

Springer Aerospace Technology

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Designing Aircraft Simulators

 Springer

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Introduction

This book is written on the basis of many years of personal experience of the authors in the development of complex space-controlled dynamic systems (extra-atmospheric complexes for astrophysical research, aviation and space simulators). The book formulates methodological principles of their design, based on system methodology. The monograph presents a hierarchy of criteria for the effectiveness of the system. At its upper (first) level are the usefulness of the system, the quality of functioning, the organization of the system, evolutionary efficiency, economic efficiency, etc. Among the criteria of the second level, the initial characteristics of the system, importance, relevance, prospects, scope, noise immunity, accuracy, reliability, sensitivity, quality of management, perfection of structure, complexity, feasibility, resources, possibilities of modifications, etc. are considered.

For each criterion, quantitative indicators, characterizing them and methods of measuring them (calculated, experimental or expert assessments) are indicated.

Empirical dependencies between criteria are determined (by methods of factor analysis and mathematical statistics, using procedures for evaluating hypotheses and weighing factors, etc.).

Based on the introduced hierarchy of criteria and selected rational complexes of solved problems, the monograph presents a hierarchy of systems with estimates of its elements, which serves as the basis for long-term planning of the entire complex of developments and design of individual systems.

In the synthesis of these systems, associated with the choice of constructive schemes and control laws, an iterative method is used to optimize the parameters of the system from the minimum condition of the objective function.

The objective function is formed, taking into account both the reactions of the system to the test effects and synchronous measurements of phase coordinates and control actions during normal operation.

Taking into account the high order of system models and the extreme difficulty of determining the influence of technical parameters on the parameters of control actions, especially with operator control, the spectral composition of control actions is determined during normal maintenance.

The characteristic frequencies in the selected spectral range of interest and their relationship with constructive or other parameters (for example, the operator) are established.

Further, the structural and mathematical models of the system are refined, followed by the identification of parameters from the conditions for obtaining minimum of quality functionals.

Gradient methods are used to optimize the parameters, including methods of experiment planning (both machine and real).

The correctness and prospects of the proposed methodological principles of designing complex dynamic systems, controlled in space, have been confirmed by their repeated use in the design of these objects for various purposes.

The implementation of the proposed principles is carried out on the example of the development of aviation simulators. The solution of a number of important tasks at various stages of aviation simulator construction is given.

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Abbreviations

AC	Aircraft
ACS	Automatic control systems
AE	Aviation equipment
ANCS	Aircraft navigation computer system
API	Application programming interface
APP	Auxiliary power plant
AS	Aviation simulators
ASC	The altitude-speed characteristics
ASE	Automatic start of the engine
ASI	The air situation indicator
ATM	Air traffic management
ATS	Air traffic support
CAS	Complex aviation simulators
CMS	Content management system
CS	Control system
CS-APP	Control system of the auxiliary power plant
CSTC	Computer system for traction control
DDT	Device for data transmission
DOS VS	Data output system of visualization system
EDS	Electronic display system
EER	The electronic part of the system for engine regulation
EFS	Engine fire sign
FAR	Federal Aviation Regulations
FCCS	Flight control computer system
FDS	Flight dynamics simulator
FMI	Functional module interfaces
FPM	Flight performance modeling
GMT	Greenwich Mean Time
ICAO	International Civil Aviation Organization
IES	The information exchange system
IPF	Indication of the presence of fuel at the engine input

IS	Indication of surging
IVAO	International Virtual Aviation Organisation
IW	Instructor's workplace
LS APP	Launch sign of APP
MNRS	The meteorological navigation radar station
NSS	Navigation situation simulator
PCM	The pilot's control movements
PCS	A potential conflict situation
PPS	Power plant simulator
RAM	Operating memory
SCFW	Sign of control by front wheel
SCS	Sign of compression of the chassis
SD	Switching device
SF	A sign of a fire
SF APP	A sign of a fire of auxiliary power plant
SOES	Sign of the operation of the electric starter
SOHS	Sign of operation for hydraulic system
SPB	Sign of parking brake
SR	Sign of reverse
SVT	A sign of the presence of voltage on the tires
TC	True course
TTS	Text-to-speech
TTT	Technical training tools
VATSIM Network	Virtual Air Traffic Simulation Network
VEE	A virtual educational environment
VS	Visualization system
USA	United States of America
UTC	Universal Coordinated Time

Chapter 1

The State and Prospects of Development of Aviation Simulator Construction



The basis of flight safety is the high professional qualification of flight personnel. At the same time, as the complexity of both the aircraft itself and its maintenance conditions increases, training and retraining flight personnel become an increasingly difficult task. If we also take into account the need for personnel training at minimal cost, it will become obvious that the required high quality of training can be achieved only with the intensification of training on high-performance aviation simulators (AS) of various types [1–9]. So, *the relevance* of creating AS and their application is obvious.

The indisputable proof of the high efficiency of AS is the increasing demand for them. Numerous orders for the production of AS are systematically received from airlines around the world [1, 2]. The large-scale introduction of AS in the US Air Force allowed, according to American experts, by the mid-90 s to reduce the cost of one hour of pilot training up to 30 times. At the end of the 80 s, the total cost of training a highly qualified fighter pilot of the US Air Force was 800 thousand dollars.

Training pilots on an airplane imposes certain restrictions on the amount of training. When the training course is conducted on an airplane, it is limited to those exercises, the performance of which does not pose a danger to the crew, and yet with such training there are numerous accidents. Training on the simulator not only eliminates the risk but also allows you to significantly expand the range of training exercises.

When training on AS, there is no need to use airspace and large areas at landfills, a negative impact on the environment is excluded; there is no threat of accidents and catastrophes, as when training in the air.

A modern aviation simulator is an effective and safe means not only for initial training but also for improving flight skills, retraining and maintaining qualifications. The instructor has the ability to reproduce most of the situations that occur during takeoff, flight and landing.

The development of aircraft construction in recent years has been accompanied by a sharp complication of objects, affecting almost all systems of the designed product, which inevitably led to the complication of AS. This has led to an increase in both

the cost of development and the time spent on searching for an acceptable design solution.

One of the most difficult and important stages of creating an AS is the development of a flight dynamics simulator.

The improvement of flight dynamics models can be carried out only through the construction of rational mathematical models, based on flexible modular software and mathematical support, automation of design and testing and development of methods for identifying and correcting these mathematical models [7].

All this leads to the need to solve a number of problems [4, 8, 10]. Among them are.

- development and implementation of a unified automated system for collecting and processing information, necessary for modeling flight dynamics; formation of a source data bank, creation of technology for its formation and correction;
- development of rational mathematical models, based on unified modular-block software and mathematical support for aircraft (AC) classes;
- development of a methodology for identifying and adjusting the flight characteristics of the simulator for a real aircraft with the development of criteria for all flight modes;
- development and implementation of a system for automated testing of flight dynamics simulator.

A unified automated system for collecting, processing and correcting source information should provide for

- the use of effective technology for collecting source information;
- the use of initial information, including aerodynamic coefficients, geometric data, moments of inertia, flight performance characteristics and other data, necessary to create a mathematical model of the flight dynamics of various aircraft classes;
- the use of effective methods that ensure acceptable timing of the correction of the source data bank;
- using a mathematical description of aerodynamic coefficients with an indication of the coordinate system;
- the form of presentation of the data bank to the AS developer in the cycle of aircraft and aviation equipment (AE) creation;
- the presence of a responsible contractor, responsible for the completeness and reliability of the data bank.

The mathematical model of flight dynamics should be built on the principle of a flexible modular structure [11–13] with the optimization of interconnections, ensuring the maximum possible unification, and consist of unified modules for AC classes and original modules for a specific AC. The mathematical model of the modules should be executed on the basis of a data bank. Modules must have complete software and mathematical support. The structure of the model should make it possible to adjust the output characteristics of the AS to the actual flight characteristics of the AC and «for the pilot». The structure of the model provides for

the content of a module that provides automated verification of the correct operation of the model. The allowable calculation time must be determined for each module.

The justification of the required accuracy of flight dynamics modeling should be carried out, based on the conditions for the formation of full-fledged professional skills during AS training.

Objective indicators of the simulation properties of AS should allow an assessment of

- the resulting characteristics of piloting;
- quality of control;
- basic ergonomic characteristics.

The increasing role of simulators in the training of aircraft crews causes high requirements for their simulation characteristics. The tasks of the simulation are to reproduce the standard flight conditions, the prerequisites for special situations and their course, both with correct and incorrect actions of the crew, in the entire range of expected maintenance conditions, accepted for simulation.

Flight dynamics simulator [10] simulates AC movement both in the air and on the ground, including in case of engine failures, aircraft systems and flight navigation equipment. The atmospheric environment (wind, turbulence, wind shear, temperature, pressure, icing and strip condition) is usually simulated by a separate simulator. When creating it, as a rule, aerodynamic characteristics, obtained either by calculation or by the results of wind tunnel purges, are used. The initial information in the simulator is given in the form of tables or empirical formulas.

It should be noted that the level of initial information is insufficient to ensure the completeness of the mathematical model in the simulator: the used computing complexes do not fully correspond to the required level; the accuracy of flight performance modeling (FPM) is insufficient; it is necessary to reduce the time of one cycle of integration and calculation of the coefficients of the equations of motion to reduce the distortion of dynamic processes [14]. The need to reduce the integration error has not disappeared. The available AS lacks mathematical models for some critical flight modes; the time of transmission of information to the indicating devices is long. This naturally affects the subjective assessments of experts.

As for the theory of the question, at the present time there are practically no effective methods of identifying ergatic systems, although satisfactory methods of mathematical modeling of purely technical systems exist. This is due to the action of *the organismic* principle in ergatic systems, according to which the *operator completes himself in an integral ergatic system in an organismically optimal way*. In other words, the object determines the behavior of the operator. Therefore, according to the data of the normal maintenance of a real human-machine system, only generalized dynamic characteristics of both a person and an object can be determined. The problem arises of determining the dynamic characteristics of the object by the method of trial effects, which is not possible for all flight modes.

The use of *applicative modeling* of flight dynamics is expanding, which is a hardware simulation, using hybrid real-time computers with a special mixed form of machine variables and a parallel distributed hybrid computing network.

In *hybrid systems*, the digital part is assigned to solve the equations of progressive movement with high accuracy, but with relatively low speed. The analog part simulates the equations of rotational motion relative to the center of mass and the control system, where accuracy requirements are limited, but high speed is required.

In an aggregative system, unlike a hybrid one, in the conventional sense, hybridization is carried out at the lower element level, while simultaneously dividing information along the stream of its processing into digital and analog. The digital part defines the node values of the operands. Continuous filling between node values is carried out by a parallel hybrid computing part. Naturally, the introduction of aggregative simulators leads to the use of specialized computing systems.

The simulator of the information complex of altitude-speed parameters [1–8] is designed to simulate the operation of a real complex. It includes simulators of the air pressure receiver, air signal systems (indicators of altitude, speed, number M and outdoor temperature), variometer, altimeter, alarm systems for dangerous ground approach speed and some others. It is built on the basis of hybrid computing systems. The docking of the analog part of the simulator with a digital computer is carried out using an information exchange system.

The simulator of aerobatic instruments [1, 3, 11, 15] allows you to acquire the skills of using aerobatic instruments in various flight conditions and provides a visual indication of the angles of roll, pitch, current and critical angles of attack, vertical overload and angular velocity of turn and slip.

Communication of devices and pointers with the control system of the simulator is carried out through the modules of the information exchange system. The connection of this simulator with others, the power supply of the aircraft, as well as with the display module of the instructor's console, is carried out, using an intramachine exchange.

The simulator of the aircraft control system [11, 15, 16] is designed to instill control skills. It simulates the operation of the elevator, ailerons and spoilers, rudder, as well as the effects, created by the loading and decoupling springs in normal and special cases of operation. The forces on the control levers are reproduced by hydraulic steering units, according to the characteristics, modeled by a digital–analog computing complex. The simulator makes it possible for pilots to perform all the actions, provided for in the flight operations manual for the preparation, inspection and maintenance of control systems both in normal operation mode, and when simulating the most characteristic failures.

The control system forms a tactile–kinetic source of information [17], which forms the shortest feedback loop in the «pilot–AC» system. This feature of the simulated system imposes strict requirements on the quality of modeling the components of the load of the control levers and increasing the speed of the simulator, which can be done using an analog computer.

The simulator also simulates the stabilization of the aircraft relative to the center of gravity, climb, descent, turns, automatic exit of the aircraft to a given echelon, indication of roll angles, pitch, course, track angle, drift angles, slip, etc.

Overload simulators are designed to create effects of acceleration on the crew, using a six-stage dynamic stand. They simulate AC vibrations, when moving along

the strip from the joints between concrete slabs, impact during landing, shaking, when AC exits at critical angles of attack, etc.

The characteristics of these simulators mainly meet the requirements. However, the issues of improving the dynamic chair, reducing energy consumption, and forming criteria for an objective assessment of acceleration simulators remain relevant.

The visual environment simulator [1–3] is designed to create a visual image out of the cabin space for crew members at all stages of the flight, starting with taxiing to the start from the parking lot, during takeoff, flying along the route with visibility of the earth's surface and ground landmarks and ending with running along the runway and taxiways. The simulator allows you to simulate the visual environment during the day, at dusk and at night, as well as simulate the starry sky, images of lights, including flashing lights, etc.

Synthesized visual environment simulators are structurally a set of hardware and software tools, including a generator and an image reproduction device. The latter contains an indicator on a cathode ray tube and an image transfer device to infinity, using spherical and translucent mirrors.

A visualization system with a delay time of up to 80 ms has great capabilities, providing for the display of translucent objects and modeling the texture of some surfaces.

Further development of image synthesis systems with modeling of small-sized and remote objects is possible on the basis of creating simulators with increased resolution (1024×1024 elements).

One of the directions in the creation of these simulators is the development of a system with stereoscopic display devices and the use of a stereo pair of two channels of image generators. Great prospects are also opening up for the development of adaptive type simulators. In such systems, the image is formed in the direction associated with the position (rotation) of the observer's head.

An example of such an adaptive system is a helmet-mounted information display system that allows you to get an image in the range of $200 \times 100^\circ$ or more.

The steering control system simulator [1–8] is designed to simulate the operation of an automatic system for stability and controllability, as well as a trimming system.

The simulator of the navigation situation [3] provides the calculation of navigation parameters for their use in simulators of radio engineering, navigation and landing systems; and the choice of the initial situation.

The acoustic noise simulator [1–3] serves to create a noise environment in the cockpit and develop skills in working under the conditions of the psychophysiological effects of aircraft noise.

We also note the *simulators of takeoff and landing facilities, de-icing system, power supply system, high-altitude equipment and air conditioning system, oxygen equipment, power plant, fuel system and auxiliary equipment*.

Due to the fact that *an aircraft simulator is a complex simulation system*, one of the most important issues is the definition of *criteria for evaluating its quality* [18, 19]. The central place among them is occupied by indicators of simulation

qualities that assess the compliance of the simulator with the original AC. These include the resulting characteristics of flight simulation modes, control parameters and psychophysiological indicators. The methodology for evaluating simulation characteristics is based on comparing the indicators, implemented in the simulator with the specified indicators.

The specified indicators are formed, according to *the airworthiness standards* of the aircraft and *the flight maintenance manual*, based on the characteristics, regulating the safety conditions. The characteristics of piloting [20], obtained in flight, for example, during certification tests, are used. An important place is occupied by indicators of high-class piloting, implemented by experienced crews. *The requirements for the implementation of piloting indicators on the simulator are the most stringent.*

The requirements of the identity of the types of special situations for AC and implemented on the simulator are natural. Criteria, defining the types of special situations, are used in conformity assessment during *AC certification tests*. For example, in the mode of interrupted takeoff, the defining indicators, characterizing the type of special situation, are the distance of the interrupted takeoff and the lateral withdrawal of the aircraft.

These conditions are also used in determining the types of special situations, implemented on the simulator. The comparison of the piloting characteristics, obtained on the simulator and the real AC, is carried out, taking into account the tolerances for the adequacy of the simulator. Unfortunately, the use of AC piloting data in the framework of simulator adequacy assessments is often limited only to their qualitative analysis. For example, this applies to the indicators of «reference» piloting. A subjective comparative assessment of the conditions and results of piloting the AC and the simulator, formulated by flight crews, is also of value.

Thus, the structure of assessments of simulator characteristics assumes the use of both objective and expert indicators of adequacy. The resulting assessment is determined, based on their joint analysis. Objective indicators of properties of the simulator allow us to assess the resulting characteristics of piloting, control and ergonomic characteristics. The method of forming objective estimates provides for a comparison of these groups of indicators for AS and AC conditions.

Expert assessments are formed in the form of a rank gradation on the selected scale. Based on them, an expert assessment of the simulator as a whole is determined for each of the modes: For example, in the form of the ratio of the number of calculated AC cases, acceptable from the point of view of *the simulator reproduction* crews to the number of calculated cases, specified by the *AC certification program*. Experts, as a rule, evaluate the dynamic and informational adequacy, including the simulation of overload, visual environment, acoustic effects, the operation of the flight and navigation complex.

The evaluation of the AC and the simulator is carried out with the participation of the same crew. The procedure for these tests involves training the crew on a simulator before performing a flight task on AC.

The direct use of an aircraft in coupled studies, which is *a standard for evaluating the simulation characteristics and training qualities of the simulator*, as well as the

participation of experienced testers in research, allows providing for a high level of assessment of the adequacy of the simulator.

It should be noted that at the stage of creation and testing of the simulator, the training qualities cannot be determined directly, but *should be predicted*, based on the materials of objective control and expert assessments.

The hierarchical **structure of the AS quality criteria system** includes three levels: *at the highest level—criteria for evaluating training capabilities; at the second—criteria for evaluating simulation properties, evaluation of the system for training control and maintenance indicators; at the third—evaluation of the main components of the simulator, providing the acquisition of skills and abilities, equivalent to the formed ones in real conditions.*

General acquaintance with piloting techniques, takeoff and landing training at night and periodic check of the pilot's level of training in these modes are possible on the aviation simulators of *the first* group. Simulators of this group simulate flight near the ground ($H < 90$ m) in takeoff and landing modes at night. Dynamics is simulated, taking into account the proximity of the earth. Traction reversal, atmospheric conditions, acoustic effects, and failures of the control system and power plant are reproduced. The night flight is realized by a television visualization system with a delay of 0.3 s and a 45° view. Simulator of acceleration effects—with the number of degrees of freedom of at least three, with a delay of up to 0.15 s.

The simulators of *the second* group provide retraining of the commander of the ship to work on AC of this type and training of the co-pilot to the level of the commander of the AC of the same type. They simulate takeoff and landing in night conditions, including taxiing on the ground, handling characteristics both in flight and when moving on the ground; loads on the control levers, the influence of wind; reproduce the operation of the control system, avionics and navigation system; snow, rain, hail, flying in clouds, movement on a dry and precipitation-covered runway, etc. The visualization system synthesizes the image on the computer with a delay of up to 0.15 s, with an overview of up to 70° horizontally and 30° vertically. The mobility system has at least six degrees of freedom (with a delay of up to 0.15 s). The simulator uses a high-speed computer and high-level problem-oriented languages, and provides means for quickly checking programs and correcting the characteristics of the simulator.

Simulators of *the third* group, in addition to the listed functions, provide initial training of pilots and retraining of crew members for another type of aircraft, modeling of all piloting operations, dynamics, changes in aerodynamic characteristics during icing, etc. The visualization system with a delay of up to 0.075 s recreates an almost real picture of the review; mobility system—with a delay of up to 0.1 s. Automatic verification of the simulator for compliance with the requirements is provided, as well as the possibility of changeover in accordance with AC modifications.

Further improvement of the level of simulator construction requires an increase in computer performance and the use of high-level machine-oriented languages, an improvement in the quality of the synthesized visualization system [21], the power

of the information exchange system between the computer and peripheral devices and a full assessment of the actions of the crew at the instructor's workplace [22].

We will indicate some areas of improvement of simulator construction: the implementation of means for the formation of equivalent piloting skills in standard and calculated cases, ensuring greater adequacy of the simulator and AC [23], objectification of evaluation, improving the maintenance qualities and characteristics of visual and acceleration environment simulators, automation of training management, improving the source data bank and using the modular structure of the simulator to improve the information exchange system.

There are new directions in the field of flight crew training and flight condition modeling tasks. Two of them relate to hardware, and the others to the methods of their application.

The first direction is the transition from individual simulators to the creation of «total» systems, in which simulators are only part of it. The purpose of such systems is to ensure that cadets successfully master the full training program up to a certain level of flight qualifications.

Such functions are performed under contracts, concluded with airlines, Flight Safety firms and International Flight Simulation (USA).

These companies have a full range of educational and methodological equipment, which includes classes for theoretical training, simulators of various complexity and various functional purposes, as well as training aircraft and helicopters. A characteristic feature here is that the contractor company not only builds and operates a system of training equipment but also provides the customer with teachers and instructors.

The second direction consists in the use of more advanced simulators that allow you to work out the implementation of all stages and flight modes.

To realize the positive aspects of this trend, two problems are solved: in order to achieve high modeling accuracy, a complete package of reliable information is created in a form convenient for modeling and it is provided with newly received information within 1–2 days after its appearance. The solution to these problems in the US Air Force was carried out within the framework of «Project 2851» [2], which involves the creation of a central database for simulators.

The third direction is characterized by increased attention to the standardization and modularity of the execution of simulators, the creation of networks and linking individual simulators and other technical training devices to each other for integrated use.

This trend was most fully manifested in the Modular Simulator Design program, which was commissioned by the US Air Force department of the Boeing-Military Airplane Simulation Training System. The specialists of this company have compiled technical requirements that define the necessary features of the architecture and interface of the computing system so that it is possible to bring together the modules of the simulator, created during the implementation of the modular principle of its construction. In addition, the technical conditions for the creation of modules provide for the possibility to manufacture individual simulator modules at various enterprises, and then combine these parts into a single simulator structure. This concept of building

and using simulators allows you to reduce the design time, and facilitate the work on the formation of large networks, consisting of many interconnected simulators.

The fourth direction is to increase attention to the formation of databases.

This was reflected in the results of research, conducted by the order of the US Air Force by department Simtec in the program «Database integrity research in simulators» [2]. These studies have shown the high importance of such databases. It was also convincingly shown that the use of incomplete and insufficiently reliable databases in simulators leads to poor training quality, unreasonably high complexity and high cost of simulators and the high cost of their life cycle.

It should also be noted the evolution of flight training, which consists in the fact, that the emphasis is not on honing the individual skills of pilots, but on training the crew as a whole (with a linear orientation). Briefly, this method can be described as a flight training program in a linear environment, providing for the participation of the entire crew in overcoming flight situations, leading to accidents, with the main focus on the mobilization of human resources. The ultimate goal of the method is to prevent errors, related to the human factor, with constant effective control of resources.

In the system of flight training, in improving flight safety, firms, that have assumed the functions of training flight personnel in ground conditions, pay great importance to the role of instructors.

The principles and methods, based on the results of the latest research in a relatively new field of psychiatry, are being implemented. Recently, *methods of automated and adaptive training*, based on proven training programs, have been widely used for more effective use of simulators.

The possibility of this modernization is determined by the basic concept of the last decade—*the modularity of the design and software and mathematical support*, which allows avoiding large costs for the development and modernization of simulators and is adopted by all advanced supplier firms.

The modular approach to creating a simulator should be considered at the software and physical levels, the final goal of which is to reduce costs for development, and increase reliability and flexibility. At the same time, the use of a computer on a single board guarantees a high speed of performing computational operations. Such simulators can be supplied in various configurations: with a movable cabin or stationary, with or without a visual environment simulator. The main parts of such a simulator are *the crew cabin, the computer system and the instructor's workplace with an electronic terminal* that provides control and control over the learning process.

The advantage of *the described concept* is the possibility of using such macromodules as a dynamic platform, a visualization system. The components of the computer system can be manufactured at various enterprises, and a specific configuration of the simulator can be completed from them at the supplier enterprise.

The greatest *potential danger* in the implementation of the modular approach is the desire to choose a highly specialized approach *that rigidly defines the architecture of the computer, the structure of the interface and dictates the use of a special programming language*. In addition, if the modules are allocated and standardized incorrectly, the ability of the industry to introduce technical innovations will be reduced. Ideally, from the point of view of introducing novelties, interface devices

should be standardized in the modular design, and complete freedom of technical thought should be provided inside the modules.

The creation of aviation simulators, that are competitive on the world market, requires the creation of a non-departmental state certification system for simulators.

The main *scientific and technical problems of simulator construction* are development of reliable models of external conditions and dynamics of the aircraft; creation of highly efficient systems for the synthesis of visual environment; development of optimal laws for controlling the mobility system and criteria for its evaluation; creation of a data bank of complete aeronautical information; development of a unified module for reproducing loads on control bodies, using a specialized computer and electrohydraulic actuators; creation of highly efficient computing systems, based on the latest achievements of microelectronics and computer technology; development of objective methods for assessing the degree of compliance of simulator characteristics with the simulated object, as well as evaluating the quality of operator training; minimization of time delays of visual and acceleration information, presented to the pilot's workplaces in the simulator cabin; development of adaptive systems, teaching methods and scenarios; formalization of instructor's knowledge; development of highly effective diagnostic systems for simulator components and systems for automated design and processing of results.

Let us dwell a little more on the state and choice of ways to simulate flight dynamics [24].

It is quite obvious that these issues are considered taking into account the structure of AS.

Until recently, the choice of the simulator structure was dominated by the architecture of the control digital computing system and its computing power. In this regard, the distribution of this power among consumers [25] was in contradiction with the main criteria of the technical level of simulators, such as training qualities, reliability, terms of development and mastering of simulators in the series, the level of unification and the possibility of modernization. The shortage of computing power led to the fact that each consumer overestimated his need, which led to an increase in the number of computing complexes in the system with all the ensuing negative consequences.

The solution to this problem is to create a distributed computing system, based on functional microprocessors with *their parallel operation* and the use of reflective memory. In this case, the issue of power distribution disappears by itself, as well as the problem of flexibility due to the possibility of simply increasing the number of microprocessors according to need.

The main advantage of the modular design of simulators is the flexibility of application, the possibility of independent development and ease of modernization.

The modular approach does not imply mandatory standardization of hardware. The main thing is the standardization of functional module interfaces (FMI).

The interface should allow you to easily combine any number of modules into a complex or specialized simulator or simulator with a limited number of flight information modules, for example, without a visual environment simulator (an aerobatic

simulator for practicing blind flight) or without an acceleration effects simulator (a navigator simulator or a flight engineer simulator).

The standardized interface, software and hardware, control devices and information display are part of the functional modules and flight information modules.

The physical implementation of functional modules should provide conditions, under which changes at one level do not entail changes at another. The physical connections of the modules should be unified, long-term, flexible and improved. To improve the modules, it is assumed that only the interface devices will be standardized.

The modular approach requires additional data processing to bring them into a compatible form. But it is worth it, because the inherent flexibility of the modular principle compensates for the additional costs of data processing and reprogramming.

The introduction of modularity requires clear coordination of work to determine the functions of the modules and the interface, as well as the implementation of their verification and certification. So, *the simulator of cabin procedures* consists of 4 modules: the crew cabin, the instructor's workplace, simplified flight aerodynamics modules and the power plant. A more complex simulator, in addition to the crew cabin and the instructor's workplace, should include complete modules of aerodynamics and power plant, as well as additional modules of the visualization system, acceleration effects, aircraft and atmospheric noise. For *the operator of on-board systems*, a specialized simulator is usually created, based on the cabin module and simulators of on-board systems, supplemented with simplified modules of aerodynamics and power plant, as well as a module of a specialized instructor workplace. Naturally, *the complex simulator* will include all the modules that are included in *specialized simulators*. Here, the simplified modules of aerodynamics and power plant are completely replaced.

A separate functional module of the simulator can be developed as a complete product, which is the basis of a *specialized simulator* (pilot, flight engineer, navigator and operator). The possibility of using this module as part of a complex simulator is provided.

To combine the training modules into a complex simulator, functional connections are determined, i.e. the composition and structure of the arrays of information that the modules should exchange. Interface devices should be standardized.

Functional modules and flight information modules (control system simulator, flight dynamics simulator, acceleration effects simulator and visual environment simulator) should be implemented in one computing complex [8].

The connections between the modules (Fig. 1.1) are stored as *unified* for all types of simulators. The number of computing complexes in a module should not change the structure of connections between modules.

The following abbreviations are shown in the diagram:

DDT—device for data transmission;

FDS—flight dynamics simulator;

PPS—power plant simulator;

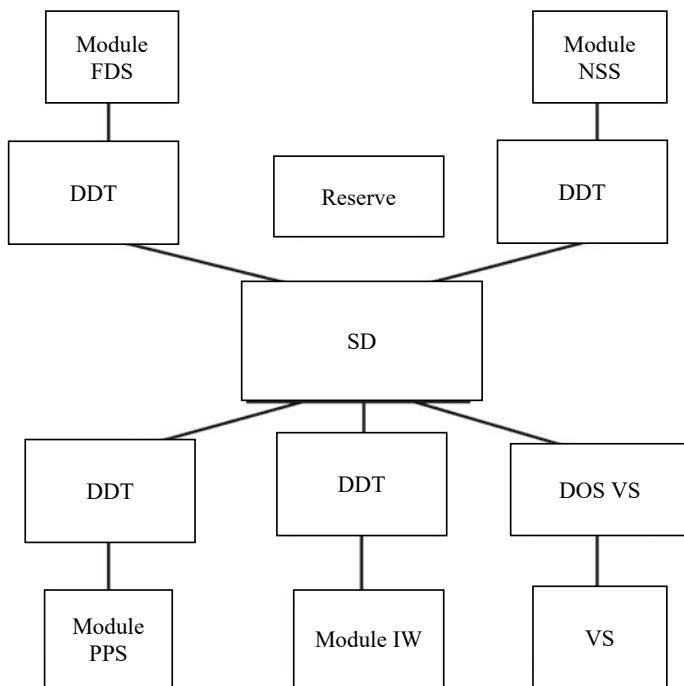


Fig. 1.1 Structural diagram of the interface of flight and navigation simulator simulators

NSS—navigation situation simulator;

IW—instructor's workplace;

VS—visualization system;

DOS VS—data output system of visualization system;

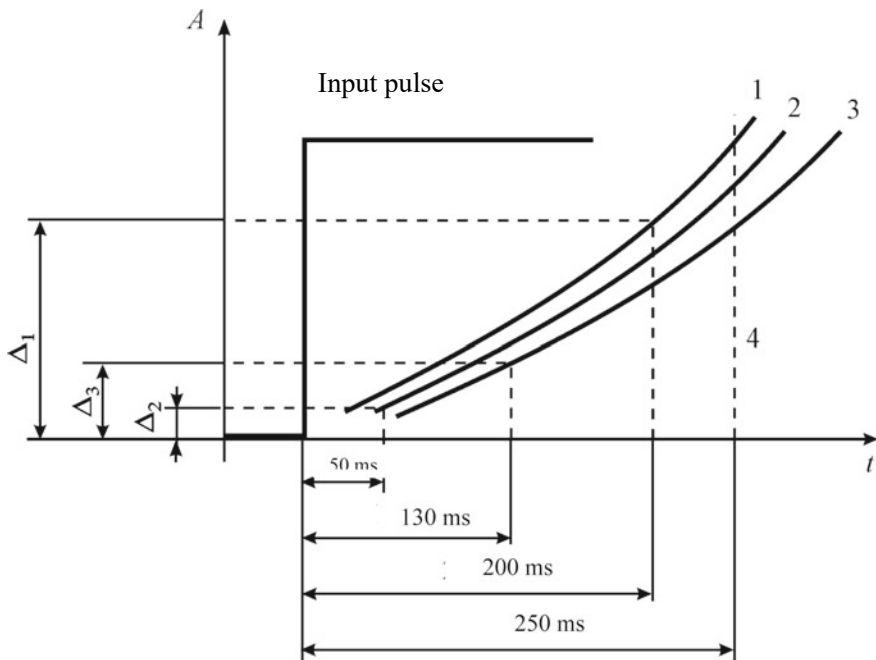
SD—switching device.

In the absence of a reserve in switching device (SD), the additional computing complexes, introduced with the main computer of the module, are connected, using autonomous means (for example, using an additional in-module switching device).

The formation of meaningful professional skills is possible only if the adequacy of the model is ensured, taking into account the psychophysiology of flight work, when developing functional flight information modules.

When modeling flight information, the developer needs not only to form a signal, that is correct in nature, but also to meet the acceptable range of delay (Fig. 1.2) with the required alternation of signals (perception of information through visual, auditory and tactile channels).

The most stringent requirements are imposed on the formation of parameters of short-period motion, because it determines the adequacy of the «piloting» of the simulator.



- 1 – visual analyzer;
- 2 – auditory analyzer;
- 3 – tactile analyzer;
- 4 – the limit of permissible delays of information flows

Fig. 1.2 Delay of information perception

First of all, this applies to modules (Fig. 1.3) for simulating

- aircraft control, when moving on the ground and in the air;
- flight dynamics;
- acceleration motion effects;
- visual environment;
- acoustic noise in the cockpit;
- engine thrust.

Formalized models of the perception of the flight crew and their professional activities in flight constitute a fundamental, extremely difficult problem of aviation engineering psychology, ergonomics. So far, there are no methods of formalized solution to the problem of AS synthesis with the determination of their effectiveness indicators already at the design stage, although the scientific foundations of the construction of training subsystems and training complexes, in general, already

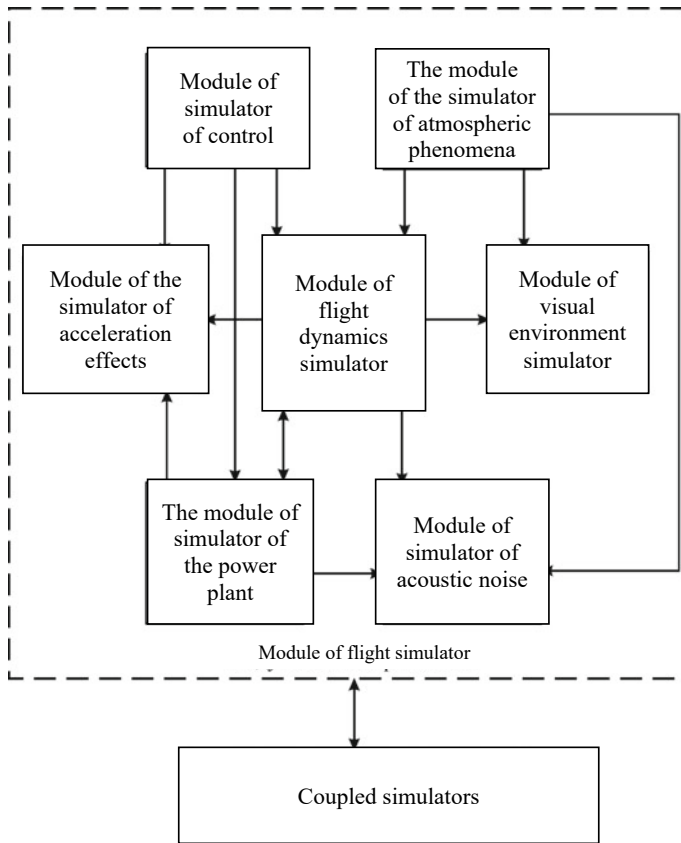


Fig. 1.3 Block diagram of the flight simulator module

constitute in their modern form a fairly coherent and practically important scientific discipline. Based on this, in this book, a significant place is given to theoretical and applied issues, related to the technical means of flight training on AC.

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Chapter 2

Complex Simulators of Transport Aircraft



2.1 General View of the Aircraft Simulator

Based on the above-mentioned methodological design principles, with *the direct participation and guidance of the authors*, a number of complex simulators have been developed that are *widely used for training flight personnel in training centers of both civil and military aviation*.

Highly effective complex computerization of complete professional training requires huge expenditures of intellectual labor, including the labor of highly qualified professionals. With a centralized architecture of computer TTT (technical training tools) and a centralized system for their development (even by powerful specialized firms) this may not be feasible, at least for a limited period of time. This dictates the modular construction of a complex of technical training tools with a thorough development of the structure of a computing system with the implementation of *parallel and distributed computing*.

The general view of the aircraft simulator is shown in Fig. 2.1.

The simulator consists of the following:

- a cabin with two pilots, a flight engineer and an instructor, installed on a six-stage simulator of acceleration effects;
- a basic computer, including five computers and a complexing device;
- information exchange module, consisting of five racks of subscriber terminals, located outside the cabin and two racks in the cabin;
- a visual environment simulator module, including an image generator and a visual environment display module;
- the instructor's workplace outside the simulator cabin;
- two racks with on-board digital computers;
- the simulator module of the meteorological navigation radar station MNRS-85;
- general power supply module with an autonomous set of primary power supply for the basic simulator computer;
- air conditioning module in the simulator cabin.

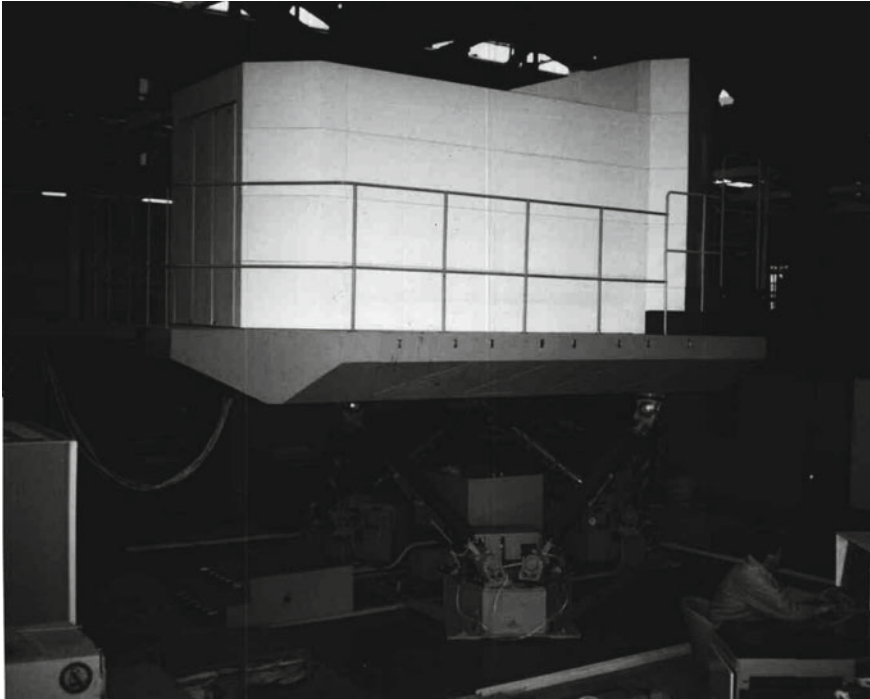


Fig. 2.1 General view of the aircraft simulator

2.2 Flight Dynamics Simulator Module

The flight dynamics simulator reproduces the movement of the aircraft in space, based on the solution of a *closed* system of *nonlinear* differential equations, the *input* parameters of which are the *control actions* of the crew in the cockpit of the aircraft, and the *output*—*parameters of the flight*. The following tasks are provided in the simulator:

- taxiing on the runway and taxiways;
- takeoff and height gain;
- en-route flight, descent and approach;
- going to the second round, using the means of a complex of standard flight and navigation equipment;
- emergency descent of aircraft;
- flying in a circle, landing approach and landing;
- run on the runway, using all means of braking;
- flight in conditions of dangerous external influences.

The tasks are performed, taking into account the influence of the following conditions on the flight characteristics:

- air temperature and atmospheric pressure;
- altitude (terrain level) of the airfield;
- horizontal component of wind speed;
- vertical and horizontal «wind shear»;
- horizontal and vertical wind gusts;
- effects of icing on aerodynamic characteristics;
- the effects of different runway conditions;
- the mass of the aircraft in the form of three components;
- aircraft alignment;
- operating modes of the power plant, including reverse thrust mode;
- positions of control surfaces, wing mechanization and landing gear;
- aeroelasticity.

When driving on the ground, the flight dynamics simulator takes into account the impact on the stability and controllability characteristics of the landing gear parameters (the coefficient of wheel withdrawal, the coefficient of wheel friction, depending on the condition of the runway, as well as wheel braking and anti-skid device operation).

The simulation of flight dynamics is based on the continuous calculation of its parameters in real time, in order to create a similarity of the simulated «flight» to the flight process of the aircraft.

The «flight» is carried out by providing the crew with visual, acoustic and acceleration information, as well as reproducing the readings of the instruments of the flight navigation complex and the position of the controls, including loads on the controls. All the specified information is formed in accordance with the parameters, obtained in the flight dynamics simulator.

The block diagram of the flight dynamics simulator module and its relationship with other simulators is shown in Fig. 2.2.

The designations are accepted here:

ANCS—aircraft navigation computer system;

FCCS—flight control computer system;

CSTC—computer system for traction control;

EDS—electronic display system;

TC—true course;

P_{RTS} —runway touch sign;

G —weight of aircraft;

G_f —weight of fuel;

H —altitude of flight;

H_{af} —height of airfield;

H_s —height of seat level;

M —Max number;

N_a, N_p, N_n —compression force of the chassis racks;

N_{hp}, N_{lp} —high and low pressure rotor speed;

n_x, n_y, n_z —components of the overload vector in the associated coordinate system;

P_{td}, P_{tp} —brake system pressure;

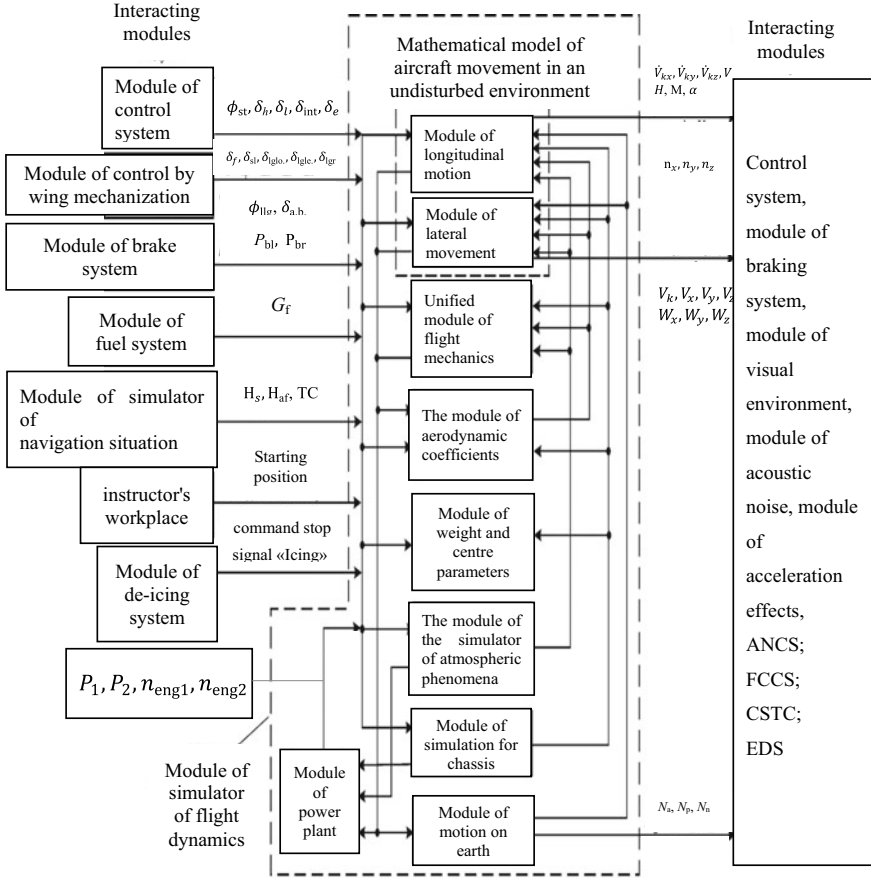


Fig. 2.2 Block diagram of the module of simulator for flight dynamics

- P_1, P_2 —engine thrust force;
- V —AC speed, relative to the air environment;
- V_k —ground speed AC;
- $\phi_{n.w.}$ —angular position of the nose wheel;
- ϕ_{st} —angular position of the stabilizer;
- ψ —yaw angle AC;
- $\dot{\psi}$ —angular velocity of the return;
- $\omega_x, \omega_y, \omega_z$ —components of the angular velocity in the associated coordinate system;
- $\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z$ —components of angular acceleration in the associated coordinate system;

V_{kx}, V_{ky}, V_{kz} —the components of the earth's velocity AC in the normal coordinate system;

V_x, V_y, V_z —components of the velocity AC in the associated coordinate system;

α —angle of attack;

γ —angle of AC roll;

ϑ —pitch angle of AC;

$\delta_f, \delta_{sl}, \delta_{lgl0}, \delta_{lgle}, \delta_{lgr}, \delta_{a.b.}$ —deflections of flaps, slats, landing gear and air brakes;

$\delta_a, \delta_r, \delta_e, \delta_{int}$ —deflections of ailerons, rudders, elevators and interceptors.

The following parameters are determined in the simulator of flight dynamics:

$$\dot{V}_{kx}, \dot{V}_{ky}, \dot{V}_{kz}, V_{kx}, V_{ky}, V_{kz}, V_k, V_x, V_y, V_z, V, W_x, W_y, W_z,$$

$$\Delta\psi_w, \dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z, \omega_x, \omega_y, \omega_z, R_x, R_y, R_z, M_{py}, M_{pz}, M_{Gy}, M_{Gz},$$

$$V_{yg}, H, M, q, n_x, n_y, n_z, G, \Delta X, C_x, C_y, C_z, X_{lg}, Y_{lg}, Z_{lg},$$

$$M_{lgx}, M_{lgy}, M_{lgz}, N_{low}, N_{le}, N, Z_{low}, Z_{le}, Z, \dot{\gamma}, \dot{\nu}, \dot{\psi}, \gamma, \nu, \psi,$$

$$\alpha, \beta, \beta, m_x, m_y, m_z.$$

The simulator uses signals and parameters from the following simulators and systems:

- control systems ($\phi_{st}, \delta_h, \delta_l, \delta_{int}, \delta_e$);
- power plant ($G, P_1, P_2, n_{eng1}, n_{eng2}$);
- takeoff and landing facilities ($\delta_f, \delta_{sl}, \delta_{lgl0}, \delta_{lgle}, \delta_{lgr}, \phi_{llg}, \delta_{a.b.}$);
- brake system (P_{bl}, P_{br});
- simulator of navigation situation (H_s, H_{af}, TC);
- simulator of atmospheric phenomena ($\alpha, \rho, W_{yg}, W_{xgzgsd}, W_{xgzg}, W_{gp}, W_{hp}$, runway condition);
- de-icing system (activation de-icing system, «Icing»);
- from the instructor's workplace (signals «Starting place» and «Stop»).

The equations, solved in the simulator of flight dynamics, are conditionally divided into four groups:

- longitude movement;
- lateral movement;
- movement on the earth;
- aerodynamic coefficients.

All parameters are contained in the calculator in the form of codes.

The step of solving the flight dynamics equations is selected from the condition of ensuring the stability of the system and should be ≤ 50 ms (25 ms is accepted) [1]. Communication of the module of flight dynamics simulator with modules of other

systems is carried out on the basis of intramachine and inter-machine information exchange.

For operational modification of the flight dynamics model, the possibility of program input and output of physical parameter values to the display module is provided.

2.3 Control System Simulator Module

The module of simulator of the aircraft control system is designed to control the flight on the simulator and instill in the crew the skills to use the control system [2], issuing information to the interacting systems. This module provides the following:

- calculated determination of the position of the steering surfaces (controls), depending on the signals from the sensors of the position of the control levers, on the flight parameters and on the parameters generated by the simulator of the flight control computer system (FCCS);
- determination of the required flight stability and controllability characteristics by implementing algorithms of control by simulator;
- automatic limitation of the maximum flight modes, as well as a warning to the crew about going beyond the main operational flight modes;
- playback of control lever downloads [3, 4];
- simulation of individual characteristic failures.

The construction of the module of simulator of control system is based on semi-natural modeling, using on-board equipment. The module simulates the operation of the control channels of the stabilizer, elevator, ailerons, interceptors, air brakes and rudder.

As part of the stabilizer control channel, there is an executive balancing electromechanism, mechanically connected to a manual balancing wheel, and a stabilizer position sensor.

The channel of stabilizer control provides simulation of the following:

- automatic balancing of flight in the longitudinal channel and trimming of forces on the control columns with the main steering and automatic (from the FCCS simulator module) control of the movement of the manual balancing steering wheels in accordance with the deviations of the stabilizer;
- manual balancing of flight and trimming of forces on the columns from the trimming switches, when simulating failures of the automatic balancing mode;
- emergency pitch flight control from the manual balancing wheel;
- emergency pitch control in case of failure of the elevator control channel.

The *elevator control* channel provides simulation of the following:

- flight control from the control columns during normal operation through the main electric control circuit and a backup electric control circuit, when simulating the

main electric control circuit or a mechanical emergency control circuit and when simulating failures of the main and backup electric control circuits;

- testing of FCCS control signals in automatic control modes without moving the control columns;
- emergency control and flight balancing in case of failure of the stabilizer control system.

The aileron control channel (by means of transverse controllability) provides simulation of the following:

- roll flight control from the steering wheels through the main electric control circuit during normal operation or through a backup electric control circuit, when simulating the failure of the main electric control circuit;
- testing of FCCS control signals in automatic control modes without moving the steering wheels;
- roll balancing in flight with steering and automatic control.

The channel of control of *the interceptors and air brakes* provides simulation of the following:

- control of the interceptors by roll from the steering wheels through the main electric control circuit during normal operation, through the backup electric control circuit, when simulating the failure of the main electric control circuit or through the emergency mechanical control circuit, when simulating failures of the main and backup electric control circuits;
- testing of FCCS control signals in automatic control modes without moving the steering wheels;
- symmetrical release of interceptors in flight (on descent) for braking from the brake control handle (BCH);
- symmetrical automatic or manual release of interceptors and air brakes on the run from BCH (in case of refusal of automatic one).

The channel of *rudder control* provides simulation of flight control along the course from the control pedals through the main electric control circuit during normal operation, through the backup electric control circuit, when simulating failures of the main electric control circuit or through the emergency mechanical control circuit, when simulating failures of the main and backup electric control circuits.

The module simulates probable failures of the real control system of the aircraft:

- failure of the electrical balancing system of the stabilizer control channel;
- failure of the electric station system for regulating the range of connection of additional loading of speakers;
- failures of electric control circuits;
- jamming of emergency mechanical wiring;
- failures in electrical and hydraulic systems.

2.4 The Module of Power Plant Simulator

The module of power plant simulator (PPS) provides reproduction of the operation of the following:

- two main engines;
- auxiliary power plant (APP);
- control system of the auxiliary power plant (CS-APP);
- automatic control systems (ACS);
- automatic start of the engine (ASE);
- electronic controller of the operating modes of the auxiliary power plant.

The simulation is carried out on the basis of mathematical modeling of the characteristics of the power plant in the basic computer of the simulator, using the block-modular structure of the software.

The communication of the basic computer with the controls and controls in the cabin is carried out, using the information exchange system (IES), and communication with the interacting simulation modules is carried out, using the intramachine and inter-machine information exchange of the basic computer.

The block diagram of the PPS module is shown in Fig. 2.3, where the designations are adopted:

- APP—auxiliary power plant;
- LS APP—launch sign of APP;
- IPF—indication of the presence of fuel at the engine input;
- SVT—a sign of the presence of voltage on the tires;
- SCS—sign of compression of the chassis;
- SF—a sign of a fire;
- SF APP—a sign of a fire of auxiliary power plant;
- EFS—engine fire sign;
- SOHS—sign of operation for hydraulic system;
- SOES—sign of the operation of the electric starter;
- SPB—sign of parking brake;
- SCFW—sign of control by front wheel;
- α_{ecl} —position of the engine control levers;
- δ_{sl} —angular position of slats;
- Δ_{cor} —value of correction of engine mode value;
- IS—a indication of surging;
- SR—sign of reverse;
- $\dot{G}_{\text{a tak}}$ —the amount of air, taken from the engine of reverse;
- G_f —hourly consumption of fuel;
- $I_{\text{gen}}, I_{\text{st}}$ —load currents of generator and starter;
- M—number of Max;
- $N_{\text{hp. progr.}}$ —the program value of the rotation speed of the high-pressure rotor;
- N_{APP} —speed of rotation of APP;
- n_i —speed of rotation of rotor engine;

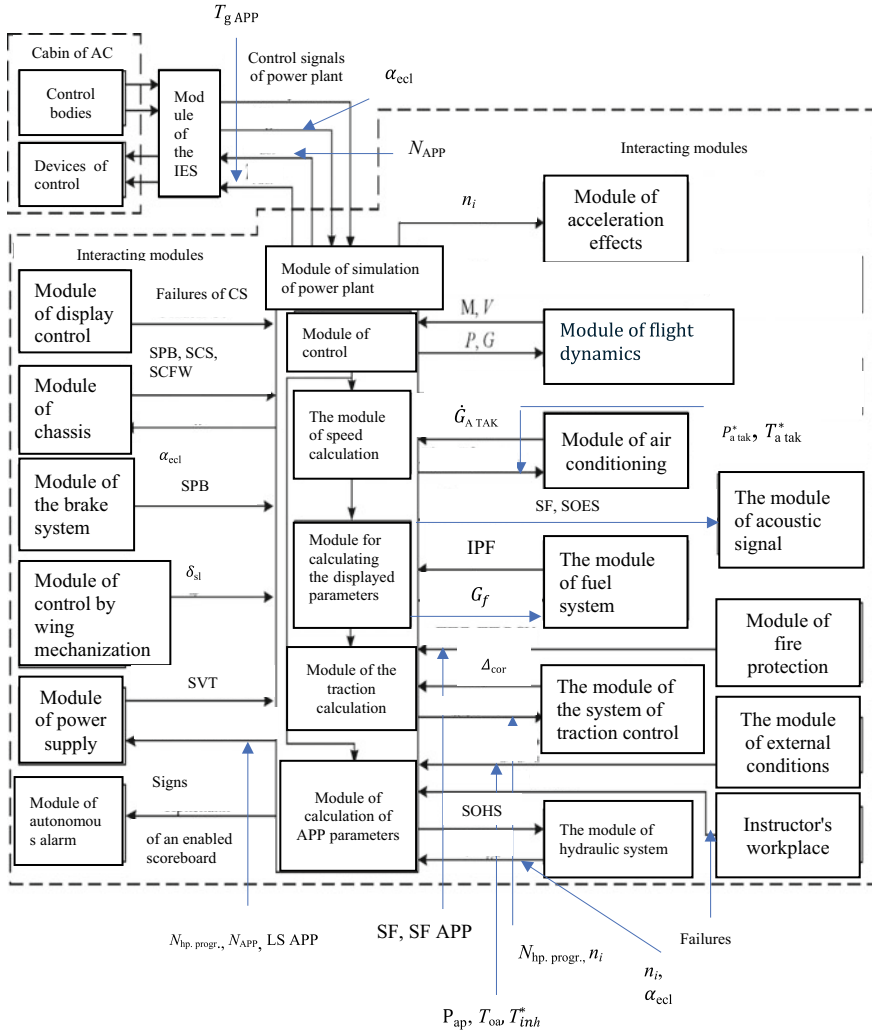


Fig. 2.3 Block diagram of the power plant simulator module

- P —engine thrust force;
- $P_{a tak}^*, T_{a tak}^*$ —pressure and temperature of the air, taken from the engines;
- P_{ap} —atmospheric pressure;
- $T_{g APP}$ —gas temperature APP;
- T_{oa} —temperature of outdoor air;
- T_{inh}^* —inhibited air temperature at the engine input.

Simulation of the auxiliary power plant operation allows the reproduction of the following operating modes of the real system:

- preflight check in the cockpit;
- preparation for starting the engines;
- cold scrolling of engines;
- starting engines on the ground from ground and on-board power sources;
- operation of the auxiliary power plant on the ground and in flight in the maintenance range of heights and speeds;
- stopping engines on the ground and in flight;
- starting engines in the air with and without blocking;
- selection of power for generator loading modes $P_{\text{gen}} = 60 \text{ kVA}$ and air extraction $\dot{G}_{\text{a ext}} = 0\text{--}2.5 \text{ kg/s}$;
- simulation of an unsuccessful launch of the power plant in the range of heights and speeds, not specified in the «Flight operation manual» (gas temperature rise above 570°C , engine speed freeze $n_{\text{APP}} = (30 + 5)\%$ и $(30 - 5)\%$, automatic termination of start-up, when protection is turned on, according to the limit parameters).

When simulating the operation of main engines, the following operating modes of real engines are reproduced:

- preparation of engines for launch;
- checking the electronic part of the system for engine regulation (EER), using the built-in control;
- cold scrolling of engines;
- false start of engines;
- starting engines on the ground from ground and on-board power sources;
- warming up and testing of engines;
- operation of engines in steady-state and transient modes in the entire operational speed range (in the altitude range 0–13000 m);
- normal and emergency stop of engines on the ground and in flight, as well as stop by a fire crane;
- autorotation;
- reverse thrust;
- automatic control from the system;
- simulation of an unsuccessful engine start in the range of heights and speeds, not specified in the «Flight operation manual» (an increase in gas temperature above the permissible, engine speed freeze, automatic start-up termination with a functioning engine control system, accelerated engine start at a signal from the instructor's workplace).

The following failures are simulated: engine stop, engine surge, gas temperature rise, engine control system failure, engine vibration increase, gas temperature limit of the auxiliary power plant, engine control disconnection, support overheating and oil pressure drop.

The chosen modeling method provides an error in reproducing dynamic characteristics no worse than recommended in the document «Standards of suitability of aviation simulators for the training of air transport personnel» [5].

2.5 Acceleration Effects Simulator Module

The acceleration effects simulator module is designed to reproduce the components of the acceleration field of information, related to changes in linear overloads, angular accelerations and displacements, which are typical for various flight modes, as well as to reproduce the following effects:

- constant non-directional «background» movement during the entire flight;
- vibrations from running engines;
- noises from the release and removing of the chassis;
- compression of the landing gear struts during braking, disinhibition, as well as effects, caused by touching their runways, including separate touching of the nose, left and right struts;
- shaking, when moving along the runway, when the permissible angle of attack is reached, taking into account the state of wing mechanization;
- stalling.

The acceleration effects simulator module provides simultaneous simulation of the spatial movement of maneuvering and movement, caused by external influences. The block diagram of the accelerator effects simulator module is shown in Fig. 2.4, where the designations are adopted:

$IH \neq 0$ —indication of height;

n_i —rotation speed of engine rotor;

P_i —engine thrust force;

V_{vib} —speed of vibration of engine;

α_{acc} —acceptable angle of attack;

α_{stal} —stall angle of attack;

α_b —deflection angle of ball;

$\delta_f, \delta_{sl}, \delta_{lgl}, \delta_{lgle}, \delta_{lgr}$ —deflections of flaps, slats and landing gear;

$F_{lgl}, F_{lgle}, F_{lgr}$ —impact force of landing gear struts during release and removing.

The requirements for dynamic stands and their control systems are determined on the basis of a compromise between the most accurate modeling (in ground conditions) acceleration sensations of the pilot and strict technical restrictions on the maximum linear (to a lesser extent, angular) movements of the booth cabin (many times smaller than the actual movements of the aircraft). On a dynamic stand, accurate modeling of the pilot's acceleration sensations in the entire frequency range is fundamentally impossible in linear degrees of freedom, and in angular—difficult. The contradiction is eliminated by using the peculiarities of the pilot's perception of overloads (as a link in the aircraft control system, the pilot is able to use acceleration information to

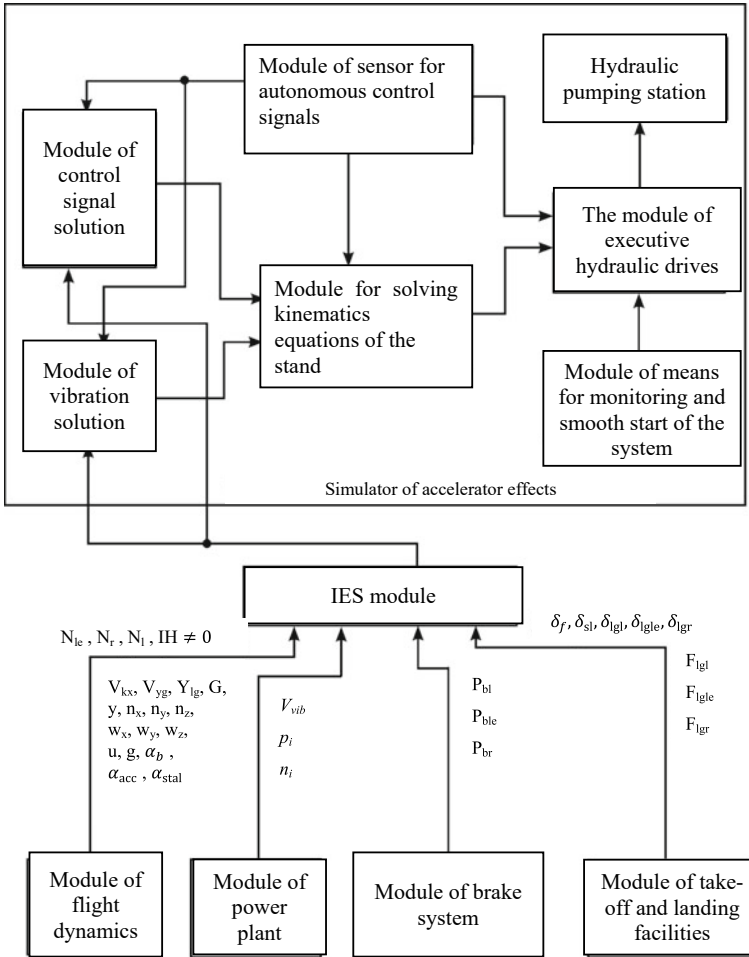


Fig. 2.4 Block diagram of the module of simulator for accelerator effects

control in a fairly narrow frequency range from 0.2 to 0.7 Hz) [6]. At lower frequencies, the pilot manages to adapt to changes in overloads and angular movements of the aircraft. The higher part of the overload spectrum is perceived as shaking or vibration. It creates an important additional similarity effect, to which the pilot does not have time to react through the controls.

So, to simulate overloads, high-pass filters can be used, the structure and parameters of which are selected so as to simulate acceleration information in the frequency range of 0.2–0.7 Hz with minimal distortion. Technically implemented ranges of linear and angular movements of the cabin are formed due to the cutoff of the lower frequencies of the real movement of the aircraft, accompanied by its large movements.

The specified movements take into account their permissible range (regardless of the method of generating control signals).

In order to simulate very low frequencies in a change of overload, the simulator cabin not only does not go beyond the restrictions on movement, but also tries to return to the middle, neutral position, unnoticed by the pilot, and high-pass filters are used [7, 8].

Reproduction on the stand of long-acting linear, lateral and longitudinal overloads (in the range of no more than $\pm 0.5 g$) is possible by tilting the cabin at the appropriate angle. As it is known, the sensations of inclinations in humans appear, according to the functions of otoliths, tactile and kinesthetic receptors. This happens due to a change in the position of the body, relative to the line of action of the resulting mass forces, or in the case, when the body does not change orientation in space, but the direction of the line of action of the resulting mass forces changes due to the appearance, for example, of inertia forces [7], described by the expression.

$$\vartheta = \arctg(n_x) \approx n_x,$$

where ϑ —pitch angle; n_x —longitudinal overload.

During acceleration and deceleration of the aircraft, the resulting mass forces, acting on the receptors, cause a feeling of tilt at the pitch angle. In this case, the simulator cabin tilts to this pitch angle. At the same time, the horizon line on the screen in front of the pilot does not change its position.

Similarly, the simulator works under the action of overload in the lateral direction [7].

The resulting mass force at any slope is 1 g. Therefore, with a range of angular displacements of $\pm 30^\circ$, it is possible to simulate long-term longitudinal and lateral overloads in the range of no more than $\pm 0.5 g$. At the same time, the apparent value of the longitudinal overload.

$$n_{x \text{ app.}} = g \cdot \sin\vartheta,$$

and the apparent value of lateral overload.

$$n_{y \text{ app.}} = g \cdot \cos\vartheta.$$

An important requirement, when simulating long-term overloads, is the location of the center of rotation at the location of the human otolith apparatus, i.e. in the area of the pilot's head. Otherwise, the otoliths will perceive the effect of overloads from centrifugal forces, arising during rotation [8, 9].

Simultaneously with the rotation of the mobile platform, linear displacements are set, the value of which is calculated in accordance with [9]

$$\Delta X = R_x \cos x + R_y \cos \gamma \cdot \sin x - R_x;$$

$$\Delta Y = R_x \sin x - R_y \cos \gamma \cdot \cos x + R_y;$$

$$\Delta Z = R_y \sin \gamma,$$

where ΔX , ΔY , ΔZ —linear movements along the axes X, Y, Z ; R_x , R_y —projections of forces on the axis X, Y .

The low-frequency signal is perceived mainly by changes in the longitudinal and lateral overloads. Signals with a frequency of more than 1–3 Hz are perceived as vibrations [10].

Thus, the frequency band, reproduced in the accelerator effects simulator, is 0.3–3 Hz.

For roll motion, such a condition is met by a filter with a transfer function.

$$W(p) = \frac{0.5 \cdot p}{(0.5p + 1)(0.05p + 1)}.$$

Similarly, for pitch motion,

$$W(p) = \frac{0.5 \cdot p}{(0.5p + 1)(0.05p + 1)}.$$

Since the same drive mechanisms are used to reproduce the movements along each of the degrees of freedom, the problem of scaling the generated signals arises. If you scale, based on the limit values of each of the displacements, then when moving simultaneously through several degrees of freedom, the output of the drive links to the stops is possible. At the same time, there are accelerations by unauthorized degrees of freedom (false acceleration information occurs). If we scale, based on the possibility of simultaneous movement through all degrees of freedom, then the entire available stroke of the mechanism will not be used, when moving only one degree of freedom. The contradiction is resolved, when using self-scaling: the scale of movement for each degree of freedom is selected, depending on the formed values of movements for the remaining degrees of freedom. For example, each signal is multiplied by a coefficient equal to 1, if there are no movements along the remaining degrees of freedom, and by a coefficient equal to 0.5, if there is a maximum movement along one of the degrees of freedom, etc. There is a change in the maximum allowable displacement amplitude for a given degree of freedom without distortion of the displacement shape itself [11, 12].

The main components of the acceleration effects simulator module are as follows:

- a dynamic stand with six degrees of freedom;
- control stand;
- microprocessor calculators;
- hydraulic pumping station.

Table 2.1 Performance characteristics of the dynamic stand

Parameter of movement	Range of movement	Maximum speed	Maximum acceleration
Longitudinal (X)	± 0.6 m	± 0.8 m/s	± 8 m/s ²
Vertical (Y)	± 0.5 m	$+0.7 \dots 0.8$ m/s	± 8 m/s ²
Lateral (Z)	± 0.6 m	± 0.8 m/s	± 8 m/s ²
Roll (γ)	± 24 deg	± 20 deg/s	± 100 deg/s ²
Pitch (ϑ)	± 21 deg	± 20 deg/s	± 100 deg/s ²
Yaw (ψ)	± 23 deg	± 20 deg/s	± 100 deg/s ²

The operating characteristics of the dynamic stand are determined by the increments of the lengths of the drive links, their mutual arrangement and linearity of movement; for hydraulic cylinders with a design stroke of 1100 mm, performance characteristics are given in Table 2.1.

Main parameters of the stand:

- power consumption—130 кVA,
- amount of refilled oil—500 l,
- load capacity not more than—5000 кг,
- height—2212 mm,
- fencing area— 8×8 M.

2.6 The Module of the Simulator of Atmospheric Phenomena

The module of the simulator of atmospheric phenomena is designed to solve problems of reproducing the parameters of the atmosphere and the condition of the runway. At the same time, the following are reproduced:

- the air temperature in the area of the airfield t_{aa} , depending on the height of the airfield H_{aif} , tropopause temperatures t_{tr} and the temperature at sea level t_{ls} in accordance with GOST 4401–81;
- outdoor air temperature T_{oa} , depending on flight altitude H , tropopause temperature t_{tr} and sea level temperature t_{ls} in accordance with NS 4401–81;
- the temperature of the inhibited air flow T_{ia}^* , depending on the change in outdoor air temperature T_{oa} and Max number;
- the speed of sound a , depending on the change in outdoor temperature T_{oa} according to NS 4401–81;
- pressure in the area of the airfield P_{aif} , depending on the height of the airfield H_{aif} , tropopause temperature t_{tr} , sea level temperature t_{ls} and pressure at sea level P_0 ;

- atmospheric pressure P_{ap} at altitude, depending on flight altitude H , tropopause temperature t_{tr} , temperature at sea level t_{ls} and pressure at sea level P_0 in accordance with NS 4401–81;
- the mass density of air ρ , depending on the change in atmospheric pressure of the P_{ap} and the temperature of the outdoor air T_{oa} in accordance with NS 4401–81;
- the condition of the runway (dry concrete, wet concrete and iced concrete), depending on the temperature t_{aa} in the area of the airfield and the pressure in the area of the airfield P_{aif} ;
- icing, which occurs in the presence of a temperature of inhibited air flow T_{ia}^* from -30° to $+2^\circ\text{C}$;
- the wind speed W_{xgzg} by flight altitude (determined by linear interpolation from the values at the nodal points; the wind can be set in the range from 0 to 128 m/s, with direction from 0 to 360°);
- wind shear W_{xgzg} is implemented in three profile variants, which vary in height H_{re} ; wind shear occurs at a speed of 26 m/s;
- vertical wind is realized in two profiles, varying in height H_{re} ;
- atmospheric turbulence ΔW_{hg} , ΔW_{vg} (horizontal ΔW_{hg} and vertical ΔW_{vg} gusts of wind) is reproduced, according to a random law, depending on the turbulence scale and flight altitude (it is provided the possibility to set different turbulence intensities).

The simulation is performed with continuous calculations of atmospheric parameters, based on mathematical dependencies in the basic calculator.

Mathematical modeling of atmospheric phenomena uses equations, describing the state of the atmosphere [13]:

$$t_a = t_{ls} - \frac{t_{ls} - t_{tr}}{32768} \cdot H_{if},$$

$$T_{oa} = t_{ls} - \frac{t_{ls} - t_{tr}}{11} \cdot f_1(H),$$

$$T_{ia}^* = T_{oa} \cdot (1 + 0.2m^2),$$

where t_{aa} —air temperature in the area of airfield; T_{oa} —temperature of outdoor air; T_{ia}^* —the temperature of the inhibited air flow.

The air density and atmospheric pressure are defined as

$$\rho = \frac{P_{ap}}{0.073566 \cdot R \cdot T_{oa}}, \dots P_{aif} = P_0 \left(1 + \frac{\beta_1}{T_{ls}} \cdot h \right)^{\frac{-q}{\beta_1 \cdot R}},$$

$$h_{aif} = \frac{r \cdot H_{aif}}{r + H_{aif}}, P = P_0 \left(1 + \frac{\beta_1}{T_{ls}} \cdot h \right)^{\frac{-q}{\beta_1 \cdot R}}.$$

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Chapter 3

Identification of Systems and Control Tasks in the Development of Aviation Simulators



In the development of aviation simulators, the solution of system problems and modeling problems is of particular importance.

A. G. Butkovsky formulated two general principles within the framework of the general philosophical and methodological understanding of the «Unified geometric theory of management» and the «Theory of management structures». One of them is the **«Principle (law) of 100% efficiency of mathematics»**: «For any reality (phenomenon, process, etc.) and any predetermined (but *not absolute*) accuracy, there is a mathematical structure, that *describes* this reality with this accuracy, and vice versa, for any mathematical structure and any accuracy, there is a reality, that is described by this structure with this accuracy». Identification is actually understood here as the development of an accurate *language* for describing reality and corresponding high-level *concepts* and *categories*. It is assumed, that any *effective identification methodology includes the process of human choice*. This is discussed in more detail in the analytical review of the reports, made at SICPRO'03. The importance of *creating system methodologies* is especially emphasized [1, 2].

This technique is used in the development of rational design concepts for complex technical systems, controlled in space with an extremely high order of their mathematical models, containing an element of uncertainty, including a large number of significant nonlinearities and being multi-contour, multi-connected.

With the participation of a human operator in the control process, as in the case of aircraft simulators (AS), the task is greatly complicated by the difficulties of formalizing the operator's activities for managing the object, assessing the transfer of control skills from AS to an aircraft (AC), etc.

The development of a unified design concept for complex space-controlled technical systems involves solving a complex of interrelated tasks.

Among them:

- formalization of the operator's control movements;
- determination of the parameters of the pilot's control movements as continuous, discrete, pulse processes, flow of applications, time series, etc.

- development of quality functionals that allows the operator to conduct an objective assessment of the flight characteristics of the aircraft;
- determination of the relationship between the technical parameters of the object and the quality functionals;
- development of a methodology for selecting the structure of aviation simulators in terms of the formation of the required piloting skills in the trainees;
- synthesis and adjustment of parameters of various types of AS;
- development of methods for selecting parameters of the computational cycle, taking into account the influence of time lag in simulators, dynamic characteristics of aircraft, etc.
- solving approximation problems in the preparation of databases, etc.

As already noted, the rapid increase in the cost of simulators, and the difficulties of their reconfiguration and modernization, naturally, led to the main concept of the last decade—the modularity of the design and software and mathematical support.

Here, at the first level of the hierarchy, there are complex aviation simulators (CAS), and at the second level, specialized navigator, flight engineer, etc., represent structurally finished products consisting of modules (the third level). It provides for the possibility of any combination of modules, for example, into a simulator for practicing blind flight (without simulating a visual environment); into a navigator's simulator or a flight engineer's simulator for the operator of onboard systems, etc.

The decomposition under consideration within the specified hierarchy is carried out until the elements, which are obtained at the lower level and belonging to the developed types, or the formulation of technical conditions for the creation of new necessary elements.

High requirements for the accuracy of the reproduction of AC flight characteristics on simulators impose their own characteristics when considering aircraft as objects of simulation. This is due to the development of the design and flight data of modern vehicles when models have to take into account a wide range of changes in flight speed and altitude, the increasing importance of unsteady modes, greater thrust-to-weight ratio, new structural elements, the influence of hull deformations, the influence of liquid fuel, etc. [3, 4].

The flight dynamics simulator should allow the reproduction of all stages of the AC flight (takeoff, climb, acceleration, braking, etc.) [5]. The system for calculating the characteristics of an aircraft simulator should ensure that the accuracy and adequacy of reproducing the flight characteristics of the AC are checked. At the same time, each module of the system should be a mathematical description of a separate AS system [6]. This approach makes it possible to achieve equivalence of system checks both autonomously while reducing the system adjustment time, and directly as part of the AS.

3.1 The Movement of the Aircraft in an Undisturbed Environment

An aircraft is a body of variable mass. Since the equations of motion of a body of variable mass formally coincide with the equations of motion of a body of constant mass, when considering an aircraft as an object of modeling, it is considered to be a solid body, whose position in space is determined by six coordinates. As a result, the motion of the aircraft is described by a system of differential equations, some of which are equations of dynamics of a solid body [7]. The type of equations of motion depends on the choice of coordinate systems [8–12].

When modeling the motion of AC in AS, the following coordinate systems are used:

- earth coordinate system $X_0Y_0Z_0$;
- high-speed coordinate system $X_aY_aZ_a$;
- related coordinate system XYZ .

In the unified mathematical model of motion AC, when solving a system of equations of motion by known right-hand sides, six parameters V_x, V_y, V_z and $\omega_x, \omega_y, \omega_z$ are determined. They can be used to calculate the linear and angular coordinates of AC by kinematic relations for the derivatives of the coordinates of the center of gravity, relative to the earth coordinate system. Linear coordinates X_g, Y_g, Z_g , obtained by the specified kinematic ratios, are used to reproduce the navigation situation in the simulation systems of flight navigation complexes, as well as to simulate the altitude of the flight.

The angular coordinates ϑ, γ, ψ in the simulator are used to simulate the operation of the corresponding flight and navigation devices (air horizon, course vertical, course system, etc.), automatic control systems, etc. [13].

From the expressions for the projections of the velocity vector on the axis of the associated coordinate system, it is possible to obtain relations that determine the airspeed of flight V , angles of attack α and slip β . The longitudinal overload n_x , vertical overload n_y and lateral overload n_z in projections on the axis of the associated coordinate system are found in the obvious expressions [12]

$$n_x = \frac{\sum F_x}{G}, \quad n_y = \frac{\sum F_y}{G}, \quad n_z = \frac{\sum F_z}{G}.$$

The motion parameters of AC are determined in projections on the axis of the associated coordinate system. This makes it possible to somewhat reduce the number of coordinate transformations from one system to another when constructing the computational process as a whole, and when there is a need to link the flight dynamics simulator with other systems.

3.1.1 *Decomposition of Longitudinal and Lateral Movements of the Aircraft*

As it is known, the characteristic equation of the fourth order corresponds to the longitudinal motion

$$p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = 0,$$

where a_0, a_1, a_2, a_3 are coefficients of the characteristic equation.

Consider the case of an oscillatory system, where the short-period component corresponds to a polynomial $p^2 + 2\alpha p + \delta$, and a long-period motion corresponds to a polynomial $p^2 + 2\mu p + \gamma$.

Then fair enough

$$p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = (p^2 + 2\alpha p + \delta)(p^2 + 2\mu p + \gamma).$$

In this case, the short-period and long-period components, respectively, are determined by the roots

$$\begin{aligned} p_{1,2} &= -\alpha \pm \beta_i, & \beta &= \sqrt{\delta - \alpha^2}, \\ p_{3,4} &= -\mu \pm v_i, & v &= \sqrt{\gamma - \mu^2}. \end{aligned}$$

It is fair, that

$$\begin{aligned} a_3 &= 2(\alpha + \mu), \\ a_2 &= \gamma + \delta + 4\alpha\mu, \\ a_1 &= 2(\alpha\gamma + \mu\delta), \\ a_0 &= \gamma\delta. \end{aligned}$$

Let's introduce $\tau = 4\alpha\mu$.

Then, we have

$$\gamma = a_2 - \frac{a_0}{\gamma} - \tau,$$

or

$$\gamma^2 - (a_2 - \tau)\gamma + a_0 = 0. \tag{3.1}$$

From the expressions of a_3, a_1, a_0 , it follows that

$$\mu = \frac{a_1 - a_3\gamma}{2(a_0 - \gamma^2)}\gamma.$$

From the expressions of a_3 , a_2 , it follows that

$$\begin{aligned}\alpha &= \frac{a_3}{2} - \mu, \\ \delta &= (a_2 - \tau) - \gamma.\end{aligned}$$

From the expression of Eq. (3.1), it follows that

$$\gamma_{1,2} = \frac{1}{2} \left[(a_2 - \tau) \mp \sqrt{(a_2 - \tau)^2 - 4a_0} \right].$$

Taking into account that $\gamma + \delta = a_2 - 4\alpha\mu$ should always be executed (see expression for a_2), it is enough to limit ourselves to considering one root

$$\gamma = \frac{1}{2} \left[(a_2 - \tau) - \sqrt{(a_2 - \tau)^2 - 4a_0} \right].$$

Thus, for the decomposition of longitudinal motion into short-period and long-period components, it is necessary to determine the values α , δ , μ , γ by solving a system of nonlinear equations

$$\begin{aligned}\alpha &= \frac{a_3}{2} - \mu, \\ \delta &= (a_2 - 4\alpha\mu) - \gamma, \\ \mu &= \frac{a_1 - a_3\gamma}{2(a_0 - \gamma^2)}\gamma, \\ \gamma &= \frac{1}{2} \left[(a_2 - 4\alpha\mu) - \sqrt{(a_2 - 4\alpha\mu)^2 - 4a_0} \right].\end{aligned}\tag{3.2}$$

Note, that in [13] is accepted

$$\alpha = \frac{a_3}{2}, \quad \delta = a_2, \quad \mu = \frac{1}{2} \frac{a_1}{a_2}, \quad \gamma = \frac{a_0}{a_2}.$$

The solution of the system of Eq. (3.2) can be determined by any of the known methods, for example, the method of simple iterations, the Seidel method, the Newton method, etc.

It seems appropriate to use the following method of approximate solution of the system of Eq. (3.2), taking into account the specifics of the problem being solved.

For the zero approximation of the root, we take

$$\begin{aligned}\alpha_0 &= \frac{a_3}{2} - \mu_0, \\ \delta_0 &= a_2,\end{aligned}$$

$$\mu_0 = \frac{a_1 - a_3\gamma_0}{2(\delta_0 - \gamma_0)},$$

$$\gamma_0 = \frac{a_0}{a_2}.$$

The first approximation is calculated using formulas (3.2).

If $\gamma + \frac{a_0}{\gamma} + 4\alpha\mu \neq a_2$, then the root is refined by the iteration method. The simplest refinement of the root is carried out by the method of dividing in half when the calculated γ is taken for γ_1 , and the calculated such γ is taken as γ_2 that the inequality $\gamma_2 + \frac{a_0}{\gamma_2} + 4\alpha\mu \neq a_2$ will be opposite in sign than at γ_1 . At the same time, α , μ are calculated for the corresponding values γ .

Calculations were stopped when the specified accuracy of equality

$$\gamma + \frac{a_0}{\gamma} + 4\alpha\mu = a_2.$$

is obtained.

We give sufficient conditions for the absence of real roots in the characteristic equation

$$\Delta(P) = p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = 0.$$

It is assumed that $a_i > 0$, $i = \overline{0,3}$.

The theorem is valid:

if one of the conditions is fulfilled:

- (1) $a_0 > a_1$, $a_2 > \frac{a_3^2 + a_1}{4}$;
- (2) $a_0 > a_1$, $a_2 > \max\left\{\frac{3}{8}a_3^2, \frac{a_3^3 + 3a_0^2a_3}{2}\right\}$.

Then equation has no real roots, i.e.

$$p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 > 0 \text{ for } p \in (-\infty, \infty).$$

Indeed, inequality $a_2 > \frac{a_3^2 + a_1}{4}$ is equivalent to inequality $p^2 + a_3p + a_2 > \frac{a_1}{4}$ for all $p \in (-\infty, \infty)$, which is easy to see. Further, from $a_0 > a_1$, it follows that $a_1p + a_0 > a_1p + a_1$.

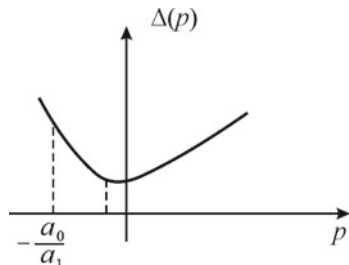
Let us now write down an obvious chain of inequalities

$$\begin{aligned} p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 &> p^2(p^2 + a_3p + a_2) + a_1p + a_1 > \\ &> \frac{a_1p^2}{4} + a_1p + a_1 = p^2(p+2)^2\frac{a_1}{4} \geq 0. \end{aligned}$$

Thus, if condition 1 is satisfied, then the statement of the theorem is proved.

Let condition 2 be fulfilled. Write the polynomial in the form

Fig. 3.1 Type of dependency $\Delta(P)$



$$p^2(p^2 + a_3p + a_2) + a_1p + a_0.$$

From $a_2 > \frac{3}{8}a_3^2$, it follows that $p^2 + a_3p + a_2 > 0$ for $p \in (-\infty, \infty)$.

Further, it is obvious that for $p \geq -\frac{a_0}{a_1}$ have $p^2(p^2 + a_3p + a_2) + a_1p + a_0 > 0$.

From $a_2 > \frac{3}{8}a_3^2$, it follows that $\Delta''(p) = 12p^2 + 6a_3p + 2a_2 > 0$.

We show that $\frac{\Delta'(P)}{-\frac{a_0}{a_1}} = \Delta'\left(-\frac{a_0}{a_1}\right) < 0$.

Really, $\Delta'\left(-\frac{a_0}{a_1}\right) = a_1 - \frac{a_0}{a_1}\left(4\frac{a_0^2}{a_1^2} - 3a_3\frac{a_0}{a_1} + 2a_2\right)$.

It is easy to directly verify that from the condition $a_2 > \frac{(a_1^3 + 3a_0^2a_3)}{2}$, it follows that $\Delta'\left(-\frac{a_0}{a_1}\right) < 0$.

Thus, the minimum $\Delta(P)$ is to the right of the point $p = -\frac{a_0}{a_1}$.

But it was shown above that when $P > -\frac{a_0}{a_1}$, then $\Delta(P) > 0$ (Fig. 3.1.), that was, what needed to be proved.

Note: from the fact that $a_3 > 0$, it follows that at least one of the two pairs of complex-conjugate roots has a negative real part.

Let p_i —roots of equation

$$p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = 0,$$

$$p_{1,2} = -\alpha \pm \beta i, \quad \delta = \alpha^2 + \beta^2, \quad p_{3,4} = -\mu \pm \vartheta i, \quad \gamma = \mu^2 + \vartheta^2.$$

Then

$$p^4 + a_3p^3 + a_2p^2 + a_1p + a_0 = (p^2 + 2\alpha p + \delta) \cdot (p^2 + 2\mu p + \gamma). \quad (3.3)$$

From here

$$a_3 = 2(\alpha + \mu), \quad a_2 = \gamma + \delta + 4\alpha\mu, \quad a_1 = 2(\alpha\gamma + \mu\delta), \quad a_0 = \gamma\delta. \quad (3.4)$$

Therefore, the positivity of a_3 corresponds to the fact that either α , or μ (more precisely, the largest of them by module) is positive, or both α and μ are positive.

Next, we will study the issues of motion stability.

Let the coefficients of the characteristic equation satisfy the conditions

$$a_i > 0, \quad a_3 a_2 > a_1, \quad a_1 < a_2, \quad a_2^2 > a_0,$$

and the decomposition is made according to Eq. (3.3).

From the dependencies (3.4), it follows that

$$a_2 = \gamma + \frac{a_0}{\gamma} + 4\alpha\mu$$

or

$$a_2\gamma = a_0 + \gamma^2 + 4\alpha\mu\gamma.$$

Since γ is small for a long-period movement, then $\gamma \approx \frac{a_0}{a_2}$.

From the root calculation algorithm, we have

$$\mu = \frac{a_1 - a_3\gamma}{2\left(\frac{a_0}{\gamma} - \gamma\right)} \approx \frac{a_1 a_2 - a_3 a_0}{2(a_2^2 - a_0)},$$

from where follows $\mu > 0$, if and only if, when $a_1 a_2 > a_0 a_3$.

Thus, the condition $a_1 a_2 > a_0 a_3$ can be considered the main condition for the stability of long-period motion, since the remaining conditions

$$a_i > 0, \quad a_3 a_2 > a_1, \quad a_2^2 > a_0$$

for the characteristic equation of longitudinal motion are almost always fulfilled. Finally, we note that it is possible to specify other decomposition methods which are also based on the approximate calculation of the roots of the characteristic equation.

For example, let, as above, $a_i > 0$, and also fulfilled

$$a_3 a_2 > a_1, \quad a_1 < a_2.$$

Let the condition of stability of long-period motion be fulfilled

$$a_1 a_2 > a_0 a_3.$$

Then, let's put

$$\mu = \frac{a_1}{2a_2}, \quad \alpha = \frac{a_3}{2} - \frac{a_1}{2a_2}, \quad \delta = \frac{a_0}{\gamma},$$

$$\tau = \frac{a_1 a_3}{a_2} - \frac{a_1^2}{a_2^2}, \quad \gamma = \frac{1}{2} \left(a_2 - \tau - \sqrt{(a_2 - \tau)^2 - 4a_0} \right)$$

and we write the decomposition in the form (3.3).

If the stability condition of the long-period motion is not met, i.e. $a_1 a_2 < a_0 a_3$, then let's put

$$\mu = \frac{-a_1}{2a_2}, \quad \alpha = \frac{a_3}{2} + \frac{a_1}{2a_2}, \quad \delta = \frac{a_0}{\gamma},$$

$$\tau = -\left(\frac{a_1a_3}{a_2} + \frac{a_1^2}{a_2^2}\right), \quad \gamma = \frac{1}{2}\left(a_2 - \tau - \sqrt{(a_2 - \tau)^2 - 4a_0}\right)$$

and we will write down the decomposition in the form (3.3) in the same way as above.

The following theorems on the existence of real roots of the polynomial turned out to be very useful for the study of longitudinal and lateral movements, especially for predicting the results of AS tuning

$$P_4(x) = x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$

3.1.1.1 Real Roots of the Polynomial

Theorem 3.1 *Let the coefficients $P_4(x)$ be non-negative, and $a_0 > a_1$, $4a_2 \geq a_1 + a_3^2$.*

Then the polynomial has no real roots.

Indeed, inequality $4a_2 \geq a_1 + a_3^2$ is equivalent to the inequality $x^2 + a_3x + a_2 \geq \frac{a_1}{4}$ for all x (it is easy to verify this by calculating the discriminant of the equation $x^2 + a_3x + a_2 = \frac{a_1}{4}$).

Further, the following chain of obvious inequalities leads to inequality that is equivalent to the statement of the theorem. Really

$$P_4(x) = x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = x^2(x^2 + a_3x + a_2) + a_1x + a_0 \geq$$

$$\geq \frac{a_1}{4}x^2 + a_1x + a_0 > \frac{a_1}{4}(x^2 + 4x + 4) > 0,$$

that was, what needed to be proved.

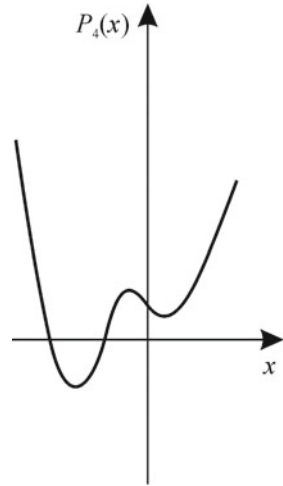
Theorem 3.2 *Let the inequality $3a_3^2 > 8a_1$ be satisfied for the coefficients of the polynomial $P_4(x)$, and x_1, x_2, x_3 —real distinct (in ascending order) roots of the polynomial*

$$x^3 + 0.75a_3x^2 + 0.5a_2x + 0.25a_1.$$

Then:

- (1) *if $P_4(x_1) > 0$ and $P_4(x_3) > 0$, then the polynomial $P_4(x)$ has no real roots;*
- (2) *if $P_4(x_2) < 0$, then $P_4(x)$ has two different real roots;*
- (3) *if $P_4(x_1) \leq 0$, $P_4(x_2) \geq 0$, $P_4(x_3) \leq 0$, then $P_4(x)$ has four real roots (possibly multiples, if at least one of the inequalities is non-strict).*

Fig. 3.2 Dependencies $P_4(x)$, when $3a_3^2 > 8a_2$



Theorem 3.3 Let the inequality $3a_3^2 \leq 8a_2$ be satisfied, and x_1 —the real root of the polynomial

$$x^3 + 0.75a_3x^2 + 0.5a_2x + 0.25a_1$$

(in this case, the real root is the only one, possibly a multiple one).

Then:

- (1) if $P_4(x_1) \leq 0$, then $P_4(x)$ has either two different real roots or one multiple;
- (2) if $P_4(x_1) > 0$, then $P_4(x)$ has no real roots.

The proofs of theorems 2 and 3 are not difficult if we refer to the graphs in Figs. 3.2, 3.3 and 3.4.

Remarks

1. There are three fundamentally different graphs shown in Figs. 3.2, 3.3 and 3.4. However, the graphs shown in Figs. 3.2 and 3.3 should be attributed to the same type, determined by the condition $3a_3^2 > 8a_2$ that corresponds to the presence of two different inflection points. However, by the number of extremes (by the number of real distinct roots of the polynomial $P'_4(x)$), they are significantly different.

The graph, shown in Fig. 3.4, corresponds to the condition $3a_3^2 \leq 8a_2$ (there is no concavity interval). There can be either three internal extremes (two minimums, one maximum) or one (minimum).

2. By making a variable replacement, assuming $x = x + \frac{a_3}{4}$, we get

$$P_4(x) = x^4 + px^2 + qx + z,$$

Fig. 3.3 Dependencies $P_4(x)$, when $3a_3^2 > 8a_2$

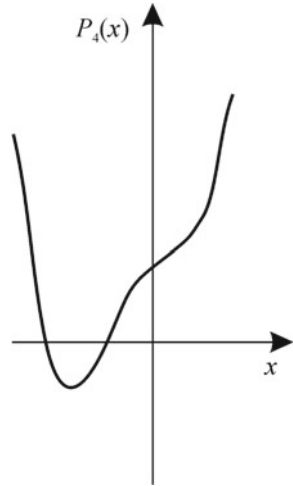
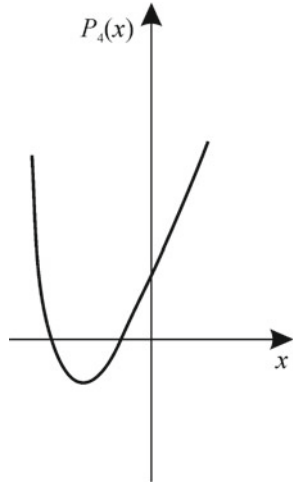


Fig. 3.4 Dependencies $P_4(x)$, when $3a_3^2 \leq 8a_2$



where $p = a_2 - \frac{3}{8}a_3^2$, $q = a_1 - \frac{a_1a_3}{2} + \frac{a_3^3}{8}$, $z = a_0 - \frac{a_1a_3}{4} + \frac{a_2a_3^2}{16} - \frac{3a_3^4}{256}$.

In this case $P'_4(x) = 4x^3 + 2px + q$, finding (approximate) roots $P'_4(x) = 0$ is easy to implement graphically as the intersection points of the curve $y = x^3$ and a straight line $y = -\frac{p}{2}x - \frac{q}{4}$.

By finding the roots of the polynomial and using theorems 2 and 3, we get information about the roots of the polynomial $P_4(x)$.

3.1.1.2 Representations of the Polynomial

The polynomial $P_4(x)$ can be written in various ways.

Here are some of them, the simplest and most useful for the tasks under consideration.

Let's use the Taylor–Maclaurin formula and write down the polynomial in degrees of the binomial $x - \alpha$

$$P_4(x) = p_4(\alpha) + p_4'(\alpha)(x - \alpha) + \frac{1}{2}p_4''(\alpha)(x - \alpha)^2 + \frac{1}{6}p_4'''(\alpha)(x - \alpha)^3 + (x - \alpha)^4.$$

Choose α from the condition $p_4'''(\alpha) = 0$, and we get $\alpha = -\frac{a_3}{4}$.

In this case, the view will take the form

$$P_4(z) = z^4 + pz^2 + qz + r, \quad (3.5)$$

$$z = x + \frac{a_3}{4},$$

where $p = a_2 - \frac{3}{8}a_3^2$, $q = a_1 - \frac{a_2a_3}{2} + \frac{a_3^3}{8}$, $r = a_0 - \frac{a_1a_3}{4} + \frac{a_2a_3^2}{16} - \frac{3a_3^4}{256}$.

In some important special cases, the expression $P_4(z)$ is simplified by vanishing some of the coefficients p , q , r . However, the representation $P_4(z)$ is already very convenient, for example, for the graphical calculation of real roots (or establishing the fact of their absence).

Indeed, by constructing a curve $y = z^4 + pz^2 + r$ and straight line $y = -qz$ on plane zOy , we find the abscissa of their intersection points—these will be real roots (Fig. 3.5), and if the curve and the straight line do not intersect, then there are no real roots (Fig. 3.6).

The polynomial can be transformed by selecting the full square from $p z^2 + qz$, after which, we get

$$z^4 + p\left(z + \frac{q}{2p}\right)^2 + r - \frac{q^2}{4p}, \quad \text{when } p \neq 0. \quad (3.6)$$

In particular, it is easy to obtain various criteria about the presence or absence of real roots from the representation (3.6).

The original polynomial $P_4(x)$, in this case, can be represented as

$$P_4(x) = \left(x + \frac{a_3}{4}\right)^4 + p(x + \beta)^2 + S, \quad (3.7)$$

$$p = a_2 - \frac{3}{8}a_3^2, \quad \beta = \frac{a_3}{4} + \frac{q}{2p}, \quad S = r - \frac{q^2}{4p},$$

where $q = a_1 - \frac{a_2a_3}{2} + \frac{a_3^3}{8}$, $r = a_0 - \frac{a_1a_3}{4} + \frac{a_2a_3^2}{16} - \frac{3a_3^4}{256}$.

Fig. 3.5 Illustration of graphoanalytic definition of the roots of a polynomial $P_4(z)$ ($p, q < 0, r > 0$)

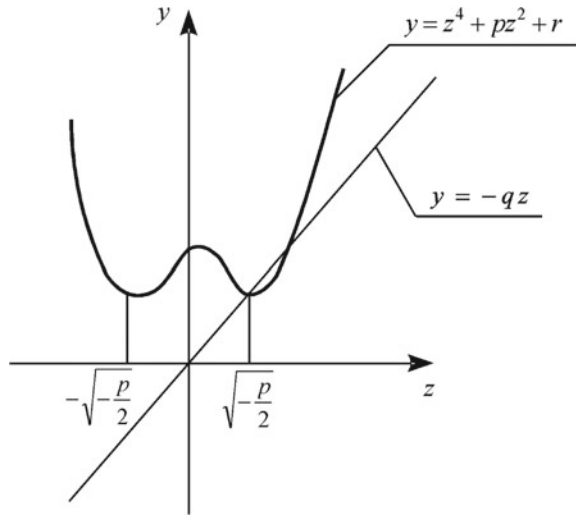
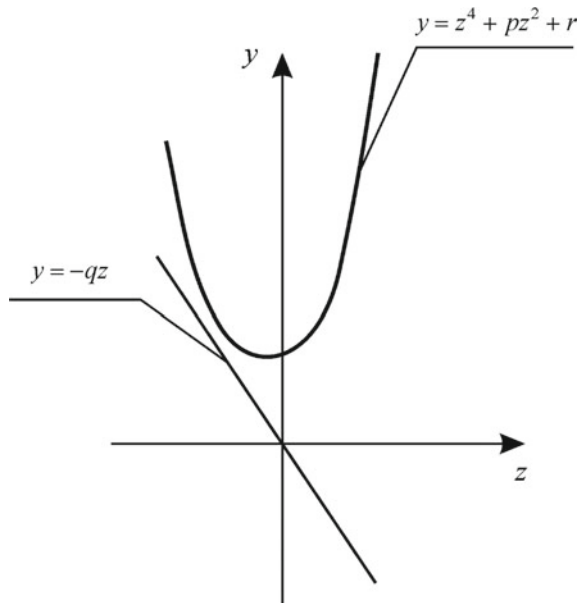


Fig. 3.6. Illustration of graphoanalytic definition of the roots of a polynomial $P_4(z)$ ($p, q, r > 0$)



3.1.1.3 Decomposition of the Characteristic Polynomial

Based on the representation (3.5), we will look for the decomposition in the form

$$z^4 + pz^2 + qz + r = (z^2 + bz + \gamma) \left(z^2 - bz + \frac{r}{\gamma} \right). \quad (3.8)$$

At the same time, we assume that $q \neq 0$. Otherwise, it is easy to find all four roots and write the expression (3.8). This obvious fact does not require detailed discussion, as and the case $r = 0$.

Comparing the coefficients of the polynomials, standing on the left and right in expression (3.8), we obtain the relations (a system of equations for determining b and γ)

$$\begin{aligned} b &= q \frac{\gamma}{r - \gamma^2}, \\ p + q^2 \frac{\gamma^2}{(r - \gamma^2)^2} &= \gamma + \frac{r}{\gamma}. \end{aligned} \quad (3.9)$$

In order to obtain an algorithm for solving the second equation from the system of Eq. (3.9) (algorithm of calculation γ), consider graphs of functions

$$\begin{aligned} f_1(\gamma) &= \gamma + \frac{r}{\gamma}, \\ f_2(\gamma) &= p + q^2 \frac{\gamma^2}{(r - \gamma^2)^2}. \end{aligned}$$

Consider the cases:

- (1) $r > 0$;
- (2) $r < 0$.

The study of functions presents no difficulties. Here are just the basic facts:

1. $f_1'(\gamma) = 1 - \frac{r}{\gamma^2}$, $f_1''(\gamma) = \frac{2r}{\gamma^3}$,

$$f_2'(\gamma) = q^2 \frac{2\gamma(\gamma^2 + r)}{(r - \gamma^2)^3}, \quad f_2''(\gamma) = q^2 \frac{6(r + \gamma^2)^2 - 2r(r - \gamma^2)}{(r - \gamma^2)^4}.$$

2. $f_1(\gamma)$ has two asymptotes: vertical $\gamma = 0$ and inclined $y = \gamma$. $f_2(\gamma)$, when $r > 0$, there are vertical $\gamma = \pm\sqrt{r}$ and horizontal $y = p$ asymptotes, and when $r < 0$, there is only a horizontal asymptote $y = p$.
3. $f_1(\gamma)$ has a maximum at $\gamma = \sqrt{r}$, is equal $2\sqrt{r}$, and has a minimum at $\gamma = -\sqrt{r}$, is equal $-2\sqrt{r}$, ($r > 0$). $f_2(\gamma)$ has a maximum at $\gamma = \pm\sqrt{-r}$, is equal $p - \frac{q^2}{4r}$ (when $r < 0$).

3.1.1.4 Approximate Graphoanalytical Calculation γ

The coefficient γ , as shown above, is the root of the equation $f_1(\gamma) = f_2(\gamma)$ (it is enough to find a value γ , smaller in modulus).

We will search γ in the form

$$\begin{aligned}
\gamma &= \alpha\sqrt{r}, \text{ if } r > 0, \quad (0 < \alpha < 1), \\
\gamma &= \alpha\sqrt{-r}, \text{ if } r < 0 \text{ and } p < \frac{q^2}{4r}, \quad (0 < \alpha < 1), \\
\gamma &= -\alpha\sqrt{-r}, \text{ if } r < 0 \text{ and } p > \frac{q^2}{4r}, \quad (0 < \alpha < 1).
\end{aligned} \tag{3.10}$$

(In the case, if $r < 0$ and $p = \frac{q^2}{4r}$, $\gamma = \pm\sqrt{-r}$, it is not difficult to verify this by looking at the graph in Fig. 3.8 or by checking directly).

Let's introduce the functions into consideration

$$\begin{aligned}
g_1(\alpha) &= \frac{1}{\sqrt{\pm r}} f_1(\pm\alpha\sqrt{\pm r}), \\
g_2(\alpha) &= \frac{1}{\sqrt{\pm r}} f_2(\pm\alpha\sqrt{\pm r}),
\end{aligned}$$

where the signs are selected as indicated in (3.10), and $\alpha \in (0, 1)$.

When $r > 0$, we get

$$g_1(\alpha) = \frac{1 + \alpha^2}{\alpha}, \quad g_2(\alpha) = p_r + q_r \frac{\alpha^2}{(1 - \alpha^2)^2}, \tag{3.11}$$

here $p_r = \frac{p}{\sqrt{r}}$, $q_r = \frac{q^2}{r\sqrt{r}}$.

When $r < 0$ and $p_r < \frac{q_r}{4}$ ($p_r = \frac{p}{\sqrt{-r}}$, $q_r = \frac{q^2}{r\sqrt{-r}}$, $q_r < 0$)

$$g_1(\alpha) = \frac{\alpha^2 - 1}{\alpha}, \quad g_2(\alpha) = p_r + q_r \frac{\alpha^2}{(1 + \alpha^2)^2}. \tag{3.12}$$

When $r < 0$ and $p_r > \frac{q_r}{4}$ ($p_r = \frac{p}{\sqrt{-r}}$, $q_r = \frac{q^2}{r\sqrt{-r}}$, $q_r < 0$)

$$g_1(\alpha) = \frac{1 - \alpha^2}{\alpha}, \quad g_2(\alpha) = p_r + q_r \frac{\alpha^2}{(1 + \alpha^2)^2}. \tag{3.13}$$

Graphs of functions $g_1(\alpha)$ and $g_2(\alpha)$ are presented in Figs. 3.7, 3.8 and 3.9 for cases (3.11–3.13), respectively.

Remarks

1. When solving the equation $g_1(\alpha) = g_2(\alpha)$ (when $r > 0$), it is useful to have a table of function values.
2. The equation $g_1(\alpha) = g_2(\alpha)$ can be simplified by dividing both parts into q_r , that is, replacing with the equation

Fig. 3.7 Functions $g_1(\alpha)$ and $g_2(\alpha)$, when $r > 0$

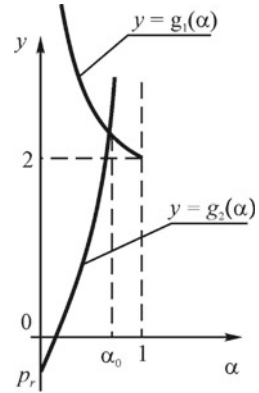


Fig. 3.8 Functions $g_1(\alpha)$ and $g_2(\alpha)$, when $r < 0$ and $p_r < \frac{q_r}{4}$

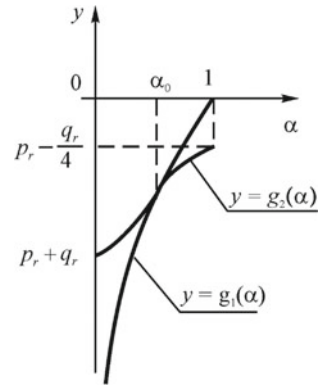
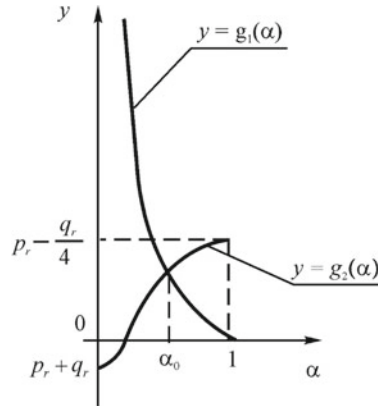


Fig. 3.9 Functions $g_1(\alpha)$ and $g_2(\alpha)$, when $r < 0$ and $p_r > \frac{q_r}{4}$



$$AS_1(\alpha) = B + S_2(\alpha), \text{ where } A = \frac{1}{q_r}, \quad B = \frac{p_r}{q_r}. \quad (3.14)$$

In this case, it is not difficult to calculate the root verbally, using a table of values for $S_1(\alpha)$ and $S_2(\alpha)$.

3. Similar tables of value for functions $\sigma_1(\alpha) = \alpha - \frac{1}{\alpha}$ and $\sigma_2(\alpha) = \left(\frac{\alpha}{1+\alpha^2}\right)^2$ useful to have, when calculating γ , in the case $r < 0$.

Algorithm of calculation γ .

1. Construct graphs $y = g_1(\alpha)$, $y = g_2(\alpha)$, specifying the values of the functions, when $\alpha = 0.2; 0.5; 0.7$.
2. Determine the abscissa α_0 of the intersection of graphs.
3. Define γ_0 in accordance with the expressions (3.10), i.e.

$$\begin{aligned} \gamma_0 &= \alpha_0\sqrt{r}, \text{ if } r > 0, \\ \gamma_0 &= \alpha_0\sqrt{-r}, \text{ if } r < 0, \quad p_r < \frac{q_r}{4}, \\ \vartheta_0 &= -\alpha_0\sqrt{-r}, \text{ if } r < 0, \quad p_r > \frac{q_r}{4}. \end{aligned}$$

Clarification γ can be done in many ways. In particular, linear or quadratic approximations of the function

$$f(\gamma) = f_2(\gamma) - f_1(\gamma) \text{ in the vicinity of the point } \gamma_0 \text{ can be used.}$$

The corresponding formulas are given below.

Using the Taylor–Maclaurin formula, a quadratic approximation of a function $f(\gamma)$ in the vicinity of a point γ_0 can be represented as

$$f(\gamma) \approx f(\gamma_0) + f'(\gamma_0)(\gamma - \gamma_0) + \frac{f''(\gamma_0)}{2}(\gamma - \gamma_0)^2 \quad (3.15)$$

(the first two terms of formula (3.15) give, respectively, a linear approximation).

$$\text{For function } f(\gamma) = f_2(\gamma) - f_1(\gamma) = p + q^2 \frac{\gamma^2}{(r-\gamma^2)^2} - \left(\gamma + \frac{r}{\gamma}\right).$$

we have

$$\begin{aligned} f(\gamma) &= p + q^2 \frac{\gamma_0^2}{(r - \gamma_0^2)^2} - \frac{\gamma_0^2 + r}{\gamma_0} + \left(2q^2 \frac{\gamma_0(\gamma_0^2 + r)}{(r - \gamma_0^2)^3} - \frac{r - \gamma_0^2}{\gamma_0^2} \right) (\gamma - \gamma_0) \\ &+ \left(q^2 \frac{3(r + \gamma_0^2)^2 - 2r(r - \gamma_0^2)}{(r - \gamma_0^2)^4} - \frac{r}{\gamma_0^3} \right) (\gamma - \gamma_0)^3. \end{aligned} \quad (3.16)$$

When correction γ by the partial linear approximation method, we leave the function $f_1(\gamma) = \gamma + \frac{r}{\gamma}$ unchanged (due to its simplicity), and replace the function $f_2(\gamma)$ with a linear approximation. We obtain a quadratic equation for determining γ

$$p + q^2 \frac{\gamma_0^2}{(r - \gamma_0^2)^2} + 2q^2 \frac{\gamma_0(\gamma_0^2 + r)}{(r - \gamma_0^2)^3} (\gamma - \gamma_0) = \gamma + \frac{r}{\gamma}. \quad (3.17)$$

To illustrate, consider AC, whose characteristics are taken in accordance with table 2 [13] for the case M = 0.9; H = 12 km.

First, consider the longitudinal motion of an aircraft with a characteristic polynomial

$$x^4 + 1.88x^3 + 8.53x^2 + 3.68x + 3.6.$$

By replacing the variable $z = x + 0.47$, we get a polynomial

$$z^4 + 7.22z^2 - 3.507z + 3.604$$

$$(p = 7.22; \quad q = 3.507; \quad z = 3.604; \quad p_r = \frac{p}{\sqrt{r}} = 3.8; \quad q_r = \frac{q^2}{r\sqrt{r}} = 1.8).$$

The polynomial has no real roots.

The values of functions

$$g_1(\alpha) = \alpha + \frac{1}{\alpha}, \quad g_2(\alpha) = 3.8 + 1.8 \left(\frac{\alpha}{1 - \alpha^2} \right)^2$$

are given in Table 3.1.

Thus, already without plotting $g_1(\alpha)$ and $g_1(\alpha)$ with accuracy of 0.002, it is possible to accept $\alpha_0 = 0.275$.

From here

$$\gamma_0 = \alpha_0 \sqrt{r} = 0.275 \sqrt{3.604} = 0.5225;$$

$$b_0 = \frac{q\gamma_0}{(r - \gamma_0^2)} = \frac{3.507 \cdot 0.5225}{3.604 - 0.5225^2} = -0.535.$$

Therefore

$$z^4 + 7.22z^2 - 3.507z + 3.604 = (z^2 - 0.535z + 0.5225)(z^2 + 0.535z + 6.917),$$

Table 3.1 The values of functions $g_1(\alpha)$ and $g_2(\alpha)$

α	$g_1(\alpha)$	$g_2(\alpha)$
0	–	3.8
0.2	5.2	3.88
0.3	3.63	4.01
0.27	3.97	3.95

$$z_{1,2} = 0.267 \pm \sqrt{0.0713 - 0.5225} = 0.267 \pm 0.67j,$$

$$z_{3,4} = -0.267 \pm \sqrt{0.0713 - 6.917} = -0.267 \pm 2.62j,$$

$$x_{1,2} = z_{1,2} - 0.47 = -0.203 \pm 0.67j,$$

$$x_{3,4} = z_{3,4} - 0.47 = -0.737 \pm 2.62j.$$

Next, consider *the lateral movement of AC*. Having calculated the coefficients of the characteristic polynomial, according to Table 3.1 [13], we obtain

$$x^4 + 1.48x^3 - 6.46x^2 - 7.7x + 0.087$$

or

$$z^4 - 7.26z^2 - 2.56z + 2.04, \quad z = x + 0.37, \quad \left(z = x + \frac{a_3}{4} \right).$$

By plotting graphs of function $y = 2.56z$, $y = z^4 - 7.26z^2 + 2.04$, we make sure that all the roots of the polynomial are real (Fig. 3.10).

$$z_1 = -2.41, \quad z_2 = -0.781, \quad z_3 = 0.381, \quad z_4 = 2.81.$$

Accordingly

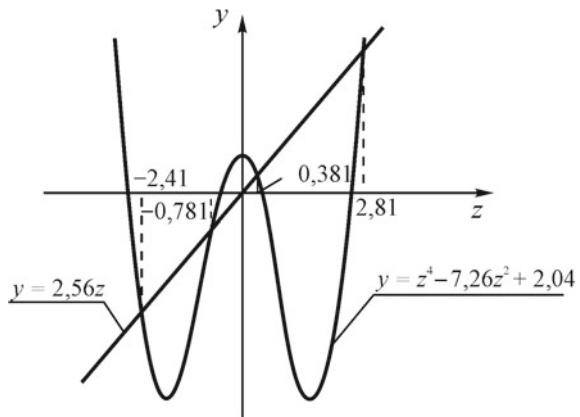
$$x_1 = -2.78, \quad x_2 = -1.151, \quad x_3 = 0.011, \quad x_4 = 2.44.$$

In the equation

$$AS_1(\alpha) = B + S_2(\alpha)$$

we have

Fig. 3.10 Illustration of the decomposition of the lateral movement of the aircraft



$$A = \frac{r\sqrt{r}}{q_r} = 0.445, \quad B = \frac{pr}{q_r} = -2.26.$$

Using the table of values $S_1(\alpha)$ and $S_2(\alpha)$, we find the solution of the equation $\alpha = 0.76$, accordingly, $\gamma = 1.088$, $b = -3.26$.

The decomposition of the polynomial is represented as

$$z^4 - 7,26z^2 - 2,56z + 2.04 = (z^2 - 3,26z + 1.088)(z^2 + 3,26z + 1,875).$$

Solving quadratic equations with an accuracy of 10^{-2} , we get the same roots z_i and x_i , respectively.

Notes

1. The decomposition of the considered polynomial can be carried out in principle, using the well-known Ferrari formulas for solving equations of the fourth degree. However, due to the complexity of the transformations, used in this case, the use of this algorithm causes significant difficulties.
2. Consideration of stability conditions is possible by using the Rauss–Hurwitz criterion, as well as the frequency method or other methods.

However, the algorithms, proposed above, turned out to be simpler in practical implementation.

3.2 Mathematical Modeling of the Motion of an Aircraft, Taking into Account the Influence of Wind

To reproduce the motion of an aircraft in a disturbing atmosphere, the components of the wind influence are introduced into the equations [14]. The wind is set in the earth coordinate system and can have any direction in the vertical and horizontal planes. When composing equations with the influence of wind, one should take into account the fact that inertial forces depend on the ground speed V_k and acceleration, and aerodynamic forces depend on air velocity V . Therefore, in the system, taking into account the influence of the wind, as a result of the solution, we obtain a ground speed V_k , and the equations for determining the angle of attack and slip should be supplemented with additions for the angles of attack and slip.

3.3 Mathematical Model of Aircraft Flight

The mathematical model of an airplane flight does not depend on the AC design. The flight parameters are determined in the following sequence:

$$\left. \begin{aligned} M_x^* &= M_x - [(J_z - J_y) \omega_y + J_{xy} \omega_x] \omega_z \\ M_y^* &= M_y - [(J_x - J_z) \omega_x - J_{xy} \omega_y] \omega_z \end{aligned} \right\}, \quad (3.18)$$

$$\left. \begin{aligned} \dot{\omega}_x &= \frac{1}{J_x \left(1 - \frac{J_{xy}^2}{J_x J_y}\right)} \left(M_x^* + \frac{J_{xy}}{J_y} M_y^* \right) \\ \dot{\omega}_y &= \frac{1}{J_y \left(1 - \frac{J_{xy}^2}{J_x J_y}\right)} \left(M_y^* + \frac{J_{xy}}{J_x} M_x^* \right) \\ \dot{\omega}_z &= \frac{1}{J_z} [J_{xy} (\omega_x^2 - \omega_y^2) - (J_y - J_x) \omega_x \omega_y + M_z] \end{aligned} \right\}, \quad (3.19)$$

$$\left. \begin{aligned} \omega_x &= \int \dot{\omega}_x dt \\ \omega_y &= \int \dot{\omega}_y dt \\ \omega_z &= \int \dot{\omega}_z dt \end{aligned} \right\}, \quad (3.20)$$

$$\left. \begin{aligned} \dot{\vartheta} &= \omega_y \sin \gamma + \omega_z \cos \gamma - A_t(\delta) \\ \dot{\psi} &= (\omega_x - \dot{\gamma}) \sin \vartheta + (\omega_y \cos \gamma - \omega_z \sin \gamma) \cos \vartheta \\ \dot{\gamma} &= \omega_x - \dot{\psi} \sin \vartheta + B_t(\delta) \end{aligned} \right\}, \quad (3.21)$$

$$\left. \begin{aligned} \vartheta &= \int \dot{\vartheta} dt \\ \psi &= \int \dot{\psi} dt \\ \gamma &= \int \dot{\gamma} dt \end{aligned} \right\}, \quad (3.22)$$

$$\left. \begin{aligned} i_x &= \cos \psi \cos \vartheta \\ i_y &= \sin \vartheta \\ i_z &= -\sin \psi \cos \vartheta \end{aligned} \right\}, \quad (3.23)$$

$$\left. \begin{aligned} j_\xi &= -\cos \psi \cos \gamma \sin \vartheta + \sin \psi \sin \gamma \\ j_y &= \cos \vartheta \cos \gamma \\ j_z &= \cos \psi \sin \gamma + \sin \psi \sin \vartheta \cos \gamma \end{aligned} \right\}, \quad (3.24)$$

$$\left. \begin{aligned} k_x &= \cos \psi \sin \vartheta \\ k_y &= -\cos \vartheta \sin \gamma \\ k_z &= \cos \psi \cos \gamma - \sin \psi \sin \vartheta \sin \gamma \end{aligned} \right\}, \quad (3.25)$$

$$\left. \begin{aligned} X &= c_x \cdot S \cdot q \\ Y &= c_y \cdot S \cdot q \\ Z &= c_z \cdot S \cdot q \end{aligned} \right\}, \quad (3.26)$$

$$\left. \begin{aligned} n_x &= \frac{1}{G} (P_x - X - X_{ch}) \\ n_y &= \frac{1}{G} (P_y + Y + Y_{ch}) \\ n_z &= \frac{1}{G} (P_z + Z + Z_{ch}) \end{aligned} \right\}, \quad (3.27)$$

$$\left. \begin{aligned} \dot{V}_{kx} &= g(n_x - i_y) - V_{kz}\omega_y + V_{ky}\omega_z \\ \dot{V}_{ky} &= g(n_y - j_y) - V_{kx}\omega_z + V_{kz}\omega_x \\ \dot{V}_{kz} &= g(n_z - k_y) - V_{ky}\omega_x + V_{kx}\omega_y \end{aligned} \right\}, \quad (3.28)$$

$$\left. \begin{aligned} V_{kx} &= \int \dot{V}_{kx} dt \\ V_{ky} &= \int \dot{V}_{ky} dt \\ V_{kz} &= \int \dot{V}_{kz} dt \\ V_{kyg} &= V_{kx} i_y + V_{ky} j_y + V_{kz} k_y \end{aligned} \right\}, \quad (3.29)$$

$$\left. \begin{aligned} H &= H_m + \int_0^t V_{kyg} dt \\ H_{tr} &= H - H_M \end{aligned} \right\}, \quad (3.30)$$

$$\Delta\Psi_W^\circ = \Psi_W^\circ - TC + 180^\circ, \quad (3.31)$$

where Ψ_W° —meteorological direction of wind; TC—the true course of the aircraft

$$\left. \begin{aligned} W_{xgzg} &= W_{xgzg_0} + W_{xgzgws} \\ W_{yg} &= W_{yg_0} + W_{vg} \\ W_x &= (W_{xgzg_0} \cos \Delta\Psi_W + W_{hg}) \cos \vartheta + W_{yg} \sin \vartheta \\ W_y &= (W_{xgzg} \sin \Delta\Psi_W + W_{hg}) \sin \gamma - (W_{xgzg} \cos \Delta\Psi_W + W_{hg}) \\ &\quad \times \sin \vartheta \cos \gamma + W_{yg} \cos \vartheta \cos \gamma \\ W_z &= (W_{xgzg} \sin \Delta\Psi_W + W_{hg}) \cos \gamma + (W_{xgzg} \cos \Delta\Psi_W + W_{hg}) \\ &\quad \times \sin \vartheta \sin \gamma - W_{yg} \cos \vartheta \sin \gamma \end{aligned} \right\}, \quad (3.32)$$

$$\left. \begin{aligned} V_x &= V_{kx} - W_x \\ V_y &= V_{ky} - W_y \\ V_z &= V_{kz} - W_z \\ V &= \sqrt{V_x^2 + V_y^2 + V_z^2} \\ M &= \frac{V}{a} \end{aligned} \right\}, \quad (3.33)$$

$$\left. \begin{aligned} \alpha_f &= \arctg\left(-\frac{V_y}{V_x}\right), \quad \alpha = \alpha_f + \alpha_{est}, \quad \alpha_n \\ \beta &= \arcsin\left(\frac{V_z}{V}\right), \quad \beta_n \end{aligned} \right\}, \quad (3.34)$$

$$q = \frac{\rho V^2}{2}, \quad (3.35)$$

$$\left. \begin{aligned} \dot{\alpha} &= \frac{\alpha_n - \alpha_{n-1}}{t_c}, \quad \alpha_{n-1} \\ \dot{\beta} &= \frac{\beta_n - \beta_{n-1}}{t_c}, \quad \beta_{n-1} \end{aligned} \right\}. \quad (3.36)$$

In Eq. (3.21), the parameters $A_r(\delta)$ and $B_r(\delta)$ are introduced in order to reduce the reaction time to the actions of the controls in the pitch and roll channel. The parameters W_{xgzgws} , W_{vg} , W_{hg} are, respectively, components of wind speed, simulating wind shear, vertical and horizontal gusts of atmospheric turbulence.

In Eq. (3.27), X_{ch} , Y_{ch} , Z_{ch} determine the corresponding reactions of the chassis on the axis of the associated coordinate system.

In Eq. (3.34) V_x and V are not equal to zero. For small values, V_x and V are limited to the minimum allowable values.

The integration of the equations is carried out, using numerical integration methods, mainly Runge–Kutta.

When forming software and mathematical support, the input and output of parameters in the module are carried out through a common memory area. In order to unify the module, the labels of the transmitted parameters remain unchanged.

3.4 Mathematical Modeling of Aerodynamic Forces and Moments

Aerodynamic forces and moments [11] in steady-state flight depend on the parameters of the aircraft: shape, size, layout, orientation, relative to the airspeed vector, the magnitude of the airspeed of flight, air density and viscosity, the speed of sound, the speed of rotation of the aircraft around the center of mass, deviations of controls, etc.

The following coefficients of aerodynamic forces and moments are determined:

- frontal resistance C_x ;
- lifting force C_y ;
- lateral force C_z ;
- transverse aerodynamic moment m_x ;
- track aerodynamic moment m_y ;
- longitudinal aerodynamic moment m_z .

Traditionally, the initial aerodynamic information is a set of tables, graphs, formula dependencies and a text description. At the same time, the processing of initial information is understood as bringing it to a form, convenient for modeling in real time in compliance with accuracy requirements. For these purposes, the function is divided by arguments, groups, having the same points of split and other actions.

Among the ways of presenting information, the tabular method is the most widely used.

Particular attention is paid to the reduction of computing time.

The search for sections of splitting the arguments of the function is carried out:

- with a uniform partition with a step multiple of two;
- with a uniform partition with a step not a multiple of two;
- binary search method (method of half division);

- according to a «hard» algorithm (the algorithm of search is defined in advance and does not change during the counting process);
- buffering the value of the argument of the previous step.

The most effective, from the point of view of speed, is the method of uniform partitioning with a step, multiple of two (due to the use of bitwise shift operations). However, due to partitioning with step multiples of two, the number of node points can be significant, and therefore, large amounts of RAM are required.

The method of uniform partitioning with a step not multiple of two has somewhat worse characteristics. The disadvantage of the method is the redundancy of the initial data.

The binary search method does not require data redundancy. The nodal point is located in several comparison steps, and the number of steps depends on the number of nodal points of this argument and on the input value. The disadvantage of the method is the need for a comparison procedure (in some cases—repeatedly), so it is inferior in speed to the above methods. In addition, an «unfavorable» nodal point may occur during the counting process. When calculating the search time, you should focus on this case.

The search method by «hard» algorithm is based on the fact that the developer determines in advance the sequence of comparing the current value with the values of the nodal points. The disadvantages of the method include all the listed disadvantages of the binary search method. However, the time, spent for the «worst» case of determining the nodal point, is greater than the binary search method, and depends on the developer's qualifications.

The essence of the search method with buffering the value of the argument of the previous step [15] is that the search begins with the value of the previous step. The search starts from the middle node. The method gives a gain in time if the argument changes within 2–3 nearest nodal points during the cycle. In this case, the search is performed in one or two comparison operations. The only exception is the initial search during initialization, but the time, spent on this, is not fundamental. The disadvantage of the method is that the time costs are greater than in the previous methods (if the values of the argument change by 10 or more nodal points during the cycle). However, this can be avoided, if, with a rapid change in the argument, the vector of the rate of change of the argument is periodically found and the nodal point is determined, taking into account this forecast.

From what has been said, the expediency of using the method of search with a uniform step or the half-division method with subsequent buffering of the nodal point follows.

The expediency of constructing a structurally unified module of aerodynamic coefficients, consisting of the following blocks is obvious:

- search, interpolation and determination of aerodynamic coefficients;
- operational storage of data on the components of the aerodynamic coefficients;
- long-term data storage.

The module has common memory areas for input and output variables, as well as for internal parameter exchange. Sharing parameters across general areas saves RAM and time. It also provides the initial initialization of the data of the operational storage unit according to the components of the aerodynamic coefficients. The long-term data storage block is a file with the components of the aerodynamic coefficients.

3.5 Mathematical Modeling of the Power Plant, Weight and Centering Characteristics, Chassis and Atmospheric Phenomena

The power plant module outputs engine thrust values over the entire flight range. Engine failures, autorotation and reverse are simulated; possible thrust losses are taken into account.

The thrust values are calculated on the basis of the altitude-speed characteristics (ASC) of the engine, which are, as a rule, functions of three variables: speed (number of M), altitude and engine speed. Taking into account the structural unification, it is possible to build this module in an automated way.

In the modules, discussed above, it was assumed that the mass, moments of inertia and center of gravity of the aircraft do not change. However, during the AC flight, these parameters change. Of particular importance is the change in the center of gravity, since the equations of motion of the aircraft are usually compiled in a related coordinate system, the beginning of which coincides with its center of gravity. Therefore, the coefficients of these equations are recalculated relative to the current position of the center of gravity. In addition to fuel production, the change in alignment is also affected by the release of the chassis and the discharge of cargo.

The module of chassis simulation uses motion parameters from the flight mechanics module and outputs projections of forces and moments, acting on AC from the chassis side. The mathematical model of the landing gear describes the operation of each landing gear of the aircraft separately. The mathematical description of the operation of a rack with a turning wheel [16] is somewhat different from a rack without turning wheels.

The atmospheric phenomena simulator module is designed to determine temperature, pressure, air density, sound speed, wind speed, horizontal and vertical wind shear, atmospheric turbulence and icing. Due to the bulkiness, the relations, connecting them are not given.

3.6 Establishing the Correspondence of the Parameters of the Object and the Subjective Assessments of the Operator. Description of Areas of Equal Assessment. Functional of Quality

When developing an aviation simulator, the task is to achieve the adequacy of human behavior in the «Pilot—AS», «Pilot—AC» systems, when solving the same tasks. Therefore, the evaluation of the quality of the «Pilot—AS» system is reduced by comparing the pilot's control actions in both systems. The fundamental task of quantifying the quality of the system is to match the concepts of «better» and «worse» with the concepts of «more» and «less».

For this purpose, quality criteria are used, which are quantitative indicators, the numerical values of which are a measure of the quality of the system. If we find a scientifically based quantitative criterion for the quality of an ergatic control system, then the study can be carried out through a formalized apparatus. *If there is no such criterion, then an alternative to the formalized apparatus is unsubstantiated judgments, that allow ambiguous interpretations* [17].

It is known from the theory of synthesis of structures that, taking into account the properties of the object and the purpose of the system, its structure is determined by the choice of the quality criterion laid in the basis of the synthesis [18].

When developing AS, the choice of the system structure is complicated by the fact that elements, heterogeneous in physical properties, are combined: a person and a machine. This circumstance entails the requirement of the presence of two systems of particular criteria in the generalized quality criterion:

- system, reflecting the technical aspect of the synthesis of the AS structure;
- psychophysiological, reflecting the features of the functional interaction of man and machine.

Note that despite the similarity of the task of constructing AS with the task of synthesizing the ergamate structure, there is a significant difference between these two tasks.

When synthesizing the structure of ergamate, we proceed from the compatibility of man and ergamate expressed in a general organismic principle: «A person completes his organism in an optimal way». When developing AS, however, it is required, that a person behaves in the «Pilot—AS» system in the same way as in the «Pilot—AC» system, if homogeneous tasks are solved in them.

However, conclusions, essential for the development of AS, follow from the general organismic organizational principle, namely:

- quality criteria that determine the functional behavior of a person inside a purposeful ergatic system and the quality criteria of an ergatic system must coincide (the principle of compatibility of the first kind);
- the quality criterion that determines the functional behavior of a person in the ergatic system should be part of the general quality criterion, which determines

the functional behavior of the whole system (the principle of compatibility of the second kind) [19–22].

From the principle of compatibility of *the first kind*, it follows that, if the quality of AS is determined by the «similarity» of object controls, when solving the same tasks assigned to the pilot on the plane and AS, then the assessment of the pilot's training to solve any specific task on the plane during his training on AS can be made only if the optimal labor skills at solving this problem in the conditions of AS and aircraft are close.

From the principle of compatibility of *the second kind*, it follows that, if the general quality criterion determining the functional behavior of the entire «Pilot–AS» system is to choose the «similarity» of controls on an airplane and AS, then when solving the same tasks, the optimal controls on an airplane and AS should be close.

Thus, it follows from the principles of compatibility of the first and second kind that the criterion for the quality of AS can be taken to be the «similarity» of controls on an airplane and AS, *if and only if* they are the same to consider optimal controls on an airplane and AS, when performing the same tasks, are close (a necessary and sufficient condition) [19, 23–26].

As already noted, in the existing AS, the optimal control movements of the pilot are very far from those control movements that take place on an airplane. The task of obtaining the mentioned «similarity» of controls is equivalent to the task of approximating the optimal control on AS to the optimal control of the pilot of the aircraft. The solution to the second problem is preferable because the class of optimal controls is already the class of all controls.

The development of AS in its theoretical part is reduced to the task of identifying an ergatic system, and during identification, it must be obtained for the implementation (or evaluation of the quality of implementation) of all private operators not only of aircraft, but also of human [27–29].

Models of human operator models are a fairly natural consequence of models of individual human organs. In typical tasks, perception can be taken as the input of a human operator. Decision-making is carried out by the central nervous system. The way out is the movement of the limbs. When performing their functions, the human operator acts in a pulse mode, performs pattern recognition for forecasting and uses maximum control action during each time period [7, 30–32].

The stage of human operator training eliminates the need for conscious strategic decision-making (with low information bandwidth of conscious thinking). In this case, the adaptive actions of the operator become subconscious. The human operator is so exposed to the need to achieve the goal that even extremely sharp structural changes, such as changes in the polarity of the elements, may not affect the overall quality of the function of the entire system, and a person may not even remember this change in the structure of the object [33].

In order to determine the dynamic characteristics of the system during operation and maintain optimal control (even with a significant change in the parameters and

structure of the object), the human operator surveys the object; periodically sending pulse-like signals [34].

It is important to note that a human operator is not able to find the extremum of a multi-connected goal criterion if the number of variables in it is more than six.

It is assumed [35] that the operator uses the difference between the observed change in the error rate and the predicted change caused by the movement of control devices (steering wheel, pedals, handles, etc.) for control. If this difference is small, the operator assumes that the dynamics of the system have not changed; if it is large, the operator can assume that there has been a change in dynamics.

In order to predict the reaction of the system, the human operator must actually have an «internal» representation of the dynamics of the system (a conceptual model) [36].

Based on the above, when analyzing random control signals, it is advisable [37] to present a model of an adaptive process from 4 phases:

- identification by the operator of changes in the dynamics of the system;
- recognition of changes in dynamics;
- changing the operator's own dynamic characteristics (in order to achieve stability and the ability to control the object);
- optimization of operator characteristics (in order to achieve good working characteristics of the system).

When considering the issue of AS quality here, as in the work as a whole, it is assumed that the assessment of quality is possible only from the point of view of the possibility of forming a piloting style, close to the style, available in the «Pilot-AC» system for AS trainees. Note that for the existing AS and the class of flight tasks under consideration, the control actions $x(t)$ are in a certain «corridor», determined by the specified values of mathematical expectation $m^*(t)$ and variance $D^*(t)$, i.e.

$$\begin{aligned} |M[x(t)] - m^*(t)| &< \delta_1(t), \\ |D[x(t)] - D^*(t)| &< \delta_2(t). \end{aligned}$$

Then, as characteristics of the piloting style, some parameters can be taken that determine the internal structure of the centered random function

$$\overset{o}{x}(t) = x(t) - M[x(t)].$$

This structure is sufficiently characterized by a correlation function or a set of distribution laws of the first order, second order, etc. However, they do not have sufficient clarity and simplicity of practical definition. This book offers indicators that have a simple physical meaning convenient for practical use.

It is known [7, 9, 30, 38, 39] that the pilot's assessment of the aerobic characteristics of the aircraft, i.e. the pilot's control movements (PCM) and their structure, depend on the natural oscillation frequencies ω_i and damping coefficients ξ_i .

Directly from the physics of the control process, it is clear that, when controlled by the i -th channel, the control movements of the pilot, adapted to this aircraft, as the dominant component will contain a harmonic with a frequency close to ω_i . The presence of cross-links between the control channels can lead to a noticeable appearance in the PCM of the i -th channel of a harmonic with a frequency ω_j , $j \neq i$.

The above makes it possible to include points $\hat{\omega}$ of local maximums of spectral density $S_x^o(\omega)$ in the set of objective characteristics of the PCM structure. For practical determination of the assessment $\hat{S}_x^o(\omega)$, the fast Fourier transform method can be used. Smoothing can be performed, for example, by Hann [4]. It is quite obvious that the less ξ_i and the better the pilot has adapted to the value ω_i , the more $S_x^o(\hat{\omega})$. From this, it follows that the pilot's adaptation to ω_i and ξ_i can be approximately estimated by the values

$$D_i = \int_{\hat{\omega}_i - \Delta\omega'_i}^{\hat{\omega}_i + \Delta\omega''_i} \hat{S}_x^o(\omega) d\omega,$$

$$P_i = P(\hat{\omega}_i - \Delta\omega'_i < \omega < \hat{\omega}_i + \Delta\omega''_i),$$

where D_i represents the variance per site

$$\hat{\omega}_i - \Delta\omega'_i < \omega < \hat{\omega}_i + \Delta\omega''_i,$$

and P_i is the probability of hitting the frequency ω in the PCM on this site.

It is advisable to make a practical determination P_i by taking into account the relationship, indicated in [7, 30, 40], between the density of frequency distribution in the process $x(t)$ and the spectral density of this process. Values $\hat{\omega}_i$, $\Delta\omega'_i$, $\Delta\omega''_i$ should be determined by the spectral density $\hat{S}_x^o(\Delta\omega)$ graph.

It is not difficult to see that the question of assessing the quality of the control style and the quality of AS, which is discussed below, is directly related to the issue under consideration.

Let the objective indicators $\hat{\omega}_{i0}$, \hat{P}_{i0} , \hat{D}_{i0} correspond to the «ideal» pilot-standard. Then

$$\Delta\omega_i = \hat{\omega}_i - \hat{\omega}_{i0},$$

$$\Delta P_i = \hat{P}_i - \hat{P}_{i0},$$

$$\Delta D_i = \hat{D}_i - \hat{D}_{i0}$$

will be objective indicators of the formation of the required control style among the trainees. This is a consequence of the organismic principle, as well as the principle of least action [7, 30, 38]. As an additive global criterion, then we can take

$$K_y = \sum_i c_i K_i = \sum_i c_i \left(\alpha_i |\Delta\tilde{\omega}_i| + \beta_i |\Delta\tilde{P}_i| + \gamma_i |\Delta\tilde{D}_i| \right),$$

where $\Delta\tilde{\omega}_i$, $\Delta\tilde{P}_i$, $\Delta\tilde{D}_i$ —the normalized values, accordingly, $\Delta\omega_i$, ΔP_i , ΔD_i .

If $\Delta\hat{\omega}_{i0}^s$, $\Delta\hat{P}_{i0}^s$, $\Delta\hat{D}_{i0}^s$ correspond to the «ideal» pilot in the «Pilot-AS» system, then based on the organizational principle, as a criterion for the quality of AS in the sense, indicated above, we can take

$$K_s = \sum c_i^s K_i^s = \sum_i c_i^s \left(\alpha_i^s |\Delta\tilde{\omega}_i^s| + \beta_i^s |\Delta\tilde{P}_i^s| + \gamma_i^s |\Delta\tilde{D}_i^s| \right),$$

where

$$\begin{aligned} \Delta\tilde{\omega}_i^s &= (\hat{\omega}_{i0}^s - \hat{\omega}_{i0}) / \sigma[\omega_{i0}^s], \quad \Delta\tilde{P}_i^s = (\hat{P}_{i0}^s - \hat{P}_{i0}) / \sigma[P_{i0}^s], \\ \Delta\tilde{D}_i^s &= (\hat{D}_{i0}^s - \hat{D}_{i0}) / \sigma[D_{i0}^s]. \end{aligned}$$

Determination of weight constants α_i^s , β_i^s , γ_i^s can be made based on the analysis of the correlation between particular criteria $\Delta\omega_i^s$, ΔP_i^s , ΔD_i^s and group criteria— K_i^s , and determination of constants c_i^s can be made by correlation relationship between K_s and K_i^s [7, 17].

Based on many years of research for systems of the type

$$\dot{x} = Ax + Bu + f(t)$$

in order to assess the quality of AC (AS), it is proposed to use the functional [7]

$$\Phi_0(S) = \frac{a}{\max_i |\alpha_i|} + b \cdot \max_i \left| \frac{\beta_i}{\alpha_i} \right| + c \cdot \max_i |\beta_i| + \frac{d}{\max_i |\beta_i|},$$

where x , u , $f(t)$ —the vectors of the phase coordinates of the controlling and disturbing effects, accordingly; $\lambda_i = \alpha_i + j\beta_i$ —own numbers of matrix A; a , b , c , d —weight constants.

Using experimental data for longitudinal motion, the proposed functional can be represented as

$$\Phi_0(S) = \left(\frac{0.1}{\omega_0} + 0.2 \right) \sqrt{\left| 1 - \frac{1}{\xi^2} \right|} + \frac{12}{\omega_0} + \omega_0,$$

where ξ —dimensionless attenuation coefficient; $\omega_0 \xi = \left| \frac{\lambda_1 + \lambda_2}{2} \right|$.

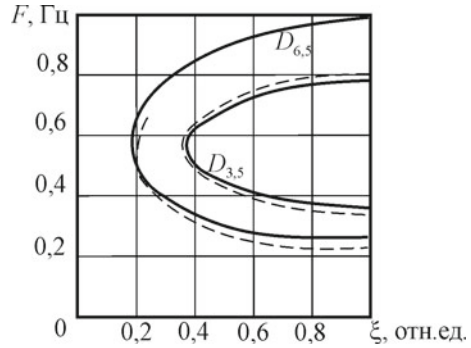
For oscillatory systems, $\omega_c = \omega_0 \sqrt{1 - \xi^2}$ coincides with the own frequency. For aperiodic systems, $\omega_0 = T^{-1}$. In this case, the characteristic equation has the form

$$T^2 p^2 + 2\xi \cdot T \cdot p + 1 = 0.$$

Areas of equal assessments are introduced in the form

$$D_k = \{ (\xi, \omega_0) | d_{k-1} \leq \Phi_0(S) < d_k \}.$$

Fig. 3.11 Areas of equal assessment



The classes with grades 3.5 and 6.5 on the Cooper–Harper scale are shown in Fig. 3.11 (solid lines are assessments, obtained by the proposed functional; dotted lines are experimentally obtained areas) [41]. However, it should be noted that the selectivity of the considered functional in determining AC classes is insufficient (for class 3.5 value $\Phi_0 \leq 7.5$; for class 6.5 – $\Phi_0 \leq 8.25$). Therefore, along with the use of $\Phi_0(S)$ to evaluate AC, it is proposed to use the values themselves ξ and ω_0 , as private criteria.

For an oscillatory system

$$\Phi_0(S) = \left(\frac{a}{\omega_0 \sqrt{1 - \xi^2}} + b \right) \cdot \sqrt{\frac{1}{\xi^2} - 1} + \omega_0 \cdot \sqrt{1 - \xi^2} + \frac{d}{\omega_0 \sqrt{1 - \xi^2}},$$

or

$$\Phi_0(S) = \frac{a}{\xi \omega_0} + b \cdot \sqrt{\frac{1}{\xi^2} - 1} + \xi \omega_0 \sqrt{\frac{1}{\xi^2} - 1} + \frac{d}{\xi \omega_0 \sqrt{\frac{1}{\xi^2} - 1}}.$$

Let us introduce into consideration the characteristics of control, proposed in [7, 11], namely

$$t_{sp} = \frac{3}{\xi \omega_0}, \quad n_{sp} = 0.5 \sqrt{\frac{1}{\xi^2} - 1}, \quad T_c = \frac{2\pi}{\omega_c} = \frac{2\pi}{6} \frac{t_{sp}}{n_{sp}}.$$

At the same time $\frac{1}{T_c} = f_c = \frac{6}{2\pi} \frac{n_{sp}}{t_{sp}}$.

Then $\Phi_0(S)$ can be written as

$$\Phi_0(S) = \frac{a}{3} t_{sp} + 2b n_{sp} + 6 \frac{n_{sp}}{t_{sp}} + \frac{d}{6} \frac{t_{sp}}{n_{sp}},$$

or

$$\Phi_0(S) = \frac{a}{3}t_{sp} + 2bn_{sp} + 2\pi f_c + \frac{d}{2\pi}T_c,$$

or

$$\Phi_0(S) = At_{sp} + Bn_{sp} + Cf_c + DT_c.$$

At the accepted values of the weight constants $a = 0.1$; $b = 0.2$; $c = 1$; $d = 12$.

$$\Phi_0(S) = 0.033t_{sp} + 0.4n_{sp} + 6.28f_c + 1.91T_c.$$

When $\xi = 0.6$; $f_c = 0.5$ we have

$$\omega_c = 2\pi f_c = 3.14; \quad \omega_0 \approx 3.92; \quad T_c = 2; \quad t_{sp} = 1.28; \quad n_{sp} = 0.667;$$

$$\Phi_0(S) \approx 7.27.$$

When $\xi = 0.35$; $f_c = 0.6$, we have

$$\omega_c = 3.77; \quad \omega_0 = 4.02; \quad T_c = 1.67; \quad t_{sp} = 2.13; \quad n_{sp} = 1.34;$$

$$\Phi_0(S) = 7.55.$$

The selection of weight constants is checked according to Fig. 3.11 [36].

1. When $\xi = 0.6$; $f_c = 0.5$;

$$\Phi_0(S) = 7.27 < 7.5; \quad (\xi, f_c) \in D_{3,5}.$$

2. $\xi = 1$; $f_c = 0.8$;

$$\Phi_0(S) = 7.42 < 7.5; \quad (\xi, f_c) \in D_{3,5}.$$

3. $\xi = 1$; $f_c = 0.4$;

$$\Phi_0(S) = 7.29 < 7.5; \quad (\xi, f_c) \in D_{3,5}.$$

4. $\xi = 0.4$; $f_c = 0.5$;

$$\Phi_0(S) = 7.49 < 7.5; \quad (\xi, f_c) \in D_{3,5}.$$

5. $\xi = 0.35$; $f_c = 0.6$;

$$\Phi_0(S) = 7.55 > 7.5; \quad (\xi, f_c) \in D_{6,5}.$$

Further

$$\frac{\partial \Phi_0}{\partial \omega_c} = -\frac{0.1}{\omega_c^2} \sqrt{\frac{1}{\xi^2} - 1} - \frac{12}{\omega^2} + 1 = 1 - \gamma,$$

$$\gamma = \frac{1}{\omega_c^2} \left(0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12 \right).$$

$$\frac{\partial \Phi_0}{\partial \omega_c} = \left(\frac{0.1}{\omega_c} + 0.2 \right) \frac{1}{2 \cdot \sqrt{\frac{1}{\xi} - 1}} \left(-\frac{2}{\xi^3} \right) = -\left(\frac{0.1}{\omega_c} + 0.2 \right) \frac{1}{\xi^3 \cdot \sqrt{\frac{1}{\xi^2} - 1}} < 0.$$

Thus, with a fixed ω_c , the value $\Phi_0(S)$ decreases with an increase in ξ . Hence, the desire to increase ξ in AS conditions is understandable.

As ξ increases, the value $\Phi_0(S)$ decreases, and the class of the system improves.

When $\gamma = 1$ $\frac{\partial \Phi_0}{\partial \omega_c} = 0$.

When $\gamma < 1$ $\frac{\partial \Phi_0}{\partial \omega_c} > 0$, when $\gamma > 1$ $\frac{\partial \Phi_0}{\partial \omega_c} < 0$, i.e. when $\gamma = 1$ $\omega_{co}^2 = 0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12$, or $\omega_{co} = \sqrt{0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12}$, $\Phi_0(\omega_c)$ reaches a minimum.

To reduce $\Phi_0(S)$ (or to improve the class of system) with a fixed ξ , first you need to calculate $\omega_{co} = \sqrt{0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12}$, then compare ω_c with the value ω_{co} .

Then, if $\omega_c < \omega_{co}$, then it is necessary to move in the direction of increasing ω_c to the value ω_{co} , if $\omega_c > \omega_{co}$, then it is necessary to move in the direction of decreasing ω_c to the value ω_{co} .

This is illustrated in Fig. 3.12.

For example, let $\xi = 1$.

Consider $f_c = 0.8$; $\omega_c = 5.024$.

At the same time $\Phi_0(S) = 7.42$.

Here $\gamma = 1$ when $\omega_{co} = \sqrt{0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12} = 3.46$.

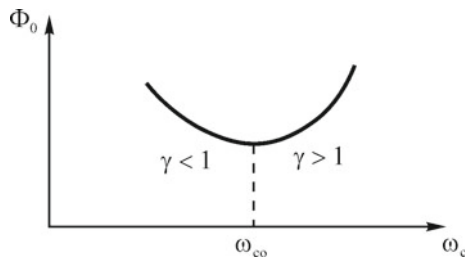
As we see, $\omega_c > \omega_{co}$, and it is necessary to move in the direction of reduction ω_c .

The lowest value $\Phi_0(S)$ will be, when $\omega_c = 3.46$.

At the same time $\Phi_0(S) = 6.93$.

A further decrease in frequency already leads to an increase in $\Phi_0(S)$.

Fig. 3.12 Graph of functional of quality



For example, $\Phi_0(1; 0.4) = 7.29 > 6.93$.

For this, the values ω_c would have $\gamma = 1.9 > 1$, and it would be necessary to move in the direction of increase ω_c .

Note

$$\frac{d\omega_{c0}}{d\xi} = \frac{0.1 \frac{1}{2 \cdot \sqrt{\frac{1}{\xi^2} - 1}} \cdot \left(-\frac{2}{\xi^3}\right)}{2 \cdot \sqrt{0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12}} < 0,$$

i.e. with increasing ξ , ω_{c0} is decreasing.

The direction of the fastest descent coincides with the direction

$$-\overline{\text{grad } \Phi_0(\xi, \omega_c)} = \left(-\frac{\partial \Phi_0}{\partial \xi}, -\frac{\partial \Phi_0}{\partial \omega_c} \right).$$

3.7 Methodology for Setting Dynamic Parameters of an Aircraft Simulator

It is necessary to improve the class of a system having parameters (ξ, ω_c) .

To do this, based on the above equations, it is necessary to:

calculate $\omega_{c0} = \sqrt{0.1 \cdot \sqrt{\frac{1}{\xi^2} - 1} + 12}$;

- compare ω_c with the value ω_{c0} ;
- select a step $\pm \Delta \omega_c$ and take instead ω_c the value $\omega_c \pm \Delta \omega_c$;
- calculate $\frac{\partial \Phi(\xi, \omega_c)}{\partial \omega_c}$;
- calculate $\frac{\partial \Phi(\xi, \omega_c)}{\partial \xi}$;
- calculate step by ξ :

$$\Delta \xi = \frac{\frac{\partial \Phi}{\partial \omega_c}}{\frac{\partial \Phi}{\partial \xi}};$$

- move in this direction while $\Phi_0(S)$ decreasing;
- specify the direction of the gradient vector at the penultimate point (ξ, ω_c) , where the value $\Phi_0(S)$ was less than the last one;
- repeat the procedure from the beginning.

3.8 Investigation of the Dependence of the Areas of Equal Ratings on the Gain

Consider a system of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ u &= -Px \end{aligned},$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = b_1 \begin{bmatrix} 1 \\ \gamma \end{bmatrix}$, $P = p_1 \begin{bmatrix} 1 & p \end{bmatrix}$.

It can be represented as

$$\dot{x} = \tilde{A}x,$$

where $\tilde{A} = A - BP = \begin{bmatrix} a_{11} + \beta & a_{12} + \beta p \\ a_{21} + \beta\gamma & a_{22} + \beta\gamma p \end{bmatrix}$.

Coefficient $\beta = -b_1 p_1$, in essence, there is a gain that depends on both the design features of the system and the operator. *Due to the organismic principle, it is not easy to change β , because the operator tries to compensate for the change b_1 by changing p_1 .*

It is clear, that $\beta < 0$, $\gamma > 0$, $p > 0$.

Invariants

$$\begin{aligned} \tilde{\sigma} &= tr\tilde{A}, \\ \tilde{\Delta} &= det\tilde{A} \end{aligned}$$

can be easily obtained through the invariants of the matrix A: $\sigma = trA$, $\Delta = detA$ and the gain β , namely:

$$\begin{aligned} \tilde{\sigma} &= \sigma + \alpha_1\beta, \\ \tilde{\Delta} &= \Delta - \alpha_2\beta, \end{aligned}$$

where

$$\begin{aligned} \alpha_1 &= 1 + \gamma p, \\ \alpha_2 &= a_{12}\gamma + a_{21}p - a_{22} - a_{11}\gamma p. \end{aligned}$$

Usually $\alpha_1 \leq 1$, and with great accuracy $\alpha_1 \approx 1$, $\alpha_2 > 0$ and $\alpha_2 \approx a_{21}p - a_{22} \approx 1$, because $(a_{12} - a_{11}p)\gamma \approx 0$.

Let's introduce $\beta_m = \alpha_2\beta$. It is not difficult to see that the gain β is simply scaled with a scaling coefficient α_2 . Then

$$\begin{aligned}\tilde{\Delta} &= \Delta - \beta_m, \\ \tilde{\sigma} &= \sigma + \alpha\beta_m,\end{aligned}$$

where $\alpha = \frac{\alpha_1}{\alpha_2}$ and by virtue of the above $\alpha \approx 1$.

Based on the functional $\Phi_0(S)$ for the system $\dot{x} = Ax$, you can write a similar functional for the system \tilde{S}

$$\dot{x} = \tilde{A}x,$$

exactly

$$\begin{aligned}\Phi(\tilde{S}) &= \left(\frac{\alpha}{\tilde{\omega}_c} + b\right) \sqrt{\frac{1}{\tilde{\xi}^2} - 1} + \tilde{\omega}_c + \frac{d}{\tilde{\omega}_c}, \\ \tilde{\omega}_c &= \tilde{\omega}_0 \sqrt{1 - \tilde{\xi}^2} = \sqrt{\tilde{\Delta}} \sqrt{1 - \tilde{\xi}^2}, \\ \tilde{\xi} &= -\frac{\tilde{\sigma}}{2\sqrt{\tilde{\Delta}}},\end{aligned}$$

where $\tilde{\omega}_c, \tilde{\xi}$ —own frequency and damping coefficient of a *closed* system \tilde{S} [42].

Following [7, 43], we will try to describe both oscillatory and aperiodic systems with one functional. To do this, instead of the functional $\Phi_0(S)$, describing oscillatory systems ($0 < \xi < 1$), we introduce the functional

$$\Phi_0(\tilde{S}) = \left(\frac{\alpha}{\tilde{\omega}_c} + b\right) \sqrt{\left|\frac{1}{\tilde{\xi}^2} - 1\right|} + \tilde{\omega}_c + \frac{d}{\tilde{\omega}_c},$$

where $\tilde{\omega}_c = \sqrt{\tilde{\Delta}} \sqrt{1 - \tilde{\xi}^2}$, $\tilde{\xi} = -\frac{\tilde{\sigma}}{2\sqrt{\tilde{\Delta}}}$.

Here $\tilde{\xi}$ can take any valid non-negative values and can also be used for aperiodic systems with $\tilde{\xi} > 1$.

We have

$$\begin{aligned}\tilde{\xi} &= \frac{\xi + \tau_k}{\sqrt{1 + k^2}}, \\ \tilde{\omega}_c &= \sqrt{\left|\frac{1 + k^2 - (\xi + \tau_k)^2}{1 - \xi^2}\right|} \omega_c,\end{aligned}$$

where $k = \sqrt{-\frac{\beta_m}{\Delta}}$, $\tau = \frac{\alpha\sqrt{-\beta_m}}{2}$.

We note

$$\xi'_k = \frac{\tau - \xi k}{(1 + k^2)^{\frac{3}{2}}},$$

where from $\tilde{\xi}' > 0$ (or < 0), if, respectively, $(\tau - \xi k) > 0$ (or < 0),
 or $\tau > \xi k$ (or $<$), $\tau^2 > \xi^2 k^2$, $-\alpha^2 \beta > -\frac{\beta \sigma^2}{4\Delta^2}$, $4\alpha^2 \Delta^2 > \sigma^2$,
 or $2\alpha\Delta + \sigma > 0$ (< 0).

When $\alpha = 0.5$, we get $\tilde{\xi}'_k > 0$ (< 0), if, respectively, $\Delta + \sigma > 0$ (< 0).

Taking into account the above said, it follows that for small k , with an increase of the coefficient k , the system shifts on the plane (ξ, ω_c) .

- right-up, if $2\alpha\Delta + \sigma > 0$, $\mu = \left| 1 - k^2 \frac{1 - \tau^2 - 2\xi k}{1 - \xi^2 k^2} \right| > 1$,
- left-up, if $2\alpha\Delta + \sigma < 0$, $\mu > 1$,
- right-down, if $2\alpha\Delta + \sigma > 0$, $\mu < 1$,
- left-down, if $2\alpha\Delta + \sigma < 0$, $\mu < 1$.

The results are shown in Figs. 3.13 and 3.14.

Fig. 3.13 Type of dependence $\omega_c(\xi)$ for different values of d

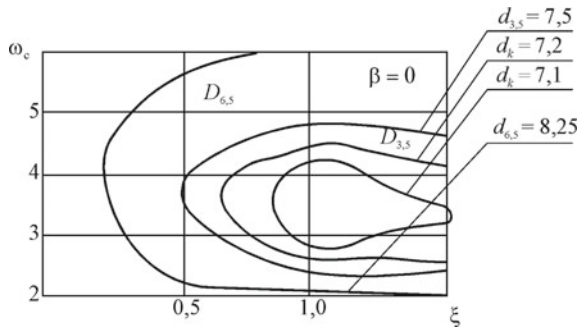
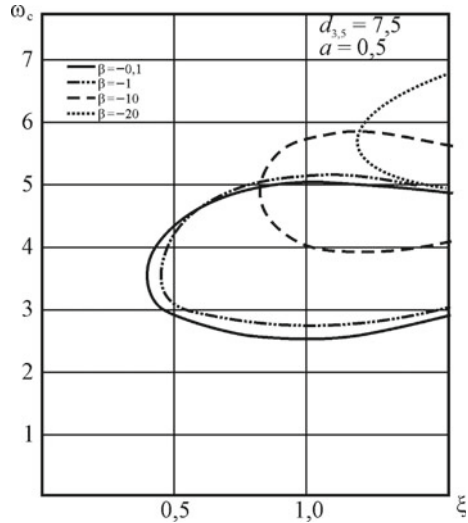


Fig. 3.14 Type of dependence $\omega_c(\xi)$ for different values of β



3.9 Investigation of the Dependence of the Areas of Equal Ratings on the Technical Parameters of the Aircraft

Let us consider the linearized equations of the longitudinal motion of a rigid aircraft in a calm atmosphere [44] in the vicinity of a steady rectilinear horizontal flight

$$(H = H_0, \quad V = V_0, \quad \vartheta_0 = \alpha_0, \quad \omega_{z0} = \theta_0 = 0),$$

neglecting the change in air density with height variations. The equations of longitudinal motion in accordance with [7, 13] have the form

$$\begin{aligned} \dot{V} &= -a_x^v V - a_x^\alpha \alpha - g \vartheta + \frac{P}{m}, \\ \dot{\theta} &= a_y^{v^*} V + a_y^{\alpha^*} \alpha + a_y^{\varphi^*} \varphi, \\ \dot{\omega}_z &= -a_{m_z}^v V - a_{m_z}^\alpha \alpha - a_{m_z}^{\dot{\alpha}} \dot{\alpha} - a_{m_z}^{\omega_z} \omega_z - a_{m_z}^\varphi \varphi, \\ \dot{\vartheta} &= \omega_z, \quad \vartheta = \theta + \alpha. \end{aligned}$$

It is assumed that the thrust P and deflection φ of the elevator (stabilizer) are the specified control actions.

The parameters of the initial motion of the aircraft are determined from the conditions of balancing forces and moments

$$P = C_x \frac{\rho V_0^2}{2} S, \quad G = C_y \frac{\rho V_0^2}{2} S.$$

Then, the determination of the dependence of the areas D_k of equal estimates on the technical parameters (coefficients of the equations of motion) can be carried out on the basis of the method of decomposition of longitudinal motion into fugoid and short-period components, given in Sect. 3.1.1.

3.10 Mathematical Model of Pilot Control Movements

Consider the equations of longitudinal motion (short-period component)

$$\begin{aligned} \dot{\omega}_z(t) &= a_{11} \omega_z(t) + a_{12} \alpha(t) + b_1 u(t), \\ \dot{\alpha}(t) &= a_{21} \omega_z(t) + a_{22} \alpha(t) + b_2 u(t), \\ u_1 &= k_1 x_b(t) + k_2 \dot{x}_b(t), \\ u_2(t) &= p_1 \omega_z(t) + p_2 \alpha(t) + q_1 \omega_z(t - \tau_1) + q_2 \alpha(t - \tau_1) + r_1 \dot{\omega}_z(t - \tau_2) \\ &\quad + r_2 \dot{\alpha}(t - \tau_2), \\ u(t) &= u_1(t) + u_2(t). \end{aligned}$$

The coefficients of the equations can be determined by synchronous measurements ω_z , α , u , x_b . We take as a generalized vector of control

$$U = \text{col}(k_1, k_2, p_1, p_2, q_1, q_2, r_1, r_2, \hat{\omega}_c, P, D, A_u, t_u).$$

$$\text{Let } \sigma = a_{11} + a_{22}, \quad \Delta = a_{11}a_{22} - a_{12}a_{21}.$$

Suppose

$$\begin{aligned} k_1 &= \xi_{10} + \xi_{1\sigma}\sigma + \xi_{1\Delta}\Delta + \xi_{11}b_1 + \xi_{12}b_2, \\ k_2 &= \xi_{20} + \xi_{2\sigma}\sigma + \xi_{2\Delta}\Delta + \xi_{21}b_1 + \xi_{22}b_2, \\ p_1 &= \xi_{30} + \xi_{3\sigma}\sigma + \xi_{3\Delta}\Delta + \xi_{31}b_1 + \xi_{32}b_2, \\ p_2 &= \xi_{40} + \xi_{4\sigma}\sigma + \xi_{4\Delta}\Delta + \xi_{41}b_1 + \xi_{42}b_2, \\ D &= \xi_{110} + \xi_{11\sigma}\sigma + \xi_{11\Delta}\Delta + \xi_{111}b_1 + \xi_{112}b_2, \\ A_u &= \xi_{120} + \xi_{12\sigma}\sigma + \xi_{12\Delta}\Delta + \xi_{121}b_1 + \xi_{122}b_2, \\ t_u &= \xi_{130} + \xi_{13\sigma}\sigma + \xi_{13\Delta}\Delta + \xi_{131}b_1 + \xi_{132}b_2. \end{aligned}$$

If for N systems with known matrices A and B (i.e. with known σ , Δ , b_1 , b_2) the components U , i.e. the right-hand sides of the equations are known, then the coefficients ξ_{10} , $\xi_{1\sigma}$, $\xi_{1\Delta}$, ξ_{11} , ξ_{12} , \dots , ξ_{130} , \dots , ξ_{132} are determined by the method of least squares.

Let

$$\begin{aligned} \bar{k}_1 &= \frac{1}{N} \sum_{i=1}^N k_1^i, \\ \bar{k}_2 &= \frac{1}{N} \sum_{i=1}^N k_2^i, \\ &\dots \quad \dots \quad \dots \\ \bar{A}_u &= \frac{1}{N} \sum_{i=1}^N A_u^i, \\ \bar{t}_u &= \frac{1}{N} \sum_{i=1}^N t_u^i, \\ \bar{\sigma} &= \frac{1}{N} \sum_{i=1}^N \sigma^i, \\ \bar{\Delta} &= \frac{1}{N} \sum_{i=1}^N \Delta^i, \\ \bar{b}_1 &= \frac{1}{N} \sum_{i=1}^N b_1^i, \quad \bar{b}_2 = \frac{1}{N} \sum_{i=1}^N b_2^i. \end{aligned}$$

Let's introduce

$$\begin{aligned} k_{10} &= k_1 - \bar{k}_1, \\ k_{20} &= k_2 - \bar{k}_2, \\ &\dots \quad \dots \quad \dots \\ b_{10} &= b_1 - \bar{b}_1, \\ b_{20} &= b_2 - \bar{b}_2. \end{aligned}$$

Then, similarly to the above, it is possible to write dependencies in normalized variables [7, 45]

$$\begin{aligned} k_{10} &= \xi_{1\sigma}\sigma_0 + \xi_{1\Delta}\Delta_0 + \xi_{11}b_{10} + \xi_{12}b_{20}, \\ k_{20} &= \xi_{2\sigma}\sigma_0 + \xi_{2\Delta}\Delta_0 + \xi_{21}b_{10} + \xi_{22}b_{20}, \\ &\dots \quad \dots \quad \dots \\ t_{u0} &= \xi_{13\sigma}\sigma_0 + \xi_{13\Delta}\Delta_0 + \xi_{131}b_{10} + \xi_{132}b_{20}. \end{aligned}$$

Let's introduce

$$\begin{aligned} X_0 &= \begin{bmatrix} k_{10}^1 & k_{12}^1 & \dots & A_{uo}^1 & t_{uo}^1 \\ & & & \dots & \\ k_{10}^N & k_{20}^N & \dots & A_{uo}^N & t_{uo}^N \end{bmatrix}^T, \\ V &= \begin{bmatrix} \sigma_0^1 & \Delta_0^1 & b_{10}^1 & b_{20}^1 \\ \sigma_0^2 & \Delta_0^2 & b_{10}^2 & b_{20}^2 \\ & & \dots & \dots \\ \sigma_0^N & \Delta_0^N & b_{10}^N & b_{20}^N \end{bmatrix}, \\ \xi &= \begin{bmatrix} \xi_{1\sigma} & \xi_{1\Delta} & \xi_{11} & \xi_{12} \\ \xi_{2\sigma} & \xi_{2\Delta} & \xi_{21} & \xi_{22} \\ & & \dots & \dots \\ \xi_{13\sigma} & \xi_{13\Delta} & \xi_{131} & \xi_{132} \end{bmatrix}. \end{aligned}$$

Then ξ must satisfy the ratio

$$\xi = (V^T V)^{-1} V^T X_0.$$

It allows you to determine the dependencies by which you can determine the influence of the technical parameters of the object on the structure of control actions.

Based on the processing of statistical materials, it was possible to establish that it is fair for the AC classes under consideration

$$A_u = 18 + 7.1\tilde{k} + 5.3\tilde{\omega}_c + 16.4\tilde{\xi};$$

$$\begin{aligned}
 t_u &= 1.86 + 0.09\tilde{k} - 0.43\tilde{\omega}_c - 0.65\tilde{\xi}; \\
 P &= 0.45 + 0.09\tilde{k} - 0.16\tilde{\omega}_c + 0.05\tilde{\xi}; \\
 D &= 2.4 + 2.5\tilde{k} - 5.1\tilde{\omega}_c - 2.3\tilde{\xi}; \\
 \omega &= 2.7 + 1.3\tilde{k} + 0.9\tilde{\omega}_c - 0.7\tilde{\xi}; \\
 \tilde{k} &= \frac{k - \bar{k}}{\sigma_k}, \quad \tilde{\omega}_c = \frac{\omega_c - \bar{\omega}_c}{\sigma_{\omega_c}}, \quad \tilde{\xi} = \frac{\xi - \bar{\xi}}{\sigma_{\xi}}; \\
 \bar{k} &= -0.083, \quad \bar{\omega}_c = 1.78, \quad \bar{\xi} = 0.48; \\
 \sigma_k &= 0.02, \quad \sigma_{\omega_c} = 0.13, \quad \sigma_{\xi} = 0.14.
 \end{aligned}$$

3.11 Determination of Equal Control Areas

The areas of equal control are proposed to be determined based on the functional

$$K_y = \sum_i c_i \left[\left(\alpha_i |\Delta \tilde{\omega}_i| + \beta_i |\Delta \tilde{P}_i| + \gamma_i |\Delta \tilde{D}_i| \right) + \left(\delta_i |\Delta \tilde{A}_i| + \mu_i |\Delta \tilde{I}_i| \right) \right].$$

Classes of control are constructed according to the double inequality

$$L_{K-1} \leq K_y < L_K.$$

It follows from $K_y = L_K$ that the area of the control, which is no worse than the K -th class, is a part of the first quadrant cut off by a hyperplane $K_y = L_K$.

For $i = 1$; $\alpha, \beta, \gamma > 0$ and the remaining zero constants, the view of the areas of equal controls is shown in Fig. 3.15.

Optimal control corresponds to the origin of coordinates. If you used the functional of the form $K_y = \alpha(\Delta\omega)^2 + \beta(\Delta P)^2 + \gamma(\Delta D)^2$, then the areas of equal control would be ellipsoids.

Fig. 3.15 Areas of equal control

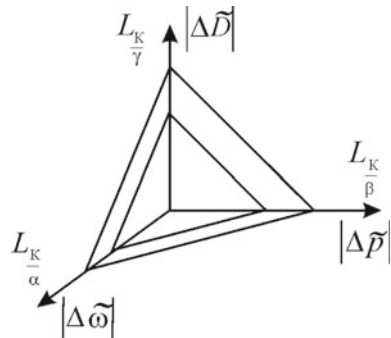
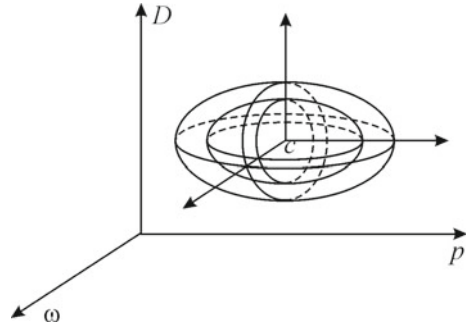


Fig. 3.16 The point of optimal control



The optimal control will correspond to the point $C(\omega_0, P_0, D_0)$ (Fig. 3.16). Here, ω_0 determines the own frequency of the short-period component of the aircraft, and P_0 and D_0 characterize the degree of adaptation of the pilot to this frequency by taking into account the damping coefficient.

3.12 Adjustment of Dynamic Parameters of the Aircraft Simulator

The tuning technique is based on the proposed criteria of dynamic similarity. AS and AC

$$K_y^{AS} \approx K_y^{AC},$$

$$\Phi_0^{AS} \approx \Phi_0^{AC}$$

or

$$L_{K-1}^{AS} \approx L_{K-1}^{AC},$$

$$L_K^{AS} \approx L_K^{AC},$$

$$d_{K-1}^{AS} \approx d_{K-1}^{AC},$$

$$d_K^{AS} \approx d_K^{AC}.$$

At the same time, in accordance with the above methodology, dependencies

$$K_y = K_y(a_1, a_2, \dots, a_5),$$

$$\Phi_0 = \Phi_0(a_1, a_2, \dots, a_5),$$

are obtained

where a_i —technical parameters of AS, which allow us to solve the problem of AS synthesis to the end.

In some cases, the models of the pilot's control action were corrected and replaced with nonlinear ones.

From the above, the AS certification algorithm obviously follows from the point of view of the formation of the required control skills in the trainees, namely: the control class is determined by the corresponding values of K_y and Φ_0 and their location in the areas of equal ratings.

3.13 Other Criteria for Evaluating Aerobatic Properties

According to [13], piloting can be divided into «large» control and «small» control. The criteria and assessments of dynamic aerobatic performance, discussed below, relate to «small» control.

For such control, the linear models, described in [13, 46] and the division of the aircraft movement into longitudinal and lateral, are valid. Therefore, dynamic aerobatic characteristics are considered separately for longitudinal and lateral movements. Moreover, the main role in assessing the stability and controllability of the aircraft is played by short-period motion. In view of this, the main estimates of dynamic aerobatic characteristics are based on linear models of short-period longitudinal and lateral movements.

The transient process of the short-period component of the longitudinal motion “small” is completely determined by the location in the complex plane of the roots of the characteristic equation, which we will write as

$$p^2 + 2\xi\omega p + \omega^2 = 0.$$

Consequently, the controllability of AC, based on the short-period component, will be determined by the location of the roots, which is the basis for the development of controllability criteria.

Let the roots be complex conjugate. Let's denote them

$$p_{1,2} = -\alpha \pm \beta_i.$$

Note that $\alpha = \xi\omega$, $\beta^2 + \alpha^2 = \omega^2$, $\alpha, \beta > 0$ when $0 < \xi < 1$, $\omega > 0$, and vice versa, $\omega > 0$, $0 < \xi < 1$, if $\alpha, \beta > 0$.

The quality of the transition process is determined by stability (positivity α); the frequency of oscillations (magnitude β); the actual duration of the oscillation of the transition process (which can be characterized by the ratio $\frac{\beta}{\alpha}$).

Thus, the values of α , β , $\frac{\beta}{\alpha}$ should be in some sense average for well-controlled AC, i.e. the criteria of controllability should be based on estimates of the parameter α , β , $\frac{\beta}{\alpha}$. In addition, the criteria should be simple enough, so that they can be used not only to easily assess the controllability of real AC, but also to calculate

the coefficients ξ , ω for the constructed AC, according to a given assessment of controllability [47].

Note that the criteria may not necessarily include all three parameters α , β , $\frac{\beta}{\alpha}$, but only a part of them.

Four quality criteria are given below (three scalar and one vector), which are functions of parameters ξ , ω or α , β , respectively.

$$1. \quad K_1(\xi, \omega) = \frac{a}{\xi^2} \left(\frac{1}{\omega^2} + b \right) \left(\text{or, respectively, } \tilde{K}_1(\alpha, \beta) = \frac{a}{\alpha^2} + ab \left(1 + \frac{\beta^2}{\alpha^2} \right) \right).$$

Lines of level G_m : $K_1(\xi, \omega) = m$, i.e. $\xi = \sqrt{\frac{a}{m}} \sqrt{\frac{1}{\omega^2} + b}$, $m \geq 1$.

Areas of equal assessments G_m : $m < K_1(\xi, \omega) < m + 1$, $m \geq 0$.

$$2. \quad K_2(\xi, \omega) = \frac{a}{\xi} \left(\frac{1}{\omega} + b \right) \left(\text{or, respectively, } \tilde{K}_2(\alpha, \beta) = \frac{a}{\alpha} + b \left(\alpha + \frac{\beta}{\alpha} \beta \right) \right).$$

Lines of level G_m : $K_2(\xi, \omega) = \sqrt{m}$, $m \geq 1$, $\xi = \frac{a}{\sqrt{m}} \left(\frac{1}{\omega} + b \right)$.

Areas of equal assessments G_m : $\sqrt{m} < K_2(\xi, \omega) < \sqrt{m+1}$, $m \geq 0$.

$$3. \quad K_3(\xi, \omega) = \frac{a}{\xi\omega} + b \frac{\omega}{\xi} + \frac{c}{\omega\sqrt{1-\xi^2}} \left(\text{or, respectively, } \tilde{K}_3(\alpha, \beta) = \frac{a}{\alpha} + b \left(\alpha + \frac{\beta}{\alpha} \beta \right) + \frac{c}{\beta} \right).$$

$$\text{Lines of level } G_m: K_3(\xi, \omega) = \sqrt{m}, \quad \omega_{1,2} = \frac{1}{2} \left(\frac{\sqrt{m}}{b} \xi \pm \sqrt{\frac{m}{b^2} \xi^2 - \frac{4}{b} \left(a + \frac{c\xi}{\sqrt{1-\xi^2}} \right)} \right).$$

Areas of equal assessments G_m : $\sqrt{m} < K_3(\xi, \omega) < \sqrt{m+1}$.

$$4. \quad \text{Vector criterion } K_4(\xi, \omega) = \left(\xi\omega; \omega\sqrt{1-\xi^2} \right) \left(\text{or, } \tilde{K}_4(\alpha, \beta) = (\alpha, \beta), \text{ respectively} \right).$$

Lines of level G_m : $\xi\omega = \frac{a_1}{\sqrt{m}}$, $\omega\sqrt{1-\xi^2} = \frac{b_1}{\sqrt{m}}$, $\omega\sqrt{1-\xi^2} = b_2\sqrt{m}$.

Areas of equal assessments $G_1 = D_1$, $G_m = D_m - D_{m-1}$, $m \geq 2$, where $D_m: \frac{a_1}{\sqrt{m}} < \xi m$, $\frac{b_1}{\sqrt{m}} < \omega\sqrt{1-\xi^2} < b_2\sqrt{m}$.

The lines of level and areas of equal assessments corresponding to criteria of quality 1–4 are shown in Figs. 3.17, 3.18, 3.19 and 3.20. The lines of level correspond to the classification parameter m , with an increase in which the quality of the transition process deteriorates.

Note that the criteria $K_1(\xi, \omega)$ and $K_2(\xi, \omega)$ do not limit the frequency of oscillations of the transient process from below, and the criterion $K_4(\xi, \omega)$ does not limit the attenuation rate from above and, accordingly, the duration of the transient process from below. Such restrictions can be made either additionally, or they naturally arise from design considerations for real dynamic systems.

The criterion $K_3(\xi, \omega)$ gives restrictions from below and from above on all the parameters of the quality of the transition process (i.e. on α , β , $\frac{\beta}{\alpha}$). With the increase $K_3(\xi, \omega)$, the quality of the transition process deteriorates.

Fig. 3.17 Lines of level in the area of equal assessments

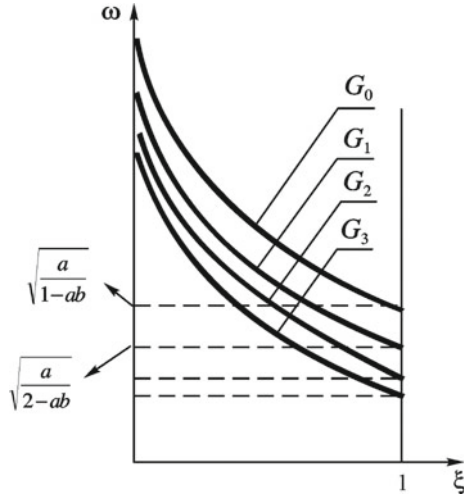
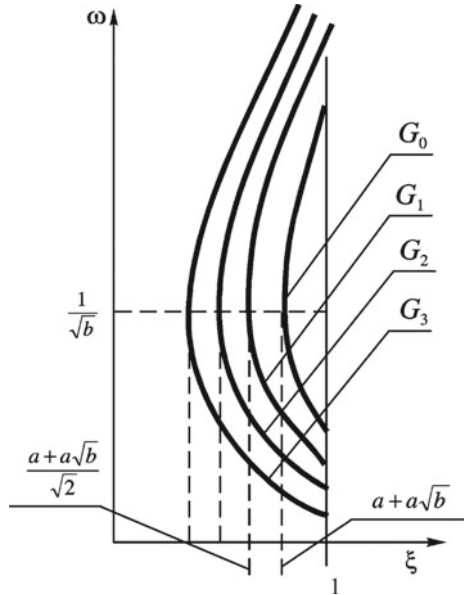


Fig. 3.18 Lines of level in the area of equal assessments



Note that for the criteria $K_2(\xi, \omega)$, $K_3(\xi, \omega)$ and $K_4(\xi, \omega)$, \sqrt{m} is selected as a classification parameter in such a way that the areas of equal assessments really correspond to transition processes, which are approximately equal in quality, if the parameters are taken from the same area G_m (starting from $m \geq 1$).

Fig. 3.19 Lines of level in the area of equal assessments

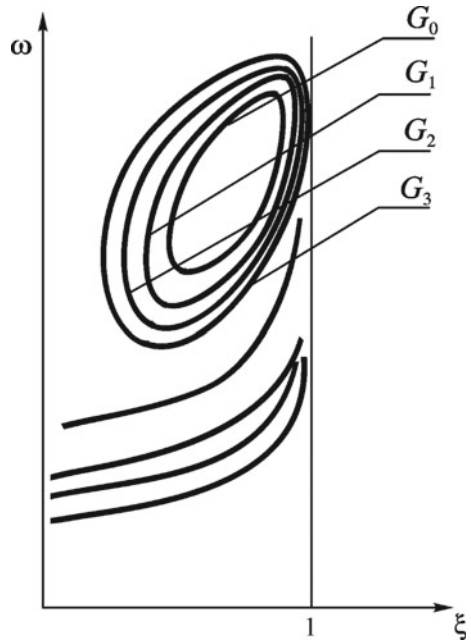
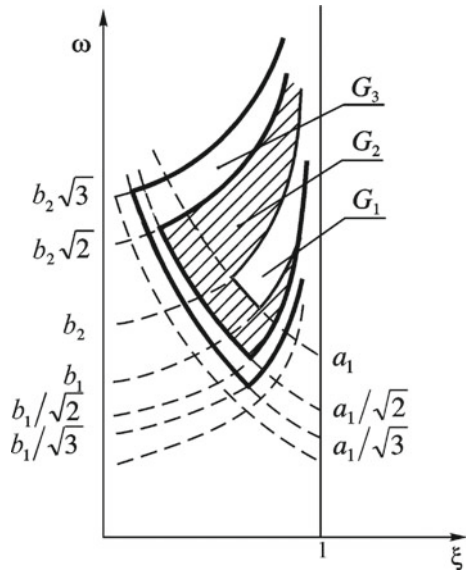


Fig. 3.20 Lines of level in the area of equal assessments



3.14 Selection of the Criterion for the Quality of Control by Energy Costs of the System

When training operators to control systems and evaluate training complexes (in particular, simulators), it is necessary to have criteria of quality for control (operator's control actions on the system, when performing a specific task) quite simple (numerical or graphical with a small amount of easily perceived information), quickly implemented, perhaps more fully expressing the essence of the control process, quite understandable to the student.

In order to develop such criteria, it is necessary to identify the main factors, that fully characterize the operator's control actions.

Such factors include:

- the quality of completing the task (regardless of the state of the system after completing the task);
- the cost of energy resources of the system to complete the task;
- clarity of the operator's actions (minimum unnecessary impacts on the system, when performing the task).

It is difficult to formalize the assessment of the first factor in the general case. This can only be done very roughly on a two-point system: the task is completed, the task is not completed. In specific situations, it is possible to take into account some specific features of the task, and therefore, give a more differentiated assessment. Therefore, the first factor is taken into account further as follows: the quality of control is considered only if the task is completed, and only in this case, the value of the quality criterion for the corresponding control is calculated. It should be noted that for the purposes of training, it is important just to complete the task with the rational use of systems, i.e. the energy costs of the system for completing the task play an important role. It is also important to develop a clear skill in working with the system from the operator.

Thus, when developing a criterion of quality for control (implementing the completion of a task), the main attention is paid to the second and third factors.

The energy costs of the system can be described by a functional E_u (it is assumed that it is possible to calculate control actions $u(t)$ at any time t in the interval $[T_0, T_1]$: T_0 —start time, T_1 —end time of the task completion)

$$E_u = A \int_{T_0}^{T_1} (|u(t)| + |\dot{u}(t)| + |\ddot{u}(t)|) dt,$$

here A is a certain normalizing multiplier, the choice of such a multiplier will be discussed below.

The value $u(t)$ characterizes the energy costs, and $|\dot{u}(t)|$, $|\ddot{u}(t)|$ —dynamic loads, arising in the system due to the actions of the operator. In general, the functional E_u characterizes the energy costs of the system.

The fuzziness of the operator's actions is characterized by unnecessary impacts on the system, which can be fully assessed by the functional

$$Z_u = B(Z_u + Z_{\dot{u}}),$$

where Z_u —length of curve $u(t)$, $T_0 \leq t \leq T_1$; $Z_{\dot{u}}$ —length of curve $\dot{u}(t)$; B —the normalizing multiplier.

The functional Z_u also characterizes the cost of system resources, taking into account the duration of the completion of task. Indeed, the functional Z_u can be written as

$$Z_u = B \int_{T_0}^{T_1} \left(\sqrt{1 + (\dot{u}(t))^2} + \sqrt{1 + (\ddot{u}(t))^2} \right) dt.$$

Since $\frac{(1+|\alpha|)}{\sqrt{2}} \leq \sqrt{1 + \alpha^2} \leq 1 + |\alpha|$ (it is not difficult to prove by squaring), then Z_u equivalently can be implemented in the form

$$B_1 \int_{T_0}^{T_1} (2 + |\dot{u}(t)| + |\ddot{u}(t)|) dt.$$

By virtue of all the above, the general criterion for the quality of control is defined by the formula

$$R = C \int_{T_0}^{T_1} (2 + |u(t)| + 2|\dot{u}(t)| + 2|\ddot{u}(t)|) dt$$

or

$$R = C (R_T + R_u + R_{\dot{u}} + R_{\ddot{u}}),$$

where $R_T = 2(T_1 - T_0)$, $R_u = \int_{T_0}^{T_1} |u(t)| dt$, $R_{\dot{u}} = 2 \int_{T_0}^{T_1} |\dot{u}(t)| dt$, $R_{\ddot{u}} = \int_{T_0}^{T_1} |\ddot{u}(t)| dt$.

The normalizing factor C is defined so that, for example, well-trained operators receive a value R in the interval $[0, 4]$ —operators of the 1st and 2nd class (for operators of the 1st class— $R \leq 2$, for operators of the 2nd class— $2 < R \leq 4$).

In the case of a discrete setting of control $u(t)$ with a step h (in practice, it can be implemented graphically; in this case, removing the values $u(t)$, one should take into account the scale and, possibly, change the normalizing factor C), we obtain

$$R_u = h \sum_{i=2}^n |u_i|, \quad u_i = u(t_i), \quad t_{i+1} - t_i = h,$$

$$R_{\dot{u}} = 2 \sum_{i=2}^n |u_i - u_{i-1}|, \quad R_{\ddot{u}} = \frac{2}{h} \sum_{i=2}^{n-1} |u_{i+1} - 2u_i + u_{i+1}|.$$

In the functional R , the component R_T characterizes the duration of the task completion; R_u is the energy costs of the system for the task completion; R_{ii} is fluctuations in the system; R_{ij} is dynamic overloads.

3.15 Description of the Functional of Quality of Control as an Impulse Process

When describing the functional, we limit ourselves to the case, when the generalized vector of control, as its components, contains:

- A_i — amplitudes of control pulse;
- t_i — pulse duration;
- n — the number of pulses for a given duration of implementations;
- T — duration of implementations.

When forming a generalized vector of control, the well-known hypothesis of Milsam and Miller [34, 37] about the impulsive nature of control of human operator is assumed to be valid.

When a fixed duration T of implementation, the longer n , the obviously higher the activity and, accordingly, the psychophysiological tension of the operator. It is clear that the activity of a human operator cannot be zero, which follows from the hypothesis of Milsam and Miller, according to which the operator in the process of control not only acts in a pulse mode, but also interrogates the object in the process of control. Human activity cannot also be infinite. Therefore, the ratio $\frac{T}{n}$ must belong to some optimal interval of the numeric axis. If we assume that all the weight constants of the additive global criterion of quality are positive, then it follows from what has been said that the particular criteria of quality should be taken as $\frac{T}{n}$, so and $\frac{n}{T}$. Then both the increase and decrease n are limited, and its optimal value will be determined for a given duration T .

It is not difficult to see further that the energy costs for program control stabilization should be minimal. Since these costs are most simply set by the value $\sum_i S_i$, where S_i is the area of the i -th pulse, it is obvious that it is necessary to include $\sum_i S_i$ in the global criterion as a particular. However, it is clear that the energy costs should be optimal: at very low costs, the system will be «sluggish», and too high costs are also not acceptable. If we consider that

$$S_i \approx A_i t_i,$$

then from this, similarly to the previous one, it follows that it is necessary to include the value $\frac{A_i}{t_i}$ in the number of particular criteria (once again, we emphasize the positivity of weight constants!). In a certain sense, $\frac{A_i}{t_i}$ characterizes overloads in the system. Therefore, in the first approximation, we can limit ourselves to the inclusion of a particular criterion $\max_i \frac{A_i}{t_i}$.

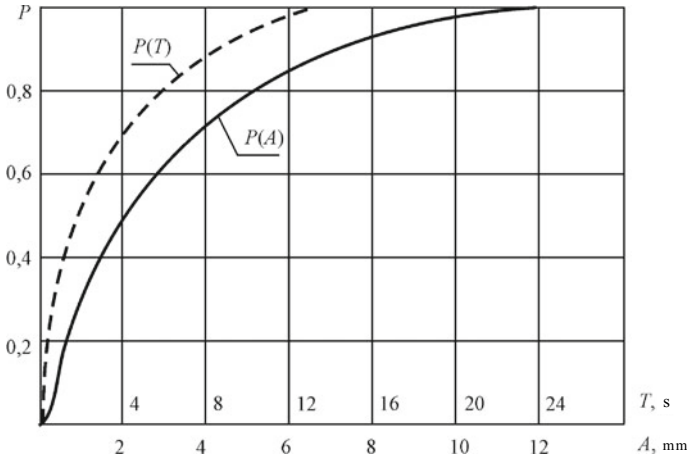


Fig. 3.21 Probabilities $P(A)$ and $P(T)$ obtained as a result of processing the operator’s control movements as random functions

Based on the above, it is recommended to use the functional

$$F(AS) = a_1 \max_i \frac{A_i}{t_i} + a_2 \sum_i S_i + a_3 \frac{T}{n} + a_4 \frac{n}{T}$$

to assess the quality of AS

Of course, this functional does not pretend to be complete. It can always be corrected by introducing additional private criteria.

It is assumed that $\sum_i a_i = 1$. The choice of weight constants is easy to implement in the previously proposed way by taking into account the degree of correlation between $F(AS)$ and particular criteria of quality. In the first approximation, the weight constants can be assumed to be the same. Figure 3.21 illustrates the probabilities $P(A)$ and $P(T)$ obtained as a result of statistical processing of the operator’s control movements as random functions.

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Chapter 4

Some Issues of Practical Development of Flight Dynamics Simulators



4.1 Analysis of Existing Methods of Setting Up an Aircraft Simulator

Let's limit ourselves to the consideration of the short-period component of the longitudinal motion:

$$\begin{aligned} \dot{\omega}_z &= a_{11}\omega_z - a'_{11}\dot{\omega}_z + a_{12}\alpha + a'_{12}\dot{\alpha} + a_{13}\delta_b + a'_{13}\dot{\delta}_b, \\ \dot{\alpha} &= \omega_z + a_{42}\alpha + a_{43}\delta_b. \end{aligned} \tag{4.1}$$

We study the dependence of the output coordinates $\omega_z(t)$, $\alpha(t)$ on parameters a_{11} , a'_{11} , a'_{12} , a'_{13} .

To simplify subsequent entries and transformations for output coordinates and controls, we introduce the notation

$$x_1 = \omega_z(t), \quad x_2 = \alpha(t), \quad \delta = \delta_b(t).$$

Initial values of parameters a_{11} , a_{12} , a_{42} , a_{13} , a_{43} (without variations) denote, respectively, a_{11} , a_{12} , a_{22} , b_1 , b_2 .

The system of equations of motion

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + b_1\delta, \\ \dot{x}_2 &= x_1 + a_{22}x_2 + b_2\delta \end{aligned} \tag{4.2}$$

let's call the initial system (without variations of parameters). Variations of parameters a'_{11} , a'_{12} , a'_{13} denote, respectively, ε_1 , ε_2 , ε_3 , ε_4 .

Initial values of parameters a'_{11} , a'_{12} , a'_{13} take equal to zero.

It is clear that a system with non-zero initial values a'_{11} , a'_{12} , a'_{13} is easy to reduce to a system (4.1) by simple algebraic transformations.

In accordance with the entered designations, we will rewrite the system (4.1) in the form of

$$\begin{aligned}\dot{x}_1 &= \frac{a_{11} + \varepsilon_1 + \varepsilon_3}{1 + \varepsilon_2} x_1 + \frac{a_{12} + a_{22}\varepsilon_3}{1 + \varepsilon_2} x_2 + \frac{b_2\varepsilon_3 + b_1}{1 + \varepsilon_2} + \frac{\dot{\varepsilon}_4}{1 + \dot{\varepsilon}_2} \dot{\delta}, \\ \dot{x}_2 &= x_1 + a_{22}x_2 + b_2\dot{\delta}.\end{aligned}\quad (4.3)$$

Denote by σ the trace of system matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ 1 & a_{22} \end{bmatrix}$; by σ_1 —trace of the corresponding matrix of the system (4.3).

Then

$$\begin{aligned}\sigma &= a_{11} + a_{22}, \\ \sigma_1 &= \frac{a_{11} + \varepsilon_1 + \varepsilon_3}{1 + \varepsilon_2} + a_{22} = \sigma + \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_2 a_{11}}{1 + \varepsilon_2} = \sigma + \varepsilon,\end{aligned}$$

where $\varepsilon = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_2 a_{11}}{1 + \varepsilon_2}$.

To study the dependencies of the output coordinates on the technical parameters, we take second-order approximation formulas. This will allow you to get fairly accurate estimates without complicating the results with technical details.

These formulas have the form

$$\begin{aligned}\dot{x}_i(t) &= x_i(t_0) + \int_{t_0}^t \left(t - S + \frac{\sigma}{2}(t - S)^2 \right) f_i(S) dS, \quad (i = 1, 2), \\ f_1(S) &= a_{12}b_2\dot{\delta} + b_1\dot{\delta}, \quad f_2(S) = b_1\dot{\delta} + b_2\dot{\delta}.\end{aligned}$$

For the system

$$\begin{aligned}\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + b_1\dot{\delta} + c_1\ddot{\delta}, \\ \dot{x}_2 &= x_1 + a_{22}x_2 + b_2\dot{\delta}\end{aligned}$$

functions $f_1(S)$, $f_2(S)$, respectively, equal

$$f_1(S) = a_{12}b\dot{\delta} + b_1\dot{\delta} + c_1\ddot{\delta}, \quad f_2(S) = b_1\dot{\delta} + (c_1 + b_2)\dot{\delta}.\quad (4.4)$$

It is easy to compare the output coordinates of systems (4.2) and (4.3).

Indeed, denoting the output coordinates of the system (4.2) by $x_1(t)$, $x_2(t)$, and the output coordinates of the system (4.3) by $\tilde{x}_1(t)$, $\tilde{x}_2(t)$ (functions $f_i(S)$ for the system (4.3) denote, respectively, $\tilde{f}_i(S)$), under the same initial conditions for systems (4.2) and (4.3), we obtain

$$\begin{aligned} \tilde{x}_1(t) &= x_i(t) + (\tilde{x}_i(t) - x_i(t)) = x_i(t) + \int_{t_0}^t (t-S) \left(\tilde{f}_i(S) - f_i(S) \right) dS + \\ &+ \int_{t_0}^t \frac{(t-S)^2}{2} \left(\sigma_1 \tilde{f}_i(S) - \sigma f_i(S) \right) dS = x_i(t) + \int_{t_0}^t (t-S) \left(\tilde{f}_i(S) - f_i(S) \right) dS + \\ &+ \frac{\sigma}{2} \int_{t_0}^t (t-S)^2 \left(\tilde{f}_i(S) - f_i(S) \right) dS + \frac{\varepsilon}{2} \int_{t_0}^t (t-S)^2 \tilde{f}_i(S) dS. \end{aligned}$$

Thus,

$$\begin{aligned} \tilde{x}_i(t) - x_i(t) &= \int_{t_0}^t \left[(t-S) + \frac{\sigma}{2} (t-S)^2 \right] \left(\tilde{f}_i(S) - f_i(S) \right) dS + \frac{\varepsilon}{2} \int_{t_0}^t (t-S)^2 \tilde{f}_i(S) dS, \\ \sigma &= a_{11} + a_{22}, \quad \varepsilon = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_2 a_{11}}{1 + \varepsilon_2}, \\ f_1(S) &= a_{12} b_2 \delta + b_1 \dot{\delta}, \quad f_2(S) = b_1 \delta + b_2 \dot{\delta}, \\ \tilde{f}_1(S) &= \frac{a_{12} + a_{22}}{1 + \varepsilon_2} b_2 \delta + \frac{b_2 \varepsilon_3 + b_1}{1 + \varepsilon_2} \dot{\delta} + \frac{\varepsilon_4}{1 + \varepsilon_2} \ddot{\delta}, \\ \tilde{f}_2(S) &= \frac{1}{1 + \varepsilon_2} \left[\delta (b_2 \varepsilon_3 + b_1) + \dot{\delta} (\varepsilon_4 + b_2 (1 + \varepsilon_2)) \right]. \end{aligned} \quad (4.5)$$

Formulas (4.5) allow us to estimate the change of output coordinates $x_1(t)$, $x_2(t)$ (i.e. $\omega_z(t)$, $\alpha(t)$) with the given variations of parameters a_{11} , a'_{11} , a'_{12} , a'_{13} .

Obviously, this technique allows you to study the dependence $\omega_z(t)$, $\alpha(t)$ and from the rest of the system parameters.

4.2 Approximate Methods for Estimating the Effect of Delay on the Vector of Output Coordinates

Let, as before,

$$\dot{x}(t) = Ax(t) + Bu(t),$$

A , B —matrices of dimension $n \times n$, $n \times m$, accordingly. And let $u(t)$ be implemented in one of the ways:

$$u(t) = Px(t)$$

or

$$u(t) = Px(t - \tau).$$

Let's compare two types of movements $x(t)$ and $y(t)$, which are solutions of initial problems:

$$\begin{aligned} \text{and } \dot{x}(t) &= A_1x(t), \quad x(t_0) = x_0 \\ \dot{y}(t) &= Ay(t) + B_1y(t - \tau), \\ y(t - \tau) &\equiv x_0 \text{ when } t - \tau \leq t_0. \end{aligned}$$

Here $B_1 = BP$, $A_1 = A + B_1 = A + BP$.

Let's estimate the difference $x(t) - y(t)$ when $t \geq \tau$.

4.2.1 Operator Method for Solving the Differential-Difference Equation

Assuming τ sufficiently small and neglecting to simplify the calculations of the values by order τ^2 , we write down the solutions of the problems in the form

$$x(t) = x(t - \tau) + \tau A_1x(t), \quad x(t - \tau) \equiv x_0 \text{ when } t - \tau \leq 0, \quad (4.6)$$

$$y(t) = y(t - \tau) + \tau(Ay(t) + B_1y(t - \tau)), \quad y(t - \tau) \equiv x_0 \text{ when } t - \tau \leq 0. \quad (4.7)$$

Since for small τ there are operators $(E - \tau A_1)^{-1}$ and $(E - \tau A)^{-1}$, then (4.6) and (4.7) can be rewritten as

$$x(t) = (E - \tau A_1)^{-1}x(t - \tau), \quad x(t - \tau) \equiv x_0 \text{ when } t - \tau \leq t_0,$$

$$y(t) = (E - \tau A)^{-1}(E + \tau B_1)y(t - \tau), \quad y(t - \tau) \equiv x_0 \text{ when } t - \tau \leq t_0.$$

For $t_k = t_0 + k\tau$ let's introduce $x_k = x(t_k)$, $y_k = y(t_k)$, we will get

$$x_k = (E - \tau A_1)^{-k}x_0, \quad k \geq 1.$$

Note also that

$$\begin{aligned} x_1 &= x_0 + \tau A_1x_0, \quad \tau A_1x_0 = x_1 - x_0, \\ y_1 &= y_0 + \tau(Ay_0 + B_1y(t_0 - \tau)) = x_0 + \tau A_1x_0 = x_1. \end{aligned}$$

Thus, $x_0 = y_0$, $x_1 = y_1$.

Up to τ^2 , an approximation of the resolvents and operators, standing in the right-hand sides for x_k and y_k , gives

$$\begin{aligned}
x_k &= (E - \tau A)^{-k} x_0 = x_0 + k\tau A_1 x_0 + \frac{k(k+1)}{2} \tau A_1 \tau A_1 x_0 = \\
&= x_0 + k(x_1 - x_0) + \frac{k(k+1)}{2} \tau A_1 (x_1 - x_0) \\
y_k &= [(E - \tau A)^{-1} (E + \tau B_1)]^k x_0 = [E + \tau A_1 + \tau A \cdot \tau A_1]^k x_0 \\
&= [E + (E + \tau A) \tau A_1]^k x_0 = \\
&= x_0 + k(E + \tau A)(x_1 - x_0) + \frac{k(k+1)}{2} (E + \tau A)^2 \tau^2 A_1^2 x_0
\end{aligned}$$

or $y_k = x_0 + k(x_1 - x_0) + \frac{k(k+1)}{2} \tau(A + BP)(x_1 - x_0) + \tau kBP(x_1 - x_0)$.

Thus, $x_k - y_k = \tau kBP(x_1 - x_0)$ when $k \geq 2$ or, finally,

$$x_k - y_k = \tau^2 kBP(A + BP)x_0 \text{ when } k \geq 2. \quad (4.8)$$

4.2.2 Euler's Method with an Integration Step of τ

Fair

$$\begin{aligned}
x_1 &= x_0 + \tau(A + B)x_0, & y_1 &= x_0 + \tau(A + B_1)x_0, \\
x_2 &= x_1 + \tau(A + B_1)x_1, & y_2 &= x_1 + \tau(Ax_1 + B_1x_0), \\
x_3 &= x_2 + \tau(A + B_1)x_2, & y_3 &= y_2 + \tau(Ay_2 + B_1x_1), \\
x_1 &= y_1, & x_2 - y_2 &= \tau B_1(x_1 - x_0).
\end{aligned}$$

Since

$$\begin{aligned}
\tau A(x_2 - y_2) &= \tau^2 AB_1(x_1 - x_0), \\
\tau B_1(x_2 - x_1) &= \tau B_1(x_1 - x_0 + \tau A_1(x_1 - x_0)), \\
x_3 - y_3 &= (x_2 - y_2) + \tau A(x_2 - y_2) + \tau B_1(x_2 - x_1),
\end{aligned}$$

then $x_3 - y_3 = 2\tau B_1(x_1 - x_0) + 0(\tau^2)$.

Similarly,

$$x_k - y_k = \tau(k-1)B_1(x_1 - x_0) = \tau^2 BP(k-1)(A + BP)x_0. \quad (4.9)$$

As you can see, there is a slight difference in the estimates (4.8) and (4.9) (instead of k in the expression (4.9) we made a replacement $(k-1)$). This is due to the nature of the approximation of the solution.

Of course, more accurate estimates are possible, taking into account the values of the order τ^2 .

To do this, the solutions should be approximated at each step as follows:

$$x(t) = x(t - \tau) + \tau A_1 x(t) + \frac{\tau^2}{2} A_1^2 x(t) + 0(\tau^3).$$

Accordingly,

$$y(t) = y(t - \tau) + \tau(Ay(t) + B_1 y(t - \tau)) + \frac{\tau^2}{2}(A(Ay(t) + B_1 y(t - \tau)) + B_1(Ay(t - \tau) + B_1 y(t - 2\tau) + 0(\tau^3))).$$

However, this only complicates the calculations and, in essence, gives the same result.

4.2.3 The Euler Method with an Integration Step of $\delta = \tau/m$

In this case $x_1 = \delta(A + B_1)x_0 + x_0$, $\delta A_1 x_0 = x_1 - x_0$, $y_1 = \delta(A + B_1)x_0 + x_0$, I.E. $x_1 = y_1$.

Taking into account, that for $k \leq m$ rightly $y(t_k - \tau) \equiv x_0$, we will get

$$\begin{aligned} x_k &= x_{k-1} + \delta(A + B_1)x_{k-1}, \\ y_k &= y_{k-1} + \delta A y_{k-1} + \delta B_1 x_0. \end{aligned}$$

Therefore, for $k \leq m$ will be executed.

$$x_2 - y_2 = \delta B_1(x_1 - x_0),$$

$$\begin{aligned} x_3 - y_3 &= x_2 - y_2 + \delta A(x_2 - y_2) + \delta B_1(x_2 - x_0) = \\ &= \delta B_1(x_1 - x_0) + \delta B_1(x_2 - x_0) + 0(\delta^2) = \delta B_1(x_1 + x_2 - 2x_0) + 0(\delta^2), \\ x_4 - y_4 &= x_3 - y_3 + \delta B_1(x_3 - x_0) = \delta B_1(x_1 + x_2 + x_3 - 2x_0) + 0(\delta^2), \\ x_k - y_k &= \delta B_1(x_1 + x_2 + \dots + x_{k-1} - (k-1)x_0) + 0(\delta^2), \quad k \leq m. \end{aligned} \quad (4.10)$$

Since $x_j = (E + \delta A_1)^j x_0$, $j \geq 1$, then up to $0(\delta^2)$ rightly

$$\begin{aligned} x_j &= (E + j\delta A_1)x_0 \approx x_0 + j(x_1 - x_0), \\ (x_1 - x_0) &= \delta A_1 x_0. \end{aligned}$$

Therefore,

$$\sum_{j=1}^{k=1} x_j = (k-1)x_0 + \frac{k(k-1)}{2}(x_1 - x_0). \quad (4.11)$$

Substituting $\sum x_j$ from (4.11) to (4.10), we will get.

$$x_k - y_k = \delta B_1 \frac{k(k-1)}{2} (x_1 - x_0), \quad k \leq m,$$

or, finally,

$$x_k - y_k = \frac{k(k-1)}{2} \delta^2 B P (A + B P) x_0, \quad k \leq m. \quad (4.12)$$

Note that we can get estimates and for $k > m$. However, it is quite difficult to record a general law. So, when $k = m + 1$

$$y(t_k - \tau) = y_1 = x_1,$$

where from

$$\begin{aligned} x_{m+1} - y_{m+1} &= x_m - y_m + \delta A(x_m - y_m) + \delta B_1(x_m - x_1) = \\ &= \frac{m(m-1)}{2} \delta B_1(x_1 - x_0) + 0(\delta^2) + \delta B_1(x_0 + m(x_1 - x_0) - x_1) = \\ &= \left(\frac{m(m-1)}{2} + (m-1) \right) \delta B_1(x_1 - x_0) \approx \frac{m(m+1)}{2} \delta B P(x_1 - x_0). \end{aligned} \quad (4.13)$$

Для $1 < j \leq m - 1$ we will get

$$\begin{aligned} x_{m+j+1} - y_{m+j+1} &= x_{m+j} - y_{m+j} + \delta A(x_{m+j} - y_{m+j}) + \delta B_1(x_{m+j} - y_j) = \\ &= \delta \left(\frac{m(m-1)}{2} + jm \right) B P(x_1 - x_0). \end{aligned} \quad (4.14)$$

4.3 An Accurate Method for Assessing the Effect of Lag in the Systems of Simulation of Physical Factors of Flight on Aerobatic Performance

Consider the system of differential equations of motion AC.

$$\left. \begin{aligned} \alpha'(t) &= a_{11} \alpha(t) + a_{12} \omega(t) + b_1 u(t) \\ \omega'(t) &= a_{21} \alpha(t) + a_{22} \omega(t) + b_2 u(t) \\ c u'(t) &= p_1 \alpha(t - \tau_1) + p_2 \omega(t - \tau_2) + p_3 \theta(t - \tau_2) - u(t) \\ \theta'(t) &= \omega(t) \\ (t) &\geq 0 \end{aligned} \right\},$$

where $\tau_1, \tau_2 > 0$ and

$$\left. \begin{aligned} \alpha(t) &= S_1(t), \quad t \leq 0 \\ \omega(t) &= S_2(t), \quad t \leq 0 \\ \alpha(+0) &= S_1(0) \\ \omega(+0) &= S_2(0) \\ u(+0) &= U_0 \\ \theta(+0) &= \int_{-\tau_2}^0 S_2(\eta) d\eta \end{aligned} \right\}.$$

To solve the problem, we apply algebraic operational calculation—the method of perfect operators [1–3].

The product $\varphi * \psi$ of two functions (turns to zero each to the left of some point, and one of the functions ψ is locally absolutely continuous) we will understand as a differentiated convolution:

$$\begin{aligned} \varphi * \psi &= \frac{d}{dt} \int_{-\infty}^{\infty} \varphi(t - \eta) \psi(\eta) d\eta = \frac{d}{dt}(\varphi \cdot \psi) = \\ &= \int_{-\infty}^{\infty} \varphi(t - \eta) \psi'(\eta) d\eta = \varphi \cdot \psi'. \end{aligned}$$

Let's introduce $(\varphi(t))_+ = \{\varphi(t); t > 0, 0, t \leq 0\}$, $\psi = [\psi(t)]$ —the operator of the function type $\psi(t)$.

Numerical operators γ_+ , i.e. the operator of the function type $\gamma_+ = \{\gamma; t > 0, 0, t \leq 0\}$, $\gamma_+ = \{\gamma; t > 0, 0, t \leq 0\}$, we will denote, when it does not cause misunderstandings, as numbers γ . The value of an operator over a function and the product of two operators is denoted by $*$.

D —operator of differentiation:

$$\phi' = D * (\dot{\phi} - \phi(+0)), \quad \text{where } \phi = [(\phi(t))_+].$$

T_τ —operator of shift:

$$T_\tau * \psi = \psi(t - \tau) \quad (T_0 = 1_+).$$

In the operator form, the problem after the elimination of the last equation will be written as follows:

$$\left. \begin{aligned} D * (\alpha - S_1(0)) &= a_{11} \alpha + a_{12} \omega + b_1 u \\ D * (\omega - S_2(0)) &= a_{21} \alpha + a_{22} \omega + b_2 u \\ cD * (u - u_0) &= p_1 T_{\tau_1} * (S_1 + \alpha) + p_2 T_{\tau_2} * (S_2 + \omega) + \\ &\quad + p_3 T_{\tau_2} * D^{*-1} * (S_2 + \omega) - u \end{aligned} \right\}.$$

Here

$$\alpha = [(\alpha(t))_+], \quad \omega = [(\omega(t))_+], \quad u = [(u(t))_+], \\ S_i = [0, t < -\tau_i; S_i(t), -\tau \leq t \leq 0; 0, t > 0] \quad (i = 1, 2).$$

From the third equation, we find

$$u = (cD + 1)^{*-1} *$$

$$*(c u_0 D + p_1 T_{\tau_1} * (S_1 + \alpha) + p_2 T_{\tau_2} * (S_2 + \omega) + p_3 T_{\tau_2} * D^{*-1} * (S_2 + \omega))$$

and substitute in the first and second equations:

$$\left. \begin{aligned} D * (\alpha - S_1(0)) &= a_{11} \alpha + a_{12} \omega + (cD + 1)^{*-1} * \\ *(c b_1 u_0 D + b_1 p_1 T_{\tau_1} * (S_1 + \alpha) + b_1 p_2 T_{\tau_2} * (S_2 + \omega) + b_1 p_3 T_{\tau_2} * D^{*-1} * (S_2 + \omega)) \\ D * (\omega - S_2(0)) &= a_{21} \alpha + a_{22} \omega + (cD + 1)^{*-1} * \\ *(c b_2 u_0 D + b_2 p_1 T_{\tau_1} * (S_1 + \alpha) + b_2 p_2 T_{\tau_2} * (S_2 + \omega) + b_2 p_3 T_{\tau_2} * D^{*-1} * (S_2 + \omega)) \end{aligned} \right\},$$

or

$$\left. \begin{aligned} (cD^{*3} + D^{*2}) * (\alpha - S_1(0)) &= (cD^{*2} + D) * (a_{11} \alpha + a_{12} \omega) + c b_1 u_0 D^{*2} + \\ + b_1 p_1 T_{\tau_1} * D * (S_1 + \alpha) + b_1 p_2 T_{\tau_2} * D * (S_2 + \omega) + b_1 p_3 T_{\tau_2} * (S_2 + \omega) \\ (cD^{*3} + D^{*2}) * (\omega - S_2(0)) &= (cD^{*2} + D) * (a_{21} \alpha + a_{22} \omega) + c b_2 u_0 D^{*2} + \\ + b_2 p_1 T_{\tau_1} * D * (S_1 + \alpha) + b_2 p_2 T_{\tau_2} * D * (S_2 + \omega) + b_2 p_3 T_{\tau_2} * (S_2 + \omega) \end{aligned} \right\},$$

or

$$\left. \begin{aligned} (cD^{*3} + D^{*2} - c a_{11} D^{*2} - a_{11} D - b_1 p_1 T_{\tau_1} * D) * \alpha + \\ + (-c a_{12} D^{*2} - a_{12} D - b_1 p_2 T_{\tau_2} * D - b_1 p_3 T_{\tau_2}) * \omega = \\ = c S_1(0) D^{*3} + (S_1(0) + c b_1 u_0) D^{*2} + D * (b_1 p_1 T_{\tau_1} * S_1 + b_1 p_2 T_{\tau_2} * S_2) + \\ + b_1 p_3 T_{\tau_2} * S_2 \\ (-c a_{21} D^{*2} - a_{21} D - b_2 p_1 T_{\tau_1} * D) * \alpha + \\ + (cD^{*3} + D^{*2} - c a_{22} D^{*2} - a_{22} D - b_2 p_2 T_{\tau_2} * D - b_2 p_3 T_{\tau_2}) * \omega = \\ = c S_2(0) D^{*3} + (S_2(0) + c b_2 u_0) D^{*2} + D * (b_2 p_1 T_{\tau_1} * S_1 + b_2 p_2 T_{\tau_2} * S_2) + \\ + b_2 p_3 T_{\tau_2} * S_2 \end{aligned} \right\}.$$

Let's move on to the matrix-vector notation. We have

$$\begin{aligned} & \left[\begin{array}{l} cD^{*3} + D^{*2} - ca_{11}D^{*2} - a_{11}D - b_1p_1T_{\tau_1} * D - ca_{12}D^{*2} - a_{12}D - b_1p_2T_{\tau_2} * D - b_1p_3T_{\tau_1} \\ -ca_{21}D^{*2} - a_{21}D - b_2p_1T_{\tau_1} * D - cD^{*3} + D^{*2} - ca_{22}D^{*2} - a_{22}D - b_2p_2T_{\tau_2} * D - b_2p_3T_{\tau_2} \end{array} \right] * \\ & * \begin{bmatrix} \alpha \\ \omega \end{bmatrix} = \begin{bmatrix} cS_1(0)D^{*3} + (S_1(0) + cb_1u_0)D^{*2} + D * (b_1p_1T_{\tau_1} * S_1 + b_1p_2T_{\tau_2} * S_2) + b_1p_3T_{\tau_2} * S_2 \\ cS_2(0)D^{*3} + (S_2(0) + cb_2u_0)D^{*2} + D * (b_2p_1T_{\tau_1} * S_1 + b_2p_2T_{\tau_2} * S_2) + b_2p_3T_{\tau_2} * S_2 \end{bmatrix} \end{aligned}$$

or

$$\begin{aligned} & \left(cD^{*3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + D^{*2} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - c \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \right) - D * \left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_1p_1 & b_1p_2 \\ b_2p_1 & b_2p_2 \end{bmatrix} \begin{bmatrix} T_{\tau_1} & 0 \\ 0 & T_{\tau_2} \end{bmatrix} \right) - \right. \\ & \left. - p_3T_{\tau_2} \begin{bmatrix} 0 & b_1 \\ 0 & b_2 \end{bmatrix} \right) * \begin{bmatrix} \alpha \\ \omega \end{bmatrix} = cD^{*3} \begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix} + D^{*2} \left(\begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix} + cu_0 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \right) + \\ & + D * \begin{bmatrix} b_1p_1 & b_1p_2 \\ b_2p_1 & b_2p_2 \end{bmatrix} \begin{bmatrix} T_{\tau_1} & 0 \\ 0 & T_{\tau_2} \end{bmatrix} * \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + p_3T_{\tau_2} * S_2 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}. \end{aligned}$$

Let's introduce the notation:

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{—the unit matrix, } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, Q = \\ & \begin{bmatrix} b_1p_1 & b_1p_2 \\ b_2p_1 & b_2p_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} [p_1 \ p_2] = bp, \end{aligned}$$

where $p = [p_1 \ p_2]$ —vector-string, corresponding to a vector-column $\bar{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$.

Note that Q —a degenerate matrix: its determinant $|Q| = 0$.

$$\bar{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, T_{\bar{\tau}} = \begin{bmatrix} T_{\tau_1} & 0 \\ 0 & T_{\tau_2} \end{bmatrix}, \bar{x} = \begin{bmatrix} \alpha \\ \omega \end{bmatrix}, \bar{S} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \bar{S}_0 = \begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix}.$$

We will get

$$\begin{aligned} & (cD^{*3}E + D^{*2}(E - cA) - D * (A + QT_{\bar{\tau}}) - p_3T_{\tau_2}\bar{b}[0 \ 1])^{*-1} * \bar{x} = \\ & = cD^{*3} * \bar{S}_0 + D^{*2}(\bar{S}_0 + cu_0\bar{b}) + D * QT_{\bar{\tau}} * \bar{S} + p_3T_{\tau_2} * S_2\bar{b}. \end{aligned}$$

The determinant of a matrix that stands as a multiplier for \bar{x} , in general, is a quasi-polynomial of the sixth degree.

Hence, the solution of the problem will be in the form

$$\begin{aligned} \bar{x} &= (cD^{*3}E + D^{*2}(E - cA) - D * (A + QT_{\bar{\tau}}) - p_3T_{\tau_2}\bar{b}[0 \ 1]) * \\ & * (cD^{*3} * \bar{S}_0 + D^{*2}(\bar{S}_0 + cu_0\bar{b}) + D * QT_{\bar{\tau}} * \bar{S} + p_3T_{\tau_2} * S_2\bar{b}). \end{aligned}$$

Next, we will limit ourselves to considering the case $c = 0$, $p_3 = 0$.

Then

$$\bar{x} = (DE - A - QT_{\bar{\tau}})^{*-1} * (D\bar{S}_0 + QT_{\bar{\tau}} * \bar{S}).$$

In particular, if there is no delay, i.e. when $\tau_1 = \tau_2 = 0$,

$$\bar{x}|_{\bar{\tau}=0} = (DE - A - Q)^{*^{-1}} * D\bar{S}_0.$$

Since in this case $T_{\bar{\tau}} = E$, \bar{S} is an operator of the vector function type, which can be different from zero only at a point $t = 0$, then

$$(DE - A - Q)^{*^{-1}} * QT_{\bar{\tau}} * \bar{S} = ((DE - A - Q)^{*^{-1}} * Q) * \bar{S} = 0.$$

To implement the resulting solution, you need to reverse the matrix of the form

$$DE - K, \text{ where } K = \begin{vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{vmatrix}.$$

We have

$$(DE - K)^{*^{-1}} = (D^{*2} - (k_{11} + k_{22}) * D + |K|)^{*^{-1}} * (DE - \hat{K}),$$

$$\text{where } |K| \text{—determinant of matrix } K, \hat{K} = \begin{bmatrix} k_{22} & -k_{12} \\ -k_{21} & k_{11} \end{bmatrix} \text{—attached matrix.}$$

Then

$$\bar{x} = (D^{*2} - (k_{11} + k_{22}) * D + |K|)^{*^{-1}} * (DE - \hat{K}) * (D\bar{S}_0 + QT_{\bar{\tau}} * \bar{S})$$

or

$$\bar{x} = (D^{*2} - (k_{11} + k_{22}) * D + |K|)^{*^{-1}} * (D^{*2}\bar{S}_0 + D * (QT_{\bar{\tau}} * \bar{S} - \hat{K}\bar{S}_0) + \hat{K}QT_{\bar{\tau}} * \bar{S}),$$

$$\text{where } K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = A + QT_{\bar{\tau}};$$

$$\bar{x}|_{\bar{\tau}=0} = (D^{*2} - (k_{11}^0 + k_{22}^0) * D + |K^0|)^{*^{-1}} * (D^{*2}\bar{S}_0 - D\hat{K}\bar{S}_0),$$

$$\text{where } K^0 = \begin{bmatrix} k_{11}^0 & k_{12}^0 \\ k_{21}^0 & k_{22}^0 \end{bmatrix} = A + Q;$$

$$|K| = \begin{vmatrix} a_{11} + b_1 p_1 T_{\tau_1} & a_{12} + b_1 p_2 T_{\tau_2} \\ a_{21} + b_2 p_1 T_{\tau_1} & a_{22} + b_2 p_2 T_{\tau_2} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + p_1 \begin{vmatrix} b_1 a_{12} \\ b_2 a_{22} \end{vmatrix} T_{\tau_1} + p_2 \begin{vmatrix} a_{11} b_1 \\ a_{21} b_2 \end{vmatrix} T_{\tau_2},$$

$$\text{because the coefficient at } T_{\tau_1} * T_{\tau_2} \text{ turned out to be equal to } |Q| = \begin{vmatrix} b_1 p_1 & b_1 p_2 \\ b_2 p_1 & b_2 p_2 \end{vmatrix} = 0.$$

Similarly,

$$|K^0| = \begin{vmatrix} a_{11} + b_1 p_1 & a_{12} + b_1 p_2 \\ a_{21} + b_2 p_1 & a_{22} + b_2 p_2 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + p_1 \begin{vmatrix} b_1 a_{12} \\ b_2 a_{22} \end{vmatrix} + p_2 \begin{vmatrix} a_{11} b_1 \\ a_{21} b_2 \end{vmatrix}.$$

Denote

$$\Delta = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \Delta_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix},$$

$$\sigma = a_{11} + a_{22}.$$

Then

$$\begin{aligned}
|K| &= \Delta + p_1 \Delta_1 T_{\tau_1} + p_2 \Delta_2 T_{\tau_2}, \\
|K^0| &= \Delta + p_1 \Delta_1 + p_2 \Delta_2, \\
k_{11} + k_{22} &= \sigma + b_1 p_1 T_{\tau_1} + b_2 p_2 T_{\tau_2}, \\
k_{11}^0 + k_{22}^0 &= \sigma + b_1 p_1 + b_2 p_2, \\
\hat{K} &= \hat{A} + \hat{T}_{\bar{\tau}} \hat{Q}, \\
\hat{K}^0 &= \hat{A} + \hat{Q}.
\end{aligned}$$

Rightly

$$\begin{aligned}
\bar{x} &= (D^{*2} - (\sigma + b_1 p_1 T_{\tau_1} + b_2 p_2 T_{\tau_2}) * D + \Delta + p_1 \Delta_1 T_{\tau_1} + p_2 \Delta_2 T_{\tau_2})^{*-1} * \\
&\quad * (D^{*2} \bar{S}_0 + D * (QT_{\bar{\tau}} * \bar{S}) - \hat{A} \bar{S}_0 - \hat{T}_{\bar{\tau}} \hat{Q} \bar{S}_0) - \hat{A} QT_{\bar{\tau}} * \bar{S}), \\
\bar{x}|_{\bar{\tau}=0} &= (D^{*2} - (\sigma + b_1 p_1 + b_2 p_2) * D + \Delta + p_1 \Delta_1 + p_2 \Delta_2)^{*-1} \\
&\quad * (D^{*2} \bar{S}_0 - D(\hat{A} + \hat{Q}) \hat{S}_0).
\end{aligned}$$

Calculate

$$\bar{v} = [\bar{v}(t)] = QT_{\bar{\tau}} * \bar{S} - \hat{T}_{\bar{\tau}} \hat{Q} \hat{S}_0 \text{ and } \bar{w} = [\bar{w}(t)] = -\hat{A} QT_{\bar{\tau}} * \hat{S}.$$

$$\begin{aligned}
\bar{v} &= \begin{bmatrix} b_1 p_1 & b_1 p_2 \\ b_2 p_2 & b_2 p_2 \end{bmatrix} \begin{bmatrix} T_{\bar{\tau}} & 0 \\ 0 & T_{\tau_2} \end{bmatrix} * \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} - \begin{bmatrix} T_{\tau_1} & 0 \\ 0 & T_{\tau_2} \end{bmatrix} \begin{bmatrix} b_2 p_2 & -b_1 p_2 \\ -b_2 p_1 & b_1 p_1 \end{bmatrix} \begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix} = \\
&= \begin{bmatrix} b_1 p_1 T_{\tau_1} * S_1 + b_1 p_2 T_{\tau_2} * S_2 \\ b_2 p_1 T_{\tau_1} * S_1 + b_2 p_2 T_{\tau_2} * S_2 \end{bmatrix} - \begin{bmatrix} b_2 p_2 T_{\tau_2} S_1(0) & -b_1 p_2 T_{\tau_2} S_2(0) \\ -b_2 p_1 T_{\tau_1} S_1(0) & b_1 p_1 T_{\tau_1} S_2(0) \end{bmatrix}.
\end{aligned}$$

So,

$$\begin{aligned}
\bar{v} &= \begin{bmatrix} b_1 p_1 T_{\tau_1} * S_1 + b_1 p_2 T_{\tau_2} * S_2 + p_2 (b_1 S_2(0) - b_2 S_1(0)) T_{\tau_2} \\ b_2 p_1 T_{\tau_1} * S_1 + b_2 p_2 T_{\tau_2} * S_2 + p_1 (b_2 S_1(0) - b_1 S_2(0)) T_{\tau_1} \end{bmatrix}. \\
\bar{w} &= \begin{bmatrix} -a_{22} & a_{12} \\ a_{21} & -a_{11} \end{bmatrix} \begin{bmatrix} b_1 p_1 & b_1 p_2 \\ b_2 p_1 & b_2 p_2 \end{bmatrix} \begin{bmatrix} T_{\tau_1} & 0 \\ 0 & T_{\tau_2} \end{bmatrix} * \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \\
&= \begin{bmatrix} -a_{22} b_1 p_1 + a_{12} b_2 p_1 & a_{22} b_1 p_2 + a_{12} b_2 p_2 \\ a_{21} b_1 p_1 - a_{21} b_2 p_1 & a_{21} b_1 p_2 - a_{11} b_2 p_2 \end{bmatrix} \begin{bmatrix} T_{\tau_1} * S_1 \\ T_{\tau_2} * S_2 \end{bmatrix} = \\
&= \begin{bmatrix} -p_1 \Delta_1 & -p_2 \Delta_1 \\ -p_1 \Delta_2 & -p_2 \Delta_2 \end{bmatrix} \begin{bmatrix} T_{\tau_1} * S_1 \\ T_{\tau_2} * S_2 \end{bmatrix}.
\end{aligned}$$

And finally

$$\bar{w} = - \begin{bmatrix} p_1 \Delta_1 T_{\tau_1} * S_1 + p_2 \Delta_1 T_{\tau_2} * S_2 \\ p_1 \Delta_2 T_{\tau_1} * S_1 + p_2 \Delta_2 T_{\tau_2} * S_2 \end{bmatrix}.$$

Here

$$\begin{aligned} T_{\tau_1} * S_1 &= [0, t < 0; S_1(t - \tau_1), 0 \leq t \leq \tau_1; 0, t > \tau_1], \\ T_{\tau_2} * S_2 &= [0, t < 0; S_2(t - \tau_2), 0 \leq t \leq \tau_2; 0, t > \tau_2]. \end{aligned}$$

When $\tau_i = 0$ ($i = 1, 2$), $T_{\tau_i} * S_i = S_i$ is a operator function type that can be non-zero only at the point $t = 0$. Further $\bar{v}, \bar{\omega}$ will meet only in expressions of the form $\psi * \bar{v}$, $\psi * \bar{\omega}$. Therefore, in the case of $\tau_i = 0$, terms, containing a S_i multiplier in expressions v, ω , can be omitted.

When $\tau_1 = \tau_2 = \tau > 0$, we will get

$$\begin{aligned} \bar{v} &= T_{\tau} * \left[\begin{array}{l} b_1(p_1 S_1 + p_2 S_2) + p_2(b_1 S_2(0) - b_2 S_1(0)) \\ b_2(p_1 S_1 + p_2 S_2) + p_1(b_2 S_1(0) - b_1 S_2(0)) \end{array} \right], \\ \bar{\omega} &= -T_{\tau} * \left[\begin{array}{l} p_1 \Delta_1 S_1 + p_2 \Delta_1 S_2 \\ p_1 \Delta_2 S_1 + p_2 \Delta_2 S_2 \end{array} \right]. \end{aligned}$$

From the above, follows

$$\begin{aligned} \bar{x} &= (D^{*2} - (\sigma + b_1 p_1 T_{\tau_1} + b_2 p_2 T_{\tau_2}) * D + \Delta + p_1 \Delta_1 T_{\tau_1} + p_2 \Delta_2 T_{\tau_2})^{*-1} * \\ &\quad * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{\omega}). \end{aligned}$$

When $\bar{\tau} \neq 0$, there are three possible cases:

1. $\tau_1 > 0, \tau_2 > 0$.

Then

$$\begin{aligned} \bar{x} &= (D^{*2} - \sigma D + \Delta)^{*-1} * \left[1 - (D^{*2} - \sigma D + \Delta)^{*-1} * (T_{\tau_1} * (b_1 p_1 D - p_1 \Delta_1) + \right. \\ &\quad \left. + T_{\tau_2} * (b_2 p_2 D - p_2 \Delta_2)) \right]^{*-1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{\omega}). \end{aligned}$$

Hence

$$\begin{aligned} \bar{x} &= (D^{*2} - \sigma D + \Delta)^{*-1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) \bar{\omega}) \\ &\quad + \sum_{k=1}^{\infty} \sum_{m=0}^k \binom{k}{m} T_{m\tau_1 + (k-m)\tau_2} * \\ &\quad * (D^{*2} - \sigma D + \Delta)^{*-k} * (b_1 p_1 D - p_1 \Delta_1)^{*m} * (b_2 p_2 D - p_2 \Delta_2)^{*k-m} * \\ &\quad * (D^{*2} - \sigma D + \Delta)^{*-1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{\omega}). \end{aligned}$$

2. $\tau_1 > 0, \tau_2 = 0$.

In this case

$$\bar{x} = (D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2)^{* -1} * (1 - (D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2)^{* -1} * T_{\tau_1} * (b_1 p_1 D - p_1 \Delta_1))^{* -1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{w}).$$

Hence

$$\begin{aligned} \bar{x} &= (D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2)^{* -1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{w}) + \\ &+ \sum_{k=1}^{\infty} T_{\mu \tau_1} * (D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2)^{* -k} * (b_1 p_1 D - p_1 \Delta_1)^{* k} * \\ &* (D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2)^{* -1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{w}). \end{aligned}$$

3. $\tau_1 = 0, \tau_2 > 0$.

Similarly to the previous one, we get

$$\begin{aligned} \bar{x} &= (D^{*2} - (\sigma + b_1 p_1)D + \Delta + p_1 \Delta_1)^{* -1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{w}) + \\ &+ \sum_{k=1}^{\infty} T_{k \tau_2} * (D^{*2} - (\sigma + b_1 p_1)D + \Delta + p_1 \Delta_1)^{* -k} * (b_2 p_2 D - p_2 \Delta_2)^{* k} * \\ &* (D^{*2} - (\sigma + b_1 p_1)D + \Delta + p_1 \Delta_1)^{* -1} * (D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{w}). \end{aligned}$$

If $\bar{\tau} = 0$, i.e. $\tau_1 = \tau_2 = 0$, then, as it was shown,

$$\begin{aligned} \bar{x}|_{\bar{\tau}=0} &= (D^{*2} - (\sigma + b_1 p_1 + b_2 p_2)D + \Delta + p_1 \Delta_1 + p_2 \Delta_2)^{* -1} * \\ &* (D^{*2} \bar{S}_0 - D(\hat{A} + \hat{Q}) \bar{S}_0). \end{aligned}$$

Next, we will use the well-known formulas of operational calculus:

$$\begin{aligned} (D - \gamma)^{* -m-1} * D &= \left(\frac{t^m}{m!} e^{\gamma t} \right)_+ \quad (m = 0, 1, 2, \dots). \\ ((D - \gamma)^{*2} - \lambda^2)^{* -1} * D * (D - \gamma) &= (e^{\gamma t} ch(\lambda t))_+, \\ ((D - \gamma)^{*2} - \lambda)^{* -1} * \lambda D &= (e^{\gamma t} sh(\lambda t))_+, \\ ((D - \gamma)^{*2} + \mu^2)^{* -1} * D * (D - \gamma) &= (e^{\gamma t} \cos(\mu t))_+, \\ ((D - \gamma)^{*2} + \mu^2)^{* -1} + \mu D &= (e^{\gamma t} \sin(\mu t))_+; \end{aligned}$$

if $\phi(t)$ is locally absolutely continuous, then

$$\varphi * \psi = (D * \varphi) \times \psi, \quad \varphi * \varphi * \psi = (D * \varphi) \times (D * \varphi) \times \psi, \dots,$$

$$\underbrace{\varphi * \varphi * \dots * \varphi}_m * \psi = \underbrace{(D * \varphi) \times (D * \varphi) \times \dots \times (D * \varphi)}_m * \psi;$$

$$T_\tau * \psi = \psi(t - \tau).$$

Let $F = D^{*2} + a_1 D + a_2$.

Then

$$F = \left(D^{*2} + 2\frac{a_1}{2}D + \frac{a_1^2}{4} \right) + \left(a_2 - \frac{a_1^2}{4} \right) = \left(D + \frac{a_1}{2} \right)^{*2} + \left(a_2 - \frac{a_1^2}{4} \right) = \begin{cases} (D - \gamma)^{*2}, & \lambda^2 = 0 \\ (D - \gamma)^{*2} - \lambda^2, & \lambda^2 > 0 \\ (D - \gamma)^{*2} + \mu^2, & \lambda^2 = -\mu^2 < 0, \end{cases}$$

where $\gamma = -\frac{a_1}{2}$, $\lambda^2 = \frac{a_1^2}{4} - a_2$, $\mu^2 = a_2 - \frac{a_1^2}{4}$.

Let ζ, ξ —numerical operators, scalar or vector.

Then

$$\begin{aligned} F^{*-1} * (D^{*2}\zeta + D\xi) &= (D - \gamma)^{*2} * (D^{*2}\zeta + D\xi) = \\ &= (D - \gamma)^{*2} * (D * (D - \gamma)\zeta + D(\gamma\zeta + \xi)) = (e^{\gamma t}(\zeta + t(\gamma\zeta + \xi)))_+, \quad \lambda^2 = 0; \\ F^{*-1} * (D^{*2}\zeta + D\xi) &= ((D - \gamma)^{*2} + \mu^2)^{*1} * (D^{*2}\zeta + D\xi) = \\ &= ((D - \gamma)^{*2} + \mu^2)^{*1} * \left(D * (D - \gamma)\zeta + \mu D \left(\frac{\gamma}{\mu}\zeta + \frac{1}{\mu}\xi \right) \right) = \\ &= \left(e^{\gamma t}(\cos(\mu t))\zeta + \frac{1}{\mu} \sin(\mu t)(\gamma\zeta + \xi) \right)_+, \quad \lambda^2 = -\mu^2 < 0. \end{aligned}$$

Hence, in particular,

$$F^{*-1} * D^{*2} = (D - \gamma)^{*2} * D^{*2} = ((1 + \gamma t)e^{\gamma t})_+ \text{ when } \lambda^2 = 0;$$

$$F^{*-1} * D^{*2} = ((D - \gamma)^{*2} - \lambda^2)^{*1} * D^{*2} = (e^{\gamma t}(ch(\lambda t) + \frac{\gamma}{\lambda}sh(\lambda t)))_+ \text{ when } \lambda^2 > 0;$$

$$F^{*-1} * D^{*2} = ((D - \gamma)^{*2} + \mu^2)^{*1} * D^{*2} = \left(e^{\gamma t} \left(\cos(\mu t) + \frac{\gamma}{\mu} \sin(\mu t) \right) \right)_+$$

when $\lambda^2 = -\mu^2 < 0$.

$$F^{*-1} * D = (D - \gamma)^{*2} * D = (te^{\gamma t})_+ \text{ when } \lambda^2 = 0;$$

$$F^{*-1} * D = ((D - \gamma)^{*2} - \lambda^2)^{*1} * D = \left(\frac{1}{\lambda} e^{\gamma t} sh(\lambda t) \right)_+ \text{ when } \lambda^2 > 0;$$

$$F^{*-1} * D = ((D - \gamma)^{*2} + \mu^2)^{*1} * D = \left(\frac{1}{\mu} e^{\gamma t} \sin(\mu t) \right)_+ \text{ when } \lambda^2 = -\mu^2 < 0.$$

Applying the above formulas:

– to the operator $F = D^{*2} - \sigma D + \Delta$, when $\tau_1 > 0$, $\tau_2 > 0$, we will get

$$\gamma = \frac{\sigma}{2}, \lambda^2 = \frac{\sigma^2}{4} - \Delta = -\mu^2;$$

– to the operator $F = D^{*2} - (\sigma + b_2 p_2)D + \Delta + p_2 \Delta_2$, when $\tau_1 > 0$, $\tau_2 = 0$, we will get

$$\gamma = \frac{\sigma + b_2 p_2}{2}, \lambda^2 = \frac{(\sigma + b_2 p_2)^2}{4} - \Delta - p_2 \Delta_2 = -\mu^2;$$

– to the operator $F = D^{*2} - (\sigma + b_1 p_1)D + \Delta + p_1 \Delta_1$, when $\tau_1 = 0$, $\tau_2 > 0$, we will get

$$\gamma = \frac{\sigma + b_1 p_1}{2}, \lambda^2 = \frac{(\sigma + b_1 p_1)^2}{4} - \Delta - p_1 \Delta_1 = -\mu^2.$$

In all three cases

$$\begin{aligned} \bar{f} &= F^{*-1} * \left(D^{*2} \bar{S}_0 + D * (\bar{v} - \hat{A} \bar{S}_0) + \bar{\omega} \right) = \\ &= F^{*-1} * \left(D^{*2} \bar{S}_0 - D \hat{A} \bar{S}_0 \right) + F^{*-1} * D * \bar{v} + F^{*-1} * \bar{\omega} = \\ &= F^{*-1} * \left(D^{*2} \bar{S}_0 - D \hat{A} \bar{S}_0 \right) + (F^{*-1} * D^{*2}) \times \bar{v} + (F^{*-1} * D) \times \bar{\omega}. \end{aligned}$$

And, after the transformations, we get

$$\begin{aligned} \bar{f} &= \left(e^{\gamma t} \left(E + t(\gamma E - \hat{A}) \right) \bar{S}_0 + \int_0^t (1 + \gamma(t - \eta)) e^{\gamma(t-\eta)} \bar{v}(\eta) d\eta + \right. \\ &\quad \left. + \int_0^t (t - \eta) e^{\lambda(t-\eta)} \bar{\omega}(\eta) d\eta \right) \quad \text{when } \lambda^2 = 0, \\ \bar{f} &= \left(e^{\gamma t} \left(ch(\lambda t) E + \frac{1}{\lambda} sh(\lambda t) (\lambda E - \hat{A}) \right) \bar{S}_0 \right. \\ &\quad \left. + \int_0^t e^{\gamma(t-\eta)} \left(ch(\lambda(t-\eta)) + \frac{\gamma}{\lambda} sh(\lambda(t-\eta)) \right) \times \right. \\ &\quad \left. \times \bar{v}(\eta) d\eta + \frac{1}{\lambda} \int_0^t e^{\gamma(t-\eta)} sh(\lambda(t-\eta)) \bar{\omega}(\eta) d\eta \right) \quad \text{when } \lambda^2 > 0, \\ \bar{f} &= \left(e^{\gamma t} \left(\cos(\mu t) E + \frac{1}{\mu} \sin(\mu t) (\gamma E - \hat{A}) \right) \bar{S}_0 \right. \end{aligned}$$

$$\begin{aligned}
 & + \int_0^t e^{\gamma(t-\eta)} \left(\cos(\mu(t-\eta)) + \frac{\gamma}{\mu} \sin(\mu(t-\eta)) \right) \times \\
 & \times \bar{v}(\eta) d\eta + \frac{1}{\mu} \int_0^t e^{\gamma(t-\eta)} \sin(\mu(t-\eta)) \bar{w}(\eta) d\eta \Bigg)_+ \quad \text{when } \lambda^2 = -\mu^2 < 0.
 \end{aligned}$$

For $i = 1, 2$ also fair

$$\begin{aligned}
 f_i & = F^{*-1} * (b_i p_i D - p_i \Delta_i) * D = \\
 & \left(p_i e^{\gamma t} (b_i + (\gamma b_i - \Delta_i) t) \right)_+, \quad \lambda^2 = 0 \\
 & \left(p_i e^{\gamma t} \left(b_i ch(\lambda t) + \frac{1}{\lambda} (\gamma b_i - \Delta_i) sh(\lambda t) \right) \right)_+, \quad \lambda^2 > 0 \\
 & \left(p_i e^{\gamma t} \left(b_i \cos(\mu t) + \frac{1}{\mu} (\gamma b_i - \Delta_i) \sin(\mu t) \right) \right)_+, \quad \lambda^2 = -\mu^2 < 0.
 \end{aligned}$$

Finally, we get.

1. When $\tau_1 > 0, \tau_2 > 0$

$$\begin{aligned}
 \bar{x} & = \bar{f} + \sum_{k=1}^{\infty} \sum_{m=0}^k \binom{k}{m} T_{m\tau_1+(k-m)\tau_2} * \left(\underbrace{f_1 \times \dots \times f_1}_m \times \underbrace{f_2 \times \dots \times f_2}_{k-m} \times \bar{f} \right) = \\
 & = f + \sum_{k=1}^{\infty} \sum_{m=0}^k \binom{k}{m} \left\{ \int_0^{t-m\tau_1-(k-m)\tau_2} f_1(t-m\tau_1-(k-m)\tau_2-\xi_m) d\xi_m \dots \right. \\
 & \dots \int_0^{\xi_2} f_1(\xi_2-\xi_1) d\xi_1 \int_0^{\xi_1} f_2(\xi_1-\eta_{k-m}) d\eta_{k-m} \dots \int_0^{\eta_2} f_2(\eta_2-\eta_1) \bar{f}(\eta_1) d\eta_1, \\
 & \left. t-m\tau_1-(k-m)\tau_2 > 0; \quad 0, t-m\tau_1-(k-m)\tau_2 \leq 0 \right\}.
 \end{aligned}$$

2. When $\tau_1 > 0, \tau_2 = 0$

$$\begin{aligned}
 \bar{x} & = \bar{f} + \sum_{k=1}^{\infty} T_{k\tau_1} * \left(\underbrace{f_1 \times \dots \times f_1}_k \times \bar{f} \right) = \\
 & = \bar{f} + \sum_{k=1}^{\infty} \left\{ \int_0^{t-k\tau_1} f_1(t-k\tau_1-\eta_k) d\eta_k \dots \int_0^{\eta_2} f_1(\eta_2-\eta_1) \bar{f}(\eta_1) d\eta_1, \right. \\
 & \left. t-k\tau_1 > 0; \quad 0, t-k\tau_1 \leq 0 \right\}.
 \end{aligned}$$

3. When $\tau_1 = 0, \tau_2 > 0$

$$\begin{aligned} \bar{x} &= \bar{f} + \sum_{k=1}^{\infty} T_{k\tau_2} * \left(\underbrace{f_2 \times \dots \times f_2}_{k} \times \bar{f} \right) = \\ &= \bar{f} + \sum_{k=1}^{\infty} \left\{ \begin{array}{l} \int_0^{t-k\tau_2} f_2(t-k\tau_2-\eta_k) d\eta_k \dots \int_0^{\eta_2} f_2(\eta_2-\eta_1) \bar{f}(\eta_1) d\eta_1, \\ t-k\tau_2 > 0; \quad 0, \quad t-k\tau_2 \leq 0 \end{array} \right\}. \end{aligned}$$

4. When $\bar{\tau} = 0$, i.e. $\tau_1 = \tau_2 = 0$,

$$F = D^{*2} - (\sigma + b_1 p_1 + b_2 p_2) D + \Delta + p_1 \Delta_1 + p_2 \Delta_2,$$

$$\gamma = \frac{\sigma + b_1 p_1 + b_2 p_2}{2}, \quad \lambda^2 = \frac{(\sigma + b_1 p_1 + b_2 p_2)^2}{4} - \Delta - p_1 \Delta_1 - p_2 \Delta_2 = -\mu^2;$$

$$\bar{x} = \begin{cases} \left(e^{\gamma t} \left(E + t \left(\gamma E - \left(\hat{A} + \hat{Q} \right) \right) \right) \right) \bar{S}_0, & \lambda^2 = 0 \\ \left(e^{\gamma t} \left(ch(\lambda t) E + \frac{1}{\lambda} sh(\lambda t) \left(\gamma E - \left(\hat{A} + \hat{Q} \right) \right) \right) \right) \bar{S}_0, & \lambda^2 > 0 \\ \left(e^{\gamma t} \left(\cos(\mu t) E + \frac{1}{\mu} \sin(\mu t) \left(\gamma E - \left(\hat{A} + \hat{Q} \right) \right) \right) \right) \bar{S}_0, & \lambda^2 = -\mu^2 < 0. \end{cases}$$

We have obtained an accurate analytical solution to the posed problem (when $c = 0, p_3 = 0$). When $\bar{\tau} = 0$, the solution is presented in the final form. When $\bar{\tau} \neq 0$, the infinite series, included in the solution, is a series of shifts (i.e. each bounded domain of a variable t contains only a finite number of terms). In particular, in the area of $0 \leq t \leq \tau$, where

$$\tau = \begin{cases} \min(\tau_1, \tau_2) & \text{when } \tau_1 > 0, \tau_2 > 0 \\ \tau_1 & \text{when } \tau_1 > 0, \tau_2 = 0 \\ \tau_2 & \text{when } \tau_1 = 0, \tau_2 > 0 \end{cases}$$

the series contains only the initial term.

By numerical values γ and $\lambda^2 (\mu^2)$, corresponding to the cases $\bar{\tau} = 0$ and $\bar{\tau} \neq 0$, it is easy to establish the nature of the process in each of these cases and the difference between them.

To illustrate, consider the case of a *short-period longitudinal motion* of an aircraft, described by a system of differential equations:

$$\begin{aligned} \alpha'(t) &= (-a_y^\alpha) \alpha(t) + \omega(t) + (-a_y^u) u(t), \\ \omega'(t) &= (-a_{mz}^\alpha) \alpha(t) + (-a_{mz}^{\omega_z}) \omega(t) + (-a_{mz}^\alpha) \alpha'(t) + (-a_{mz}^u) u(t), \quad (t \geq 0) \end{aligned}$$

$$u(t) = p_1\alpha(t - \tau_1) + p_2\omega(t - \tau_2),$$

where $\tau_1, \tau_2 \geq 0$, under the conditions

$$\begin{aligned} \alpha(t) &= S_1(t), \quad t \leq 0; \\ \omega(t) &= S_2(t), \quad t \leq 0; \\ \alpha(+0) &= S_1(0); \\ \bar{\omega}(+0) &= S_2(0); \\ u(+0) &= u_0; \\ (\omega &= \omega_z). \end{aligned}$$

Substituting into the second equation $\alpha'(t)$ from the first equation, under the same conditions, we will come to the system

$$\begin{aligned} \alpha'(t) &= (-a_y^\alpha) \alpha(t) + \omega(t) + (-a_y^u) u(t), \\ \omega'(t) &= (a_{mz}^{\dot{\alpha}} a_y^\alpha - a_{mz}^\alpha) \alpha(t) + (-a_{mz}^{\dot{\omega}} - a_{mz}^{\omega_z}) \omega(t) + (a_{mz}^{\dot{\alpha}} a_y^u - a_{mz}^u) u(t), \\ u(t) &= p_1\alpha(t - \tau_1) + p_2\omega(t - \tau_2), \\ (t &\geq 0). \end{aligned}$$

Thus, $c = 0, p_3 = 0$.

$$\begin{aligned} A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -\alpha_y^\alpha & 1 \\ a_{mz}^{\dot{\alpha}} a_y^\alpha - a_{mz}^\alpha & -a_{mz}^{\dot{\omega}} - a_{mz}^{\omega_z} \end{bmatrix}, \\ \bar{b} &= \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -a_y^u \\ a_{mz}^{\dot{\alpha}} - a_{mz}^u \end{bmatrix}, p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}. \end{aligned}$$

As you can see, the task completely fits into the considered scheme.

Let's study the effect of lag for the case of horizontal flight with automated control [3].

Let us have a system of equations

$$\left. \begin{aligned} \alpha'(t) &= -0,0117\alpha(t) + \omega(t) + 0,000502 u(t) \\ \omega'(t) &= 0,0076\alpha(t) - 0,589\omega(t) - 0,0332 u(t) \\ u(t) &= \alpha(t - \tau_1) + \frac{\pi}{360} \omega(t - \tau_2) \\ (t &\geq 0) \end{aligned} \right\}.$$

Let's introduce

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -0,0117 & 1 \\ 0,0076 & -0,589 \end{bmatrix},$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0,000502 \\ -0,0332 \end{bmatrix}, \quad \bar{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{\pi}{360} \end{bmatrix},$$

$$S_1(t) = 1, \quad S_2(t) = 0$$

$$\sigma = a_{11} + a_{22} = -0,6007.$$

$$\Delta = a_{11}a_{22} - a_{21}a_{12} = -0,0007087$$

$$\Delta_1 = b_1a_{22} - b_2a_{12} = 0,032904322$$

$$\Delta_2 = a_{11}b_2 - a_{21}b_1 = 0,0003846248.$$

$$\hat{A} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} -0,589 & -1 \\ -0,0076 & -0,0117 \end{bmatrix},$$

$$Q = \begin{bmatrix} b_1p_1 & b_1p_2 \\ b_2p_1 & b_2p_2 \end{bmatrix} = \begin{bmatrix} 0,000502 & 0,0000043 \\ -0,0332 & -0,0002897 \end{bmatrix},$$

$$\hat{Q} = \begin{bmatrix} b_2p_2 & -b_1p_2 \\ -b_2p_1 & b_1p_1 \end{bmatrix} = \begin{bmatrix} -0,0002897 & -0,0000043 \\ 0,0332 & 0,000502 \end{bmatrix}.$$

Let's make calculations for two cases:

1. $\tau_1 = \tau_2 = 0.$

$$\gamma = \frac{\sigma + b_1p_1 + b_2p_2}{2} = -0,30024.$$

$$\lambda^2 = \gamma^2 - \Delta - p_1\Delta_1 - p_2\Delta_2 = 0,0589475 > 0.$$

$$\lambda = 0,2406.$$

$$S_1(0) = [0, t \leq 0; \quad 1, t > 0], \quad S_2(0) = 0, \quad \bar{S}_0 = \begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix}.$$

$$\begin{aligned} \gamma E - (\hat{A} + \hat{Q}) &= \frac{a_{11} + a_{22} + b_1 p_1 + b_2 p_2}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{22} + b_2 p_2 & -a_{12} - b_1 p_2 \\ -a_{21} - b_2 p_1 & a_{11} + b_1 p_1 \end{bmatrix} = \\ &= \begin{bmatrix} \frac{a_{11} - a_{22} + b_1 p_1 - b_2 p_2}{2} & a_{12} + b_1 p_2, & t > 0 \\ a_{21} + b_2 p_1 & \frac{a_{22} - a_{11} + b_2 p_2 - b_1 p_1}{2}, & t > 0 \end{bmatrix}. \end{aligned}$$

$$(\gamma E - (\hat{A} + \hat{Q})) \bar{S}_0 = \begin{bmatrix} 0, t \leq 0; & \frac{a_{11} - a_{22} + b_1 p_1 - b_2 p_2}{2}, t > 0 \\ 0, t \leq 0; & a_{21} + b_2 p_1, & t > 0 \end{bmatrix}.$$

$$\begin{aligned} \bar{x} &= \begin{bmatrix} \alpha \\ \omega \end{bmatrix} \\ &= \left\{ \begin{array}{l} 0, t \leq 0; \\ e^{\gamma t} ch(\lambda t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\lambda} e^{\gamma t} sh(\lambda t) \begin{bmatrix} \frac{a_{11} - a_{22} + b_1 p_1 - b_2 p_2}{2} \\ a_{21} + b_2 p_1 \end{bmatrix}, t > 0 \end{array} \right\} \\ &\quad \frac{1}{2\lambda} (a_{11} - a_{22} + b_1 p_1 - b_2 p_2) = 1, 202354, \\ &\quad \frac{1}{\lambda} (a_{21} + b_2 p_1) = -0, 10631. \end{aligned}$$

$$\begin{aligned} \bar{x} &= \begin{bmatrix} \alpha \\ \omega \end{bmatrix} \\ &= \left\{ \begin{array}{l} 0, t \leq 0; \\ e^{-0,30024t} ch(0, 2406t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-0,30024t} sh(0, 2406t) \begin{bmatrix} 1, 202354 \\ -0, 10631 \end{bmatrix}, t > 0 \end{array} \right\} \end{aligned}$$

2. $\tau_1 = \tau_2 = \tau > 0, \tau = 0, 15$.

$$\gamma = \frac{\sigma}{2} = -0, 3035.$$

$$\lambda^2 = \gamma^2 - \Delta = 0, 090902 > 0.$$

$$\lambda = 0, 3015.$$

$$S_1(0) = [0, t \leq 0; 1, t > 0], \quad S_2(0) = 0, \quad \bar{S}_0 = \begin{bmatrix} S_1(0) \\ S_2(0) \end{bmatrix}.$$

$$S_1 = [0, t < -0, 15; 1, -0, 15 \leq t \leq 0; 0, t > 0], \quad S_2 = 0, \quad \bar{S} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}.$$

$$T_{\tau_1} * S_1 = [0, t < 0; \quad 1, 0, \quad 0 \leq t \leq 0, 15; \quad 0, t > 0, 15],$$

$$T_{\tau_2} * S_2 = 0.$$

$$\bar{v} = T_{\tau} * \begin{bmatrix} b_1 p_1 S_1 - b_2 p_2 S_1(0) \\ b_2 p_1 S_1 + b_2 p_1 S_1(0) \end{bmatrix},$$

$$\bar{\omega} = -T_{\tau} * p_1 S_1 \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}.$$

$$\gamma E - \hat{A} = \frac{a_{11} + a_{22}}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{22} & -a_{12} \\ a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} \frac{a_{11} - a_{22}}{2} & a_{12} \\ a_{21} & \frac{a_{22} - a_{11}}{2} \end{bmatrix}.$$

$$(\gamma E - A) \bar{S}_0 = \begin{bmatrix} 0, t \leq 0; & \frac{a_{11} - a_{22}}{2}, t > 0 \\ 0, t \leq 0; & a_{21}, t > 0 \end{bmatrix}.$$

$$\bar{f} = \{0, t \leq 0;$$

$$e^{\gamma t} ch(\lambda t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\lambda} e^{\gamma t} sh(\lambda t) \begin{bmatrix} \frac{a_{11} - a_{22}}{2} \\ a_{21} \end{bmatrix} + \int_0^t e^{\gamma \eta} (ch(\lambda \eta) + \frac{\gamma}{\lambda} sh(\lambda \eta)) d\eta \cdot p_1 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} -$$

$$- \frac{1}{\lambda} \int_0^t e^{\gamma \eta} sh(\lambda \eta) d\eta \cdot p_1 \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}, \quad 0 < t \leq \tau;$$

$$e^{\gamma t} ch(\lambda t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\lambda} e^{\gamma t} sh(\lambda t) \begin{bmatrix} \frac{a_{11} - a_{22}}{2} \\ a_{21} \end{bmatrix} +$$

$$+ \left[\begin{array}{l} b_1 p_1 \int_{t-\tau}^t e^{\gamma \eta} (ch(\lambda \eta) + \frac{\gamma}{\lambda} sh(\lambda \eta)) d\eta - b_2 p_2 \int_0^{t-\tau} e^{\gamma \eta} (ch(\lambda \eta) + \frac{\gamma}{\lambda} sh(\lambda \eta)) d\eta \\ b_2 p_1 \int_0^t e^{\gamma \eta} (ch(\lambda \eta) + \frac{\gamma}{\lambda} sh(\lambda \eta)) d\eta \end{array} \right] -$$

$$- \frac{1}{\lambda} \int_{t-\tau}^t e^{\gamma \mu} sh(\lambda \mu) d\mu \cdot p_1 \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}, \quad t > \tau \}.$$

$$\int_{\tau_1}^{\tau_2} e^{\gamma \eta} (A ch(\lambda \eta) + B sh(\lambda \eta)) d\eta$$

$$= \frac{1}{\lambda^2 - \gamma^2} e^{\gamma \eta} ((\lambda B - \gamma A) ch(\lambda \eta) + (\lambda A - \gamma B) sh(\lambda \eta)) \Big|_{\eta_1}^{\eta_2},$$

$$\lambda^2 - \gamma^2 = -\Delta.$$

$$\begin{aligned} \bar{f} &= \{0, t \leq 0; \\ e^{\gamma t} ch(\lambda t) &\begin{bmatrix} 1 + p_1 \frac{\Delta_1}{\Delta} \\ p_1 \frac{\Delta_2}{\Delta} \end{bmatrix} + \frac{1}{\lambda} e^{\gamma t} sh(\lambda t) \begin{bmatrix} \frac{a_{11} - a_{22}}{2} + b_1 p_1 - \gamma p_1 \frac{\Delta_1}{\Delta} \\ a_{21} + b_2 p_1 - \gamma p_1 \frac{\Delta_2}{\Delta} \end{bmatrix} - \frac{p_1}{\Delta} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}, \\ 0 < t \leq \tau; \\ \gamma^t ch(\lambda t) &\begin{bmatrix} 1 + (b_1 p_1 + b_2 p_2) sh(\lambda \tau) + p_1 \frac{\Delta_1}{\Delta} \end{bmatrix} + \\ + \frac{1}{\lambda} e^{\gamma t} sh(\lambda t) &\begin{bmatrix} \frac{a_{11} - a_{22}}{2} + b_1 p_1 - (b_1 p_1 + b_2 p_2) ch(\lambda \tau) - \gamma p_1 \frac{\Delta_1}{\Delta} \\ a_{21} + b_2 p_1 - \gamma p_1 \frac{\Delta_2}{\Delta} \end{bmatrix} - \frac{p_1}{\Delta} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}, t > \tau \}. \end{aligned}$$

$$\tau = 0, 15, \quad \lambda \tau = 0, 045225, \quad ch(\lambda \tau) = 1, 0010226, \quad sh(\lambda \tau) = 0, 04523735.$$

$$\begin{aligned} \bar{f} &= \{0, t \leq 0; \\ e^{-0,30035t} ch(0, 3015t) &\begin{bmatrix} -0, 45429518 \\ -0, 54271878 \end{bmatrix} + e^{-0,30035t} sh(0, 3015t) \begin{bmatrix} -45, 29965 \\ -0, 62552262 \end{bmatrix} + \\ &\begin{bmatrix} 46, 429518 \\ 0, 54271878 \end{bmatrix}, \quad 0 < t \leq 0, 15; \\ e^{-0,30035t} ch(0, 3015t) &\begin{bmatrix} -47, 429527 \\ -0, 54271878 \end{bmatrix} + e^{-0,30035t} sh(0, 3015t) \begin{bmatrix} -45, 295688 \\ -0, 62552262 \end{bmatrix} + \\ &\begin{bmatrix} 46, 429518 \\ 0, 5427878 \end{bmatrix}, \quad t > 0, 15\}. \\ f_1 &= \left\{ 0, t \leq 0; \quad p_1 e^{\gamma t} \left(b_1 ch(\lambda t) + \frac{1}{\lambda} (\gamma b_1 - \Delta_1) sh(\lambda t) \right), \quad t > 0 \right\} = \\ = \left\{ 0, t \leq 0; \quad e^{-0,30035t} (0, 000502 ch(0, 3015t) - 0, 1096325 sh(0, 3015t)), \quad t > 0 \right\}. \\ f_2 &= \left\{ 0, t \leq 0; \quad p_2 e^{\gamma t} \left(b_2 ch(\lambda t) + \frac{1}{\lambda} (\gamma b_2 - \Delta_2) sh(\lambda t) \right), \quad t > 0 \right\} = \\ = \left\{ 0, t \leq 0; \quad e^{-0,30035t} (-0, 002897 ch(0, 3015t) + 0, 0340167 sh(0, 3015t)), \quad t > 0 \right\}. \end{aligned}$$

$$\begin{aligned} \bar{x} = \begin{bmatrix} \alpha \\ \omega \end{bmatrix} &= \bar{f} + \sum_{k=1}^{\infty} \sum_{m=0}^k \binom{k}{m} \{0, t - 0, 15k \leq 0\}; \\ &\int_0^{t-0,15k} f_1(t - 0, 15k - \xi_m) d\xi_m \dots \\ &\dots \int_0^{\xi_2} f_1(\xi_2 - \xi_1) d\xi_1 \int_0^{\xi} f_2(\xi - \eta_{k-m}) d\eta_{k-m} \dots \end{aligned}$$

$$\dots \int_0^{\eta_2} f_2(\eta_2 - \eta_1) \bar{f}(\eta_1) d\eta_1, \quad t = 0, 15k > 0\}.$$

Rightly

$$\begin{aligned} \begin{bmatrix} \alpha(t) \\ \omega(t) \end{bmatrix} &= e^{-0,30035t} \left(ch(0, 3015t) \begin{bmatrix} -45, 429518 \\ -0, 54271878 \end{bmatrix} + sh(0, 3015t) \begin{bmatrix} -45, 292955 \\ -0, 62552262 \end{bmatrix} \right) + \\ &+ \begin{bmatrix} 46, 429518 \\ 0, 54271878 \end{bmatrix}, \quad 0 \leq t \leq 0, 5. \end{aligned}$$

4.4 Assessment of the Dependence of the Required Duration Computing Cycle on the Own Frequency of the Aircraft

The dependences of the required cycle duration on the aircraft’s own frequency (determined by the system parameters) are analyzed below.

Let’s limit ourselves to the consideration of the short-period component of the longitudinal motion.

Let the task $\bar{x}' = A\bar{x}$, $\bar{x}(0) = \bar{x}_0$ be set, where $\bar{x} = \bar{x}(t)$ —two-dimensional vector function; A—square matrix. The problem is defined by six parameters: the components of the matrix A and the vector \bar{x}_0 .

As it is known, in the case of imaginary roots $-\frac{\tilde{\sigma}}{2} \pm i\omega = \frac{\sigma}{2} \pm i\omega$ ($\tilde{\sigma} \geq 0, \omega > 0$) of the characteristic equation, the solution of the task has form

$$\bar{x} = e^{-\frac{\tilde{\sigma}}{2}t} (\bar{p} \cos \omega t + \bar{q} \sin \omega t),$$

where \bar{p}, \bar{q} —linearly independent vectors.

If the task is solved by an approximate numerical method with a step h , then the calculation error, for example, for the Runge–Kutta method of the second order of accuracy

$$\bar{r}_k = \bar{x}_k - \bar{x}(t_k) = \frac{-1}{24} h^2 f''(\xi_k),$$

where $\bar{f}(t) = A\bar{x}$, $0 < \xi_k < t_k$.

Since $\bar{f}(t) = \bar{x}'$, then $\bar{r}_k = \frac{-1}{24} h^2 \bar{x}''(\xi_k)$, $0 < \xi_k < t_k$.

We have

$$\bar{x}' = e^{-\frac{\tilde{\sigma}}{2}t} \left(\bar{p} \left(-\frac{\tilde{\sigma}}{2} \cos \omega t - \omega \sin \omega t \right) + \bar{q} \left(\omega \cos \omega t - \frac{\tilde{\sigma}}{2} \sin \omega t \right) \right),$$

$$\begin{aligned}\bar{x}'' &= e^{-\frac{\tilde{\sigma}}{2}t} \left(\begin{aligned} &\bar{p} \left(\left(\frac{\tilde{\sigma}^2}{4} - \omega^2 \right) \cos \omega t + \tilde{\sigma} \omega \sin \omega t \right) \\ &+ \bar{q} \left(-\tilde{\sigma} \omega \cos \omega t + \left(\frac{\tilde{\sigma}^2}{4} - \omega^2 \right) \sin \omega t \right) \end{aligned} \right), \\ \bar{x}''' &= e^{-\frac{\tilde{\sigma}}{2}t} \left(\begin{aligned} &\bar{p} \left(\left(\frac{3}{2} \tilde{\sigma} \omega^2 - \frac{\tilde{\sigma}^3}{8} \right) \cos \omega t + \left(\omega^3 - \frac{3}{4} \tilde{\sigma}^2 \omega \right) \sin \omega t \right) + \\ &+ \bar{q} \left(\left(\frac{3}{4} \tilde{\sigma}^2 \omega - \omega^3 \right) \cos \omega t + \left(\frac{3}{2} \tilde{\sigma} \omega^2 - \frac{\tilde{\sigma}^3}{8} \right) \sin \omega t \right) \end{aligned} \right).\end{aligned}$$

Obviously, $\bar{p} = \bar{x}_0$, $\omega \bar{q} - \frac{\tilde{\sigma}}{2} \bar{p} = A \bar{x}_0$. Therefore, we can take the following six parameters as parameters, defining the system: $\tilde{\sigma}$, ω and components of vectors \bar{p} , \bar{q} .

Since

$$|\bar{x}'''(t)| \leq e^{-\frac{\tilde{\sigma}}{2}t} \left(\frac{\tilde{\sigma}}{2} + \omega \right)^3 (|\bar{p}| + |\bar{q}|),$$

then

$$|\bar{r}_k| \leq \frac{h^2}{24} e^{-\frac{\tilde{\sigma}}{2} \xi_k} \left(\frac{\tilde{\sigma}}{2} + \omega \right)^3 (|\bar{p}| + |\bar{q}|) \leq \frac{h^2}{24} \left(\frac{\tilde{\sigma}}{2} + \omega \right)^3 (|\bar{p}| + |\bar{q}|).$$

From the condition $|\bar{r}_k| < \varepsilon$, we get

$$h^2 < \frac{24 \varepsilon}{(|\bar{p}| + |\bar{q}|) \left(\frac{\tilde{\sigma}}{2} + \omega \right)^3}.$$

From the six parameters, defining the task, we fix four, namely components of vectors \bar{p} and \bar{q} , i.e. we will consider different tasks with the same values \bar{p} and \bar{q} . We obtain the dependence of the step h on the attenuation decrement $\frac{\tilde{\sigma}}{2}$ and frequency ω .

$$h^2 < \frac{B}{\left(\frac{\tilde{\sigma}}{2} + \omega \right)^3}, \quad B = \frac{24 \varepsilon}{|\bar{p}| + |\bar{q}|}.$$

When $B = 1$, the curve has the form, shown in Fig. 4.1.

The obtained results indicate that the studied dependence should be taken into account when developing AS.

When determining the area of acceptable values h , depending on ω and the specified accuracy ε , it seems advisable to take into account the following factors.

First of all, we note that the operating frequency band is limited to $\approx 10 \text{ s}^{-1}$ from above and $\approx 2 \text{ s}^{-1}$ from below. In this case, the dimensionless attenuation coefficient ξ varies from 0.4 to 0.9.

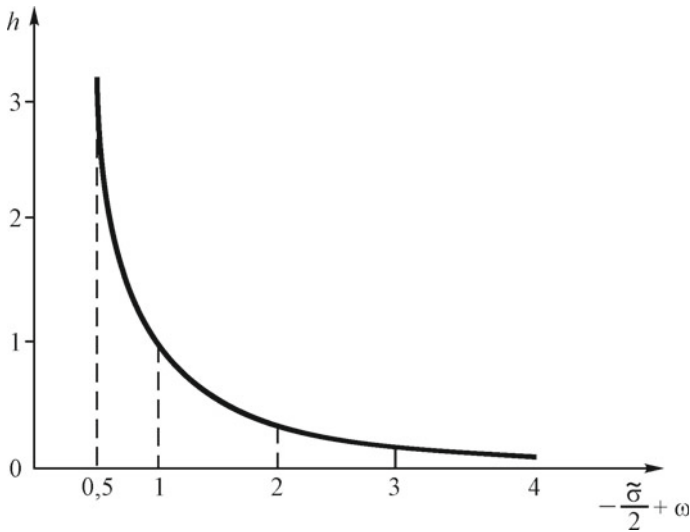


Fig. 4.1 Graph of dependence of step h from $(-\frac{\sigma}{2} + \omega)$

With the growth ξ , the value

$$-\frac{\sigma}{2} + \omega = -\frac{\xi\omega}{\sqrt{1-\xi^2}} + \omega = \omega_0(\sqrt{1-\xi^2} - \xi)$$

decreases, because

$$\frac{\partial(-\frac{\sigma}{2} + \omega)}{\partial\xi} = -\omega_0\left(1 + \frac{\xi}{\sqrt{1-\xi^2}}\right) < 0,$$

from where, in accordance with the previous, follows that an increase ξ leads to an increase in the permissible value h .

This explains to a large extent the desire in practice to increase the damping coefficient, when setting AS, at sufficiently large real values h . However, this leads to a distortion of the characteristics of the aircraft, because an increase ξ causes a corresponding decrease in the own frequency ω , and this, in turn, shifts the point (ξ, ω) in the areas of equal assessments of aerobatic characteristics. The evaluation of the aerobatic characteristics of AS improves, and the correspondence of the aircraft and AS deteriorates [1, 2]. In the special case, when $\xi = 0$ (system without attenuation!), we will have

$$h^2 < \frac{B}{\omega^3}.$$

For systems with own frequencies ω and $\Omega = k\omega$, the ratio of the maximum allowable steps (estimated from above!) will be defined as

$$\frac{h_{\Omega}}{h_{\omega}} = \sqrt{\frac{\omega^3}{\Omega^3}} = \frac{1}{\sqrt{k^3}}.$$

As you can see, in a given operating frequency band from 2 rad/s to 10 rad/s, the permissible step of integration varies by more than 10 times.

A similar estimate for the step h can be obtained, based on the condition

$$|\bar{R}_k| < \delta,$$

where $\bar{R}_k = \bar{x}'_k - \bar{x}'(t_k) = r'_k$.

Indeed,

$$\begin{aligned} \bar{R}_k = \bar{r}'_k = A\bar{r}_k &= A\left(-\frac{h^2}{24}\bar{x}'''(\xi_k)\right) = -\frac{h^2}{24}\bar{x}^{IV}(\xi_k), \quad 0 < \xi_k < t_k. \\ \bar{x}^{IV} &= e^{-\frac{\tilde{\sigma}}{2}t} \left(\begin{aligned} &\bar{p}\left(\left(\frac{\tilde{\sigma}^4}{16} - \frac{3}{2}\tilde{\sigma}^2\omega^2 + \omega^4\right)\cos\omega t + \left(\frac{\tilde{\sigma}^3\omega}{2} - 2\tilde{\sigma}\omega^3\right)\sin\omega t\right) + \\ &+\bar{q}\left(\left(2\tilde{\sigma}\omega^3 - \frac{\tilde{\sigma}^3\omega}{2}\right)\cos\omega t + \left(\frac{\tilde{\sigma}^4}{16} - \frac{3}{2}\tilde{\sigma}^2\omega^2 + \omega^4\right)\sin\omega t\right) \end{aligned} \right). \end{aligned}$$

Since

$$|\bar{x}^{IV}(t)| \leq e^{-\frac{\tilde{\sigma}}{2}t} \left(\frac{\tilde{\sigma}}{2} + \omega\right)^4 (|\bar{p}| + |\bar{q}|),$$

then

$$|\bar{R}_k| \leq \frac{h^2}{24} e^{-\frac{\tilde{\sigma}}{2}\xi_k} \left(\frac{\tilde{\sigma}}{2} + \omega\right)^4 (|\bar{p}| + |\bar{q}|) \leq \frac{h^2}{24} \left(\frac{\tilde{\sigma}}{2} + \omega\right)^4 (|\bar{p}| + |\bar{q}|).$$

From the condition $|\bar{R}_k| < \delta$, we will get

$$h^2 < \frac{24\delta}{(|\bar{p}| + |\bar{q}|) \left(\frac{\tilde{\sigma}}{2} + \omega\right)^4} \text{ or } h^2 < \frac{c}{\left(\frac{\tilde{\sigma}}{2} + \omega\right)^4}, \quad C = \frac{24\delta}{|\bar{p}| + |\bar{q}|}.$$

If $\delta = \varepsilon$, then $C = B$.

Figure 4.2. shows graphs for h_{ε} and h_{δ} , where h_{ε} —maximum value of step h on condition $|\bar{r}_k| < \varepsilon$; h_{δ} —maximum value of step h on condition $|\bar{R}_k| = |\bar{r}'_k| < \delta$ (when $B = C = 1$).

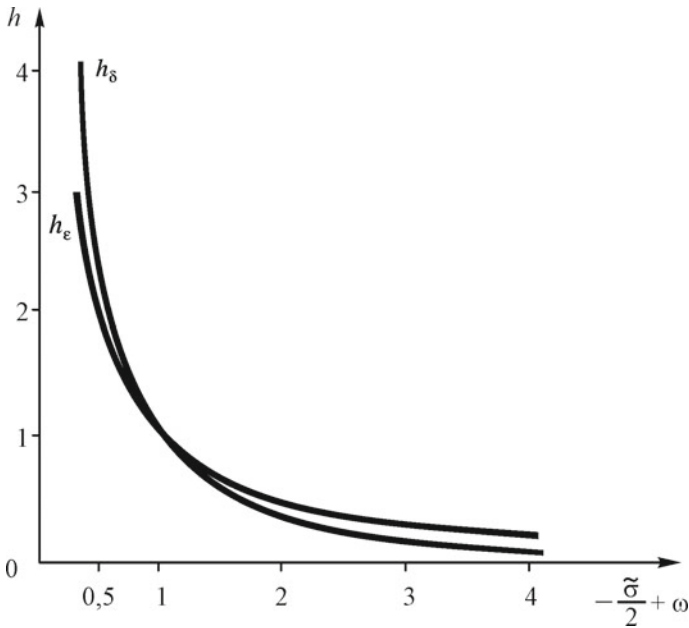


Fig. 4.2 Dependence of the integration step h from ϵ and δ

4.5 The Choice of the Integration Step in the Approximate Calculation of a Linear System with a Delay

Let the task:

$$\left. \begin{aligned} x'(t) &= a_{11}x(t) + a_{12}y(t) + b_1(p_1x(t - \tau_1) + p_2y(t - \tau_2)) \\ y'(t) &= a_{21}x(t) + a_{22}y(t) + b_2(p_1x(t - \tau_1) + p_2y(t - \tau_2)) \end{aligned} \right\} (t \geq 0),$$

$$\begin{aligned} x(t) &= S_1(t), \quad t \leq 0; \quad x(+0) = S_1(0), \\ y(t) &= S_2(t), \quad t \leq 0; \quad y(+0) = S_2(0), \end{aligned} \quad (\tau_1, \tau_2 \geq 0).$$

be set.

If the delays τ_1, τ_2 are so small that they can be neglected, then the problem is solved without delay:

$$\left. \begin{aligned} x'(t) &= (a_{11} + b_1p_1)x(t) + (a_{12} + b_1p_2)y(t) \\ y'(t) &= (a_{21} + b_2p_1)x(t) + (a_{22} + b_2p_2)y(t) \end{aligned} \right\} (t \geq 0);$$

$$x(+0) = S_1(0); \quad y(+0) = S_2(0).$$

This is acceptable if already the step $h > \max(\tau_1, \tau_2)$ gives sufficient accuracy (due to the continuity of the dependence on the delay).

If it is impossible to neglect the delay, then the step should be proportionate with the delay. Let $\tau_1 = \tau_2 = \tau$. Then, it is accepted $\tau = mh$, where $m \in \{1, 2, \dots\}$.

The task in vector–matrix form has the form

$$\begin{aligned}\bar{x}'(t) &= A\bar{x}(t) + QT\bar{x}(t) \quad (t \geq 0), \\ \bar{x}(t) &= \bar{S}(t), \quad t \leq 0, \bar{x}(+0) = \bar{S}(0)\end{aligned}$$

Here

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad Q = \begin{bmatrix} b_1 p_1 & b_1 p_2 \\ b_2 p_1 & b_2 p_2 \end{bmatrix}, \quad T = \begin{bmatrix} T_\tau & 0 \\ 0 & T_\tau \end{bmatrix} = T_\tau E$$

$$T_\tau \text{—operator of shift: } T_\tau z(t) = z(t - \tau), \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \bar{S}(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \end{bmatrix}.$$

On the segment $0 \leq t \leq \tau = mh$, it is necessary to solve approximately the problem

$$\bar{x}(t) = \bar{S}(t), \quad t \leq 0, \bar{x}(+0) = \bar{S}(0).$$

At the same time, it is advisable to reduce the step against the accepted h , namely take a step h^* , where $h = nh^*$, $n \in \{1, 2, \dots\}$;

$$\tau = mh = mn h^*, \quad 0 \leq t \leq \tau = mn h^*.$$

Here, the Runge–Kutta method of the second order of accuracy will take the form

$$\begin{aligned}\bar{x}_{j+1} &= \bar{x}_j + h * A\bar{x}_j + \frac{(h^*)^2}{2} A^2 \bar{x}_j + h * Q\bar{S}_{j-mn+\frac{1}{2}} \\ (j &= 0, 1, \dots, mn - 1), \quad \bar{x}_j = \bar{x}(jh^*), \quad \bar{S}_j = S(jh^*).\end{aligned}$$

Next for $t \geq \tau = mh$ the initial problem with a delay with a step h is solved approximately. Here, the Runge–Kutta method will take the form

$$\begin{aligned}\bar{x}_{k+1} &= \bar{x}_k + hA\bar{x}_k + \frac{h^2}{2} A^2 \bar{x}_k + hQ\bar{x}_{k-m} + \frac{h^2}{2} Q^2 \bar{x}_{k-2m} \quad (k = 0, 1, \dots), \\ \bar{x}_k &= \bar{x}(\tau + kh), \quad \bar{S}_k = \bar{S}(\tau + kh).\end{aligned}$$

Due to the continuity of the dependence of the correctly set task on the delay, with small values of the delay, the dependence between the step h , frequency ω , attenuation decrement $-\sigma/2$ and the specified accuracy ε (when fixing the remaining parameters of the system) remains close to the dependence that exists for systems without delay.

In the final form, the dependence h on τ is not given due to the inefficiency of the large formula.

4.6 Selection of the Simplest Means of Smooth Approximation

Let be given a finite strictly increasing sequence x_1, x_2, \dots, x_m of the values x , where m —the number of values is quite large, and the sequence y_1, y_2, \dots, y_m corresponding values of some function $y = y(x)$. Let also be given an acceptable error $\varepsilon > 0$ of uniform approximation of the given values of this function by some smooth function.

It is required to split the sequence x_1, x_2, \dots, x_m into possibly longer sequences $x_1, x_2, \dots, x_{l_1}; x_{l_1+1}, \dots, x_{l_2}; x_{l_2+1}, \dots, x_{l_h}; x_{l_h+1}, \dots, x_m$ (each of which, with the possible exception of the last one, would contain more than three points), so that on each such sequence it is possible to approximate the function $y = y(x)$ by the least squares method with a polynomial of degree no higher than the third with an acceptable error. If the last sequence contains only three points $x_{l_h+1}, x_{l_h+2}, x_m$, then perform a parabolic interpolation of the function $y = y(x)$ on it; if the last sequence contains only two points x_m , then perform a linear interpolation of the function $y = y(x)$ on it; if the last sequence contains only one point x_m , then it is necessary to give a given value y_m of the approximated function to the approximating function at this point. After performing this approximation, join the polynomials at the joints $[x_{\ell_1}, x_{\ell_1+1}], [x_{\ell_2}, x_{\ell_2+1}], \dots, [x_{\ell_h}, x_{\ell_h+1}]$ in a smooth way, using polynomials of the third degree, and, if the last sequence contains only one point $x_{\ell_h+1} = x_m$, then at the last joint $[x_{\ell_h}, x_m]$ join a polynomial and a value y_m in the final point x_m in a smooth way, using a polynomial of the second degree.

As a result, we obtain a uniform approximation with an acceptable error by a spline no higher than the third degree.

4.6.1 The Composition of the Software

Software for smooth approximation of functions is a combination of information-related modules:

1. Module D: Data entry, preparation of the first cycle.
2. Module A: Approximation by least squares method by a polynomial of degree no higher than the third with an acceptable error on the longest as possible (more than three points) sequence, starting from this point (cycle) (Fig. 4.3.).
3. Modulus I: Interpolation on the last sequence, if it contains no more than three points.
4. Module R: Redirection, in order to move from one sequence to the next within the entire given sequence of values of the independent variable (i.e. from one cycle A to the next) (Fig. 4.4.).
5. Module J: Joint of the approximating function at the joints in a smooth manner by the polynomials of the third degree, and at the last junction, possibly, second degree.

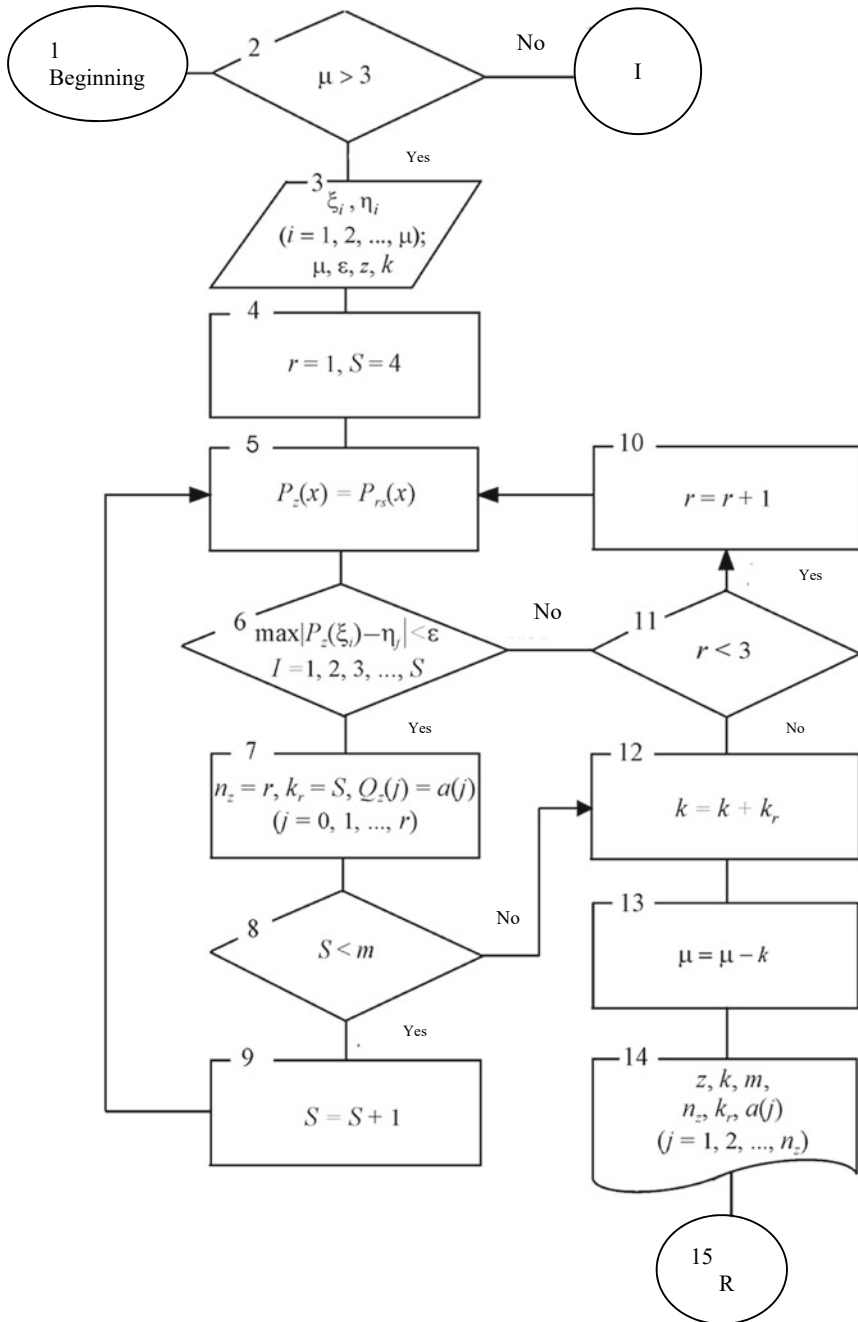
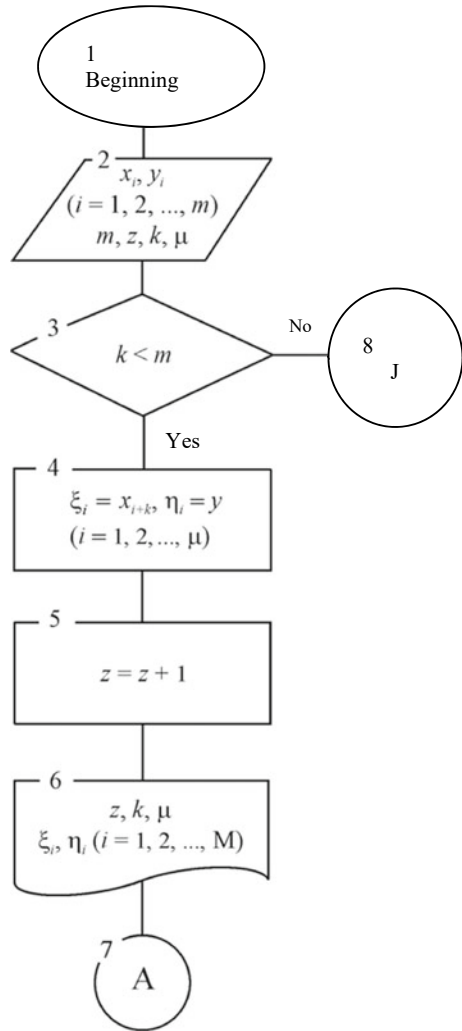


Fig. 4.3 Block diagram of the algorithm of the functioning of module A

Fig. 4.4 Block diagram of the algorithm of module R operation



The program is developed, using a programming system in the FORTRAN algorithmic language, and used in the creation of AS software.

4.6.2 Implementation of the Program

The program of smooth approximation of functions is based on the following logic circuits.

Module D

1. Entering pairs of values $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$; the numbers m of these values ($m > 3$); permissible error ε ($\varepsilon > 0$) of uniform approximation.
2. Passing values to the first cycle:

$$\xi_i = x_i, \quad \eta_i = y_i \quad (i = 1, 2, \dots, m); \quad \mu = m;$$

$z = 1$ (start of cycle counting); $k = 0$ (the beginning of the count of passed points).

3. Transition to the module A.

Модуль А

Let $P_z(x) = \sum_{j=0}^n a_z(j) x^j$ —approximating polynomial on the z -th cycle; $P_{nk}(x) = \sum_{j=0}^n a_{nk}(j) x^j$ —a polynomial of degree n , constructed by the least squares method by values y_1, y_2, \dots, y_k of function $y = y(x)$ at the points x_1, x_2, \dots, x_k .

Module I

Let $Q(x) = ax^2 + bx + c$ —interpolation polynomial.

1. Input μ :

- if $\mu = 1$, then η_1 is introduced, and we believe

$$a = 0, \quad b = 0, \quad c = \eta_1;$$

- if $\mu = 2$, then $\xi_1, \xi_2, \eta_1, \eta_2$ are introduced, and we believe

$$a = 0, \quad b = \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1}, \quad c = \frac{\xi_2 \eta_1 - \xi_1 \eta_2}{\xi_2 - \xi_1};$$

- if $\mu = 3$, then $\xi_1, \xi_2, \xi_3, \eta_1, \eta_2, \eta_3$ are introduced, and we believe

$$a = \frac{\eta_3 - 2\eta_2 + \eta_1}{2(\xi_2 - \xi_1)^2}, \quad b = \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1} - \frac{(\eta_3 - 2\eta_2 + \eta_1)(\xi_2 + \xi_1)}{2(\xi_2 - \xi_1)^2},$$

$$c = \frac{(\eta_3 - 2\eta_2 + \eta_1)\xi_2\xi_1}{2(\xi_2 - \xi_1)^2} - \frac{\eta_2 - \eta_1}{\xi_2 - \xi_1}.$$

2. Output of calculation results a, b, c and transition to the module J.

Module J

1. For Each z -th Joint $[\xi_1, \xi_2]$, Where $\xi_1 = x_{k_1+k_2+\dots+k_z}$, $\xi_2 = x_{k_1+k_2+\dots+k_z+1}$, a Joint Polynomial $R_z(x)$ is Constructed:
 - if $\xi_2 < x_m$, then $R_z(x) = b_0(z)x^3 + b_1(z)x^2 + b_2(z)x + b_3(z)$; coefficients $b_0(z), b_1(z), b_2(z), b_3(z)$ are found from the system of equations in the only

way, because the determinant of the system $(\xi_2 - \xi_1)^4$ is different from zero for any different points;

- if $\xi_2 = x_m$, then $R_z(x) = b_1(z)x^2 + b_2(z)x + b_3(z)$, ($b_0(z) = 0$); coefficients $b_1(z)$, $b_2(z)$, $b_3(z)$ are found from the system of equations

$$\left. \begin{aligned} \xi_1^2 b_1(z) + \xi_1 b_2(z) + b_3(z) &= P_z(\xi_1) \\ 2\xi_1 b_1(z) + b_2(z) &= P'_z(\xi_1) \\ \xi_2^2 b_1(z) + \xi_2 b_2(z) + b_3(z) &= P_{z+1}(\xi_2) \end{aligned} \right\}$$

in the only way, because the determinant of the system $(\xi_2 - \xi_1)^2$ is different from zero for any different points.

2. The results of the calculations $b_0(z)$, $b_1(z)$, $b_2(z)$, $b_3(z)$ are given.

The calculations are finished.

4.7 Approximation Problems in the Formation of Databases

We present software and algorithmic support for the efficient construction of best approximation polynomials, using I.I. Etherman aggregates for tabular and graphical assignment of functions of two and three variables.

To approximate the function $\varphi(z)$ on the segment $-1 \leq z \leq 1$, asymptotic polynomials, listed below, are used:

$$Q_n(z) = b_0 T_0(z) + \sum_{r=1}^n b_r T_r(z),$$

where

$$b_0 = \frac{1}{n+1} \sum_{j=0}^{n+1} \varphi(\eta_j^{(n)}),$$

$$b_r = \frac{r}{n+1} \sum_{j=0}^{n+1} \varphi(\eta_j^{(n)}) T_r(\eta_j^{(n)}), \quad r = \overline{1, n};$$

symbol \sum ' means

$$\sum_{j=0}^{n+1} a_j = \frac{a_0}{2} + \sum_{j=0}^n a_j + \frac{a_{n+1}}{2},$$

$T_r(z) = \cos r \cdot \arccos z$, $r = 0, 1, \dots$ —Chebyshev polynomials of the first kind;

$$\begin{aligned} T_0(z) &= 1, \\ T_1(z) &= z, \\ T_2(z) &= 2z^2 - 1, \dots \end{aligned}$$

$\eta_j^{(n)} = \cos \frac{j\pi}{n+1}$, $j = \overline{0, n+1}$ —points, asymptotically close at $n \rightarrow \infty$ to the points of alternance $\xi_j^{(n)}$, i.e. to the points, at which the difference between a continuous function and its polynomial of the best approximation of a given degree n reaches values $\pm E_n(\varphi)$ alternately, where $E_n(\varphi) = \min_{P_n} \max_{-1 \leq z \leq 1} |\varphi(z) - P_n(z)|$ —the best approximation of a polynomial P_n .

There are assessments

$$\begin{aligned} |\varphi(z) - Q_n(z)| &\leq E_n(\varphi) \left(E_n(\varphi) + 9 + \frac{2}{\pi} \ln(n+1) \right), \\ \left| \varphi \left(\eta_j^{(n)} \right) - Q_n \left(\eta_j^{(n)} \right) \right| &\leq \frac{1}{2^n (n+1)!} \varphi^{(n+1)}(\theta), \\ (-1 < \theta < 1, \quad j &= \overline{0, n+1}). \end{aligned}$$

So, by applying a sequential approximation by an asymptotic polynomial for each of the two variables, we obtain the required form of the asymptotic polynomial:

$$\begin{aligned} R(u, v) &= A_{00} + A_{10}u + A_{01}v + A_{20}u^2 + A_{11}uv + A_{02}v^2 + A_{21}u^2v \\ &\quad + A_{12}uv^2 + A_{22}u^2v^2 \end{aligned}$$

for the function $F(u, v)$ (variable substitution $F(u, v) = f(x, y)$ is used, where $x = ru - q$, $y = sv - h$, when $-1 \leq x \leq 1$, $-1 \leq y \leq 1$; $a \leq u \leq b$, $c \leq v \leq d$).

Similarly, to the case $n = 2$, the representation is obtained:

$$\begin{aligned} f(x, y) &\approx Q_n(x, y) = \\ &= \frac{1}{(n+1)^2} \sum_{k=0}^{n+1} \sum_{l=0}^{n+1} f_{kl} \left(1 + 2 \sum_{p=1}^n T_p \left(\eta_k^{(n)} \right) T_p(x) + 2 \sum_{q=1}^n T_q \left(\eta_l^{(n)} \right) T_q(y) + \right. \\ &\quad \left. + 4 \sum_{p=1}^n \sum_{q=1}^n T_p \left(\eta_k^{(n)} \right) T_q \left(\eta_l^{(n)} \right) T_p(x) T_q(y) \right), \end{aligned}$$

where

$$f_{kl} = f\left(\eta_k^{(n)}, \eta_l^{(n)}\right)$$

$$\eta_j^{(n)} = \cos \frac{j\pi}{n+1}, \quad (j = \overline{0, n+1})$$

If function $F(u, v, w)$ set on a parallelepiped

$$a \leq u \leq b,$$

$$c \leq v \leq d,$$

$$e \leq w \leq f$$

in a tabular-graphical way (a finite set of graphs, giving a dependence $F(u, v, w)$ from the first variable u for each of the given pairwise different sets (v_j, w_j) of the values of the other two variables v, w ($i = \overline{0, l}, j = \overline{0, m}$)), then, by making a variable replacement

$$x = ru - q \quad (-1 \leq x \leq 1),$$

let's move to the function $\varphi(x, v, w) = F(u, v, w)$, where $r = \frac{2}{b-a}$, $q = \frac{b+a}{b-a}$.

Choosing a sufficiently large n will determine a set of points $x = \eta_k^{(n)} = \cos \frac{k\pi}{n+1}$ ($k = \overline{0, n+1}$) on the segment $-1 \leq x \leq 1$.

Replacing the functions $\varphi(\eta_k^{(n)}, v, w)$ with interpolation polynomials, we get a polynomial, approximating the given function:

$$F(u, v, w) = \varphi(ru - q, v, w) = \varphi(x, v, w),$$

$$Q_n(x, v, w) = C_0(v, w) T_0(x) + \sum_{p=1}^n C_p(v, w) T_p(x),$$

where

$$C_0(v, w) = \frac{1}{n+1} \sum_{k=0}^{n+1} \sum_{j=0}^m \sum_{i=0}^l {}^k \alpha_i^j v^i w^j,$$

$$C_p(v, w) = \frac{2}{n+1} \sum_{k=0}^{n+1} \sum_{j=0}^m \sum_{i=0}^l {}^k \alpha_i^j T_p(\eta_k^{(n)}) v^i w^j.$$

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Chapter 5

Creation of a Virtual Educational Environment for Training Air Traffic Management Dispatchers



The work of an air traffic management (ATM) dispatcher is an intellectual work, associated with processing significant amounts of information, coming from a variety of sources, as well as with large psychophysiological overloads, fatigue and stress.

Fatigue is a temporary disintegration of the functional systems of the body (especially the neurons of the cerebral cortex), leading to a decrease in the reserve capabilities of a person and his working capacity. In a state of fatigue, the number of radio exchanges decreases, the number of errors in the perception and transmission of speech messages increases, the vocabulary is depleted and the clarity of articulation and the pace of speech decreases [1].

Stress is a state of the body that occurs as a result of intense or prolonged exposure (regardless of its nature) and is characterized by the stress of nonspecific adaptive mechanisms [1].

Summarizing the above, it can be noted that the stress and fatigue of the dispatcher is a serious threat to flight safety. What is the nature of stress? To answer this question, it is necessary to understand how the process of receiving and analyzing incoming information goes, as well as subsequent decision-making, based on this information. In the modern field of psychology, labor psychology, engineering psychology and ergonomics, these processes are described as follows.

When information is received through visual, auditory and other channels, the human intellect begins to analyze the received information, and the process of searching for a familiar image in an «informational» or «permanent conceptual» model begins.

The concept of the «conceptual model» was proposed by the English psychologist A. T. Welford in 1961 at the XIV International congress on applied psychology. The conceptual model is revealed as a global image, formed in the operator's head. In the understanding of A. T. Welford, the conceptual model gives the operator a holistic picture of a human-machine control system and therefore provides an opportunity to correlate different parts of the process with the whole, and, accordingly, to act effectively [2].

According to G. V. Sukhodolsky, the conceptual model is the main internal means of professional activity, created in the process of his professional training [3]. A. A. Oboznov connects the content of the conceptual model with the systematized knowledge that a specialist needs to navigate the processes, occurring in technological complexes [4].

If such an image is not found in the information model of a human operator, the process of independent search for a solution, an active phase of thinking, begins. This phase consumes a large amount of energy, which leads to fatigue, and psychophysiological exhaustion.

The absence of certain rules and norms of behavior in critical non-standard situations determines significant emotional overload, which can significantly reduce the reliability of the dispatcher work [1].

Based on the above, it can be concluded that the expansion of the conceptual model, the accumulation of images, is one of the most important aspects of the educational process. That is why it is necessary to pay great attention to the formation of a powerful theoretical base, the foundation of the trainee's professional knowledge, as well as practical exercises on air traffic support (ATS) simulators. At the same time, a qualitative conceptual model should be structured; it should not be a set of disparate elements of information, but an integral structure of knowledge.

In order to provide trainees with opportunities to expand the conceptual model, the development of a virtual educational environment for training ATM dispatchers was started.

Separately, it is worth noting the relevance of the development of virtual educational systems in the year 2020. The importance of distance learning systems in the context of a viral infection pandemic is obvious and does not need additional assessment. This will allow adding the educational process and will enable trainees to acquire the practical skills and abilities, necessary for the implementation of ATS, regardless of the availability of access to the dispatcher training center.

5.1 Analysis of the Technical Possibilities of Creating a Virtual Educational Environment. Development of the Concept of the Virtual Educational Environment Application Software Package

5.1.1 Analysis of the Technical Possibilities of Creating a Virtual Educational Environment

A virtual educational environment (VEE) is a software and telecommunications environment that provides the management of the educational process, its information support and documentation in electronic networks, using unified technological means [5].

A virtual educational environment is a rapidly developing, multi-level and multifunctional system that unites.

- innovative and traditional technologies, specific to the interaction of participants in the educational process within the framework of an open model of asynchronous individual learning;
- information resources: databases and knowledge, libraries, electronic educational materials, etc.;
- modern software tools: software shells, electronic communication tools [6].

To analyze the technical possibilities of a VEE ATM creation, it is necessary to determine the purpose and tasks of the intended educational environment.

The purpose of the educational environment is to prepare ATM dispatchers for the direct implementation of ATS.

Tasks of the educational environment:

1. Providing trainees with a convenient and easily accessible form of a certain amount of relevant educational and methodological materials is necessary for the direct implementation of ATS.
2. Providing trainees with the opportunity to familiarize themselves with the working conditions of the ATM dispatcher.
3. Providing trainees with the opportunity, within the framework of self-training, using this educational environment, to obtain theoretical knowledge, practical skills and skills, necessary for conducting work in a Unified Air Traffic Management System, including for the implementation of direct ATS.

It should be noted that the term «trainee» means anyone who wants to use this VEE for informational and educational purposes.

Educational and methodological materials are materials that contain both theoretical information on an academic discipline (or its section) and materials on the methodology of its independent study and practical mastering [7]. Educational and methodological materials can be provided in the following types:

- on physical (paper) media;
- in electronic form.

The technical sphere, which, of course, includes ATS, imposes additional requirements on the educational process. The trainee must be fully confident that the information he receives is the most relevant at the current time. When organizing the process of training ATM dispatchers, it is necessary to take into account the pace of technological progress in the twenty-first century, the speed of technology development and the continuity of modernization of technological systems. The information provided needs constant updating. The procedure for updating information on physical (paper) media is technologically much more complicated, requires large financial resources and takes a longer period of time, compared to a similar procedure for information, presented in electronic form. Also, the disadvantages of physical media include aspects, such as

- accumulation of a large number of different editions of the same teaching materials;
- difficulties, associated with obtaining, storing and returning paper media to the university library;
- fragility of paper media, the need for careful handling of them;
- the need for transportation, the inability to access information, if the paper medium is not near the person.

Based on this, it can be concluded that the distribution of educational and methodological materials in electronic form is more optimal. Teaching materials can be presented in the form of electronic educational articles on various topics, covering the ATS process.

Next, it is necessary to consider options for electronic information platforms, in which electronic educational articles can be placed:

- a group in a social network («In contact», «Facebook»);
- free information service, blog platform («Yandex Zen», «Live journal», etc.);
- electronic portal (website), developed directly for VEE.

Posting educational articles on social networks and blog platforms does not require financial and significant time resources, however, an electronic portal, designed for a specific VEE, has significant advantages:

- the opportunity to develop and debug in detail for specific educational purposes the information space, in which educational and methodological materials will be placed;
- the possibility of implementing a publicly available rating system, open to trainees, teachers and ATM specialists. Such a system can create a competitive environment for trainees, which will also increase motivation to study;
- the possibility of placing special tests and application programs to consolidate the acquired knowledge, and the formation of skills and abilities, necessary for the implementation of ATS. This opportunity plays a crucial role, since it is necessary to fulfill the second and third highlighted tasks of the educational environment.

Application programs are programs designed to solve specific tasks and interact directly with the user. Within the framework of designing a virtual educational environment, the purpose of application programs is to interact with the trainee: the trainee enters data at the input, and at the output he receives the result in the form of processed data, thus contributing to the formation of certain skills, necessary for him in further activities in the area of ATM.

The publicly available application programs that significantly contribute to the formation of ATS skills include virtual aviation simulators and computer networks, designed to recreate real procedures for interaction between aircraft crews and ATM controllers on the Internet.

These include

- IVAO («The International Virtual Aviation Organisation»);
- «VATSIM Network» (Virtual Air Traffic Simulation Network) and others.

Such simulators and networks accurately simulate the real working conditions of aircraft crews and ATM dispatchers, however, they are quite difficult to install and configure, and also involve a preliminary study of a large amount of information (including in English) to interact with real people, who already have a certain level of knowledge, skills and abilities. This may be a psychological barrier for a trainee with an initial level of knowledge, as a result of which he will not dare to install and study such a simulator.

In this case, the best option may be those applications that are most easily accessible and convenient for a wide audience. To consolidate the theoretical knowledge, obtained from the teaching materials on the VEE website, application programs can be used that are available without prior installation and configuration. Such an opportunity is provided by programs in the format of browser games, created using the HTML markup language (in particular, HTML5) and the JavaScript programming language.

Advantages of HTML5 application programs:

- there is no need to install additional software modules;
- open and free software;
- compatible with various operating systems, computers and mobile devices;
- local storage «HTML5 Local Storage» has the ability to store a large amount of information, limited only by the amount of storage on the computer of use and browser settings;
- multi-user mode of support.

Lack of application programs in the format of browser games:

- limited resources available.

This disadvantage must be taken into account, when developing and implementing the concept of application programs, however, for a virtual educational environment, the browser format of application programs remains optimal, since the goals of the online modules of the dispatcher simulator are educational and do not involve recreating the full scope of functions, available in the software of real computer system ATS.

Special attention should be paid to the aspect of adapting VEE for use on mobile devices. Mobile devices are an integral part of the life of most modern people; the volume of distribution and use of touch tablets and phones do not need additional assessment. These devices provide wide opportunities to access the necessary information, wherever the user is. For this reason, all educational and methodological materials, available in VEE, as well as application programs, should be maximally adapted to mobile devices, if this does not interfere with the achievement of the educational goals, prescribed by them.

5.1.2 The Concept of the Virtual Educational Environment Application Software Package

Skill is the ability of an activity, formed by repetition and bringing it to automatism. Any new mode of action, proceeding initially as some independent, expanded and conscious, then as a result of repeated repetitions, can already be carried out as an automatically performed component of activity [8].

Flexible (soft) skills are a complex of non-specialized, important for career, supra-professional skills that are responsible for successful participation in the workflow, high productivity and are end-to-end, that is, not related to a specific subject area [9].

Specialized skills—a set of professional skills, necessary for the implementation of labor activity within a specific specialty.

The following skills, needed for an ATM dispatcher, were highlighted:

- (1) specialized skills (vectorization, recognition of potential conflict situations, forecasting of the air situation (individual aircraft flows) and the skill of conducting phraseology of radio exchange in English);
- (2) auditory perception of English;
- (3) concentration and distribution of attention;
- (4) short-term memory;
- (5) reaction speed;
- (6) quick and accurate analysis of the situation and subsequent decision-making.

The development of concepts of application programs takes place in accordance with the task of forming these skills, according to the following plan:

- definition of the name;
- description of the program;
- purpose of the program;
- necessary equipment to use the program;
- program interface;
- functionality of the program;
- standard scenario of using.

A necessary component of the ATS simulator is a high degree of interactivity, interaction between the user and objects. The interactivity of the ATS simulator brings simulated working conditions closer to real ones, which can increase the user's motivation for the learning process, his interest in practical classes. For this reason, voice messages from virtual aircraft crews are planned to implement in the ATS simulator. Technical capabilities allow you to create various interaction scenarios and implement the issuance of requests and «receipts» (confirmations of issued instructions) by virtual crews of the aircraft. The introduction of different types of voices for the crews of the aircraft, as well as the introduction of differences in the pace of speech, is also very important.

In order to increase the interactivity of the simulator, the information notification area was also included in the concept of the ATS simulator. Messages about incorrect

user actions can be displayed here. As an example, untimely acceptance of the aircraft under control and transfer of the aircraft to the next ATS sector can be cited. The exact number of implemented variants of displayed messages can be determined at the stage of implementation.

Possible interaction scenarios:

- (1) The crew's report on the location and acceptance of the aircraft by the trainee under the control of the «ASSUME» function;
- (2) Requesting a change of echelon from the crew and introducing the requested echelon by the trainee for the corresponding aircraft;
- (3) Request to change the frequency and switch to radio communication with another ATS body from the crew, the subsequent actions of the trainee, using the appropriate interface functions.

5.2 Implementation of the Developed Concepts of the Application Software Package

The implementation of the developed concepts of applied training programs, as well as the development of educational and methodological material, should be based on Federal Aviation Regulations (FAR) and specialized methodological literature [10, 11].

The implementation of the programs is based on the concepts, presented in the previous paragraph.

Performance indicators were selected in the following programs:

- «Solution of a potential conflict situation (PCS)»;
- «Provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data»;
- «Numbers»;
- «Identification of the aircraft».

Let's give a brief description of each of these programs.

«Solution of potential conflict situations (PCS)»

Task: to solve 36 potential conflict situations in the safest, most efficient and economical way.

PCS solution, when the aircraft is moving

- on opposite courses;
- on passing courses;
- on crossing courses.

There are 6 types of PCS available:

- one aircraft is on the horizon, and the other aircraft is in the climb;
- one aircraft is on the horizon, and the other aircraft is in the decline;

- both aircraft are on the horizon;
- one aircraft is in the climb, and the other aircraft is in the decline;
- both aircraft are in the climb;
- both aircraft are in the decline.

Each type of PCS is implemented in 2 variants:

- it is necessary to separate the trajectories, in order to avoid dangerous convergence;
- it is enough to control the vertical speed (acceleration of the climb/descent).

To issue instructions, you need to go to the control panel at the bottom of the screen:

- select the aircraft;
- select the required instruction in the list;
- enter the initial data (echelon, speed in the form of a Mach number and course or turn angle);
- if necessary, select the «immediately» option and additional data;
- press the corresponding input button on the right side of the control panel.

If the instruction is drawn up correctly and corresponds to the current air situation, it will be entered in the list of issued instructions on the right side of the control panel, and the crew will confirm it by issuing a «receipt». Incorrect instructions will not be accepted (for example, a descending aircraft cannot accelerate the «climb») [10].

«Provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data»

The training complex contributes to the formation of practical ATS skills, in particular, the skill of provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data of aircraft.

The program contains 8 different ATS sectors (exercises). Each sector has unique restriction zones, the location of the airfield and the controlled aircraft. After 3 s. after starting the exercise, the crew of a given aircraft requests a voice command from the ATM dispatcher.

The trainee task:

- identify the aircraft on the air situation indicator (ASI);
- analyze the air situation and mentally draw up the shortest route from the current location of the aircraft to the airfield, bypassing the restricted areas. It is necessary to maintain a minimum distance to the zones, but do not allow the aircraft to enter the restricted areas;
- control the aircraft by issuing instructions, using the corresponding panel in the lower right corner of the screen;
- the course of the aircraft upon reaching the airfield should be in the range of ± 45 degrees from the course of the runway of the airfield. In this case, the «Fly by your own means» command will be available in the control panel;

- finish the procedure of provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data of aircraft by issuing the above command.

The complex «Numbers» contributes to the formation of a stable skill of perception of numbers in English. Working with numbers in English is a significant part of the radio exchange process, so this skill is critically necessary for effective ATS.

The program generates random numbers with the specified parameters and reproduces them in different voices. The pronunciation of numbers occurs, according to the rules of aviation English.

It contains 3 modes.

1. Continuous dictation

It is the mode of the continuous output of numbers with the specified number of characters (2-, 3- or 4-digit), with a given pace of speech. It is recommended to record the voiced numbers manually. As the perception of numbers improves, it is necessary to set a higher rate of speech.

2. Single input

It is entering numbers into the input window with the ability to evaluate the speed of perception. We have 3 indicators:

- speed (number of inputs per minute);
- average speed for the last 15 inputs;
- the best series of 15 inputs (the best average speed for 15 inputs).

3. Comparison mode

The mode is designed to increase the level of awareness and the quality of perception of numbers.

Two random numbers are given.

Task: Select a larger or smaller number, depending on the selected installation. Three installations are provided:

- more;
- less;
- random.

If a random setting is selected, the program randomly determines whether a larger or smaller number needs to be entered. The sign «>» (more) or «<» (less) appears after 3 s. after voicing the numbers.

«Identification of the aircraft»

The training complex «Identification of the aircraft» promotes the development of short-term memory, reaction speed and concentration skills; forms speech perception skills in English, skills of effective radio communication with foreign crews.

The Air Situation Indicator (ASI) displays 24 aircraft in cruising flight mode. The radio exchange is conducted in English. One of the virtual crews requests a descent or climb through the radio. Examples of requests:

- Moscow Control, AFL1277, request climbing to flight level 320;
- Moscow Control, AFL1498, request descending to flight level 130.
- Task:
- promptly identify the aircraft, whose crew requested a change of echelon (find on ASI);
- open the echelon window by clicking on the CFL and select the requested echelon;
- give the instructions for the crew for changing the echelon in the window that appears for issuing instructions. This window is an analogue of issuing instructions by voice to the radio. In this regard, it is recommended to pronounce the instructions, issued out loud;
- listen to the response («receipt») of the crew.

The number of scores varies, depending on the scenario. The scores are displayed in the upper right corner of the screen.

Possible scenarios:

1. The aircraft is correctly identified, and the instruction contains the correct direction (decrease or climb) and the echelon. An example of the crew's response: «Roger, to descend to FL170, AFL1252». 1 score is added.
2. The aircraft was correctly identified, the correct echelon and the direction was incorrectly selected. An example of the crew's response: «Negative, we have to descend, AFL1252». 1 score is deducted.
3. The aircraft was correctly identified, the wrong echelon. An example of the crew's response: «Negative, we have to descend to FL170, AFL1252». 2 scores are deducted.
4. The aircraft was incorrectly identified. Example of the crew response: «Our callsign is AFL1252, we have to descend to FL170». 3 scores are deducted.

Additional features:

- «Say again» function. The crew will repeat the request. Example of repeat: «I will say again. Request descending to FL170, AFL1252»;
- speech tempo adjustment. We have 4 speeds, the switch is displayed in the upper right corner of the ASI;
- pause;
- reset statistics;
- change of the aircraft callsigns to ASI.

5.3 Development of the Electronic Portal of the Virtual Educational Environment and Implementation of the Developed Educational Materials. Evaluation of the Effect of the Use of a Virtual Educational Environment

5.3.1 Development of the Electronic Portal of the Virtual Educational Environment

We will highlight the following stages of the development of the electronic portal of the virtual educational environment:

- choosing a VEE name and acquiring a domain (site address);
- selection of the hosting service;
- choosing a site content management system (CMS), installing CMS on web hosting and configuring CMS and MySQL databases;
- development of website sections, placement of theoretical educational materials and applied programs, developed at the second stage of research works;
- development of the electronic portal design, using CMS and manual editing of HTML, CSS code;
- development and placement of a feedback form;
- development and placement of documents «Policy on personal data processing», «User agreement»;
- development of an adaptive mobile version of the electronic portal.

When choosing the name VEE, the following criteria were taken into account:

- connection with aviation activities and aviation technical terms;
- ease of listening and writing;
- memorability;
- possibilities of visual registration.

As a result of the work at this stage of development, the name VEE «Zulu Project», abbreviated as «Zulu» (Zulu) was chosen.

The term «zulu» in aviation has two main meanings.

1. «Zulu» is the official pronunciation of the English letter «Z» in the phonetic alphabet of the International Civil Aviation Organization (ICAO) [10]. The phonetic or aviation alphabet underlies many areas of professional activity in the area of aviation. In particular, it is used in radio exchange, the transmission of writing words, which are difficult to hear, abbreviations, call signs, etc. in order to avoid mistakes. In this regard, for a person, associated with this area of activity, the name VEE «Zulu» will be extremely easy to remember and write down.

2. «Zulu time»—this term was previously used in aviation to mean Greenwich Mean Time (GMT), and currently stands for Universal Coordinated Time (UTC). These concepts denote the generally accepted moment of time counting throughout the world. They are also associated with aviation and, in addition, contain a symbolic connotation VEE «Zulu», which can become for future air traffic controllers as a starting point during training.

At this stage of development, a domain was purchased in the Russian national domain zone: «zuluproject.ru».

Hosting is leasing to the user a part of their server space, maintaining the functionality of the user's website on their web servers [12]. The Moscow hosting company «Ruweb» was chosen as the hosting service. 1 gigabyte of physical memory, creation of MySQL databases, mail service for mailbox maintenance on the domain zuluproject.ru, round-the-clock technical support, as well as the connection of a certificate of security over the SSL protocol, which will improve the operation of VEE in all web browsers, are available on the tariff, chosen for VEE.

SSL is a cryptographic protocol that implies more secure communication. It uses asymmetric cryptography to authenticate exchange keys, symmetric encryption to preserve confidentiality and message authentication codes for message integrity [13].

The process of developing and placement of an electronic portal at this hosting provider takes place using the DirectAdmin control panel. DirectAdmin is a web hosting control panel, created in 2003 by the Canadian company JBMC Software [14]. By using this platform to the website <http://zuluproject.ru>, an SSL security certificate was installed, as a result of which the web address was changed to <https://zuluproject.ru>.

A WordPress site content management system was chosen for the development of the electronic portal.

WordPress is a system for the management of site content with an open-source code; written in PHP; database server—MySQL; released under the GNU GPL license. The scope of application ranges from blogs to fairly complex news resources. The built-in system of «themes» and «plugins» together with a successful architecture allows us to design projects of wide functional complexity [15, 16]. According to the data, published on the official website of this system, up to 39% of all sites on the Internet are based on WordPress.

WordPress provides a wide range of functionality for the development, modernization and maintenance of an electronic portal, using the control panel. These capabilities are necessary to further expand the list of theoretical materials and improve the work of ATM dispatcher training application programs.

Using the DirectAdmin control panel, the WordPress system was installed and configured at this stage of development, and the electronic portal database was added to the MySQL database management system. MySQL is a database management system (RDMS), based on a relational model, with an open-source code with a client-server model. An RDMS is a software or service used to create and manage databases, based on a relational model [17].

Using the WordPress addition for «Contact Form 7», a feedback form was added to the electronic portal. Trainees can use it for communication, in order to clarify information on the work of VEE, in case of technical difficulties, as well as in case of requests and suggestions. At the bottom of the contact form, there is a file of attachment function, if there is such a need [18].

5.3.2 Evaluation of the Effect of the Virtual Educational Environment

In order to control the learning outcomes, objective indicators were identified for five developed programs. For the convenience of further use, the performance indicators are numbered (Table 5.1).

5.4 Possibilities of Using a Voice Assistant in the Process of Professional Training of Air Traffic Management Dispatchers

To improve the conceptual model of an ATM dispatcher, both theoretical and intensive practical training and practical experience in ATS are necessary.

Table 5.1 Performance indicators

Name of the application program	Performance indicators
1. «Solution of PCS»	№1.1 The average number of errors and recommendations in 6 random exercises (by 2 from each section «Counter courses», «Passing courses» and « Intersecting courses») №1.2 The number of TCAS triggers in 6 random exercises (by 2 from each section «Counter courses», «Passing courses» and «Intersecting courses»)
2. «Provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data»	№2.1 The average efficiency of ATS when provision of navigation guidance of aircraft is by means of instructions to the crews of certain courses, based on the use of radar data, in 4 random exercises
3. «Numbers»	№3.1 The best result of the average speed for the last 15 inputs within 2 min
4. «Identification of the aircraft»	№4.1 The number of points scored in the first 2 min (may be negative in case of errors in the maintenance of air traffic)

Public applications that contribute to the acquisition of ATS knowledge, skills and abilities include virtual aviation simulators and computer networks, designed to recreate real procedures for interaction between aircraft crews and ATM controllers on the Internet. As an example of such flight simulators, one can cite IVAO («The International Virtual Aviation Organization»), «VATSIM Network» (Virtual Air Traffic Simulation Network), etc.

These simulators and networks simulate the real working conditions of aircraft crews and ATM dispatchers, however, they are difficult to install and configure, and also involve a preliminary study of a large amount of information for interaction with hundreds of people, who already have a certain level of training and perform ATS functions in real time. These factors are a psychological barrier for a trainee with an initial level of knowledge, as a result of which he may refuse to install and use such a simulator.

The development of special software—virtual ATS training simulators, simulating various configurations of the air situation, as well as special application programs for obtaining and improving individual professional skills of an ATM dispatcher—can contribute to solving this problem.

Examples of areas of development of training applications:

- Solving individual potential conflict situations;
- Development of the aircraft identification skill (in particular, the rapid visual finding of the aircraft mark on the air situation indicator) in the condition of busy ATS sector;
- Development of speech perception skills in English (queries, instructions, numbers, aviation alphabet, etc.);
- Development of provision of navigation guidance of aircraft by means of instructions to the crews of certain courses, based on the use of radar data vector and skills (providing navigation guidance of aircraft through instructions to the crews of certain courses);
- Development of skills in the application of zonal navigation procedures.

A significant part of the ATM dispatcher's professional activity is occupied by radio exchange—voice exchange of information with aircraft crews via radio communication. In the above flight simulators, the functions of aircraft crews and ATM controllers are performed by real people, users of these programs. A voice assistant can perform the functions of the virtual crew of the aircraft within the training simulator complex.

The voice assistant contains speech recognition modules, processing of received requests and response playback, using Text-To-Speech (TTS) technology. The voice assistant interacts with other programs and software components via the API.

Application programming interface (API) is a description of the ways (a set of classes, procedures, functions, structures or constants) that one computer program can interact with another program.

Speech recognition is an automatic process of converting a speech signal into digital information (for example, text data). The inverse problem is speech synthesis.

Speech synthesis is the formation of a speech signal, based on printed text. A speech synthesizer is a structure, capable of translating text/images into speech, in software and/or hardware. The voice (speech) engine is directly the system/core for converting text/commands into speech.

The principle of operation of voice assistants:

1. Passive sound reading, by activating a function with a built-in code word;
2. Signal filtering—the stage of eliminating noise and interference that occurs when recording a voice request;
3. Digitization of sound—the audio signal is converted into a digital form, understandable to the computer;
4. Signal analysis—areas with speech are highlighted, and parameters are evaluated, such as part of speech, word form and communication in one query;
5. Search for template data—the program collects different pronunciations of a word, compares it with templates and outputs the result [voice assistant].

As part of the assigned task to improve the conceptual model of the ATM dispatcher, the voice assistant can perform the following functions of the virtual crew of the aircraft:

- independent transmission of requests and information messages (for example, when a virtual aircraft passes the ATS sector transmission boundary);
- recognition of instructions issued to trainees;
- transfer of the «receipt» (repetition of the instruction, received from the trainee), etc. (Fig. 5.1).

The main elements of the virtual training complex in this model are theoretical materials, applied training programs, a personal account of trainee, as well as a general rating table of users of the complex.

Theoretical materials include educational articles, using infographics (tables, diagrams, drawings and other elements). The programs simulate various configurations of the air situation, and contribute to the formation and improvement of the professional skills of an ATM dispatcher.

Application programs and a module for testing trainees on the obtained theoretical knowledge interact with the developed voice assistant, using the API.

Application programs calculate the trainee's performance indicators and upload them to the database of the virtual training complex. The same happens with the test results—the module of testing loads the information into the corresponding database.

The personal account contains a display of the trainee's academic performance dynamics. Academic performance statistics are visualized, using the corresponding module and displayed as graphs.

A public rating table of trainees will create a competitive environment, and, as a result, can help increase motivation to use this virtual training complex. The lists of trainees for the rating table can be formed as general (a global table, containing the academic performance statistics of all users of the training complex), or according to

A model of a virtual training complex, using a voice assistant

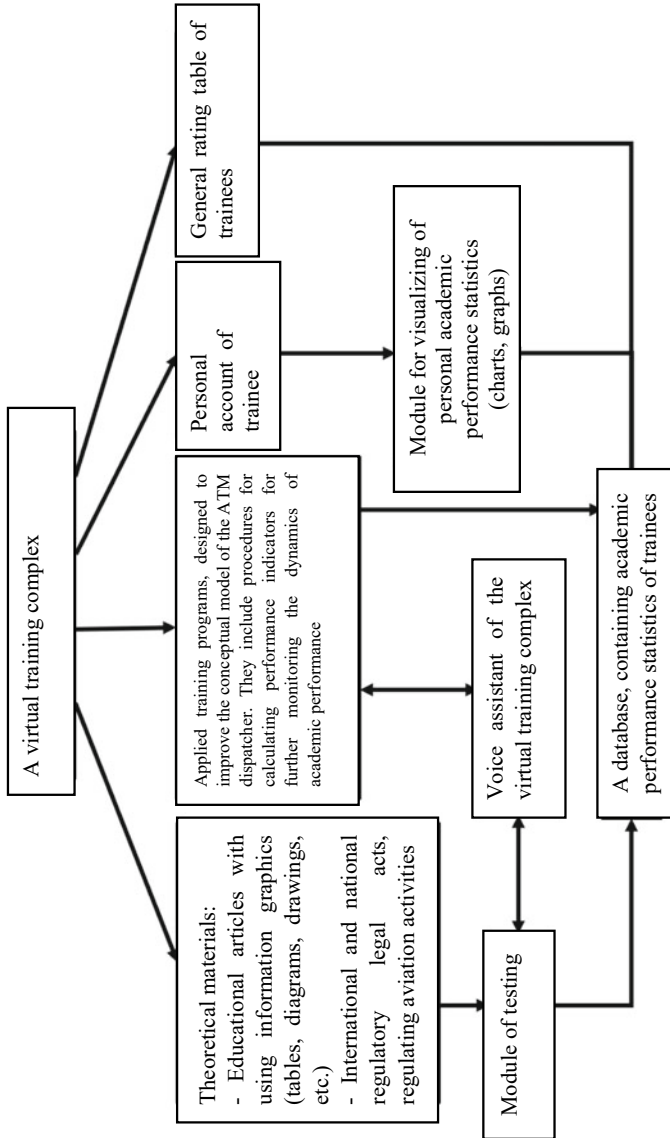


Fig. 5.1 A model of a virtual training complex, using the functions of a voice assistant

certain criteria (for example, in relation to the university—by courses, study groups, etc.). Information about the academic performance of trainees is loaded from the general database into the rating table and into the visualization module of personal statistics.

5.5 Classification of Air Traffic Management Simulators

Analysis of the methodology for organizing practical training for ATM dispatchers allows us *to identify the following main classes of simulators for ATM dispatchers:*

- (1) high-quality (high-precision) complex simulators for group training: simulators are characterized by the presence of exact copies of the dispatcher's workplaces of the corresponding automated system (AS) ATM, including all equipment and computer programs, necessary to present a complete list of tasks in the management sectors or on the tower and their environment. In the case of the workplaces of the dispatchers of the start and taxiing of the airfield, it includes a model of the visual airfield situation;
- (2) simulators of individual training: simulate the most important characteristics (properties) of a real situation to the trainee and reproduce maintenance conditions that allow practicing (work out) tasks, directly in real time;
- (3) procedure simulators: training devices (personal computers) that allow the trainee to work out some operational (working) functions independently of other functions that are not presented there, although they are necessarily associated with the first in solving production tasks;
- (4) other training devices are computer hardware tools that provide the trainee with some operational (working) functions on unrealistic reproduction of working devices. As a rule, these are generating computers or workstations, connected by a network or autonomous, designed for a single trainee or a small group. At the same time, equipment (hardware and software) of wide application and not deeply modified for specific purposes of air traffic management is used (Table 5.2).

5.6 Collection and Evaluation of Information in the Learning Process

In Fig. 5.2, the algorithm for collecting and evaluating information in the process of training on aviation simulators is given.

Below are the requirements for the controlled parameters, used in AS ATM simulators.

For each group of activity parameters, a set of controlled parameters is created, each of which must be characterized by physical, quantitative values or represented

Table 5.2 Characteristics of training equipment, used in Eurocontrol states

The problem being solved	Training devices	Procedure devices	Individual simulators	Complex high-precision simulators
Acquisition of knowledge (skills)	Best using	No need	No need	No need
Working out private tasks	Not enough	Best using	Best using	No need
Individual simulation	Not enough	Not enough	Best using	Best using
Team simulation	Not enough	Not enough	Best using	Best using
Group simulation	Not enough	Not enough	Best using	Best using

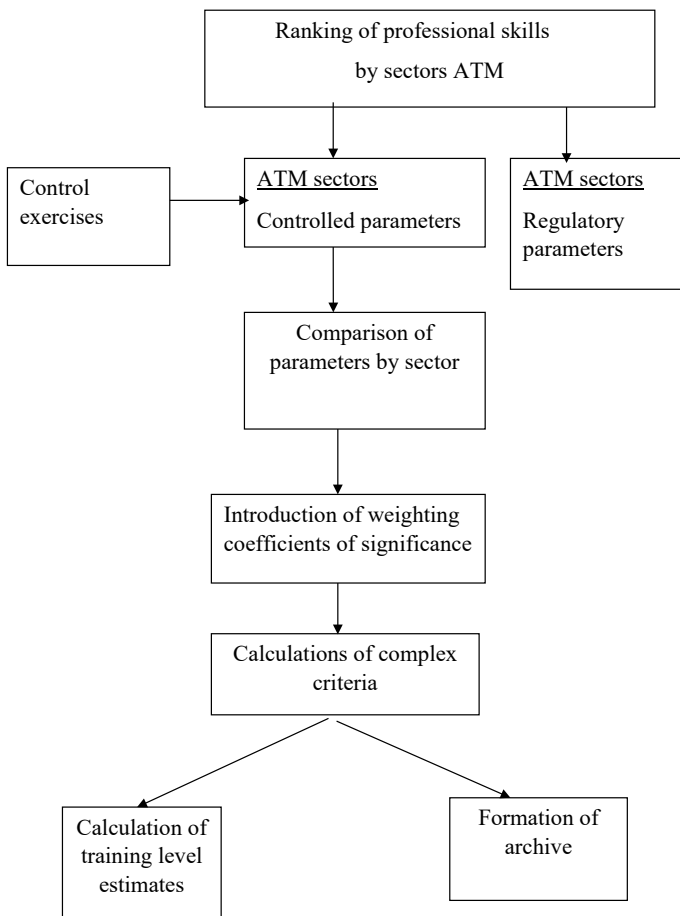


Fig. 5.2 Algorithm for collecting and evaluating information in the process of training on aviation simulators

by some algorithm for its description. The controlled parameters, used in AS ATM dispatcher simulators, must meet a number of requirements, **the main of which are**

- (1) completeness—all basic skills should be monitored, i.e. a formalized relationship between the main skills from their list and the controlled parameters should be established;
- (2) reliability—with the help of the found estimates, it is necessary adequately to show the corresponding parameter;
- (3) accuracy—controlled parameters must be measurable and evaluated with reasonable accuracy;
- (4) purposefulness—the requirements of adequacy of the skills, being worked out, must be met;
- (5) scalability—the normative values of the controlled parameters should be determined in such a way that they relate to the real activity of the dispatcher, i.e. that the requirement of adequacy of the accepted norms to the real indicators of the dispatcher's work is met;
- (6) feasibility—measurements and calculations of controlled parameters should not cause fundamental difficulties, when using standard means of modern training equipment;
- (7) simplicity—the number of controlled parameters should be minimal and optimal, provided that the previous requirements are met.

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Appendix

Data from an Independent Examination of the Quality of Simulation of Elements of Flight Training Exercises on Russian Aviation Simulators

The above methodological principles of designing aircraft simulators, the proposed algorithm for the synthesis of AS and the considered methods for solving individual problems of dynamics and control are used in the development of a number of aircraft simulators for both civil and military aircraft.

The results of the testing and maintenance of the AS showed the correctness and prospects of the principles, laid down for the construction of the AS. This is evidenced by the assessment of a number of organizations, including the indicators of military-economic efficiency of the AS and the quality of imitation of elements of flight training exercises, given in Table 1.

The proposed design principles allow us to make the modernization of the AS, based on new hardware, technologies and element base.

Table 1 The quality of simulation of elements of the flight training exercise

Elements of the exercise	Average score from 0 to 1	Average score in points: 5, 4, 3, 2
Average scores		
– on piloting techniques	0.54	3.5
– by airplane navigation	0.70	4.3
– for special flight occasions	0.78	4.0
General on the simulator	0.56	3.6
Actions with cabin equipment and preparation of as for flight	0.93	4.8
Taxing on the runway	0.15	2.3
Takeoff to H = 50 m	0.40	2.9
Climb, Descent	0.75	3.9

(continued)

Table 1 (continued)

Elements of the exercise	Average score from 0 to 1	Average score in points: 5, 4, 3, 2
Horizontal flight:		
– at low and extremely low altitudes	0.35	2.8
– at medium and high altitudes	0.70	3.6
Turns (Turns):		
– with a roll angle of 30°	0.60	3.5
– with a roll angle of 60°	0.40	3.0
Acceleration and deceleration	0.60	3.6
Spirals (Ascending, descending)	0.77	3.9
Horizontal barrels	0.38	3.0
Aerobatics:		
– Nesterov loop, oblique loop, half loop	0.40	3.1
– coup, combat U-turn		
Flying to the practical ceiling	0.60	4.0
Flight along the route, using flight navigation systems	0.70	4.3
Landing approach (Visually)	0.30	2.7
Landing	0.15	2.1
Actions in special of flight	0.78	4.0