market scenario. **Conditional value at risk** (CVaR), or expected loss, measures the average loss over a specific time period conditional on that loss exceeding the VaR threshold. While computationally more complex and beyond the scope of this lesson, CVaR is often measured using historical simulation or Monte Carlo techniques. Two related measures of portfolio VaR include incremental and relative measures. For example, an analyst seeking to measure the impact of adding or removing a portfolio position might use an **incremental VaR** (or **partial VaR**) calculation for this purpose. As mentioned in an earlier lesson, an investor could use **relative VaR** to measure the expected tracking error versus a benchmark portfolio by calculating VaR (or CVaR) based on a portfolio containing the active positions *minus* the benchmark holdings under a market stress scenario.

EXAMPLE 21 VaR Measures

An active fixed-income manager is considering increasing an overweight portfolio allocation to BBB rated health care issuers versus a targeted index. Which of the following VaR measures is the most appropriate to evaluate the impact of this decision on overall portfolio VaR?

A. Incremental VaR
B. Relative VaR
C. CVaR

Solution: The correct answer is A. Incremental VaR measures the impact of a specific portfolio position change on VaR, while relative VaR in answer B evaluates all active portfolio positions versus the benchmark index and could be important for an active fixed-income mandate that aims to beat an index once the portfolio change has been made. CVaR in C measures a portfolio's average loss over a specific time period conditional on that loss exceeding the VaR threshold.

Tail risk assessment is typically conducted using one of the three methods summarized in Exhibit 24.

EXHIBIT 24 Methods to Assess Portfolio Tail Risk

Method	Description	Advantages	Disadvantages
Parametric Method	Uses expected value and standard deviation of risk factors assuming normal distribution	Simple and transparent calculation	Not well suited for non-normally distributed returns or option-based portfolios
Historical Simulation	Prices existing portfolio using historical parameters and ranking results	Actual results, accommodates options, with no probability distribution assumed	Highly dependent on historical period and repetition of historical market trends
Monte Carlo Analysis	Involves generating random outcomes using portfolio measures and sensitivities	Randomly generated results from a probability distribution, accommodates options	Highly dependent on model assumptions and less transparent

Hypothetical scenario analyses are often used to supplement these three methods of analysis to test portfolio vulnerabilities to specific portfolio parameter changes over time.

In addition to portfolio measures of duration and convexity as a basis for portfolio value changes, analytical models often rely on implied volatility parameters for benchmark interest rates and currencies, such as swaption volatility or currency option volatility, respectively, while reduced form or structural credit models incorporating CDS or equity volatility can be used to model expected spread volatility. Finally, term structure models introduced in an earlier lesson that incorporate interest rate volatility and drift in an equilibrium or arbitrage-free framework are frequently incorporated to simulate term structure changes over time.

Once tail risk under an extreme market scenario has been quantified, it is important to weigh this exposure against other binding portfolio constraints and to take steps to manage the downside risk. For example, a leveraged portfolio might face forced liquidation of certain bond positions beyond a certain tail risk threshold. Alternatively, a defined-benefit pension fund manager might be required to increase plan contributions if extreme market moves cause plan funding status to fall below a statutory minimum. Finally, a bank treasury officer could face increased regulatory capital requirements if adverse market changes under a stress test show significant portfolio losses.

A fixed-income portfolio manager can reduce tail risk by establishing position limits, risk budgeting, or using similar techniques designed to reduce portfolio concentration or to cap portfolio risk exposure to certain issuers, credit ratings, or regions. Alternatively, a portfolio manager might consider the use of derivatives to protect against downside portfolio risk. For example, the manager could consider purchasing a swaption (or the right to enter an interest rate swap at a pre-agreed rate in the future) or a credit default swaption (the right to purchase credit protection on an issuer or index at a strike rate in the future) to protect against the risk of benchmark YTM changes or credit spread changes, respectively. However, each of these strategies requires an upfront premium that will reduce excess portfolio spread over time. In addition, establishing these hedges in a distressed market will greatly increase hedging cost because of higher option volatility, so the manager must weigh these hedging costs against a risk mitigation strategy to determine the best course of action.

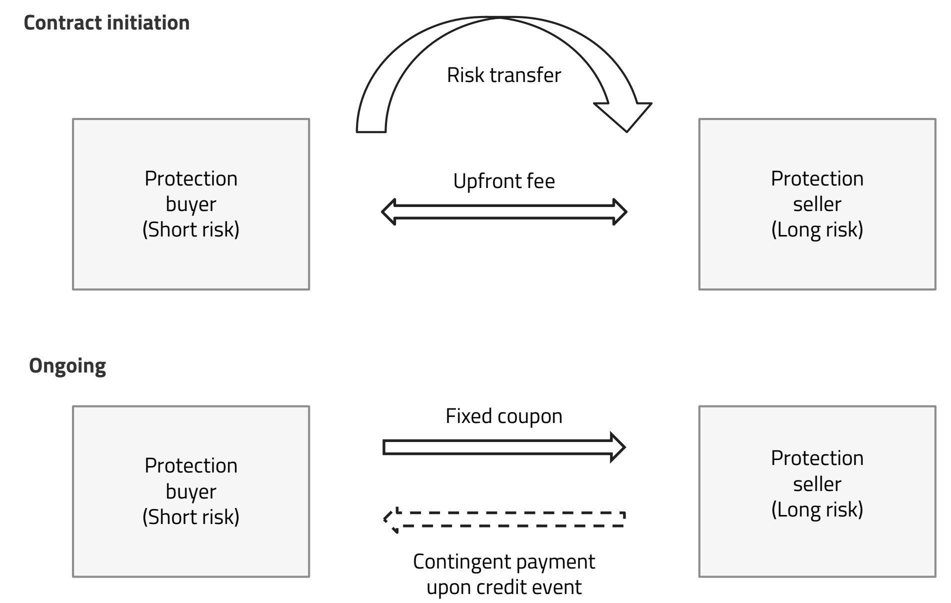
5. SYNTHETIC CREDIT STRATEGIES

- **discuss the use of credit default swap strategies in active fixed-income portfolio management**

As outlined in an earlier lesson, a CDS is the basic building block for strategies to manage credit risk separately from interest rate risk. CDS are often more liquid than an issuer's underlying bonds, enabling investors to take long or short positions, access maturities, and establish other exposures unavailable in cash markets with a smaller cash outlay than direct bond purchases.

Exhibit 25 shows CDS contract mechanics under which a protection "buyer" purchases credit protection from a protection "seller." Each contract references a specific issuer (or issuers) as well as credit event terms that, when triggered, lead to a settlement payment equal to the LGD multiplied by the contract notional amount from the seller to the buyer.

EXHIBIT 25 CDS Mechanics



CDS contracts are usually quoted on an issuer's CDS spread, which corresponds to a price equal to the present value difference between the CDS spread and a fixed coupon rate on the notional amount over the contract life. Fixed CDS coupon rates of 1% for investment-grade issuers and 5% for high-yield issuers were established when the International Swaps and Derivatives Association standardized CDS market conventions following the 2008 financial crisis. CDS pricing models discount future payments by the swap zero curve multiplied by the **hazard rate**, or the likelihood that an issuer credit event will occur given that it has not already occurred in a prior period.

The CDS price at contract inception or on a coupon payment date as a percentage of notional can be approximated using Equation 14:

$$\text{CDS Price} \approx ((\text{Fixed Coupon} - \text{CDS Spread}) \times \text{EffSpreadDur}_{CDS}) \quad (14)$$

where CDS Spread is the issuer's current CDS market spread and $\text{EffSpreadDur}_{CDS}$ is the CDS contract's effective spread duration (sometimes referred to as CDS DV01). At contract inception, the protection buyer must either make a payment to or receive a payment from the protection seller equal to the CDS contract price difference from par, as shown in Exhibit 26.

EXHIBIT 26 Upfront Payment at CDS Contract Inception

Description	Upfront Premium
CDS Spread = Fixed Coupon	None
CDS Spread < Fixed Coupon	Protection buyer *receives* ((Fixed Coupon – CDS Spread) × $\text{EffSpreadDur}_{\text{CDS}}$)
CDS Spread > Fixed Coupon	Protection buyer *pays* ((CDS Spread – Fixed Coupon) × $\text{EffSpreadDur}_{\text{CDS}}$)

CDS contracts have similarities to both bonds and interest rate swaps. As with a cash bond priced at a discount when its coupon is below current market rates, the protection seller is entitled to an upfront payment in exchange for accepting a fixed coupon below the CDS market spread. As with a standard interest rate swap, a CDS contract priced at par has a zero net present value, and the notional is not exchanged but rather serves as a basis for spread and settlement calculations.

EXAMPLE 22 CDS Price and Price Changes

An investor seeks to purchase credit protection under a five-year CDS contract at a CDS market spread of 0.50% p.a. for an investment-grade issuer with an estimated effective spread duration ($\text{EffSpreadDur}_{\text{CDS}}$) of 4.75.

1. Determine whether the investor must pay or receive an upfront amount upon CDS contract inception and calculate the difference from par.
2. Calculate the change in contract price if the CDS spread rises to 0.60% p.a. and interpret the impact of the change on the protection buyer.

Solution to 1: Because investment-grade CDS contracts have a fixed coupon of 1.00% p.a. versus the 0.50% p.a. CDS market spread, the investor buying protection should receive the difference from par upfront in exchange for paying an "above market" coupon under the contract. Calculate the estimated difference using Equation 14 ((Fixed Coupon – CDS Spread) × $\text{EffSpreadDur}_{\text{CDS}}$)) with CDS Spread of 0.50%, Fixed Coupon of 1.00%, and $\text{EffSpreadDur}_{\text{CDS}}$ equal to 4.75.

Upfront premium: 2.375% of CDS notional (= (1.00% – 0.50%) × 4.75).

Solution to 2: Calculate the upfront premium using Equation 14 and a 0.60% spread.

Upfront premium: 1.90% of CDS notional (= (1.00% – 0.60%) × 4.75).

The protection buyer realizes a mark-to-market gain equal to 0.475% (2.375% – 1.90%) of the CDS contract notional because of the wider CDS spread.

CDS price changes for a given CDS spread change can be quantified using the contract's effective spread duration:

$$\Delta(\text{CDS Price})/\Delta(\text{CDS Spread}) = -(\Delta(\text{CDS Spread}) \times \text{EffSpreadDur}_{\text{CDS}}) \qquad (15)$$

Active fixed-income portfolio managers buy or sell CDS protection across issuers, maturities, and/or sectors to alter portfolio exposure, as illustrated in the following example.

EXAMPLE 23 Credit Underweight Using CDS

A European-based fixed-income manager intends to underweight exposure to a BBB rated French media and telecommunications issuer. She observes that the issuer's current on-the-run five-year CDS contract is trading at a spread of 110 bps p.a. with an $\text{EffSpreadDur}_{\text{CDS}}$ of 4.595. Which position should she take in the CDS market? Calculate the result if spreads widen to 125 bps for a €10 million notional position.

Solution: The manager can underweight the issuer's credit by *purchasing* protection in the CDS market. This short risk position will realize a gain if the issuer's spreads widen. For example, if the issuer's credit spreads widen from 110 bps p.a. to 125 bps p.a., we can estimate the change in CDS contract value by multiplying ($-\Delta$(CDS Spread) $\times$ $\text{EffSpreadDur}_{\text{CDS}}$) from Equation 15 by the CDS notional to get €68,925 (= –€10,000,000 × (–0.15% × 4.595)).

While CDS contracts are available across maturities, the five-year tenor is generally the most frequently traded contract. Exhibit 27 summarizes the most common CDS strategies used in practice.

EXHIBIT 27 Credit Derivative–Based Alternatives to Corporate Bonds

Instrument	Description	Targeted Return	Portfolio Impact
Single-Name CDS	Protection buyer pays premium to seller in exchange for payment if credit event occurs	Buyer gains and seller loses if single-name credit spread widens or credit event occurs	Short (buyer) or long (seller) single-name credit spread exposure
Index-Based CDS	Protection buyer pays premium in exchange for partial payment if credit event occurs for index member	Buyer gains and seller loses if index member spreads widen or if credit event occurs	Short (buyer) or long (seller) index-based credit spread exposure
Payer Option on CDS Index	Option buyer pays premium for right to buy protection ("pay" coupons) on CDS index contract at a future date	Max (CDS Credit Spread Strike – CDS Credit Spread at expiration, 0) – Option Premium	Short CDS index-based credit spread exposure
Receiver Option on CDS Index	Option buyer pays premium for right to sell protection ("receive" coupons) on CDS index contract at a future date	Max (Credit spread at expiration – CDS Credit Spread Strike, 0) – Option Premium	Long CDS index-based credit spread exposure

Single-name reference entities include both private corporations and sovereign borrowers. Several CDS indexes are available across regions and often also offer subindexes covering a particular sector or borrower type. For example, the Markit CDX North American Investment Grade index consists of 125 equally weighted CDS contracts on entities, including six subindexes (High Volatility, Consumer Cyclical, Energy, Financials, Industrial, and Telecom, Media, and Technology).

CDS strategies are commonly used by active fixed-income portfolio managers to over- or underweight credit spread exposure to individual issuers, specific sectors, or borrower types. As with benchmark yield curves, CDS portfolio positioning strategies are usually based on expected changes in the credit curve level, slope, or shape. The credit curve referred to here is the **CDS curve**, or the plot of CDS spreads across maturities for a single reference entity or index, rather than the fitted credit spread curves addressed earlier. This might involve an investor taking a long or short CDS position in one issuer or issuer type, or a long or short position overweighting one reference entity or group of entities and underweighting another. Investors using CDS strategies to hedge bond portfolios must always consider the potential impact of basis changes on the strategy over the investment horizon.

Fixed-income ETFs offer derivatives such as futures and options that are different from CDS contracts. As with bond futures, ETF futures are a contract to take future delivery of an ETF and trade on a price rather than a spread basis. Because underlying ETF prices are derived from all-in bond yields held by the fund, ETF derivative prices change with changes in both benchmark rates and credit spreads.

CDS long–short strategies based on spread level are appropriate for both bottom-up and top-down approaches. Assume, for example, that an investor believes that issuer A's credit spreads will likely narrow versus those of issuer B. To capitalize on this view in the cash market, the investor would first source A's individual bonds for purchase and then seek a duration-matched amount of issuer B's bonds to borrow and sell short. The existence of a liquid single-name CDS market for both issuers allows the investor to simply sell protection on A and purchase protection on B for the same notional and tenor.

EXAMPLE 24 CDS Long–Short Strategies

Consider the investor from the prior example who sought to underweight a French media and telecommunications issuer. Assume instead that the investor seeks to maintain a constant media and telecommunications credit allocation by overweighting a BBB rated German media and telecommunications competitor. CDS contract details are as follows:

Issuer	Tenor	CDS Spread	$EffSpreadDur_{CDS}$
French Media & Telecoms Issuer	5 years	110 bps	4.697
German Media & Telecoms Issuer	5 years	130 bps	4.669

Describe an appropriate long–short CDS strategy to meet this goal, and calculate the investor's return if the French issuer's spreads widen by 10 bps and those of the German issuer narrow by 25 bps based on €10 million notional contracts.

Solution: The manager purchases protection on the French issuer and simultaneously sells protection on the German issuer. Use $(-\Delta(\text{CDS Spread}) \times \text{EffSpreadDur}_{CDS})$ from Equation 15 multiplied by the CDS notional to solve for changes in the short and long risk positions:

Short risk (French issuer): €46,970 (= €10,000,000 × (–0.10% × –4.697))
Long risk (German issuer): €116,725 (= –€10,000,000 × (–0.25% × 4.669))

The total gain on the long–short strategy is €163,695 (= €46,970 + €116,725).

A similar long–short strategy can be applied under a top-down approach. For example, an investor might overweight (underweight) a specific sector given an expectation of narrower (wider) spread levels versus the total portfolio by selling (buying) protection on a CDS subindex contract. Alternatively, assume an active manager expects a weaker economy and a widening of high-yield versus investment-grade credit spread levels. The manager can capitalize on this view by buying five-year protection on a high-yield CDS index and selling protection on an investment-grade CDS index for the same tenor. Standardized CDS contracts eliminate the impact of duration difference, liquidity, and other factors that arise under a similar strategy in the cash bond market.

CDS long–short strategies based on expected credit curve slope changes involve CDS curve trades. For example, an upward-sloping credit curve implies relatively low near-term expected default probability that rises over time. An investor might expect an issuer's CDS curve to steepen if its near-term default probability declines as a result of higher than expected profits and stable leverage. This investor can capitalize on this view by selling short-term protection using a single-name CDS contract and buying long-term protection on that same reference entity. As shown in the following example, capitalizing on spread changes for different maturities requires duration matching of the positions, as in the case of benchmark yield curve strategies.

EXAMPLE 25 Duration-Weighted Single-Name CDS Curve Steepener

Returning to our earlier example of the German media and telecommunications issuer, the investor decides instead to position her portfolio for a steepening of the issuer's credit curve using the CDS market. Details of on-the-run 5- and 10-year CDS contracts outstanding are as follows.

Issuer	Tenor	CDS Spread	$\text{EffSpreadDur}_{CDS}$
German Media & Telecoms Issuer	5 years	130 bps	4.669
German Media & Telecoms Issuer	10 years	175 bps	8.680

Describe an appropriate long–short CDS strategy to meet this goal assuming a €10,000,000 10-year CDS contract notional. Calculate the investor's return if the 5-year spreads rise 10 bps and the 10-year spreads rise 20 bps.

Solution: A steeper credit curve implies that ((CDS Spread)$_{10yr}$ – (CDS Spread)$_{5yr}$) will increase. The appropriate long–short strategy to position for this change is to purchase protection based on the 10-year, €10,000,000 notional and to sell protection on an equivalent duration 5-year CDS contract.

1. Calculate the 5-year CDS contract notional that matches the basis point value (BPV) of a 10-year, €10,000,000 CDS (BPV_{10yr} = $EffSpreadDur_{10yrCDS}$ = notional) using the effective spread duration ratio of 1.859 ($EffSpreadDur_{10yrCDS}$/EffSpread-Dur_{5yrCDS} = 8.68/4.669) multiplied by €10,000,000 to get €18,590,000.

   Confirm this equivalence by comparing BPV_{5yr} and BPV_{10yr}:

   $$BPV_{5yr}\text{: €8,680} = \text{€18,590,000} \times 4.669/10,000$$
   $$BPV_{10yr}\text{: €8,680} = \text{€10,000,000} \times 8.68/10,000$$

2. Calculate portfolio return for a 10 bp increase in 5-year CDS spreads and a 20 bp increase in 10-year CDS spreads using Equation 15 (–Δ(CDS Spread) × EffSpread-Dur_{CDS}) multiplied by the CDS notional.

   5 year (long risk): –€86,800 (= €18,590,000 × (–0.1% × 4.669))
   10 year (short risk): €173,600 (= –€10,000,000 × (–0.2% × 8.68))
   Net portfolio gain: €86,800 = €173,600 – €86,800

The same curve strategy just described applies to expected credit curve slope changes for a CDS index or subindex. For instance, an investor who believes the economy is nearing the end of a growth cycle might expect the CDS curve for industrial issuers to flatten amid rising near-term credit spreads. Under this expected scenario, an investor purchases short-term CDS subindex protection on industrials and sells long-term protection on the same subindex to capitalize on a flattening view. Alternatively, an investor taking a top-down approach who shares a similar bearish economic view might consider a flattening trade for an entire CDS index.

Additional CDS strategies seek to either capitalize on the basis difference between CDS and cash bonds or take advantage of specific events that affect CDS spreads and curves. As noted earlier, basis differences arise from a number of factors but are also due to differences in liquidity across derivative and cash markets, a detailed treatment of which is beyond the scope of this lesson. Corporate events that influence CDS spreads by affecting bondholders differently from shareholders include mergers and acquisitions and leveraged buyouts, both of which are addressed elsewhere in the curriculum.

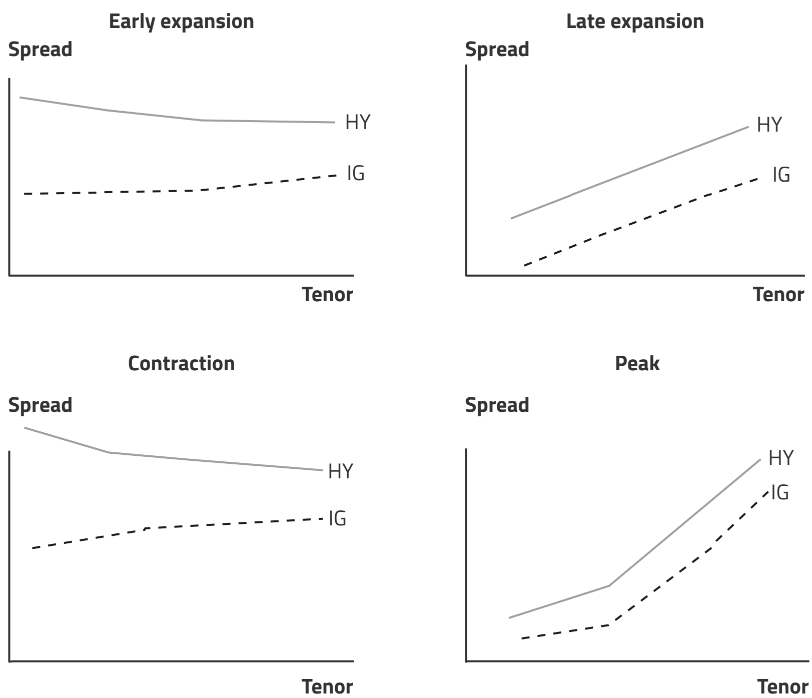
6. CREDIT SPREAD CURVE STRATEGIES

- **discuss various portfolio positioning strategies that managers can use to implement a specific credit spread view**

Earlier in the lesson, we established that the credit cycle is a key driver of credit spread changes across maturities and ratings. The probability of issuer default and severity of loss over the cycle must be considered within the context of an overall market view. For example, positively sloped credit spread curves suggest relatively low near-term default probability, a view consistent with stable or rising future inflation and relatively strong expected economic growth. Investor demand for higher credit spreads for assuming the risk of downgrade or default for longer periods also contributes to a positive slope.

The level and slope of credit curves change over the economic cycle. Early in an expansion, as profits rise and defaults remain high, high-yield spreads remain elevated and well above investment-grade spreads, which often exhibit a flat to inverted spread curve. As an expansion progresses, lower defaults and increased profits cause short-term high-yield and investment-grade spreads to decline and credit spread curves to steepen. Credit curve steepening continues as economic growth peaks amid higher leverage and inflation expectations. As economic growth slows or the economy enters a recession, credit spreads rise, and spread curves flatten, with the high-yield curve inverting in some instances amid falling profitability and rising defaults. Although no two credit cycles are alike, Exhibit 28 presents a stylized view of these credit spread curve level and shape changes for investment-grade (IG) and high-yield (HY) issuers over the economic cycle.

EXHIBIT 28 Credit Spread Curves over the Economic Cycle



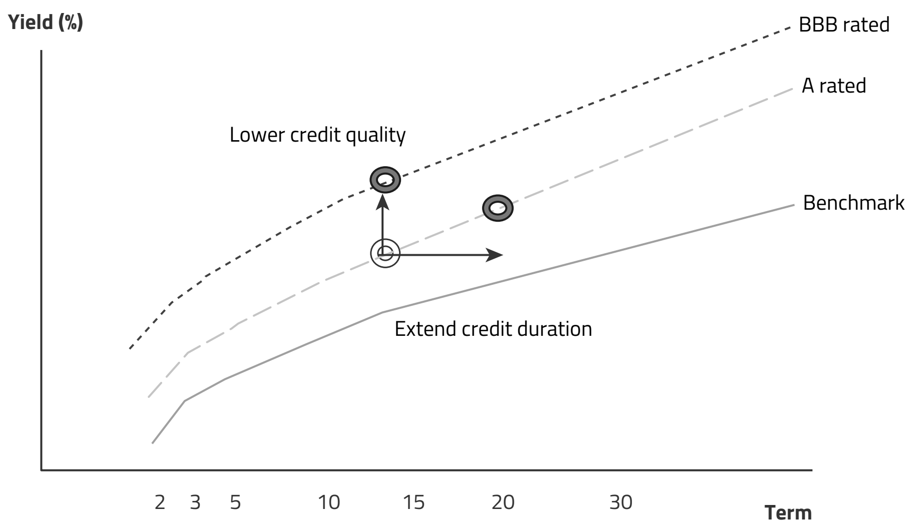
Active credit managers often incorporate the credit cycle into economic growth and inflation forecasts, which are then translated into sector- and issuer-specific views driving specific credit curve level and slope expectations using the bottom-up or top-down approaches outlined earlier. If these forecasts coincide with current credit spread curves, managers will choose active credit strategies consistent with static or stable credit market conditions. However, if an investor's views differ from what today's credit curve implies about future defaults and the severity of credit loss, they will position the portfolio to generate excess return based on this divergent view within investment mandate constraints using the cash and derivative strategies outlined in the following section.

6.1. Static Credit Spread Curve Strategies

An active credit manager might believe that current credit spreads are reasonably priced and that credit curves will remain stable or unchanged over an investment horizon while credit defaults and annual loss rates remain low. Exhibit 29 shows that a manager could position a portfolio to generate excess return in this scenario by either lowering the portfolio's average credit rating or adding credit spread duration by investing in longer-dated bonds with a similar rating to the current portfolio.

In the first case, a portfolio tilt toward lower-rated bonds will increase expected spread return, as seen in Equation 10 (E [ExcessSpread] ≈ Spread_0 −(EffSpreadDur x × ΔSpread) − (POD × LGD)) if Spread, POD, and LGD remain stable. The shift from an average A rated to BBB rated portfolio in Exhibit 29 is an extension of an earlier case (Example 10) that quantified corporate versus government bond roll-down returns as the YTM difference assuming constant spreads and default rates.

EXHIBIT 29 Buy-and-Hold Strategies under a Static Credit Curve



The increase in credit spread duration in the second case involves a "buy and hold" or "carry and roll down" approach familiar from the earlier yield curve strategies lesson. The first involves buying risky bonds with durations above the benchmark without active trading during a subsequent period. If the relationship between long- and short-term credit spreads remains stable over the investment horizon, the manager is rewarded with greater return from the higher spread duration. "Rolling down" the credit curve not only generates incremental coupon income (adjusted over time for any price difference from par) due to wider spreads but also adds return from the passage of time and the investor's ability to sell the shorter-maturity position in the future at a lower credit spread at the end of the investment horizon. The following example illustrates this second case, shown in Exhibit 29.

EXAMPLE 26 Adding Credit Duration under a Static Credit Curve

A Sydney-based investor notes the following available option-free bonds for an A rated Australian issuer:

Debt Term	Coupon	YTM	Price
5 years	1.00%	1.00%	100
10 years	1.35%	1.25%	100.937
15 years	2.00%	1.95%	100.648

The 5-year, 10-year, and 15-year Australian government bonds have YTMs and coupons of 0.50%, 0.75%, and 1.10%, respectively, and both corporate and government bonds have a semiannual coupon. As an active manager who expects stable benchmark yields and credit spreads over the next six months, the investor decides to overweight (by AUD50,000,000 in face value) the issuer's 15-year versus 10-year bond for that period. Calculate the return to the investor of the roll-down strategy in AUD and estimate the returns attributable to benchmark yield versus credit spread changes.

Solution: To estimate credit curve roll-down returns, we must solve for the first two return components from Equation 1 (Coupon income +/– Roll-down return) and separate the impact of benchmark yield versus credit spread changes.

1. Solve for the respective 5-year, 10-year, and 15-year bond credit spreads. Yield spread and G-spread are reasonable approximations because the bonds are option-free, with maturities closely aligned to par government securities.

   5-year spread: 0.50% (= 1.00% – 0.50%)
   10-year spread: 0.50% (= 1.25% – 0.75%)
   15-year spread: 0.85% (= 1.95% – 1.10%)

2. Solve for 6-month expected returns of the 10-year versus 15-year bond:
   A. Incremental coupon income = \$162,500 (= (2.00% – 1.35%)/2 × \$50 million)

Debt Tenor	Coupon	Yield Spread	Total Coupon Income	Coupon (Benchmark Yield)	Coupon (Credit Spread)
10 years	1.35%	0.50%	\$337,500	\$187,500	\$150,000
15 years	2.00%	0.85%	\$500,000	\$275,000	\$225,000

Divide incremental coupon into benchmark and credit spread components:

Income due to benchmark yields: \$87,500 = \$275,000 – \$187,500
Income due to credit spreads: \$75,000 = \$225,000 – \$150,000

B. Price appreciation is determined by the bond's price today and in six months' time based on unchanged benchmark rates. In six months, the 10-year and 15-year positions will be 9.5-year and 14.5-year bonds, respectively, at a yield and yield spread point along the curve. Estimate all-in YTMs and yield spreads using interpolation to arrive at the following results:

Debt Tenor	Date	Coupon	All-In Yield	Benchmark Yield	Yield Spread
5 years	Today	1.00%	1.00%	0.50%	0.50%
10 years	Today	1.35%	1.25%	0.75%	0.50%
15 years	Today	2.00%	1.95%	1.10%	0.85%
9.5 years	In six months	1.35%	1.225%	0.725%	0.50%
14.5 years	In six months	2.00%	1.88%	1.065%	0.815%

Calculate price appreciation using the difference between current bond prices and those in six months using the Excel PV function (= –PV(rate, nper, pmt, FV, type)) where "rate" is the interest rate per period (0.01225/2), "nper" is the number of periods (19), "pmt" is the periodic coupon (1.35/2), "FV" is future value (100), and "type" (0) involves payments made at the end of each period.

10-year: Initial price: 100.937
Price in six months: 101.118 (= –PV (0.01225/2, 19, 1.35/2, 100, 0))
Price appreciation: \$89,660 (= (101.118 – 100.937)/100.937 × \$50 million)

Because the yield spread curve is flat at 0.50%, the full \$89,660 price change in the 10-year is benchmark yield curve roll down.

15-year: Initial price: 100.648
Price in six months: 101.517 (= –PV (0.0188/2, 29, 1, 100, 0))
Price appreciation: \$431,700 (= (101.517 – 100.648)/100.648 × \$50 million)

Because the 0.07% decline in YTM is estimated to be equally attributable to benchmark yield and yield spread changes, each is assumed equal to \$215,850.

3. Incremental income due to price appreciation is therefore \$342,040 (= \$431,700 – \$89,660), of which \$215,850 is attributable to credit spread changes.

In total, the incremental roll-down strategy generates \$504,540 (= \$342,040 + 162,500), of which \$290,850 (= \$215,850 + \$75,000) is estimated to be due to credit spread curve roll down.

Derivative-based credit strategies to add credit spread duration or increase credit exposure include selling CDS single-name or index protection for longer maturities or lower credit quality or using a long–short approach to achieve a similar objective.

EXAMPLE 27 Using CDS for a Static Fixed-Income Credit Strategy

Returning to our earlier example of the investment-grade German media and telecommunications issuer, the investor decides instead to overweight exposure to this name by taking a long risk position in the single-name 10-year CDS market for one year. Details of today's 5-year and 10-year CDS contracts are as follows.

Issuer	Tenor	CDS Spread	$\text{EffSpreadDur}_{CDS}$
German Media & Telecoms Issuer	5 years	130 bps	4.669
German Media & Telecoms Issuer	10 years	175 bps	8.680

Describe the roll-down strategy using CDS and calculate the one-year return in euros on a €10,000,000 position assuming an annual coupon payment and a 9-year $\text{EffSpreadDur}_{CDS}$ of 7.91.

Solution: The investor sells 10-year CDS protection on the German issuer to overweight exposure and terminates the position in one year. As with the bond example, the sold protection strategy generates coupon income if the issuer does not default and price appreciation if credit spreads decline over time.

1. The fixed coupon received at the end of one year equals the notional multiplied by the standard 1% investment-grade coupon for the period, or €100,000, or €10,000,000 × 1.00% for one year.
2. Estimate the CDS price change over one year by interpolating the 9-year issuer spread under a static credit curve assumption.

Solve for the 5-year and 10-year CDS spread weights in the 9-year spread interpolation calculation.

$$\text{5-year CDS weight} = w_5 = 20\% \ (= (10 - 9)/(10 - 5))$$
$$\text{10-year CDS weight} = w_{10} = 80\% \ (\text{or } (1 - w_5))$$
$$\text{Note that } (w_5 \times 5) + (w_{10} \times 10) = 9$$

The 9-year spread is a weighted average of 5- and 10-year CDS spreads.

$$\text{CDS Spread}_{9yr} = w_5 \times \text{CDS Spread}_{5yr} + w_{10} \times \text{CDS Spread}_{10yr}$$
$$1.66\% \ (= 1.30\% \times 0.2 + 1.75\% \times 0.8)$$

Estimate the CDS contract price change by multiplying the change in CDS price from Equation 14 (CDS Price ≈ 1 + ((Fixed Coupon − CDS Spread) × $\text{EffSpreadDur}_{CDS}$)) by the CDS notional.

$$\text{10-year CDS per €1 par: } 0.934 = (1 + (-0.75\% \times 8.68))$$
$$\text{9-year CDS per €1 par: } 0.947794 = (1 + (-0.66\% \times 7.91))$$

Calculate the price appreciation by multiplying the price change by the contract notional to get €128,940 (= (0.947794 – 0.9349) × €10,000,000).

Total return equals the sum of the coupon income and price appreciation, or €228,940 (= €100,000 + €128,940).

6.2. Dynamic Credit Spread Curve Strategies

Active credit managers seek to capitalize on divergent market views using cash-based or derivative strategies related to specific issuers, sectors, or the overall credit market over the credit cycle given anticipated credit curve changes across both maturities and rating categories. The following examples demonstrate how an active manager might position a credit portfolio in anticipation of these changes to generate excess return.

EXAMPLE 28 Tactical Credit Strategies – Economic Slowdown Scenario

An active credit portfolio manager considers the following corporate bond portfolio choices familiar from an earlier example:

Rating Category	Current OAS	Expected Loss (POD × LGD)	EffSpreadDur
A	1.05%	0.06%	5.5
Baa	1.35%	0.30%	6.0
Ba	2.45%	0.60%	4.5
B	3.50%	3.00%	4.0

The investor anticipates an economic slowdown in the next year that will have a greater adverse impact on lower-rated issuers. Assume that an index portfolio is equally allocated across all four rating categories, while the investor chooses a tactical portfolio combining equal long positions in the investment-grade (A and Baa) bonds and short positions in the high-yield (Ba and B) bonds.

1. Calculate excess spread on the index and tactical portfolios assuming no change in spreads over the next year (ignoring spread duration changes).
2. Calculate excess spread under an economic downturn scenario for the index and tactical portfolios where both OAS and expected loss rise 50% for investment-grade bonds and double for high-yield bonds.

Solution to 1: The following table summarizes expected excess returns E [ExcessSpread] ≈ Spread_0 – (EffSpreadDur × ΔSpread) – (POD × LGD) for each of the four rating categories with no change in spreads. For example, expected excess return for rating category A is 0.99% (= 1.05% – (5.5 × 0) – 0.06%).

Rating Category	Excess Spread
A	0.99%
Baa	1.05%
Ba	1.85%
B	0.50%

Solve for the equally weighted versus tactical portfolios as follows:

Equally weighted index: 1.10% (= (0.99% + 1.05% + 1.85% + 0.50%)/4)
Tactical portfolio: −0.16% (= (0.99% + 1.05%)/2 − (1.85% + 0.50%)/2)

Solution to 2: The following table summarizes expected excess returns E [ExcessSpread] = Spread_0 − (EffSpreadDur × ΔSpread) − (POD × LGD) for each of the four rating categories with the expected 50% increase in both OAS and expected loss under the slowdown scenario. For example, expected excess return for rating category A is −1.928% (= 1.05% − (5.5 × 0.525%) − 0.09%).

Rating Category	E(OAS)	E(Expected Loss)	E(Excess Spread)
A	1.575%	0.09%	−1.928%
Baa	2.025%	0.45%	−3.150%
Ba	4.900%	1.20%	−9.775%
B	7.000%	6.00%	−16.500%

Solve for the equally weighted versus tactical portfolios as follows.

Equally weighted index: −7.84% = (−1.928% − 3.150% − 9.775% − 16.500%)/4)
Tactical portfolio: +10.6% = (−1.928% − 3.150%)/2 − (−9.775% − 16.500%)/2)

This example assumes that an active manager is able to source and borrow the necessary Ba- and B rated bonds to sell short at no cost. However, in practice, the availability and cost of shorting bonds vary over the economic cycle, and shorting bonds is often far more difficult and costly during an economic slowdown. The synthetic, CDS-based strategy in the following example targets a similar objective.

EXAMPLE 29 Synthetic Credit Strategies: Economic Slowdown Scenario

As in the prior example, an active fixed-income manager anticipates an economic slowdown in the next year with a greater adverse impact on lower-rated issuers. The manager chooses a tactical CDX (credit default swap index) strategy combining positions in investment-grade and high-yield CDX contracts to capitalize on this view. The current market information for investment-grade and high-yield CDX contracts is as follows:

CDX Contract	Tenor	CDS Spread	$EffSpreadDur_{CDS}$
CDX IG Index	5 years	120 bps	4.67
CDX HY Index	5 years	300 bps	4.65

Assume that both CDX contracts have a $10,000,000 notional with premiums paid annually, and that the $EffSpreadDur_{CDS}$ for the CDX IG and CDX HY contracts in one year are 3.78 and 3.76, respectively.

1. Describe the appropriate tactical CDX strategy, and calculate the one-year return assuming no change in credit spread levels.
2. Calculate the one-year return on the tactical CDX strategy under an economic downturn scenario in which investment-grade credit spreads rise by 50% and high-yield credit spreads double.

Solution to 1: The investor should buy protection on the CDX IG Index and sell protection on the CDX HY Index. Current CDS prices are estimated by multiplying $EffSpreadDur_{CDS}$ by the spread difference from the standard rates of 1% and 5%, respectively:

CDX IG: 99.066 per $100 face value, or 0.99066 (= 1 + (–0.20% × 4.67))
CDX HY: 109.3 per $100 face value, or 1.093 (= 1 + (2.00% × 4.65))

In one year, the return is measured by combining the coupon income with the price appreciation assuming no spread change. Because the investor is long CDX HY and short CDX IG, the net annual premium received is $400,000 (= $10,000,000 × (5.00% – 1.00%). The respective CDS prices in one year are as follows:

CDX IG: 99.244 per $100 face value, or 0.99244 (= 1 + (–0.20% × 3.78))
CDX HY: 107.52 per $100 face value, or 1.0752 (= 1 + (2.00% × 3.76))

The investor has a $17,800 gain from the CDX IG position (= (0.99244 – 0.99066) × $10,000,000) and a $178,000 gain from the short CDX HY position (= (1.0752 – 1.093) × –$10,000,000). Adding the $400,000 coupon income results in a one-year gain from the strategy of $595,800 with constant spreads.

Solution to 2: Initial CDS prices are derived exactly as in Question 1:

CDX IG: 99.066 per $100 face value, or 0.99066 (= 1 – (4.67 × 0.2%))
CDX HY: 109.3 per $100 face value, or 1.093 (= 1 + (4.65 × 2.00%))

In one year, the return is measured by combining the coupon income with the price appreciation given the expected rise in the CDX IG spread to 1.80% and the CDX HY spread to 6.00%. In this case, the investor takes the opposite position to that of Question 1, namely long CDX HY and short CDX IG, so the net annual premium received is \$400,000 (= \$10,000,000 × (5.00% − 1.00%)). Respective CDS prices in one year are as follows:

CDX IG: 96.976 per \$100 face value, or 0.96976 (= 1 − (3.78 × 0.8%))
CDX HY: 96.24 per \$100 face value, or 0.9624 (= 1 − (3.76 × 1.00%))

The investor has a \$209,000 loss from the CDX IG position (= (0.96976 − 0.99066) × \$10,000,000) and a \$1,306,000 gain from the short CDX HY position (= (0.9624 − 1.093) × −\$10,000,000). Adding the \$400,000 net premium results in a one-year gain from the strategy of \$1,497,000 under this scenario.

The early expansion phase of the credit cycle is usually characterized by rising profits and falling leverage, as shown earlier in Exhibit 8, increasing cash flow coverage available to service outstanding debt. This reduction in the likelihood of near-term financial distress leads to both lower credit spread levels and a steeper credit curve, an effect that is more pronounced for lower-rated issuers in cyclical industries. The following examples illustrate how an active manager might capitalize on this credit cycle view in cash and synthetic markets.

EXAMPLE 30 Tactical Credit Strategies: Economic Recovery Scenario

A long-only active credit manager faces similar corporate bond portfolio choices to those in an earlier example:

Rating Category	OAS	EffSpreadDur	Expected Loss
A	1.40%	5.5	0.10%
Baa	2.00%	6.0	0.30%
Ba	3.75%	4.5	1.00%
B	5.50%	4.0	4.50%

Given an expectation that an economic rebound will cause both credit spreads and expected loss rates to fall by one-third, an active manager decides to tilt her credit portfolio toward high yield. Compare the impact of this rebound scenario on an active portfolio (33.3% invested in each of the Ba and B bond categories, with the remaining 33.3% split evenly between A and Baa) versus on an equally weighted passive portfolio.

Solution: The economic rebound scenario results in the following new OAS and expected losses, with expected excess returns E [ExcessSpread] ≈ Spread_0 − (EffSpread-Dur × ΔSpread) − (POD × LGD) in the far right column:

Rating Category	E(OAS)	E(Expected Loss)	E(Excess Spread)
A	1.40%	0.07%	4.03%
Baa	2.00%	0.20%	6.20%
Ba	3.75%	0.67%	10.04%
B	5.50%	3.00%	15.83%

Solve for the passive (equally weighted) portfolio returns versus tactical portfolio returns.

Passive portfolio return: 9.03% (= (4.03% + 6.20% + 10.04% + 15.83%)/4)
Tactical portfolio return: 10.33% (= 15.83%/3 + 10.04%/3 + 6.20%/6 + 4.03%/6).

EXAMPLE 31 Synthetic Credit Strategies: Economic Recovery Scenario

As in the prior example, an active fixed-income manager anticipates an economic rebound that is expected to cause high-yield credit curve steepening. The manager chooses a tactical CDX strategy combining 5-year and 10-year credit positions to capitalize on this view. Current market information for these high-yield CDX contracts is as follows:

CDX Contract	Tenor	CDS Spread	$EffSpreadDur_{CDS}$
CDX HY Index	5 years	450 bps	4.637
CDX HY Index	10 years	375 bps	8.656

Describe an appropriate duration-neutral portfolio positioning strategy to capitalize on this view using these CDX HY contracts. Calculate the return assuming that 5-year CDX spreads immediately fall by 175 bps and 10-year spreads decline by 25 bps for an equivalent $10,000,000 notional on the 10-year CDX index contract.

Solution: The appropriate strategy is to sell protection on the 5-year CDX HY and buy protection on the 10-year CDX HY.

1. Calculate the 5-year CDS contract notional that matches the BPV of a 10-year, $10,000,000 CDS ($BPV_{10yr}$ = $EffSpreadDur_{10yrCDS}$ × notional) using the effective spread duration ratio of 1.8667 ($EffSpreadDur_{10yrCDS}$/$EffSpreadDur_{5yrCDS}$ = 8.656/4.637) multiplied by $10,000,000 to get $18,667,000.

   Confirm this equivalence by comparing BPV_{5yr} and BPV_{10yr}:

   BPV_{5yr}: $8,656 = $18,667,000 × 4.637/10,000
   BPV_{10yr}: $8,656 = $10,000,000 × 8.656/10,000

2. Calculate portfolio return for a 175 bp decline in 5-year CDX HY spreads and a 25 bp decline in 10-year CDX HY spreads using Equation 15 (–Δ(CDS Spread) × $EffSpreadDur_{CDS}$) multiplied by the CDS notional as follows:

   CDX HY 5 year: $1,514,780 = (–1.75% × 4.637) × $18,667,000
   CDX HY 10-year: –$216,400 = (0.25% × 8.656) × –$10,000,000 Portfolio gain:
   $1,298,380 = $1,514,780 – $216,400.

   Note that this equals the contract BPV of $8,656 multiplied by the 150 bp credit curve steepening.

7. GLOBAL CREDIT STRATEGIES

- **discuss considerations in constructing and managing portfolios across international credit markets**

While yield curve strategies across currencies were covered in an earlier lesson, we now turn to cross-border fixed-income investments in which investors face the risk that they will not receive interest and principal cash flows as expected. Investors distinguish between international credit markets in developed market countries versus emerging or frontier markets. Fixed-income markets in developed countries usually have well-established and liquid derivative and other capital markets and feature a broad range of private and public debt issuers with bonds denominated in a freely floating domestic or other major currency. Emerging or frontier fixed-income markets on the other hand are often dominated by sovereign issuers, state-owned or controlled enterprises, banks, and producers operating in a dominant domestic industry such as basic commodities. As some emerging economies face concentrated risk to a particular commodity or industry, investments across sovereign, bank, and private sector debt could offer little to no diversification. While many emerging-market bonds are denominated in a restricted domestic currency with varying degrees of liquidity, the sovereign government and a select few domestic issuers often issue global bonds in a major foreign currency such as US dollars or euros.

Credit strategies across countries must take these and other individual market differences into consideration. For example, in the case of developed markets, sector composition differences exist. A far higher percentage of the US fixed-income market (and roughly one-third of the Bloomberg Barclays US Aggregate Bond Index) comprises mortgage-backed and other asset-backed instruments versus other developed markets. Investors in developed European and Asian markets seeking commercial or residential real estate exposure might instead consider covered bonds or indirect exposure via bank bonds in markets where securitization is less prevalent. International accounting standards differences between the International Accounting Standards Board's International Financial Reporting Standards and US GAAP in such areas as inventory recognition, restricted cash, and cash flow definitions require adjustment for financial ratio comparisons across jurisdictions. Finally, while most developed markets face common macroeconomic factors that influence the bond term premium and expected returns, such as inflation, monetary policy, and economic growth, differences in the timing and magnitude of market changes, as well as the credit cycle across countries, are often reflected in interest rate differentials, exchange rates, and credit spreads.

EXAMPLE 32 Credit Strategies across Developed Markets

An active United States–based credit manager is offered similar US corporate bond portfolio choices to those in an earlier example:

Rating Category	OAS	EffSpreadDur	Expected Loss
A	1.40%	5.5	0.10%
Baa	2.00%	6.0	0.30%
Ba	3.75%	4.5	1.00%
B	5.50%	4.0	4.50%

As in the earlier case, the manager expects an economic rebound but now believes that European economies will experience a stronger recovery than the United States.

In particular, European high-yield credit spreads are expected to narrow by 25% in the near term, the euro is expected to appreciate 1% against the US dollar, and all US credit spreads are expected to decline just 10% over the same period. The euro-denominated 5-year European iTraxx Crossover index (iTraxx-Xover) of liquid high-yield issuers (with a 5% fixed premium) is currently trading at 400 bps with an $EffSpreadDur_{CDS}$ of 4.25.

Describe the position the manager would take in iTraxx-Xover to capitalize on the stronger European rebound, and calculate the expected excess return percentage assuming an equally weighted allocation to US corporate bonds and an iTraxx-Xover position that matches that of the US high-yield bond allocation.

Solution: To capitalize on expected greater euro spread tightening, the manager would sell protection on the iTraxx-Xover index. To calculate expected return, first consider the US corporate bond portfolio. The economic rebound scenario results in the following new OAS and expected losses for the portfolio, with expected excess returns E [ExcessSpread] $\approx Spread_0 - (EffSpreadDur \times \Delta Spread) - (POD \times LGD)$ in the far right column:

Rating Category	E(OAS)	E(Expected Loss)	E(Excess Spread)
A	1.26%	0.09%	2.26%
Baa	1.80%	0.27%	3.47%
Ba	3.38%	0.90%	6.34%
B	4.95%	4.05%	11.75%

Return on the equally weighted portfolio is equal to 5.95% (= (2.26% + 3.47% + 6.34% + 11.75%)/4). We can estimate the initial iTraxx-Xover price by subtracting the product of $EffSpreadDur_{CDS}$ and the difference between the standard coupon (5%) from the market premium of 400 bps as follows:

Original iTraxx-Xover 5-year: 95.75 per \$100, or 0.9575 (= 1 − (4.25 × 1.00%))

If European high-yield spreads tighten by 25%, the iTraxx-Xover premium narrows by 100 bps to 300 bps, and the protection seller realizes a gain:

New iTraxx-Xover 5-year: 91.50 per \$100, or 0.9150 (= 1 − (4.25 × 2.00%))

We can calculate the percentage return on the iTraxx-Xover investment in euro terms by dividing the price change by the initial price to get 4.439% (= (95.75 − 91.50)/95.75). For a United States–based investor, we must convert the euro return to US dollars as described in an earlier lesson:

$$R_{DC} = (1 + R_{FC})(1 + R_{FX}) - 1$$

R_{DC} and R_{FC} are the domestic and foreign currency returns in percent, and R_{FX} is the percentage change of the domestic versus foreign currency.

We solve for US dollar iTraxx-Xover returns as 5.483% (= (1 + 4.439%) × (1 + 1.00%) − 1). Given that iTraxx-Xover carries a weight equal to one-half of the US corporate bond portfolio, the strategy returns 8.696% (or 5.95% + 5.483%/2).

Emerging markets are characterized by higher, more volatile, and less balanced economic growth than developed markets, often in addition to greater geopolitical risk, currency restrictions, and capital controls. Sovereign credit risk is therefore a critical starting point in considering fixed-income investments in emerging markets, where both the ability and willingness of issuers to repay debt is of importance. An earlier lesson outlined in detail sovereign credit risk considerations such as a country's institutional and economic profile, use of monetary and fiscal policy, the exchange rate regime, and external debt status and outlook.

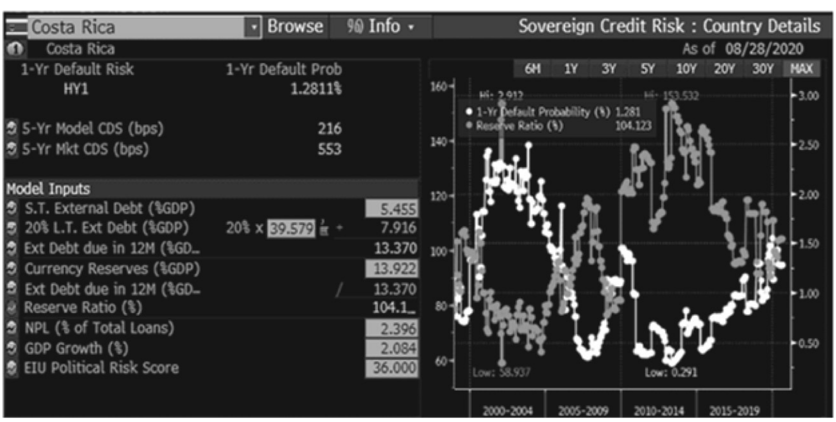
Institutional considerations include political stability, institutional transparency, and adherence to property rights and contract law. Geopolitical risks include such factors as potential conflicts and trade relations, which in some instances could have a greater impact on emerging markets whose economies are highly dependent on energy or other commodity exports. As mentioned earlier, ESG factors are key elements for sustainable, balanced, long-term economic growth.

As sovereign governments tax economic activity within their borders to repay interest and principal, key financial ratios used to assess and compare sovereign creditworthiness are usually measured as a percentage of GDP. For example, government debt to GDP and the annual government budget deficit (or surplus) as a percentage of GDP are common measures of indebtedness and fiscal stability, respectively, for both developed and emerging markets.

Finally, a country's exchange rate regime is a critical element of monetary and external flexibility. Freely floating currency regimes that allow a currency to be held in reserve outside the country enable sovereign governments to pursue an independent and flexible monetary policy. Restrictive or fixed-rate regimes limit policy effectiveness, magnifying the impact of economic crises and increasing the likelihood of financial distress. Emerging markets are usually characterized by non-reserve currency regimes with significant external debt denominated in major foreign currencies, leading analysts to incorporate external debt to GDP and currency reserves as a percentage of GDP as key leverage and liquidity measures of creditworthiness, respectively.

The Bloomberg Sovereign Risk (SRSK) model shown in Exhibit 30 combines quantitative and qualitative factors such as external debt to GDP, currency reserves, GDP growth, and political risk to estimate a sovereign issuer's one-year POD. Similar to the DRSK model discussed earlier, the SRSK model allows users to change model inputs and also derives a "model" CDS spread, which could be compared to the market CDS spread.

EXHIBIT 30 Bloomberg SRSK Screen



In this example, Costa Rica has a 1.28% one-year default risk and a model CDS spread well below the market CDS spread.

EXAMPLE 33 Sovereign Risk Factors for Emerging Markets

A financial analyst is considering the likelihood that an emerging market sovereign issuer of US dollar–denominated bonds is able to meet its interest payments over the next 12 months. Which of the following financial ratios is most appropriate to assess the sovereign borrower's liquidity position?

A. Government budget deficit/GDP
B. External debt/GDP
C. Currency reserves/GDP

Solution: The correct answer is C. The government budget deficit as a percentage of GDP is a gauge of fiscal stability for the domestic economy, while the external debt-to-GDP ratio is a measure of financial leverage to foreign lenders. Currency reserves as a percentage of GDP measure the available liquidity in foreign currency to meet external obligations.

Several additional considerations are important for investors in emerging market bonds issued by private companies. First, although some local companies might have partial private ownership and publicly traded equity, the sovereign government could exercise controlling influence on the business, including replacing management or ownership groups.

Credit quality in the emerging market credit universe exhibits a high concentration in lower investment-grade and upper high-yield ratings categories. This concentration of credit ratings is largely a reflection of the sovereign ratings of emerging markets but also reflects the fact that a "sovereign ceiling" is usually applied to corporate issuers globally. This ceiling implies that a company's rating is typically no higher than the sovereign credit rating of its domicile.

Finally, relative liquidity conditions and currency volatility are key considerations for international credit investors. In emerging markets, liquidity is often constrained because of a relatively small number of bonds that trade regularly, resulting in investors demanding higher premiums for holding emerging market credit securities. Local bond markets might seem highly liquid and can exceed the trading volume of the local stock exchanges, but such high trading volume could also be inflated by interbank trading by local banks and retail investors. Currency volatility can be particularly significant in emerging markets as a result of restrictive currency regimes and derivative markets. Higher YTMs available in emerging market currencies versus developed markets typically suggest that these emerging currencies will depreciate over time. That said, emerging markets offer investors the opportunity to exploit divergence from interest rate parity conditions (known as the forward rate bias) by investing in higher-yielding currencies, as addressed in earlier lessons. Although temporary deviations from a fixed exchange rate are possible under such regimes, what is more common during economic crises is exchange rate regime change, central bank intervention, and/or devaluation. The following example demonstrates how such factors are considered in emerging market credit strategies.

EXAMPLE 34 Emerging Market Credit Strategy

An active United States–based investor is considering a portfolio allocation to the bonds of a major commodities producer headquartered in an emerging market economy. The issuer is a major exporter, and commodity exports comprise a significant proportion of the country's economic growth. Describe how the investor would decide between purchasing a higher-yielding, local-currency-denominated bond and a lower-yielding, US-dollar-denominated bond with otherwise similar features.

Solution: A United States–based investor seeking to maximize US-dollar-denominated return must consider the relationship between the higher local currency bond YTM, the lower US dollar bond YTM, and the local currency's expected depreciation (or appreciation) versus the US dollar over the investment horizon. While uncovered interest rate parity suggests that local currency depreciation versus the US dollar would offset any benefit of a higher YTM, an investor with a bullish view of the emerging economy's growth prospects would benefit from forward rate bias and earn a higher return in US dollar terms from an unhedged investment in the local currency bond if the local currency were to depreciate less than expected under interest rate parity conditions.

8. STRUCTURED CREDIT

- **describe the use of structured financial instruments as an alternative to corporate bonds in credit portfolios**

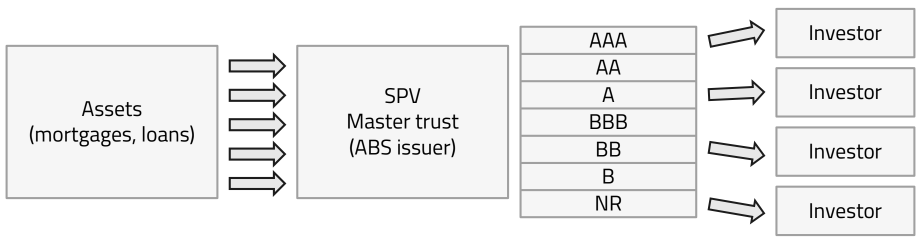
Active managers have access to a wide array of credit management tools beyond individual fixed-income securities that include structured financial instruments. These alternatives to direct bond investments in corporate bonds introduced in earlier lessons are summarized in Exhibit 31.

EXHIBIT 31 Structured Alternatives to Individual Bonds

Instrument	Description	Exposure	Portfolio Applications
Collateralized Debt Obligations (CDOs)	Fixed-income securities backed by a diversified pool of debt obligations	Redistribute portfolio debt cash flows across ratings spectrum	Create tailored portfolio-based debt exposure categories/profiles unavailable in the cash bond market
Collateralized Loan Obligations (CLOs)	Fixed-income securities backed by a diversified pool of floating-rate leveraged loan obligations	Redistribute portfolio loan cash flows across ratings spectrum	Create tailored portfolio-based loan and interest rate exposure profiles unavailable in the cash bond market
Mortgage-Backed Securities (MBS)	Fixed-income securities backed by a pool of commercial or residential mortgage loans	Provide portfolio-based exposure to real estate cash flows	Offer active managers exposure to real estate and to volatility (prepayment/extension risk) unavailable in the cash bond market
Asset-Backed Securities (ABS)	Fixed-income securities backed by a pool of credit card, auto, and other loans	Provide portfolio-based exposure to consumer loan cash flows	Offer active managers direct exposure to consumer loans and to volatility unavailable in the cash bond market
Covered Bonds	Senior debt obligations backed by pool of commercial/residential mortgages or public sector assets	Provide portfolio-based exposure to real estate cash flows with recourse to issuer	Offer active managers direct exposure to consumer loans and to real estate/public sector cash flows unavailable in the cash bond market

Structured financial instruments can offer active credit managers the ability to access fixed-income cash flows such as commercial or residential real estate, enhance returns by increasing portfolio exposure to interest rate volatility (via mortgage prepayment and extension risk), and add debt exposure created by the redistribution of default risk into different tranches across the credit spectrum. Exhibit 32 shows an illustrative example of the tranching that characterizes ABS and CDO transactions. In this case, the ABS issuer is a special purpose vehicle (SPV) that owns the underlying asset pool and issues debt across several tranches backed by the asset pool cash flows.

EXHIBIT 32 Illustrative Tranching Example



An earlier lesson addressed the redistribution of default risk from the underlying asset pool. This is achieved by establishing higher-rated tranches via internal credit enhancement or overcollateralization, with successively lower-rated tranches absorbing a greater proportion of the associated default risk. An active investor might overweight default risk by choosing a lower-rated ABS tranche based on a tactical view. For example, such an investor might anticipate lower-than-expected defaults or believe the credit cycle is in recovery mode and that lower-rated tranches will experience greater spread tightening than higher-rated tranches. Alternatively, a portfolio manager might underweight credit exposure using a higher-rated tranche in a downturn.

While covered bonds offer real estate cash flow exposure similar to that of ABS, given the dual recourse (i.e., to both the issuing financial institution and the underlying asset pool), as well as the substitution of non-performing assets, covered bonds usually involve lower credit risk and a lower yield. The following examples demonstrate the role of structured products in active credit portfolios.

EXAMPLE 35 The Role of Structured Products in Active Credit Management

1. An active credit manager anticipates an economic slowdown led by a decline in residential housing prices. Which of the following portfolio positioning strategies involving structured products is the most appropriate to consider under this scenario?
   A. Shift exposure from an A rated tranche of a credit card ABS transaction to a BB rated tranche
   B. Increase exposure to an A rated CDO tranche and reduce exposure to a BBB rated CDO tranche
   C. Increase exposure to an A rated MBS tranche and decrease exposure to a BBB rated MBS tranche

2. An active fixed-income portfolio manager expects an economic recovery in the near term to be accompanied by rising short-term rates and a flatter benchmark yield curve. Which of the following strategies best positions an active manager to capitalize on this scenario?
   A. Increase exposure to covered bonds and decrease exposure to MBS
   B. Shift exposure from an A rated CDO tranche to a BBB rated CLO tranche
   C. Shift exposure from a BB rated tranche of an automotive ABS transaction to a A rated tranche

Solution to 1: The correct answer is C. As the housing sector slows and default rates rise, credit spreads of lower-rated MBS tend to widen by more than those of higher-rated MBS. The investor retains exposure to real estate cash flows while reducing exposure to spread widening. The shift to a BB rated credit card ABS tranche increases credit exposure, while the switch from BBB rated to A rated CDOs represents a reduction in overall market risk rather than a more targeted underweight, as in C.

Solution to 2: The correct answer is B. Economic recovery is typically associated with lower defaults and greater credit spread tightening among lower-rated issuers and debt tranches. CLO tranches benefit more from short-term rate rises than CDOs because CLOs comprise leveraged loans based on MRRs plus a credit spread. As for A, a shift to covered bonds from MBS reduces credit risk because of the dual recourse and substitutability of collateral characteristics of covered bonds. In C, credit exposure is reduced, limiting the benefit from credit spread reduction within the portfolio.

9. FIXED-INCOME ANALYTICS

- **describe key inputs, outputs, and considerations in using analytical tools to manage fixed-income portfolios**

Fixed-income analytical tools continue to adapt not only to technological change but also to the market and regulatory environment within which active fixed-income practitioners operate. The inputs and outputs of these models have become more complex as market participants integrate tasks across operational duties and portfolio decision making and execution. These tasks include portfolio construction, risk analytics, trading and settlement, cash and collateral management, daily valuation, portfolio accounting, and regulatory reporting.

Primary inputs for fixed-income models include all long and short cash bond and derivative positions, repurchase agreements, and cash across currencies. Fixed-income security inputs use CUSIP or ISIN identifiers to capture all relevant features such as interest and principal payment dates, day count conventions, and put–call features. Portfolio derivative and repo position inputs also include details of such agreements, such as settlement dates, option strike prices, and collateral terms necessary to satisfy derivative counterparty or clearing requirements based on market changes.

Real-time market data feeds usually sourced from vendors via application programming interfaces include spot and forward rates, credit curves, implied volatilities, and exchange rates that are used to value historical, existing, and potential future new portfolio positions. These tools value inactively traded fixed-income instruments using matrix pricing (or evaluated pricing) based on observable liquid benchmark YTMs of similar maturity and duration and credit spreads of actively traded bonds with comparable times to maturity, credit quality, and sector.

Additional model inputs include index subscriptions, ESG and credit ratings, and issuer balance sheet data. In contrast to more static equity indexes, fixed-income indexes are subject to constant change as a result of both new debt issuance and bond maturities as well as ratings changes, bond callability, and prepayment.

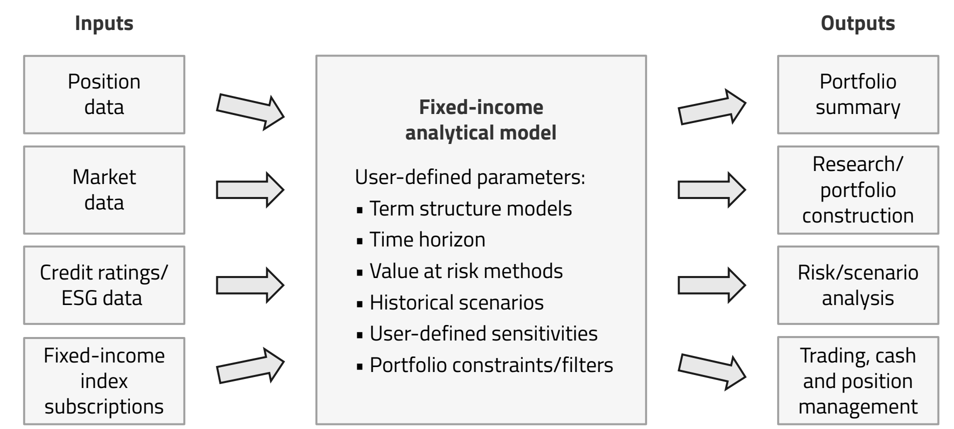
Model assumptions include user-defined parameters such as term structure models, investment time horizon, VaR methodology, historical and/or specific market scenarios, and portfolio filters that could involve inclusion or exclusion of specific sectors or a minimum ESG rating threshold for consideration.

Fixed-income analytical model outputs support each stage of the active portfolio management process, namely portfolio selection and construction, risk analysis of existing and prospective portfolio positions, and trading and position management. A portfolio summary or landing page typically aggregates current portfolio risk and return across sectors, ratings, and currencies versus the benchmark index. Model applications supporting research and portfolio construction allow managers to assess the expected change in portfolio performance by including incremental long or short cash bond, derivative, or structured product positions. Portfolio risk dashboards embedded in these tools provide detailed insight into portfolio duration and convexity as well as tail risk. These statistics are often further disaggregated into key rate duration measures for benchmark rates and credit spreads by maturity. VaR and expected shortfall (or CVaR) are calculated based on user threshold and methodology settings. Finally, trading, cash, and position management outputs quantify existing cash positions, anticipated cash inflows and outflows from existing positions, and liquidity risk. Exhibit 33 summarizes the key elements of a fixed-income portfolio analytics tool.

Key considerations for fixed-income analytical tools include both the accuracy of model inputs and assumptions and the degree of alignment between model outputs and specific fixed-income manager objectives.

Bond price and YTM calculations are affected by assumptions related to the term structure of benchmark rates and volatilities and how they change over time based on term structure models. Model outputs are often tailored to match an active manager's objectives. For example, an index fund manager might seek to minimize the tracking error defined earlier as the deviation of portfolio returns from an index. An active fixed-income manager with fewer constraints might maximize risk-adjusted returns, while estimating and categorizing how each position

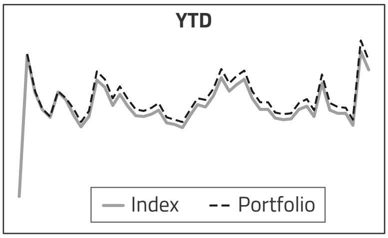
EXHIBIT 33 Illustrative Portfolio Analytics Tool



(continued)

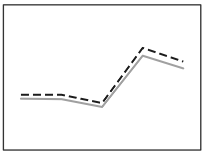
EXHIBIT 33 (Continued)

Portfolio summary

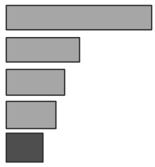


Summary Statistics	Portfolio	Index
Returns (YTD)	x.xx%	x.xx%
Returns (30d)	x.xx%	x.xx%
OAS (bps)	x.xx	x.Xx
Effective duration	x.xx	x.xx
Effective convexity	x.xx	x.Xx
Tracking error	x.xx%	—

5-day return



Top 5	%
10Y UST	x.xx%
ABC	x.xx%
XYZ	x.xx%
CDE	x.xx%
30Y FUT	-x.xx%



Currency	Portfolio	Index
USD	x.xx%	x.xx%
EUR	x.xx%	x.xx%
GBP	x.xx%	x.xx%
JPY	x.xx%	x.xx%
BRL	x.xx%	x.xx%

Rating	Portfolio	Index
AAA-AA	x.xx%	x.xx%
A	x.xx%	x.xx%
BBB	x.xx%	x.xx%
BB	x.xx%	x.xx%
B-CCC	x.xx%	x.xx%

Segment	Portfolio	Index
Inv grade corporate	x.xx%	x.xx%
High yield corporate	x.xx%	x.xx%
Sovereign DM	x.xx%	x.xx%
Sovereign EM	x.xx%	x.xx%
Securitized	x.xx%	x.xx%

contributes to active risk taking. For example, performance attribution measures returns from credit, duration, sector, and currency tilts, among other factors. Finally, an active manager facing liability constraints usually models the fixed-income characteristics of obligations to maximize the expected surplus of assets over liabilities. Practitioners applying these tools must both recognize their limitations and anticipate and interpret model results, as in the following example.

EXAMPLE 36 Applying Fixed-Income Analytical Tools

An active fixed-income manager is conducting scenario analysis for the MBS component of a portfolio. Which of the following analytical model input changes is most likely to reduce the future value of the MBS subportfolio versus similar option-free bond holdings?

A. An increase in benchmark yield curve volatility
B. A decrease in benchmark yield curve volatility
C. Upward parallel shift in the benchmark yield curve

Solution: The correct answer is A. The value of a bond with an embedded option is equal to the sum of the value of an option-free bond plus the value to the embedded option. The value of the embedded call option owned by the issuer will increase as volatility rises, reducing the value of the MBS versus a similar option-free bond. Answers B and C are more likely to result in an increase in the value of MBS versus an option-free bond.

SUMMARY

Active spread-based, fixed-income portfolio management involves taking positions in credit and other risk factors that differ from those of an index to generate excess return. The main points of the reading are as follows:

- Yield spreads compensate investors for the risk that they will not receive expected interest and principal cash flows and for the bid–offer cost of buying or selling a bond under current market conditions.
- Two key components of a bond's credit risk are the POD and the LGD.
- Credit spread changes are driven by the credit cycle, or the expansion and contraction of credit over the business cycle, which causes asset prices to change based on default and recovery expectations.
- High-yield issuers experience greater changes in the POD over the credit cycle than investment-grade issuers, with bond prices approaching the recovery rate for distressed debt.
- While fixed-rate bond yield spread measures use actual, interpolated, or zero curve–based benchmark rates to capture relative credit risk, OAS allow comparison between risky option-free bonds and bonds with embedded options.
- FRNs pay periodic interest based on an MRR plus a yield spread.
- Spread duration measures the change in a bond's price for a given change in yield spread, while spread changes for lower-rated bonds tend to be proportional on a percentage rather than an absolute basis.
- Bottom-up credit strategies include the use of financial ratio analysis, reduced form credit models (such as the Z-score model), and structural credit models, including Bloomberg's DRSK model.
- Top-down credit strategies are often based on macro factors and group investment choices by credit rating and industry sector categories.
- Fixed-income factor investing incorporates such factors as size, value, and momentum to target active returns and also increasingly include ESG factors.
- Liquidity risk in credit markets is higher than in equities because of market structure differences and is often addressed using liquid bonds for short-term tactical positioning, less liquid positions for buy-and-hold strategies, and liquid alternatives where active management adds little value.
- Credit market tail risk is usually quantified using VaR or expected shortfall measures and is frequently managed using position limits, risk budgeting, or derivative strategies.
- Credit derivative strategies offer a synthetic liquid alternative to active portfolio managers as a means of over- or underweighting issuers, sectors, and/or maturities across the credit spectrum.
- Credit spread levels and curve slopes change over the credit cycle, with credit curve steepening usually indicating low near-term default expectations and higher growth expectations, while curve flattening, or inversion, suggests rising default expectations and lower future growth.
- Active credit managers can benefit under a stable credit curve scenario by adding spread duration for existing exposures and/or increasing average portfolio credit risk and can capitalize on divergent market views using cash- or derivative-based strategies related to specific issuers, sectors, or the overall credit market.
- Investors in international credit markets distinguish between developed and emerging markets. Developed markets face common macro factors, with market and credit cycle differences affecting relative interest rates, foreign exchange rates, and credit spreads. Emerging markets usually exhibit higher growth combined with greater sovereign and geopolitical risk, currency restrictions, and capital controls.

- Structured financial instruments offer active credit managers access to liquid bond portfolios, fixed-income cash flows derived from real estate and consumer loans, and enhanced returns by adding volatility and/or debt exposure via tranching across the credit spectrum.
- Key considerations for fixed-income analytical tools include the accuracy of model inputs and assumptions as well as alignment between model outputs and fixed-income manager objectives.

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PRACTICE PROBLEMS

1. Which of the following statements best describes empirical duration?
   A. A common way to calculate a bond's empirical duration is to run a regression of its price returns on changes in a benchmark interest rate.
   B. A bond's empirical duration tends to be larger than its effective duration.
   C. The price sensitivity of high-yield bonds to interest rate changes is typically higher than that of investment-grade bonds.
2. A junior analyst considers a 10-year high-yield bond issued by EKN Corporation (EKN) position in a high-yield portfolio. The bond has a price of 91.82, a modified duration of 8.47, and a spread duration of 8.47. The analyst speculates on the effects of an interest rate increase of 20 bps and, because of a change in its credit risk, an increase in the EKN bond's credit spread of 20 bps. The analyst comments that because the modified duration and the credit spread duration of the EKN bond are equal, the bond's price will not change (all else being equal) in response to the interest rate and credit spread changes.

Is the analyst's prediction correct that the EKN bond price will not change in response to the interest rate and credit spread changes, all else being equal?

A. Yes.
B. No, the bond price should decrease.
C. No, the bond price should increase.

3. Which of the following outcomes is most likely if the junior analyst revises the bond's original recovery rate higher?
   A. An increase in the bond's POD
   B. A decrease in the bond's POD
   C. A decrease in the bond's credit spread
4. Which of the following observations on the risks of spread-based fixed-income portfolios is the most accurate?
   A. Because credit spreads equal the product of the LGD and the POD, distinguishing between the credit risk and liquidity risk components of yield spread across all market scenarios is straightforward.
   B. Given that frequent issuers with many bonds outstanding across maturities have their own issuer-specific credit curve, distinguishing between the credit spread and liquidity spread of all bonds for these issuers is straightforward.
   C. The yield spread of a particular bond comprises both credit and liquidity risk and depends on market conditions and the specific supply-and-demand dynamics of each fixed-income security.

The following information relates to Questions 5–8

An active portfolio manager observes the following market information related to an outstanding corporate bond and two on-the-run government bonds that pay annual coupons:

Issuer	Term	Coupon	Yield	ModDur
Corporate	12y	3.00%	2.80%	9.99
Government	10y	1.75%	1.85%	9.09
Government	20y	2.25%	2.30%	15.94

The portfolio manager also observes 10-year and 20-year swap spreads of 0.20% and 0.25%, respectively.

5. Calculate the G-spread of the corporate bond.
   A. 0.860%
   B. 0.725%
   C. 0.950%
6. Calculate the I-spread of the corporate bond.
   A. 0.85%
   B. 0.65%
   C. 0.95%
7. Calculate the ASW of the corporate bond.
   A. 0.65%
   B. 0.95%
   C. 0.85%

8. Estimate the corporate bond's percentage price change if the government yield curve steepens, assuming a 0.20% increase in the 20-year YTM and no change to the 10-year government YTM or corporate G-spread.
   A. –0.40%
   B. 0.40%
   C. –0.04%

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9. Which of the following statements about credit spread measures is most accurate?
   A. The DM is the yield spread over the MRR established upon issuance to compensate investors for assuming an issuer's credit risk.
   B. The Z-DM will be above the DM if the MRR is expected to remain constant over time.
   C. The yield spread for a corporate bond will be equal to the G-spread if the government benchmark yield curve is flat.

The following information relates to Questions 10–12

An active fixed-income manager is considering two corporate bond positions for an active portfolio. The first bond has a BBB rating with a credit spread of 2.75% and an effective spread duration of 6, and the second bond has a BB rating with a credit spread of 3.50% and an effective spread duration of five years.

10. What is the approximate excess return if the BBB rated bond is held for six months and the credit spread narrows by 40 bps, ignoring spread duration changes and assuming no default losses?
    A. 3.775%
    B. 2.35%
    C. 2.40%
11. What is the instantaneous (holding period of zero) excess return for the BB rated bond if the spread widens by 50 bps?
    A. 3.00%
    B. –2.50%
    C. 2.50%
12. What is the expected excess spread of the BBB rated bond for an instantaneous 50 bp decline in yields if the bond's LGD is 40% and the POD is 0.75%?
    A. 1.95%
    B. 2.45%
    C. 2.70%

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13. An active manager is considering the senior bonds of one of several corporate issuers. Holding other factors constant, which of the following key financial ratio changes would lead the manager to expect a decrease in the POD for that issuer?
    A. An increase in the issuer's coverage ratio
    B. An increase in the issuer's stock price volatility
    C. An increase in the issuer's leverage ratio
14. Which of the following statements about statistical credit analysis models is most accurate?
    A. Structural credit models solve for the POD using observable company-specific variables such as financial ratios and macroeconomic variables.

B. Reduced-form credit models use market-based variables to estimate an issuer's asset value and the volatility of asset value.
C. Structural credit models define the likelihood of default as the probability of the asset value falling below that of liabilities.

The following information relates to Questions 15–17

An investor is faced with an active portfolio decision across three bond rating categories based on the following current market information:

Rating Category	Current OAS	Expected Loss (POD × LGD)	EffSpreadDur
A	1.00%	0.10%	7
BBB	1.75%	0.75%	6
BB	2.75%	2.50%	5

15. Which bond rating category offers the highest expected excess return if credit spreads remain stable under current market conditions?
    A. A rated bond category
    B. BBB rated bond category
    C. BB rated bond category
16. Which active bond portfolio maximizes expected excess return under a stable credit market assumption versus an equally weighted benchmark portfolio across the three rating categories?
    A. 50% A rated bonds, 50% BBB rated bonds
    B. 50% BBB rated bonds, 50% BB rated bonds
    C. 50% A rated bonds, 50% B rated bonds
17. Which bond rating category offers the highest expected excess return if spreads instantaneously rise 10% across all ratings categories?
    A. A rated bond category
    B. BBB rated bond category
    C. BB rated bond category

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18. Which of the following strategies best addresses the liquidity risk of a less frequently traded bond position in an active manager's portfolio?
    A. Enter into a receive fixed, pay floating asset swap, unwinding the swap position once the illiquid bond position is sold.
    B. Sell single-name CDS protection on the illiquid bond issuer, unwinding the CDS contract when the bond is sold.
    C. Allocate the illiquid bond to the buy-and-hold portion of the investment portfolio.
19. Which of the following statements best describes methods for assessing portfolio tail risk?
    A. Parametric methods use expected value and standard deviation of risk factors under a normal distribution and are well suited for option-based portfolios.
    B. Historical simulation methods use historical parameters and ranking results and are not well suited for option-based portfolios.
    C. Monte Carlo methods generate random outcomes using portfolio measures and sensitivities and are well suited for option-based portfolios.

The following information relates to Questions 20 and 21

An investor is considering the portfolio impact of a new 12-year corporate bond position with a $75 million face value, a 3.25% coupon, current YTM of 2.85%, modified duration of 9.887, and a price of 104.0175 per 100 of face value.

20. Which of the following VaR measures is most appropriate for the portfolio manager to use to evaluate how this position would affect portfolio tail risk?
    A. CVaR
    B. Relative VaR
    C. Incremental VaR
21. What is the approximate VaR for the bond position at a 99% confidence interval (equal to 2.33 standard deviations) for one month (with 21 trading days) if daily yield volatility is 1.50% and returns are normally distributed?
    A. $1,234,105
    B. $2,468,210
    C. $5,413,133

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22. Which of the following statements best describes how a single-name CDS contract is priced at inception?
    A. If the reference entity's credit spread trades below the standard coupon rate, the CDS contract will be priced at a premium above par because the protection buyer pays a "below market" periodic coupon.
    B. If the reference entity's credit spread trades above the standard coupon rate, the CDS contract will be priced at a discount to par because the protection seller effectively receives a "below market" periodic premium.
    C. Similar to fixed-rate bonds, CDS contracts are initially priced at par with a fixed coupon and a price that changes over time as the reference entity's credit spreads change.

The following information relates to Questions 23 and 24

An active portfolio manager seeking to purchase single-name CDS protection observes a 1.75% 10-year market credit spread for a private investment-grade issuer. The effective spread duration is 8.75 and CDS basis is close to zero.

23. What should the protection buyer expect to pay or receive to enter a new 10-year CDS contract?
    A. The buyer should receive approximately 6.5625% of the notional.
    B. The buyer should pay approximately 15.3125% of the notional.
    C. The buyer should pay approximately 6.5625% of the notional.
24. Once the manager purchases CDS protection, the issuer's CDS spread immediately falls to 1.60%. What is the investor's approximate mark-to-market gain or loss for a contract notional of €10,000,000?
    A. The manager realizes an approximate loss of €131,250.
    B. The manager realizes an approximate gain of €131,250.
    C. The manager realizes an approximate gain of €525,000.

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25. Which of the following credit portfolio positioning strategies is the most appropriate to underweight the financial sector versus an index?
    A. Purchase protection on the CDX and sell protection on the CDX Financials subindex.

B. Sell protection on the CDX and purchase protection on the CDX Financials subindex.
C. Purchase a payer option on the CDX and sell protection on the CDX Financials subindex.

26. Which of the following phases of the credit cycle typically involves a decline in the number of issuer defaults?
    A. Late expansion phase
    B. Early expansion phase
    C. Peak phase
27. Which of the following regarding the shape of the credit spread curve for high-yield issuers is most accurate?
    A. High-yield credit spread curves change shape more over the cycle than investment-grade ones do and usually invert during the peak phase.
    B. Investors should exercise caution in interpreting credit spread curve shape for distressed debt issuers because their bonds tend to trade on a price rather than credit spread basis as the likelihood of default increases.
    C. High-yield credit spread curves often invert because of the empirical observation that DTS is the best way to measure high-yield bond price changes.
28. Which of the following statements best describes a credit curve roll-down strategy?
    A. Returns from a credit curve roll-down strategy can be estimated by combining the incremental coupon from a longer maturity corporate bond with price appreciation due to the passage of time.
    B. A synthetic credit curve roll-down strategy involves purchasing protection using a single-name CDS contract for a longer maturity.
    C. A credit curve roll-down strategy is expected to generate a positive return if the credit spread curve is upward sloping.

The following information relates to Questions 29 and 30

An investor observes the following current CDS market information:

CDX Contract	Tenor (years)	CDS Spread	$\text{EffSpreadDur}_{CDS}$
CDX IG Index	5	85 bps	4.9
CDX IG Index	10	135 bps	8.9
CDX HY Index	5	175 bps	4.7
CDX HY Index	10	275 bps	8.7

29. Select the most appropriate credit portfolio positioning strategy to capitalize on an expected steepening of the investment-grade credit spread curve.
    A. Sell protection on the 10-year CDX IG index and purchase protection on the 5-year CDX IG index using contracts of equal notional value.
    B. Sell protection on the 10-year CDX IG index and purchase protection on the 5-year CDX IG index using a contract with a notional amount equal to 1.82 times that of the 10-year contract.
    C. Buy protection on the 10-year CDX IG index and sell protection on the 5-year CDX IG index using a contract with a notional amount equal to 1.82 times that of the 10-year contract.

30. Which of the following is the most appropriate credit portfolio positioning strategy to capitalize on an expected economic contraction?
    A. Buy protection on the 5-year CDX HY index and sell protection on the 5-year CDX IG index in approximately equal notional amounts.
    B. Buy protection on the 10-year CDX IG index and sell protection on the 5-year CDX IG index using a contract with a notional amount equal to 1.82 times that of the 10-year contract.
    C. Buy protection on the 10-year CDX HY index and sell protection on the 5-year CDX HY index using a contract with a notional amount equal to 1.85 times that of the 10-year contract.

31. Which of the following is the most accurate statement related to international credit markets?
    A. Fixed exchange-rate regimes among emerging markets usually reduce the likelihood of financial distress because the domestic currency is tied to a major foreign currency.
    B. Although many emerging economies have domestic bond markets that include sovereign, financial, and corporate issuers, investments across these bonds offer less diversification than similar investments in developed markets.
    C. Higher domestic currency YTMs among emerging versus developed markets are due to expected currency appreciation resulting from higher economic growth.

The following information relates to Questions 32 and 33

An active United States–based credit manager faces the following US and European investment-grade and high-yield corporate bond portfolio choices:

Rating Category	OAS	EffSpreadDur	Expected Loss
USD IG	1.25%	4.50	0.40%
USD HY	3.00%	5.50	2.25%
EUR IG	1.15%	4.75	0.50%
EUR HY	3.25%	6.00	2.50%

The EUR IG and EUR HY allocations are denominated in euros, and the euro is expected to depreciate by 2% versus the US dollar over the next year.

32. What is the approximate unhedged excess return to the United States–based credit manager for an international credit portfolio index equally weighted across the four portfolio choices, assuming no change to spread duration and no default losses occur?
    A. –0.257%
    B. –0.850%
    C. 0.750%
33. Which of the following active portfolios is expected to have the highest excess return versus the index if European economies are expected to experience an earlier and much stronger credit cycle recovery than the United States?
    A. EUR HY 50.0%, EUR IG 25.0%, USD IG 12.5%, USD HY 12.5%
    B. EUR IG 50.0%, EUR HY 25.0%, USD IG 12.5%, USD HY 12.5%
    C. EUR HY 33.3%, US HY 33.3%, EUR IG 16.7%, USD IG 16.7%

34. Which of the following statements about the role of structured products in an active credit portfolio is most accurate?
    A. Covered bonds perform relatively well in a downturn versus other fixed-income bonds with real estate exposure because a covered bond investor also has recourse to the issuer.
    B. Higher-rated ABS tranches are attractive for active investors seeking to overweight default risk when the credit cycle is in recovery.
    C. CLO tranches are more advantageous than CDO tranches with similar ratings under an economic slowdown scenario.
35. An active fixed-income manager is evaluating the relative performance of an investment-grade corporate versus a high-yield corporate debt allocation in a fixed-income portfolio. Which of the following analytical model assumption changes is most likely to reduce the future value of the high-yield portfolio relative to the investment-grade holdings?
    A. Steepening of the benchmark yield volatility curve
    B. Decreased likelihood of an economic slowdown
    C. Increased likelihood of a flight to quality associated with bullish benchmark yield curve flattening (long-term rates fall by more than short-term rates do)

GLOSSARY

Accounting defeasance Also called in-substance defeasance, accounting defeasance is a way of extinguishing a debt obligation by setting aside sufficient high-quality securities to repay the liability.

Accrued interest Interest earned but not yet paid.

Active management A portfolio management approach that allows risk factor mismatches relative to a benchmark index causing potentially significant return differences between the active portfolio and the underlying benchmark.

Active return Portfolio return minus benchmark return.

Active risk The annualized standard deviation of active returns, also referred to as *tracking error* (also sometimes called *tracking risk*).

Add-on rates Bank certificates of deposit, repos, and indexes such as Libor and Euribor are quoted on an add-on rate basis (bond equivalent yield basis).

Agency bond See *quasi-government bond*.

Agency RMBS In the United States, securities backed by residential mortgage loans and guaranteed by a federal agency or guaranteed by either of the two GSEs (Fannie Mae and Freddie Mac).

Amortizing bond Bond with a payment schedule that calls for periodic payments of interest and repayments of principal.

Amortizing loans Loans with a payment schedule that calls for periodic payments of interest and repayments of principal.

Analytical duration The use of mathematical formulas to estimate the impact of benchmark yield-to-maturity changes on bond prices.

Arbitrage opportunity An opportunity to earn an expected positive net profit without risk and with no net investment of money.

Arbitrage-free models Term structure models that project future interest rate paths that emanate from the existing term structure. Resulting prices are based on a no-arbitrage condition.

Arbitrage-free valuation An approach to valuation that determines security values consistent with the absence of any opportunity to earn riskless profits without any net investment of money.

Asset swap Converts the periodic fixed coupon of a specific bond to an MRR plus or minus a spread.

Asset swap spread (ASW) The spread over MRR on an interest rate swap for the remaining life of the bond that is equivalent to the bond's fixed coupon.

Asset-backed securities A type of bond issued by a legal entity called a *special purpose entity* (SPE) on a collection of assets that the SPE owns. Also, securities backed by receivables and loans other than mortgages.

Auction A type of bond issuing mechanism often used for sovereign bonds that involves bidding.

Authorized participants Institutional investors who create and redeem ETF shares using an OTC primary market with an ETF sponsor.

Average life See *weighted average life*.

Backup line of credit A type of credit enhancement provided by a bank to an issuer of commercial paper to ensure that the issuer will have access to sufficient liquidity to repay maturing commercial paper if issuing new paper is not a viable option.

Balloon payment Large payment required at maturity to retire a bond's outstanding principal amount.

Bankruptcy A declaration provided for by a country's laws that typically involves the establishment of a legal procedure that forces creditors to defer their claims.

Barbell A fixed-income investment strategy combining short- and long-term bond positions.

Barbell portfolio Fixed-income portfolio that combines short and long maturities.

Basis point Used in stating yield spreads, one basis point equals one-hundredth of a percentage point, or 0.01%.

Basis trade A trade based on the pricing of credit in the bond market versus the price of the same credit in the CDS market. To execute a basis trade, go long the "underpriced" credit and short the "overpriced" credit. A profit is realized as the implied credit prices converge.

Bear flattening A decrease in the yield spread between long- and short-term maturities across the yield curve, which is largely driven by a rise in short-term bond yields-to-maturity.

Bear steepening An increase in the yield spread between long- and short-term maturities across the yield curve, which is largely driven by a rise in long-term bond yields-to-maturity.

Bearer bonds Bonds for which ownership is not recorded; only the clearing system knows who the bond owner is.

Bearish flattening Term structure shift in which short-term bond yields rise more than long-term bond yields, resulting in a flatter yield curve.

Benchmark A comparison portfolio; a point of reference or comparison.

Benchmark issue The latest sovereign bond issue for a given maturity. It serves as a benchmark against which to compare bonds that have the same features but that are issued by another type of issuer.

Benchmark rate Typically the yield-to-maturity on a government bond having the same or close to the same time-to-maturity.

Benchmark spread The yield spread over a specific benchmark, usually measured in basis points.

Best-efforts offering An offering of a security using an investment bank in which the investment bank, as agent for the issuer, promises to use its best efforts to sell the offering but does not guarantee that a specific amount will be sold.

Bid–ask spread The difference between the prices at which dealers will buy from a customer (bid) and sell to a customer (offer or ask). It is often used as an indicator of liquidity.

Bid–offer spread The difference between the prices at which dealers will buy from a customer (bid) and sell to a customer (offer or ask). It is often used as an indicator of liquidity.

Bilateral loan A loan from a single lender to a single borrower.

Bond Contractual agreement between the issuer and the bondholders.

Bond equivalent yield A calculation of yield that is annualized using the ratio of 365 to the number of days to maturity. Bond equivalent yield allows for the restatement and comparison of securities with different compounding periods.

Bond indenture The governing legal credit agreement, typically incorporated by reference in the prospectus. Also called *trust deed*.

Bond risk premium The expected excess return of a default-free long-term bond less that of an equivalent short-term bond.

Bootstrapping The use of a forward substitution process to determine zero-coupon rates by using the par yields and solving for the zero-coupon rates one by one, from the shortest to longest maturities.

Bridge financing Interim financing that provides funds until permanent financing can be arranged.

Bull flattening A decrease in the yield spread between long- and short-term maturities across the yield curve, which is largely driven by a decline in long-term bond yields-to-maturity.

Bull steepening An increase in the yield spread between long- and short-term maturities across the yield curve, which is largely driven by a decline in short-term bond yields-to-maturity.

Bullet A fixed-income investment strategy that focuses on the intermediate term (or "belly") of the yield curve.

Bullet bond Bond in which the principal repayment is made entirely at maturity.

Bullet portfolio A fixed-income portfolio concentrated in a single maturity.

Bullish flattening Term structure change in which the yield curve flattens in response to a greater decline in long-term rates than short-term rates.

Bullish steepening Term structure change in which short-term rates fall by more than long-term yields, resulting in a steeper term structure.

Butterfly spread A measure of yield curve shape or curvature equal to double the intermediate yield-to-maturity less the sum of short- and long-term yields-to-maturity.

Butterfly strategy A common yield curve shape strategy that combines a long or short bullet position with a barbell portfolio in the opposite direction to capitalize on expected yield curve shape changes.

Call protection The time during which the issuer of the bond is not allowed to exercise the call option.

Callable bond A bond containing an embedded call option that gives the issuer the right to buy the bond back from the investor at specified prices on pre-determined dates.

Capacity The ability of the borrower to make its debt payments on time.

Capital market securities Securities with maturities at issuance longer than one year.

Capital structure The mix of debt and equity that a company uses to finance its business; a company's specific mixture of long-term financing.

Capped floater Floating-rate bond with a cap provision that prevents the coupon rate from increasing above a specified maximum rate. It protects the issuer against rising interest rates.

Carry trade across currencies A strategy seeking to benefit from a positive interest rate differential across currencies by combining a short position (or borrowing) in a low-yielding currency and a long position (or lending) in a high-yielding currency.

Carrying value The net amount shown for an asset or liability on the balance sheet; book value may also refer to the company's excess of total assets over total liabilities. For a bond, the purchase price plus (or minus) the amortized amount of the discount (or premium).

Cash collateral account Form of external credit enhancement whereby the issuer immediately borrows the credit-enhancement amount and then invests that amount, usually in highly rated short-term commercial paper.

Cash flow matching Immunization approach that attempts to ensure that all future liability payouts are matched precisely by cash flows from bonds or fixed-income derivatives.

Cash flow yield The internal rate of return on a series of cash flows.

Cash market securities Money market securities settled on a "same day" or "cash settlement" basis.

Cash settlement A procedure used in certain derivative transactions that specifies that the long and short parties settle the derivative's difference in value between them by making a cash payment.

CDS curve Plot of CDS spreads across maturities for a single reference entity or group of reference entities in an index.

CDS spread A periodic premium paid by the buyer to the seller that serves as a return over a market reference rate required to protect against credit risk.

Cell approach See *stratified sampling*.

Central bank funds market The market in which deposit-taking banks that have an excess reserve with their national central bank can lend money to banks that need funds for maturities ranging from overnight to one year. Called the Federal or Fed funds market in the United States.

Central bank funds rate Interest rate at which central bank funds are bought (borrowed) and sold (lent) for maturities ranging from overnight to one year. Called Federal or Fed funds rate in the United States.

Certificate of deposit An instrument that represents a specified amount of funds on deposit with a bank for a specified maturity and interest rate. CDs are issued in various denominations and can be negotiable or non-negotiable.

Change of control put A covenant giving bondholders the right to require the issuer to buy back their debt, often at par or at some small premium to par value, in the event that the borrower is acquired.

Character The quality of a debt issuer's management.

Cheapest-to-deliver The debt instrument that can be purchased and delivered at the lowest cost yet has the same seniority as the reference obligation.

Collateral Assets or financial guarantees underlying a debt obligation that are above and beyond the issuer's promise to pay.

Collateral manager Buys and sells debt obligations for and from the CDO's portfolio of assets (i.e., the collateral) to generate sufficient cash flows to meet the obligations to the CDO bondholders.

Collateral trust bonds Bonds secured by securities, such as common shares, other bonds, or other financial assets.

Collateralized debt obligation Generic term used to describe a security backed by a diversified pool of one or more debt obligations.

Collateralized mortgage obligations Securities created through the securitization of a pool of mortgage-related products (mortgage pass-through securities or pools of loans).

Collaterals Assets or financial guarantees underlying a debt obligation that are above and beyond the issuer's promise to pay.

Commercial paper A short-term, negotiable, unsecured promissory note that represents a debt obligation of the issuer.

Conditional pass-through covered bonds Covered bonds that convert to pass-through securities after the original maturity date if all bond payments have not yet been made and the sponsor is in default.

Conditional value at risk (CVaR) Also known as expected loss. The average portfolio loss over a specific time period conditional on that loss exceeding the value at risk (VaR) threshold.

Constant-yield price trajectory A graph that illustrates the change in the price of a fixed-income bond over time assuming no change in yield-to-maturity. The trajectory shows the "pull to par" effect on the price of a bond trading at a premium or a discount to par value.

Contingency provision Clause in a legal document that allows for some action if a specific event or circumstance occurs.

Contingent convertible bonds Bonds that automatically convert into equity if a specific event or circumstance occurs, such as the issuer's equity capital falling below the minimum requirement set by the regulators. Also called *CoCos*.

Contingent immunization Hybrid approach that combines immunization with an active management approach when the asset portfolio's value exceeds the present value of the liability portfolio.

Contract rate See *mortgage rate*.

Contraction risk The risk that when interest rates decline, the security will have a shorter maturity than was anticipated at the time of purchase because borrowers refinance at the new, lower interest rates.

Conventional bond See *plain vanilla bond*.

Conversion period For a convertible bond, the period during which bondholders have the right to convert their bonds into shares.

Conversion premium The difference between the convertible bond's price and its conversion value.

Conversion price For a convertible bond, the price per share at which the bond can be converted into shares.

Conversion rate (or ratio) For a convertible bond, the number of shares of common stock that a bondholder receives from converting the bond into shares.

Conversion value For a convertible bond, the current share price multiplied by the conversion ratio.

Convertible bond Bond that gives the bondholder the right to exchange the bond for a specified number of common shares in the issuing company.

Convexity adjustment For a bond, one-half of the annual or approximate convexity statistic multiplied by the change in the yield-to-maturity squared.

Covenants The terms and conditions of lending agreements that the issuer must comply with; they specify the actions that an issuer is obligated to perform (affirmative covenant) or prohibited from performing (negative covenant).

Covered bond Debt obligation secured by a segregated pool of assets called the cover pool. The issuer must maintain the value of the cover pool. In the event of default, bondholders have recourse against both the issuer and the cover pool.

Covered interest rate parity The relationship among the spot exchange rate, the forward exchange rate, and the interest rate in two currencies that ensures that the return on a hedged (i.e., covered) foreign risk-free investment is the same as the return on a domestic risk-free investment. Also called *interest rate parity*.

Cox-Ingersoll-Ross model A general equilibrium term structure model that assumes interest rates are mean reverting and interest rate volatility is directly related to the level of interest rates.

Credit correlation The correlation of credit (or default) risks of the underlying single-name CDS contained in an index CDS.
Credit curve The credit spreads for a range of maturities of a company's debt.
Credit cycle The expansion and contraction of credit over the business cycle, which translates into asset price changes based on default and recovery expectations across maturities and rating categories.
Credit default swap (CDS) basis Yield spread on a bond, as compared to CDS spread of same tenor.
Credit default swap A derivative contract between two parties in which the buyer makes a series of cash payments to the seller and receives a promise of compensation for credit losses resulting from the default.
Credit derivative A derivative instrument in which the underlying is a measure of the credit quality of a borrower.
Credit enhancements Provisions that may be used to reduce the credit risk of a bond issue.
Credit event The event that triggers a payment from the credit protection seller to the credit protection buyer.
Credit loss rate The realized percentage of par value lost to default for a group of bonds equal to the bonds' default rate multiplied by the loss severity.
Credit migration risk The risk that a bond issuer's creditworthiness deteriorates, or migrates lower, leading investors to believe the risk of default is higher. Also called *downgrade risk*.
Credit migration The change in a bond's credit rating over a certain period.
Credit protection buyer One party to a credit default swap; the buyer makes a series of cash payments to the seller and receives a promise of compensation for credit losses resulting from the default.
Credit protection seller One party to a credit default swap; the seller makes a promise to pay compensation for credit losses resulting from the default.
Credit risk The risk that the borrower will not repay principal and interest. Also called *default risk*.
Credit tranching A structure used to redistribute the credit risk associated with the collateral; a set of bond classes created to allow investors a choice in the amount of credit risk that they prefer to bear.
Credit valuation adjustment (CVA) The present value of credit risk for a loan, bond, or derivative obligation.
Credit-linked coupon bond Bond for which the coupon changes when the bond's credit rating changes.
Credit-linked note (CLN) Fixed-income security in which the holder of the security has the right to withhold payment of the full amount due at maturity if a credit event occurs.
Cross-currency basis swap An interest rate swap involving the periodic exchange of floating payments in one currency for another based upon respective market reference rates with an initial and final exchange of notional principal.
Cross-default Covenant or contract clause that specifies a borrower is considered in default if they default on another debt obligation.
Cross-default provisions Provisions whereby events of default, such as non-payment of interest on one bond, trigger default on all outstanding debt; implies the same default probability for all issues.
Currency option bonds Bonds that give bondholders the right to choose the currency in which they want to receive interest payments and principal repayments.
Current yield The sum of the coupon payments received over the year divided by the flat price; also called the *income* or *interest yield* or *running yield*.
Curvature One of the three factors (the other two are level and steepness) that empirically explain most of the changes in the shape of the yield curve. A shock to the curvature factor affects mid-maturity interest rates, resulting in the term structure becoming either more or less hump-shaped.
Curve duration The sensitivity of the bond price (or the market value of a financial asset or liability) with respect to a benchmark yield curve.
Curve trade Buying a CDS of one maturity and selling a CDS on the same reference entity with a different maturity.
Debentures Type of bond that can be secured or unsecured.
Default intensity POD over a specified time period in a reduced form credit model.

Default probability The probability that a borrower defaults or fails to meet its obligation to make full and timely payments of principal and interest, according to the terms of the debt security. Also called *default risk*.

Default risk Likelihood that a borrower will default or fail to meet its obligation to make full and timely payments of principal and interest according to the terms of a debt obligation.

Deferred coupon bond Bond that pays no coupons for its first few years but then pays a higher coupon than it otherwise normally would for the remainder of its life. Also called *split coupon bond*.

Depository Trust and Clearinghouse Corporation A US-headquartered entity providing post-trade clearing, settlement, and information services.

Discount factor The present value or price of a risk-free single-unit payment when discounted using the appropriate spot rate.

Discount function Discount factors for the range of all possible maturities. The spot curve can be derived from the discount function and vice versa.

Discount margin See *required margin*.

Discount rates In general, the interest rates used to calculate present values. In the money market, however, a discount rate is a specific type of quoted rate.

Discount To reduce the value of a future payment in allowance for how far away it is in time; to calculate the present value of some future amount. Also, the amount by which an instrument is priced below its face (par) value.

Dominance An arbitrage opportunity when a financial asset with a risk-free payoff in the future must have a positive price today.

Downgrade risk The risk that a bond issuer's creditworthiness deteriorates, or migrates lower, leading investors to believe the risk of default is higher. Also called *credit migration risk*.

Dual-currency bonds Bonds that make coupon payments in one currency and pay the par value at maturity in another currency.

Duration gap A bond's Macaulay duration minus the investment horizon.

Duration matching Immunization approach based on the duration of assets and liabilities. Ideally, the liabilities being matched (the liability portfolio) and the portfolio of assets (the bond portfolio) should be affected similarly by a change in interest rates.

Duration times spread (DTS) Weighting of spread duration by credit spread in order to incorporate the empirical observation that spread changes for lower-rated bonds tend to be consistent on a percentage, rather than absolute, basis.

Early repayment option See *prepayment option*.

Effective annual rate The amount by which a unit of currency will grow in a year with interest on interest included.

Effective convexity A *curve convexity* statistic that measures the secondary effect of a change in a benchmark yield curve on a bond's price.

Effective duration Sensitivity of the bond's price to a 100 bps parallel shift of the benchmark yield curve, assuming no change in the bond's credit spread.

Embedded option Contingency provisions that provide the issuer or the bondholders the right, but not the obligation, to take action. These options are not part of the security and cannot be traded separately.

Empirical duration Estimation of the price-yield relationship using historical bond market data in statistical models.

Enhanced indexing approach Maintains a close link to the benchmark but attempts to generate a modest amount of outperformance relative to the benchmark.

Enhanced indexing strategy Method investors use to match an underlying market index in which the investor purchases fewer securities than the full set of index constituents but matches primary risk factors reflected in the index.

Equipment trust certificates Bonds secured by specific types of equipment or physical assets.

Eurobonds Type of bond issued internationally, outside the jurisdiction of the country in whose currency the bond is denominated.

Evaluated pricing See *matrix pricing*.
Excess spread Credit spread return measure that incorporates both changes in spread and expected credit losses for a given period.
Expected exposure The projected amount of money an investor could lose if an event of default occurs, before factoring in possible recovery.
Expected loss Default probability times loss severity given default.
Extendible bond Bond with an embedded option that gives the bondholder the right to keep the bond for a number of years after maturity, possibly with a different coupon.
Extension risk The risk that when interest rates rise, fewer prepayments will occur because homeowners are reluctant to give up the benefits of a contractual interest rate that now looks low. As a result, the security becomes longer in maturity than anticipated at the time of purchase.
Failure to pay When a borrower does not make a scheduled payment of principal or interest on any outstanding obligations after a grace period.
Fiduciary An entity designated to represent the rights and responsibilities of a beneficiary whose assets they are managing, such as a bond trustee acting on behalf of fixed-income investors.
Firm commitment offering See *underwritten offering*.
First lien debt Debt secured by a pledge of certain assets that could include buildings, but it may also include property and equipment, licenses, patents, brands, etc.
First mortgage debt Debt secured by a pledge of a specific property.
Flat price The full price of a bond minus the accrued interest; also called the *quoted* or *clean* price.
Flight to quality During times of market stress, investors sell higher-risk asset classes such as stocks and commodities in favor of default-risk-free government bonds.
Floaters See *floating-rate notes*.
Floating-rate notes Notes on which interest payments are not fixed but instead vary from period to period depending on the current level of a reference interest rate.
Floored floater Floating-rate bond with a floor provision that prevents the coupon rate from decreasing below a specified minimum rate. It protects the investor against declining interest rates.
Forced conversion For a convertible bond, when the issuer calls the bond and forces bondholders to convert their bonds into shares, which typically happens when the underlying share price increases above the conversion price.
Foreclosure Allows the lender to take possession of a mortgaged property if the borrower defaults and then sell it to recover funds.
Forward curve A series of forward rates, each having the same time frame.
Forward market For future delivery, beyond the usual settlement time period in the cash market.
Forward pricing model The model that describes the valuation of forward contracts.
Forward rate An interest rate determined today for a loan that will be initiated in a future period.
Forward rate bias An empirically observed divergence from interest rate parity conditions that active investors seek to benefit from by borrowing in a lower-yield currency and investing in a higher-yield currency.
Forward rate model The forward pricing model expressed in terms of spot and forward interest rates.
Full price The price of a security with accrued interest; also called the *invoice* or *dirty* price.
Full replication approach When every issue in an index is represented in the portfolio, and each portfolio position has approximately the same weight in the fund as in the index.
G-spread The yield spread in basis points over an actual or interpolated government bond.
Government equivalent yield A yield that restates a yield-to-maturity based on a 30/360 day count to one based on actual/actual.
Green bonds Fixed-income instruments issued by private or public sector borrowers that directly fund ESG initiatives.
Grey market The forward market for bonds about to be issued. Also called "when issued" market.
Guarantee certificate A type of structured financial instrument that provides investors with capital protection. It combines a zero-coupon bond and a call option on some underlying asset.
Haircut See *repo margin*.

Hard-bullet covered bonds Covered bonds for which a bond default is triggered and bond payments are accelerated in the event of sponsor default if payments do not occur according to the original maturity schedule.

Hazard rate The conditional POD, or the likelihood that default will occur given that it has not already occurred in a prior period.

Ho–Lee model The first arbitrage-free term structure model. The model is calibrated to market data and uses a binomial lattice approach to generate a distribution of possible future interest rates.

Horizon yield The internal rate of return between the total return (the sum of reinvested coupon payments and the sale price or redemption amount) and the purchase price of the bond.

I-spread (interpolated spread) Yield spread measure using swaps or constant maturity Treasury YTMs as a benchmark.

I-spreads Shortened form of "interpolated spreads" and a reference to a linearly interpolated yield.

Immunization An asset/liability management approach that structures investments in bonds to match (offset) liabilities' weighted-average duration; a type of dedication strategy.

Implied forward rates Calculated from spot rates, an implied forward rate is a breakeven reinvestment rate that links the return on an investment in a shorter-term zero-coupon bond to the return on an investment in a longer-term zero-coupon bond.

Incremental VaR (or partial VaR) The change in the minimum portfolio loss expected to occur over a given time period at a specific confidence level resulting from increasing or decreasing a portfolio position.

Indenture Legal contract that describes the form of a bond, the obligations of the issuer, and the rights of the bondholders. Also called the *trust deed*.

Index CDS A type of credit default swap that involves a combination of borrowers.

Index-linked bond Bond for which coupon payments and/or principal repayment are linked to a specified index.

Inflation-linked bond Type of index-linked bond that offers investors protection against inflation by linking the bond's coupon payments and/or the principal repayment to an index of consumer prices. Also called *linkers*.

Interbank market The market of loans and deposits between banks for maturities ranging from overnight to one year.

Interest rate risk The risk that interest rates will rise and therefore the market value of current portfolio holdings will fall so that their current yields to maturity then match comparable instruments in the marketplace.

Interest-only mortgage A loan in which no scheduled principal repayment is specified for a certain number of years.

Interpolated spread The yield spread of a specific bond over the standard swap rate in that currency of the same tenor.

Inverse floater A type of leveraged structured financial instrument. The cash flows are adjusted periodically and move in the opposite direction of changes in the reference rate.

ISDA Master Agreement A standard or "master" agreement published by the International Swaps and Derivatives Association. The master agreement establishes the terms for each party involved in the transaction.

Kalotay–Williams–Fabozzi (KWF) model An arbitrage-free term structure model that describes the dynamics of the log of the short rate and assumes constant drift, no mean reversion, and constant volatility.

Key rate durations Sensitivity of a bond's price to changes in specific maturities on the benchmark yield curve. Also called *partial durations*.

Law of one price A principle that states that if two investments have the same or equivalent future cash flows regardless of what will happen in the future, then these two investments should have the same current price.

Letter of credit Form of external credit enhancement whereby a financial institution provides the issuer with a credit line to reimburse any cash flow shortfalls from the assets backing the issue.

Level One of the three factors (the other two are steepness and curvature) that empirically explains most yield curve shape changes. A shock to the level factor changes the yield for all maturities by an almost identical amount.

Liability-based mandates Mandates managed to match or cover expected liability payments (future cash outflows) with future projected cash inflows.

Libor–OIS spread The difference between Libor and the overnight indexed swap rate.

Limitations on liens Meant to put limits on how much secured debt an issuer can have.

Linker See *inflation-linked bond*.

Liquidity preference theory A term structure theory that asserts liquidity premiums exist to compensate investors for the added interest rate risk they face when lending long term.

Liquidity premium The premium or incrementally higher yield that investors demand for lending long term.

Loan-to-value ratio The amount of the property's mortgage divided by the purchase price

Local expectations theory A term structure theory that contends the return for all bonds over short periods is the risk-free rate.

London interbank offered rate (Libor) Collective name for multiple rates at which a select set of banks believes they could borrow unsecured funds from other banks in the London interbank market for different currencies and different borrowing periods ranging from overnight to one year.

Long/short credit trade A credit protection seller with respect to one entity combined with a credit protection buyer with respect to another entity.

Loss given default The amount that will be lost if a default occurs.

Loss severity Also known as loss given default (LGD). The amount of loss if a default occurs, usually expressed as a percentage in annual terms.

Macaulay duration The approximate amount of time a bond would have to be held for the market discount rate at purchase to be realized if there is a single change in interest rate. It indicates the point in time when the coupon reinvestment and price effects of a change in yield-to-maturity offset each other.

Maintenance covenants Covenants in bank loan agreements that require the borrower to satisfy certain financial ratio tests while the loan is outstanding.

Market conversion premium per share For a convertible bond, the difference between the market conversion price and the underlying share price, which allows investors to identify the premium or discount payable when buying a convertible bond rather than the underlying common stock.

Market conversion premium ratio For a convertible bond, the market conversion premium per share expressed as a percentage of the current market price of the shares.

Market discount rate The rate of return required by investors given the risk of the investment in a bond; also called the *required yield* or the *required rate of return*.

Market liquidity risk The risk that the price at which investors can actually transact—buying or selling—may differ from the price indicated in the market.

Market reference rate (MRR) Collective name for a set of rates covering different currencies for different maturities, ranging from overnight to one year.

Matrix pricing An approach for estimating the prices of thinly traded securities based on the prices of securities with similar attributions, such as similar credit rating, maturity, or economic sector. Also called *evaluated pricing*.

Maturity structure A factor explaining the differences in yields on similar bonds; also called *term structure*.

Medium-term note A corporate bond offered continuously to investors by an agent of the issuer, designed to fill the funding gap between commercial paper and long-term bonds.

Modified duration A measure of the percentage price change of a bond given a change in its yield-to-maturity.

Monetizing Unwinding a position to either capture a gain or realize a loss.

Money convexity For a bond, the annual or approximate convexity multiplied by the full price.

Money duration A measure of the price change in units of the currency in which the bond is denominated given a change in its yield-to-maturity.

Money market securities Fixed-income securities with maturities at issuance of one year or less.

Mortgage loan A loan secured by the collateral of some specified real estate property that obliges the borrower to make a predetermined series of payments to the lender.

Mortgage pass-through security A security created when one or more holders of mortgages form a pool of mortgages and sell shares or participation certificates in the pool.

Mortgage rate The interest rate on a mortgage loan; also called *contract rate* or *note rate*.

Mortgage-backed securities Debt obligations that represent claims to the cash flows from pools of mortgage loans, most commonly on residential property.

Muni See *municipal bond*.

Municipal bond A type of non-sovereign bond issued by a state or local government in the United States. It very often (but not always) offers income tax exemptions.

Naked credit default swap A position where the owner of the CDS does not have a position in the underlying credit.

Negative butterfly An increase in the butterfly spread due to lower short- and long-term yields-to-maturity and a higher intermediate yield-to-maturity.

Non-agency RMBS In the United States, securities issued by private entities that are not guaranteed by a federal agency or a GSE.

Non-recourse loan A loan in which the lender does not have a shortfall claim against the borrower, so the lender can look only to the property to recover the outstanding mortgage balance.

Non-sovereign bond A bond issued by a government below the national level, such as a province, region, state, or city.

Notching Ratings adjustment methodology where specific issues from the same borrower may be assigned different credit ratings.

Note rate See *mortgage rate*.

Notional amount The amount of protection being purchased in a CDS.

OAS duration The change in bond price for a given change in OAS.

Off-the-run A series of securities or indexes that were issued/created prior to the most recently issued/created series.

On-the-run The most recently issued and most actively traded sovereign securities.

One-sided durations Effective durations when interest rates go up or down, which are better at capturing the interest rate sensitivity of bonds with embedded options that do not react symmetrically to positive and negative changes in interest rates of the same magnitude.

Open market operations The purchase or sale of bonds by the national central bank to implement monetary policy. The bonds traded are usually sovereign bonds issued by the national government.

Option-adjusted price The value of the embedded option plus the flat price of the bond.

Option-adjusted spread (OAS) A generalization of the Z-spread yield spread calculation that incorporates bond option pricing based on assumed interest rate volatility.

Option-adjusted yield The required market discount rate whereby the price is adjusted for the value of the embedded option.

Options on bond futures contracts Instruments that involve the right, but not the obligation, to enter into a bond futures contract at a pre-determined strike (bond price) on a future date in exchange for an up-front premium.

Organized exchange A securities marketplace where buyers and seller can meet to arrange their trades.

Over-the-counter (OTC) market A decentralized market where buy and sell orders initiated from various locations are matched through a communications network.

Overcollateralization Form of internal credit enhancement that refers to the process of posting more collateral than needed to obtain or secure financing.

Overnight indexed swap (OIS) rate An interest rate swap in which the periodic floating rate of the swap equals the geometric average of a daily unsecured overnight rate (or overnight index rate).

Par curve A hypothetical yield curve for coupon-paying Treasury securities that assumes all securities are priced at par.

Par swap A swap in which the fixed rate is set so that no money is exchanged at contract initiation.

Par value The amount of principal on a bond.

Parallel shift A parallel yield curve shift implies that all rates change by the same amount in the same direction.

Pari passu Covenant or contract clause that ensures a debt obligation is treated the same as the borrower's other senior debt instruments and is not subordinated to similar obligations.

Partial duration See *key rate duration*.

Pass-through rate The coupon rate of a mortgage pass-through security.

Passive investment In the fixed-income context, it is investment that seeks to mimic the prevailing characteristics of the overall investments available in terms of credit quality, type of borrower, maturity, and duration rather than express a specific market view.

Payout amount The loss given default times the notional.

Periodicity The assumed number of periods in the year; typically matches the frequency of coupon payments.

Perpetual bonds Bonds with no stated maturity date.

Perpetuity A perpetual annuity, or a set of never-ending level sequential cash flows, with the first cash flow occurring one period from now. A bond that does not mature.

Physical settlement Involves actual delivery of the debt instrument in exchange for a payment by the credit protection seller of the notional amount of the contract.

Plain vanilla bond Bond that makes periodic, fixed coupon payments during the bond's life and a lump-sum payment of principal at maturity. Also called *conventional bond*.

Positive butterfly A decrease in the butterfly spread due to higher short- and long-term yields-to-maturity and a lower intermediate yield-to-maturity.

Preferred habitat theory A term structure theory that contends that investors have maturity preferences and require yield incentives before they will buy bonds outside of their preferred maturities.

Premium In the case of bonds, premium refers to the amount by which a bond is priced above its face (par) value. In the case of an option, the amount paid for the option contract.

Premium leg The series of payments the credit protection buyer promises to make to the credit protection seller.

Prepayment option Contractual provision that entitles the borrower to prepay all or part of the outstanding mortgage principal prior to the scheduled due date when the principal must be repaid. Also called *early repayment option*.

Prepayment penalty mortgages Mortgages that stipulate a monetary penalty if a borrower prepays within a certain time period after the mortgage is originated.

Prepayment risk The uncertainty that the timing of the actual cash flows will be different from the scheduled cash flows as set forth in the loan agreement due to the borrowers' ability to alter payments, usually to take advantage of interest rate movements.

Present value of distribution of cash flows methodology Method used to address a portfolio's sensitivity to rate changes along the yield curve. This approach seeks to approximate and match the yield curve risk of an index over discrete time periods.

Price value of a basis point A version of money duration, it is an estimate of the change in the full price of a bond given a 1 basis point change in the yield-to-maturity.

Primary bond market A market in which issuers first sell bonds to investors to raise capital.

Primary dealer Financial institution that is authorized to deal in new issues of sovereign bonds and that serves primarily as a trading counterparty of the office responsible for issuing sovereign bonds.

Principal amount Amount that an issuer agrees to repay the debtholders on the maturity date.

Principal The amount of funds originally invested in a project or instrument; the face value to be paid at maturity.

Principal value Amount that an issuer agrees to repay the debtholders on the maturity date.

Principle of no arbitrage In well-functioning markets, prices will adjust until there are no arbitrage opportunities.

Priority of claims Priority of payment, with the most senior or highest ranking debt having the first claim on the cash flows and assets of the issuer.

Private placement Typically, a non-underwritten, unregistered offering of securities that are sold only to an investor or a small group of investors. It can be accomplished directly between the issuer and the investor(s) or through an investment bank.

Probability of default The likelihood that a borrower defaults or fails to meet its obligation to make full and timely payments of principal and interest.

Probability of survival The probability that a bond issuer will meet its contractual obligations on schedule.

Prospectus The document that describes the terms of a new bond issue and helps investors perform their analysis on the issue.

Protection leg The contingent payment that the credit protection seller may have to make to the credit protection buyer.

Protection period Period during which a bond's issuer cannot call the bond.

Public offer See *public offering*.

Public offering An offering of securities in which any member of the public may buy the securities. Also called *public offer*.

Pure discount bonds See *zero-coupon bond*.

Pure expectations theory A term structure theory that contends the forward rate is an unbiased predictor of the future spot rate. Also called the *unbiased expectations theory*.

Pure indexing Attempts to replicate a bond index as closely as possible, targeting zero active return and zero active risk.

Putable bond Bond that includes an embedded put option, which gives the bondholder the right to put back the bonds to the issuer prior to maturity, typically when interest rates have risen and higher-yielding bonds are available.

Quasi-government bond A bond issued by an entity that is either owned or sponsored by a national government. Also called *agency bond*.

Quoted margin The specified yield spread over the reference rate, used to compensate an investor for the difference in the credit risk of the issuer and that implied by the reference rate.

Rebate rate The portion of the collateral earnings rate that is repaid to the security borrower by the security lender.

Reconstitution When dealers recombine appropriate individual zero-coupon securities and reproduce an underlying coupon Treasury.

Recourse loan A loan in which the lender has a claim against the borrower for any shortfall between the outstanding mortgage balance and the proceeds received from the sale of the property.

Recovery rate The percentage of the loss recovered.

Redemption yield See *yield-to-maturity*.

Reduced form credit models Credit models that solve for default probability over a specific time period using observable company-specific variables such as financial ratios and macroeconomic variables.

Reference entity The borrower (debt issuer) covered by a single-name CDS.

Reference obligation A particular debt instrument issued by the borrower that is the designated instrument being covered.

Registered bonds Bonds for which ownership is recorded by either name or serial number.

Relative value A concept that describes the selection of the most attractive individual securities to populate the portfolio with, using ranking and comparing.

Relative VaR The minimum portfolio loss expected to occur over a given time period at a specific confidence level based on a portfolio containing active positions minus benchmark holdings.

Repo A form of collateralized loan involving the sale of a security with a simultaneous agreement by the seller to buy back the same security from the purchaser at an agreed-on price and future date. The party who sells the security at the inception of the repurchase agreement and buys it back at maturity is borrowing money from the other party, and the security sold and subsequently repurchased represents the collateral.

Repo margin The difference between the market value of the security used as collateral and the value of the loan. Also called *haircut*.

Repo rate The interest rate on a repurchase agreement.

Repurchase agreement A form of collateralized loan involving the sale of a security with a simultaneous agreement by the seller to buy the same security back from the purchaser at an agreed-on price and future date. The party who sells the security at the inception of the repurchase agreement and buys it back at maturity is borrowing money from the other party, and the security sold and subsequently repurchased represents the collateral.

Repurchase date The date when the party who sold the security at the inception of a repurchase agreement buys back the security from the cash lending counterparty.

Repurchase price The price at which the party who sold the security at the inception of the repurchase agreement buys back the security from the cash lending counterparty.

Required margin The yield spread over or under the reference rate such that an FRN is priced at par value on a rate reset date.

Required rate of return See *market discount rate*.

Required yield See *market discount rate*.

Required yield spread The difference between the yield-to-maturity on a new bond and the benchmark rate; additional compensation required by investors for the difference in risk and tax status of a bond relative to a government bond. Sometimes called the *spread over the benchmark*.

Reserve accounts Form of internal credit enhancement that relies on creating accounts and depositing in these accounts cash that can be used to absorb losses. Also called *reserve funds*.

Reserve funds See *reserve accounts*.

Restricted payments A bond covenant meant to protect creditors by limiting how much cash can be paid out to shareholders over time.

Restructuring Reorganizing the capital structure of a firm.

Reverse repo A repurchase agreement viewed from the perspective of the cash lending counterparty.

Reverse repurchase agreement A repurchase agreement viewed from the perspective of the cash lending counterparty.

Riding the yield curve See *rolling down the yield curve*.

Roll When an investor moves its investment position from an older series to the most current series.

Rolling down the yield curve A maturity trading strategy that involves buying bonds with a maturity longer than the intended investment horizon. Also called *riding the yield curve*.

Running yield See *current yield*.

Scenario analysis What-if analysis that involves changing multiple assumptions at the same time in order to evaluate the change in an investment's value.

Second lien A secured interest in the pledged assets that ranks below first lien debt in both collateral protection and priority of payment.

Secondary bond markets Markets in which existing bonds are traded among investors.

Secured bonds Bonds secured by assets or financial guarantees pledged to ensure debt repayment in case of default.

Secured debt Debt in which the debtholder has a direct claim—a pledge from the issuer—on certain assets and their associated cash flows.

Secured overnight financing rate (SOFR) A daily volume-weighted index of rates on qualified cash borrowings collateralized by US Treasuries that is expected to replace Libor as a floating reference rate for swaps.

Securitization A process that involves moving assets into a special legal entity, which then uses the assets as guarantees to secure a bond issue.

Securitized assets Assets that are typically used to create asset-backed bonds; for example, when a bank securitizes a pool of loans, the loans are said to be securitized.

Segmented markets theory A term structure theory that contends yields are solely a function of the supply and demand for funds of a particular maturity.

Semiannual bond basis yield An annual rate having a periodicity of two; also known as a *semiannual bond equivalent yield*.
Semiannual bond equivalent yield See *semiannual bond basis yield*.
Seniority ranking Priority of payment of various debt obligations.
Serial maturity structure Structure for a bond issue in which the maturity dates are spread out during the bond's life; a stated number of bonds mature and are paid off each year before final maturity.
Settlement date Date when the buyer makes cash payment and the seller delivers the security.
Settlement In the case of a credit event, the process by which the two parties to a CDS contract satisfy their respective obligations.
Shaping risk The sensitivity of a bond's price to the changing shape of the yield curve.
Shelf registration A type of public offering that allows the issuer to file a single, all-encompassing offering circular that covers a series of bond issues.
Simple yield The sum of the coupon payments plus the straight-line amortized share of the gain or loss, divided by the flat price.
Single-name CDS Credit default swap on one specific borrower.
Sinking fund arrangement Provision that reduces the credit risk of a bond issue by requiring the issuer to retire a portion of the bond's principal outstanding each year.
Sinking fund bond A bond that requires the issuer to set aside funds over time to retire the bond issue, thus reducing credit risk.
Smart beta Involves the use of transparent, rules-based strategies as a basis for investment decisions.
Soft-bullet covered bonds Covered bonds for which bond default and payment acceleration of bond cash flows may be delayed upon sponsor default until a new final maturity date is reached.
Sovereign bond A bond issued by a national government. Also called "Sovereign."
Special purpose entity A non-operating entity created to carry out a specified purpose, such as leasing assets or securitizing receivables; can be a corporation, partnership, trust, or limited liability partnership formed to facilitate a specific type of business activity. Also called *special purpose vehicle*, *special purpose company*, or *variable interest entity*.
Split coupon bond See *deferred coupon bond*.
Spot curve A sequence of yields-to-maturity on zero-coupon bonds. Sometimes called *zero* or *strip curve* (because coupon payments are "stripped" off the bonds).
Spot rates A sequence of market discount rates that correspond to the cash flow dates; yields-to-maturity on zero-coupon bonds maturing at the date of each cash flow.
Spot yield curve The term structure of spot rates for loans made today.
Spread duration The change in bond price for a given change in yield spread. Also referred to as *OAS duration* when the option-adjusted spread (OAS) is the yield measure used.
Spread In general, the difference in yield between different fixed-income securities. Often used to refer to the difference between the yield-to-maturity and the benchmark.
Spread over the benchmark See *required yield spread*.
Spread risk Bond price risk arising from changes in the yield spread on credit-risky bonds; reflects changes in the market's assessment and/or pricing of credit migration (or downgrade) risk and market liquidity risk.
Steepness The difference between long-term and short-term yields that constitutes one of the three factors (the other two are level and curvature) that empirically explain most of the changes in the shape of the yield curve.
Step-up coupon bond Bond for which the coupon, fixed or floating, increases by specified margins at specified dates.
Straight bond An underlying option-free bond with a specified issuer, issue date, maturity date, principal amount and repayment structure, coupon rate and payment structure, and currency denomination.
Stratified sampling A sampling method that guarantees that subpopulations of interest are represented in the sample. Also called *representative sampling* or *cell approach*.

Street convention A yield measure that neglects weekends and holidays; the internal rate of return on cash flows assuming payments are made on the scheduled dates, even when the scheduled date falls on a weekend or holiday.

Stripping A dealer's ability to separate a bond's individual cash flows and trade them as zero-coupon securities.

Structural credit models Credit models that apply market-based variables to estimate the value of an issuer's assets and the volatility of asset value.

Structural risk Risk that arises from portfolio design, particularly the choice of the portfolio allocations.

Structural subordination Arises in a holding company structure when the debt of operating subsidiaries is serviced by the cash flow and assets of the subsidiaries before funds can be passed to the holding company to service debt at the parent level.

Structured financial instrument A financial instrument that shares the common attribute of repackaging risks. Structured financial instruments include asset-backed securities, collateralized debt obligations, and other structured financial instruments such as capital protected, yield enhancement, participation, and leveraged instruments.

Subordinated debt A class of unsecured debt that ranks below a firm's senior unsecured obligations.

Subordination Form of internal credit enhancement that relies on creating more than one bond tranche and ordering the claim priorities for ownership or interest in an asset between the tranches. The ordering of the claim priorities is called a senior/subordinated structure, where the tranches of highest seniority are called senior followed by subordinated or junior tranches. Also called *credit tranching*.

Succession event A change of corporate structure of the reference entity, such as through a merger, a divestiture, a spinoff, or any similar action, in which ultimate responsibility for the debt in question is unclear.

Support tranches Classes or tranches in CMOs that protect PAC tranches from prepayment risk.

Supranational bond A bond issued by a supranational agency such as the World Bank.

Surety bond Form of external credit enhancement whereby a rated and regulated insurance company guarantees to reimburse bondholders for any losses incurred up to a maximum amount if the issuer defaults.

Surplus The difference between the value of assets and the present value of liabilities. With respect to an insurance company, the net difference between the total assets and total liabilities (equivalent to policyholders' surplus for a mutual insurance company and stockholders' equity for a stock company).

Swap curve The term structure of swap rates.

Swap rate curve The term structure of swap rates.

Swap rate The "price" that swap traders quote among one another. It is the rate at which the present value of all the expected floating-rate payments received over the life of the floating-rate bond equals the present value of all the expected fixed-rate payments made over the life of the fixed-rate bond.

Swap spread The difference between the fixed rate on an interest rate swap and the rate on a Treasury note with equivalent maturity; it reflects the general level of credit risk in the market.

Swaption This instrument grants a party the right, but not the obligation, to enter into an interest rate swap at a pre-determined strike (fixed swap rate) on a future date in exchange for an up-front premium.

Syndicated loan A loan from a group of lenders to a single borrower.

Syndicated offering A bond issue underwritten by a group of investment banks.

TED spread A measure of perceived credit risk determined as the difference between Libor and the T-bill yield of matching maturity.

Tenor The time-to-maturity for a bond or derivative contract. Also called *term to maturity*.

Term maturity structure Structure for a bond issue in which the bond's notional principal is paid off in a lump sum at maturity.

Term premium The additional return required by lenders to invest in a bond to maturity net of the expected return from continually reinvesting at the short-term rate over that same time horizon.

Term structure of credit spreads The relationship between the spreads over the "risk-free" (or benchmark) rates and times-to-maturity.

Term structure of yield volatility The relationship between the volatility of bond yields-to-maturity and times-to-maturity.

Term structure See *maturity structure*.

Term to maturity See *tenor*.

Time tranching The creation of classes or tranches in an ABS/MBS that possess different (expected) maturities.

Total return payer Party responsible for paying the reference obligation cash flows and return to the receiver but that is also compensated by the receiver for any depreciation in the index or default losses incurred by the portfolio.

Total return receiver Receives both the cash flows from the underlying index and any appreciation in the index over the period in exchange for paying the MRR plus a predetermined spread.

Total return swap A swap in which one party agrees to pay the total return on a security. Often used as a credit derivative, in which the underlying is a bond.

Tracking error The standard deviation of the differences between a portfolio's returns and its benchmark's returns; a synonym of *active risk*. Also called *tracking risk*.

Tracking risk The standard deviation of the differences between a portfolio's returns and its benchmark's returns; a synonym of *active risk*. Also called *tracking error*.

Tranche CDS A type of credit default swap that covers a combination of borrowers but only up to pre-specified levels of losses.

True yield The internal rate of return on cash flows using the actual calendar, including weekends and bank holidays.

Trust deed The governing legal credit agreement, typically incorporated by reference in the prospectus. Also called *bond indenture*.

Unbiased expectations theory A term structure theory that contends the forward rate is an unbiased predictor of the future spot rate. Also called the *pure expectations theory*.

Uncovered interest rate parity The proposition that the expected return on an uncovered (i.e., unhedged) foreign currency (risk-free) investment should equal the return on a comparable domestic currency investment.

Underwriter A firm, usually an investment bank, that takes the risk of buying the newly issued securities from the issuer and then reselling them to investors or to dealers, thus guaranteeing the sale of the securities at the offering price negotiated with the issuer.

Underwritten offering A type of securities issue mechanism in which the investment bank guarantees the sale of the securities at an offering price that is negotiated with the issuer. Also known as *firm commitment offering*.

Unsecured debt Debt that gives the debtholder only a general claim on an issuer's assets and cash flow.

Upfront payment The difference between the credit spread and the standard rate paid by the protection buyer if the standard rate is insufficient to compensate the protection seller. Also called *upfront premium*.

Upfront premium See *upfront payment*.

Value additivity An arbitrage opportunity when the value of the whole equals the sum of the values of the parts.

Value at risk (VaR) A measure of the minimum portfolio loss expected to occur over a given time period at a specific confidence level.

Vasicek model A partial equilibrium term structure model that assumes interest rates are mean reverting and interest rate volatility is constant.

Warrant Attached option that gives its holder the right to buy the underlying stock of the issuing company at a fixed exercise price until the expiration date.

Weighted average coupon rate Weighting the mortgage rate of each mortgage loan in the pool by the percentage of the mortgage outstanding relative to the outstanding amount of all the mortgages in the pool.

Weighted average life A measure that gives investors an indication of how long they can expect to hold the MBS before it is paid off; the convention-based average time to receipt of all principal repayments. Also called *average life*.

Weighted average maturity Weighting the remaining number of months to maturity for each mortgage loan in the pool by the amount of the outstanding mortgage balance.

Yield curve factor model A model or a description of yield curve movements that can be considered realistic when compared with historical data.

Yield duration The sensitivity of the bond price with respect to the bond's own yield-to-maturity.

Yield spread The simple difference between a bond's YTM and the YTM of an on-the-run government bond of similar maturity.

Yield-to-maturity Annual return that an investor earns on a bond if the investor purchases the bond today and holds it until maturity. It is the discount rate that equates the present value of the bond's expected cash flows until maturity with the bond's price. Also called *yield-to-redemption* or *redemption yield*.

Yield-to-redemption See *yield-to-maturity*.

Yield-to-worst The lowest of the sequence of yields-to-call and the yield-to-maturity.

Z-score Credit risk model that uses financial ratios and market-based information weighted by coefficients to create a composite score used to classify firms based on the likelihood of financial distress.

Zero A bond that does not pay a coupon but is priced at a discount and pays its full face value at maturity.

Zero-coupon bond A bond that does not pay a coupon but is priced at a discount and pays its full face value at maturity.

Zero-discount margin (Z-DM) A yield spread calculation for FRNs that incorporates forward MRR.

Zero-volatility spread (Z-spread) Calculates a constant yield spread over a government (or interest rate swap) spot curve.

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