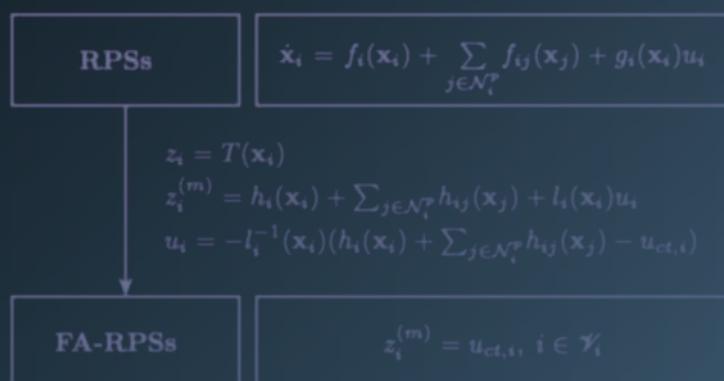


Yi Yu · Guo-Ping Liu · Josep M. Guerrero

Control of Renewable Power Systems

A Fully Actuated System Approach

Series of Fully Actuated System
Approach for Control and AI



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Preface

Driven by the ongoing energy transition, the high penetration of renewable energy sources is fundamentally reshaping the dynamic characteristics of modern power systems. The inherent variability of wind and photovoltaic generation, the reduction in synchronous inertia associated with converter-interfaced resources, the fast and multiscale dynamics of power electronic devices, and the complex coupling among generation, grid infrastructure, loads, and energy storage systems pose substantial challenges to conventional modeling, stability analysis, and control methodologies. Concurrently, these developments are accelerating the evolution of power networks toward lower-carbon, more distributed, and increasingly converter-dominated architectures.

Within this transformation, microgrids have become an important and flexible architecture for integrating and coordinating distributed generation, energy storage systems, and local loads. Their distributed structure enables the coordinated utilization of decentralized resources but also imposes increasingly stringent requirements on control performance, communication, and system-level coordination. Local feedback loops and linearized models remain indispensable for device-level regulation; however, they often need to be complemented by distributed control frameworks capable of supporting multi-node coordination, system-wide performance optimization, and enhanced dynamic resilience. Additional challenges arise from the growing penetration of power electronic loads, including electric vehicle charging systems, industrial motor drives, and artificial intelligence data centers. Depending on their converter controls and operating conditions, these loads may exhibit constant-power characteristics, nonlinear dynamics, rapid demand variations, and strong interactions with network and energy storage dynamics. These features motivate the development of more advanced, systematic, and robust control methodologies.

To address these challenges, this monograph introduces the fully actuated system (FAS) approach as a powerful control-oriented modeling and controller design framework for renewable power systems. The FAS methodology provides a systematic complement to conventional state-space and nonlinear control approaches and is particularly well suited to systems with nonlinear, coupled,

and heterogeneous dynamics. Starting from the original physical and control-level models, it constructs fully actuated representations that expose the actuation structure of selected input–output or regulation-error dynamics. With its mathematical foundations in differential equations and differential geometric control theory, particularly in the use of diffeomorphic transformations, the FAS approach offers distinctive theoretical advantages and provides a powerful framework for engineering control applications. Under appropriate structural and feasibility conditions, these representations provide substantial flexibility in assigning desired closed-loop characteristics and can greatly simplify controller synthesis and stability analysis. Compared with conventional trial-and-error design procedures, the FAS approach can reduce reliance on heuristic parameter tuning by establishing a more direct relationship between controller parameters and desired dynamic performance. It also facilitates the development of controllers with transparent structures, flexible parameter configurations, and strong extensibility to predictive, learning-based, distributed, and resilient control. These features make the FAS methodology a particularly promising theoretical and engineering tool for the modeling and control of contemporary renewable power systems.

The particular value of the FAS methodology for renewable power systems lies in its ability to establish a direct and structured connection between physical-system dynamics, control objectives, and closed-loop controller design. Converter-interfaced renewable power systems commonly involve heterogeneous component dynamics, nonlinear operating characteristics, hierarchical control loops, and distributed interactions among generation, networks, loads, and energy storage units. Within this context, FAS representations provide systematic means of reorganizing complex and coupled dynamics into forms that are more transparent and tractable for controller synthesis. This feature can facilitate the coordinated treatment of voltage, frequency, current, and power regulation objectives while supporting the integration of predictive, learning-based, distributed, and resilient control mechanisms.

More importantly, the FAS approach offers a coherent methodological perspective rather than an isolated controller design technique. By retaining the relevant physical structure of the original system while exposing additional freedom for assigning selected closed-loop dynamics, it provides a promising bridge between advanced nonlinear control theory and practical power electronic applications. Its extensibility across different converter configurations, operating conditions, and control layers makes it particularly attractive for renewable power systems, whose structures and control requirements continue to evolve rapidly. Accordingly, a central purpose of this monograph is not merely to present a collection of individual control algorithms, but to demonstrate how FAS-based modeling can provide a common foundation for the systematic analysis, design, and coordination of modern renewable power systems.

This monograph develops a systematic framework for applying the FAS methodology to the modeling and control of renewable power systems. It brings together theoretical foundations, control-oriented model transformation, controller synthesis, stability analysis, and simulation or experimental validation. Rather than treating the

individual control methods as isolated techniques, the book demonstrates how FAS-based representations can provide a coherent methodological thread linking physical system dynamics, regulation objectives, and closed-loop controller design.

The presentation progresses from the fundamental modeling and control principles of renewable power systems to predictive, learning-based, high-order, and distributed control of converter-interfaced microgrids. It then extends the FAS framework to cyber-physical security and resilient coordination. Through this progression, the monograph highlights the distinctive value of the FAS methodology in reorganizing nonlinear, coupled, and heterogeneous system dynamics into control-oriented forms that facilitate systematic analysis and controller design.

Specifically, the monograph is organized into 11 chapters.

Chapter 1 introduces the research motivation and reviews the development of renewable power systems and their associated control challenges. It summarizes recent advances in renewable power system control, presents the fundamental concepts of the FAS methodology, and introduces the basic framework of interconnected multi-agent systems used in the subsequent chapters.

Chapter 2 presents the control-oriented dynamic models that provide the physical and mathematical foundation of the book. It covers representative DC, AC, and hybrid renewable power system architectures, together with the device-level models and control structures of grid-following and grid-forming converters. The discussion of their topologies, operating modes, and dynamic characteristics establishes the basis for the controller designs developed in later chapters.

Chapter 3 investigates coordinated voltage regulation in DC microgrids under communication delays and packet dropouts. Based on a coupled discrete-time system model, a consensus-based proportional–integral predictive control strategy and a distributed observer are developed to compensate for the effects of imperfect communication and improve multi-DG voltage tracking.

Chapter 4 develops an online learning-based adaptive secondary controller for AC microgrids. Two neural networks are updated online to compensate for uncertain system dynamics, while an extended state observer is employed to estimate and reject external disturbances. The resulting framework coordinates voltage regulation and reactive-power sharing without requiring complete knowledge of the microgrid dynamics.

Chapter 5 addresses the nonlinear dynamics of constant power load-integrated DC systems with representative buck- and boost-converter configurations. Through state reconstruction, dimension reduction, and order elevation, the chapter derives high-order fully actuated models and develops estimator-assisted voltage controllers. The corresponding stability and control performance properties are analyzed under the stated modeling and operating assumptions.

Chapter 6 presents a distributed predictive fully actuated secondary control strategy for multi-bus DC microgrids. Built upon a control-oriented FAS model, the proposed method coordinates bounded voltage restoration and proportional current sharing while exploiting predictive information to improve the closed-loop response.

Chapter 7 examines the inherent trade-off between voltage regulation and proportional current sharing in converter-interfaced multi-bus DC microgrids. A distributed learning-based FAS secondary control framework is developed to coordinate these objectives. Its online learning mechanism supports controller adaptation while retaining a comparatively simple structure suitable for hardware implementation.

Chapter 8 addresses secondary control of DC microgrids with resistive–inductive tie lines. An integrated circuit-and-control model is first established, from which a fully actuated current-output representation is derived. Dynamic compensation and finite-horizon predictive control are then combined to improve transient performance while maintaining PCC voltages within their admissible ranges and achieving proportional current sharing.

Chapter 9 develops a composite regulation error model for multi-bus DC microgrids by integrating converter and network dynamics, local PI controllers, primary droop control, and distributed average voltage estimation. Based on the resulting fully actuated error representation, a model-based distributed secondary controller is designed to coordinate average voltage restoration and proportional current sharing, together with a quadratic performance interpretation.

Chapter 10 focuses on cybersecurity vulnerabilities within cyber-physical microgrid systems. It introduces a blockchain-driven secondary security control strategy to counter potential network attacks. To facilitate this model-based security design, the chapter establishes an FAS model for the cyber-physical microgrid. This modeling approach balances the accurate characterization of electrical dynamics with overall controller simplicity.

Chapter 11 summarizes the core contributions of the monograph. It concludes with a discussion on the future prospects and open challenges of applying the FAS approach to the modeling, control, and optimization of modern renewable power systems.

Hong Kong, China
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The theoretical foundation of this book draws on the FAS control theory developed by Professor Guang-Ren Duan. As an original and systematic control-theoretic framework for system modeling, closed-loop dynamic assignment, and controller synthesis, FAS theory represents an important advance in the development of modern control theory and has opened a new avenue for addressing complex engineering control problems. Its emphasis on fully actuated and high-order fully actuated representations provides enhanced flexibility in shaping closed-loop dynamics and offers considerable potential for nonlinear, multivariable, and large-scale systems. The concepts and methodological principles of FAS theory have provided the central intellectual foundation for the control-oriented modeling and controller design problems investigated throughout this monograph. We are deeply grateful to Professor Duan for his pioneering theoretical contributions, profound scholarly inspiration, and continued guidance.

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Contents

1	Introduction	1
1.1	Overview	1
1.2	Development Status of Renewable Power Systems	2
1.3	Control of Renewable Power Systems	4
1.4	Basic Concept of Fully Actuated System Approaches	5
1.5	Interconnected Multi-Agent Systems	10
1.6	About the Book	12
1.6.1	Structure of the Book	12
1.6.2	Origins of the Book	13
 Part I Secondary Control of Renewable Power Systems		
2	Models of Renewable Power Systems	17
2.1	Overview	17
2.2	Models of DC Renewable Power Systems	18
2.3	Models of AC Renewable Power Systems	20
2.4	Models of Hybrid Renewable Power Systems	22
2.5	Models of Grid-Following and Grid-Forming Converter Systems	26
2.6	Notes and References	29
3	Predictive Control of DC Renewable Power Systems	31
3.1	Overview	31
3.2	Problem Formulation	33
3.2.1	Dynamics of DC MGs With Disturbance	34
3.2.2	Electrical and Communication Networks	35
3.2.3	Problem of Interest	36
3.3	Predictive Control of DC MG Systems	37
3.4	Analysis of Stability and Consensus of DC MGs With CPIP Control	41
3.5	Experiment Verification	45
3.5.1	Hardware Setup Details	46

3.5.2	Performance Comparison and Evaluation	49
3.5.3	Robustness Evaluation Against Load Changes	55
3.5.4	Plug-and-Play Physical Capability	56
3.5.5	Performance Assessment for a Large-Scale DC MG	58
3.6	Notes and References	59
4	Learning-Based Control of AC Renewable Power Systems	61
4.1	Overview	61
4.2	Problem Formulation	62
4.3	Predictive Control of AC Microgrid Systems	65
4.3.1	Achievement of Average Regulation of Voltages of All DGs	66
4.3.2	Design of Online Learning-Based Distributed Secondary Controller	68
4.3.3	Active Disturbance Rejection via Extended State Observer	70
4.4	Simulation Verification and Result Analysis	72
4.4.1	Simulation Setup and Parameter Configuration	72
4.4.2	Scenario 1: Evaluation of Control Objectives and Model Independence	73
4.4.3	Scenario 2: Robustness Against Dynamic Load Fluctuations	75
4.5	Notes and References	77
 Part II Fully Actuated Control of Renewable Power Systems		
5	Fully Actuated Control of Renewable Power Systems with Disturbances	81
5.1	Overview	81
5.2	Problem Formulation	83
5.2.1	Description of DC MGs	83
5.2.2	Regulation Tasks	84
5.3	HOFA-VC for DC MGs With Known CPLs	86
5.3.1	Buck Converter-Based MGs With Known CPLs	86
5.3.2	Boost Converter-Based MGs With Known CPLs	88
5.4	HOFA-VC for DC MGs With Unknown CPLs	89
5.4.1	HOFA-VC for Buck Converter–Interfaced DC MGs With Unknown CPLs	90
5.4.2	HOFA-VC for Boost Converter–Interfaced DC MGs With Unknown CPLs	92
5.5	Experiment Verification	94
5.5.1	Hardware Description and Experimental Setup	94
5.5.2	Performance Test of a Buck Converter–Based MG With the HOFA-VC	96
5.5.3	Performance Test of a Boost Converter–Based MG With the HOFA-VC	97

5.5.4	Performance Test of an MG With Unknown Loads	98
5.6	Notes and References	98
6	Distributed Predictive Fully Actuated Control of Renewable Power Systems	101
6.1	Overview	101
6.2	Problem Formulation	103
6.2.1	Control Objectives	103
6.2.2	DC MG Modeling Based on HOFA System Approaches	104
6.3	Fully Actuated Predictive Control of DC MG Systems	109
6.3.1	Setting of Communication Network Based on Graph Theory	109
6.3.2	DPFA Coordinated Secondary Control	110
6.3.3	Stability and Consensus Analysis of DC MGs With DPFA Control	114
6.4	Experiment Verification	116
6.4.1	Hardware Description and Experimental Procedure	117
6.4.2	Performance Comparison and Assessment	119
6.4.3	Performance Assessment Under Different Control Parameters	123
6.4.4	Performance Assessment Under Communication Delays	125
6.4.5	Robustness to Load Variations	126
6.4.6	Evaluation of Plug-and-Play Capabilities	128
6.5	Notes and References	130
7	Learning-Based Fully Actuated Predictive Control of Renewable Power Systems	133
7.1	Overview	133
7.2	Problem Formulation	135
7.2.1	Ring-bus Multi-bus DC MGs	135
7.2.2	Preliminaries on Graph Theory	135
7.3	Voltage Recovery and Current Allocation in DC MGs	136
7.3.1	Modeling of Ring-bus Multi-bus DC MGs	137
7.3.2	DL-HOFA Secondary Controller Design	139
7.3.3	Stability and Consensus Analysis of DC MGs	144
7.4	Experimental Validation	147
7.4.1	Hardware Description and Experimental Procedure	147
7.4.2	Performance Comparison and Analysis	148
7.4.3	Performance Evaluation for Input-Voltage Uncertainty	152
7.4.4	Performance Evaluation for Load Demand Variations	152
7.4.5	Plug-and-Play Capability	153
7.5	Notes and References	155

8	Fully Actuated Predictive Control of Renewable Power Systems with Inductive Lines	157
8.1	Overview	157
8.2	Problem Formulation	159
8.3	DC MGs Modeling with HOFA System Approaches	160
8.4	Design of DOFA Predictive Control	164
8.5	Stability and Consensus Analysis of DC MGs	168
8.6	Frequency Domain Model Development	174
8.7	Experimental Verification	175
8.7.1	Performance Comparison and Analysis	176
8.7.2	Fundamental Performance Evaluation	177
8.7.3	Plug-and-Play Feature Test	180
8.8	Notes and References	181
 Part III Fully Actuated Control of Cyber-Physical Renewable Power Systems		
9	Distributed Control of Renewable Power Systems via a Fully Actuated Error Model	185
9.1	Overview	185
9.2	Problem Formulation	187
9.2.1	Structure of Multi-bus DC MGs	187
9.2.2	Dynamics of Converter Interface-Based Ring-bus Multi-bus DC MGs	188
9.2.3	Current Allocation and Voltage Balancing Requirements in DC MGs	190
9.3	Dynamics Model of Multi-bus DC MGs	191
9.3.1	Dynamic Characteristics of Multi-bus DC MGs	191
9.3.2	First-Order Fully Actuated Model of Multi-bus DC MGs	193
9.4	Distributed Optimal Secondary Control for DC MGs	196
9.4.1	Structure of the Optimal Controller for Multi-bus DC MGs	196
9.4.2	Parameters Specification for Optimal Cooperative Secondary Control	197
9.5	Experimental Results	203
9.5.1	Implementation of MG Prototype and Experimental Setup	203
9.5.2	Comparison with a Recent Study	205
9.5.3	Regulation Results Under Load Changes	209
9.5.4	Regulation Results Under PnP Operation	211
9.6	Notes and References	213
10	Blockchain-Based Fully Actuated Secure Control of Renewable Power Systems	215
10.1	Overview	215

10.2	Problem Formulation.....	217
10.3	Blockchain-Based Fully Actuated Secure Control of DC MG Systems.....	218
10.3.1	Large-Signal Characterization of Droop-Controlled MGs.....	218
10.3.2	Error System Modeling for Droop-Controlled MGs.....	219
10.3.3	Blockchain-Assisted Fully Actuated MG Model	221
10.3.4	Blockchain-Based Fully Actuated Secure Control of MGs	226
10.4	Simulation Verification.....	228
10.4.1	Simulation System Setup.....	228
10.4.2	Communication Security Visualization.....	232
10.5	Notes and References	233
11	Conclusions and Future Research Directions	235
11.1	Conclusions.....	235
11.2	Future Research Directions	236
	References.....	239

Acronyms

AC	Alternating current
ACS	Accurate current sharing
ADC	Analog-to-digital converter
AI	Artificial intelligence
CCM	Continuous conduction mode
CPL	Constant-power load
CPIP	Consensus-based proportional-integral predictive
CPPS	Cyber-physical power system
CPS	Cyber-physical system
CSM	Comprehensive system model
CVO	Consensus voltage observer
CVR	Consensus voltage restoration
DAB	Dual-active-bridge converter
DC	Direct current
DER	Distributed energy resource
DG	Distributed generator
DL-HOFA	Distributed learning-based high-order fully actuated
DOFA	Distributed optimal fully actuated
DPFA	Distributed predictive fully actuated
DVO	Distributed voltage observer
EPT	Electronic power transformer
EV	Electric vehicle
FA	Fully actuated
FAS	Fully actuated system
FDIA	False data injection attack
GFL	Grid-following
GFM	Grid-forming
HIL	Hardware-in-the-loop
HOFA	High-order fully actuated
HOFA-VC	High-order fully actuated voltage control
IGBT	Insulated-gate bipolar transistor

ILC	Interlinking converter
KCL	Kirchhoff's current law
KVL	Kirchhoff's voltage law
LC	Inductor-capacitor
LESO	Linear extended state observer
LLM	Large language model
LMI	Linear matrix inequality
LQR	Linear quadratic regulator
LTi	Linear time-invariant
MG	Microgrid
MPPT	Maximum power point tracking
NCC	Networked cooperative control
NN	Neural network
P2P	Peer-to-peer
PBFT	Practical Byzantine fault tolerance
PCC	Point of common coupling
PI	Proportional-integral control
PID	Proportional-integral-derivative control
PLL	Phase-locked loop
PoS	Proof of stake
PoW	Proof of work
PV	Photovoltaic
PWM	Pulse-width modulation
QSL	Quasi-stationary line
RBNN	Radial basis function neural network
RES	Renewable energy source
RLC	Resistor-inductor-capacitor
RPS	Renewable power system
SMC	Sliding mode control
UD	Unknown or uncertain dynamics
UUB	Uniformly ultimately bounded
V2G	Vehicle-to-grid
VSM	Virtual synchronous machine

Chapter 1

Introduction



Abstract This chapter introduces the motivation, system characteristics, and control challenges of renewable power systems, with particular emphasis on microgrids as representative converter-dominated energy systems. It first reviews the development status and operational features of renewable power systems, then discusses the modeling and control challenges introduced by distributed renewable generation, power electronic interfaces, communication networks, and cyber-physical coupling. The chapter subsequently presents the fundamental concepts of the fully actuated system approach, including fully actuated and high-order fully actuated representations, closed-loop dynamic assignment, and stabilizing parameter design, thereby establishing its relevance to microgrid modeling and control. Finally, the chapter extends the discussion to interconnected multi-agent systems, where physically coupled subsystems interact through sparse communication networks. This perspective provides the basis for the distributed, predictive, learning-based, and cybersecure microgrid control strategies developed in the remainder of the book. The chapter concludes by presenting the organization, scope, and intellectual origins of the monograph within the broader fully actuated system research program.

1.1 Overview

The global transition toward renewable power systems (RPSs) represents a fundamental structural shift in modern energy infrastructure. Historically reliant on finite fossil-fuel resources, the power sector is now rapidly integrating renewable generation assets to address pressing climate constraints and carbon-neutrality targets. This momentum is largely driven by the technological maturation of power electronics, energy storage, and grid integration techniques. Solar photovoltaics and high-capacity wind turbines, in particular, have experienced a continuous decline in the levelized cost of energy, enabling their deployment at an unprecedented scale and solidifying their position as highly cost-effective electricity sources.

Beyond technological drivers, this energy transition is deeply rooted in international agreements, such as the Paris Agreement and strategic government policies. These top-down initiatives have accelerated the decommissioning of conventional

synchronous generators in favor of low-carbon alternatives, incentivizing investments across the entire clean energy supply chain. Consequently, RPSs have transcended their initial role as supplementary environmental measures. Today, they form the core foundation of national energy strategies, functioning as an indispensable component for achieving long-term energy security and economic resilience in a modernized grid.

Within this broader context, this book centers on the advanced modeling and control methodologies for RPSs, utilizing the fully actuated system (FAS) approach. The primary physical systems considered in this book are renewable-dominated microgrids (MGs). As a highly flexible, decentralized energy architecture, MGs seamlessly integrate local generation, storage, and loads within a localized energy-management framework. Their inherent capability to operate in both grid-connected and islanded modes provides critical operational resilience against external disruptions. While this localized paradigm effectively reduces transmission losses and enhances regional power reliability, the heavy reliance on converter-interfaced resources introduces low-inertia characteristics and highly coupled dynamic behaviors. These control complexities motivate the use of the FAS framework.

To lay a solid foundation for the subsequent chapters, the remainder of this overview is structured around four essential aspects. We first examine the current development status and physical architectures of RPSs, followed by a review of their established control hierarchies. Subsequently, we mathematically formalize the fundamental concepts of the FAS methodology. Finally, we extend this approach to interconnected multi-agent systems, providing the theoretical prerequisites for the distributed coordination of modern MGs.

1.2 Development Status of Renewable Power Systems

The global renewable energy landscape is undergoing a major structural transformation, as illustrated in Table 1.1. Quantitatively, by the end of 2023, the cumulative global installed renewable capacity reached approximately 3870 gigawatts (GW). Solar and wind power dominated this expansion, contributing roughly 1419 GW and 1017 GW, respectively, alongside 1268 GW from hydroelectricity. Consequently,

Table 1.1 Global Renewable Power Capacity (2023)

Technology	Installed capacity	Annual growth	Key regional leader
Solar photovoltaic (PV)	~1419 GW	~32.2%	China (>50% of new adds)
Wind	~1017 GW	~13%	China (followed by USA/EU)
Hydropower	~1268 GW	~2%	China (followed by Brazil)
Bioenergy	~150 GW	~3.1 %	China/Brazil

Note: Data sourced from IRENA’s 2024 report (International Renewable Energy Agency, 2024). The symbol “~” denotes approximation

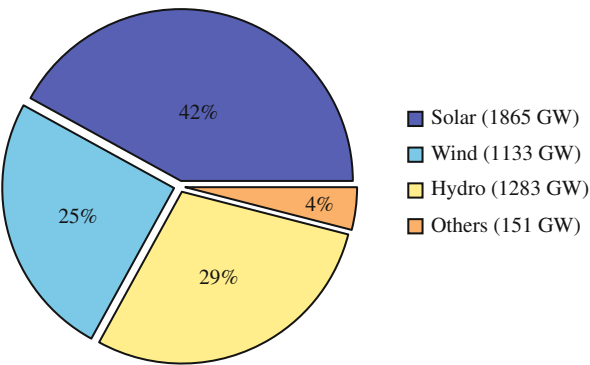


Fig. 1.1 Renewable Capacity Statistics 2025 (IRENA, 2025)

wind and solar power collectively accounted for 13.4% of global electricity generation, surpassing nuclear power for the first time (Ember, 2024; International Renewable Energy Agency, 2024). This rapid capacity expansion is projected to continue, with Fig. 1.1 detailing the 2025 global renewable power statistics. Beyond quantitative growth, this transition is equally characterized by qualitative technological advancements. For instance, perovskite-silicon tandem solar cells continue to achieve record-breaking conversion efficiencies in laboratory settings, while floating offshore wind platforms, such as Norway’s 88-MW “Hywind Tampen” project, successfully harness previously inaccessible deep-water wind resources.

As an important architecture for distributed energy systems, MGs are rapidly transcending traditional geographical and application boundaries. This evolution enables highly customized deployments across diverse and extreme operational environments. For example, in the Arctic Svalbard archipelago, advanced scientific expedition MGs integrate direct-drive wind turbines, vanadium redox flow batteries (VRFBs), and methanol fuel cells to strictly minimize the reliance on diesel backup generators (Nordregio Projects, 2019; United States Arctic Research Commission & Arctic Executive Steering Committee, 2016). Similarly, the innovative “Zero-Carbon Hydrogen Microgrid Cluster” planned for the NEOM development in Saudi Arabia utilizes off-grid electrolysis, facilitating the complete utilization of surplus photovoltaic generation to produce green hydrogen (NEOM, 2023; Inside Saudi, 2025). Pushing the boundaries further into aerospace, the Japan Aerospace Exploration Agency (JAXA) is conceptualizing wireless power transfer–based MGs for lunar base applications (Saha et al., 2021; Japan Aerospace Exploration Agency, 2025). Ultimately, these pioneering breakthroughs demonstrate that MGs serve as the critical physical interface that reconciles operational resilience with system determinism, leveraging spatial decoupling to customize renewable integration for specific scenarios.

1.3 Control of Renewable Power Systems

The transition toward RPSs is reshaping traditional power-system paradigms, driven by several distinct operational and structural characteristics. First, the inherent pattern of centralized generation is replaced by a highly decentralized architecture, characterized by a proliferation of spatially distributed energy resources (DERs). Second, the power output of these systems is strongly affected by stochastic meteorological conditions, introducing severe intermittency and volatility. For instance, photovoltaic (PV) outputs fluctuate instantaneously with solar irradiance, continuously perturbing the system's power balance. Third, modern RPSs require substantial operational flexibility. Microgrids, a quintessential RPS subsystem, exemplify this by seamlessly transitioning between grid-connected and islanded modes while autonomously maintaining local voltage and frequency stability. Furthermore, structural complexity escalates with the integration of diverse energy carriers (e.g., hydrogen, biomass) and hybrid AC/DC infrastructures. This complexity is compounded by the plug-and-play nature of modern DERs and the integration of flexible loads, particularly electric vehicles (EVs) with bidirectional power flow capabilities. These highly dynamic edge devices induce nonlinear load fluctuations and frequent topological changes, rendering conventional control paradigms based on predefined parameters and fixed architectures largely obsolete.

These unique physical and operational attributes not only define the boundaries of modern energy systems but also underscore the critical necessity for advanced modeling and control frameworks. From a modeling perspective, accurate mathematical representations are prerequisites for unraveling the cross-domain coupling mechanisms of multi-energy systems and quantifying the propagation of uncertainties. Such models provide the theoretical underpinnings for rigorous system planning and dynamic risk assessment. From a control perspective, sophisticated strategies are paramount to transitioning the grid from a state of passive disturbance tolerance to one of proactive resilience. Effective control influences the renewable energy hosting capacity and the expansion of the grid's operational security margins. Ultimately, the deep interdependence among physical energy flows, power electronic interfaces, and communication networks makes modern renewable systems highly complex cyber-physical systems (CPSs).

Consequently, RPSs present formidable modeling and control challenges that span multiple domains. In terms of modeling, the high heterogeneity of system components necessitates the mathematical integration of ultrafast power-electronic switching dynamics with the slower inertial responses of rotating machines, resulting in profound multi-timescale coupling phenomena that permeate the entire system. In the control domain, strategies must continuously reconcile conflicting objectives, such as balancing decentralized voltage and frequency regulation with the optimal, economic coordination of multiple energy sources. At the device level, specific issues such as the inherent active-reactive power coupling and power sharing inaccuracies of traditional droop control in resistive low-voltage networks remain pressing. Meanwhile, achieving true plug-and-play capability demands

controllers equipped with online topology identification and adaptive parameter self-tuning. Furthermore, at the system level, the reliance on communication networks for distributed coordination introduces vulnerabilities; network-induced imperfections, such as communication delays, packet losses, and cyber threats (e.g., false data injection attacks) can severely destabilize consensus algorithms. Addressing these vulnerabilities requires the development of resilient frameworks that deeply integrate physical dynamics with cybersecurity protection mechanisms.

Moreover, in practical RPSs, control objectives are commonly organized into hierarchical layers. Primary control provides fast local voltage, frequency, and power-sharing regulation; secondary control restores voltage and frequency deviations and improves power-sharing accuracy; tertiary control coordinates economic dispatch, grid interaction, and energy management. The increasing penetration of converter-interfaced DERs challenges this hierarchy by introducing low inertia, communication dependence, and multi-timescale coupling. Addressing these multi-faceted challenges is expected to reshape the operational paradigms of future energy networks. The maturation of distributed, autonomous coordination algorithms will propel microgrids from isolated pilot projects to interconnected, city-level MG clusters, potentially paving the way for decentralized transactive energy markets. On a macro level, the coevolution of advanced modeling and control techniques is an indispensable enabling force for global carbon neutrality strategies. By reducing reliance on centralized dispatch, empowering off-grid electrification in remote areas, and optimizing multi-timescale resource allocation to minimize fossil-fuel reserves, these technologies will accelerate the deep decarbonization of the energy sector. Nevertheless, persistent open problems remain, particularly regarding the interdisciplinary integration of nonlinear control with artificial intelligence (AI) and the establishment of unified, standardized communication protocols. These unresolved issues not only underscore the urgency of continued research but also offer vast frontiers for innovation across academia and industry, ultimately determining whether RPSs can successfully transition from local autonomy to a paradigm of global grid intelligence.

Sections 1.2 and 1.3 show that renewable power systems, especially MGs, exhibit converter-dominated, multi-timescale, and cyber-physically coupled dynamics. These features challenge conventional modeling and control methods that rely on fixed structures, centralized coordination, or simplified decoupling assumptions. The following section introduces the fully actuated system approach as a control-oriented framework for restructuring system dynamics and assigning desired closed-loop characteristics.

1.4 Basic Concept of Fully Actuated System Approaches

Paradigm Shift and Theoretical Development of FAS Theory

The emergence of FASs theory in the history of control science marks the establishment of a novel methodological framework. The founding research team, led by

Professor Guang-Ren Duan, has methodically constructed this groundbreaking theoretical architecture through prolonged systematic exploration of the mathematical essence of control problems, fundamentally redefining the analytical paradigm for dynamic system control.

Critical Reconstruction of the Classical Paradigm

Within the classical control theory framework, the state-space method has long held a dominant position. Rooted in the mathematical perspective of differential equation solving, this methodology prioritizes state variables as its central focus while relegating control variables to a secondary role. Professor Guang-Ren Duan's research team points out that this "state-centered" perspective, while theoretically consistent at the level of solving system states, reveals limitations when applied to solving control problems. Specifically, the central role of control variables as system inputs is weakened, providing no practical convenience for addressing control problems of dynamic systems.

By delving deeply into this fundamental contradiction, Professor Duan's research team pioneered the FASs theory. Its pivotal insight lies in reestablishing the dominant position of control variables: by restructuring the mathematical model to elevate control variables as first-principle objectives, FASs theory enables a control-oriented restructuring of open-loop dynamics and systematic assignment of closed-loop characteristics. This innovation achieves a seamless alignment between control theory and engineering demands. Remarkably, even for complex nonlinear systems, this methodology facilitates the derivation of time-invariant linear closed-loop control models, transcending conventional limitations of feedback linearization techniques.

Dialectical Unity of Physical Intuition and Mathematical Abstraction

Professor Duan's research team extracted core characteristics from typical full actuation systems like robotic arms (robotic arms with independent actuators per degree of freedom): the direct control capability that globally covers system dynamics. This physical property is abstracted at the mathematical level as a correspondence between the number of control variables and system state variables. Particularly noteworthy is that this methodology provides a systematic route to linear closed-loop representations for certain classes of nonlinear systems, breaking through the stringent requirements of traditional feedback linearization on system structure.

Explorations of Two Fundamental Research Questions

However, this idea faced two challenges in its early development: the concept of full actuation was often interpreted narrowly as a physical property of special systems, and the rapid growth of research on underactuated systems diverted attention from fully actuated formulations. While most researchers focused on the linearization of first-order state-space models and parameter optimization, Professor Duan's team pursued a different theoretical route by identifying two fundamental questions:

- Existence of mathematically constructed fully actuated systems: Can nonphysical fully actuated systems inherit full control characteristics through mathematical construction?
- Mapping underactuated systems in a physical sense to fully actuated paradigms: Is there a systematic methodology to transform physically underactuated systems into mathematically favored FASs to easily achieve global stabilization?

During the years 2020–now, a series of papers published in “Acta Automatica Sinica” and “International Journal of System Sciences” successfully confirmed these two major propositions from a rigorous mathematical perspective (Duan, 2021a, 2021b, 2021c, 2021d, 2021e, 2021f, 2021g, 2022a, 2022b, 2022c, 2024). The theoretical validation demonstrated that the methodology is not only compatible with the analysis of nonlinear, time-varying, and time-delay systems but also exhibits a simplicity in control law design dimensions that surpasses what state-space methods can achieve. This dual breakthrough positions the fully actuated system theory as a universally adaptable counterpart to classical state-space methods, actively reshaping the knowledge landscape of the control discipline.

FASs Theory

To facilitate the introduction of the basic principles of FAS theory, assume a nonlinear dynamics of the following form:

$$\dot{x}^{(m)} = f(x^{(0\sim m-1)}, \zeta) + g(x^{(0\sim m-1)}, \zeta)u \quad (1.1)$$

where $x \in \mathbb{R}^n$, $x^{(0\sim m-1)} = \text{col}(x, \dot{x}, \dots, x^{(m-1)})$, $u \in \mathbb{R}^r$, $f(\cdot) \in \mathbb{R}^n$, $g(\cdot) \in \mathbb{R}^{n \times r}$.

Assumption 1.4.1 For any $x^{(0\sim m-1)}$, ζ , there is

$$\text{rank}(g(x^{(0\sim m-1)}, \zeta)) = r \leq n. \quad (1.2)$$

This assumption indicates that the number of control inputs of the system is not greater than the dimension of x . Regarding (1.1), the following will discuss both the fully actuated and underactuated scenarios separately. First, if $r = n$, then (1.1) is already an FAS. According to FAS theory, its controller can be quickly designed in the following form:

$$\begin{aligned} w &= - \sum_{i=0}^{m-1} A_i x^{(i)} - f(x^{(0\sim m-1)}, \zeta) + v \\ u &= g^{-1}(x^{(0\sim m-1)}, \zeta)w \end{aligned} \quad (1.3)$$

where A_i , $i = 0, 1, \dots, m-1$, are the control parameters, and v is the reference input. At this point, substituting the abovementioned controller into (1.1), the

closed-loop system can be obtained as follows:

$$x^{(m)} + \sum_{i=0}^{m-1} A_i x^{(i)} = v. \quad (1.4)$$

Second, if $r < n$, then (1.1) is an underactuated system. At this point, assuming further that $m = 1$ and ζ does not exist, the system (1.1) reduces to the well-known affine nonlinear system

$$\dot{x} = f(x) + g(x)u. \quad (1.5)$$

According to Assumption 1.4.1, the following state transformation can be performed:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = Qx, \quad z_1 \in R^{n-r} \quad z_2 \in R^r. \quad (1.6)$$

Partition the matrix Q into $Q = [Q_1^T, Q_2^T]^T$, $Q_1 \in R^{(n-r) \times n}$ and $Q_2 \in R^{r \times n}$. Further, it is assumed that there exists a matrix with a nonzero constant determinant such that

$$Qg(x) = \begin{bmatrix} 0 \\ G(x) \end{bmatrix}$$

where $G(x) \in \mathbb{R}^{r \times r}$, $\det G(x) \neq 0$. Multiplying both sides of Eq. (1.5) by Q , there is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} Q_1 f(x) \\ Q_2 f(x) \end{bmatrix} + \begin{bmatrix} 0 \\ Q_2 g(x) \end{bmatrix} u. \quad (1.7)$$

It is obtained that

$$\begin{aligned} \dot{z}_1 &= \tilde{f}_1(z) \\ \dot{z}_2 &= f_2(z) + G(z)u. \end{aligned} \quad (1.8)$$

Further, assume that $n - r > r$ and that z_1 can be decomposed as follows:

$$\begin{aligned} \dot{z}_{1,0} &= g(z_{1,0}) \\ \dot{z}_{1,1} &= f_1(z_{1,0}, z_{1,1}) + B_1 z_{1,2}, \quad z_{1,2} \in R^r \\ \dot{z}_{1,2} &= f_2(z_{1,0}, z_{1,1}, z_{1,2}) + B_2 z_{1,3}, \quad z_{1,3} \in R^r \\ &\vdots \\ \dot{z}_{1,q-1} &= f_{q-1}(z_{1,0}, z_{1,1}, z_{1,2}, \dots, z_{1,q-1}) + B_{q-1} z_{1,q} \\ \dot{z}_{1,q} &= f_q(z_{1,0}, z_{1,1}, z_{1,2}, \dots, z_{1,q}, z_2) + B_q u. \end{aligned} \quad (1.9)$$

In summary, a pseudo-strict feedback system can be obtained

$$\begin{aligned}\dot{z}_{1,0} &= g(z_{1,0}) \\ z_{1,1}^{(q)} &= h_1(z_{1,0}, z_{1,1}, z_{1,2}, \dots, z_{1,q}, z_2) + Bu\end{aligned}\quad (1.10)$$

where $B \in R^{r \times r}$ denotes the equivalent input matrix obtained after the pseudo-strict-feedback transformation, and $\det B \neq 0$. Assuming that n is an integer multiple of r , the first subsystem in (1.10) vanishes at this point. According to Duan (2021b), the abovementioned pseudo-strict feedback system is equivalent to the following high-order system:

$$z_{1,1}^{(q)} = h_1(z_{1,1}, z_{1,2}, \dots, z_{1,q}, z_2) + Bu. \quad (1.11)$$

According to the FAS theory, the controller can be designed as

$$\begin{aligned}u &= -B^{-1} \left(\sum_{i=0}^{q-1} A_i z_{1,1}^{(i)} + u^* \right) \\ u^* &= h_1(z_{1,1}, z_{1,2}, \dots, z_{1,q}, z_2) - v.\end{aligned}\quad (1.12)$$

Substituting this controller into (1.11) yields the closed-loop system

$$z_{1,1}^{(q)} + \sum_{i=0}^{q-1} A_i z_{1,1}^{(i)} = v. \quad (1.13)$$

Examining (1.4) and (1.13), when the external input is 0, their closed-loop systems under the FAS approach both have the following form:

$$x^{(m)} + \sum_{i=0}^{m-1} A_i x^{(i)} = 0. \quad (1.14)$$

By defining the state variables as $\tilde{x} = [x, \dot{x}, \dots, x^{(m-1)}]^T$, the abovementioned system can be represented in the following state-space form:

$$\dot{\tilde{x}} = \Phi \tilde{x} \quad (1.15)$$

where

$$\Phi = \begin{bmatrix} 0 & I_n & 0 & \cdots & 0 \\ 0 & 0 & I_n & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_0 & -A_1 & -A_2 & \cdots & -A_{m-1} \end{bmatrix}.$$

The asymptotic stability of the closed-loop system is equivalent to identifying a stabilizing matrix $[A_0, A_1, \dots, A_{m-1}]$. For this problem's solution, the following theorem governs the parameter design framework.

Theorem 1.4.1 (Duan, 2021a) *For an arbitrary selected $F \in R^{mn \times mn}$, all matrices A_i ($i = 0, 1, \dots, m-1$) that can render the closed-loop system (1.15) asymptotically stable are given as follows:*

$$[A_0, A_1, \dots, A_{m-1}] = -ZF^n V^{-1}$$

$$V = \begin{bmatrix} Z \\ ZF \\ \vdots \\ ZF^{n-1} \end{bmatrix} \quad (1.16)$$

where $Z \in R^{m \times mn}$ is an arbitrary parameter matrix satisfying $\det V \neq 0$. V is nonsingular and satisfying $\Phi = VFV^{-1}$.

The significance of the FAS approach for renewable power systems lies in its ability to transform selected classes of nonlinear and coupled dynamics into closed-loop forms with assignable characteristics. This property is particularly relevant for converter-interfaced MGs, where voltage, frequency, and power-sharing dynamics are coupled through both physical networks and control loops. The subsequent chapters build on this theoretical foundation to develop modeling and control methods for renewable-energy-dominated MGs.

1.5 Interconnected Multi-Agent Systems

In the context of RPSs, control architectures can be fundamentally bifurcated into two paradigms: standalone independent systems and interconnected multi-agent systems. Standalone systems are characterized by strict individual autonomy, with a typical example being single grid-connected PV unit, as illustrated in Fig. 1.2. Mathematically, these systems are conventionally modeled using a lumped-parameter centralized state-space formulation, which integrates all localized components, such as PV arrays, DC/AC inverters, energy storage units, and local loads, into a monolithic dynamic model. The defining feature of this paradigm is its reliance on self-contained local control loops, devoid of bidirectional peer-to-peer energy or information exchanges with other subsystems. While this centralized modeling approach transparently captures the intricate internal dynamics among localized components, its structural limitations become apparent upon system expansion. As the grid scales or incorporates highly heterogeneous devices, the dimension of the centralized state-space model increases substantially, which can lead to high computational burden and reduced scalability. Furthermore, such rigid centralized frameworks are highly susceptible to single points of failure and are fundamen-

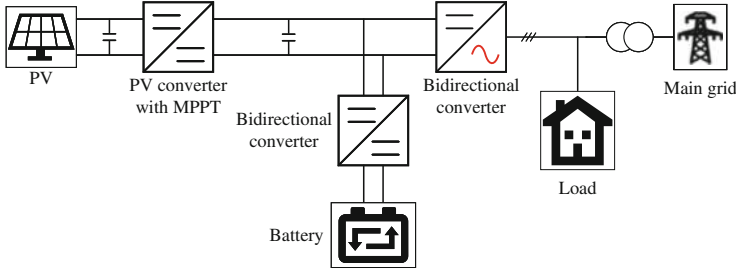


Fig. 1.2 Grid-connected PV generation system

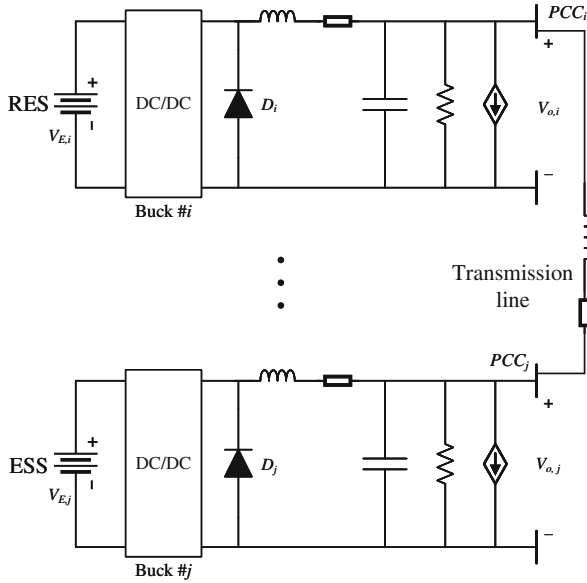


Fig. 1.3 Interconnected DC microgrid system

tally incompatible with the inherent topological flexibility required for modern distributed coordination.

To overcome these structural bottlenecks, interconnected multi-agent systems link autonomous subsystems into a cohesive cooperative network through physical electrical infrastructures. Each subsystem within this network retains its localized decision-making capabilities. A representative architecture is the MG depicted in Fig. 1.3. Here, distributed energy units operate simultaneously as independent regulatory nodes and as power-coupled entities bound by network impedances. The dynamical equations governing these systems are inherently complex. They must strictly account for both the internal nonlinear dynamics of local converters, such as phase-locked loops and LC filter transients, and the cross-coupling network terms governed by Kirchhoff's laws. This dual coupling of physical electrical flows and

distributed information exchanges introduces multidimensional challenges to the control design. First, the controllers must execute decentralized decision-making without relying on global system awareness. They are required to ensure fast tracking of local objectives, like bus voltage regulation, while simultaneously coordinating with neighboring nodes via sparse communication to correct power sharing through droop mechanisms. Second, the heterogeneous impedance characteristics of the physical network cause spatiotemporally coupled transient interactions during local power fluctuations. Traditional stability analysis frameworks often rely on strict decoupling assumptions, which may be insufficient for capturing these modal interactions. Consequently, stability evaluation paradigms must be radically reconstructed using advanced mathematical tools like algebraic graph theory and multi-agent consensus protocols.

The operational complexity of interconnected multi-agent systems is further exacerbated by their intrinsic reliance on cyber-physical integration. The electrical grid constitutes the foundational physical layer, but advanced cooperative functionalities are entirely contingent upon data exchange across communication networks. Typical examples of these functionalities include distributed secondary frequency restoration and virtual synchronous machine (VSM) inertia sharing. This deep convergence functionally transforms the modern MG into a highly sophisticated cyber-physical power system (CPPS). Within this domain, network constraints like communication delays, data packet dropouts, and bandwidth limitations introduce detrimental phase lags into the control loops. If left uncompensated, these delays severely degrade the system phase margin and frequently trigger low-frequency oscillatory instability. This heavy reliance on communication also broadens the system attack surface, allowing cyber threats to propagate directly into the physical execution layer. For instance, malicious actors orchestrating false data injection (FDI) attacks could deliberately tamper with the consensus reference values of neighboring nodes to induce severe voltage deviations or destabilizing transients. This multidimensional risk landscape compels control engineers to adopt a holistic design philosophy. They must concurrently optimize the robust margins of the physical topology and fortify the cybersecurity of the information network, ultimately yielding a system with proactive and self-healing operational capabilities under extreme perturbation scenarios.

1.6 About the Book

1.6.1 *Structure of the Book*

This book takes MGs, which are representative renewable power systems, as its primary example to systematically explore the subject. It covers a wide range of topics, including the modeling and control of MGs, as well as the control of cyber-physical MG systems, and provides a detailed discussion of distributed control

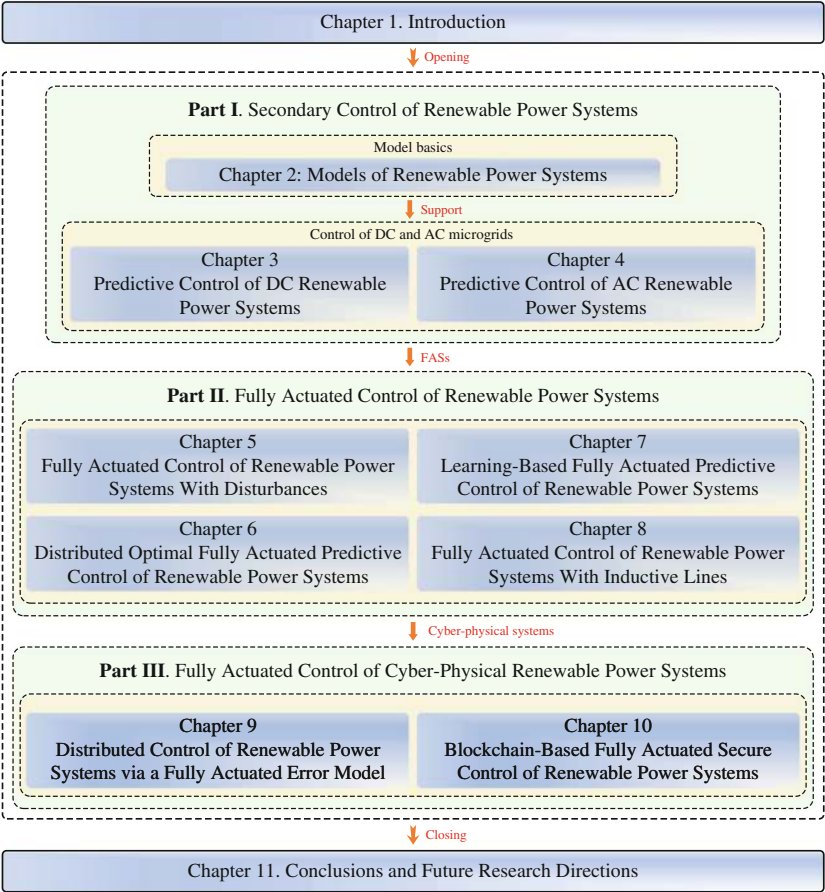


Fig. 1.4 Organizational structure of the book

strategies for MGs. Special emphasis is placed on the significant role and practical applications of the FAS approach at both primary and secondary control levels. The book is organized into three main parts, comprising a total of 11 chapters, with a clear and logical structure. The overall framework is illustrated in Fig. 1.4.

1.6.2 Origins of the Book

This book is one of the six volumes in the book series titled *Fully Actuated System Approach for Control and AI*, which will be published by Springer (see <http://www.fasta.org.cn> for the proposal). The series is scheduled to run from 2025 to 2030.

In parallel with the development of this book series, the FASTA conference serves as a dedicated academic forum for related theoretical and applied research. To date, this conference has been successfully organized two times. The conference details are as follows:

Title *International Conference on Fully Actuated System Theory and Applications*

Websites

<http://fasta2024.fasta.org.cn>; <http://fasta2025.scimeeting.cn>.

As planned, four to six books on this topic will be published annually during the publication period. The following titles are scheduled for 2025:

1. *Fully Actuated System Approach: A New Framework for Control*
 Volume I. Global Fully Actuated Systems
 Volume II. Sub-fully Actuated Systems
 Volume III. Unidirectionally Connected FASs
 by Guang-Ren Duan.
2. *Predictive Cooperative Control for High-Order Fully Actuated Multiagent Systems*
 by Da-Wei Zhang, Guo-Ping Liu.
3. *Fault-Tolerant Control for Fully Actuated Systems*
 by Dong-Hua Zhou, Miao Cai.
4. *Adaptive Constrained Control for High-Order Fully Actuated Nonlinear Systems*
 by Chang-Chun Hua, Liu-Liu Zhang, Peng-Ju Ning.
5. *Cooperative Safety Control of Multiagent Systems: A Fully Actuated System Approach*
 by Ke Zhang, Bin Jiang, Yonghao Ma, Yuhang Xu, Yuan Lu.
6. *Control of Renewable Power Systems: A Fully Actuated System Approach*
 by Yi Yu, Guo-Ping Liu, Josep M. Guerrero.

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Part I
Secondary Control of Renewable Power
Systems

Chapter 2

Models of Renewable Power Systems



Abstract This chapter introduces representative models of renewable power systems, including DC microgrids, AC microgrids, hybrid AC/DC microgrids, and grid-connected converter systems. It first discusses the topology and operational characteristics of DC and AC microgrids, then examines hybrid AC/DC architectures and their interlinking converter functions. Finally, it presents grid-following and grid-forming converter models, highlighting their synchronization mechanisms, control structures, and relevance to power-flow regulation and stability analysis. These models provide the architectural and control-oriented foundation for the subsequent chapters on renewable-power-system control using the fully actuated system approach.

2.1 Overview

In this chapter, several key models of RPSs will be explored in depth to help the reader understand the basic architecture and operational characteristics of these systems. It first discusses the topology and operational characteristics of DC and AC RPSs, then examines hybrid AC/DC architectures and their interlinking converter functions. Finally, this chapter presents models of grid-forming and grid-following grid-connected converters, which are key devices for interconnecting renewable energy sources to the grid to effectively manage power flow and stability. By analyzing these typical models, the reader can gain a comprehensive understanding of the topology and classical control aspects of current RPSs, thus laying a solid foundation for further research and application.

The chapter is organized as follows. Section 2.2 presents models of DC RPSs. Section 2.3 introduces models of AC RPSs. Section 2.4 presents models of hybrid RPSs. Section 2.5 briefly discusses models of grid-following and grid-forming converter systems.

2.2 Models of DC Renewable Power Systems

This section uses DC microgrids as representative systems to outline the model architecture and core characteristics of typical DC RPSs. A DC microgrid is primarily composed of generation units, energy storage devices, power electronic converters, and loads. The network topology can be either single-bus or multi-bus depending on the application scenario; the corresponding structures are shown in Figs. 2.1 and 2.2, respectively. The single-bus system employs a centralized bus

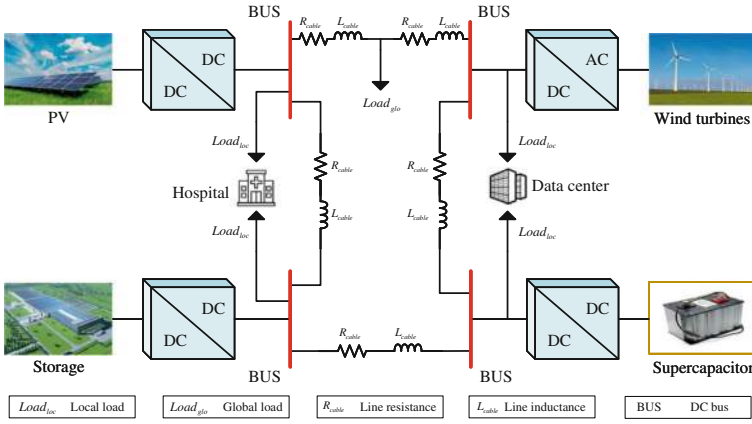


Fig. 2.1 DC microgrids with ring bus topology

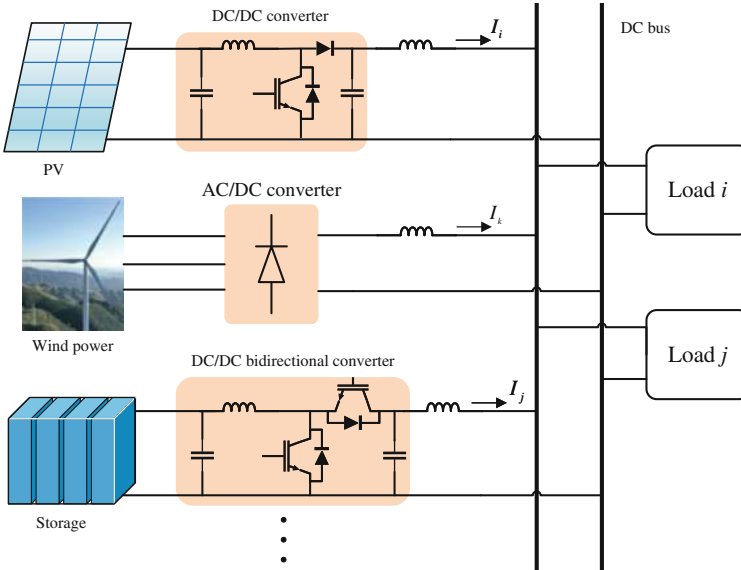


Fig. 2.2 DC microgrids with single bus topology

architecture where all generation units and loads are connected to a common coupling point via power electronic interfaces, forming a star-shaped radial structure. The core advantage of this design is that only a single-bus voltage needs to be regulated, simplifying the complexity of the dynamic equations. Consequently, the complexity of dynamic tracking and stable regulation is relatively low, making it suitable for small-scale off-grid power supply scenarios. In contrast, the multi-bus architecture establishes multiple independent controllable point of common couplings (PCCs) through power electronic converters, creating a meshed or layered structure. Each PCC typically links to critical sensitive loads or specific generation units, such as the medical equipment or data center server clusters highlighted in Fig. 2.1. While this multi-bus structure improves system flexibility and power supply reliability, the coordinated control of voltage at each node becomes a key challenge. The choice between these topologies hinges on balancing system scale, control real-time requirements, and fault tolerance. For scenarios demanding high power supply reliability like data centers, the multi-bus architecture can achieve fault isolation through redundant design, though this comes at the expense of heightened control logic complexity and computational overhead.

Load modeling is a fundamental step in the design of multi-bus microgrid systems and can be categorized into local and global types based on their coupling characteristics and scope of influence. Local loads are typically connected directly to a specific PCC via a dedicated line, and their dynamic behavior is primarily influenced by the electrical parameters at the connection point, as illustrated by the local load in Fig. 2.1. On the other hand, global loads, here referring to inter-bus loads connected between two buses, are electrically coupled to the voltage states of both terminal buses. Their power variations therefore introduce coupled current disturbances at the connected buses and may propagate voltage deviations through the network. It is noteworthy that the distinction between these two categories becomes less pronounced with the adoption of source-grid-load co-modeling methodology. By coupling and solving the network flow equations with power dynamics equations, system-level models can comprehensively capture the interactive effects of loads at different locations, thereby reducing the complexity of load characterization while enhancing the fidelity of stability analyses.

At the operational level, achieving dynamic current sharing among distributed generation units remains a critical technical challenge for both single-bus and multi-bus architectures. Due to the impact of the randomness of renewable energy generation and the nonlinear characteristics of power electronic devices, instantaneous power outputs from distributed sources like solar arrays and fuel cells may differ, potentially leading to bus-current imbalances. To address this issue, modern DC microgrids integrate adaptive droop control algorithms into hierarchical control architectures. By embedding tunable virtual impedance parameters into the converter control loops, these systems enable autonomous current balancing between generation units while minimizing reliance on the communication infrastructure required by traditional master–slave control schemes. Furthermore, the FAS approach has emerged as a critical technical pathway in DC microgrid modeling and control due to its unique advantages in dealing with nonlinearities. This approach

not only maintains voltage stability but also effectively coordinates multisource power flows, providing essential technical support for the high penetration of renewable energy resources.

2.3 Models of AC Renewable Power Systems

As the core platform for distributed AC renewable energy systems, AC microgrids' architecture design and dynamic characteristics are directly linked to power quality and system stability. While AC microgrids share fundamental similarities with DC microgrids in terms of topology—both adopting multi-bus and single-bus configurations (as shown in Figs. 2.3 and 2.4, respectively)—they exhibit distinct differences in terms of dynamic behavior and control requirements. Current research

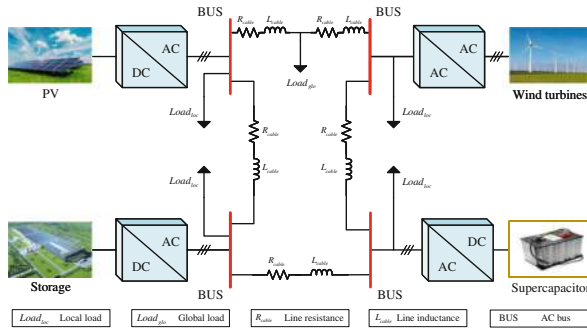


Fig. 2.3 AC microgrids with ring bus topology

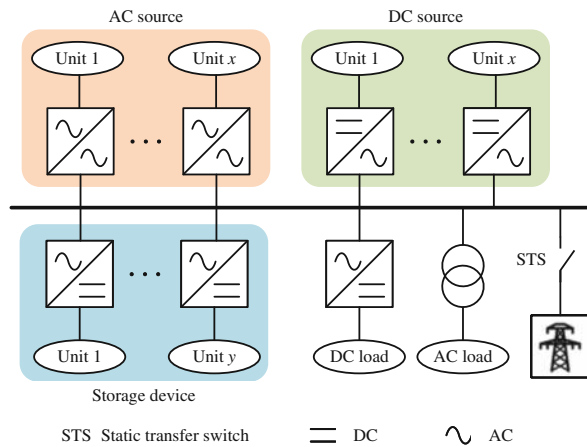


Fig. 2.4 AC microgrids with single bus topology

trends highlight the superior architectural adaptability of multi-bus AC microgrids, which effectively support industrial park-scale implementations. These systems have received considerable research attention in areas such as the coordinated operation of electric vehicle charging/swapping stations and solar/storage hybrid systems. In contrast, single-bus architecture studies predominantly focus on low-capacity residential applications and remain less standardized in terms of design paradigms.

In terms of structural design, multi-bus systems enable localized accommodation of sensitive loads and flexible integration of distributed generation through deploying PCCs in partitioned areas. As exemplified in Fig. 2.3, where loads like office buildings and manufacturing workshops connect to separate buses, this architecture allows critical loads to be served locally while facilitating inter-regional power exchange. Each node must fulfill the dual function of supplying power to local critical loads and engaging in power interactions with adjacent regions. The control complexity stems not only from the phase synchronization requirements among multiple nodes but also from the reactive power dynamics inherent to AC systems. Energy exchange between buses is facilitated through interconnecting inverters, while the heterogeneous distribution of line impedance amplifies topology-dependent power transmission losses. Furthermore, unlike the linear voltage transmission characteristics of DC systems, multi-bus AC microgrids face the compounded effects of three-phase voltage imbalance, making phase sequence coordination between buses particularly critical. The load modeling in such systems maintains the previously mentioned classification into local and global loads. For example, the local loads marked in Fig. 2.3 require independent modeling, whereas cross-bus loads necessitate the construction of a globally coupled model.

The core control framework of AC microgrids focuses on dual stability objectives of frequency and voltage, while also necessitating the decoupled coordination of active and reactive power. System frequency is directly linked to the synchronous operation capability of distributed energy resources. For instance, inverters employ virtual synchronous machine techniques to mimic the frequency control characteristics of traditional generators, ensuring that frequency deviations remain within an allowable threshold under the impact of wind and solar power fluctuations. Voltage stability control, on the other hand, requires a comprehensive consideration of the spatial distribution characteristics of reactive power, particularly in multi-bus architectures with asynchronous motors and reactive compensation devices. This requires distributed cooperative algorithms to enable dynamic reactive power compensation across individual buses. Notably, the increasing penetration of power electronic devices has brought new control challenges such as subsynchronous oscillations induced by inverter output harmonics. These phenomena are driving the development of advanced control strategies—including harmonic damping techniques and broadband impedance reshaping—to mitigate instability risks, thereby laying a solid foundation for next-generation smart grids characterized by high renewable integration and enhanced resilience.

2.4 Models of Hybrid Renewable Power Systems

The preceding sections show that DC and AC MGs have distinct modeling advantages and control challenges. DC MGs simplify voltage regulation and reduce unnecessary conversion stages for DC sources and loads, whereas AC MGs remain compatible with existing distribution infrastructures and conventional AC loads. Hybrid AC/DC MGs combine these two architectures by integrating AC and DC buses through bidirectional interlinking converters. This section focuses on their architectural features and application value in modern power systems.

With the increasing diversification of distributed energy resources—such as DC-based photovoltaic systems and fuel cells, as well as AC-based wind turbines—the need for coordinated AC/DC integration has become increasingly important. Concurrently, emerging loads such as data centers and electric vehicle charging stations impose heterogeneous requirements on power quality, supply continuity, and energy-conversion efficiency, exposing the limitations of single-mode AC or DC microgrids. As shown in Fig. 2.5, hybrid architectures create flexible energy-transfer pathways between AC and DC subgrids. By matching resources and loads to their native AC or DC forms, these systems can improve resource utilization, reduce unnecessary conversion stages, and enhance operational flexibility, thereby providing an important architectural basis for intelligent distribution networks. Their core advantages include:

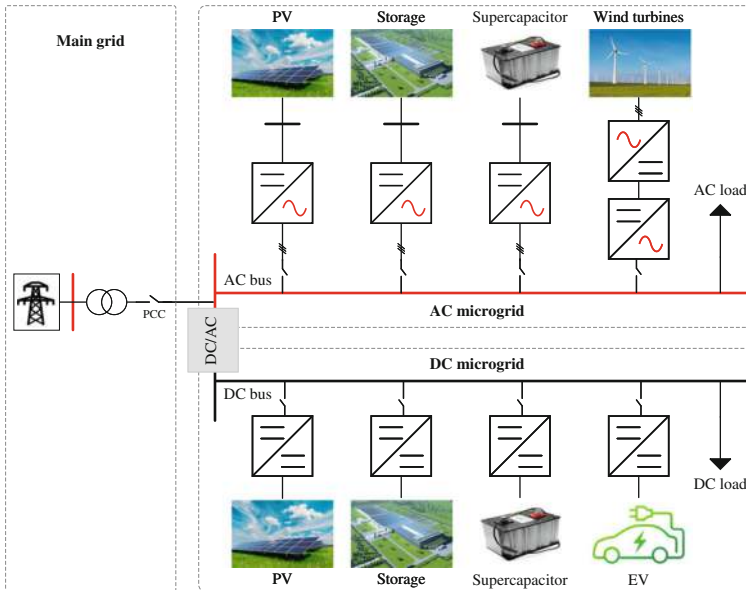


Fig. 2.5 Structure of the hybrid microgrid

1. Hybrid systems reduce redundant conversion steps through intelligent adaptation of power supply modes. DC power sources (such as solar arrays and storage batteries) and loads are directly connected to the DC bus, avoiding some of the efficiency loss incurred by two-stage AC/DC conversion. AC sources and loads are closely connected to the AC subgrid, minimizing unnecessary DC/AC inversion requirements.
2. Using flexible control of bidirectional interlinking converters, dual-bus parallel operation enables autonomous power path adjustment based on real-time conditions. For example, during peak photovoltaic generation, the DC bus prioritizes power delivery to local data centers, while the system switches to absorbing grid support via the AC subgrid when the state of charge (SOC) of energy storage falls below critical thresholds. Under fault conditions, the system dynamically reconfigures supply paths using its modal conversion capability. For example, in the event of an AC-side short circuit, critical loads can be sustained through the DC interconnection channel. This bidirectional adaptive mechanism not only enhances source–load matching efficiency but also provides a physical foundation for multi-timescale energy scheduling. This allows the system to maintain AC-side frequency and voltage stability while regulating DC-bus voltage and accommodating economic dispatch objectives under carbon-emission constraints.

The interconnection architecture of hybrid AC/DC microgrids can be classified into three typical topologies according to the location and implementation of the utility-grid interface, or point of common coupling (PCC). As shown in Fig. 2.6, these topologies differ in how the main grid is coupled to the AC and DC subgrids, and therefore exhibit different design principles, power-routing paths, and control responsibilities. (1) In the AC-coupled architecture (Topology I), the main grid is connected to the AC bus through an AC-side PCC, usually implemented with a transformer and a circuit breaker. This topology is suitable for systems dominated by AC-coupled distributed energy resources and AC loads, while the DC subgrid is interfaced through an interlinking converter (ILC). (2) In the DC-coupled architecture (Topology II), the DC bus acts as the main energy-routing hub and is connected to the utility grid through a bidirectional grid-side AC/DC converter. This topology is suitable for newly developed distribution systems or local energy hubs with high penetration of DC-compatible resources and loads, such as photovoltaic generation, battery energy storage, electric vehicle charging facilities, and data centers. (3) In the electronic power transformer (EPT)-integrated architecture (Topology III), the EPT serves as a multiport power-electronic interface between the utility grid, an internal DC-link or DC-bus hub, and the downstream AC and DC buses. This topology supports flexible interconnection among multiple voltage levels and is particularly suitable for active distribution networks with high-density renewable energy integration and heterogeneous AC/DC loads.

In terms of operational modes, the three hybrid AC/DC topologies exhibit different control characteristics because the utility interface is placed at different electrical ports. In the AC-coupled architecture (Topology I), the main grid is connected to

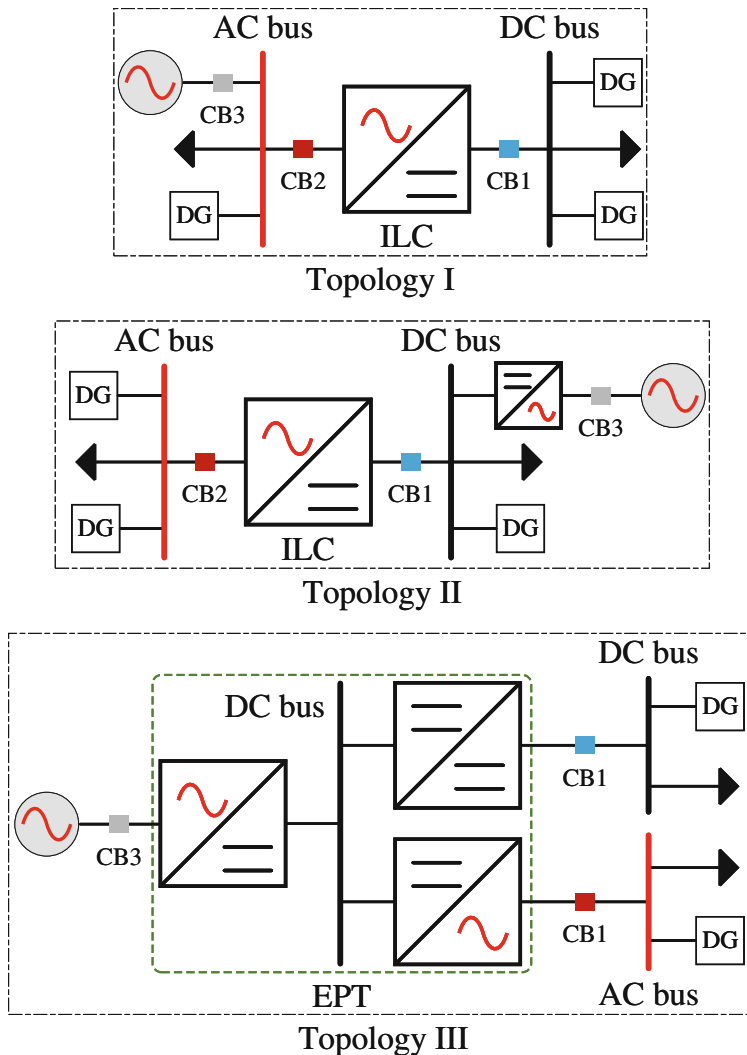


Fig. 2.6 Typical topologies of the hybrid microgrid

the AC bus through the AC-side PCC. During grid-connected operation, the grid normally provides the dominant voltage and frequency reference for the AC bus, while the interlinking converter (ILC) regulates the DC-bus voltage or controls the scheduled power exchange between the AC and DC subgrids. Depending on the operating objective, the ILC may operate in a fixed-power mode for scheduled energy exchange, a DC-voltage-regulation mode for maintaining DC-side power balance, or a flexible power-support mode for dynamic dispatch and ancillary service provision. In islanded operation, the loss of the utility reference requires local

grid-forming capability. The AC-bus voltage and frequency must be established by designated grid-forming converters, distributed generators, energy storage units, or the ILC, while the DC-bus voltage is maintained through coordinated control of DC-side sources, storage, and the ILC. Therefore, the ILC serves as the key coordination interface that balances power between the AC and DC subgrids rather than merely acting as a passive power-transfer device.

In the DC-coupled architecture (Topology II), the main grid is interfaced through the DC bus by a grid-side AC/DC converter. During grid-connected operation, this grid-side converter typically regulates the DC-bus voltage and manages the power exchange with the external grid. The AC bus is not directly supported by the main grid; therefore, its voltage and frequency must be maintained by the ILC and/or local AC-side grid-forming units, such as converter-interfaced distributed generators and energy storage systems. In this topology, the ILC transfers power between the regulated DC bus and the AC subgrid, and it may also provide AC-side voltage and frequency support when operating in a grid-forming mode. Under islanded conditions, the grid-side converter is disconnected from the utility grid, and the DC-bus voltage must be supported by local DC sources, storage devices, or coordinated converter controls. Meanwhile, the AC-side voltage and frequency are maintained through the coordinated operation of the ILC and AC-side generation units. Thus, compared with Topology I, Topology II shifts the primary grid interface from the AC bus to the DC bus, which changes the allocation of voltage-regulation and frequency-support responsibilities.

The EPT-coupled architecture (Topology III) further extends this idea by using an electronic power transformer as a multiport power-electronic interface between the utility grid, an internal DC-link or DC-bus hub, and the downstream AC and DC buses. In grid-connected operation, the grid-side stage of the EPT controls the power exchange with the utility grid and maintains the internal DC-link voltage, while the downstream converter stages regulate the DC-bus voltage, support the AC-bus voltage and frequency, and coordinate power routing among different voltage levels. Unlike a conventional transformer, the EPT does not rely on mechanical tap changing; instead, it adjusts converter modulation commands, voltage-conversion ratios, and port-level power references to realize flexible AC/DC power transfer. In islanded operation, the EPT can reconfigure its converter stages to support autonomous operation, provided that sufficient local generation or storage is available to maintain the internal DC link and supply the AC and DC loads. Owing to its multiport structure, the EPT-coupled topology provides the highest degree of controllability among the three configurations, but it also introduces greater modeling complexity, tighter coordination requirements, and stronger dependence on converter-level control reliability. Overall, these three topologies illustrate the principle that the location of the grid interface determines the dominant voltage and frequency references, while the topology of the AC/DC coupling determines the control responsibilities of the ILC, grid-side converter, and local distributed resources.

2.5 Models of Grid-Following and Grid-Forming Converter Systems

In addition to microgrid-level architectures, grid-connected converters constitute a key modeling object in RPSs because they determine how renewable energy resources exchange power with the external grid and participate in voltage and frequency regulation. According to their synchronization mechanisms and grid-support functions, grid-connected converters are commonly classified into grid-following (GFL) and grid-forming (GFM) types. As shown in Figs. 2.7 and 2.8, GFL converters rely on an existing grid voltage as the synchronization reference and primarily regulate injected current or power, whereas GFM converters establish their own voltage phase, magnitude, and frequency through internal control mechanisms. Therefore, GFL converters are commonly modeled as controlled current sources, while GFM converters are commonly modeled as controlled voltage sources. The following subsections introduce simplified models of these two converter types and highlight the dominant dynamic mechanisms retained in classical stability-oriented analysis.

Grid-Following Inverter Model

The modeling of GFL inverters focuses on the deep coupling effects between the phase-locked loop dynamics and the grid impedance. The primary controlled output of GFL inverters is typically the injected current I . It involves a closed-loop current control architecture that synchronizes to the grid voltage phase angle δ via the

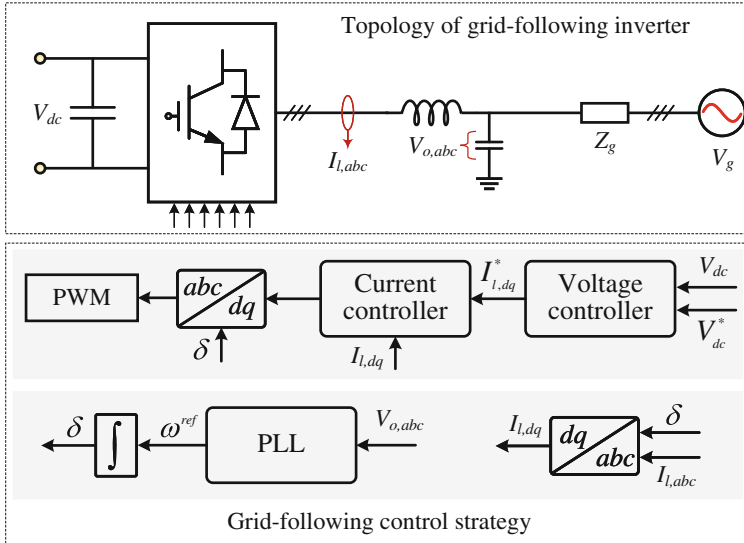


Fig. 2.7 Typical topology of the GFL inverter. Note: PWM stands for pulse width modulation

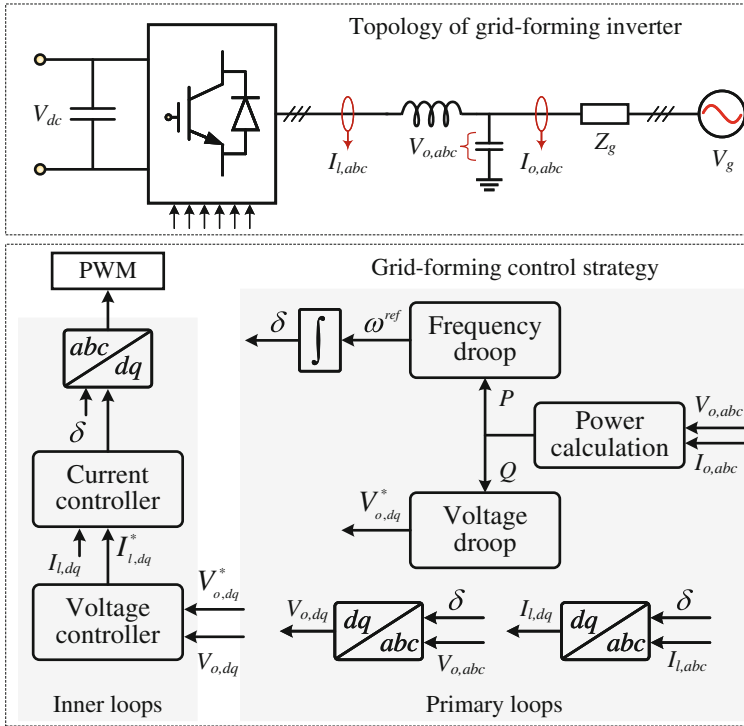


Fig. 2.8 Typical topology of the GFM inverter

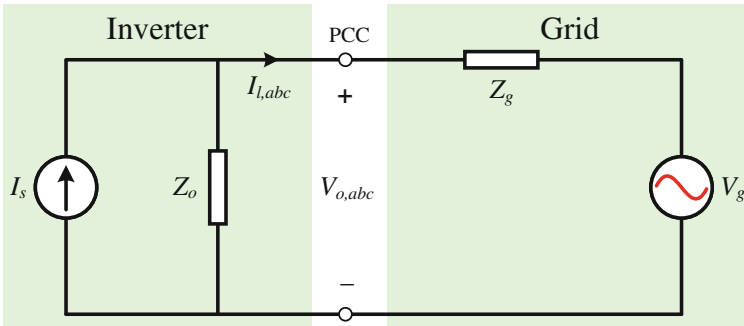


Fig. 2.9 Equivalent circuit of the GFL inverter

phase-locked loop (PLL). When grid-tied, their dynamic behavior approximates that of a controlled current source (as shown in Fig. 2.9); hence, they are also known as current-controlled inverters. Since the bandwidth of the current inner loop is typically tens of times that of the PLL, the analysis of low frequency oscillations dominated by phase desynchronization often simplifies the current control block to

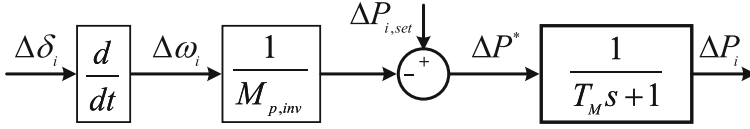


Fig. 2.10 Simplified model of the GFL inverter presented in (Lasseter et al., 2019)

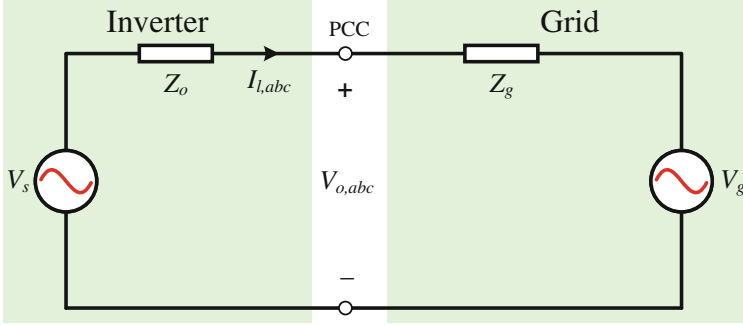


Fig. 2.11 Equivalent circuit of the GFM inverter

a pure gain function, thereby disregarding the influence of fast dynamic loops. At this point, the inertial characteristics of the power response can be represented by a first-order inertia element with a single time constant (Zhang and Xu, 2017), as shown in Fig. 2.10, which effectively captures the nonlinear relationship between PLL-induced phase deviations and power fluctuations.

Grid-Forming Inverter Model

The common technical features of GFM inverters are as follows: (1) GFM inverters operate as controlled voltage sources based on power orientation (as shown in Fig. 2.11), rather than as controlled current sources; (2) synchronization with the grid is achieved through output power regulation instead of traditional phase-locked methods. This dual nature equips them with the ability to autonomously establish voltage and ensure the feasibility of islanded operation. Existing control strategies have evolved along four primary paths: basic droop control, direct power control, filtered droop control, and virtual synchronous machine emulation, all of which focus on power–frequency dynamic interactions. Typically, only the frequency/power dynamics of GFM inverters are modeled (Lasseter et al., 2019), incorporating first-order power measurement filters as shown in Fig. 2.12. Voltage control dynamics and other fast transients are generally neglected, while current-limiting controls are omitted from theoretical models. Due to limitations in transient stability analysis tools for high-order nonlinear systems, traditional methodologies often employ corresponding quasi-steady-state GFM converter models, utilizing phase portraits or system trajectories to assess lower-order transient dynamics.

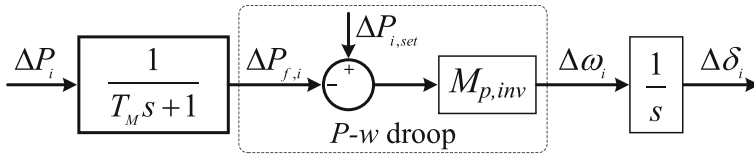


Fig. 2.12 Simplified model of the GFM inverter presented in (Lasseter et al., 2019)

It should be noted that a fundamental challenge shared by both grid-following and grid-forming inverters lies in their inherent nonlinear dynamics. Conventional linearization techniques, while effective for small-signal stability analysis, struggle to ensure convergence under extreme operating conditions, such as rapidly switching operating states or large-scale grid disturbances. In contrast, the FAS methodology provides a promising framework for handling nonlinear converter dynamics through structured model transformation and closed-loop characteristic assignment. This capability positions FAS as a promising candidate for next-generation control frameworks aiming to address stability–complexity trade-offs in power-electronics-dominated grids.

2.6 Notes and References

This chapter systematically introduces the core modeling paradigms of renewable power systems. It provides a detailed introduction to the topological structures of DC microgrids, AC microgrids, and hybrid AC/DC microgrids, as well as the model and control principles of grid-following and grid-forming inverters. It is worth emphasizing that, in addition to the system models discussed in this book, the modeling of energy storage control systems, an essential support for enhancing system flexibility, holds significant research value. Readers are referred to the comprehensive review in (Faisal et al., 2018; Babu et al., 2020; Chaudhary et al., 2021) for recent advances in this area. Renewable power systems represented by microgrids are becoming a research focus due to their natural compatibility with grid-forming control and vehicle-to-grid (V2G) technologies. With the large-scale integration of electric vehicles, V2G not only facilitates the absorption of variable renewable energy but also significantly enhances system stability by providing virtual inertia and fast frequency regulation capabilities. The global surge in research activity further underscores the strategic importance of this technological pathway. In this context, the subsequent chapters of this book will focus on microgrids as the primary research subject. The book will conduct an in-depth analysis of how the FAS methodology can address selected limitations of conventional control approaches and provide systematic control-design tools for power systems with high renewable energy penetration.

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Chapter 3

Predictive Control of DC Renewable Power Systems



Abstract This chapter is concerned with the voltage tracking problem of DC microgrids subject to communication delays and packet losses, for which existing work commonly adopts passive fault-tolerant approaches. To accurately compensate for the abovementioned communication constraints experienced by DC microgrids, a coupled discrete microgrid model is developed on the basis of the physical laws of actual DC microgrids. Unlike the practice of passively tolerating communication delays, through the physical model established, this chapter suggests a consensus-based proportional–integral predictive control strategy, which can actively compensate for communication delays and consecutive packet dropouts encountered by DC microgrids. Under the observer-based distributed networked predictive control framework, each distributed generation subsystem in the microgrid exchanges its own measurements over the network and integrates information from controllers of other units to achieve output voltage consensus in the presence of time delays and packet losses. Furthermore, to demonstrate the generality of the proposed method, the sufficient and necessary conditions for the DC microgrid system to accomplish voltage tracking are given. These conditions are rendered in the form of matrix eigenvalues associated with the physical connections and communication couplings between distributed generation units. Additionally, the assurance of both stability and convergence of the closed-loop microgrid system is established through a linear matrix inequality formulation, predicated upon selecting an appropriate Lyapunov function. Finally, the performance of the proposed control scheme is evaluated in terms of its convergence, robustness to load variations, and plug-and-play functionality through the built photovoltaic cell-based (with battery banks) DC microgrid hardware system.

3.1 Overview

Addressing constant communication delay in DC MGs, the authors of Ding et al. (2018) introduced a distributed control strategy founded upon a multiobjective optimization framework. This approach aimed to achieve voltage regulation and

explicitly determined the tolerated delay margin of the system, highlighting the critical influence of latency on overall MG system stability. Continuing this line of research, the work in Lou et al. (2020) focused on time-delayed microgrids. Leveraging the strong correlation between convergence and algebraic connectivity, this paper developed an optimal design for the directed communication topology and pinning locations, simultaneously defining the requisite secondary control and calculating distinct individual delay limits. Furthermore, the study in Wang et al. (2021) assessed how communication delays impact the consensus-based secondary controller, successfully establishing the maximum delay threshold the system can endure before becoming unstable. While valuable for guiding parameter selection in DC MG applications, this particular analysis does not incorporate any mechanism for positive delay mitigation. Conversely, a methodology detailed in Lou et al. (2017) proposed a distributed secondary control technique utilizing MPC, integrating a finite-time observer to accurately estimate global reference data.

It is noted that while the abovementioned types of research findings have withstood the communication delays in MGs to some extent, these studies have rarely devoted themselves to actively compensating for these constraints experienced by DC MGs. Acknowledging that prior research did not proactively mitigate communication delays, the study in Yao et al. (2022) introduced the distributed weighted average prediction (WAP) control for islanded microgrids. This method effectively handles time delays by utilizing predicted states, which are derived from a complex discrete equation that incorporates a time-delayed differential element. Although WAP successfully counteracts these delays, its prediction model dependent on the differential component exhibits vulnerability to disturbances such as load switching events. Furthermore, a state estimator-based controller was proposed in Gu et al. (2017) to implement voltage regulation of islanded MGs, which has the potential to handle the communication delays. However, the distributed control with the observer in this study cannot actively compensate for packet losses experienced by the system and is not applicable for random and heterogeneous delays. To the best of our knowledge, the problem of ensuring that the terminal voltage is at the desired level by active ways has not been well addressed for DC MGs subject to communication delays and packet losses.

Inspired by the above discussion, this study presents a novel consensus-based proportional-integral predictive (CPIP) control scheme to actively compensate for communication delays and packet losses suffered by autonomous DC MGs, thus completing voltage regulation. It is essential to acknowledge existing literature dedicated to solving communication constraints through the use of predictive control. For example, the authors of Liu (2020) introduced a networked predictive control technique designed for uncoupled multi-agent systems. However, this specific approach is unsuitable for the DC MGs analyzed in this chapter, as DC MGs are coupled systems rather than the standalone configurations addressed in Liu (2020). Consequently, to confirm the effectiveness and operational viability

of the new voltage tracking strategy, a comprehensive photovoltaic DC MG setup integrating energy storage units has been constructed, forming the basis for extensive experimental case studies.

The primary novelty of this chapter, in comparison to the current state-of-the-art literature, resides in the three key advantages detailed below.

1. An approach seldom documented in previous studies involves developing a physical model of the actual DC MG to generate predictions. This strategy allows for precise compensation of both time delays and continuous packet dropouts observed within the system.
2. Through the established real interconnected physical model from duty cycle to output voltage, a CPIP control strategy is proposed by integrating the consensus-based distributed control method and networked predictive control concept, which can actively compensate for the communication delays and consecutive packet dropouts encountered by DC MGs.
3. Combined with the consensus and stability analysis, the sufficient conditions for the DC MG system to complete voltage tracking are given by rigorous mathematical derivations. Furthermore, the Lyapunov function is utilized to derive the conditions for the stability and convergence of the MG system under the developed CPIP control, which are articulated as a linear matrix inequality (LMI). It makes practical sense, such as guiding the tuning of control parameters and determining the physical topology.

The notation used in this chapter is typical. In particular, matrices I and O are the identity matrix and the zero matrix with appropriate dimensions, respectively. The symbol A^T denotes the transpose of matrix A . For $x = (x_1, x_2, \dots, x_n)^T \in R^n$, the norm of x is $\|x\| := (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$.

The remainder of this chapter is structured into six sections. Section 3.2 then outlines the system description and formally defines the problem formulation. The subsequent Section 3.3 details the design methodology for the predictive control applied to the DC RPSs. Following this, a comprehensive stability and consensus analysis for the DC MG is offered in Sect. 3.4, utilizing the CPIP approach. These theoretical findings are validated in Sect. 3.5 by experimental results. Section 3.6 provides the concluding remarks for the chapter.

3.2 Problem Formulation

In this section, the system model and communication structure considered in this chapter are given, and the control issues of DC MG with communication delays are clarified.

3.2.1 Dynamics of DC MGs With Disturbance

Consider a DC MG composed of N distributed generator (DG) units interconnected through power lines. The electrical structure of the i th DG unit is illustrated in Fig. 3.1. As can be seen in Fig. 3.1, each generation unit is composed of a DC voltage source charged by a renewable energy source, a DC/DC converter, an RLC filter, and a resistive load R_{Li} . In addition, unknown disturbances on the load side are considered and modeled as current sources represented by I_{Li} . It is worth noting that a DC/DC buck circuit (shown in Fig. 3.1) is employed in this chapter, but the applicable circuit topologies are not limited to this, as both boost circuit and buck-boost circuit topologies can be used as converters for DC MG systems. In particular, the resistor R_{fi} in the RLC filter is not necessary, but is used more to regulate the system damping. It is assumed that this DC MG can use the quasi-stationary line (QSL) approximation (Han et al., 2019a), i.e., $dI_{ij}/dt = 0$. This assumption is typical and reasonable since line inductance L_{ij} in DC MG systems is generally pretty small. The inherent uncertainty and intermittency associated with renewable energy sources pose a challenge to the reliability of DGs. This issue is typically mitigated in practice using solutions like energy storage systems or complementary energy alternatives, which force the DGs into an approximate steady-state operation. Consistent with this established practice, this chapter treats the DG as a stable power supply when considering the control design of the converters. Based on the abovementioned facts and with the help of Kirchhoff's voltage and current laws, the dynamic characteristics of i th DG can be given as

$$\begin{aligned}\dot{V}_i(t) &= \frac{I_{fi}(t)}{C_{fi}} + \sum_{j \in \mathcal{N}_i^p} \frac{V_j(t) - V_i(t)}{C_{fi} R_{ij}} - \frac{V_i}{C_{fi} R_{Li}} - \frac{I_{Li}(t)}{C_{fi}} \\ \dot{I}_{fi}(t) &= -\frac{V_i(t)}{L_{fi}} - \frac{R_{fi}}{L_{fi}} I_{fi}(t) + \frac{U_{0,i} d_{buck,i}(t)}{L_{fi}}\end{aligned}\quad (3.1)$$

where V_i and I_{fi} are the capacitor voltage and the inductor current of the i th DG, respectively. $V_j \in \mathcal{N}_i^p$ are the voltage at the port interconnected with the i th DG, and $\mathcal{N}_i^p$ is the set comprising the neighboring DG units connected to i th DG

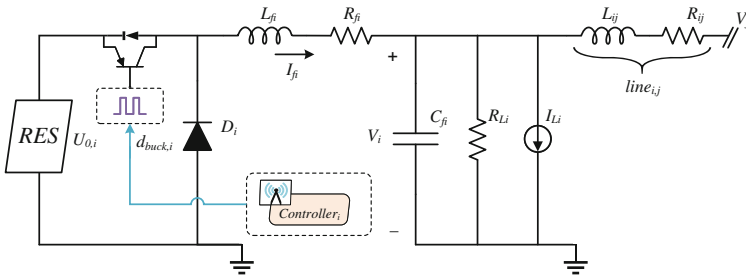


Fig. 3.1 Configuration of the i th DG incorporating a resistive load

through the power line. $d_{buck,i}$ represents the duty cycle used to generate the PWM for driving the IGBT. R_{fi} , L_{fi} and C_{fi} denote the electrical parameters of the filter.

Similar to the control methodology of Tucci et al. (2018), $y_{[i]} = V_i$ is taken as output, $u_{[i]} = d_{buck,i}$ as input, $x_{[i]} = [V_i, I_{fi}]^T$ as state, and $\omega_{[i]} = I_{Li}$ as perturbation. Thus, the i th DG can be described by the following state-space model

$$\begin{aligned}\dot{x}_{[i]}(t) &= A_{ii}^c x_{[i]}(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}^c x_{[j]}(t) + B_i^c u_{[i]}(t) + D_i^c \omega_{[i]}(t) \\ y_{[i]}(t) &= C_i x_{[i]}(t)\end{aligned}\quad (3.2)$$

where $C_i = [1 \ 0]$,

$$\begin{aligned}A_{ii}^c &= \begin{bmatrix} \sum_{j \in \mathcal{N}_i^p} -\frac{1}{C_{fi}R_{ij}} - \frac{1}{C_{fi}R_{Li}} & \frac{1}{C_{fi}} \\ -\frac{1}{L_{fi}} & -\frac{R_{fi}}{L_{fi}} \end{bmatrix}, \\ A_{ij}^c &= \begin{bmatrix} \frac{1}{C_{fi}R_{ij}} & 0 \\ 0 & 0 \end{bmatrix}, B_i^c = \begin{bmatrix} 0 \\ \frac{U_{0,i}}{L_{fi}} \end{bmatrix}, D_i^c = \begin{bmatrix} -\frac{1}{C_{fi}} \\ 0 \end{bmatrix}.\end{aligned}$$

3.2.2 Electrical and Communication Networks

In the physical layer, the electrical interactions of the networked MG system are modeled as a weighted graph $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{E}_p, \mathcal{A}_p)$, where $\mathcal{V}_p = \{v_1, \dots, v_N\}$ is the vertex set containing N DGs and $\mathcal{E}_p \subseteq \mathcal{V}_p \times \mathcal{V}_p$ is the edge set whose entry denotes the power line. $\mathcal{N}_i^p := \{v_j \in \mathcal{V}_p : (v_i, v_j) \in \mathcal{E}_p\}$ is already defined above, i.e., the set of neighbors of the i th DG. The adjacency matrix $\mathcal{A}_p$ is the conductance matrix of power lines, whose elements are given as $a_{ij}^p = 1/R_{ij}$ if $v_j \in \mathcal{N}_i^p$ and $a_{ij}^p = 0$ otherwise. The representation of the physical layer of the MG system is completed.

DGs share information with each other through communication networks, which constitute the cyber layer in the MG system. The communication topology between DGs is represented as a graph $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n, \mathcal{A}_n)$, where $\mathcal{V}_n = \{v_1, \dots, v_N\}$ stands for the set of DGs, $\mathcal{E}_n \subseteq \mathcal{V}_n \times \mathcal{V}_n$ is the edge set of network links. The set of neighbors of the i th vertex is denoted as $\mathcal{N}_i^n := \{v_j \in \mathcal{V}_n : (v_i, v_j) \in \mathcal{E}_n\}$. The elements of the adjacency matrix $\mathcal{A}_n$ are defined as $a_{ij}^n = 1$ if $v_j \in \mathcal{N}_i^n$, $a_{ij}^n = 0$ otherwise. The degree d_i of vertex i is the number of its neighbors, i.e., $d_i = \sum_{j \in \mathcal{N}_i^n} a_{ij}^n$.

The communication infrastructure is an essential and integral part of the DC MG, and its introduction brings many benefits to the MG, such as making it possible to operate and deploy in a distributed manner, giving it plug-and-play capability, and allowing it to be highly robust to single points of failure. On the other hand, the establishment of communication networks also poses some negative

impacts on MG systems. For example, communication constraints such as time delays, packet losses, and malicious corruption of the information layer could significantly deteriorate the control performance of the MG system, and in severe cases, it may even appear to make the grid unstable. In this way, these inevitable communication constraints cannot be ignored in the controller design. As a result, the communication constraint problem encountered in DC MGs is of passionate concern in this chapter.

Assumption 3.2.1 *The delay of the measurements/signals of neighboring DGs to the i th DG induced by the communication network has an upper bound f_i , and the number of consecutive packet dropouts on these channels has an upper bound s_i .*

Assumption 3.2.2 *Without loss of the generality, let $\tau_i = s_i + f_i + 1$. It is assumed that the units of s_i and f_i are the sampling period used for system discretization, which guarantees that τ_i is a positive integer.*

The abovementioned two assumptions are popular and typical in the study of networked control systems (Liu, 2020, 2017). For the random time delays encountered by MGs, caching techniques can be used in practice to ensure that the delay upper bound is reasonable.

3.2.3 Problem of Interest

Before elaborating on the problem to be addressed in this chapter, the following definition of stability and consensus for the DC MG systems is given first.

Definition 3.2.1 *The DC MG system comprising N DG units (3.2) with a controller $u_{[i]}$ is input–output stable and reaches the output consensus if and only if the following conditions hold:*

1. $\lim_{t \rightarrow \infty} \|y_{[i]}(t)\| < \infty$, if $\|v_{ref}\| < \infty$, for $t \geq 0$
2. $\lim_{t \rightarrow \infty} \|y_{[j]}(t) - y_{[i]}(t)\| = 0$, for $\forall i, j \in \mathcal{V}_p$

where v_{ref} is a constant reference input, as shown in Fig. 3.2.

The DC MG achieves voltage tracking consensus if

$$\lim_{k \rightarrow \infty} \|y_i(k) - v_{ref}\| = 0, \quad \forall i \in \mathcal{V}_p.$$

Equivalently, this requires both $y_1(k) \rightarrow v_{ref}$ and $y_i(k) - y_1(k) \rightarrow 0$ for all $i \neq 1$.

The problem of interest in this chapter can then be formulated as follows: the tracking problem of the output voltage of a DC MG subject to communication constraints, considering the control architecture shown in Fig. 3.2. It is worth mentioning that the proposed CPIP control method offers dual functionality. It is applicable not only to voltage restoration but also directly to accurate current sharing

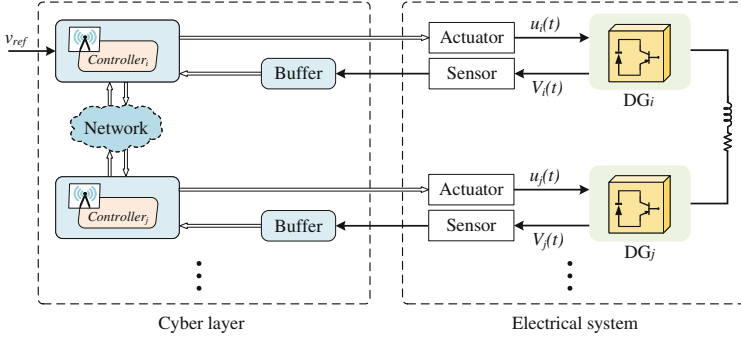


Fig. 3.2 System configuration of the DC MG including its controllers and communication network

in DC microgrids. Nonetheless, given that a series-connected DC microgrid is under consideration, employing this predictive control to achieve current sharing does not inherently ensure that the output voltage of every DG will converge to an identical reference value.

3.3 Predictive Control of DC MG Systems

In this section, this chapter considers the voltage restoration problem of the DC MG system under communication delays and packet dropouts. As a solution, a consensus-based proportional integral predictive control scheme with an observer is proposed and introduced in detail.

By means of direct discretization (Ren and Cao, 2010), the DC MG (3.2) in continuous time can be discretized as

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(k) + B_i u_i(k) + D_i \omega_i(k) \\ y_i(k) &= C_i x_i(k) \end{aligned} \quad (3.3)$$

where $A_{ii} = I + A_{ii}^c T_s$, $A_{ij} = A_{ij}^c T_s$, $B_i = B_i^c T_s$, $D_i = D_i^c T_s$, and T_s is the sampling period. For all DGs, it is assumed that the pair (A_{ii}, B_i) is controllable, and the pair (A_{ii}, C_i) is observable, which is reasonable in practice and widely adopted in related works. Moreover, it is assumed that each DG unit has precise knowledge of its dynamics, namely, it knows system matrices A_{ii} , A_{ij} , B_i , and C_i . Specifically, A_{ij} represents the local information describing the power tie line coupling between the i th DG and its neighboring DGs.

In the subsequent predictive controller design and stability analysis, the nominal case without external current disturbances is considered, i.e., $\omega_i(k) = 0$ for all $i \in \mathcal{V}_p$. Therefore, the disturbance term is omitted in the compact model used for closed-loop analysis.

Further, the DC MG system composed of all interconnected DGs can be given in the compact form as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \quad (3.4)$$

where

$$\begin{aligned} x(k) &= [x_1^T(k) \ x_2^T(k) \ \cdots \ x_N^T(k)]^T \\ u(k) &= [u_1(k) \ u_2(k) \ \cdots \ u_N(k)]^T \\ y(k) &= [y_1(k) \ y_2(k) \ \cdots \ y_N(k)]^T \end{aligned}$$

are the state, input, and output vectors of the overall system, $A \in R^{2N \times 2N}$ consists of A_{ii} on the diagonal and A_{ij} on the off-diagonal, $B \in R^{2N \times N}$ and $C \in R^{N \times 2N}$ are the block-diagonal with the elements B_i and C_i , respectively. For the entire DC MG system, assume that the tuples (A, B) and (A, C) are controllable and observable pairs, respectively.

It is worth mentioning that under the studied communication structure, it is assumed that only the first DG unit has direct access to the voltage reference values v_{ref} sent by the upper control (e.g., economic dispatch layer). Then, for the DC MG system with consideration of communication latency and packet loss, a common consensus-based distributed controller can be given as follows:

$$u_i(k) = k_i^{p,y} y_i(k - \tau_i) + k_i^{p,e} \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (y_j(k - \tau_i) - y_i(k - \tau_i)) + k_i^z z_i^T(k) \quad (3.5)$$

where $k_i^{p,y}$, $k_i^{p,e}$ and k_i^z are the control parameters to be designed, and the integration variables are defined as follows:

$$\begin{aligned} z_1^T(k+1) &= z_1^T(k) + y_1(k - \tau_1) - v_{ref} \\ z_i^T(k+1) &= z_i^T(k) + y_i(k - \tau_i) - y_1(k - \tau_1) \end{aligned} \quad (3.6)$$

for $i \in \mathcal{V}_p / \{1\}$. It is well known that the performance of the system will always be affected by the controller $u_i(k)$ given by (3.5) that does not have any treatment or intervention for latency. Even if the system can be stabilized, communication delays and packet dropouts can degrade the system performance to some extent.

Based on the preceding assumption, the system states are observable, even though they cannot be completely measured by sensors. Therefore, using this property, an estimate of the state vector of each DG can be obtained. In this way, based on the available information of the networked controller, including its own measurement information $y_i(k - \tau_i)$, neighbor's output voltage information $y_j(k - \tau_i)$ and other DGs' information coming from the network, the following observer can be designed for i th DG

$$\begin{aligned}
\hat{x}_l(k - \tau_i + 1|k - \tau_i) = & L_i A \hat{x}(k - \tau_i|k - \tau_i - 1) \\
& + L_i B u(k - \tau_i) + L_i L_{obs,i}(y(k - \tau_i) \\
& - C \hat{x}(k - \tau_i|k - \tau_i - 1))
\end{aligned} \quad (3.7)$$

for $l = 1, 2, \dots, N$, where $\hat{x}(k - \tau_i|k - \tau_i - 1) = [\hat{x}_1^T(k - \tau_i|k - \tau_i - 1), \hat{x}_2^T(k - \tau_i|k - \tau_i - 1), \dots, \hat{x}_N^T(k - \tau_i|k - \tau_i - 1)]^T$, $\hat{y}(k - \tau_i|k - \tau_i - 1) = [\hat{y}_1(k - \tau_i|k - \tau_i - 1), \hat{y}_2(k - \tau_i|k - \tau_i - 1), \dots, \hat{y}_N(k - \tau_i|k - \tau_i - 1)]^T$, $L_{obs,i}$ is the observer gain to be designed, and $L_i = [0_{2 \times 2(i-1)}, I_n, 0_{2 \times 2(N-i)}]$. To enhance the robustness of the predictive controller against measurement noise and errors in the DC MG, the observer specified in (3.7) could be substituted with a Kalman filter, as suggested by Gu et al. (2017). Furthermore, the existing observer not only calculates the observed state vector for each DG but also serves as the basis for a deliberate design process, ensuring that the DC MG meets specific performance indexes.

Standing on the shoulders of the observer (3.7) and using model prediction, the state vector prediction from $k - \tau_i + 2$ to k can be obtained for i th DG as follows:

$$\begin{aligned}
\hat{x}_i(k - \tau_i + m|k - \tau_i) = & L_i B u(k - \tau_i + m - 1) \\
& + L_i A \hat{x}(k - \tau_i + m - 1|k - \tau_i)
\end{aligned} \quad (3.8)$$

where $m = 2, 3, \dots, \tau_i$. Combining the observers (3.7) and (3.8), it can be obtained by repeated iteration

$$\begin{aligned}
\hat{x}_i(k|k - \tau_i) = & L_i A^{\tau_i-1} \hat{x}(k - \tau_i + 1|k - \tau_i) + L_i \sum_{l=2}^{\tau_i} A^{\tau_i-l} B u(k - \tau_i + l) \\
= & L_i A^{\tau_i-1} (A - L_{obs,i} C) \hat{x}(k - \tau_i|k - \tau_i - 1) \\
& + L_i \sum_{l=1}^{\tau_i} A^{\tau_i-l} B u(k - \tau_i + l) \\
& + L_i A^{\tau_i-1} L_{obs,i} C x(k - \tau_i).
\end{aligned} \quad (3.9)$$

Moreover, integrating the MG model (3.3) into it leads to

$$\begin{aligned}
\hat{y}_i(k|k - \tau_i) = & C_i L_i A^{\tau_i-1} L_{obs,i} C x(k - \tau_i) + C_i L_i \sum_{l=1}^{\tau_i} A^{\tau_i-l} B u(k - \tau_i + l) \\
& + C_i L_i A^{\tau_i-1} (A - L_{obs,i} C) \hat{x}(k - \tau_i|k - \tau_i - 1).
\end{aligned} \quad (3.10)$$

It can be seen from the time of $\hat{y}_i(k|k - \tau_i)$ that, together with the elaborate design of the observer and model prediction, the delay of the output signals received by each DG over the network can be well compensated.

Based on the $\hat{y}_i(k|k - \tau_i)$ obtained from (3.10), similar to the integral variable in controller (3.5), a novel integral variable $z_i(k)$ based on the predictor variable can be redefined as follows:

$$\begin{aligned} z_1(k+1) &= z_1(k) + \hat{y}_1(k|k - \tau_1) - v_{ref} \\ z_i(k+1) &= z_i(k) + \hat{y}_i(k|k - \tau_i) - \hat{y}_1(k|k - \tau_1) \end{aligned} \quad (3.11)$$

where $i \in \mathcal{V}_p / \{1\}$. Therefore, from (3.10) and (3.11), for the problem presented in Sect. 3.2, the following CPIP controller can be designed

$$\begin{aligned} u_i(k) &= k_i^{p,y} \hat{y}_i(k|k - \tau_i) + k_i^z z_i(k) \\ &\quad + k_i^{p,e} \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (\hat{y}_j(k|k - \tau_i) - \hat{y}_i(k|k - \tau_i)) \end{aligned} \quad (3.12)$$

for $i = 1, 2, \dots, N$. It is noted that the novel CPIP controller (3.12) contains three terms. The first and third terms are proportional control of the output voltage of i th DG and cooperation errors between outputs of DGs, while the second term is a purpose-designed integration term to eliminate static errors as well as the difference between the output voltage and the desired reference. The most important feature of the proposed CPIP is that all the output variables appearing in the controller (3.12) are variables obtained by forward τ_i step prediction based on time delay data. This feature, while analogous to the derivative action found in conventional PID control, is fundamentally distinct because the look-ahead variables presented in (3.12) are constructed from multi-step future predictions, unlike the single-step extrapolation typical of PID. Consequently, the new control scheme effectively preserves the inherent benefits of PI control, such as simplified tuning and robust fault tolerance.

The implementation of the proposed CPIP control scheme for interconnected DC MG can be basically divided into the following two steps.

- (i) Each DG_i , $i \in \mathcal{V}_p$, is tasked with packaging and transmitting its local output voltage and control input across the network to all neighboring DG_j , $j \in \mathcal{N}_i^n$. A highly effective technique for converting the problem of consecutive packet loss into an equivalent time delay issue is the adoption of a signal sequence transmission strategy. This strategy ensures that the transmitted data include historical values as the vectors $[y_i(k), y_i(k-1), \dots, y_i(k-f_i)]$, $[u_i(k), u_i(k-1), \dots, u_i(k-f_i)]$, alongside local system knowledge. Even though packet losses and time delays are fundamentally different failures, this approach enables predictive control to compensate for the former in the same manner as the latter.
- (ii) The controller of i th DG will use the available output voltage information $y_i(k - \tau_i)$ for $i = 1, \dots, N$ and dynamics knowledge of other DGs to obtain a prediction vector of system output voltages $\hat{y}_i(k|k - \tau_i)$ based on the model predictions (3.10). Then, the control input $u_i(k)$ for i th DG unit can be calculated according to the designed controller (3.12).

At this point, as long as the control gains $k_i^{p,y}$, $k_i^{p,e}$, and k_i^z in the proposed CPIP controller (3.12) are reasonably well determined, the problem of voltage tracking in the presence of communication constraints is solved.

3.4 Analysis of Stability and Consensus of DC MGs With CPIP Control

In this section, the stability and consensus analysis of the DC MG systems driven by the novel CPIP controller is given. Also, the reasonable determination of the control parameters utilized in the CPIP controller (3.12) under the supervision of voltage tracking, which may be of interest to engineers in applications, is given in detail.

Similar to the model prediction, a recursive iteration of the dynamics (3.3) of the i th DG gives

$$x_i(k) = L_i A^{\tau_i} x(k - \tau_i) + L_i \sum_{l=1}^{\tau_i} A^{\tau_i-l} B u(k - \tau_i + l). \quad (3.13)$$

Subtracting (3.13) from (3.9) eliminates $u_i(k)$ and leads to

$$\begin{aligned} \hat{x}_i(k|k - \tau_i) - x_i(k) = & L_i A^{\tau_i-1} (A - L_{obs,i} C) \hat{x}(k - \tau_i|k - \tau_i - 1) \\ & + L_i A^{\tau_i-1} L_{obs,i} C x(k - \tau_i) - L_i A^{\tau_i} x(k - \tau_i). \end{aligned} \quad (3.14)$$

The above equation can be reorganized to obtain

$$\begin{aligned} \hat{x}_i(k|k - \tau_i) = & x_i(k) + L_i A^{\tau_i-1} (A \\ & - L_{obs,i} C) (\hat{x}(k - \tau_i|k - \tau_i - 1) - x(k - \tau_i)). \end{aligned} \quad (3.15)$$

Define $\tilde{x}(k) = [\tilde{x}_1^T(k), \tilde{x}_2^T(k), \dots, \tilde{x}_N^T(k)]^T$, where $\tilde{x}_i(k) = x_i(k) - \hat{x}_i(k|k - 1)$. Then (3.15) can be rewritten as

$$\hat{x}_i(k|k - \tau_i) = x_i(k) - L_i A^{\tau_i-1} (A - L_{obs,i} C) \tilde{x}(k - \tau_i). \quad (3.16)$$

On the other hand, shifting the time in the observer (3.7) from k to $k + \tau_i$ yields

$$\hat{x}_i(k + 1|k) = L_i A \hat{x}(k|k - 1) + L_i B u(k) + L_i L_{obs,i} (y(k) - C \hat{x}(k|k - 1)). \quad (3.17)$$

Subtracting the state of i th DG (3.3) from the state of the above observer (3.17), it is not difficult to derive

$$\tilde{x}_i(k + 1) = L_i (A - L_{obs,i} C) \tilde{x}(k). \quad (3.18)$$

Substituting (3.18) into Eq. (3.15) results in

$$\hat{x}_i(k|k - \tau_i) = x_i(k) - L_i A^{\tau_i-1} \tilde{x}(k - \tau_i + 1). \quad (3.19)$$

Further, by substituting (3.19) in the controller (3.12), one has

$$\begin{aligned}
 u_i(k) = & k_i^{p,y} C_i x_i(k) - k_i^{p,y} C_i L_i A^{\tau_i-1} \tilde{x}(k - \tau_i + 1) \\
 & + k_i^z z_i(k) + k_i^{p,e} \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (C_j x_j(k) - C_i x_i(k)) \\
 & + k_i^{p,e} \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (C_i L_i A^{\tau_i-1} \tilde{x}(k - \tau_i + 1) \\
 & - C_j L_j A^{\tau_i-1} \tilde{x}(k - \tau_i + 1)).
 \end{aligned} \tag{3.20}$$

Define the increments of the state variables of i th DG as $\Delta x_i(k) = x_i(k) - x_i(k-1)$. From the system dynamics of i th DG _{i} and substituting the controller (3.20) into increments dynamics, it is easy to obtain

$$\begin{aligned}
 \Delta x_i(k+1) = & k_i^{p,e} B_i \sum_{j \in \mathcal{N}_i^n} a_{ij}^n C_j \Delta x_j(k) + (k_i^{p,y} - k_i^{p,e} d_i) B_i C_i \Delta x_i(k) \\
 & + k_i^z B_i \Delta z_i(k) + k_i^{p,e} B_i \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (C_i L_i A^{\tau_i-1} \Delta \tilde{x}(k - \tau_i + 1) \\
 & - C_j L_j A^{\tau_i-1} \Delta \tilde{x}(k - \tau_i + 1)) + L_i A \Delta x(k) \\
 & - k_i^{p,y} B_i C_i L_i A^{\tau_i-1} \Delta \tilde{x}(k - \tau_i + 1)
 \end{aligned} \tag{3.21}$$

where $\Delta x(k) = [\Delta x_1^T(k), \Delta x_2^T(k), \dots, \Delta x_N^T(k)]^T$. Similarly, for the increment of the integral variable $z_i(k)$, there exists

$$\begin{aligned}
 \Delta z_1(k+1) = & \Delta z_1(k) + C_1 \Delta x_1(k) - C_1 L_1 A^{\tau_1-1} \Delta \tilde{x}(k - \tau_1 + 1) \\
 \Delta z_i(k+1) = & \Delta z_i(k) + C_i \Delta x_i(k) - C_i L_i A^{\tau_i-1} \Delta \tilde{x}(k - \tau_i + 1) \\
 & + C_1 L_1 A^{\tau_1-1} \Delta \tilde{x}(k - \tau_1 + 1)
 \end{aligned} \tag{3.22}$$

for $i \in \mathcal{V}_p / \{1\}$, where $\Delta z_i(k) = z_i(k) - z_i(k-1)$.

Denote

$$\begin{aligned}
 \Delta z(k) = & [\Delta z_1^T(k), \Delta z_2^T(k), \dots, \Delta z_N^T(k)]^T \\
 \tilde{x}_\tau(k) = & [\tilde{x}^T(k - \tau_1 + 1), \tilde{x}^T(k - \tau_1), \tilde{x}^T(k - \tau_2 + 1), \\
 & \tilde{x}^T(k - \tau_2), \dots, \tilde{x}^T(k - \tau_N + 1), \tilde{x}^T(k - \tau_N)]^T.
 \end{aligned}$$

Then, the compact forms of (3.18), (3.21) and (3.22) for $i = 1, 2, \dots, N$, can be presented as follows:

$$\begin{aligned}
 \Delta x(k+1) = & (O_L + M + J) \Delta x(k) + H \Delta z(k) + (F + G + D) \tilde{x}_\tau(k) \\
 \Delta z(k+1) = & H_z \tilde{x}_\tau(k) + D_z \Delta x(k) + I_N \Delta z(k) \\
 \tilde{x}_\tau(k+1) = & \bar{L}_{obs} \tilde{x}_\tau(k)
 \end{aligned} \tag{3.23}$$

where $M = \{M_{ij}\}$, $M_{ij} = k_i^{p,e} a_{ij}^n B_i C_j$, $F = \{F_{ij}\}$, $F_{ij} = k_i^{p,e} a_{ij}^n B_i C_j L_j A^{\tau_j-1}$,

$$\begin{aligned}
D_z &= [D_{z1}, D_{z2}, \dots, D_{zN}], \\
D_{z1} &= [C_1^T, 0, \dots, 0]^T, \\
D_{zi} &= [-C_1^T, 0, \dots, C_i^T, \dots, 0]^T, \text{ for } i \neq 1, \\
O_L &= \text{diag}\{L_1 A, L_2 A, \dots, L_N A\}, \\
H_z &= \text{diag}\{H_{z1}, H_{z2}, \dots, H_{zN}\}, \\
H_{z1} &= C_1 L_1 A^{\tau_1-1}, H_{zi} = C_i L_i A^{\tau_i-1} - C_i L_i A^{\tau_i-1}, \\
H &= \text{diag}\{k_1^z B_1, k_2^z B_2, \dots, k_N^z B_N\}, \\
\bar{L}_{obs} &= \text{diag}\{\bar{L}_{obs,1}, \bar{L}_{obs,2}, \dots, \bar{L}_{obs,N}\}, \\
\bar{L}_{obs,i} &= \begin{bmatrix} A - L_{obs,i} C & 0 \\ I_{2N} & 0 \end{bmatrix}, \\
D &= \text{diag}\{D_{11}, D_{22}, \dots, D_{NN}\}, \\
F &= \text{diag}\{F_{11}, F_{22}, \dots, F_{NN}\}, \\
G &= \text{diag}\{G_{11}, G_{22}, \dots, G_{NN}\}, \\
D_{ii} &= [-k_i^p B_i C_i L_i A^{\tau_i-1}, k_i^p B_i C_i L_i A^{\tau_i-1}], \\
F_{ii} &= [-k_i^p B_i C_i L_i A^{\tau_i-1}, k_i^p B_i C_i L_i A^{\tau_i-1}], \\
G_{ii} &= [k_i^{p,e} d_i C_i L_i A^{\tau_i-1}, -k_i^{p,e} d_i C_i L_i A^{\tau_i-1}], \\
J &= \{J_{ij}\}, \text{ for } i \neq j, J_{ij} = 0, J_{ii} = (k_i^{p,y} - k_i^{p,e} d_i) B_i C_i,
\end{aligned}$$

I_N and I_{2N} are the N -dimensional and $2N$ -dimensional identity matrices, respectively. To obtain the equations of the closed-loop system with the proposed CPIP control strategy for the DC MG, recalling (3.23), one can derive

$$X_{\mathcal{J}}(k+1) = \Theta_{\tau} X_{\mathcal{J}}(k) \quad (3.24)$$

where $X_{\mathcal{J}}(k) = [\Delta x^T(k), \Delta z^T(k), \tilde{x}_{\tau}^T(k)]^T$, and

$$\Theta_{\tau} = \begin{bmatrix} O_L + M + J & H & F + G + D \\ D_z & I_N & H_z \\ O & O & \bar{L}_{obs} \end{bmatrix}. \quad (3.25)$$

Therefore, the necessary and sufficient conditions for achieving consensus and stability of the closed-loop DC MG system with CPIP control scheme are presented in the following.

Theorem 3.4.1 *The DC MG system (3.3), with the proposed CPIP control strategy, is asymptotically stable and able to accomplish output consensus if and only if all eigenvalues of matrices Ξ and $A - L_{obs,i}C$, for $i = 1, 2, \dots, N$, lie within the unit circle, where*

$$\Xi = \begin{bmatrix} O_L + M + J & H \\ D_z & I_N \end{bmatrix}.$$

Proof Based on the designed CPIP control protocol, the closed-loop equation of the DC MG system is given by (3.24). It is well known that the sufficient and necessary condition for the stability of a closed-loop system (3.24) is that the system matrix Θ_τ is Schur stable. Therefore, the fact that all eigenvalues of matrices Ξ and $A - L_{obs,i}C$, for $i = 1, 2, \dots, N$, are within the unit circle guarantees that the closed-loop DC MG system is asymptotically stable, which leads to $\lim_{k \rightarrow \infty} \Delta x(k) = 0$, $\lim_{k \rightarrow \infty} \Delta z(k) = 0$, and $\lim_{k \rightarrow \infty} \tilde{x}_\tau(k) = 0$. First, it follows from $\lim_{k \rightarrow \infty} \Delta x(k) = 0$ that $x(k)$ is stable, which implies that $y(k)$ is stable, since $\Delta y_i(k) = L_i C \Delta x(k)$. Second, the incremental form of the integral variable is easily obtained from (3.11) as follows:

$$\begin{aligned} \Delta z_1(k+1) &= \hat{y}_1(k|k - \tau_1) - v_{ref} \\ \Delta z_i(k+1) &= \hat{y}_i(k|k - \tau_i) - \hat{y}_1(k|k - \tau_1) \end{aligned} \quad (3.26)$$

for $i \in \mathcal{V}/\{1\}$. Therefore, based on $\lim_{k \rightarrow \infty} \Delta z(k) = 0$, it can be obtained that

$$\begin{aligned} \hat{y}_1(k|k - \tau_1) &\rightarrow v_{ref} \\ \hat{y}_i(k|k - \tau_i) &\rightarrow \hat{y}_1(k|k - \tau_1) \end{aligned} \quad (3.27)$$

as $k \rightarrow \infty$. Furthermore, from $\lim_{k \rightarrow \infty} \tilde{x}_\tau(k) = 0$, one has

$$x_i(k) \rightarrow \hat{x}_i(k|k - \tau_i) \quad (3.28)$$

as $k \rightarrow \infty$. Similarly, due to $y_i(k) = C_i x_i(k)$, $\hat{y}_i(k|k - \tau_i) = C_i \hat{x}_i(k|k - \tau_i)$, there exist

$$y_i(k) \rightarrow \hat{y}_i(k|k - \tau_i) \quad (3.29)$$

as $k \rightarrow \infty$. Combining (3.27) and (3.29), it is not difficult to infer that

$$\begin{aligned} \lim_{k \rightarrow \infty} \|y_1(k) - v_{ref}(k)\| &= 0 \\ \lim_{k \rightarrow \infty} \|y_i(k) - y_1(k)\| &= 0 \end{aligned} \quad (3.30)$$

$\forall i \in \mathcal{V}/\{1\}$, as $k \rightarrow \infty$. At this point, both conditions in Definition 3.2.1 are satisfied. This completes the proof of Theorem 3.4.1, and the formulated problem is solved. ■

Beyond the sufficient and necessary condition for stability and convergence derived from eigenvalues of the closed-loop system matrix, the subsequent stability criteria are now formally presented as an LMI.

Corollary 3.4.1 *For the DC MG system (3.3) operating under the secondary controller (3.12), the closed-loop augmented system achieves asymptotic stability and output consensus if there exists a positive definite matrix $P \in \mathbb{R}^{n_S \times n_S}$, where $n_S = \dim(X_S)$, such that*

$$\begin{bmatrix} -P & \Theta_\tau^T P \\ P \Theta_\tau & -P \end{bmatrix} < 0, \quad (3.31)$$

where Θ_τ is specified by (3.25).

Proof Let the candidate Lyapunov function for the closed-loop system (3.24) be $V(k) = X_{\mathcal{J}}^T(k) P X_{\mathcal{J}}(k)$. The difference of $V(k)$ along the trajectory defined by the dynamics (3.24) can be deduced as

$$\begin{aligned} V(k+1) - V(k) &= X_{\mathcal{J}}^T(k+1) P X_{\mathcal{J}}(k+1) - X_{\mathcal{J}}^T(k) P X_{\mathcal{J}}(k) \\ &= X_{\mathcal{J}}^T(k) (\Theta_\tau^T P \Theta_\tau - P) X_{\mathcal{J}}(k). \end{aligned} \quad (3.32)$$

According to Lyapunov's second method for stability, the closed-loop system (3.24) is deemed asymptotically stable provided that

$$\Theta_\tau^T P \Theta_\tau - P < 0. \quad (3.33)$$

Because P is a positive definite matrix, the resulting inequality (3.33) satisfies the criteria for applying the Schur complement (Zhou and Doyle, 1998). This establishes its equivalence to the LMI (3.31). The proof is complete. ■

To conclude, this section has detailed the stability and consensus analysis for the DC MG system operating under the innovative CPIP controller. Furthermore, this section has thoroughly presented the systematic process for selecting the control parameters used in the CPIP controller (3.12) to achieve voltage tracking, offering valuable and practical insights for implementing engineers.

3.5 Experiment Verification

In order to demonstrate the performance of the proposed CPIP control scheme and to experimentally validate the theoretical results as well as the superiority over existing methods, a modified four-bus DC microgrid prototype with source voltage of about 100 V is developed in the laboratory. In addition, extensive case studies are carried out on this platform, and detailed analysis will be provided subsequently.

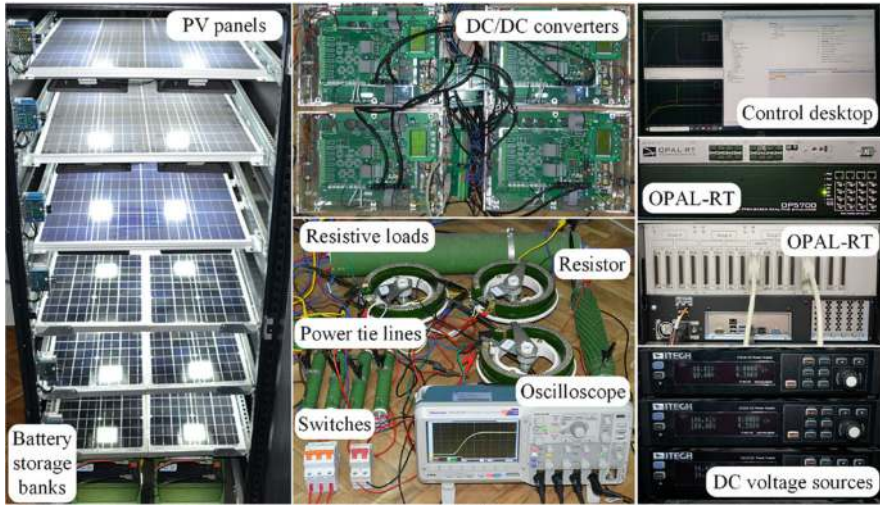


Fig. 3.3 Detailed view of the PV-connected DC MG experimental testbed

3.5.1 Hardware Setup Details

The feasibility and effectiveness of the designed CPIP approach will be verified by carrying out a popular output voltage regulation problem on a PV-connected DC microgrid experimental platform. Fig. 3.3 shows the experimental prototype of the DC MG system, containing four distributed generation units, namely three DC voltage sources and one set of PV panels with battery banks, their respective DC-DC converters with filters and local resistive loads, resistive-inductive lines connecting DGs, and a monitoring platform. The vertex set defined in Sect. 3.2 can be easily identified as $\mathcal{V}_p = \{1, 2, 3, 4\}$. In this regulation problem, the output voltage of each generation unit in the tested DC MG needs to track a reference signal v_{ref} by means of the designed control method, to adapt to different power supply scenarios. The distributed voltage regulation control algorithm is embedded in Simulink on a programming/monitoring computer, which can be used for controller design, programming, debugging, monitoring, etc., and is linked to a real-time controller OPAL-RT (OP4510) with a minimum sampling time of 10 μ s. The hardware converter module is equipped with the control board, the driver card, measurement modules, and the power stage, which includes the input power supply, the DC-DC converter, and the LC filter. Every single phase is constructed by a current-sense and temperature-sense IGBT-type inverter PM50RL1A120T01CA3G, where the nominal operating parameters are 1200 V and 50 A.

To be specific, the detailed schematic of the tested DC MG as well as the local controllers of DGs within it is illustrated in Fig. 3.4, which also shows the circuit configuration $\mathcal{G}_p$ of interconnected DGs and the corresponding communication topology $\mathcal{G}_n$ in the cyber layer. The communication topology is chosen as time-

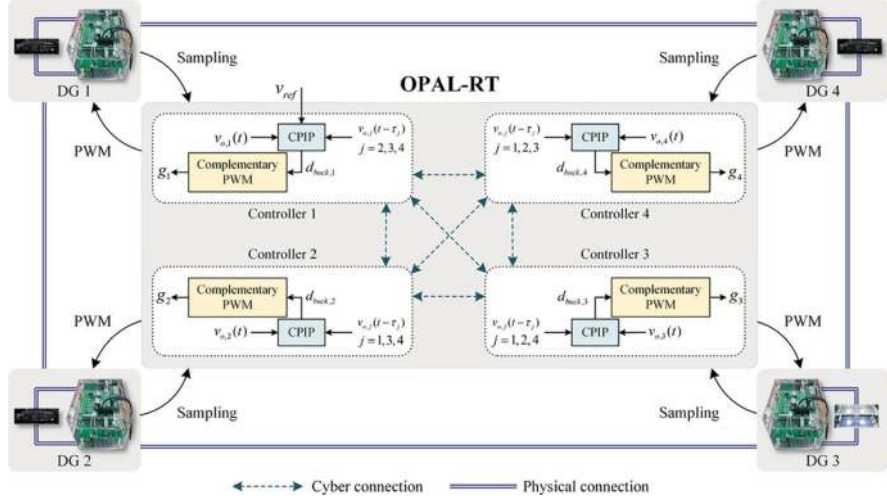


Fig. 3.4 Detailed schematic of the experimental test bench, featuring physical converter hardware controlled by local controllers implemented within the OPAL-RT

invariant and fully connected (each generator communicates with each other via, for example, ZigBee). It is worth mentioning that the OPAL-RT in the experiment acts as a hardware controller that receives the measured voltage signal fed from the hardware converter circuit to generate the pulse width modulation (PWM) signal (Vuyyuru et al., 2019), instead of a controlled target only executing a Simulink model simulating the tested system as in Hu et al. (2021b). Besides, although all the distributed controllers for each DG are implemented by one OPAL-RT to simplify hardware prototyping and save development costs, these controllers all work separately and independently, and still maintain the distributed characteristics of DC MGs.

As can be seen in Fig. 3.4, the third DG is supplied by the PV panels with battery banks, while others are supplied by DC voltage sources, representing the penetration of distributed renewable energy sources into MGs. As a necessary step for applying the proposed method, the uncertainty and intermittency inherent to renewable energy sources, as previously referenced in Sect. 3.2.1, must be addressed. To this end, the PV panels serving as the third DG have been augmented with batteries (refer to Figs. 3.3 and 3.4), thereby implementing the energy storage system solution introduced earlier. Only the output voltage $v_{o,i}$ of the i th DG is measured, where $i \in \mathcal{V}_p$, and these voltage signals are scaled down by a factor of 10 due to the limitation of the maximum voltage that can be accepted by ADC channels of the OPAL-RT. The PWM signal is generated by the built-in module of OPAL-RT, which will be connected to the converter through the output port to drive the corresponding IGBT. Moreover, the duty cycle for each DG calculated independently in OPAL-RT is recorded. It is worth mentioning that the output voltages will also be measured

Table 3.1 Parameters for the DC microgrid

Parameters	Experiment
Switching frequency	10 kHz
DC source ($U_{0,i}$)	$U_{0,1} = 100 \text{ V}, U_{0,2} = 90 \text{ V}$ $U_{0,3} = 80 \text{ V}, U_{0,4} = 80 \text{ V}$
Inductor ($L_{fi}, i \in \mathcal{V}_p$)	$\approx 2 \text{ mH}$
Capacitance ($C_{fi}, i \in \mathcal{V}_p$)	$1500 \text{ }\mu\text{F}$
Load resistance ($R_{Li}, i \in \mathcal{V}_p$)	$10 \text{ }\Omega$
Alternate load for load change	$1 \text{ }\Omega$
Desired reference (v_{ref})	48 V
Tie line resistance (R_{ij})	$R_{12} = 0.5 \text{ }\Omega, R_{23} = 0.5 \text{ }\Omega$ $R_{34} = 0.3 \text{ }\Omega, R_{41} = 0.4 \text{ }\Omega$
Tie line inductance ($L_{ij}, i, j \in \mathcal{V}_p$)	$\approx 2 \text{ }\mu\text{H}$
Sampling time	1 ms
Control period	10 ms
Time delays	$\tau_{ij} = 5 \text{ steps for } a_{ij}^n \neq 0 \text{ (i.e., } 50 \text{ ms)}$

by the Tektronix voltage probe, and this measurement will then be sent to an oscilloscope (Tektronix MSO 2024B) for display.

In the experiment, the switching frequency for the IGBT is 10 kHz, which is a common choice (Tan et al., 2020) and is satisfied to obtain the accurate average model of converter systems (Lucas et al., 2021). The specifications for the hardware devices and the settings for other experimental parameters are documented in Table 3.1. It is crucial to note that selecting these specific circuit parameters is what ensures the controllability and observability of the DC microgrid system.

At the controller level, for each distributed generation unit i , the observer gain is calculated by assigning the two poles of the observers to $\{0.6 \ 0.8\}$ to provide acceptable and transient convergence for the state estimation. As a result, the obtained observer gain matrices are given as follows:

$$L_1 L_{obs,1} = \begin{bmatrix} -0.87 \\ 1.19 \end{bmatrix}, L_2 L_{obs,2} = \begin{bmatrix} -0.96 \\ 4.48 \end{bmatrix},$$

$$L_3 L_{obs,3} = \begin{bmatrix} -0.99 \\ 5.62 \end{bmatrix}, L_4 L_{obs,4} = \begin{bmatrix} -1.03 \\ 7.52 \end{bmatrix}.$$

The proportional and integral gains in the controller (3.12) are designed using a trial-and-error approach to give four uncoordinated generators with completely different dynamic responses. In this way, for the built interconnected DC MG system, the control parameters of the proposed CPIP approach are designed as

$$\begin{aligned} k_1^{p,y} &= 0.5, k_2^{p,y} = 0.7, k_3^{p,y} = 0.6, k_4^{p,y} = 0.55, \\ k_1^{p,e} &= 0.12, k_2^{p,e} = 0.14, k_3^{p,e} = 0.1, k_4^{p,e} = 0.13, \\ k_1^z &= 2.5, k_2^z = 2.3, k_3^z = 2.2, k_4^z = 2.5. \end{aligned} \quad (3.34)$$

It can be verified that all the conditions of Theorem 3.4.1 can be satisfied under this set of parameters. Therefore, the stability and output consensus of the tested interconnected DC MG system can be theoretically guaranteed.

To illustrate the sharpness and effectiveness of the CPIP control strategy, the following four groups of tests are conducted:

- performance comparison with the existing method,
- robustness against load changes,
- plug-and-play property of the DC MG,
- assessment of the operational performance for a large-scale DC MG.

3.5.2 Performance Comparison and Evaluation

Case I: Existing approach (Lai et al., 2016) for voltage regulation under non-ideal communication network

The relevant existing results on distributed voltage recovery control against communication delays can be found in Lou et al. (2020), Lai et al. (2016). The research detailed in Lou et al. (2020) goes beyond merely qualitatively analyzing the impact of the delay margin on system performance by also achieving the optimal design of the directed communication topology and pinning locations. To provide a fair assessment, we therefore conduct a direct comparison between the newly proposed CPIP approach and the existing method from Lai et al. (2016), utilizing the communication delays and packet dropouts specified in Table 3.1.

In this case, the distributed controller designed in Lai et al. (2016) is applied to the experimental platform first, with the following specific form:

$$u_i(k) = k_i^{p,y} y_i(k) + k_i^{p,e} \sum_{j \in \mathcal{N}_i^n} a_{ij}^n (y_j(k) - y_i(k)) + k_i^z z_i(k) \quad (3.35)$$

for $i \in \mathcal{V}_p$, where $z_1(k+1) = z_1(k) + y_1(k) - v_{ref}$, $z_i(k+1) = z_i(k) + y_i(k) - y_1(k)$, for $i = 2, 3, 4$. The corresponding control parameters are given in (3.34) and $v_{ref} = 48$ V. The communication topology of the test DC MG system is shown in Fig. 3.4. Derived directly from the control period T_s and Assumption 2, the DC MG test system is configured to effectively operate with a time delay of 50 ms. Under the same circuit and control parameters and communication topology, the maximum delay tolerated by the controller (3.35) is calculated to be $\tau_{max} = 33.564$ ms, using Theorem 3.4.1 from Lai et al. (2016). This demonstrates that the delay condition of 50 ms is a more stringent and challenging test scenario, as it clearly exceeds the maximum tolerable latency defined by Lai et al. (2016).

The responses of the output voltage V_i and the recorded control input signal u_i for each DG are displayed in Figs. 3.5 and 3.6, respectively. As can be seen from Fig. 3.5, the output voltage of all DGs will continue to oscillate with increasingly large amplitudes, reflecting the instability of the DC MG all the time. Obviously,

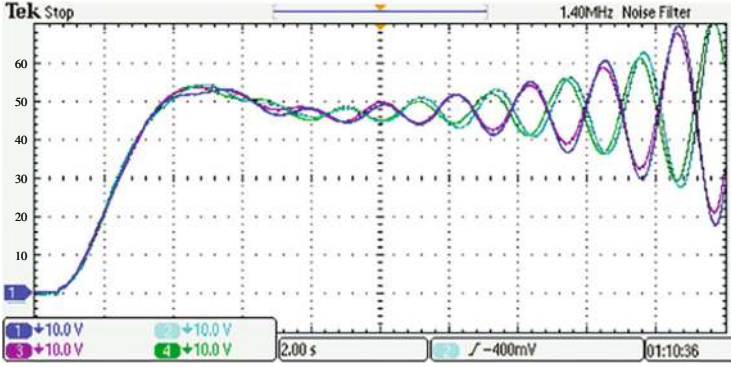


Fig. 3.5 Output voltage waveforms of all DGs under the control strategy (3.35) with communication delay

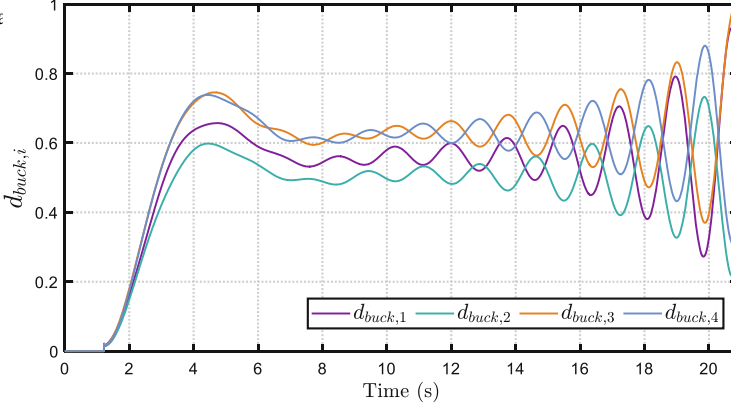


Fig. 3.6 Control input signals for each DG under control strategy (3.35) with communication delays

controller (3.35) would not be able to maintain the stability of the interconnected DC MG system with large time delays (50 ms), thus failing to achieve the tracking objective and output consensus. It can be seen that the distributed controller developed in Lai et al. (2016), as a representative control method for passively tolerating communication delays, would have poor control performance at a relatively large delay, due to the lack of active intervention and compensation of delays.

Case II: Proposed CPIP approach for voltage regulation under non-ideal communication network

To verify the superiority of the proposed active compensation method over the passive tolerance mechanism in dealing with communication delays and completing voltage regulation, comparative experiments are carried out. In this case, the proposed CPIP approach is tested on the physical experimental platform with the same scenario as Case I, and the corresponding results are depicted in Figs. 3.7 and 3.8. As seen, all of the output voltage curves of four generation units displayed

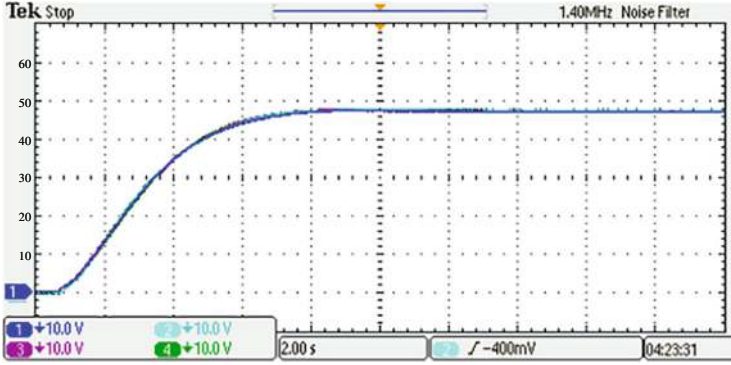


Fig. 3.7 Output voltage waveforms of all DGs under the CPIP approach with communication delays

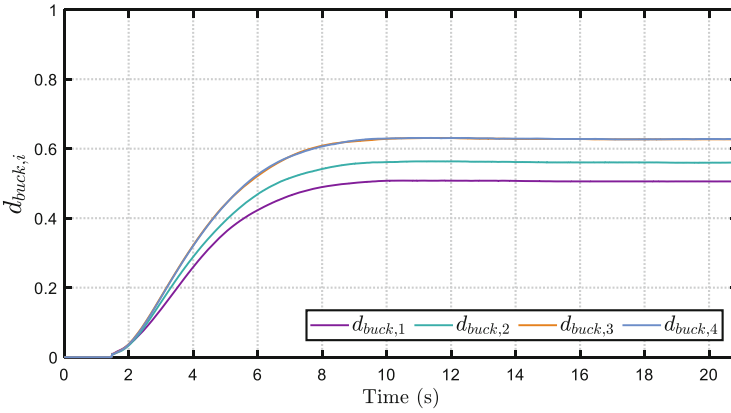


Fig. 3.8 Control input signals for each DG under the CPIP approach with communication delays

in Fig. 3.7 rapidly converge to the set point (48 V), and their control input signals can also stabilize quickly. Even for the dynamic process, the output voltages of the 4 DGs are close, that is, there is a small collaborative error among terminal voltages of the DC MG.

A clear improvement in control performance is evident when contrasting these findings with Figs. 3.5 and 3.6. In sharp contrast to Case I, the DC MG equipped with the CPIP controller achieves exceptionally fast stabilization and ensures the bus voltages track the reference signal v_{ref} quickly and accurately. This outcome effectively highlights the capacity of the proposed controller to mitigate challenging communication delays and packet dropouts.

To further validate the controller, an additional experiment investigates voltage recovery under a fully distributed communication topology. That is, the ring topology defined by $\mathcal{A}_n = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$. This architecture necessitates that a DG wait for information from all nodes before

updating its controller. More precisely, a DG at time k generates its control signal only after compensating for delayed messages (e.g., $y_i(k - \tau)$) received from all other DGs and with the common timestamp (e.g., $k - \tau$). Given the 50 ms communication delay between every pair of DGs as per Table 3.1, each DG must endure a 100 ms (10-step) latency to acquire global information in this distributed scheme. Notwithstanding this communication network, the experimental outcomes in Figs. 3.9 and 3.10 confirm that the CPIP method remains effective in achieving voltage recovery. This increased delay imposed by the technical approach mandates a much higher performance requirement for the prediction algorithm, simultaneously confirming that the fully connected network assumption introduced in Sect. 3.3 is conservative.

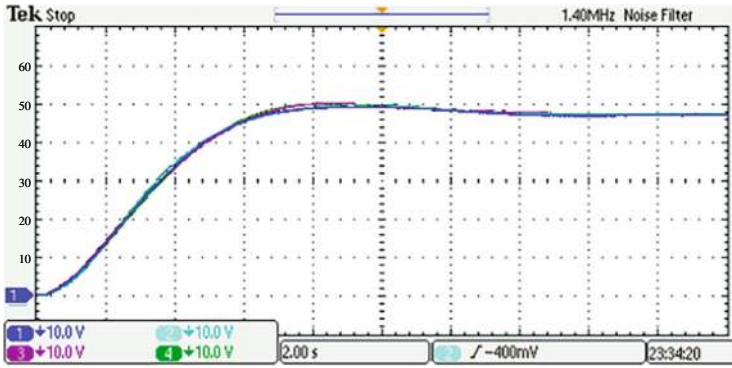


Fig. 3.9 Output voltage waveforms of all DGs under the CPIP approach with communication delays in the ring topology

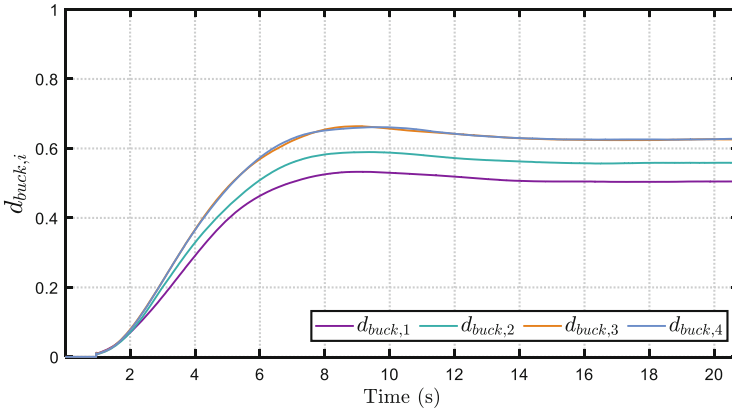


Fig. 3.10 Control input signals for each DG under the CPIP approach with communication delays in the ring topology

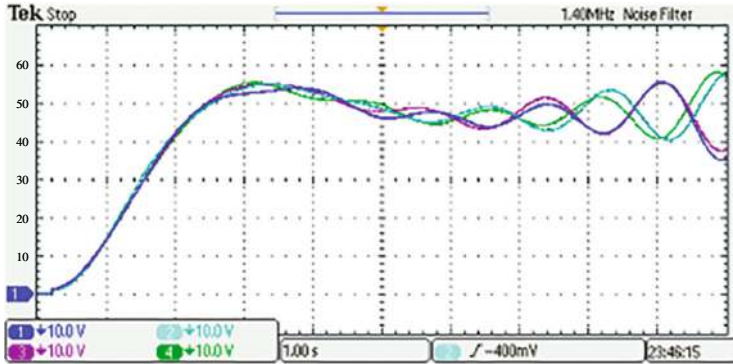


Fig. 3.11 Output voltage waveforms of all DGs under the CPIP approach with a large time delay

In conclusion, while control techniques that rely on passive delay tolerance struggle to manage large latencies and ultimately fail to stabilize the system, the proposed distributed voltage recovery control method offers a demonstrably effective solution. This approach proves its distinct superiority and efficacy in resolving the voltage regulation challenge for DC MGs operating over a non-ideal communication network.

Case III: Practical assessment for communication delay and packet loss

Theoretically, complete compensation for communication delays and packet losses is achievable, provided that the maximum limits for both are established, the system model used is highly accurate, and both measurement and process noises are small enough to be disregarded. Nevertheless, in a real-world scenario, the DC MG employing the suggested predictive control technique cannot withstand overly long time delays. To investigate this, a new experiment is conducted where all parameters remained constant, except for setting the maximum communication delay to $f_i = 7$ and the maximum consecutive packet losses to $s_i = 8$, totaling approximately 150 ms. These constraints are confirmed to satisfy the requirements of Theorem 3.4.1. As displayed in Fig. 3.11, the results indicate that under these specific communication constraints, the DC MG using the proposed controller struggles to effectively recover the voltage. This outcome arises because gaps persist between the actual hardware and the theoretical mathematical model, particularly due to parameter deviations. Such errors become magnified during the iterative process of multiple rolling predictions, which prevents the control method from fully compensating for the long time delay, ultimately causing the system to diverge.

Case IV: Impact of control gain settings

The three critical parameters governing the proposed predictive controller, $k_i^{p,y}$, k_i^e , and k_i^z , are responsible for the dynamic response speed, cooperative operation, and tracking accuracy of the system, respectively. As established in (3.12), these three parameters collectively define the overall system performance. Consequently, enhancing one specific parameter inherently diminishes the weighting or importance

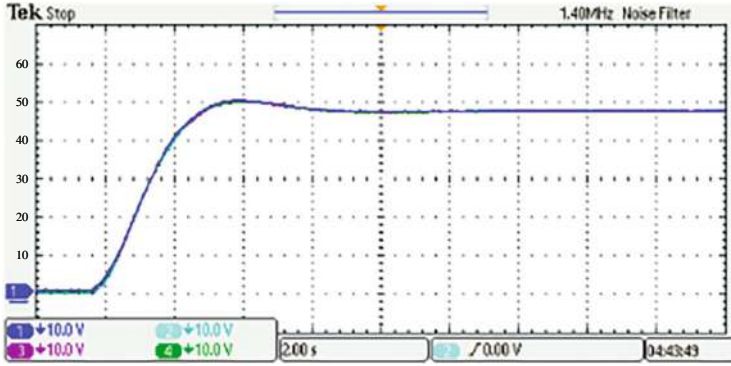


Fig. 3.12 Output voltage waveforms of all DGs under the CPIP approach with the increased control gain

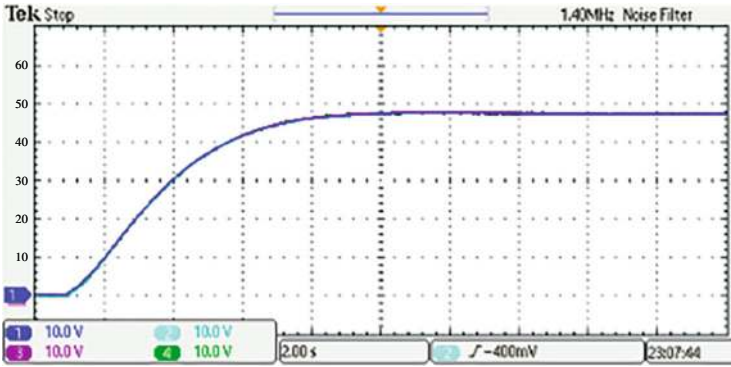


Fig. 3.13 Output voltage waveforms of all DGs under the CPIP approach with the decreased control gain

assigned to the other control variables. To analyze the relationship between these control settings and the system performance, voltage recovery under variations of the integral control gain is specifically examined. Two new experimental sets are run using settings $k_i^z = 4$ and $k_i^z = 1.5$ for $i = 1, 2, 3, 4$, respectively. Furthermore, every other parameter used during these trials is kept identical to those utilized in preceding cases. The results, illustrated in Fig. 3.12, indicate that raising the integral gain speeds up the voltage restoration process, although this action introduces a certain level of overshoot to the microgrid. Conversely, the experimental results presented in Fig. 3.13 clearly show that reducing the integral gain substantially slows the pace at which the voltage is recovered.

3.5.3 Robustness Evaluation Against Load Changes

The subsequent analysis focuses on the control performance of the proposed CPIP approach when the system is subjected to load changes. The test scenario involves locally increasing the load connected to DG₂ by 10% at time $t = t_1$, followed by a return to its initial value at $t = t_2$. This load variation is executed by manipulating a switch that inserts an auxiliary resistor in series with the local load of DG₂. The resulting output voltages of the DC MG test system and control inputs of DGs within it are presented in Figs. 3.14 and 3.15. Notably, with the implemented CPIP scheme, both the output voltages and the control inputs rapidly converge to their steady-state setpoints following each load perturbation.

Figure 3.14 offers two zoomed-in screenshots illustrating the specific evolution of the four DG output voltages at the moments of the load transitions. Specifically, when the local load of DG₂ is abruptly increased from $10\ \Omega$ to $11\ \Omega$ at t_1 , the duty cycle $d_{buck,2}$ immediately rises. This increase reflects the transient adjustment of the local converter to the load-resistance change and helps restore the output voltage $v_{o,2}$ to its reference value. Moments later, the influence propagates, causing the output voltages of the other DGs to also increase, until the system ultimately returns to its initial steady state, as seen in Fig. 3.14a. Figure 3.14b confirms a symmetric response for the load reduction from $11\ \Omega$ to $10\ \Omega$ at t_2 . The experiments collectively demonstrate that the DC MG retains stability and successfully executes voltage regulation with the coordinated distributed predictive control scheme,

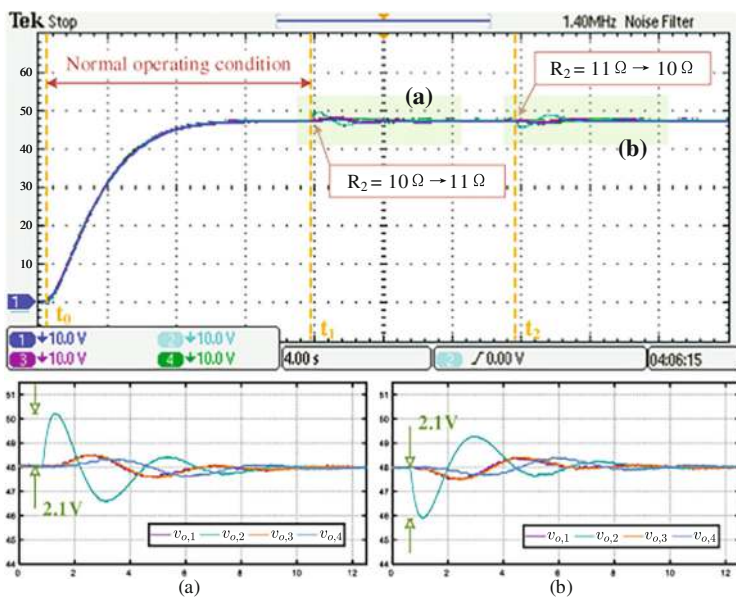


Fig. 3.14 Output voltage waveforms during the load variation test

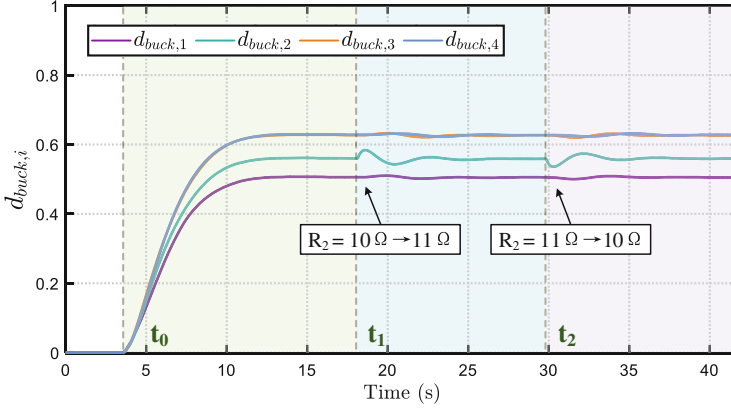


Fig. 3.15 Control input signals during the load variation test

irrespective of whether the load increases or decreases. Ultimately, the CPIP method proposed herein achieves accurate voltage regulation in DC MGs while exhibiting robustness to load changes, even when communication delays are present.

3.5.4 Plug-and-Play Physical Capability

To evaluate the robustness of the system using the proposed CPIP approach against the removal of faulty units and the re-integration of renewable energy sources, the experimental platform is subjected to the following scenario: DG₃ is simulated to fail and disconnect from the grid at $t = t_1$, and subsequently rejoins the DC MG at $t = t_3$. Throughout the isolation period $t \in [t_1, t_3)$, the physical power links *line*_{2,3} and *line*_{3,4} are treated as open circuits. Concurrently, the coordinated distributed predictive control scheme is deactivated for DG₃ during $t \in [t_1, t_2)$. Once DG₃ has recovered from its fault between t_1 and t_2 , local control is first engaged during $t \in [t_2, t_3)$ to elevate its output voltage to a level approximating the operational voltage of the DC MG before final connection. Once the output voltage reaches the desired value, DG₃ will be plugged in to the DC MG, and the proposed CPIP control will take over it. Similar to the case of load variations, the operating state of DG₃ is also manipulated by a switch. It should be noted that, DG₃, which is powered by PV panels, is chosen as the target to perform the plug-and-play capability investigation to better emulate the scenario where renewable energy sources are connected to the DC MG.

The control performance and signal evolutions in this case are presented in Figs. 3.16 and 3.17. As shown, there is a little divergence in the voltage output and control input when the physical structure of the DC MG is changed. In Fig. 3.16, after islanding a part of distributed generation units from the DC MG, it can be

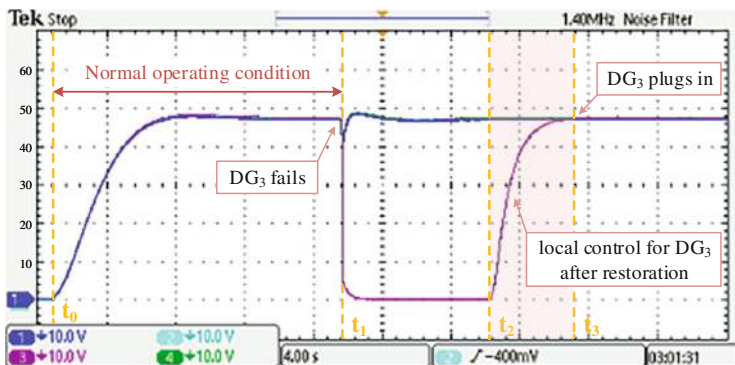


Fig. 3.16 Output voltage waveforms during the DG plug-and-play sequence

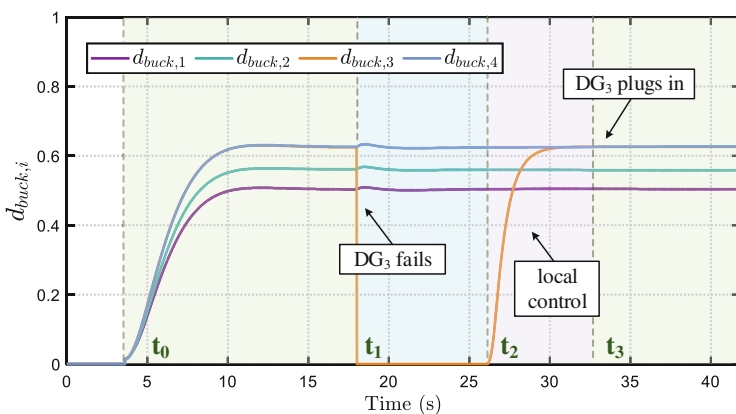


Fig. 3.17 Control input signals during the DG plug-and-play sequence

seen that the excellent voltage regulation can still be achieved for the rest system containing DG_1 , DG_2 and DG_4 , even though it may not perceive the information of DG_3 being plugged out in a very timely manner due to the communication delays. That is, the proposed CPIP method is also effective for the voltage regulation of the remaining DGs in the DC MG, albeit with communication delays. Furthermore, when DG_3 is plugged back at t_3 , the original steady-state operating mode is formed after an extremely short transient evolution with almost no fluctuations.

During the whole switching process between disconnection and reconnection of DG_3 , the experimental results indicate that the designed CPIP method enables the DC MG with plug-and-play capability and make it easier for generation units to be integrated into or out of the DC MG.

The three preceding case studies collectively confirm that the proposed CPIP controller effectively executes voltage recovery within the DC microgrid, thereby validating Theorem 3.4.1 as detailed in Sect. 3.4.

3.5.5 Performance Assessment for a Large-Scale DC MG

To rigorously evaluate the efficacy of the proposed control method under conditions of expanded scale, an experiment involving eight DGs is designed. Since the laboratory's physical hardware test bench is limited to four converters, the 8-DG DC MG experiment is implemented entirely within the OPAL-RT real-time simulator. Specifically, the controllers and physical power components are constructed using the component library of the simulator and then compiled and executed in real time using the built-in protocol. Figure 3.18 illustrates the comprehensive simulation architecture.

The test system is modeled as a ring bus DC MG interconnected by tie lines. System parameters for this large-scale MG are defined as follows. All DG units utilize a 1500 μF capacitance and an inductor of approximately 2 mH, with a load resistance of 10 Ω at each node. Source voltages exhibit heterogeneity, including $U_{0,5} = 95 \text{ V}$, $U_{0,6} = 85 \text{ V}$, $U_{0,7} = 85 \text{ V}$, $U_{0,8} = 75 \text{ V}$. Line connections feature a small inductance of approximately 2 μH , and tie line resistances vary, such as $R_{12} = 0.5 \Omega$, $R_{23} = 0.5 \Omega$, $R_{34} = 0.3 \Omega$, $R_{45} = 0.4 \Omega$, $R_{56} = 0.5 \Omega$, $R_{67} = 0.5 \Omega$, $R_{78} = 0.3 \Omega$, and $R_{81} = 0.4 \Omega$.

To further demonstrate the feasibility of the proposed method under a non-fully connected communication network, the adjacency matrix for the information layer is set as the following matrix: $\mathcal{A}_n = [0, 1, 1, 1, 1, 1, 0, 1; 1, 0, 1, 1, 0, 1, 0, 0; 1, 1, 0, 1, 1, 1, 1, 0; 0, 0, 0, 1, 1, 1, 1, 1; 1, 1, 1, 0, 0, 0, 1, 1; 1, 0, 1, 0, 1, 0, 1, 1; 0, 1, 1, 0, 0, 1, 0, 1; 1, 0, 1, 0, 1, 1, 1, 0]$. This communication topology necessitates an increased waiting time for each DG to satisfy the prerequisites of the predictive control method. Specifically, the DGs must tolerate a total delay equivalent to twice the original communication latency (i.e., 100 ms or 10 steps) to ensure that output information propagates throughout the MG. In this way, this approach guarantees a degree of distributed operation while simultaneously satisfying the application requirements of the control scheme. As evidenced by the results in Figs. 3.19 and 3.20, the method retains its capability to compensate effectively for communication constraints and achieve rapid voltage restoration in this larger-scale MG configuration.

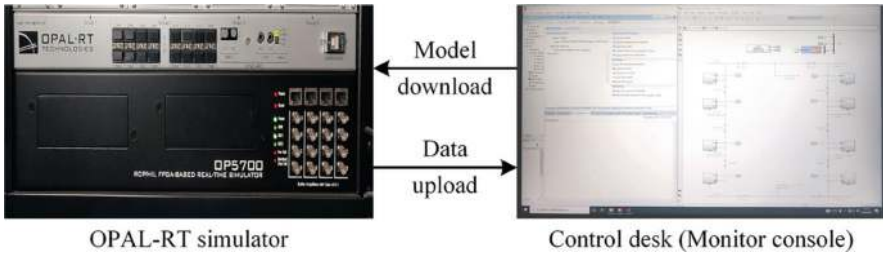


Fig. 3.18 Architecture of the real-time simulation of the 8-DG large-scale microgrid

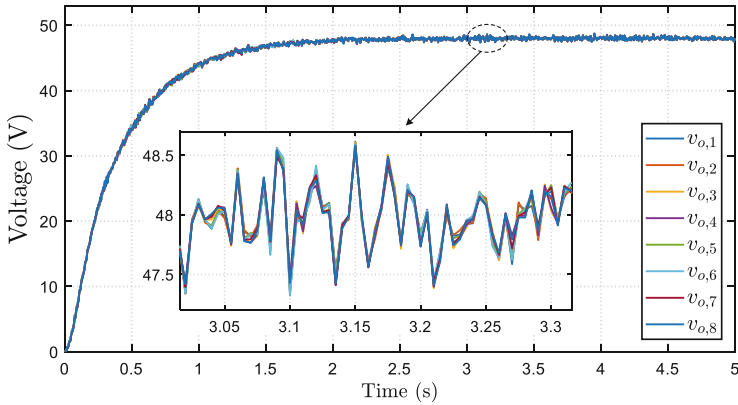


Fig. 3.19 Output voltage waveforms of all eight DGs in the large-scale DC MG

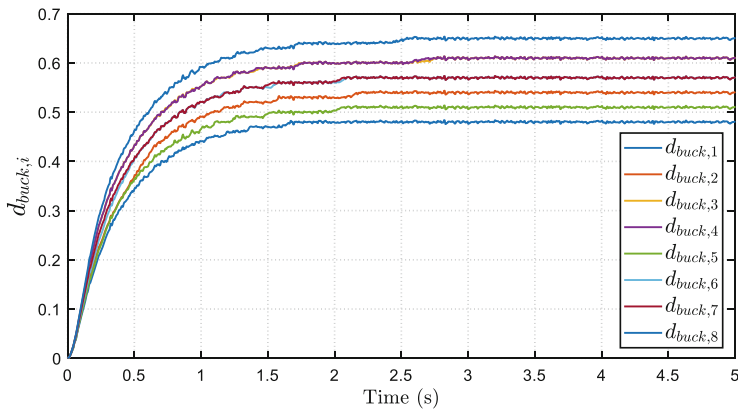


Fig. 3.20 Control input signals for all eight DGs in the large-scale DC MG

3.6 Notes and References

The core focus of this chapter is the voltage regulation challenge of the DC MG when operating under significant communication latency and continuous data packet losses. To enable precise mitigation of these network constraints, a coupled, discrete QSL model is formulated based on the electrical dynamics of the components in the microgrid, serving as the foundation for the subsequent analysis. Utilizing this model, a novel observer-based CPIP control strategy is introduced. Unlike passive control approaches that merely accommodate delays, the proposed CPIP method actively compensates for the intermittent communication delays and packet losses encountered by DC MGs. This distributed approach allows the subsystems in the DC MG to achieve output voltage recovery and consensus despite network impairments, by leveraging local DG measurements and

information received from neighboring controllers. Furthermore, the development includes providing sufficient and necessary conditions for closed-loop stability and consensus, thereby supporting the applicability of the developed method under the stated modeling and communication assumptions. The efficacy and advantage of the strategy are confirmed through extensive experimental cases executed on a DC MG hardware system featuring PV panels with battery banks. Subsequent research efforts will focus on online identification of interconnected MG systems under time delays, as well as designing voltage regulation and power sharing schemes driven by a hybrid of data and models.

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Chapter 4

Learning-Based Control of AC Renewable Power Systems



Abstract This chapter addresses distributed secondary control of islanded AC renewable power systems. An online-learning-based adaptive secondary controller integrated with a linear extended state observer (LESO) is developed to improve voltage regulation and reactive power allocation under unknown/uncertain dynamics and external disturbances. Two radial-basis-function neural networks are used to approximate unknown voltage- and reactive-power-related dynamics, while the LESO estimates lumped disturbances for feedforward compensation. A key innovation is the introduction of a vector of switching factors, which successfully unifies the objectives of critical bus voltage recovery and accurate reactive power-sharing into a singular, cohesive control framework. The proposed control approach is implemented in a distributed fashion, making it compatible with plug-and-play operation in principle, although explicit DG connection/disconnection tests are left for future validation. The effectiveness and robustness of the controller are validated through simulation studies covering two main scenarios: control-objective evaluation under reactive-power-sharing and voltage-restoration priorities, and robustness assessment under sudden and heterogeneous load fluctuations.

4.1 Overview

Despite considerable success in developing control strategies for voltage and reactive power regulation in AC microgrids, existing solutions remain constrained by inherent limitations that impede their robust deployment within real-world smart grids (Zhang et al., 2020). First, current secondary voltage controllers, often based on multi-agent consensus algorithms (Pilloni et al., 2018), critically rely on the availability of accurate microgrid model parameters. This leads to the primary restriction: given the frequent parameter variations due to factors like component aging and line impedance changes, most prior methodologies are ill-equipped to robustly manage these UD. Second, the operational safety and quality of the MG are severely challenged by random disturbances and large load variations that accompany drastic shifts in energy demand. These external factors induce significant, potentially system-destabilizing transient overshoots, a critical issue

that has yet to be comprehensively addressed in available literature (Wang et al., 2020). Third, a fundamental, inherent contradiction exists in low-voltage, lossless MGs: the inability to simultaneously achieve both precise voltage regulation and accurate reactive power-sharing (Dehkordi et al., 2019). This structural conflict means that existing works have rarely proposed algorithms capable of unifying these two crucial, yet conflicting, control objectives. These three limitations, the lack of robustness to UD, the vulnerability to transient disturbances, and the contradiction between control goals, form the core motivation for the research presented here.

To decisively overcome these shortcomings, this chapter introduces an innovative control architecture: an online learning-based adaptive secondary controller integrated with a linear extended state observer (LESO). The key features are described below. Handling UD: By utilizing the robust features of supervised learning, two meticulously designed neural networks (NNs) are incorporated. The NNs enable strong online adaptive capabilities that actively and continuously compensate for the UD, ensuring effective voltage regulation and power-sharing despite model inaccuracies. Suppressing Disturbances (LESO): We significantly enhance the microgrid's robustness against external disturbances and sudden load changes by leveraging the active disturbance rejection properties of the LESO technique (Yoo et al., 2006). The LESO functions as a powerful tool for real-time estimation of total disturbances (internal and external), providing crucial feedforward compensation to mitigate large transient overshoots (Bhowmick and Panja, 2019). Resolving Control Conflict: A crucial innovation is introduced to address the voltage/reactive power contradiction: a vector of switching factors. This vector unifies the voltage recovery for critical buses and accurate reactive power-sharing into a single, cohesive control framework. This design empowers the user, via the online learning framework, to adjust the priority between these two goals, thus resolving the conflict within a single controller, unlike previous approaches that required distinct or complex hierarchical structures (Lai et al., 2019).

The remainder of this chapter is organized as follows. The problem formulation is presented in Sect. 4.2. Section 4.3 is dedicated to detailing the design of the predictive control strategy tailored for AC MGs. Section 4.4 presents and analyzes the comprehensive simulation results. A brief conclusion to the chapter is provided in Sect. 4.5.

4.2 Problem Formulation

Consistent with the definitions introduced in Chap. 3, the communication topology inherent to the networked microgrid system is modeled as a directed graph, denoted $\mathcal{G} = (V, E, \mathcal{A})$. Let $\mathcal{N}_i$ denote the neighboring set of agent i . This set contains all agents that can directly transmit information to agent i , mathematically defined as $\mathcal{N}_i = \{j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}, j \neq i\}$. In-degree matrix ($\mathcal{D}$) is a diagonal matrix where $\mathcal{D} = \text{diag}\{d_1, \dots, d_N\}$. The diagonal element d_i represents the in-degree of

vertex i , defined as the total number of its neighbors: $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The network structure is completed by the Laplacian matrix, which is derived from the difference between the in-degree and adjacency matrices: $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

In this chapter, we consider an islanded microgrid system comprising N three-phase inverter-based generation units. Each DG unit is connected to its primary DC power sources (such as photovoltaic arrays or battery banks) via dedicated interface feeders. The inherent complexity of managing dynamic loads and ensuring power quality in this autonomous system necessitates a sophisticated, hierarchical control structure. For effective regulation of the output voltage and reactive power of the DGs, a comprehensive three-tier control architecture is employed: the internal loop (inner loop) responsible for fast current and voltage regulation; the primary control layer, which executes local control tasks, typically using droop control, to manage power-sharing; and the secondary control layer, which coordinates the system to ensure voltage restoration and accurate reactive power-sharing. For the purpose of simplified modeling and coordinated control design, the system dynamics are analyzed within the direct-quadrature (dq) reference frame. To maintain synchronization across the network, all DGs are modeled as rotating at the corresponding frequency of a designated reference DG unit. This unified frame simplifies the coordination of distributed controllers and facilitates the design of the secondary control mechanism.

In AC microgrids, the design of primary control often relies on simplifying assumptions regarding the inverter's output impedance. Whenever the output impedance of the inverter is predominantly inductive ($X \gg R$), the relationship between voltage difference and reactive power simplifies significantly, allowing for the decoupling of active (P) and reactive (Q) power flows. Under this common assumption, the reactive power output ($Q_{e,i}$) of the i -th DG unit can be accurately approximated by the following locally linearized relationship:

$$Q_{e,i}(t) = \sum_{j \in \mathcal{N}_i} |Y_{ij}| V_i^2(t) - \sum_{j \in \mathcal{N}_i} |Y_{ij}| V_i(t) V_j(t) \cos(\delta_i(t) - \delta_j(t)) \quad (4.1)$$

where V_i represents the voltage magnitude of the i -th DG unit, $|Y_{ij}|$ is the magnitude of the admittance between buses i and j , and δ_i is the voltage phase angle. For further simplification in some literature, the voltage phase angle difference ($\delta_i - \delta_j$) is often considered small, or the formula is presented in its simplified averaged form. In this way, one has

$$Q_{e,i}(t) = \sum_{j \in \mathcal{N}_i} |Y_{ij}| V_i^2(t) - \sum_{j \in \mathcal{N}_i} |Y_{ij}| V_i(t) V_j(t). \quad (4.2)$$

This simplified reactive power relationship serves as the foundational basis for implementing voltage droop control, a decentralized mechanism crucial for regulating voltage and achieving autonomous reactive power-sharing. The voltage

droop characteristic is derived directly from (4.2), establishing a relationship between the measured reactive power and the reference voltage component:

$$\begin{aligned} V_{od,i}(t) &= V_{ni}(t) - n_{Qi} Q_{e,i}(t) \\ V_{oq,i}(t) &= 0. \end{aligned} \quad (4.3)$$

In this standard primary control law, V_{ni} denotes the nominal voltage magnitude provided as a reference to the inner control loops. $Q_{e,i}(t)$ is the instantaneous measured reactive power output of DG_i , and n_{Qi} is the voltage droop gain, which determines the sensitivity of the voltage magnitude to changes in reactive power. By setting the q -axis voltage reference ($V_{oq,i}$) to zero, the control primarily influences the magnitude of the voltage, facilitating local reactive power management.

The primary control loop, while effective for local power-sharing, is inherently subject to voltage magnitude deviations from the prescribed nominal values due to the droop action and impedance effects. To effectively alleviate these steady-state errors and restore the system voltage to the desired nominal level, a secondary control layer must be strategically considered and designed. This layer acts as a coordinating authority, providing adjustments to the primary controller's setpoints. To integrate the action of the secondary controller, we analyze the time evolution of the primary droop equation (4.3). Differentiating both sides of (4.3) with respect to time yields the following dynamic relationship:

$$\begin{aligned} \dot{V}_{ni}(t) &= \dot{V}_{od,i}(t) + n_{Qi} \dot{Q}_{e,i}(t) \\ &= u_i^v(t) + n_{Qi} u_i^Q(t). \end{aligned} \quad (4.4)$$

In this derivative form, $\dot{V}_{od,i} = u_i^v$ and $\dot{Q}_{e,i} = u_i^Q$ are defined as the dynamic control input components for the voltage and reactive power adjustment, respectively, derived from the secondary layer. It is critical to note the separation of timescales principle: the primary control operates on a much faster timescale than the deliberative secondary control layer. Based on the derived relationship, the nominal input voltage (V_{ni}) required by the primary control loop can be calculated by integrating the output of the secondary control components over time:

$$V_{ni}(t) = V_{ni}(t_0) + \int_{t_0}^t \left(u_i^v(\xi) + n_{Qi} u_i^Q(\xi) \right) d\xi. \quad (4.5)$$

Here, the secondary control signals u_i^v and u_i^Q are precisely the components that will be generated and commanded by the online learning-based controller proposed in this chapter. This integrated approach ensures that the secondary control effectively and continually resets the primary droop reference, thus achieving precise global voltage restoration and reactive power coordination.

4.3 Predictive Control of AC Microgrid Systems

In the hierarchical control architecture of AC microgrids, the secondary control layer plays a pivotal role in compensating for the steady-state deviations inherent to primary droop control. To guarantee high-quality power supply and economic operation, the proposed control strategy is designed to strictly satisfy two asymptotic objectives simultaneously, while maintaining robustness against system uncertainties.

1. Global Voltage Regulation:

Due to the droop mechanism and line impedance voltage drops, the bus voltages deviate from the nominal value. The first objective is to regulate the average voltage of the microgrid to the rated value, rather than forcing every single bus to a rigid point, which allows for necessary voltage gradients for power flow. Mathematically, this asymptotic tracking objective for the set of generation nodes $\mathcal{V}$ is formulated as:

$$\lim_{t \rightarrow \infty} \left| \frac{1}{N} \sum_{i \in \mathcal{V}} V_{od,i}(t) - V_{\text{nom}} \right| = 0 \quad (4.6)$$

where $V_{od,i}(t)$ denotes the output voltage magnitude of the i -th DG unit, and V_{nom} represents the nominal voltage reference.

2. Proportional Reactive Power Allocation:

To prevent circulating currents and ensure that DGs do not exceed their power ratings, the reactive load demand must be shared among DGs according to their power capacities. This requires the normalized reactive power outputs to converge to a consensus state:

$$\lim_{t \rightarrow \infty} (k_{q,i} Q_{e,i}(t) - k_{q,j} Q_{e,j}(t)) = 0, \quad \forall i, j \in \mathcal{V} \quad (4.7)$$

where $Q_{e,i}(t)$ is the measured reactive power output, and $k_{q,i}$ is the reactive droop coefficient, which is inversely proportional to the maximum reactive capacity of the i -th DG.

Robustness under Uncertainty: The realization of these objectives is nontrivial due to the dynamic nature of microgrids. The control framework must exhibit strong resilience against external perturbations (e.g., step changes in load demand or PnP operations of DGs) and internal parametric uncertainties (e.g., drift in feeder impedance due to temperature changes or aging). Ensuring stability under these conditions is paramount for the reliable operation of the cyber-physical system.

The Inherent Conflict in Control Objectives: It is crucial to highlight a fundamental challenge in secondary control of droop-controlled AC microgrids: the coupling between voltage magnitude and reactive power. Under the conventional secondary-control setting considered here, voltage magnitudes are the main variables adjusted

for reactive power allocation, whereas phase-angle regulation and load-side coordination are not explicitly used as additional degrees of freedom. In networks with mismatched line impedances, especially multi-bus AC microgrids, reactive power flow is therefore strongly affected by voltage-magnitude differences among buses.

- If strict local voltage restoration is enforced, i.e., $V_{od,i} = V_{od,j} = V_{nom}$ for all $i, j \in \mathcal{V}$, the voltage-magnitude differences that are normally used to regulate reactive power flow are eliminated. Under the standard secondary-control framework that does not explicitly coordinate phase angles or load distribution, this removes the main control freedom needed for accurate proportional reactive power-sharing.
- Conversely, accurate proportional reactive power-sharing generally requires nonuniform voltage magnitudes across DG buses, particularly when line impedances and DG ratings are mismatched. These voltage gradients provide the necessary degrees of freedom for shaping reactive power flows, but they inevitably lead to local voltage deviations from the nominal value.

Therefore, a rigid simultaneous achievement of local voltage restoration and precise reactive power-sharing is mathematically conflicting. To resolve this, this work adopts a compromise strategy: utilizing average voltage restoration (as defined in (4.6)). By ensuring the average voltage remains at the nominal level, the system allows for slight local voltage variations necessary to drive the reactive power flow, thereby reconciling the contradiction between voltage quality and load-sharing accuracy.

4.3.1 *Achievement of Average Regulation of Voltages of All DGs*

As discussed in the preceding analysis, aiming for simultaneous strict voltage restoration at every local bus and precise reactive power-sharing is physically contradictory in AC microgrids with mismatched line impedances. To reconcile these conflicting control objectives, a compromise strategy is adopted in this framework: average voltage restoration.

The core philosophy of this approach is to regulate the global arithmetic mean of all DG output voltages to the nominal reference, rather than forcing each individual DG voltage to a rigid value. This allows for necessary local voltage deviations to drive the reactive power flow required for accurate sharing, while ensuring that the overall voltage profile of the microgrid remains within a safe operating range. To implement this, each DG needs access to the global average voltage information. Since a centralized controller is avoided to prevent single-point failures, a distributed dynamic consensus estimator is designed. We introduce an auxiliary state variable

$\hat{V}_{od,i}(t)$ for each agent to estimate the average voltage of the entire network. The dynamics of this distributed observer are formulated as follows:

$$\dot{\hat{V}}_{od,i}(t) = \dot{V}_{od,i}(t) + \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{V}_{od,j}(t) - \hat{V}_{od,i}(t)), \forall i \in \mathcal{V}. \quad (4.8)$$

The following lemma establishes the relationship between the estimated voltage and the measured voltage in steady-state conditions.

Lemma 4.3.1 (Nasirian et al., 2015) *Consider a networked AC microgrid system modeled by a graph $\mathcal{G}$. If the communication topology graph $\mathcal{G}$ is undirected and connected (or directed and weight-balanced), and with consistent initialization satisfying $\sum_i \hat{V}_{od,i}(0) = \sum_i V_{od,i}(0)$, the auxiliary state $\hat{V}_{od,i}(t)$ in the distributed observer (4.8) will asymptotically track the average value of the time-varying input signals $V_{od,i}(t)$ of all nodes. Mathematically, this convergence is expressed as:*

$$\lim_{t \rightarrow \infty} \left| \hat{V}_{od,i}(t) - \frac{1}{N} \sum_{k=1}^N V_{od,k}(t) \right| = 0, \quad \forall i \in \mathcal{V}. \quad (4.9)$$

Remark 4.3.1 Voltage Restoration Paradigms: The distributed observer formulation in (4.8), validated by the convergence properties in Lemma 4.3.1, serves as a cornerstone for achieving average voltage restoration. This mechanism allows distributed agents to estimate the global average voltage utilizing only local neighbor-to-neighbor information, a strategy that has been extensively adopted in a multitude of recent studies (Madichetty and Mishra, 2022; Zuo et al., 2020; Sadabadi et al., 2023).

However, it is pertinent to acknowledge that arithmetic average restoration is not the unique solution. Under the strict prerequisite of achieving accurate reactive power-sharing, diverse mathematical definitions of “voltage restoration” can be engineered to suit different operational requirements. For instance, the authors in De Persis et al. (2018) introduced a geometric mean voltage restoration strategy, offering a different perspective on global voltage regulation. Furthermore, advancements have also been made regarding the control structure itself. Certain modeling frameworks allow for voltage restoration without the necessity of auxiliary voltage observers, as demonstrated in the streamlined approaches studied in Trip et al. (2019).

Nevertheless, a generalized theory characterizing the full degrees of freedom available for voltage restoration, constrained by the rigorous requirement of accurate power-sharing, remains an open and nontrivial problem. Whether there exist other advantageous restoration forms beyond the arithmetic and geometric means and what the theoretical boundaries of these forms are are questions that merit significant future exploration.

4.3.2 Design of Online Learning-Based Distributed Secondary Controller

Before detailing the controller synthesis, we introduce the concept of uniformly ultimately bounded (UUB) stability, which serves as the theoretical foundation for our stability analysis (Lewis et al., 1998).

Definition 4.3.1 *The voltage states $V_{od,i}(t)$ of the microgrid system are considered UUB if there exists a compact set $\mathcal{S} \subset \mathbb{R}^n$ such that for any initial state $V_{od,i}(t_0) \in \mathcal{S}$, there exists a bound $\mu > 0$ and a time horizon $T > 0$ (independent of t_0) ensuring that $\|V_{od,i}(t) - V_{od,i}^*\| < \mu$ for all $t > t_0 + T$.*

A. Error Dynamics Formulation

To achieve the dual objectives of voltage restoration and reactive power-sharing, we define a composite error variable. Let the local voltage tracking error be $e_{V,i}$ and the reactive power-sharing error be $e_{Q,i}$, formulated as:

$$\begin{aligned} e_{V,i}(t) &= c_{i,1}(\hat{V}_{od,i}(t) - V_{\text{nom}}) + \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{V}_{od,i}(t) - \hat{V}_{od,j}(t)), \\ e_{Q,i}(t) &= \sum_{j \in \mathcal{N}_i} a_{ij}(k_{q,i}Q_i(t) - k_{q,j}Q_j(t)) \end{aligned} \quad (4.10)$$

where $c_{i,1}$ is a pinning gain. We then construct a weighted aggregate error term $E_{1,i}(t) = \alpha_i e_{V,i}(t) + \beta_i e_{Q,i}(t)$, where α_i and β_i are weighting factors. Additionally, a secondary error term based on the actual output voltage is defined as $E_{2,i}(t) = c_{i,2}(V_{od,i}(t) - V_{\text{nom}}) + \sum_{j \in \mathcal{N}_i} a_{ij}(V_{od,i}(t) - V_{od,j}(t))$.

The sliding surface variable for the i -th DG is designed as a linear combination of these errors:

$$\sigma_i(t) = \rho_{i,1}E_{1,i}(t) + \rho_{i,2}E_{2,i}(t) \quad (4.11)$$

where $\rho_{i,1}, \rho_{i,2} \in \{0, 1\}$ are binary switching parameters. The pair $(\rho_{i,1}, \rho_{i,2})$ constitutes the switching vector, denoted by S_w . Taking the time derivative of $\sigma_i(t)$, we obtain the dynamics of the sliding variable:

$$\dot{\sigma}_i(t) = \Gamma_i u_i^V(t) + f_{V,i}(t) + f_{Q,i}(t) \quad (4.12)$$

where Γ_i is a known control gain term, while $f_{V,i}$ and $f_{Q,i}$ represent complex nonlinear functions containing unknown dynamics and coupling effects related to voltage and reactive power, respectively.

B. Neural Network Approximation

To handle the unknown system dynamics encapsulated in $f_{V,i}$ and $f_{Q,i}$, we employ radial basis function neural networks (RBFNNs) due to their universal approximation capabilities. The unknown functions can be modeled as

$$\begin{aligned} f_{V,i}(t) &= \mathbf{W}_{vi}^{*T} \Phi_{vi}(x_{vi}(t)) + \epsilon_{V,i}(t) \\ f_{Q,i}(t) &= \mathbf{W}_{Qi}^{*T} \Phi_{Qi}(x_{Qi}(t)) + \epsilon_{Q,i}(t) \end{aligned} \quad (4.13)$$

where $\mathbf{W}_{vi}^{*T}$ and $\mathbf{W}_{Qi}^{*T}$ denote the optimal weight vectors, $\Phi(\cdot)$ represents the regressor vector of basis functions, x_{vi} and x_{Qi} are the inputs of NNs, and $\epsilon_{V,i}$ and $\epsilon_{Q,i}$ denote the bounded approximation errors.

In practical implementation, the optimal weights are unknown. Therefore, we utilize estimated weights $\hat{\mathbf{W}}_{V,i}$ and $\hat{\mathbf{W}}_{Q,i}$ updated via online learning laws. The estimation errors are defined as $\tilde{\mathbf{W}}_{vi} = \mathbf{W}_{vi}^* - \hat{\mathbf{W}}_{vi}$. Therefore, the approximations of f_{vi} and f_{Qi} via supervised online learning approach can be expressed as

$$\begin{aligned} \hat{f}_{vi}(t) &= \hat{\mathbf{W}}_{vi}^T(t) \Phi_{vi}(x_{vi}(t)) \\ \hat{f}_{Qi}(t) &= \hat{\mathbf{W}}_{Qi}^T(t) \Phi_{Qi}(x_{Qi}(t)). \end{aligned} \quad (4.14)$$

C. Adaptive Controller Synthesis and Stability Analysis

We propose the following Lyapunov candidate function to ensure system stability and weight convergence:

$$L_i(t) = \frac{1}{2} \sigma_i^2(t) + \frac{1}{2} \tilde{\mathbf{W}}_{vi}^T(t) F_{vi}^{-1} \tilde{\mathbf{W}}_{vi}(t) + \frac{1}{2} \tilde{\mathbf{W}}_{Qi}^T(t) F_{Qi}^{-1} \tilde{\mathbf{W}}_{Qi}(t) \quad (4.15)$$

where F_{vi} and F_{Qi} are positive definite learning rate matrices.

The proposed adaptive distributed control law is designed as:

$$\begin{aligned} u_i^v(t) &= -\Gamma_i^{-1} (\hat{\mathbf{W}}_{vi}^T(t) \Phi_{vi}(x_{vi}(t)) + \hat{\mathbf{W}}_{Qi}^T(t) \Phi_{Qi}(x_{Qi}(t)) \\ &\quad + r_i \sigma_i(t) + \varepsilon_M \text{sat}(\sigma_i(t))) \end{aligned} \quad (4.16)$$

where $r_i > 0$ is a feedback gain, and ε_M is a robust term to bound approximation errors. The saturation function $\text{sat}(\cdot)$ is utilized to mitigate control chattering.

The adaptive update laws for the neural network weights are derived as:

$$\begin{aligned} \dot{\hat{\mathbf{W}}}_{vi}(t) &= F_{vi} \Phi_{vi} \sigma_i(t) - d_{vi} F_{vi} \hat{\mathbf{W}}_{vi}(t) \\ \dot{\hat{\mathbf{W}}}_{Qi}(t) &= F_{Qi} \Phi_{Qi} \sigma_i(t) - d_{Qi} F_{Qi} \hat{\mathbf{W}}_{Qi}(t) \end{aligned} \quad (4.17)$$

with $d_{vi}, d_{Qi} > 0$ being sigma-modification parameters to prevent weight drift.

The analysis is bifurcated based on the region of the sliding variable σ_i .

Inside the Boundary Layer ($|\sigma_i| \leq \Delta$):

By substituting the control law (4.16) and adaptive laws (4.17) into the time derivative of Lyapunov function L_i , and applying standard inequalities, we can derive:

$$\dot{L}_i(t) \leq -\lambda_i L_i(t) + \Theta_i \quad (4.18)$$

where λ_i and Θ_i are positive constants depending on control gains and error bounds. This implies that the Lyapunov function is bounded, and the tracking error converges to a compact set $\Omega_{in} = \sigma_i : |\sigma_i| \leq \sqrt{\Theta_i/\lambda_i} + 2L_i(0)$.

Outside the Boundary Layer ($|\sigma_i| > \Delta$):

In this region, $\text{sat}(\sigma_i) = \text{sign}(\sigma_i)$. Based on the universal approximation property implied by the Stone-Weierstrass theorem, the ideal weight vectors defined on a compact set are inherently bounded. Consequently, there exist positive constants $\bar{w}_{vi}$ and $\bar{w}_{Qi}$ such that the inequalities $|\tilde{\mathbf{W}}_{vi}^*| \leq \bar{w}_{vi}$ and $|\tilde{\mathbf{W}}_{Qi}^*| \leq \bar{w}_{Qi}$ hold. The derivative of the Lyapunov function becomes:

$$\begin{aligned} \dot{L}_i(t) \leq & -r_i \sigma_i^2(t) + d_{vi} \left| \tilde{\mathbf{W}}_{vi}^T(t) \right| \bar{w}_{vi} + d_{Qi} \left| \tilde{\mathbf{W}}_{Qi}^T(t) \right| \bar{w}_{Qi} \\ & - d_{vi} \tilde{\mathbf{W}}_{vi}^T(t) \tilde{\mathbf{W}}_{vi}(t) - d_{Qi} \tilde{\mathbf{W}}_{Qi}^T(t) \tilde{\mathbf{W}}_{Qi}(t) - \bar{\varepsilon}_M |\sigma_i(t)| \end{aligned} \quad (4.19)$$

where $\bar{\varepsilon}_M = \varepsilon_M - \bar{\varepsilon}_{V,i} - \bar{\varepsilon}_{Q,i}$. Here, $\bar{\varepsilon}_{V,i}$ and $\bar{\varepsilon}_{Q,i}$ denote the upper bounds applicable when the sliding surface lies within the boundary layer. Specifically, this satisfies $\varepsilon_{V,i} \leq \bar{\varepsilon}_{V,i}$ and $\varepsilon_{Q,i} \leq \bar{\varepsilon}_{Q,i}$ for $|\sigma_i| \leq \Delta$. Define the state vector $\mathbf{Z} = [|\sigma_i|, |\tilde{w}_{vi}|, |\tilde{w}_{Qi}|]^T$, the inequality can be structured as $\dot{L}_i \leq -\mathbf{Z}^T \mathbf{Q} \mathbf{Z} + \mathbf{H}^T \mathbf{Z}$, where $\mathbf{H} = [-\varepsilon_{M*}, d_{vi}, d_{Qi}]^T$ and $\mathbf{Q} = \text{diag}(d_{vi}, d_{Qi}, r_i)$. Stability is guaranteed if matrix $\mathbf{Q}$ is positive definite and the states lie outside a specific bound Ω_{out} .

Combining both cases and invoking the extended Lyapunov stability criterion (Lewis et al., 1998), it is established that the voltage states $V_{od,i}$ exhibit UUB, with their trajectories eventually confined within the region $\Omega_{in} \cup \Omega_{out}$.

Remark 4.3.2 (Complexity): The computational complexity of the proposed algorithm scales as $\mathcal{O}(NL_v^3) + \mathcal{O}(NL_Q^3)$, where L_v and L_Q denote the number of neurons in the NNs for estimating functions $f_{V,i}$ and $f_{Q,i}$, respectively. While this cubic relationship suggests a computational cost, distributed implementation ensures scalability with the number of DGs (N).

4.3.3 Active Disturbance Rejection via Extended State Observer

While the neural network-based adaptive controller effectively compensates for structured uncertainties and slowly varying parameter drifts, the microgrid system is frequently subjected to high-frequency external perturbations. These include sudden load shedding, intermittent fluctuations from renewable sources, and topological

switching events. To further enhance the system's robustness, specifically to suppress voltage overshoots and accelerate transient recovery, an LESO is introduced. This module functions as a soft sensor, estimating total disturbances in real time to facilitate active feedforward compensation.

Recall the voltage dynamics of the i -th DG unit derived in (4.4). We rewrite the system by lumping all external disturbances and unmodeled dynamics into a generalized disturbance term $\Delta_i(t)$. The refined dynamic equation is expressed as:

$$\dot{V}_{od,i}(t) = u_i^V(t) - \Delta_i(t) \quad (4.20)$$

where $V_{od,i}(t)$ denotes the output voltage magnitude, $u_i^V(t)$ is the voltage-loop control input, and $\Delta_i(t)$ represents the lumped disturbance.

To design the observer, we treat the disturbance $\Delta_i(t)$ as an additional state variable. Let us define an extended state vector $\mathbf{z}_i = [z_{i,1}, z_{i,2}]^T$, where $z_{i,1} = V_{od,i}$ represents the system output and $z_{i,2} = \Delta_i$ represents the disturbance to be estimated. Assuming the disturbance is differentiable with a bounded derivative (i.e., $|\dot{\Delta}_i| \leq M_D$), the system can be expanded into a second-order state-space form:

$$\begin{aligned} \dot{z}_{i,1}(t) &= u_i^V(t) - z_{i,2}(t) \\ \dot{z}_{i,2}(t) &= h_i(t) \end{aligned} \quad (4.21)$$

where $\dot{\Delta}_i(t) = h_i(t)$ is the unknown derivative of the disturbance.

Based on the extended model (4.21), a second-order LESO is constructed to reconstruct the states. The observer dynamics are given by:

$$\begin{aligned} \dot{\hat{z}}_{i,1}(t) &= u_i^V(t) - \hat{z}_{i,2}(t) + L_{i,1}(V_{od,i}(t) - \hat{z}_{i,1}(t)) \\ \dot{\hat{z}}_{i,2}(t) &= -L_{i,2}(V_{od,i}(t) - \hat{z}_{i,1}(t)), \end{aligned} \quad (4.22)$$

where $\hat{z}_{i,1}$ and $\hat{z}_{i,2}$ denote the estimates of the voltage and the disturbance, respectively. The terms $L_{i,1}$ and $L_{i,2}$ are the observer correction gains.

Subtracting (4.22) from (4.21), the estimation error dynamics $\mathbf{e}_{obs,i} = [z_{i,1} - \hat{z}_{i,1}, z_{i,2} - \hat{z}_{i,2}]^T$ can be derived as:

$$\dot{\mathbf{e}}_{obs,i}(t) = \mathbf{A}_{obs,i} \mathbf{e}_{obs,i}(t) + \mathbf{B}_{obs,i} h_i(t) \quad (4.23)$$

where the system matrix is $\mathbf{A}_{obs,i} = [-L_{i,1}, -1; L_{i,2}, 0]$. To ensure stability and rapid convergence, the gains are typically tuned using the bandwidth parameterization method. By placing both poles of the characteristic equation $|s\mathbf{I} - \mathbf{A}_{obs,i}| = s^2 + L_{i,1}s + L_{i,2} = 0$ at $-\omega_o$, where ω_o is the observer bandwidth, the gains are configured as:

$$L_{i,1} = 2\omega_o, \quad L_{i,2} = \omega_o^2. \quad (4.24)$$

With $\omega_o > 0$, the matrix $\mathbf{A}_{obs,i}$ is Hurwitz, guaranteeing that the estimation errors converge asymptotically to a bounded neighborhood of zero, provided $h_i(t)$ is bounded.

By integrating the LESO-estimated disturbance into the previously designed adaptive law, we obtain a composite control strategy that combines learning-based adaptation with active disturbance rejection. The final control law for the voltage loop is modified to:

$$u_i^V(t) = \underbrace{\Gamma_i^{-1}(\Psi_{NN,i}(t) - r_i s_i(t) - \varepsilon_M \text{sat}(s_i(t)))}_{u_{i,1}} + \underbrace{\hat{z}_{i,2}(t)}_{u_{i,2}} \quad (4.25)$$

where $\Psi_{NN,i}(t) = \hat{f}_{V,i}(t) + \hat{f}_{Q,i}(t)$ represents the neural network approximation terms derived in Sect. 4.3.2, $u_{i,1}$ and $u_{i,2}$ are adaptive feedback and disturbance compensation term, respectively. By explicitly adding the term $\hat{z}_{i,2}$ (which tracks Δ_i), the controller proactively cancels out external disturbances before they significantly affect the voltage tracking error, thereby significantly improving the disturbance rejection capability of the closed-loop MG system.

4.4 Simulation Verification and Result Analysis

4.4.1 Simulation Setup and Parameter Configuration

To corroborate the efficacy and robustness of the proposed NN-based secondary regulation strategy, a comprehensive simulation study is conducted on a test MG system comprising four inverter-based DG units, implemented within the MATLAB/SimPowerSystems environment. The specific topology of this networked 4-DG system is depicted in Fig. 4.1. Detailed electrical specifications of the MG, along with the coefficients for the local and primary control layers, are provided in Table 4.1.

To rigorously test the controller's active disturbance rejection capability, time-varying external disturbances are injected into the voltage dynamics of the four DGs, modeled as $d_1^v = 5 \sin(t)$, $d_2^v = 4 \sin(3t)$, $d_3^v = 6 \sin(0.1t)$, and $d_4^v = 5 \sin(2t)$. The nominal voltage reference is standardized at $V_{\text{nom}} = 380$ V. The control framework is parameterized based on the UUB stability conditions derived in Sect. 4.3. Specifically, the feedback gains in the adaptive controller (4.25) are tuned to $r_i = 8$, $\lambda_{v,i} = 9$, and $\lambda_{Q,i} = 1$ for all agents. To prevent chattering, the boundary layer threshold for the saturation function is set to $\varphi = 2$, and the robust gain is $\varepsilon_M = 5$. The hyperparameters for the neural networks are configured as $\beta_1 = 50$, $\beta_2 = 700$, with learning rates $d_{vi} = d_{Qi} = 0.5$ and scaling matrices $F_{vi} = F_{Qi} = \mathbf{I}_9$, where $\mathbf{I}_9$ represents a 9×9 identity matrix. The operational timeline of the simulation is divided into distinct phases to observe the system evolution:

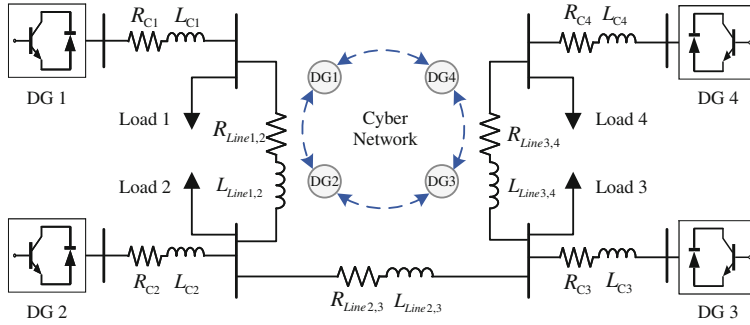


Fig. 4.1 A single-line diagram of the test microgrid

Table 4.1 Specification of the MG test system

DG's Parameters			DG ₁ & DG ₂			DG ₃ & DG ₄		
Droop control	m_p		9.4×10^{-5}			12.5×10^{-5}		
	n_Q		1.3×10^{-3}			0.9×10^{-3}		
RLC filter	R_f		0.1			0.1		
[Ω],[mH],[μ F]	L_f		0.4			0.4		
	C_f		100			100		
Coupling connector	L_l		0.35			0.35		
[Ω],[mH]	R_l		0.03			0.03		
Lines			Line 23			Line 34		
[Ω],[μ H]	R_{12}	0.1	R_{23}	0.23	R_{34}	0.1		
	L_{12}	318	L_{23}	847	L_{34}	325		
Loads			Load 2		Load 3		Load 4	
[kW],[kVar]	P_{L1}	10	P_{L2}	12	P_{L3}	15.3	P_{L4}	17
	Q_{L1}	10	Q_{L2}	11	Q_{L3}	7.6	Q_{L4}	8

- **Phase S I** ($0 \leq t < 0.5$ s): The DGs operate in standalone mode, isolated from one another.
- **Phase S II** ($0.5 \leq t < 1$ s): The physical transmission lines are connected, forming a coupled microgrid, but the secondary control remains inactive.
- **Phase S III** ($t \geq 1$ s): The proposed distributed secondary control is activated to regulate the system.

4.4.2 Scenario 1: Evaluation of Control Objectives and Model Independence

This scenario evaluates the general performance of the proposed controller under two distinct operational priorities, determined by the switching vector S_w .

4.4.2.1 Case 1: Priority on Reactive Power-Sharing ($S_w = (1, 0)$)

In this configuration, the control objective prioritizes accurate reactive power allocation. As illustrated in Fig. 4.2, once the secondary control is engaged at $t = 1$ s (Phase S III), the system exhibits clear consensus behavior. Specifically, Fig. 4.2b demonstrates that while the individual output voltages of the DGs deviate slightly to accommodate the power-sharing requirements, the average voltage magnitude of the entire microgrid successfully tracks the nominal reference V_{nom} (indicated by the dashed line). Crucially, Fig. 4.2a confirms that the reactive power outputs are allocated strictly in proportion to the DGs' capacities. Figure 4.2d further visualizes the smooth trajectories of the control inputs, confirming stability.

4.4.2.2 Case 2: Priority on Voltage Restoration ($S_w = (0, 1)$)

This case shifts the focus to strict voltage restoration. The results in Fig. 4.3a, b indicate that the proposed controller successfully forces all DG voltages to converge to the nominal value of 380 V. For comparative analysis, Fig. 4.3c, d display the performance of a traditional networked cooperative control (NCC) method. While the traditional method achieves similar objectives, it relies heavily on accurate system models. In contrast, the proposed NN-based approach, aided by

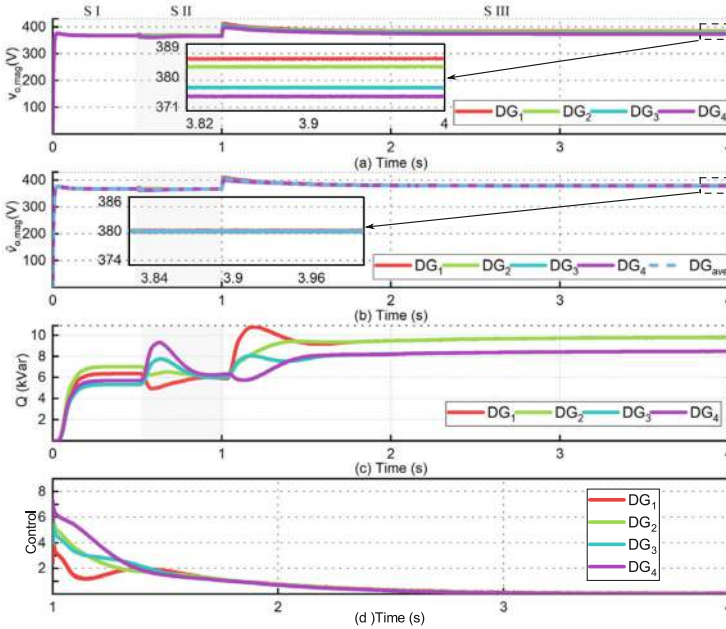


Fig. 4.2 Voltage and reactive power profiles for Case 1 in Scenario 1 under the proposed method

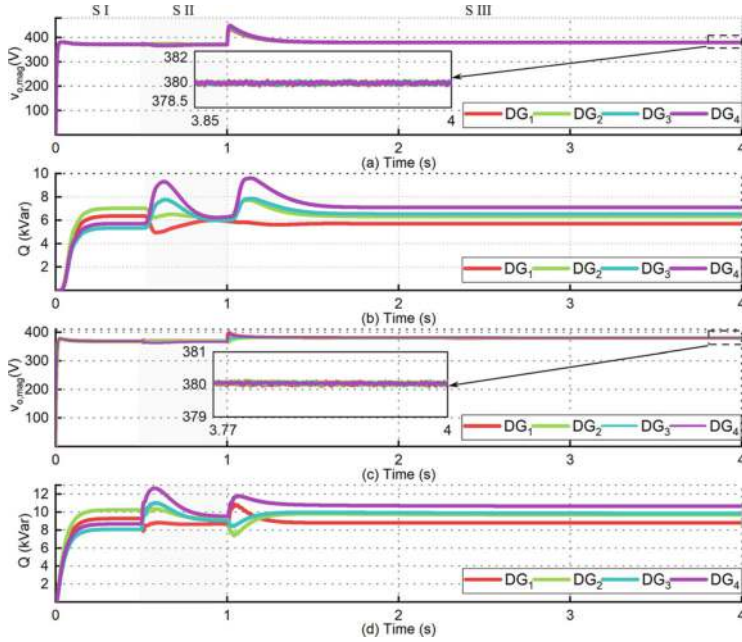


Fig. 4.3 Voltage and reactive power outputs corresponding to Case 2 in Scenario 1 are presented in (a) and (b), while those for the case adopting the NCC method (as reported in (Dehkordi et al., 2019)) are depicted in (c) and (d)

the LESO, effectively compensates for the injected sinusoidal disturbances without requiring precise knowledge of the MG parameters, thereby demonstrating superior adaptability and practical applicability compared to the model-dependent approach in Dehkordi et al. (2019).

4.4.3 Scenario 2: Robustness Against Dynamic Load Fluctuations

The second scenario investigates the controller's robustness under abrupt load variations. Following the activation of secondary control, the system is subjected to a sequence of load events:

- **Phase S IV** ($4 \leq t < 7$ s): A step increase in total load from 36.6 kVar to 40.6 kVar.
- **Phase S V** ($7 \leq t < 10$ s): The load steps back down to the original level.
- **Phase S VI** ($10 \leq t \leq 13$ s): The load undergoes heterogeneous fluctuations across the DGs, with specific reactive loads set to $Q_{L1} = 11$, $Q_{L2} = 12$, $Q_{L3} = 8.6$, and $Q_{L4} = 9$ kVar.

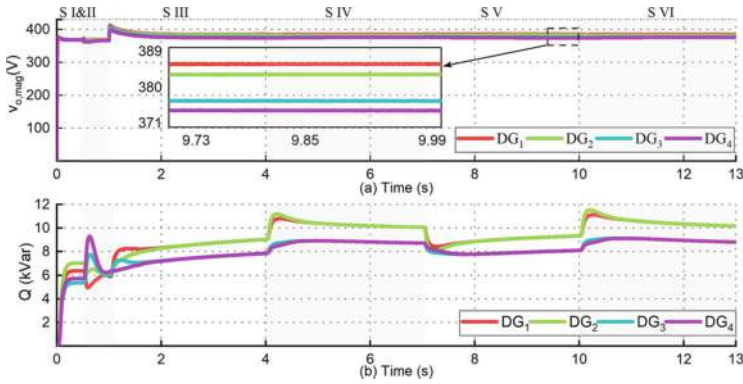


Fig. 4.4 Voltage and reactive power profiles for Case 1 in Scenario 2 under the proposed method

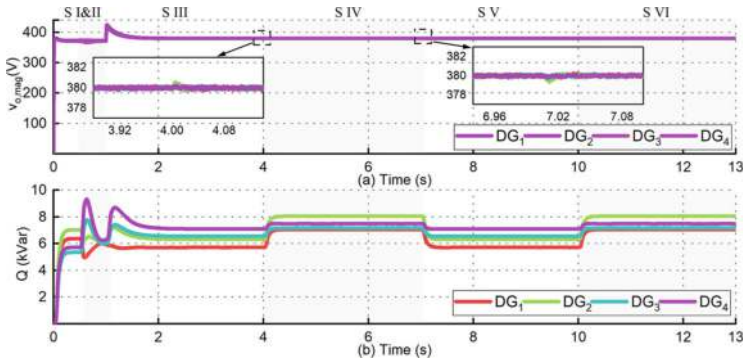


Fig. 4.5 Voltage and reactive power profiles for Case 2 in Scenario 2 under the proposed method

The system response under Case 1 settings (Power-Sharing priority) is depicted in Fig. 4.4. It is observed in Fig. 4.4b that throughout these drastic load changes, the reactive power distribution among DGs maintains the desired proportionality defined by the droop coefficients. Simultaneously, Fig. 4.4a shows that the voltage profile remains stable near the expected average value.

Conversely, under Case 2 settings (Voltage Restoration priority), as shown in Fig. 4.5a, b, the controller rapidly restores the voltage to the reference value after each load transient. However, as expected due to the fundamental conflict between the two objectives, the reactive power-sharing accuracy is compromised in this mode.

In conclusion, the extensive simulation results unequivocally demonstrate that the proposed LESO-enhanced adaptive NN controller is highly effective. It not only successfully realizes the flexible switching between voltage regulation and reactive power-sharing objectives but also exhibits strong robustness against unknown external disturbances and significant load fluctuations, validating its suitability for complex islanded microgrid applications.

4.5 Notes and References

This chapter has presented an online-learning-assisted LESO framework for distributed secondary control of islanded AC MGs. The neural-network component approximates unknown voltage- and reactive-power-related dynamics, while the LESO estimates lumped disturbances for active compensation. By introducing a switching mechanism, the controller can prioritize either average voltage restoration with reactive power-sharing or stricter local voltage restoration, depending on operational requirements. Simulation studies on a four-DG AC MG demonstrate the effectiveness of the method under disturbance injection and load variations. Several key contributions and characteristics of this methodology are worth highlighting. **Model Independence:** Unlike traditional model-based approaches, the devised controller actively identifies and compensates for unknown DG dynamics and parameter mismatches, significantly enhancing the system's robustness against modeling errors. **Unified Control Objective:** A distinct feature of this work is the reconciliation of the inherent conflict between critical bus voltage recovery and precise reactive power-sharing. By integrating a flexible switching mechanism, these competing objectives are resolved within a single, cohesive mathematical framework, thereby enabling priority adjustments to be made according to operational requirements. **Scalability and PnP Capability:** The control scheme is strictly designed for distributed implementation. This topological choice not only reduces communication burdens but also intrinsically endows the MG with PnP capabilities, ensuring seamless operation during the connection or disconnection of DG units. Extensive simulation studies have rigorously validated the efficacy of this learning-based approach, demonstrating superior transient response and disturbance rejection compared to conventional methods.

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Part II
Fully Actuated Control of Renewable
Power Systems

Chapter 5

Fully Actuated Control of Renewable Power Systems with Disturbances



Abstract The nonlinear interactions between boost converters and constant power loads (CPLs) create complex control challenges in MG systems. To address these stability issues and control design limitations, this chapter presents a high-order fully actuated (HOFA) modeling framework for CPL-integrated DC MGs interfaced through different converter topologies, such as buck and boost converters. Based on this model, an estimator-driven voltage control strategy is developed for both buck/boost converter architectures and dynamic load conditions. By applying transformation techniques (state reformulation, dimensionality compression, and order elevation), the original underactuated first-order system is systematically reconstructed into a high-dimensional fully actuated structure. Compared to conventional state space-based nonlinear controllers, the HOFA-derived control law demonstrates enhanced structural simplicity with configurable parameters while maintaining robustness. Experimental validation on a physical MG testbed confirms the method's effectiveness across multiple operational scenarios, including power fluctuations and transient disturbances. Notably, the framework eliminates traditional reliance on linearized approximations for CPL-dominated systems.

5.1 Overview

The challenges of stability and voltage regulation brought about by constant power loads (CPLs) in DC microgrids have spurred extensive research into control methodologies. These methods are broadly classified as linearization-based techniques or direct nonlinear strategies. While sliding mode control (SMC) represents a common nonlinear approach, its implementation in microgrids is hindered by high-frequency chattering from discontinuous switching dynamics. Compensatory filtering mechanisms are necessary to address this issue, albeit at the cost of reduced transient performance and destabilization of source–load impedance interactions. Emerging model-agnostic control paradigms, which leverage neural networks or adaptive coefficient tuning, offer theoretical adaptability but introduce significant computational burdens due to iterative neural weight optimization requirements.

The limitations of existing methods highlight the pressing need for innovative control frameworks capable of reconciling nonlinear dynamics with implementation efficiency. Robust control strategies in power systems often rely on linear matrix inequality (LMI)-based frameworks to impose stability constraints. Despite their mathematical rigor, these methods face computational challenges, with iterative optimization typically needed to solve LMI constraints. Recent research explores promising alternatives like hybrid controllers that combine sliding mode control (SMC) for disturbance rejection with backstepping's Lyapunov-based stability proofs, enabling voltage stabilization in CPL-intensive microgrids within sub-100 ms timeframes.

However, the integration of SMC's discontinuous switching logic with backstepping's recursive parameter tuning increases design complexity, making these controllers unwieldy for practical deployment. To bridge this gap, novel paradigms are emerging, such as data-driven controllers that train directly on operational datasets, as seen in reinforcement learning schemes optimized for transient response. Adaptive observers are also being employed to dynamically estimate fluctuating CPL parameters in real time, separating stabilization from the need for precise prior load knowledge. These approaches prioritize practicality alongside robustness, aligning theoretical guarantees with real-time feasibility. For comprehensive evaluations of CPL stabilization techniques, including assessments of oscillation damping efficacy and transient recovery speed, refer to in-depth reviews in the literature.

Addressing the escalating stabilization challenges in CPL-dominated DC microgrids, this research introduces a pioneering HOFA voltage control (HOFA-VC) framework, uniquely bridging FAS theory's nonlinear regulation capabilities (Duan, 2024; Li et al., 2024) with the dynamic realities of decentralized power grids. Diverging from conventional FAS adaptations that impose rigid theoretical constructs, our methodology strategically reconfigures MG state-space to elevate system order while deploying auxiliary decoupling logic to neutralize destabilizing voltage-CPL couplings, critical for sustaining stability amid rapid load transients and stochastic converter interactions. By rigorously embedding these operational dynamics into a high-order FAS architecture, the resulting voltage regulation law achieves exponential bus voltage convergence without computational overburden. Beyond advancing FAS's practical relevance in power electronics, this synthesis establishes a scalable template for stabilizing next-generation MGs increasingly governed by power electronic interfaces and CPL saturation. The methodology distinguishes itself through three pivotal innovations:

1. For precise characterization of MGs with CPLs, this study presents a fully actuated MG model. Creating this model involves two primary tasks: choosing innovative state variables to establish a strict feedback system and crafting virtual control quantities for a fully actuated linear system. These endeavors result in a termed mathematical fully actuated MG system, thereby sidestepping complexities in designing nonlinear controllers via traditional methods.

2. The voltage control challenges in MGs with CPLs and different converters, like boost converters, are effectively managed within the proposed framework. In specific setups such as MGs with buck converters, only the loads demonstrate nonlinearity, simplifying the design of the HOFA-VC strategy. Conversely, certain converters, such as boost converters, introduce inherent nonlinearity, resulting in a more intricate controller design process.
3. In tackling load fluctuations, this study presents an HOFA-VC strategy with an integrated disturbance estimator. Diverging from conventional methods like backstepping-dependent adaptive control, the proposed strategy combines estimation and control in a unified framework. This fusion simplifies hierarchical design complexity, facilitates a direct link from load fluctuation management to voltage regulation, and offers a hardware-friendly implementation solution.

This chapter is organized as follows. The system description and problem formulation are presented in Sect. 5.2. Section 5.3 introduces the HOFA-VC design for DC MGs with known CPLs, while Sect. 5.4 provides HOFA-VC for DC MGs with unknown CPLs. Section 5.5 shows the experimental results. Section 5.6 provides a chapter conclusion.

5.2 Problem Formulation

5.2.1 Description of DC MGs

The single-bus DC MG system is the main focus of attention in this chapter, as shown in Fig. 5.1. The proposed methodology can also be extended to the design of device-level controllers for multi-bus DC MG systems. The proposed approach can also be extended to the design of device-level controllers for DC MG systems with multi-bus architectures. In DC MG architectures, non-isolated DC-DC converter

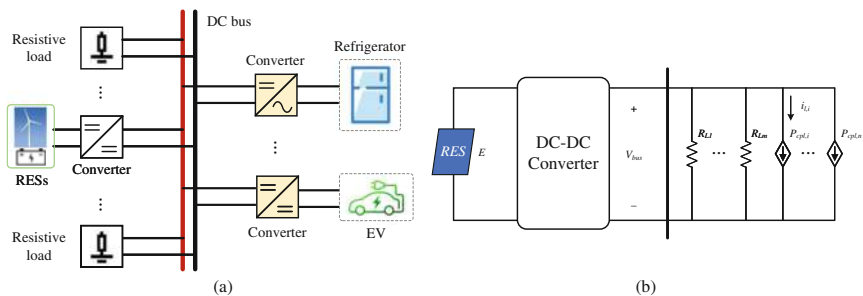


Fig. 5.1 DC MG that operates in off-grid mode: (a) system structure, (b) equivalent circuit model

topologies, especially buck converters for buck applications and boost converters for boost applications, are widely deployed power electronic interfaces.

In DC MG, a CPL refers to a load whose power consumption is regulated to remain approximately constant. In this load state, the absorbed power is regulated to remain approximately independent of voltage fluctuations within the operating range and can be expressed as $P_{\text{cpl},i} = V_{\text{bus}} \cdot i_{l,i}$ for $i \in \{1, \dots, N\}$. This behavior is common in many devices such as switched-mode power supplies, servomotor drives, LED drivers, and tightly regulated electronic circuits. In these devices, the internal feedback mechanism gives priority to stable power consumption, which leads to nonlinearity. As shown in Fig. 5.1, the nonlinear dynamics of the DC MGs studied in this chapter come from the following two main sources. First, power electronic converters governed by switching laws (e.g., boost topologies, DAB converter structures). Second, the power absorption behavior of the CPL. To facilitate the system analysis, Fig. 5.1 gives the complete picture of such a microgrid, where the converters (buck or boost) are clearly defined and the various loads are integrated. Here, the parallel CPLs and resistive loads are mathematically integrated as $P_{\text{cpl}} = \sum_{i=1}^n P_{\text{cpl},i}$ and $R_L^{-1} = \sum_{i=1}^m R_{Li}^{-1}$, respectively.

5.2.2 Regulation Tasks

In general, regardless of the type of topology, single-bus or multi-bus topology, and regardless of the type of power electronic interface, boost converter or any other type of converter, the main control goal of the MG with a single bus is voltage control, i.e.,

$$|V_{\text{bus}}(t) - V_{\text{ref}}| \rightarrow 0 \quad (5.1)$$

where V_{ref} is the desired value. From the above equations, it is easy to identify that the voltage regulation problem of a DC microgrid corresponds to a classical output regulation problem. The control task considered in this chapter is different from that of the MG with multi-bus structure, where only the voltage regulation task is of interest without considering other economic metrics.

The nonlinear dynamic characteristics of DC microgrids mainly originate from the power electronic converter devices (e.g., dual active bridge DAB converters) and specific load characteristics (especially CPLs). The negative differential impedance characteristic presented by the CPL is manifested by the fact that the load current follows the conservation relation $P_{\text{cpl},i} = V_{\text{bus}} \cdot i_{l,i}$ to increase inversely when the supply voltage decreases, and this characteristic will continue to weaken the inherent damping mechanism of the system. When the system is fully configured with a CPL, its negative feedback effect, which is contrary to the system steady state, is significantly amplified, thus triggering divergent voltage oscillations or system-level voltage instability. Experimental studies have shown (see literature Xu et al., 2025) that the stabilizing damping mechanism established by resistive

loads through passive energy dissipation can effectively suppress the nonlinear oscillations induced by CPLs. This reveals the deep connection between microgrid stability and load composition: CPLs reduce the stability margin through non-ideal power coupling, while resistive elements achieve oscillation suppression by virtue of excess energy dissipation characteristics.

Traditional proportional–integral (PI) control faces significant application bottlenecks in microgrid systems with constant power loads (CPLs), stemming from the inherent conflict between frequency domain analysis methods and nonlinear dynamics. Specifically, the challenges include the difficulty of accurately capturing the negative impedance instability characteristics induced by CPLs using the Nyquist criterion based on phase margin. Particularly in the low-frequency range, non-minimum phase crossing phenomena are prone to occur, leading to the failure of stability analysis. Moreover, controller parameter tuning heavily relies on empirical trial and error methods. In dynamic scenarios with multiple time scales, fixed-gain PI controllers fail to achieve multi-objective optimization in terms of robustness, speed, and steady-state accuracy.

Classical linear control theories (such as root locus and Bode plot design) face inherent limitations when dealing with strong nonlinear systems. The fundamental conflict arises from the theoretical foundation of linear time-invariant (LTI) assumptions and the actual nonlinear dynamic behaviors like bifurcations and chaos present in the system, exposing the inherent limitations of linearization methods. Although feedback linearization methods can achieve input–output linearization through differential homeomorphisms, their engineering feasibility is constrained by rigorous mathematical requirements. First, the relative degree completeness condition requires $L_g L_f^{r-1} h(x) \neq 0$ (where r is the relative degree, L_g and L_f are Lie derivative) to be satisfied, but CPL power disturbances can cause the r value to vary, breaking this condition. Second, zero-dynamic stability demands that the internal subsystem $\dot{z} = q(z, 0)$ be asymptotically stable at equilibrium points, yet unmodeled line impedances and distributed parameters in microgrids can lead to zero-dynamic instability. These two core constraints often render this method ineffective in engineering practice due to model uncertainties, revealing a gap between theory and practice.

These technical constraints objectively motivate the research community to turn to high-order nonlinear control theory solution paths. Based on this background, this research is dedicated to constructing a control-oriented modeling framework and developing a new control architecture for DC MGs. In particular, the control scheme constructed in this study innovatively realizes the dual breakthroughs of modular structure and transparency of feedback routes, whose core value is to significantly reduce the difficulty of controller design and engineering practice. The results are expected to provide a new technical paradigm and implementation framework for the evolution of nonlinear control strategy for MGs.

5.3 HOFA-VC for DC MGs With Known CPLs

5.3.1 Buck Converter-Based MGs With Known CPLs

Based on the standard topology configuration of the buck converter as illustrated in reference (Qi et al., 2024), this study sets the power module to operate consistently in Continuous Conduction Mode (CCM). This means maintaining a constraint where the inductor current remains nonzero continuously to avoid discontinuous conduction states. This constraint offers a quantified analytical baseline for system dynamic modeling (Li et al., 2025). Following the Kirchhoff's circuit laws (KCL/KVL), the constitutive equation set for the DC power converter unit corresponding to the buck architecture can be derived:

$$\begin{aligned} C_f \dot{V}_o &= i_L - R_L^{-1} V_o - V_o^{-1} P_{cpl} \\ L_f \dot{i}_L &= Eu - V_o - R_f i_L \end{aligned} \quad (5.2)$$

where the definitions of the variables are presented in Table 5.1. Throughout the analysis, the bus voltage is assumed to remain in a physically admissible range $V_o(t) \geq V_{\min} > 0$, and the operating region is restricted to points where the corresponding input gain is nonzero. Practical duty-cycle saturation is considered in implementation. Differentiating the output voltage twice yields

$$\ddot{V}_o = f_\alpha + g_\alpha u \quad (5.3)$$

where

$$\begin{aligned} f_\alpha &= \left(C_f^{-2} R_L^{-2} - C_f^{-1} L_f^{-1} \right) V_o + \left[C_f^{-2} \left(P_{cpl} V_o^{-2} - R_L^{-1} \right) - C_f^{-1} L_f^{-1} R_f \right] i_L \\ &\quad - C_f^{-2} P_{cpl}^2 V_o^{-3}, \\ g_\alpha &= C_f^{-1} L_f^{-1} E. \end{aligned} \quad (5.4)$$

Table 5.1 Symbols used in the system models

Symbols		Definitions
System states	i_L	Inductance current
	V_o	Capacitance voltage
Circuit parameters	R_f	Filter resistance
	L_f	Filter inductance
	C_f	Shunt capacitor
	R_L	Resistive load
	P_{cpl}	Constant power load
	E	Input voltage
	u	Duty cycle

Equation (5.3) can be interpreted as a second-order fully actuated form when $g_\alpha \neq 0$. Leveraging FAS theory, the state feedback control law can be structured as follows:

$$\begin{aligned} u &= -g_\alpha^{-1}(u_{ca} - u_{ct}) \\ u_{ca} &= f_\alpha. \end{aligned} \quad (5.5)$$

Here, u_{ca} and u_{ct} denote auxiliary and primary control terms, respectively. Applying this controller to (5.3) yields the fully actuated microgrid model for the buck converter configuration:

$$\ddot{V}_o = u_{ct} \quad (5.6)$$

formalizing its dynamic behavior under the proposed control framework.

To regulate voltage, the tracking error e is expressed as $e = v_{ref} - V_o$. From this definition, the dynamics of tracking error can be given as

$$\ddot{e} = -u_{ct}. \quad (5.7)$$

By analyzing the error dynamics of system (5.7), the primary control term u_{ct} can be constructed as

$$u_{ct} = k_p^\alpha e + k_d^\alpha \dot{e} \quad (5.8)$$

where k_p^α and k_d^α are control parameters.

Theorem 5.3.1 *For the controllers (5.5) and (5.8), if $k_p^\alpha > 0$ and $k_d^\alpha > 0$, the considered single bus DC MG system based on buck converters can accomplish the voltage tracking task in (5.1). Furthermore, asymptotic stability of the MG system is guaranteed.*

Proof Substituting u_{ct} into the error dynamics of (5.7) produces the second-order differential equation:

$$\ddot{e} + k_d^\alpha \dot{e} + k_p^\alpha e = 0. \quad (5.9)$$

The corresponding characteristic polynomial is:

$$P(s) = s^2 + k_d^\alpha s + k_p^\alpha = 0. \quad (5.10)$$

By the Routh-Hurwitz criterion, conditions $k_p^\alpha > 0$ and $k_d^\alpha > 0$ ensure that all roots of $P(s)$ lie in the left half-plane, thereby guaranteeing asymptotic stability of the error dynamics. Consequently, the voltage tracking error e converges to zero, implying V_o asymptotically follows v_{ref} . Furthermore, the inductor current i_L , governed by

$$i_L = (R_L^{-1} V_o^2 + P_{CPL})/E \quad (5.11)$$

attains a finite steady-state value as $t \rightarrow \infty$, confirming bounded and stable current dynamics. Thus, the DC MG system with the buck converter and CPLs is stabilizable under the proposed control laws. ■

5.3.2 Boost Converter-Based MGs With Known CPLs

Similar to the above section, the physical characteristics of the boost converter-based DC MG are described as follows:

$$\begin{aligned} C_f \dot{V}_o &= (1 - u)i_L - R_L^{-1}V_o - V_o^{-1}P_{cpl} \\ L_f \dot{i}_L &= -(1 - u)V_o + E \end{aligned} \quad (5.12)$$

where the meanings of the variables are presented in Table 5.1. Analysis of (5.12) reveals that the DC MG with CPLs operates as an underactuated nonlinear system. To derive the FAS model, a state transformation must first be applied to reformulate (5.12) into a strict-feedback structure. Guided by this rationale, the transformed state variables are defined as follows:

$$\begin{aligned} x_1 &= \frac{1}{2}(C_f V_o^2 + L_f i_L^2) \\ x_2 &= -R_L^{-1}V_o^2 - P_{cpl} + E I_L. \end{aligned} \quad (5.13)$$

Consequently, the FAS model for the DC MG incorporating a boost converter with CPLs is derived as

$$\ddot{x}_1 = f_\beta + g_\beta u \quad (5.14)$$

where

$$\begin{aligned} f_\beta &= L_f^{-1}V_{in}^2 - V_{in}L_f^{-1}V_o - 2R_L^{-1}C_f^{-1}V_o(t)I_L \\ &\quad + 2R_L^{-1}(R_L C_f)^{-1}V_o^2 + 2R_L^{-1}C_f^{-1}P_{cpl} \\ g_\beta &= L_f^{-1}E V_o + 2R_L^{-1}C_f^{-1}V_o I_L. \end{aligned}$$

Provided that $V_o \neq 0$, $i_L \neq 0$, $g_\beta \neq 0$ hold true, a fully actuated voltage controller can be structured as follows in regions avoiding the origin:

$$\begin{aligned} u &= -g_\beta^{-1}(u_{ca} - u_{ct}) \\ u_{ca} &= f_\beta \end{aligned} \quad (5.15)$$

where u_{ca} and u_{ct} retain the definitions outlined in Sect. 5.3.1. Under these conditions, the FAS model required for tracking control design is fully formulated as

$$\ddot{x}_1 = u_{ct}. \quad (5.16)$$

The derived system model represents a high-order FAS, contrasting with the model proposed in (Yu et al., 2024b). Transitioning to this FAS framework required the application of a state transformation, necessitating the redefinition of expected values for the transformed state variables. By leveraging the inherent voltage–current relationship, the following reference value is established:

$$x_{1,ref} = 1/2(L_f i_{L,ref}^2 + C_f v_{ref}^2) \quad (5.17)$$

where $i_{L,ref} = (R_L^{-1} v_{ref}^2 + P_{cpl})/E$.

Similarly, the tracking error is expressed as $e = x_{1,ref} - x_1$ for voltage regulation. By analyzing the FAS model (5.16) of the DC MG, the control input u_{ct} can be designed to be

$$u_{ct} = k_d^\beta (\dot{x}_{1,ref} - \dot{x}_1) + k_p^\beta (x_{1,ref} - x_1) \quad (5.18)$$

k_p^β and k_d^β are parameters to be determined.

The subsequent analysis focuses on evaluating the gain parameters of the HOFA-VC under known load conditions and examining the stability of the closed-loop MG system.

Theorem 5.3.2 *When implementing the controllers defined in (5.15) and (5.18), proper tuning of the k_p^β and k_d^β parameters ensures that the boost-converter-based DC MG system achieves accurate voltage tracking and maintains stable operation.*

Proof Following the methodology outlined in Theorem 5.3.1, this chapter demonstrates that the tracking error e converges asymptotically. This convergence ensures x_1 tracks $x_{1,ref}$. By the definition of $x_{1,ref}$, this equivalently confirms that the output voltage V_o achieves reference tracking ($V_o \rightarrow v_{ref}$). Furthermore, the stability of x_1 and V_o guarantees the existence of the limits $\lim_{t \rightarrow \infty} \frac{1}{2} (L_f i_L^2(t) + C_f V_o^2(t))$ and $\lim_{t \rightarrow \infty} V_o^2(t)$, thereby stabilizing the inductor current (i_L). Consequently, the DC MG system with CPLs achieves stabilization under the proposed control framework. This completes the proof. ■

The preceding analysis establishes an FAS model and voltage control framework for buck- and boost-converter-based DC MGs under known load conditions. We now extend this methodology to address control design for scenarios where loads are unknown.

5.4 HOFA-VC for DC MGs With Unknown CPLs

This section examines scenarios where CPLs and resistive loads are unknown.

5.4.1 HOFA-VC for Buck Converter–Interfaced DC MGs With Unknown CPLs

For buck converter–based DC MG systems operating under unknown load conditions, the high-order mathematical model can be formulated as follows:

$$\ddot{V}_o = f_{un} + g_{un}u + d_{un} \quad (5.19)$$

where $f_{un} = -C_f^{-1}L_f^{-1}V_o$, $g_{un} = C_f^{-1}L_f^{-1}E$, $d_{un} = -C_f^{-1}R_L^{-1}(i_L - R_L^{-1}V_o - V_o^{-1}P_{cpl}) - C_f^{-1}V_o^{-2}P_{cpl}(i_L - R_L^{-1}V_o - V_o^{-1}P_{cpl})$. Following FAS theory, the controller is structured as follows:

$$\begin{aligned} u &= -g_{un}^{-1}(u_{ca} - u_{ct}) \\ u_{ca} &= f_{un}. \end{aligned} \quad (5.20)$$

Substituting it into (5.19) gives the fully actuated MG model as

$$\ddot{V}_o = u_{ct} + d_{un}. \quad (5.21)$$

Let $x_1 = V_o$, $x_2 = \dot{x}_1$. It results in

$$\ddot{e} = -u_{ct} - d_{un} \quad (5.22)$$

It can be observed that the above system is an FAS with an uncertainty term. Similarly, assume that

$$\dot{d}_{un} = \theta. \quad (5.23)$$

In order to estimate the uncertainty term with unknown loads, the following observer will be designed

$$\begin{aligned} \hat{x}_2 &= u_{ct} + \hat{d}_{un} + \beta_1 e_{obs} \\ \dot{\hat{d}}_{un} &= \beta_2 e_{obs} \end{aligned} \quad (5.24)$$

where β_1, β_2 are control gains to be determined, $e_{obs} = x_2 - \hat{x}_2$. Let $\tilde{d}_{un} = d_{un} - \hat{d}_{un}$. Then, it can be obtained that

$$\dot{\tilde{d}}_{un} = \theta - \beta_2 e_{obs} \quad (5.25)$$

Based on FAS theory, u_{ct} is constructed as

$$u_{ct} = k_d^{buck}(\dot{V}_o^{ref} - \dot{V}_o) + k_p^{buck}(V_o^{ref} - V_o) - \hat{d}_{un} \quad (5.26)$$

where k_d^{buck}, k_p^{buck} are control parameters to be designed.

The stability analysis of the above closed-loop system is given below, accompanied by guidelines for the selection of control parameters.

Theorem 5.4.1 *By integrating three components, the adaptive estimator (5.24), the FA voltage control laws (5.20) and (5.26), and predefined assumptions on load uncertainty. That is, the system guarantees precise bus voltage tracking even with unpredictable load dynamics. This result holds provided positive-definite matrices P_{buck} and Q_{buck} satisfy the Lyapunov inequality*

$$A_{buck}^T P_{buck} + P_{buck} A_{buck} = -Q_{buck} \quad (5.27)$$

and

$$\lambda_{\min}(Q_{buck}) - a_{buck} \bar{\lambda} > 0 \quad (5.28)$$

where $\bar{\lambda} = \lambda_{\max}(P_{buck} E_{buck})$ and a_{buck} is a positive constant. This ensures asymptotic stability of the closed-loop error dynamics.

Proof The closed-loop dynamics of the tracking error can be obtained as

$$\ddot{e} = -k_d^{buck} \dot{e} - k_p^{buck} e - \tilde{d}_{un}. \quad (5.29)$$

Further, defining the state variables $\zeta = [e_{obs}, \tilde{d}_{un}, e, \dot{e}]^T$, the following closed-loop system in compact form can be obtained:

$$\dot{\zeta} = A_{buck} \zeta + E_{buck} \theta \quad (5.30)$$

where

$$A_{buck} = \begin{bmatrix} -\beta_1 & 1 & 0 & 0 \\ -\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -k_p^{buck} & -k_d^{buck} \end{bmatrix}, E_{buck} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Take into account the following Lyapunov function

$$V_\zeta = \frac{1}{2} \zeta^T P_{buck} \zeta. \quad (5.31)$$

With condition (5.27), taking the time derivative of V_ζ results in the following expression

$$\dot{V}_\zeta = -\frac{1}{2} \zeta^T Q_{buck} \zeta + \zeta^T P_{buck} E_{buck} \theta. \quad (5.32)$$

Recalling condition (5.28), the following inequality can be derived

$$\dot{V}_\zeta \leq -\frac{1}{2}(\lambda_{\min}(Q_{buck}) - a\bar{\lambda})\zeta(t)^T \zeta + \chi \quad (5.33)$$

where $\chi = \frac{1}{a}\theta^T P_{buck} E \theta$. This ensures convergence of the error state ζ to a neighborhood $Z_\Delta(0) = [-\sqrt{2\chi/\lambda_{\min}(Q_{buck}) - a\bar{\lambda}}, \sqrt{2\chi/\lambda_{\min}(Q_{buck}) - a\bar{\lambda}}]$.

Furthermore, by assigning

$$\begin{aligned} \psi_1(\zeta) &= \frac{1}{2}\lambda_{\min}(P_{buck})\zeta^T \zeta(t), \\ \psi_2(\zeta) &= \frac{1}{2}\lambda_{\max}(P_{buck})\zeta^T \zeta(t) \end{aligned}$$

the resulting relationship is derived as follows

$$\frac{1}{2}\lambda_{\min}(P_{buck})\zeta^T \zeta \leq V_\zeta \leq \frac{1}{2}\lambda_{\max}(P_{buck})\zeta^T \zeta. \quad (5.34)$$

The membership of $\psi_1(\zeta), \psi_2(\zeta) \in \mathcal{KL}$ is clearly established. By Lemma in (Sastry, 1999), the error dynamics are input-to-state stable, with θ as input. Under the assumption on θ , the error states asymptotically converge to zero. This completes the proof.

Furthermore, by Theorem 5.3.1's proof framework, the asymptotic convergence of the error state vector ζ confirms that the DC microgrid achieves both its voltage tracking objective and steady-state stability. ■

5.4.2 HOFA-VC for Boost Converter–Interfaced DC MGs With Unknown CPLs

As highlighted earlier, DC MGs employing boost converters exhibit greater dynamic complexity. For these systems operating with unknown CPLs, the state variables are reconfigured as follows:

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= Ei_L. \end{aligned} \quad (5.35)$$

Thus, one has

$$\dot{z}_1 = z_2 + d_{un} \quad (5.36)$$

where $d_{un} = -R_L^{-1}V_o^2 - P_{cpl}$. Moreover, differentiating $\dot{z}_1$ results in

$$\ddot{z}_1 = f_\gamma + g_\gamma u + \dot{d}_{un} \quad (5.37)$$

where $f_\gamma = EL_f^{-1}(-V_o + E)$, $g_\gamma = EL_f^{-1}V_o$. Similarly, under the condition $g_\gamma \neq 0$, the HOFA-VC strategy can be constructed as

$$\begin{aligned} u &= -g_\gamma^{-1}(u_{ca} - u_{ct}) \\ u_{ca} &= f_\gamma. \end{aligned} \quad (5.38)$$

By implementing the proposed controller in (5.37), the system dynamic can be rewritten as

$$\ddot{z}_1 = u_{ct} + \dot{d}_{un}. \quad (5.39)$$

This section describes the design of an observer for estimating the value of d_{un} . By treating d_{un} as an augmented state governed by $\dot{d}_{un} = \theta$ and assuming θ is bounded with $\lim_{t \rightarrow \infty} \theta(t) = 0$, the following observer can be formulated for MG systems with unknown loads

$$\begin{aligned} \dot{\hat{z}}_1 &= z_2 + \hat{d}_{un} + \gamma_1 e_z \\ \dot{\hat{d}}_{un} &= \gamma_2 e_z \end{aligned} \quad (5.40)$$

where $e_z = z_1 - \hat{z}_1$. Define $\tilde{d}_{un} = d_{un} - \hat{d}_{un}$ and $e = z_{1,ref} - z_1$. Then, there are

$$\dot{\tilde{d}}_{un} = \theta - \gamma_2 e_z \quad (5.41)$$

and

$$\ddot{e} = -u_{ct} - \dot{d}_{un}. \quad (5.42)$$

According to FAS theory, the u_{ct} is designed as

$$u_{ct} = \ddot{z}_1^{ref} + k_d^\gamma (\dot{z}_1^{ref} - \dot{z}_1) + k_p^\gamma (z_1^{ref} - z_1) - \dot{\hat{d}}_{un}. \quad (5.43)$$

In the following section, guidelines for parameter selection and stability assessment results for boost converter-based MGs with unknown load conditions are presented.

Theorem 5.4.2 *Under the framework of the proposed estimator (5.40), the FA voltage control strategy given by (5.38) and (5.43), and established load uncertainty assumptions, nonlinear DC MGs achieve robust bus voltage tracking and*

operational stability despite unknown load profiles. This guarantee holds if positive definite matrices P and Q satisfy the following stability criteria:

$$A^T P + P A = -Q \quad (5.44)$$

and

$$\lambda_{\min}(Q) - a\bar{\lambda} > 0 \quad (5.45)$$

where $A = [-\beta_1, 1, 0, 0; -\gamma_2, 0, 0, 0; 0, 0, 0, 1; -\gamma_2, 0, -k_P, -k_D]$, $E = [0, 1, 0, -1]^T$, $\bar{\lambda} = \lambda_{\max}(PE)$, and a is a positive constant.

Proof The proof process here is similar to that of Theorem 5.3.1 and is omitted here to avoid repetition.

Remark 5.4.1 This subsection proposes the HOFA voltage control strategy with a core design oriented toward a boost DC-DC converter microgrid system with uncertain load. It is worth noting that the framework can also be extended to other converter configurations such as dual active bridge (DAB) converters, LLC resonant converters, and other diverse topologies, provided that the state variables are appropriately modified. This adaptability emphasizes the versatility of the strategy in diverse converter architectures and its broad applicability in controller design.

5.5 Experiment Verification

In this section, experimental validations of the performance, effectiveness, and robustness of buck- and boost-converter-based MGs with the proposed fully actuated voltage control strategy are performed.

5.5.1 Hardware Description and Experimental Setup

The experimental setup (Fig. 5.2) integrates an OPAL-RT controller, a DC power supply, DC-DC converters, and two load categories: resistive loads and CPLs. CPLs emulate devices like electric vehicles by maintaining fixed power levels regulated via electronic load control. The DC microgrid operates with a 0.1-millisecond control cycle and a test voltage reference of $v_{\text{ref}} = 50$ V. The logical structure of the experimental platform is presented in Fig. 5.3. Additional hardware specifications and parameters are detailed in Table 5.2. Aligned with the hardware configuration and theoretical foundations, the HOFA-VC parameters are configured as follows:

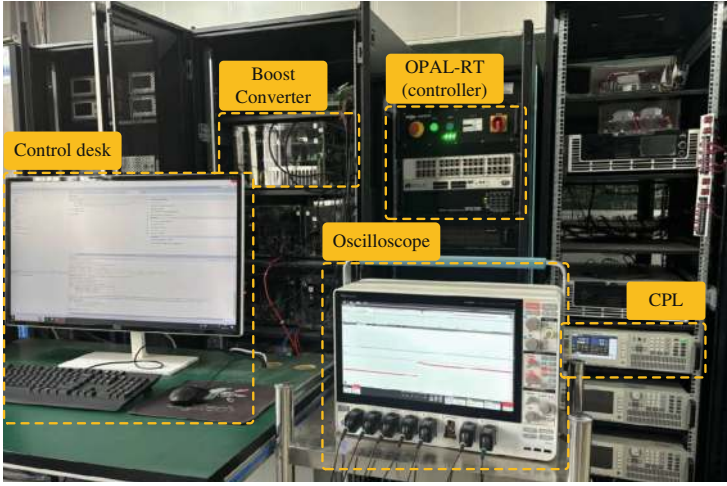


Fig. 5.2 Low voltage DC MG prototype with CPLs

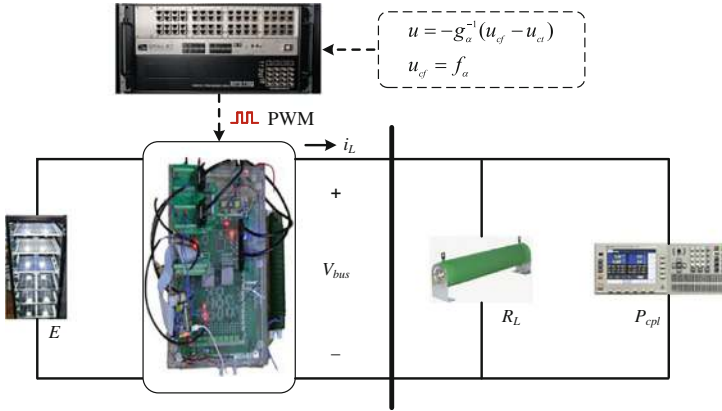


Fig. 5.3 Logical structure of the experimental platform

$$\begin{aligned}
 k_p^\alpha &= 5 \times 10^5, k_d^\alpha = 1 \times 10^4 \\
 k_p^\beta &= 5 \times 10^6, k_d^\beta = 6 \times 10^4 \\
 k_p^\gamma &= 8 \times 10^4, k_d^\gamma = 1 \times 10^1 \\
 \gamma_1 &= 5 \times 10^3, \gamma_2 = 4 \times 10^7.
 \end{aligned}$$

Three case studies were executed leveraging the experimental platform to evaluate the performance of:

1. a buck converter-based MG employing the HOFA-VC scheme;

Table 5.2 Specifications of the DC microgrid

Variable	Value [Unit]	Descriptions
/	10 [kHz]	PWM switching frequency
L_f	2 [mH]	Filter inductance
C_f	2000 [μ F]	Filter capacitance
P_{cpl}	100 [W]	Initial constant power load
R_L	50 [Ω]	Initial resistive load
E	25/100 [V]	DC source of the boost/buck converter

2. a boost converter–based MG utilizing the HOFA-VC scheme;
3. an MG operating under unknown load conditions.

Subsequent sections present the experimental results and comparative analysis of these scenarios to validate the efficacy of the proposed strategy. A detailed evaluation of the data highlights the method’s robustness and operational constraints, providing insights for targeted refinements and future investigations.

5.5.2 Performance Test of a Buck Converter–Based MG With the HOFA-VC

To verify the HOFA-VC’s efficacy in a buck converter–based MG with known loads, the operational sequence is segmented into four phases:

1. Interval $[t_0, t_1)$: Operation with resistive loads only under conventional PI control.
2. Interval $[t_1, t_2)$: Integration of a CPL into the system, activated at t_1 .
3. Interval $[t_2, t_3)$: CPL power increased to 150 W at t_2 , while maintaining HOFA-VC.
4. Post t_3 : Resistive loads are fully disconnected, leaving only the CPL powered by the MG, a critical stability-testing scenario.

As illustrated in Fig. 5.4, the HOFA-VC ensures precise output voltage tracking during both CPL activation ($[t_1, t_2]$) and power ramping ($[t_2, t_3]$). Despite peak stability challenges post- t_3 (CPL-only operation), the controller maintains robust dynamic performance and transient recovery, confirming its effectiveness in voltage regulation under varying load conditions.

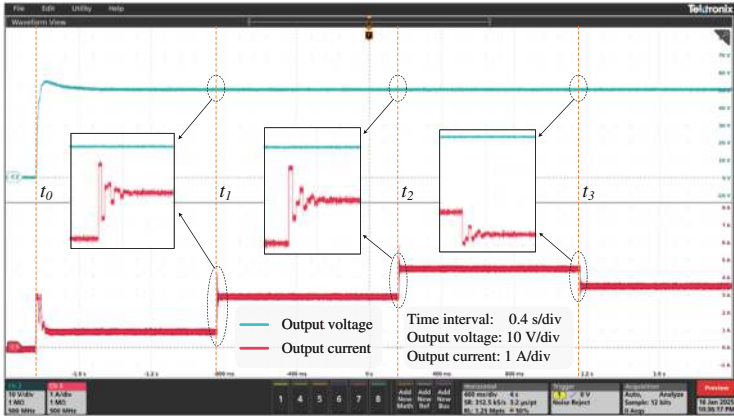


Fig. 5.4 Outputs of the buck converter–based microgrid with the proposed HOFA-VC method under specified load variations

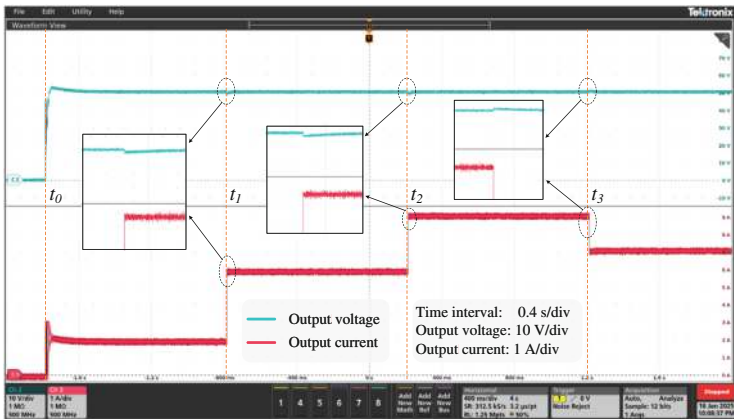


Fig. 5.5 Outputs of the boost converter–based microgrid with the proposed HOFA-VC method under specified load variations

5.5.3 Performance Test of a Boost Converter–Based MG With the HOFA-VC

This test replicates the methodology detailed in Sect. 5.5.2 of Chap. 5, with Fig. 5.5 illustrating the boost converter–interfaced MG’s response under HOFA-VC. The results demonstrate the controller’s capability to mitigate nonlinear dynamics inherent in boost converters and CPLs, achieving precise voltage regulation across operational cycles. Notably, the output current exhibits stable transient behavior despite abrupt load changes.

The proposed strategy enhances voltage tracking efficiency by suppressing non-linear interactions while maintaining seamless transient performance. Collectively, these findings validate HOFA-VC's adaptability to converter-specific challenges in practical microgrid architectures.

5.5.4 Performance Test of an MG With Unknown Loads

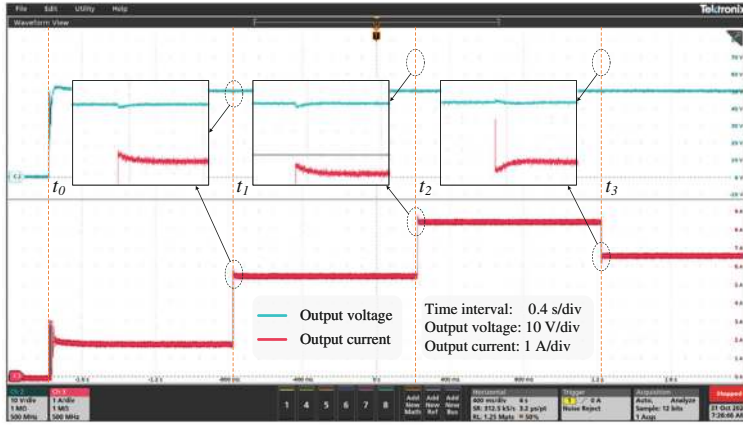
This subsection evaluates the performance of the proposed HOFA-VC in a microgrid with uncertain loads, benchmarking it against dual-loop PI control and the finite-time strategy reported by Ahmadi et al. (2023). The experimental setup follows the protocols outlined in earlier sections to ensure consistency. The method from Ahmadi et al. (2023), detailed in their Eq. (16), serves as a comparative state-of-the-art reference.

As illustrated in Fig. 5.6, the three approaches exhibit distinct dynamic behaviors under unknown loads. The PI controller induces pronounced voltage oscillations, as evidenced in Fig. 5.6b, signaling instability during load transients. While the method by Ahmadi et al. (2023) stabilizes the system (Fig. 5.6c), its convergence rate is sluggish, particularly under abrupt load transitions. In contrast, the proposed HOFA-VC, depicted in Fig. 5.6a, achieves robust voltage tracking within approximately 80 ms, excelling in scenarios with mixed resistive and CPLs, as well as in CPL-only conditions.

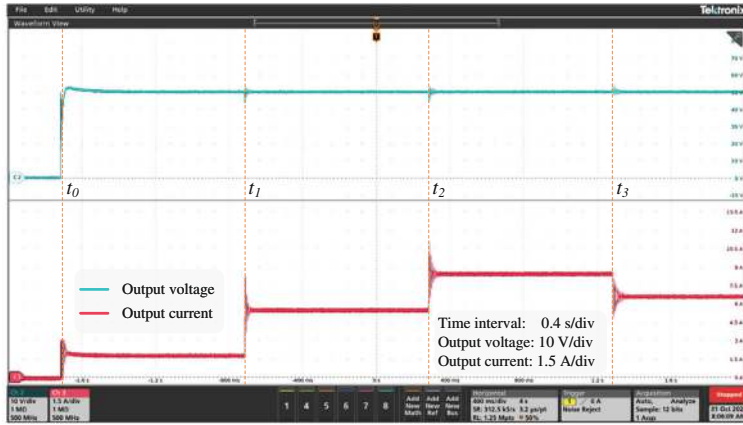
Quantitative metrics further underscore these differences. During CPL integration, the peak output voltages reach 53 V, 54 V, and 55 V for HOFA-VC, PI control, and the method in Ahmadi et al. (2023), respectively. Despite the marginally higher voltage magnitude in the latter two cases, HOFA-VC's lower peak aligns with reduced oscillations and faster stabilization. Collectively, the results confirm that the proposed controller not only accelerates transient response but also enhances voltage regulation accuracy, outperforming conventional and modern alternatives in addressing unknown load dynamics. These analysis and discussions emphasize HOFA-VC's advantages through direct comparison and quantified outcomes.

5.6 Notes and References

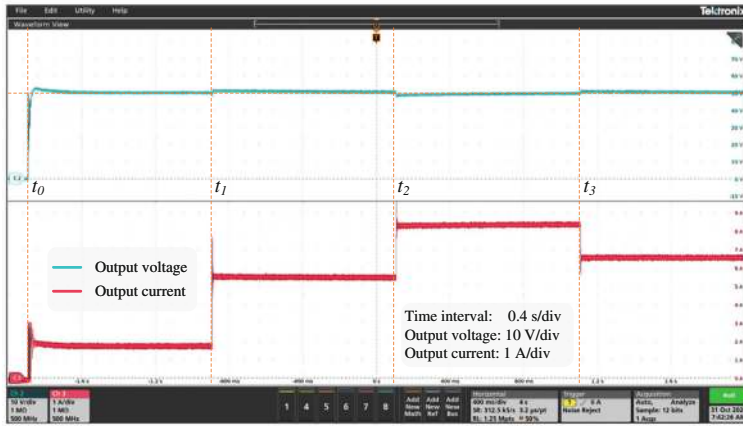
This chapter addresses the challenging issue of voltage stability control in a single-bus DC MG with CPLs and proposes an innovative solution based on the FAS theoretical framework. By designing state transformations, a unified nonlinear model with the capability to capture dynamic characteristics and compress system dimensions is established, surpassing the inherent contradiction between structural complexity and control feasibility in traditional modeling methods. The HOFA-VC strategy designed within this framework, deeply integrated with the system characteristics of FAS, eliminates redundant compensatory loops present in



(a)



(b)



(c)

Fig. 5.6 Response of the MG with (a) the proposed HOFA-VC method, (b) the PI method and (c) the method presented in (Ahmadi et al., 2023)

traditional nonlinear controls, constructing a structurally concise and dynamically transparent control architecture. Supported rigorously by Lyapunov stability theory, this approach achieves the configurability of pole locations similar to linear systems and the organic integration of nonlinear dynamics, enabling direct correlation of controller parameters with system physical properties, thereby forming an adjustment mechanism with clear engineering significance. To address uncertainties arising from load disturbances, an embedded uncertainty observer within the control strategy dynamically reconstructs load power variation trajectories through real-time coupling mechanisms, enabling adaptive compensation for unknown load fluctuations without the need for additional sensors. Theoretical analysis indicates that this mechanism ensures global stability under finite power disturbances while maintaining precise tracking of the bus voltage. Through comparative validation in typical industrial scenarios, this approach demonstrates superior dynamic response speed and disturbance rejection capabilities compared to traditional methods, particularly showcasing unique robust advantages in extreme conditions of multi-source power coupling. This methodological approach, which intricately binds the analysis of inherent system nonlinear characteristics with structured controller design, offers an expandable theoretical paradigm for addressing complex energy management issues in novel power system contexts.

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Chapter 6

Distributed Predictive Fully Actuated Control of Renewable Power Systems



Abstract The problem of voltage recovery and current balancing has been a hot research topic in the field of DC microgrids. How to build a model that reflects the circuit and component-level control (e.g., inner control loops) characteristics and design a secondary controller based on it to accomplish the abovementioned output regulations is the concern of this chapter. Accordingly, in this chapter, based on the large-signal model of microgrids, the corresponding high-order fully actuated model is developed for DC microgrids, which is not only simple in form but also able to capture the details of the actual power system, such as the relationship between voltage and current. Further, a distributed predictive fully actuated secondary control strategy is proposed based on the developed model to achieve voltage recovery and accurate current sharing. Allowing one to design the secondary controller free from the constraints of physical coupling in the microgrid is one feature of this control strategy, while the other is the arbitrarily assignable closed-loop poles. An analytical model of the closed-loop DC microgrid system with predictive fully actuated secondary control is developed, and the analysis of stability and output current consensus is presented therewith. Extensive tests on a hardware system of a DC microgrid with solar panels and an MPPT controller are performed to validate the performance of the designed controller.

6.1 Overview

The secondary controller has two main regulation tasks, namely voltage recovery and accurate current sharing (Lu et al., 2014; Long et al., 2022). Specifically, voltage restoration in a DC MG focuses on regulating the voltage at the output of each generation unit or at the PCC in order to meet the standard of electricity consumption (Alghamdi and Cañizares, 2021). Current sharing mainly indicates that the output current of each DG in the MG should satisfy some desired functional relationship between them, such as proportional distribution (Xu et al., 2022). In addition, there are two widely popular kinds of DC MGs, namely, the single bus type and the ring (series) bus type. Compared with a single-bus DC MG, output regulation of a ring-bus one is more challenging, which is the type of DC MGs that

this chapter is interested in. In a multi-bus DC MG with nonzero line resistances, strictly restoring all PCC voltages to the same reference value eliminates the voltage differences across the tie lines and hence drives their steady-state currents to zero. Consequently, each DG mainly supplies its local load, and accurate proportional current sharing cannot generally be achieved unless the local load distribution already matches the prescribed sharing ratios. Therefore, there is an inherent conflict between consistent voltage recovery and precise current sharing in a DC MG with the ring topology (Nasirian et al., 2014; Sahoo and Mishra, 2019; Yu et al., 2023b).

Regarding the abovementioned contradiction between voltage recovery and accurate current sharing, there are many proven methods (Han et al., 2019b; Du et al., 2022). For example, Vahidreza et al. in Nasirian et al. (2014) have decoupled the regulation of voltage and current through redefining the task of voltage recovery as adjusting the average value of voltages at all PCCs to the desired value. Meanwhile, modeling for DC MGs, which is highly relevant to voltage restoration and accurate current sharing, is also a hot research topic of attention (Zhou et al., 2020; Han et al., 2019a). For example, the authors in work Han et al. (2019a) considered the dynamics of the underlying circuit and used the voltage at the PCC as a control input to get a coupled multi-agent model. Such a modeling approach is widely used because it describes the electrical characteristics of DC MGs more comprehensively, but the coupled system model may cause some difficulties in the design of the secondary controller. The work in Nasirian et al. (2015) is representative of a class of modeling approaches based on the droop control interface. That is, a correction term is skillfully added to the droop control, and then using this equation to portray the relationship between the output of DC MGs and the correction term, where the correction term is generated by the secondary controller. This modeling approach is widely popular and has achieved great success because of its simplicity and ease of use (Xing et al., 2021). However, it does not actually model the underlying circuit characteristics, thus making it unsuitable for the design of model-based secondary controllers. Further, the optimal adaptive control was proposed in Jiang et al. (2021) by a constructed quadratic function of output currents of the distributed energy resource systems, which solves the voltage recovery and current sharing and cleverly unifies the secondary control and tertiary regulation. Despite these existing modeling methodologies well addressing voltage recovery and current balancing, there are still few models that can both portray the behaviors in the control layer and capture the actual underlying electrical dynamics well for the small power systems, DC MGs, in islanded operation mode.

Inspired by the above discussion and the work in Duan (2021a), this chapter develops a DC MG model that embodies the physical behavior of the DC MGs, facilitates the secondary controller design, and features a simple form. To this end, an overall description of the underlying circuit model, primary control, and droop control of the DC MG is presented, and a practical DC MG model based on HOFA system approaches is developed accordingly. Further, based on the established DC MG model, a distributed predictive secondary control strategy is proposed.

This chapter is organized as follows. The system description and design objective are presented in Sect. 6.2. In Sect. 6.3, the proposed predictive control of renewable energy power systems is presented. Experimental results of a microgrid are given in Sect. 6.4. Finally, Sect. 6.5 concludes the chapter.

6.2 Problem Formulation

In this section, the regulation tasks of interest in this chapter and a system description of a DC MG based on a droop control interface are given.

A DC MG consisting of distributed generation units interconnected by power lines in a ring topology is a typical one, and its overall structure is shown in Fig. 6.1. Although DC MGs incorporate various renewable energy sources such as wind turbines, photovoltaics, and energy-storage systems, they all contain typical components such as DC-DC converters, RLC filters, transmission lines, and local loads, as portrayed at the right of Fig. 6.1. The great fluctuations brought by the intermittent and random nature of renewable energy to the MG system are one of the characteristics that distinguishes it from the large grid, and this could affect the reliability of distributed power sources. To address this problem, energy storage schemes are mostly used in practice. In order to construct a physical model of DC MG, it is assumed here that these energy storage schemes can well solve the fluctuations of renewable energy.

6.2.1 Control Objectives

For DC MGs with ring topologies where line impedances exist between buses, there is an irreconcilable conflict between voltage restoration to a common reference value and accurate current sharing. First, it is known from the V/I characteristic of DC MGs that once the output voltage is established on a specific circuit, the

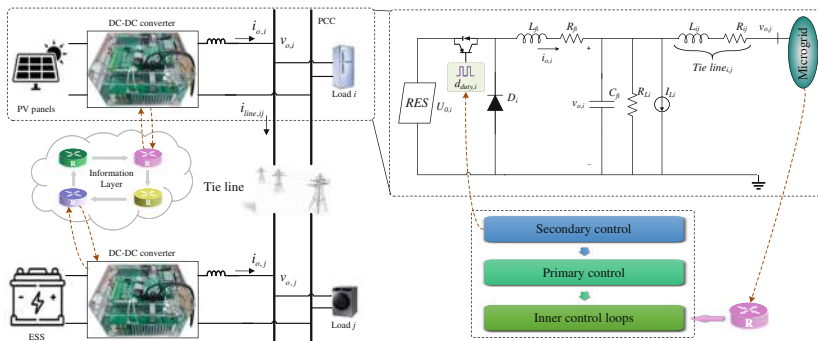


Fig. 6.1 DC MG with the ring bus topology

output current is also determined (Ding et al., 2018). Second, if the voltage at the PCC of each DG is restored to the desired value, e.g., $\lim_{t \rightarrow \infty} v_{o,i}(t) = 48$ V, for $i = 1, \dots, N$, then the MG will become a system consisting of several isolated generation units (i.e., a unit circuit comprising the power supply, converter, RLC, and local load). For such a system, although the generation subsystems are connected by power lines, there is no effective current flow on each line during actual operation. In particular, when the MG is running at the operation point (i.e., the system reaches steady state), there will be no current flow on line $line_{ij}$. Therefore, the DGs are unable to distribute current when the voltage is fully recovered to the same reference value. In addition, if only the current distribution within the MG is of concern, then it may result in the output voltage level of the MG not meeting the requirements of the distribution network.

Based on the above discussion, this chapter aims to design the secondary controller to achieve the voltage recovery to a given allowed range, and the current can be accurately shared according to a given ratio, i.e., the output voltage and current of the DC MG should satisfy

$$\lim_{t \rightarrow \infty} \left\| \frac{V_{ni}^{ref} - v_{o,i}(t)}{V_{ni}^{ref}} \right\| \leq \delta_i \quad (6.1a)$$

$$\lim_{t \rightarrow \infty} \left\| \frac{i_{o,j}(t)}{\eta_j} - \frac{i_{o,i}(t)}{\eta_i} \right\| = 0 \quad (6.1b)$$

where $\delta_i < \varepsilon$ and ε is the given allowable threshold (e.g., 5%), η_i is the desired sharing ratio of i th DG. Since the design of the secondary controller is inseparable from the system model, building a practical and simple DC MG model is a prior goal.

6.2.2 DC MG Modeling Based on HOFA System Approaches

In the following, the line-current dynamics are neglected and each tie line is represented by its effective resistance R_{ij} . Similar to the practice in Yu et al. (2023c), with the help of the QSL approximation and based on Kirchhoff's voltage and current laws, the above DC MG can be modeled as the following coupled linear system:

$$\begin{aligned} \dot{v}_{o,i}(t) &= \frac{i_{o,i}(t)}{C_{fi}} + \sum_{j \in \mathcal{N}_i^p} \frac{v_{o,j}(t) - v_{o,i}(t)}{C_{fi} R_{ij}} - \frac{v_{o,i}(t)}{C_{fi} R_{Li}} \\ \dot{i}_{o,i}(t) &= -\frac{v_{o,i}(t)}{L_{fi}} - \frac{R_{fi}}{L_{fi}} i_{o,i}(t) + \frac{U_{0,i} d_{duty,i}(t)}{L_{fi}} \end{aligned} \quad (6.2)$$

where $v_{o,i}$ and $i_{o,i}$ are respectively the voltage and current output of DG_i , $v_{o,j} \in \mathcal{N}_i^p$ are the voltage at the port linked with the i th DG , and $\mathcal{N}_i^p$ is the set of physically neighboring DG units connected to i th DG through the power lines. $d_{duty,i}$ stands for the duty cycle used to generate the PWM signal for driving the IGBT, and $U_{0,i}$ indicates the input voltage of the power supply. R_{fi} , L_{fi} and C_{fi} denote the electrical parameters of the filter.

Inspired by the previous subsection, this part is devoted to the development of the DC MG model that both captures the physical features of the MG and facilitates secondary controller design while being simple in form. For this purpose, we describe the underlying circuit model of the MG, the device-level control loops, and the droop control as a whole, and derive a large-signal model accordingly. The specific process of building the fully actuated DC MG model is elaborated in the following.

First, in order to stabilize the voltage and roughly equalize the current of each generation unit during the start-up phase, the droop control is used as follows:

$$v_{o,i}^*(t) = V_{ni}^*(t) - R_{vi}i_{o,i}(t) \quad (6.3)$$

where R_{vi} is the droop coefficient, $V_{ni}^*(t)$ is the voltage reference point and will be further designed later to accomplish the desired voltage and current regulation missions. It is common practice to employ dual closed-loop PI control for voltage and current at the device-level control layer, and here is no exception. Then, the dynamics of the outer voltage loop is given as follows:

$$\begin{aligned} \dot{\phi}_i(t) &= v_{oi}^*(t) - v_{o,i}(t) \\ i_{li}^*(t) &= k_{pvi}(v_{oi}^*(t) - v_{o,i}(t)) + k_{ivi}\phi_i(t) \end{aligned} \quad (6.4)$$

where k_{pvi} and k_{ivi} are the proportional and integral gains, respectively. Similarly, the dynamics of the inner current loop can be represented as

$$\begin{aligned} \dot{\gamma}_i(t) &= i_{li}^*(t) - i_{li}(t) \\ d_{duty,i}(t) &= k_{pci}(i_{li}^*(t) - i_{li}(t)) + k_{ici}\gamma_i(t) \end{aligned} \quad (6.5)$$

where k_{pci} and k_{ici} are the proportional and integral gains, respectively. Usually, $i_{li}(t)$ can be taken as $i_{li}(t) = i_{o,i}(t)$. Combining the abovementioned droop control (6.4), the underlying voltage controller (6.5), current controller (6.5), and the dynamics of the circuit with RLC filter (6.2), and taking the secondary voltage reference $V_{ni}^*(t)$ as the control input, the following complete state-space model of the DC MG can be obtained:

$$\begin{aligned} \dot{x}_i(t) &= A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t) + B_i u_{r,i}(t) \\ y_i(t) &= h_i^c(x_i(t)) \end{aligned} \quad (6.6)$$

where $h_i^c(x_i(t)) = C_i^c x_i(t)$, $x_i(t) = [v_{o,i}(t) \ i_{o,i}(t) \ \phi_i(t) \ \gamma_i(t)]^T$, $C_i^c = [0 \ 1 \ 0 \ 0]$, A_{ii} , and A_{ij} can be obtained from (6.2)-(6.5), which are presented in (6.2.2) shown at the bottom of the next page, $B_i = [0 \ U_{0,i}k_{pci}k_{pvi}/L_{fi} \ 1 \ k_{pvi}]^T$.

$$A_{ii} = \begin{bmatrix} -\frac{1}{C_{f,i}R_{L,i}} - \sum_{j \in \mathcal{N}_i^p} \frac{1}{C_{f,i}R_{ij}} & \frac{1}{C_{f,i}} & 0 & 0 \\ -\frac{1 + U_{0,i}k_{pc,i}k_{pv,i}}{L_{f,i}} & -\frac{R_{f,i} + U_{0,i}k_{pc,i}(k_{pv,i}R_{v,i} + 1)}{L_{f,i}} & \frac{U_{0,i}k_{pc,i}k_{iv,i}}{L_{f,i}} & \frac{U_{0,i}k_{ic,i}}{L_{f,i}} \\ -1 & -R_{v,i} & 0 & 0 \\ -k_{pv,i} & -(k_{pv,i}R_{v,i} + 1) & k_{iv,i} & 0 \end{bmatrix}.$$

$$A_{ij} = \begin{bmatrix} \frac{1}{C_{f,i}R_{ij}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.7)$$

From the above state-space model, it can be seen that $V_{ni}^*(t)$ is served as the control input and the established DC MG model is an underactuated coupled multi-agent system. Therefore, selecting current as the only controllable output of this underactuated system, and deriving the current output in the coupled large-signal model, the following high-order fully actuated DC MG model can be obtained:

$$\begin{aligned} \dot{i}_{o,i}(t) &= L_{F_i} h_i^c(x_i(t)) \\ &= C_i^c (A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t)) + b_{c,i}u_{r,i}(t) \end{aligned} \quad (6.8)$$

where $F_i(t) = A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t) + B_i u_{r,i}(t)$, $L_{F_i} h_i^c$ is the Lie derivative of h_i^c with respect to F_i (Khalil, 2015) and is defined by $L_{F_i} h_i^c = \nabla h_i^c F_i(t) = (\partial h_i^c / \partial x_i) F_i$, $b_{c,i} = C_i^c B_i = U_{0,i}K_{pc,i}K_{pv,i}/L_{fi}$. Further, since $b_{c,i} \neq 0$, the following fully actuated secondary controller can be designed:

$$u_{r,i}(t) = b_{c,i}^{-1}(u_{ca,i}(t) + u_i(t)) \quad (6.9)$$

where $u_{ca,i}(t)$ is the auxiliary control to be designed. Considering that the obtained integrated large-signal model of the DC MG (6.6) is a coupled multi-agent model, the coupling between the states of DG units would influence the design and deployment of some model-based distributed secondary controllers. To this end, the following auxiliary control for model decoupling is developed:

$$u_{ca,i}(t) = -C_i^c (A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t)). \quad (6.10)$$

Substituting (6.9) and (6.10) into (6.8), it yields the following high-order fully actuated DC MG model

$$\dot{i}_{o,i}(t) = u_i(t). \quad (6.11)$$

The subsequent predictive-control design concerns the transformed current-output dynamics. Stability of the complete physical subsystem further requires the internal dynamics associated with $v_{o,i}$, ϕ_i , and γ_i to remain stable under the selected operating conditions and primary-loop gains.

Comparing (6.6) and (6.11), it can be observed that the initially built integrated large-signal model of the DC MG is a coupled multi-agent model and is transformed into a decoupled first-order multi-agent model thereafter. In the process of model conversion, the key role is played by the design of the auxiliary control $u_{ca,i}(t)$, which characterizes the physical topology information of the DC MG as well as the dynamics of the underlying controls. First, the proposed integrated large-signal model is able to describe the circuit characteristics more comprehensively and is obtained in a way that is suitable for various application scenarios, such as the case where the resistance on the transmission line is considered, which makes the control scheme designed based on the resulting model generally not contradictory to the actual situation. In addition, the acquisition of decoupled first-order multi-agent models attenuates some potential hindrances to the design of distributed secondary controllers posed by the physical coupling of the system. Based on the above discussion, it can be seen that the proposed high-order fully actuated DC MG model is more compatible with the actual situation than the one proposed in Sahoo and Mishra (2019); Yu et al. (2023b), which is consistent with physical properties but does not directly inherit its original complex structure.

Remark 6.2.1 Fundamentally, characterizing the dynamic behavior of MGs requires a rigorous incorporation of their inherent physical constraints, grounded in fundamental electrical principles such as Kirchhoff's voltage and current laws, rather than treating them merely as abstract mathematical entities. By leveraging advanced mathematical tools like the HOFA system approach, a direct analytical bridge can be constructed between the transient voltage and current dynamics. This formulation is highly advantageous as it enables these coupled electrical states to be systematically decoupled from the perspective of controller design. In conventional control frameworks relying on algebraic equation-based DC MG models (Nasirian et al., 2015), the intrinsic physical properties of the network are typically relegated exclusively to the post-design system stability analysis phase. Conversely, the HOFA-based dynamic model (6.11) developed in this study effectively transcends these traditional limitations. It explicitly embeds the authentic electrical characteristics of the actual MG directly into the distributed controller design stage, thereby ensuring physical consistency without sacrificing analytical tractability.

Remark 6.2.2 In the existing literature concerning circuit-level modeling, researchers frequently incorporate the hardware dynamics by treating the reference values of the inner double closed-loop controllers as control inputs and the PCC voltages as system outputs (Han et al., 2019a). While this methodology generates a coupled multi-agent model capable of comprehensively capturing the network's electrical traits, it fundamentally compromises the distributed nature of the secondary control layer (Yu et al., 2023a). Specifically, when a secondary

controller is synthesized atop a coupled framework, the mutual influence among DGs mandates a global communication topology. Each local DG must acquire the full network's dynamic information to predict future state evolutions, rendering true distributed deployment computationally prohibitive.

Although the present study initially formulates a coupled large-signal model for the DC MG, it significantly advances the analytical paradigm. By applying the HOFA system methodology, this coupled formulation is mathematically transformed into a decoupled first-order multi-agent model. This crucial decoupling step not only structurally simplifies the system representation but also facilitates the fully distributed implementation of the proposed secondary predictive controller in practical real-world scenarios.

Furthermore, the comprehensive large-signal model derived herein differs from alternative contemporary approaches. For instance, the modeling strategy in Sadabadi et al. (2023) focuses solely on the physical network circuit while entirely neglecting the inner voltage/current control loops and the hierarchical control architecture. However, integrating the low-level double closed-loop controllers is virtually indispensable for mitigating circulating currents and ensuring primary operational stability (Guerrero et al., 2011). Ignoring these layers simultaneously compromises the inherent benefits of hierarchical control, notably the dynamic isolation between local device-level regulation and system-wide secondary coordination. In contrast, the framework established in this chapter unifies the physical hardware dynamics, the inner closed-loop regulators, and the primary droop mechanisms into a cohesive large-signal model, thereby offering a more holistic and practical representation than those found in Han et al. (2019a), Sadabadi et al. (2023).

Remark 6.2.3 An alternative prevailing control paradigm involves augmenting the conventional droop control mechanism with an auxiliary correction term, treating it directly as the secondary control input (Nasirian et al., 2015). Mathematically, this formulation is expressed as $v_{o,i}(t) = V_{ni}^{ref} - R_{vi}i_{o,i}(t) + \delta_i(t)$, where $\delta_i(t)$ represents the superimposed correction signal. Synthesizing this correction term based on the aforementioned algebraic equation to accomplish voltage restoration and load current sharing has been extensively adopted and proven highly successful in practice. The most prominent advantage of this methodology lies in its structural simplicity, which greatly facilitates the derivation of various distributed secondary control protocols while remaining theoretically unconstrained by the underlying physical topology of the DC MG. Furthermore, by abstracting away the actual physical dynamics, this algebraic approach naturally exhibits exceptional robustness against variations in system parameters.

Despite these distinct merits, such methods inherently suffer from specific analytical and structural limitations. For instance, while the framework in Nasirian et al. (2015) effectively designs distributed controllers for voltage and current regulation relying purely on the droop-based algebraic formulation, its rigorous stability analysis ultimately necessitates complete global knowledge of all transmission line impedances across the entire DC MG. More importantly, because

this strategy entirely bypasses the internal dynamic equations of the MG, it is classified as a model-free methodology, specifically denoting the complete absence of system model parameters in the controller design process, rather than implying a data-driven approach. Consequently, this algebraic formulation poses significant obstacles when advanced control strategies are required. In complex operational scenarios, such as those addressed in this chapter, specific dynamic performance metrics must be explicitly optimized, and optimal control laws must be analytically solved using a precise dynamic system representation. In contrast, the methodology proposed in this work establishes a comprehensive dynamic framework, thereby empowering the synthesis of an advanced, system-model-based secondary controller capable of predictive optimization.

6.3 Fully Actuated Predictive Control of DC MG Systems

In this section, a voltage limiter is introduced to address the conflict between voltage recovery and accurate current equalization, and the design of the secondary controller is carried out in this framework. In particular, a cost function considering the future output current is designed for DC MGs in order to improve the system performance, and a DPFA secondary controller is proposed based on it.

6.3.1 *Setting of Communication Network Based on Graph Theory*

In the fashion of distributed control, an undirected graph is used to represent the communication connections between the converters in the MG (Bidram et al., 2013). It should be noted that the use of a directed graph representing one-way communication is also feasible for modeling the communication network of DC MGs. In MGs, each DG is regarded as a node in the communication graph. The communication link in a communication network is represented by the edge of the corresponding graph. An undirected graph is characterized by a triple $\mathcal{G} = (\mathcal{V}_G, \mathcal{E}_G, \mathcal{A}_G)$, where $\mathcal{V}_G = \{v_1, \dots, v_N\}$ expresses the set of DGs, $\mathcal{E}_G \subseteq \mathcal{V}_G \times \mathcal{V}_G$ is a set of edges. Like the conductance matrix of an MG, the associated adjacency matrix $\mathcal{A}_G$ consists of communication weights, where $a_{ij} > 0$ if node j could transmit signal to node i , otherwise $a_{ij} = 0$. The entry a_{ij} indicates the weight of the information transferred from node j to i . These parameters can be considered as gains in data transmission and will be determined by the subsequent performance requirements of the closed-loop MG, which are assumed to be time-invariant. The set of neighbors of node i is denoted as $\mathcal{N}_i^n$. For a graph, if $j \in \mathcal{N}_i^n$, then v_j can transmit data directly through the communication network to v_i .

6.3.2 DPFA Coordinated Secondary Control

Specifically, a secondary regulation solution is brought in to achieve overall voltage and current regulation. The scheme is to add a voltage limiter to the voltage set point (given to the primary control by the secondary control layer) to guarantee that the voltage is in a certain allowed range and to design a distributed current controller for each DG to achieve accurate current sharing. With the limiter, the actual voltage reference $V_{sec,i}^*(t)$ received by the double closed-loop control will be a mapping of the control signal generated by the secondary control, i.e.,

$$V_{sec,i}^*(t) = \begin{cases} V_{ni}^{ref} - \epsilon_i, & V_{ni}^*(t) \in (-\infty, V_{ni}^{ref} - \epsilon_i] \\ V_{ni}^*(t), & V_{ni}^*(t) \in (V_{ni}^{ref} - \epsilon_i, V_{ni}^{ref} + \epsilon_i] \\ V_{ni}^{ref} + \epsilon_i, & V_{ni}^*(t) \in (V_{ni,i}^{ref} + \epsilon_i, +\infty) \end{cases} \quad (6.12)$$

where $\epsilon_i = \delta_i V_{ni}^{ref}$. It is worth mentioning that once the secondary control is activated, the reference value V_{ni}^{ref} in the droop control (6.3) switches to the voltage reference point $V_{sec}^*(t)$ generated by the secondary layer. It should be noted that the voltage limiter in (6.12) is retained as a practical safety mechanism. In the theoretical analysis and operating scenarios considered in this chapter, the controller parameters, initial conditions, and admissible disturbances are assumed to keep $V_{ni}^*(t)$ strictly within the prescribed voltage range. Consequently, the limiter remains inactive and $V_{sec,i}^*(t) = V_{ni}^*(t)$, allowing the unconstrained HOFA model to be used in the subsequent controller design and stability analysis. Unless otherwise stated, the same inactive-limiter assumption is adopted throughout the remainder of this book whenever an analogous limiting or saturation mechanism is introduced. The active-limiter case is beyond the scope of this book.

With a slight abuse of notation, the time indices in the discrete space are also denoted by t . Then, Assuming zero-order-hold implementation of $u_i(t)$ over each sampling interval, the exact discrete-time representation of (6.11) is

$$i_{o,i}(t+1) = i_{o,i}(t) + T_s u_i(t) \quad (6.13)$$

where T_s is the sampling period of DC MGs.

In order to improve the robustness to external disturbances and the dynamic performance of the system, the following cost function with future output states is proposed

$$\begin{aligned} J_i(t) = & \sum_{j \in \mathcal{N}_i^n} a_{ij} \left\| \hat{i}_{so,j}^{vec}(t + N_y|t) - \hat{i}_{so,i}^{vec}(t + N_y|t) \right\|_2^2 \\ & + b_i \left\| \hat{U}_i^{vec}(t + N_u|t) \right\|_2^2 \end{aligned} \quad (6.14)$$

where b_i is a positive penalty factor about the control cost, N_y and N_u are the lengths of the prediction horizons for the output and control input, respectively, i.e., $N_y > N_u > 0$, and $\hat{i}_{so,i}^{vec}(t + N_y|t) \in \mathbb{R}^{N_y \times 1}$ and $\hat{U}_i^{vec}(t + N_u|t) \in \mathbb{R}^{N_u \times 1}$ denote the prediction vectors for the weighted output current vector $i_{so,i}^{vec}(t + N_y|t)$ and the incremental control input vector $U_i^{vec}(t + N_u|t)$ of i th DG based on the information at the current moment t , respectively. Moreover, their specific expressions are

$$\begin{aligned}\hat{i}_{so,i}^{vec}(t + N_y|t) &= [\hat{i}_{so,i}(t + N_y|t), \hat{i}_{so,i}(t + N_y - 1|t), \dots, \hat{i}_{so,i}(t + 1|t)]^T \\ \hat{U}_i^{vec}(t + N_u|t) &= [\hat{u}_i(t + N_u|t), \hat{u}_i(t + N_u - 1|t), \dots, \hat{u}_i(t|t)]^T\end{aligned}$$

where $\hat{i}_{so,i}(t + k) = \hat{i}_{o,i}(t + k)/\eta_i$, for $k \in \mathbb{N}$.

The first term in the abovementioned cost function represents the difference between the weighted output currents of individual DGs, which play a key role in accomplishing accurate current sharing in MGs through the communication network. The another entry of the cost function describes the incremental cost of the control input of the i th DG, which acts as a penalty term to restrict the variation rate of the control input to prevent too large duty cycle changes from triggering the hardware protection of the power electronic devices. A debugging trick is that the weight factor corresponding to this item should not be too small in the experiment, because the hardware system is very sensitive to changes in duty cycle. Moreover, from the cost function (6.14), it can be seen that DG unit $\#i$ only cooperates with the neighbors whose information is successfully received by controller $\#i$. That is, if the communication link is interrupted, the packet from neighbor unit $\#j$ corresponding to this communication link will not be received by controller $\#i$, and the secondary reference $V_{sec,i}^*(t)$ will be calculated according to the same pattern as when the communication failure did not occur. At this point, the accomplishment of control objectives remains unaffected while ensuring connectivity.

Define the incremental weighted output current as $\Delta i_{so,i}(t + 1) = i_{so,i}(t + 1) - i_{so,i}(t)$, where $i_{so,i}(t) = i_{o,i}(t)/\eta_i$ and the operator Δ has the same meaning for all other variables. Based on the abovementioned HOFA discrete-time MG model (6.13), using all the input and output data available at time t , the prediction of the future value of the weighted output current and its prediction in incremental form for the i th DG can be acquired as

$$\begin{aligned}\hat{i}_{so,i}(t + 1|t) &= i_{so,i}(t) + T_s \eta_i^{-1} \hat{u}_i(t|t) \\ \Delta \hat{i}_{so,i}(t + 1|t) &= \Delta i_{so,i}(t) + T_s \eta_i^{-1} \Delta \hat{u}_i(t|t)\end{aligned}\tag{6.15}$$

where $\hat{u}_i(t|t) = u_i(t)$. Based on the above prediction of the weighted output in incremental form $i_{so,i}(t)$, a similar strategy is applied to calculate its k -step ahead prediction, resulting in

$$\hat{i}_{so,i}(t + k|t) = i_{so,i}(t) + T_s \eta_i^{-1} \sum_{l=1}^k \hat{u}_i(t + l - 1|t)\tag{6.16}$$

where $k = 2, \dots, N_u$.

Furthermore, for the time beyond the prediction window of the control input, let $\hat{u}_i(t+k|t) = \hat{u}_i(t+N_u|t)$, where $k = N_u + 1, N_u + 2, \dots, N_y$. In this case, the prediction of the incremental weighted output current of the i th DG is

$$\hat{i}_{so,i}(t+k|t) = T_s \eta_i^{-1} \sum_{l=k-N_u}^k \hat{u}_i(t+k-l|t) + i_{so,i}(t) \quad (6.17)$$

where $k = N_u + 1, N_u + 2, \dots, N_y$. Then, writing (6.16) and (6.17) in a compact form leads to

$$\hat{i}_{so,i}^{vec}(t+N_y|t) = \Lambda_i i_{so,i}(t) + \Theta_i \hat{U}_i(t+N_u|t) \quad (6.18)$$

where

$$\Lambda_i = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \Theta_i = T_s \eta_i^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ & & & \vdots & & \\ & 0 & 0 & 1 & 1 & \dots & 1 \\ & & & \vdots & & \\ & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

To search for the optimal solution of the current control increment in the secondary control, from the first-order optimality condition, let

$$\partial(\sum_{i=1}^N J_i(t))/\partial(\hat{U}_i^{vec}(t+N_u|t)) = 0. \quad (6.19)$$

Then, one has

$$\begin{aligned} & \left(\frac{\partial \hat{i}_{so,i}^{vec}(t+N_y|t)}{\partial \hat{U}_i^{vec}(t+N_u|t)} \right)^T \sum_j (a_{ji} + a_{ij}) (\hat{i}_{so,j}^{vec}(t+N_y|t) \\ & - \hat{i}_{so,i}^{vec}(t+N_y|t)) = b_i \hat{U}_i^{vec}(t+N_u|t). \end{aligned} \quad (6.20)$$

Substituting (6.18) into the above equation, one gets

$$\begin{aligned} & \Theta_i^T \sum_{j=1}^N a_{ij}^s (\Lambda(i_{so,j}(t) - i_{so,i}(t))) = \\ & (b_i I_{ii} + \tilde{a}_i \Theta_i^T \Theta_i) \hat{U}_i^{vec}(t+N_u|t) - \Theta_i^T \sum_{j=1}^N a_{ij}^s \Theta_j \hat{U}_j(t+N_u|t) \end{aligned} \quad (6.21)$$

where $\tilde{a}_i = \sum_{j=1}^N (a_{ji} + a_{ij})$, and $a_{ij}^s = a_{ij} + a_{ji}$. Writing the above equation in a compact form, one has

$$\Gamma \Delta \hat{U}(t + N_u | t) = S i_{so}^{vec}(t) \quad (6.22)$$

where

$$\Gamma = \begin{bmatrix} \vdots & \vdots \\ \cdots - \sum_j a_{ij}^s \Theta_i^T \Lambda_i \cdots & a_{ij}^s \Theta_j^T \Lambda_j \cdots \\ \vdots & \vdots \\ \cdots a_{ji}^s \Theta_i^T \Lambda_i \cdots & - \sum_l a_{jl}^s \Theta_j^T \Lambda_j \cdots \\ \vdots & \vdots \end{bmatrix}$$

$$S = \begin{bmatrix} \vdots & \vdots \\ \cdots b_i I_{ii} + \tilde{a}_i \Theta_i^T \Theta_i \cdots & -\Theta_i^T a_{ij}^s \Theta_j \cdots \\ \vdots & \vdots \\ \cdots -\Theta_j^T a_{ji}^s \Theta_i \cdots & b_j I_{jj} + \tilde{a}_j \Theta_j^T \Theta_j \cdots \\ \vdots & \vdots \end{bmatrix}.$$

Therefore, the solution of the above optimal problem for DC MG can be obtained as

$$\hat{U}(t + N_u | t) = \Gamma^{-1} S i_{so}^{vec}(t). \quad (6.23)$$

Further, for the i th DG, the optimal control increment to achieve current sharing is designed as

$$u_i(t) = g_i \Gamma^{-1} S i_{so}^{vec}(t) \quad (6.24)$$

where $g_i \in \mathbb{R}^{1 \times N_u}$ is a row vector with all entries equal to zero except for its final entry, which equals one. Therefore, combining (6.9) and (6.24), the actual control input of the i th DG can be obtained as

$$u_{r,i}(t) = b_{c,i}^{-1} (u_{ca,i}(t) + g_i \Gamma^{-1} S i_{so}^{vec}(t)). \quad (6.25)$$

In particular, the specific implementation procedure of the proposed DPFA secondary control can be summarized as Algorithm 1.

Algorithm 1 DPFA secondary control strategy

Input Current and voltage measurements, primary droop gains, desired sharing ratio, and sampling period.

Output DPFA secondary control input $u_{r,i}(t)$.

```

1: while the secondary control layer is activated do
2:   Receive the weighted output current signals  $i_{so,j}(t)$  from neighboring DGs.
3:   Predict the future output vector required for the cost function (6.14), utilizing the predictive
      model (6.18).
4:   Select the appropriate weighting factors  $a_{ij}$  and  $b_i$ , and compute the necessary coefficient
      matrices required in (6.23), such as  $\Gamma$ .
5:   Set the state and input vectors ( $i_{so,j}$ ,  $i_{so,j}^{vec}$ , and  $U_j^{vec}$ ) to zero vectors of appropriate dimen-
      sions for all non-neighboring nodes ( $j \notin \mathcal{N}_i$ ), and subsequently derive the incremental
      control input  $u_i(t)$  using (6.24).
6:   Obtain the actual DPFA secondary voltage reference  $u_{r,i}(t)$  via (6.25).
7:   Enforce saturation limits on the nominal reference input  $V_{ni}^*(t)$  by executing (6.12) to ensure
      it remains within the allowable operating voltage range, yielding the bounded reference
       $V_{sec,i}^*(t)$ .
8:   Apply the bounded secondary reference  $V_{sec,i}^*(t)$  to the primary droop control layer.
9:   Let  $t := t + 1$ .
10: end while

```

6.3.3 Stability and Consensus Analysis of DC MGs With DPFA Control

In this section, the stability and consensus analysis of DC MGs with the proposed DPFA control strategy will be given.

Without loss of generality, the sharing error between the output currents of the i th DG and the 1st one can be defined as

$$e_i^s(t) = i_{so,i}(t) - i_{so,1}(t) \quad (6.26)$$

for $i = 2, 3, \dots, N$. Recalling the developed model of the DC MG (6.11), the dynamics of the coordination error of the weighted output currents can be given as

$$\begin{aligned} e_i^s(t+1) = & e_i^s(t) + \Delta i_{so,i}(t) + T_s \eta_i^{-1} \Delta u_i(t) \\ & - \Delta i_{so,1}(t) - T_s \eta_1^{-1} \Delta u_1(t). \end{aligned} \quad (6.27)$$

With the definition of the operator Δ , the above equation can be rewritten as

$$e_i^s(t+1) = e_i^s(t) + T_s \eta_i^{-1} u_i(t) - T_s \eta_1^{-1} u_1(t). \quad (6.28)$$

Recalling (6.21), for the term associated with $i_{so,i}(t)$ in it, according to (6.26), one has

$$E i_{so,i}(t) - E i_{so,j}(t) = E e_i^s(t) - E e_j^s(t). \quad (6.29)$$

Then, it can be rewritten (6.21) as

$$\Theta_i^T \sum_{j=1}^N a_{ij}^s (I_E(e_j^s(t) - e_i^s(t))) = \gamma_{ii} \hat{U}_i^{vec}(t + N_u|t) - \sum_{j=1}^N \gamma_{ij} \hat{U}_j(t + N_u|t) \quad (6.30)$$

where $I_E = \Lambda_i$, $\gamma_{ii} = b_i I_{ii} + \tilde{a}_i \Theta_i^T \Theta_i$, $\gamma_{ij} = \Theta_i^T a_{ij}^s \Theta_j$. For the sake of analysis, define $e_E^s(t) = [Ee_2^s(t), Ee_3^s(t), \dots, Ee_N^s(t)] = KE^s(t)$, $\bar{\Lambda} = [I_E; 0] \in R^{N_y \times N}$, $K = \text{diag}\{\bar{\Lambda}, \dots, \bar{\Lambda}\} \in R^{NN_y \times N}$, and $\Delta E^s(t) = [\Delta E_2^s(t), \Delta E_3^s(t), \dots, \Delta E_N^s(t)]$. Up to this point, (6.30) can be written in the following compact form:

$$\Gamma \hat{U}(t + N_u|t) = S e_E^s(t) \quad (6.31)$$

where S is given in (6.22). Thus, the control input can be obtained as

$$u_i(t) = g_i \Gamma^{-1} S e_E^s(t) \quad (6.32)$$

where $i = 1, 2, \dots, N$. So far, the dynamics of the coordination error of the weighted currents can be expressed as

$$e_i^s(t+1) = 2e_i^s(t) - e_i^s(t-1) + T_s(g_i \eta_i^{-1} - g_1 \eta_1^{-1}) \Gamma^{-1} S e_E^s(t). \quad (6.33)$$

Further, it can be written in vector form as follows:

$$\begin{aligned} E^s(t+1) &= E^s(t) - D_g \Gamma^{-1} S K E^s(t) \\ &= (I - D_g \Gamma^{-1} S K) E^s(t) \end{aligned} \quad (6.34)$$

where $D_g = [T_s \eta_2^{-1} g_2 - T_s \eta_1^{-1} g_1; T_s \eta_3^{-1} g_3 - T_s \eta_1^{-1} g_1; \dots; T_s \eta_N^{-1} g_N - T_s \eta_1^{-1} g_1]^T$. Next, substituting (6.32) into the dynamics of $\Delta i_{o,1}$ yields

$$\begin{aligned} \Delta i_{so,1}(t+1) &= \Delta i_{so,1}(t) + T_s \eta_1^{-1} \Delta u_1(t) \\ &= \Delta i_{so,1}(t) + T_s \eta_1^{-1} g_1 \Gamma^{-1} S K (E^s(t) - E^s(t-1)). \end{aligned} \quad (6.35)$$

Define a vector $\zeta(t) = [E^s(t), E^s(t-1), \Delta i_{so,1}(t)]^T$. Combining (6.34) and (6.35), the closed-loop system equation of the DC MG under DPFA secondary control strategy can be derived as

$$\zeta(t+1) = \Omega \zeta(t) \quad (6.36)$$

where

$$\Omega = \begin{bmatrix} I - D_g \Gamma^{-1} S K & 0 & 0 \\ I & 0 & 0 \\ T_s \eta_1^{-1} g_1 \Gamma^{-1} S K & -T_s \eta_1^{-1} g_1 \Gamma^{-1} S K & I \end{bmatrix}.$$

It should be noted that the augmented matrix Ω in (6.36) contains an identity block associated with the absolute incremental-current coordinate. Therefore, strict Schur stability of the entire matrix Ω is not imposed. Instead, the convergence of the current-sharing error is determined from the reduced error subsystem in (6.34). Define

$$\Psi = I - D_g \Gamma^{-1} S K.$$

The stability condition for the current-sharing error dynamics is therefore formulated in terms of the Schur stability of Ψ . Subsequently, summarizing the above derivation and analysis, the following theorems on the stability and consensus of the DC MG are presented.

Theorem 6.3.1 *With the proposed DPFA secondary control strategy, if the reduced current-sharing error matrix $\Psi = I - D_g \Gamma^{-1} S K$ is Schur stable, then the weighted current-sharing errors converge asymptotically to zero, and the accurate current-sharing objective in (6.1b) is achieved. In addition, under the inactive-limiter assumption and the internal-dynamics stability condition stated after (6.11), if the local voltage and current PI loops stably track their bounded secondary references, the PCC voltages remain within the prescribed allowable range and the voltage-recovery objective in (6.1a) is achieved.*

Proof From (6.34), the reduced current-sharing error dynamics are given by $E^s(t+1) = \Psi E^s(t)$, where $\Psi = I - D_g \Gamma^{-1} S K$. If Ψ is Schur stable, then $\lim_{t \rightarrow \infty} E^s(t) = 0$. According to the definition of the sharing error in (6.26), this implies $\lim_{t \rightarrow \infty} \left(\frac{i_{o,j}(t)}{\eta_j} - \frac{i_{o,i}(t)}{\eta_i} \right) = 0$, for all $i, j \in 1, 2, \dots, N$, and therefore the accurate current-sharing objective in (6.1b) is achieved.

For voltage regulation, the inactive-limiter assumption ensures that the secondary reference remains within the prescribed allowable voltage range. Under the additional internal-dynamics stability condition stated after (6.11), the local voltage and current PI loops stably track their bounded references. Consequently, the PCC voltages remain within the allowable range specified in (6.1a). This completes the proof. $\blacksquare$

6.4 Experiment Verification

To demonstrate the effectiveness and superiority of the DPFA control scheme presented in this section and the correctness of the related analysis results, an

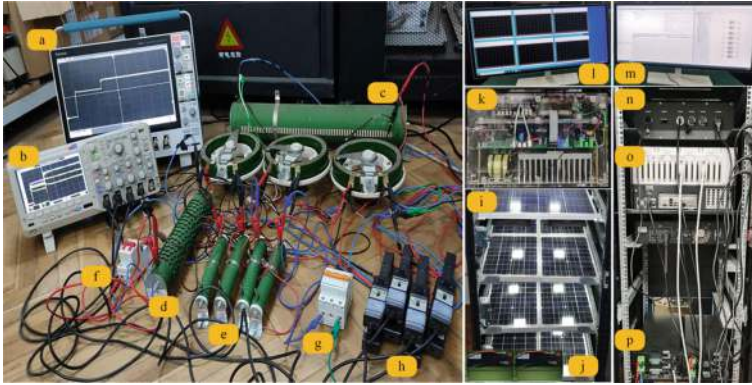


Fig. 6.2 Hardware layout of the islanded DC MG test system: (a) Oscilloscope (voltage); (b) Oscilloscope (current); (c) Resistive loads; (d) Resistance for load variation; (e) Tie lines; (f) Switch for load change; (g) Switch for plug-and-play; (h) Current probes; (i) PV panels; (j) Storage batteries; (k) DC-DC converters (4 sets); (l) Monitor; (m) Control desk; (n) Supply for OPAL-RT; (o) OPAL-RT; (p) DC sources

actual photovoltaic-based DC MG equipped with maximum power point tracking controller is constructed in the laboratory.

6.4.1 Hardware Description and Experimental Procedure

In particular, to illustrate the performance of the DPFA control strategy in practical applications in terms of steady-state output quality, fast dynamics, smooth transients, and accurate current sharing, a hardware experimental platform with an isolated DC MG shown in Fig. 6.2 is constructed. The physical diagram shows that the DC MG consists of four distributed generator sets (i.e., $\mathcal{V}_G = \{1, 2, 3, 4\}$), a hardware circuit system, a hardware controller, and a monitoring platform. In order to build a power generation unit based on renewable energy resources in a limited laboratory site, a set of photovoltaic panels with batteries act as one of the power sources, and the other three are substituted with DC voltage sources. The hardware circuit system is composed of DC-DC buck converters, LC filters, local resistive loads, and power lines connecting the DGs. The OPAL-RT (OP4510) acts as a hardware controller to receive analog signals from the voltage and current sensors of each DG. It is worth mentioning that the measured voltages and currents in the OPAL-RT need to be amplified by a factor of 10 to obtain the actual values, since its ADC channels have a maximum voltage limit. The proposed controller is implemented in the OPAL-RT. Therefore, the duty cycle signal generated by the control algorithm is eventually converted into a PWM signal by the built-in FPGA module of the OPAL-RT to drive the IGBTs in the converter.

Table 6.1 Settings of the hardware DC MG prototype

DG parameters		DG ₁ & DG ₄		DG ₂ & DG ₃			
Input voltage	$U_{0,i}$	100 80		80 100			
Droop	R_{vi}	2 2.5		1 1.5			
Voltage	k_{pvi}	0.1		0.2			
loop	k_{ivi}	1		1			
Current	k_{pci}	0.001		0.001			
loop	k_{ici}	2.2		2.2			
LC filter	L_{fi}	≈ 2		≈ 2			
[mH], [μF]	C_{fi}	1500 ± 20%		1500 ± 20%			
Line	Line 1	Line 2		Line 3		Line 4	
impedance	R_{12} 0.5	R_{23} 0.5	R_{34} 0.5		R_{41} 0.5		
[Ω], [μH]	L_{12} 318	L_{23} 318	L_{34} 325		L_{41} 325		
Loads	Local loads					Switching load	
[Ω]	R_L 11 10 10 10	$R_{L, \text{cut}}$ 1					

Note that the DC MG in the testbed is organized in a ring topology. Meanwhile, the communication topology of the information layer also adopts a ring structure, and its specific adjacency matrix is as $\mathcal{A}_G = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$. It can be confirmed that the graph satisfies the connectivity required to achieve consensus. In particular, this topology is utilized in all subsequent case studies unless otherwise specified. The control period of the DC MG is set to $T_s = 5$ ms and the current distribution factor is fixed to $\eta = [2 \ 1 \ 1 \ 2]$. The voltage setpoint in the test is chosen to be $V_{ni}^{ref} = 48$ V, which is a typical value in a low-voltage DC distribution network (Nasirian et al., 2015). The allowable threshold for voltage deviation from the given reference point is set to $\varepsilon = 5\%$, i.e., the lower and upper limits of the output voltage are 45.6 V and 50.4 V, respectively. For convenience, the specifications of the hardware devices in the DC MG and other experimental parameters not given are listed in Tables 6.1 and 6.2. They include, for example, the control parameters of the dual closed-loop PI for voltage and current. Based on the hardware and underlying control settings given in Table 6.1, it can be verified that the control parameters provided above satisfy the stability condition of Theorem 6.3.1.

Table 6.2 Main specifications of the converters

Descriptions	Value
Rated power of the converters	1~10 kW
Maximum input voltage	<750 V
Maximum output voltage	<750 V
Tolerable DC maximum current	20 A
Switching frequency	<20 kHz
Input range of the sensor	± 50 V
Dead time	2 μ s

Recalling the control objectives proposed in II.A, the voltage of each DG at the PCC is to be restored to a permissible range, while the current is to be shared precisely so that the current output of each DG matches its own capacity. To fully demonstrate that the proposed control strategy can perform these tasks, five case studies are conducted on the developed platform: (1) performance comparison with existing methods, (2) performance evaluation for control parameters, (3) performance evaluation for communication delays, (4) robustness to load variations, and (5) verification of plug-and-play functionality.

Further, the basic experimental procedure is described as follows: in the initial stage, the four DGs operate only under the droop control action; after the system is stabilized, the proposed DPFA control strategy is activated, and the system enters the secondary regulation stage of voltage recovery and accurate current sharing. It can be seen that DC MGs are basically open-loop stabilized systems, i.e., the voltage and current of the DC system can become stable only under the effect of the droop control. It should be noted that since the hardware system runs in real time, it is difficult to use absolute time as a reference, so the notations t_0, t_1, t_2, t_3, t_4 , etc., are used to represent each critical time point in the analysis of the experimental results that follow. For the above four experimental scenarios, each DG enters the start-up phase at t_0 , transitions to the secondary controller regulation phase at t_1 , and then only at the next time point is the occurrence of the connection and disconnection of a load and the unplugging and plugging of one DG.

6.4.2 Performance Comparison and Assessment

Case I: Results under the existing consensus estimator-based secondary voltage recovery and current sharing scheme for DC MGs (Nasirian et al., 2014, 2015)

To address the abovementioned contradiction between voltage and current control, the following distributed voltage observer-based (DVO) secondary control method is first proposed by the authors in (Nasirian et al., 2014, 2015)

$$v_{oi}^*(t) = V_{ni}^{ref}(t) - \delta_{c,i}(t)i_{o,i}(t) + \delta_{v,i}(t) \quad (6.37)$$

where $\delta_{c,i}(t)$ and $\delta_{v,i}(t)$ are the correction terms to be engineered. In the DVO secondary control framework, the voltage observer is designed as

$$\dot{\bar{v}}_{o,i}(t) = \dot{v}_{o,i}(t) + \sum_{j \in N_i} a_{ij}(\bar{v}_{o,j}(t) - \bar{v}_{o,i}(t)). \quad (6.38)$$

Then, the correction terms for voltage restoration and current sharing are given as

$$\begin{aligned}
\delta_{v,i}(t) &= k_i^{P,v} (V_{ni}^{ref} - \bar{v}_{o,i}(t)) + k_i^{I,v} \int (V_{ni}^{ref} - \bar{v}_{o,i}(\tau)) d\tau \\
\delta_{c,i}(t) &= k_i^{P,c} \sum_{j \in N_i} ba_{ij} e_{ji}^s(t) + k_i^{I,c} \int \sum_{j \in N_i} ba_{ij} e_{ji}^s(\tau) d\tau
\end{aligned} \tag{6.39}$$

where $e_{ji}^s(t) = e_j^s(t) - e_i^s(t)$, $k_i^{P,v}$, $k_i^{I,v}$, $k_i^{P,c}$, $k_i^{I,c}$, b , a_{ij} are the control parameters to be determined.

According to given parameters, Table 6.2, and the adjacency matrix of the used communication topology, the abovementioned control parameters can be determined by trial-and-error approach as $k_i^{P,v} = 0.1$, $k_i^{I,v} = 1.2$, $k_i^{P,c} = 0.3$, $k_i^{I,c} = 3.2$, $b = 0.5$. From the experimental results shown in Fig. 6.3, it can be seen that the DVO secondary control strategy can achieve voltage recovery to a certain range as well as precise current sharing in the MG. However, under this approach, the overall dynamics of the MG is not modeled, and therefore

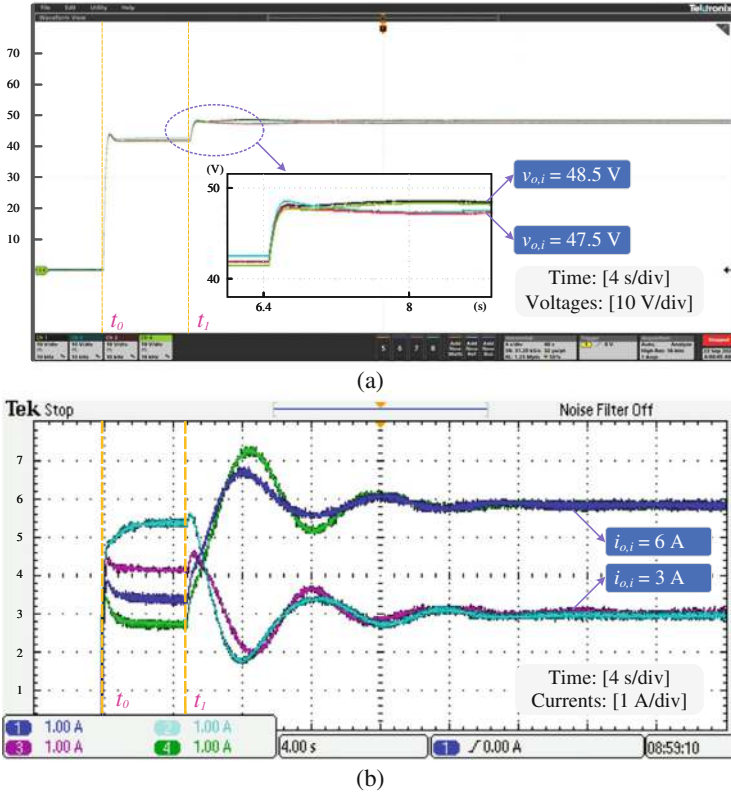


Fig. 6.3 (a) Voltage responses and (b) current responses of the MG test system under the existing DVO control method

performance metrics with future states could not be accounted for, which poses a significant limitation to the use of methods such as model predictive control. Second, in Nasirian et al. (2015), the physical relationship between voltage and current is not directly reflected in the design process of the DVO secondary control strategy, but only in the eventual stability analysis, which can also bring some limitations to its wide application. In addition, the stability analysis presented in this work is for the dynamics of the controllers designed for voltage and current, and then the relationship between voltage and current, $I = Y_{MG,bus} V$, where $Y_{MG,bus}$ is the conductance matrix of the DC MG, is used to illustrate the feasibility of achieving voltage recovery and accurate current sharing simultaneously. Accordingly, it can be seen that the underlying control loops and circuit-level dynamics of the DC MG are not considered, which are essential in small-signal analysis (Liu et al., 2021). Further, in this case corresponding to the Fig. 6.3, there is a huge overshoot in the output of the DC MG, but its regulation is still slow. In summary, the DVO method cannot achieve the trade-off between the speed of control and the smoothness of the outputs of the DC MG.

Case II: Results under the proposed DPFA secondary control approach for DC MGs

This subsection demonstrates the basic performance of the proposed DPFA secondary control method in the built DC MG. The weighting factor corresponding to the control cost in the performance metric (6.14) is chosen to be $b_i = 90$ for $i \in \mathcal{V}_G$, and the lengths of the prediction windows for the output and control input are set to $N_y = 8$ and $N_u = 7$, respectively. Figure 6.4 draws the output responses of the DC MG in this case. As can be seen from the results shown in Fig. 6.4, the rapid recovery of the voltage to the specified range and the precise current sharing are achieved after activating the proposed DPFA secondary control at the t_1 . In particular, compared to the results obtained in *Case I*, the proposed DPFA method achieves better regulation, such as faster and smoother voltage and current responses, as a result of the design process of the controller taking into account the performance index with future states. Moreover, with the DPFA control method, there is no need to worry about the discrepancy between the theoretically designed controller and the actual situation, because the physical characteristics of the DC MG have been adequately paid attention to in the course of modeling. Besides, the stability analysis for the whole DC MG supports that more insights on the system dynamics can be provided under the developed model (6.11).

Based on the experimental waveforms under the above two cases, we have the following observations and discussions.

- (i) Compared with the results obtained in *Case I*, the proposed DPFA method achieves better regulation effects, such as the current and voltage regulation process under the proposed method is both fast and smooth, unlike the DVO-based method, which could hardly achieve both fastness and smoothness. In particular, the voltage tracking and current sharing are accomplished significantly faster in *Case II* than that in *Case I*.

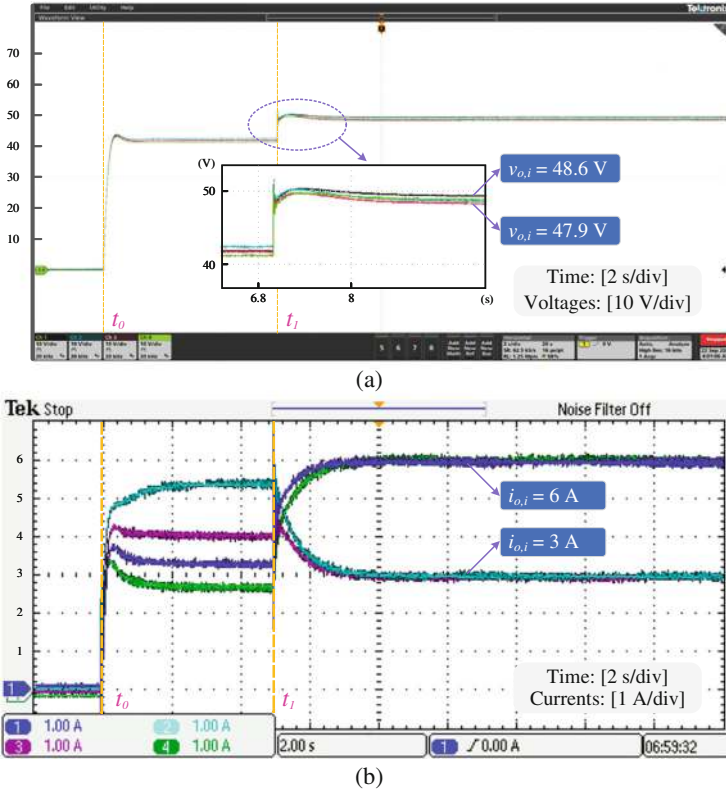


Fig. 6.4 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme

- (ii) Under the DVO-based approach, the physical coupling between the DGs and the underlying electrical characteristics are not captured in the design process of the control strategy, and these are only involved in the final stability analysis. This imposes certain limitations on its control performance. In contrast, our proposed method retains the circuit, inner-loop, and droop-control dynamics included in the adopted model.
- (iii) Due to the consideration and modeling of the complete dynamics of DC MGs, the stability analysis under the developed HOFA model in this chapter can provide additional perspectives on the relationship between the underlying system dynamics and the closed-loop system performance. This is a unique strength for possessing a wider range of application scenarios, especially for the design of performance-specific controllers.

At this point, the above results demonstrate that the proposed DPFA secondary controller allows the DGs to reach steady state and achieve accurate current sharing quickly.

6.4.3 Performance Assessment Under Different Control Parameters

In this case, the effect of control parameters on the performance of the built DC MG system is examined. To be specific, the weight parameter b_i in the cost function (6.14) is increased and decreased respectively, and the corresponding experimental results are shown in Figs. 6.5 and 6.6. As can be seen from Fig. 6.5, when b_i increases to 130, i.e., similar to the control gain is reduced, the settling time of the DC MG system is longer, but the accomplishment of the control objective is basically unaffected. As can be seen from Fig. 6.6, when b_i reduces to 50, i.e., similar to the control gain is increased, the system reaches a steady state very quickly after the MG system activates the secondary control, achieving voltage recovery and current sharing well, but with relatively large overshoot and rapid small-amplitude oscillations for a short time after the switchover. Since $\Delta u_i(t)$ is obtained in this chapter by optimizing a cost function with future predicted states,

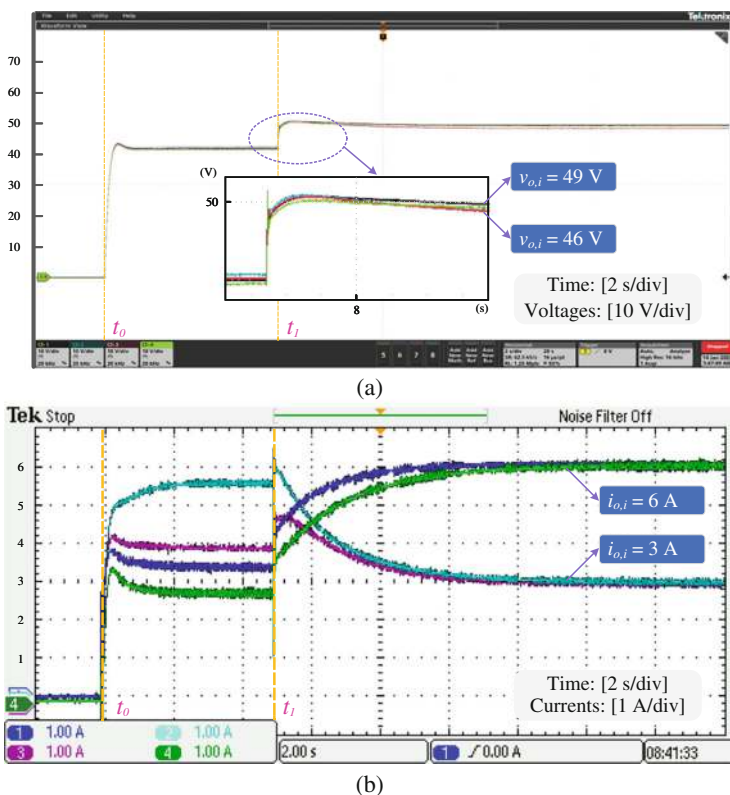


Fig. 6.5 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme as b_i increases

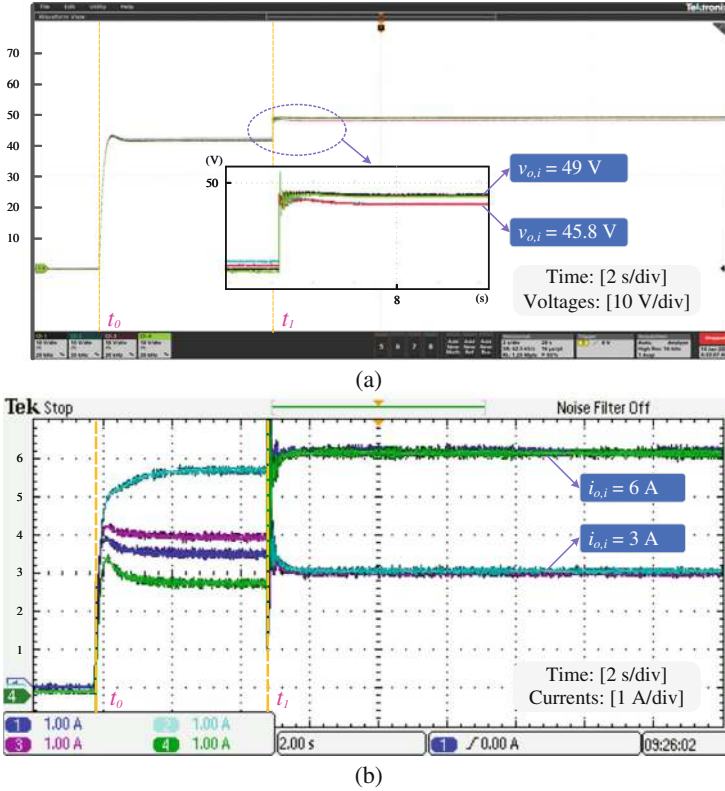


Fig. 6.6 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme as b_i decreases

its final control input $u_i(t)$, which can be freely designed in the secondary control $u_{r,i}(t)$, is similar to an integral control signal. Therefore, comparing the results in Figs. 6.4, 6.5 and 6.6, it can be concluded that the effect law of the control gain in the proposed method on the control performance is similar to that of the integration coefficient in the conventional PID on the control performance, i.e., the larger the gain, the faster the regulation process, but bringing unstable factors such as greater overshoot to the MG system. Since the secondary control method designed in this chapter has a simple form, only the weight coefficients in the cost function need to be adjusted. Therefore, the trial-and-error method can be used to make a trade-off between rapidity and smoothness of regulation according to actual requirements.

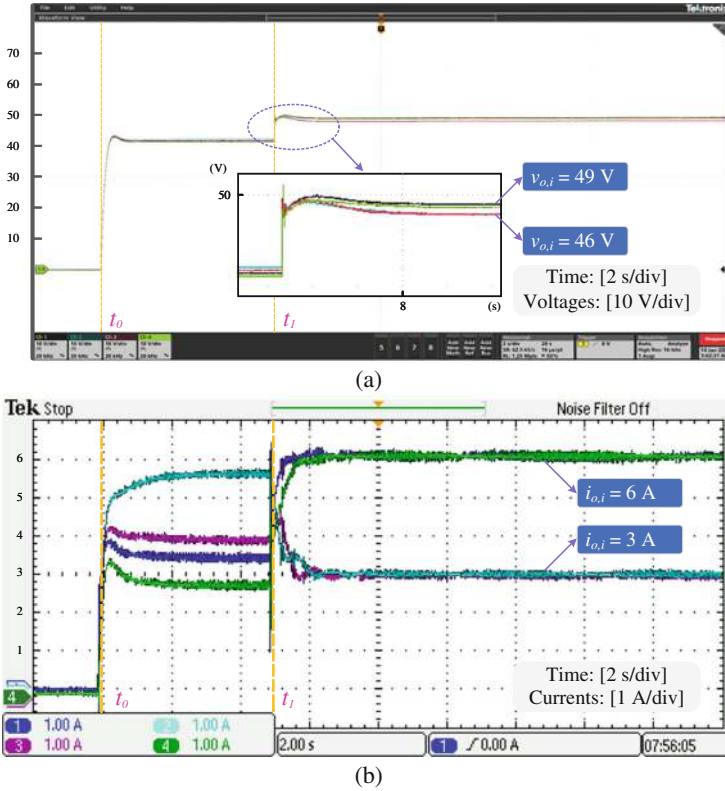


Fig. 6.7 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme with 90 ms communication delay

6.4.4 Performance Assessment Under Communication Delays

Further, the resistance of the proposed secondary control method to communication delays is tested in this case. In order to reveal the effect of different communication delays on the control performance of the proposed method, we investigate the performance variation of the built DC MG system under three communication delays, and the results are shown in Figs. 6.7, 6.8, and 6.9, respectively. In Fig. 6.7, the communication delay is 90 ms, and the output voltage and current of the MG system only mildly jitter during the dynamic process. Compared to the results given in Fig. 6.4, the regulation process of the MG under this communication delay is not as smooth as that under the ideal communication situation, but the regulation time is basically unaffected. In Fig. 6.8, the communication delay is 120 ms. From the results, it can be seen that at this point the MG system has experienced significant and prolonged oscillations, and the system is stabilizing slowly, but the control objectives can still be accomplished. In Fig. 6.9, the communication delay

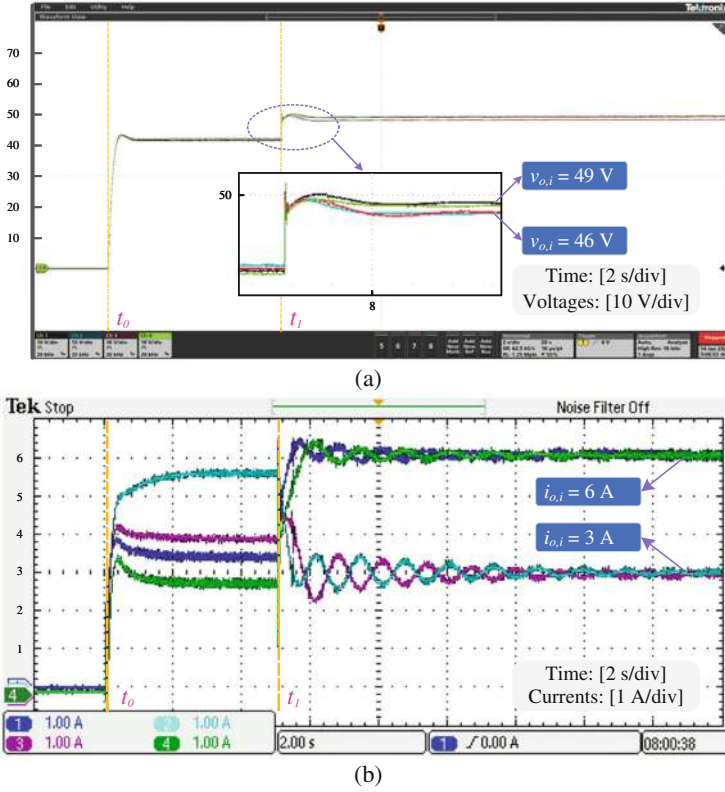


Fig. 6.8 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme with 120 ms communication delay

is further increased to 150 ms. At this point, the MG system has difficulty reaching stability, and the oscillation amplitude gradually grows until the system collapses. The above analysis on the experimental results shows that a smaller communication delay is tolerable for the MG system under the proposed control method, but a communication delay of about 120 ms or more would seriously damage the system performance and even cause the MG system to crash after 15 ms.

6.4.5 Robustness to Load Variations

Following the previous study, the robustness of the proposed control strategy against load variations will be investigated on the DC MG hardware testbed. The specific experimental steps in this case are: (1) after the DC MG achieves voltage recovery and precise current sharing, a 1 ohm resistor is shorted near DG₄ (the DG supplied

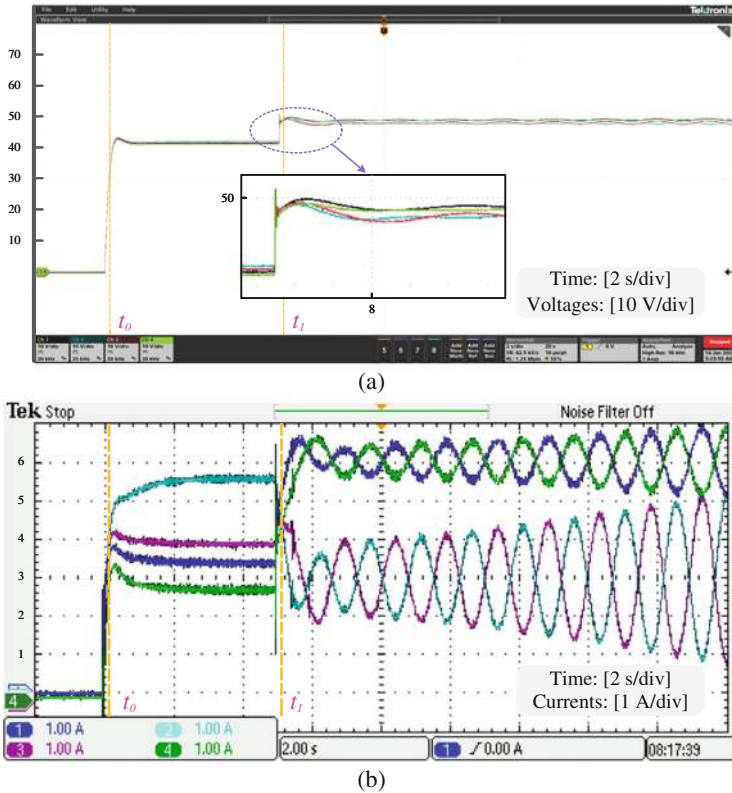


Fig. 6.9 (a) Voltage responses and (b) current responses of the MG test system under the proposed DPFA control scheme with 150 ms communication delay

by the photovoltaic cell panels) at time t_2 ; (2) after the system is stabilized, a 1 ohm resistor is connected in series to the original bus at time t_3 . The voltage and current output responses of the DC MG in this scenario can be found in Fig. 6.10. According to the above experimental steps, the output curves shown in the figures can be divided correspondingly into four phases, i.e., the grid connection phase where only the droop control is applied ($[t_0, t_1]$), the regulation phase where the secondary controller is activated ($[t_1, t_2]$), the phase where the load is reduced ($[t_2, t_3]$), and the phase where the load is restored (after t_3).

From the results shown in Fig. 6.10a, it can be seen that the output voltages of the DC MG can quickly recover to the given range after a short transient at t_2 and t_3 moments due to the change of load. As can be seen from the output responses corresponding to the last two phases in Fig. 6.10b, the output currents of the four DG units in the DC MG still stabilize rapidly at the desired values according to the prescribed distribution coefficients. Furthermore, the proposed DPFA control strategy exhibits a certain suppression effect on the fluctuations induced by load

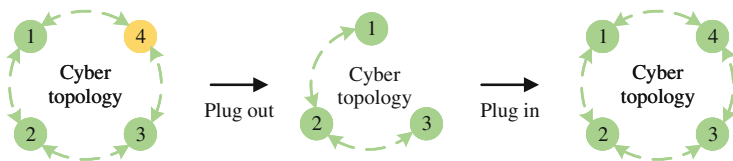


Fig. 6.11 Communication topology evolution during plug-and-play capability testing

platform. To be specific, in the experiment the following operations are performed: DG_4 is disconnected from the MG at t_4 , and it is reconnected to the MG at t_6 and shares the loads with the other DGs, where the “plugging” and “unplugging” actions are implemented by an air switch. To visualize this testing process, the evolution of the communication topology is described in Fig. 6.11. Since the proposed DPFA control takes a distributed manner, when a new DG is inserted into the DC MG, each neighbor of the newly added DG needs to reacquire the physical information of the transmission line connected to itself, i.e., update its A_{ij} . In particular, to reflect the scenario where the generation nodes supplied by renewable energy resources are involved in the DC MG, the DG_4 fed by the PV panels is selected as the fault unit to evaluate the plug-and-play capability of the proposed approach.

From the experimental results shown in Fig. 6.12a, b, it can be seen that the disconnection of DG_4 from the MG at t_4 leads to some fluctuations in the voltage output of the system, especially the voltage at the PCC of DG_4 drops to zero. However, with the DPFA, the output voltages of the remaining three DGs quickly return to stability after a brief transient and all lie within the desired interval, i.e., within $\pm 5\%$ of V_{ni}^{ref} . Similarly, the current outputs suffer a shock and fluctuate somewhat at the moment of DG detachment. Meanwhile, due to the disconnection of DG_4 , the remaining three had to take the rest of loads on the buses according to the new allocation factors. Therefore, the current response shown in the figure demonstrates that the output of DG_1 , DG_2 , and DG_3 increases in the interval from t_4 to t_6 , while still allowing for accurate current sharing.

Likewise, as DG_4 is reincorporated at time t_6 , the output voltage and current of the MG are inevitably subjected to short transients again. At the same time, the loads are still shared among the four DGs, so that the output voltage and output current of the MG are also restored to the operating state when no fault occurs. It should be noted that the responses of both transients also behave very smoothly benefiting from the prediction of future current output. Up to this point, according to the experimental results and related analysis and discussion, the plug-and-play capability of the DC MG under DPFA secondary control strategy is verified, which guarantees the practicality of the proposed method in power systems based on renewable energy resources.

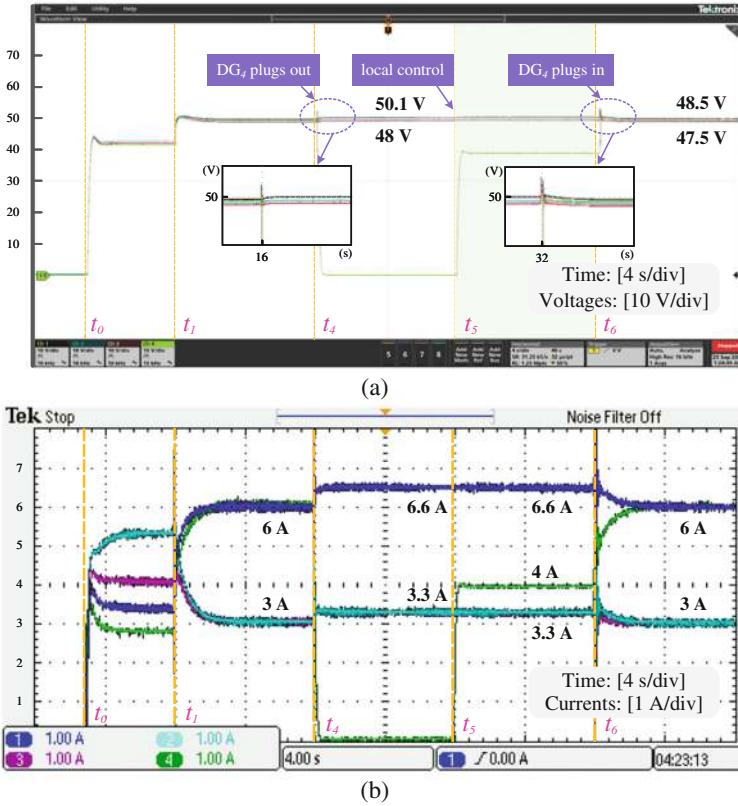


Fig. 6.12 (a) Voltage responses and (b) current responses of the MG test system in case of plug-and-play

6.5 Notes and References

Considering the line impedances in the ring distribution topology, this chapter is concerned with the achievement of voltage restoration and accurate current balancing. First, the corresponding higher-order fully actuated model is developed for DC MGs. To address the output regulation problem of DC MGs, a DPFA secondary control scheme is proposed. Benefiting from the HOFA system approaches, the proposed secondary control strategy exhibits many advantages of linear controllers, such as simplicity of structure and allowing arbitrary configuration of the poles of the closed-loop system. In addition, the optimization of the cost function with future current states achieves better steady-state output quality and smoother transient response for DC MGs. The correctness of the established model and the effectiveness of the developed secondary control scheme are verified on a DC MG prototype. As can be observed from the experimental results, the proposed method

successfully achieves voltage recovery and accurate current balancing even under load variations as well as plug-in and plug-out activities of DGs. In summary, this chapter proposes a model-based secondary control scheme for a multi-bus DC MG with a ring topology, where the suggested model facilitates the design of methods such as distributed model predictive control.

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Chapter 7

Learning-Based Fully Actuated Predictive Control of Renewable Power Systems



Abstract This chapter investigates the challenge of reconciling voltage regulation and current equalization in converter-based multi-bus DC MGs interconnected in series. To address this trade-off, a distributed learning-based high-order fully actuated (DL-HOFA) secondary control framework is proposed. Rooted in a thorough analysis of DC MG dynamics, the DL-HOFA method embeds the physical characteristics of the grid directly into its architecture, ensuring adaptability across diverse MG configurations. By leveraging the high-order fully actuated model, this approach achieves rapid dynamic response and precise current sharing, overcoming limitations of conventional strategies that often compartmentalize system physics and control logic. The DL-HOFA controller's streamlined design simplifies practical implementation without sacrificing robustness, a critical advantage for evolving grid infrastructures. Experimental validation on a photovoltaic- and battery-integrated hardware platform, equipped with maximum power point tracking (MPPT), demonstrates its efficacy in real-world scenarios. The results confirm enhanced voltage recovery and current balance even under fluctuating load and sudden plug-and-play conditions.

7.1 Overview

A critical distinction lies in the voltage regulation challenges faced by these architectures. Multi-bus DC MGs, particularly those in ring-bus multi-bus configurations, inherently grapple with competing objectives of consistent voltage regulation and precise current sharing due to line impedance effects, a conflict absent in single-bus systems (Ding et al., 2018; Yuan et al., 2022). This trade-off, extensively documented in works like Nasirian et al. (2015), Yu et al. (2023b), underscores the need for advanced control frameworks capable of balancing dynamic interactions in heterogeneous grid environments.

The challenges faced by multi-bus DC MGs have sparked extensive research in the field. One notable study by Vahidreza et al., as detailed in Nasirian et al. (2014), introduced a method that integrates a voltage controller with a consensus-

based average observer and a cooperative current regulator. This approach partially addresses the existing contradictions. However, it relies solely on the droop interface for secondary design, neglecting the actual dynamics of MGs (Han et al., 2019a; Yu et al., 2023b). Consequently, the benefits of model-based controllers, which are tailored to real-world models, remain untapped, limiting the customization of DC MG performance (Mayne, 2016). Additionally, for multi-bus DC MGs composed of various DERs, the authors in Sadabadi et al. (2023) proposed a sparsity-promoting consensus-based distributed secondary control framework. This framework minimizes the need for information exchange across the communication network. Unlike the method in Nasirian et al. (2015), it eliminates the necessity for a voltage observer, thereby removing the requirement to share voltage signals and their estimates among neighboring DERs. Despite these advancements, challenges in voltage recovery and precise current sharing for ring-bus multi-bus DC MGs persist.

Drawing inspiration from the above observations and the work in Duan (2021a), Hou and Jin (2013), Meng and Hou (2019), this chapter introduces a DL-HOFA secondary control framework designed to concurrently achieve voltage restoration at each PCC and precise current sharing in DC MGs. The proposed approach advances current methodologies through four primary contributions, addressing both theoretical and practical challenges in MG stability.

1. First, the work establishes a comprehensive system model (CSM) that holistically integrates the physical and control dynamics of DC MGs, including converter circuit behavior, dual closed-loop controller interactions, and droop control mechanisms. This model is subsequently transformed into a simplified, fully actuated representation, significantly simplifying the design and implementation of model-based control strategies.
2. Second, the derived framework introduces a virtual control layer, enabling the model to retain structural simplicity while accurately emulating real-world electrical dynamics. This enhancement bridges the gap between theoretical formulations and practical system behavior.
3. Third, the distributed learning-based secondary control enables the DC MG to handle some system uncertainties and parameter fluctuations, which can improve the adaptability of DC MG.
4. Finally, by unifying FAS theory with a learning algorithm, the DL-HOFA framework is presented, achieving the voltage recovery and accurate current sharing of the DC MG. This framework retains the theoretical rigor and predictive capabilities of model-based secondary control while introducing dynamic adaptability to real-time operational uncertainties.

The remainder of this chapter is organized as follows. Section 7.2 describes the multi-bus DC MG and formulates the voltage-restoration and proportional current-sharing objectives. Section 7.3 develops the DL-HOFA secondary controller and

presents the corresponding stability and consensus analysis. Section 7.4 reports the experimental validation. Finally, Section 7.5 concludes this chapter.

7.2 Problem Formulation

In this section, a basic description of the studied ring-bus multi-bus DC MGs and the control objectives of interest are presented.

7.2.1 Ring-bus Multi-bus DC MGs

Multi-bus DC MGs commonly interconnect DER buses through resistive distribution lines. DC MGs in series generally consist of DERs (e.g., wind turbines, photovoltaics, and energy storage), DC-DC converters, RLC filters, and local loads. To be specific, the ring-bus multi-bus DC MG examined in this chapter is presented in Fig. 6.1. Based on Kirchhoff's voltage and current laws and the QSL approximation, the circuit of the buck converter-based DC MG can be modeled as the following dynamical equations (Yu et al., 2023c)

$$\begin{aligned}\dot{V}_i(t) &= \frac{I_{fi}(t)}{C_{fi}} + \sum_{j \in \mathcal{N}_i^p} \frac{V_j(t) - V_i(t)}{C_{fi} R_{ij}} - \frac{V_i(t)}{C_{fi} R_{Li}} \\ \dot{I}_{fi}(t) &= -\frac{V_i(t)}{L_{fi}} - \frac{R_{fi}}{L_{fi}} I_{fi}(t) + \frac{V_{in,i} d_{dr,i}(t)}{L_{fi}}\end{aligned}\quad (7.1)$$

where V_i and I_{fi} are the capacitor voltage and the inductor current of the i -th DER, respectively. V_j ($j \in \mathcal{N}_i^p$) is the voltage at the port linked with the i -th DER, and $\mathcal{N}_i^p$ is the set of neighboring generation units physically connected to the i -th DER through the power line. R_{ij} is the resistance of the power line connecting DG_i and DG_j . $d_{dr,i}$ stands for the duty cycle used to generate the PWM signal for driving the IGBT, and $V_{in,i}$ indicates the voltage of the power supply for the i -th DER. R_{fi} , L_{fi} , and C_{fi} denote the electrical parameters of the filter. It is worth mentioning that the inductor current I_{fi} is also referred to as the output current in the subsequent narrative, since the inductor current is equal to the port current behind the capacitor branch at steady state, which is also commonly found in a few typical papers (Han et al., 2019a; Sadabadi et al., 2023).

7.2.2 Preliminaries on Graph Theory

In the framework of the distributed control method, it is common practice to represent the communication connections between DERs in a DC MG with a graph (Bidram et al., 2014). Since this paper does not focus on the effect of directed or

undirected graphs on the regulation performance of MGs, only an undirected graph is chosen to stand for the communication network. Admittedly, directed graphs can also be used to represent communication connections, although extending the analysis to directed graphs would require the corresponding connectivity and balance assumptions. In DC MGs, each DER is regarded as a node in the communication graph. The link in a communication network is represented by the edge of the corresponding graph. An undirected graph is represented by a triple $\mathcal{G} = (\mathcal{V}_G, \mathcal{E}_G, \mathcal{A}_G)$ consisting of a nonempty finite set of N vertices, i.e., the set of DERs, $\mathcal{V}_G = \{v_1, \dots, v_N\}$, a set of edges, $\mathcal{E}_G \subseteq \mathcal{V}_G \times \mathcal{V}_G$, and the adjacency matrix $\mathcal{A}_G$. Like the conductance matrix of an MG, the associated adjacency matrix $\mathcal{A}_G$ is composed of communication weights, where the element is defined as $a_{ij} > 0$ if node j could transmit signal to node i , otherwise $a_{ij} = 0$. The entry a_{ij} indicates the weight of the information transferred from node j to i , which can be considered as gains in data transmission and will be determined by the subsequent performance requirements of the closed-loop DC MG. The set of neighbors of the i -th DER is defined as $\mathcal{N}_i^c$. Specifically, for the graph $\mathcal{G}$, if $j \in \mathcal{N}_i^c$, then v_j can transmit data directly through the communication network to v_i . The degree matrix is defined as $D = \text{diag}\{d_1^{in}, d_2^{in}, \dots, d_N^{in}\}$, where $d_i^{in} = \sum_{j \in \mathcal{N}_i^c} a_{ij}$. Then, the Laplacian matrix is defined as $L = D - \mathcal{A}_G$.

Similar to Chap. 6, the output voltages and currents of the DC MG should, respectively, fulfill

$$\lim_{t \rightarrow \infty} \left\| \frac{V_{ni}^{ref} - V_i(t)}{V_{ni}^{ref}} \right\| = \delta_i \quad (7.2a)$$

$$\lim_{t \rightarrow \infty} \left\| \frac{I_{fj}(t)}{\eta_j} - \frac{I_{fi}(t)}{\eta_i} \right\| = 0 \quad (7.2b)$$

where the voltage reference point V_{ni}^{ref} is nominal DC voltage, $\delta_i < \varepsilon$ and ε are the given allowable threshold (e.g., 5%), and η_i is the desired current sharing ratio of the i -th DER.

7.3 Voltage Recovery and Current Allocation in DC MGs

The chief motivation of this section is to present the design procedure of an iterative learning-based secondary controller for restoring the voltage and achieving precise current sharing. Before that, the modeling of a ring-bus multi-bus DC MG will be given first, which is essential for the controller design. In addition, to illustrate the generality and feasibility of the designed secondary controller, the stability and consensus analysis of the closed-loop DC MG system is detailed.

7.3.1 Modeling of Ring-bus Multi-bus DC MGs

To facilitate secondary controller design, the established DC MG model needs to not only capture the DC MG electrical characteristics but also feature a simple form. To maximize the portrayal of the actual electrical system, a comprehensive consideration of the underlying circuit model of the DC MG, the control loops at the device level and the upper layers in the hierarchical control framework have to be taken into account.

In the hierarchical control architecture, the droop control is widely used as an interface between the upper level control and the control loops at the device level for its ability to achieve basic voltage regulation and current sharing without the aid of any communication. Then, the voltage reference $v_{o,i}^{ref}$ is generated by the droop control function as follows:

$$v_{o,i}^{ref}(t) = V_{ni}^{ref} - R_{vi} I_{fi}(t) \quad (7.3)$$

where R_{vi} is the droop gain. Notice that when the secondary control is activated, the droop control would be switched to

$$v_{o,i}^{ref}(t) = V_{ni}^*(t) - R_{vi} I_{fi}(t) \quad (7.4)$$

where $V_{ni}^*(t)$ is the secondary reference voltage to be designed to accomplish the desired voltage and current regulation. The control loop at the device level is the double closed-loop PI controller, where the voltage controller is specified as

$$\begin{aligned} \dot{\phi}_i(t) &= v_{oi}^{ref}(t) - V_i(t) \\ i_{li}^*(t) &= k_{pvi}(v_{oi}^{ref}(t) - V_i(t)) + k_{ivi}\phi_i(t), \end{aligned} \quad (7.5)$$

the current controller is designed to

$$\begin{aligned} \dot{\gamma}_i(t) &= i_{li}^*(t) - i_{li}(t) \\ d_{dr,i}(t) &= k_{pci}(i_{li}^*(t) - i_{li}(t)) + k_{ici}\gamma_i(t) \end{aligned} \quad (7.6)$$

where $\phi_i(t)$ and $\gamma_i(t)$ are the integral variables in the inner loop controllers and k_{pvi} , k_{ivi} , k_{pci} , and k_{ici} are the proportional and integral gains, respectively. It is worth mentioning that the variable $i_{li}(t)$ in the bottom current control inner loop is chosen as the output current. In this way, one gets $i_{li}(t) = i_{dg,i}(t)$, and $i_{dg,i}(t) = \sigma_i V_i(t) - \sum_{j \in \mathcal{N}_i^p} V_j(t)/R_{ij}$, where $\sigma_i = (R_{Li}^{-1} + d_{i,R}^{in})$, $d_{i,R}^{in} = \sum_{j \in \mathcal{N}_i^p} R_{ij}^{-1}$.

Referring back to the circuit model (7.1) given earlier and combining it with the above droop control (7.4), the underlying voltage controller (7.5) and current

controller (7.6), and taking the secondary voltage reference $V_{ni}^*(t)$ as the control input $u_{r,i}(t)$, the following large-signal model of the DC MG can be deduced

$$\dot{x}_i(t) = A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t) + B_i u_{r,i}(t) \quad (7.7)$$

where $x_i(t) = [V_i(t) \ I_{fi}(t) \ \phi_i(t) \ \gamma_i(t)]^T$, A_{ii} , and A_{ij} can be obtained from (7.1)–(7.6) which is omitted here due to space limitations, $B_i = [0 \ V_{in,i}k_{pci}k_{pvi}/L_{fi} \ 1 \ k_{pvi}]^T$.

The output current of the DC MG is taken as the system output of the above state-space model, i.e., $y_i(t) = h_i^c(x_i(t))$, where $h_i^c(x_i(t)) = C_i^c x_i(t)$, $C_i^c = [0 \ 1 \ 0 \ 0]$. It can be seen that the DC MG modeled with $V_{ni}^*(t)$ as the control input is an under-actuated physically interconnected multi-agent system. Deriving the output current along the coupled large-signal model (7.7), the following DC MG dynamics can be acquired

$$\begin{aligned} \dot{I}_{fi}(t) &= L_{F_i} h_i^c \\ &= C_i^c (A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t)) + b_{c,i} u_{r,i}(t) \end{aligned} \quad (7.8)$$

where $F_i = A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t) + B_i u_{r,i}(t)$, $L_{F_i} h_i^c$ is the Lie derivative of h_i^c regarding to F_i (Khalil, 2015) and is defined by $L_{F_i} h_i^c = \nabla h_i^c F_i = (\partial h_i^c / \partial x_i) F_i$, $b_{c,i} = C_i^c B_i = V_{in,i} K_{pc,i} K_{pv,i} / L_{fi}$.

Since $b_{c,i} \neq 0$, the high-order fully actuated secondary controller can be designed as

$$u_{r,i}(t) = b_{c,i}^{-1} (u_{ca,i}(t) + u_i(t)). \quad (7.9)$$

The $u_{ca,i}(t)$ is an auxiliary control for system decoupling and the design will be given next, while the $u_i(t)$ is a freely assignable auxiliary input for secondary regulation. The bus voltage and current are measured by local sensors, while the PI integrator states ϕ_i and γ_i are directly available from the digital controller. Any neighboring physical variables required by $A_{ij}x_j$ are assumed to be locally measured or communicated. The following auxiliary control can be designed

$$u_{ca,i}(t) = -C_i^c (A_{ii}x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}x_j(t)). \quad (7.10)$$

Up to this point, substituting (7.9) and (7.10) into (7.8) leads to the following high-order fully actuated DC MG model

$$\dot{I}_{fi}(t) = u_i(t). \quad (7.11)$$

In existing approaches to modeling DC MGs, the large-signal model is established by considering the dynamics of the underlying circuit (Han et al., 2019a).

This method allows a comprehensive description of the electrical characteristics of DC MGs and is therefore widely used. However, coupled multi-agent models obtained in this way may tend to impair the distributed nature of the controller (Yu et al., 2023c). Although a coupled large-signal model of DC MGs is also developed in this chapter, we use the HOFA system approach to transform it into a decoupled multi-agent model. Besides, there is another class of control patterns that adds a correction term to the droop control and takes it as a control input (Nasirian et al., 2015). For example, $V_i(t) = V_{ni}^{ref} - R_{vi} I_{fi}(t) + \delta_i(t)$, where $\delta_i(t)$ is the correction term. The idea of designing the correction term $\delta_i(t)$ to achieve secondary regulation has achieved great success. However, this approach does not facilitate the design of model-based secondary control for the DC MG. In contrast, the decoupled model established in this chapter allows for a model-based secondary controller design, which contributes to improving system performance. In particular, the developed model possesses a deep physical meaning of the DC MG system, unlike the existing work that models the dynamics of voltage and current separately as first-order systems (Sahoo & Mishra, 2019), which ignores the characteristics of the DC MG system itself.

7.3.2 DL-HOFA Secondary Controller Design

In order to achieve the objectives formulated in Sect. 7.2.2, a control framework containing a voltage limiter and a cooperative control of the output current is introduced, and a specific description of the secondary controller is given.

From (7.3), the reference value received by the double closed-loop control is $v_{o,i}^{ref}(t)$ in the absence of secondary control action. When switching to secondary control, the voltage limiter activates first, at which point the dual closed-loop control collects the actual reference value $V_{o,i}^*(t)$ as

$$V_{o,i}^*(t) = \begin{cases} V_{ni}^{ref} - \epsilon_i, & v_{o,i}^{ref}(t) \in (-\infty, V_{ni}^{ref} - \epsilon_i] \\ v_{o,i}^{ref}(t), & v_{o,i}^{ref}(t) \in (V_{ni}^{ref} - \epsilon_i, V_{ni}^{ref} + \epsilon_i] \\ V_{ni}^{ref} + \epsilon_i, & v_{o,i}^{ref}(t) \in (V_{ni}^{ref} + \epsilon_i, +\infty) \end{cases} \quad (7.12)$$

where $\epsilon_i = \varepsilon V_{ni}^{ref}$.

Before diving into the design of the secondary current cooperative controller, the established HOFA DC MG model is first exactly Eulerian discretized, i.e.,

$$I_{fi}(t+1) = I_{fi}(t) + T_s u_i(t) \quad (7.13)$$

where T_s is the sampling period of DC MGs. It should be noted that the time variables in discrete space are also denoted by t .

Define the following current cooperation tracking error for DER_i over the communication topology $\mathcal{G}$

$$\zeta_i(t) = \sum_{j \in \mathcal{N}_i^c} a_{ij}(i_{so,j}(t) - i_{so,i}(t)) \quad (7.14)$$

where $i_{so,i}(t) = I_{fi}(t)/\eta_i$.

It is assumed that an ideal distributed current cooperative controller is theoretically available to ensure that the output current of the DERs in the DC MG can be accurately distributed in a future step. In this way, without loss of generality, it can be assumed that the ideal distributed controller has the following form (Hou & Xiong, 2019; Hou et al., 2018)

$$u_i(t) = f(\zeta_i(t+1), \dots, \zeta_i(t-n_y+2), u_i(t-1), \dots, u_i(t-n_u)) \quad (7.15)$$

where $f(\cdot)$ is an unknown nonlinear function and n_y and n_u are the unknown order of the distributed coordination error and the control input in the controller, respectively. To facilitate the subsequent equivalent transformation of the ideal distributed controller, the following assumptions would be given.

Remark 7.3.1 Data-driven control is an important paradigm innovation in the field of control (Hou et al., 2018). Its core lies in eliminating the reliance on precise mathematical models by directly designing controllers using operational system data. For complex systems like microgrids, particularly nonlinear ones containing CPLs, this method offers an excellent approach for secondary control design, avoiding complicated analytical processes such as the backstepping approach.

Assumption 7.3.1 (Hou & Jin, 2013) *The second-order partial derivatives of $f(\cdot)$ with respect to the coordination error $\zeta_i(t)$ are continuous.*

Assumption 7.3.2 (Hou & Jin, 2013) *Function $f(\cdot)$ meets the generalized Lipschitz condition. That is, when $|\zeta_i(t+1) - \zeta_i(t)| \neq 0$, there exists a constant l_p such that*

$$\begin{aligned} |u_i(t) - u_i(t-1)| &\leq l_p |\zeta_i(t+1) - \zeta_i(t)| \\ |u_i(t) - u_i(t-1)| &\leq l_p |\xi_i(t+1) - \xi_i(t)| \end{aligned} \quad (7.16)$$

where $\xi_i(t+1) = \Delta \zeta_i(t+1)$ and the operator Δ have the same meaning for other variables.

Further, for the ideal distributed controller (7.15) satisfying the above assumptions, there exists an equivalent linear form

$$\Delta u_i(t) = \theta_{i,1}(t) \Delta \zeta_i(t+1) + \theta_{i,2}(t) \Delta \xi_i(t+1) \quad (7.17)$$

where $\Delta u_i(t) = u_i(t) - u_i(t-1)$ and $\theta_{i,1}(t)$ and $\theta_{i,2}(t)$ are the unknown parameters bounded by μ_θ . Since the future coordination error $\Delta \zeta_i(t+1)$ is unavailable, and the ideal controller achieves cooperation (i.e., $\zeta_i(t+1) = 0$ and $\Delta \zeta_i(t+1) = -\zeta_i(t)$),

(7.17) can be converted into the following practical form

$$\Delta u_i(t) = -(\theta_{i,1}(t) + \theta_{i,2}(t))\zeta_i(t) - \theta_{i,2}(t)\Delta\zeta_i(t). \quad (7.18)$$

Based on the derivations presented up to this point, the secondary controller synthesized in this research can be comprehensively interpreted as a distributed proportional-integral (PI) control strategy featuring time-varying feedback gains. Furthermore, when the dynamic parameters $\theta_{i,1}(t)$ and $\theta_{i,2}(t)$ are simplified to operate as fixed constant factors, the mathematical formulation in (7.18) becomes directly equivalent to the standard distributed secondary controllers that are widely implemented across contemporary microgrid architectures Nasirian et al. (2015). Ultimately, the incorporation of these dynamically adjusting, time-varying distributed control gains provides a substantial improvement in the operational performance of the secondary control layer, and this advantageous characteristic will be thoroughly explored and validated in the forthcoming discussion.

To enhance system performance, the following cost function with a one-step prediction is proposed for DC MGs

$$J_i = \frac{1}{2} \sum_{j \in \mathcal{N}_i^c} a_{ij} (\hat{i}_{so,j}(t+1) - \hat{i}_{so,i}(t+1))^2 + \frac{1}{2} b_i u_i^2(t) \quad (7.19)$$

where b_i is the weighting factor penalizing the control input to prevent practically unachievable duty cycles. For the i -th DER, to obtain the optimal control gain under the cost function (7.19), the relationship between the predicted weighted output current $\hat{i}_{so,i}(t+1)$ and $\Delta u_i(t)$ needs to be determined. This prediction is given by

$$\hat{i}_{so,i}(t+1) = i_{so,i}(t) + \Delta \hat{i}_{so,i}(t+1). \quad (7.20)$$

The discrete high-order fully actuated model of DC MGs can be written in the following incremental form

$$\Delta \hat{i}_{so,i}(t+1) = \Delta i_{so,i}(t) + T_s \eta_i^{-1} \Delta u_i(t). \quad (7.21)$$

Thus, the predicted value of the weighted output current can be expressed as

$$\hat{i}_{so,i}(t+1) = i_{so,i}(t) + T_s \eta_i^{-1} u_i(t). \quad (7.22)$$

Substituting (7.22) into the cost function of DER_{*i*} yields

$$\begin{aligned} J_i(t) = & \frac{1}{2} \sum_{j \in \mathcal{N}_i^c} a_{ij} (i_{so,j}(t) + \bar{T}_{s,i} u_j(t) \\ & - i_{so,i}(t) - \bar{T}_{s,i} u_i(t))^2 + \frac{1}{2} b_i u_i^2(t) \end{aligned} \quad (7.23)$$

where $\bar{T}_{s,i} = T_s \eta_i^{-1}$.

By utilizing the practical dynamic distributed controller presented in (7.18), the previously defined cost function (7.19) is transformed into a parameterized objective

function. In this reformulated optimization problem, the distributed learning control gains, denoted as $\theta_{i,k}(t)$ for $k \in \{1, 2\}$, act as the primary decision variables. To iteratively minimize this objective function and dynamically adjust the gains, a Newton-type optimization method is meticulously applied. Consequently, this mathematical derivation establishes the following online learning protocol for updating the control gains at each discrete time step

$$\begin{aligned} \theta_{i,k}(t+1) &= \theta_{i,k}(t) - \gamma \left(\frac{\partial^2 J_i}{\partial \theta_{i,k}^2} \right)^{-1} \frac{\partial J_i}{\partial \theta_{i,k}(t)} \\ &= \theta_{i,k}(t) - \gamma \frac{1}{(\bar{T}_{s,i}^2 d_{in} + b_i)(\zeta_i(t) + (k-1)\Delta\zeta_i(t))^2} ((\zeta_i(t) \\ &\quad + (k-1)\Delta\zeta_i(t))(\bar{T}_{s,i}\zeta_i(t) + \bar{T}_{s,i}u_i^{N_i}(t) \\ &\quad - (d_{in}\bar{T}_{s,i}^2 + b_i)u_i(t))). \end{aligned} \quad (7.24)$$

In this expression, the aggregated neighborhood control input is defined as $u_i^{N_i}(t) = \sum_{j \in \mathcal{N}_i^c} a_{ij} \bar{T}_{s,j} u_j(t)$, while the parameter γ represents the tunable learning rate that governs the convergence speed of the algorithm. It should be noted that the Newton-based gain update in (7.24) is performed only when the corresponding regression signal is sufficiently large. Define

$$\chi_{i,k}(t) = \zeta_i(t) + (k-1)\Delta\zeta_i(t), \quad k \in \{1, 2\}.$$

For a prescribed small threshold $\varepsilon_\theta > 0$, if

$$|\chi_{i,k}(t)| \leq \varepsilon_\theta,$$

the learning iteration for $\theta_{i,k}$ is suspended and the latest admissible gain is retained, i.e.,

$$\theta_{i,k}(t+1) = \theta_{i,k}(t).$$

This stopping criterion prevents the denominator in the Newton update from approaching zero and avoids unnecessary gain fluctuations when the current coordination error has already converged to a sufficiently small neighborhood of the origin. The secondary controller remains active using the latest learned gains.

To guarantee that the DC MG can eventually achieve precise proportional load current sharing with zero steady-state error under the continuously learned control gains, the system must be equipped with a specialized parameter holding mechanism. This parameter-projection or holding mechanism prevents abnormal divergence and is formulated as

$$\theta_{i,k}(t+1) = \theta_{i,k}(t), \text{ if } \theta_{i,k}(t+1) < -\mu \text{ or } \theta_{i,k}(t+1) > 0. \quad (7.25)$$

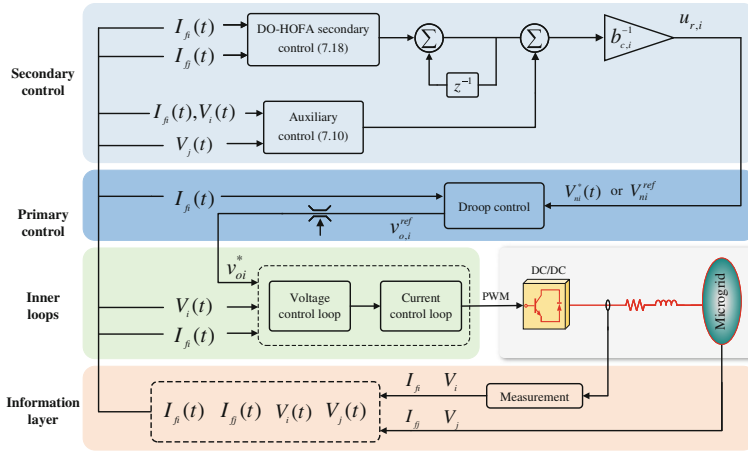


Fig. 7.1 Architecture of the proposed control scheme for voltage restoration and current sharing of the DC MG

Here, the positive constant μ serves as a predefined boundary threshold that must be carefully selected to restrict the allowable operational range of the adaptive gains. The parameter-holding mechanism constrains the learned gains to the prescribed interval and prevents numerical divergence. Current-sharing convergence additionally requires the stability condition established in Theorem 7.3.1.

In particular, the integration of the aforementioned high-order fully actuated MG modeling, the Newton-based updating protocol, and the boundary holding mechanism collectively characterizes the complete DL-HOFA secondary control strategy. For practical engineering applications, the specific execution procedures and sequential implementation steps are systematically summarized in Algorithm 1. It is important to emphasize that the selection of the learning rate and the reset threshold is highly dependent on the intrinsic dynamic characteristics of the specific MG system, which are typically determined through empirical tuning. Furthermore, a comprehensive visual representation detailing the block diagram of the proposed DL-HOFA secondary control strategy integrated within the DC MG architecture is clearly presented in Fig. 7.1.

Remark 7.3.2 While a wide array of secondary control schemes has been extensively investigated in the existing literature to successfully accomplish voltage restoration and proportional current sharing Guo et al. (2023); Wang et al. (2023), the methodology formulated in this study introduces a fundamental paradigm shift. Rather than relying on conventional approaches, we leverage the concept of the fully actuated system theory to analytically derive a rigorous first-order decoupled multi-agent dynamic model. Formulating this explicit mathematical representation is highly advantageous, as it establishes the necessary structural foundation for synthesizing an advanced model-based distributed cooperative control architecture.

Algorithm 2 DL-HOFA secondary control

Require: Real-time local measurement data, droop coefficients, learning rate γ , boundary saturation threshold μ , and communication topology.

Ensure: Secondary control input $u_i(t)$ and precise steady-state load current sharing.

- 1: **Initialization:**
 - 2: Set the discrete time step $t = 1$ and randomly initialize the control gains such that $\theta_{i,k}(1) \in [-\mu, 0)$ for $k \in \{1, 2\}$.
 - 3: **while** Secondary control layer is activated **do**
 - 4: Receive the output current information $i_{o,j}(t)$ from neighboring DERs, and simultaneously transmit local measurements to adjacent nodes.
 - 5: Compute the coordination error of the weighted output currents, and synthesize the incremental secondary control input via

$$u_i(t) = u_i(t-1) - (\theta_{i,1}(t) + \theta_{i,2}(t))\zeta_i(t) - \theta_{i,2}(t)\Delta\zeta_i(t). \quad (7.26)$$
 - 6: Obtain the actual secondary voltage reference $u_{r,i}(t)$ according to (7.9).
 - 7: Apply $V_{ni}^*(t)$ to the primary droop layer.
 - 8: Enforce saturation limits on the reference input $v_{o,i}^{ref}(t)$ generated by the droop controller, executing (7.12) to ensure it remains within the allowable operational voltage range.
 - 9: Predict the weighted output current for the subsequent time step required for the gain update, utilizing the derived HOFA MG dynamic model.
 - 10: Calculate the regression signal $\chi_{i,k}(t) = \zeta_i(t) + (k-1)\Delta\zeta_i(t)$. If $|\chi_{i,k}(t)| > \varepsilon_\theta$, update the control gains using (7.24); otherwise, suspend the learning update and retain the latest admissible gains.
 - 11: Execute the parameter boundary holding mechanism defined in (7.25) to guarantee precise steady-state current sharing, and prevent divergence.
 - 12: Let $t := t + 1$.
 - 13: **end while**
-

Building upon this theoretical advantage, a distributed iterative learning mechanism is strategically developed to optimize a predictive cost function that explicitly incorporates future operational states. Consequently, this forward-looking optimization significantly elevates the overall transient and steady-state regulation performance. Furthermore, by autonomously updating the feedback mechanisms, the proposed learning-enabled framework inherently bolsters the microgrid's resilience and adaptability to fluctuating operating conditions. This algorithmic autonomy thoroughly eliminates the traditional dependency on cumbersome, artificially tuned heuristic parameters, thereby highlighting the distinctive theoretical and practical superiority of the current work over prior literature.

7.3.3 Stability and Consensus Analysis of DC MGs

In this subsection, a comprehensive theoretical analysis concerning the closed-loop stability, state consensus, and asymptotic convergence properties of DC MGs

operating under the proposed DL-HOFA secondary control scheme is rigorously presented and mathematically proven.

Theorem 7.3.1 *By applying the proposed DL-HOFA secondary control framework specified in (7.18), driven by the online learning control gain update protocol defined in (7.24), the ring-bus multi-bus MG network formulated in (7.7) is guaranteed to be globally stabilized. Concurrently, the dual operational objectives (viz., precise global voltage restoration (7.2a) and exact proportional load current sharing (7.2b)) are successfully accomplished, provided that for any arbitrarily chosen decay constant $c \in (0, 1)$, the spectral norm of the state transition matrix satisfies the following strict inequality condition*

$$\|\Phi(t)\| \leq c \quad (7.27)$$

where the parameter matrix $\Lambda = \text{diag}\{\eta_1^{-1}, \dots, \eta_N^{-1}\}$ and $H_1 \in \mathbb{R}^{N \times N}$ represents specific indicator matrix wherein all elements are identically zero except for those in the first column, which are set to one. The system matrix $\Phi(t)$ is explicitly structured as follows:

$$\Phi(t) = \begin{bmatrix} 2I - \Gamma(\bar{\theta}_1 + \bar{\theta}_2)L & -I + \Gamma\bar{\theta}_2L \\ I & 0 \end{bmatrix}$$

where the intermediate scaling matrix is defined as $\Gamma = T_s \Lambda + T_s \eta_1^{-1} H_1$.

Proof To initiate the rigorous mathematical demonstration of the aforementioned theorem, we first construct the aggregated state vectors characterizing the MG network. Specifically, define the global weighted output current vector, the corresponding control input vector, and the cooperative tracking error vector as follows:

$$\begin{aligned} i_{so}^{vec}(t) &= [i_{so,1}(t), i_{so,2}(t), \dots, i_{so,N}(t)]^T \\ \Delta U(t) &= [\Delta u_1(t), \Delta u_2(t), \dots, \Delta u_N(t)]^T \\ \zeta(t) &= [\zeta_1(t), \zeta_2(t), \dots, \zeta_N(t)]^T. \end{aligned}$$

Furthermore, introduce an auxiliary discrepancy variable $e(t) = i_{so}^{vec}(t) - \mathbf{1}_N i_{so,1}(t)$, where $\mathbf{1}_N \in \mathbb{R}^{N \times 1}$ is a column vector of all ones and $e(t) = [e_1^s(t), e_2^s(t), \dots, e_N^s(t)]^T \in \mathbb{R}^{N \times 1}$. By leveraging the foundational high-order fully actuated MG dynamic formulation given in (7.13), the difference equation governing each localized cooperative error translates into

$$e_i^s(t+1) = 2e_i^s(t) - e_i^s(t-1) + T_s \eta_i^{-1} \Delta u_i(t) - T_s \eta_1^{-1} \Delta u_1(t). \quad (7.28)$$

According to the distributed control law (7.18), the matrix-vector representation of the incremental control input can be given as

$$\Delta U(t) = -(\bar{\theta}_1 + \bar{\theta}_2)Le(t) + \bar{\theta}_2Le(t-1) \quad (7.29)$$

where $\bar{\theta}_1 = \text{diag}\{\theta_{1,1} + \theta_{1,2}, \dots, \theta_{N,1} + \theta_{N,2}\}$, $\bar{\theta}_2 = \text{diag}\{\theta_{1,2}, \theta_{2,2}, \dots, \theta_{N,2}\}$. Incorporating the relationship (7.29) directly into the uncoupled cooperative error dynamics (7.28), one has

$$\begin{aligned} e(t+1) = & (2I - (T_s \Lambda + T_s \eta_1^{-1} H_1)(\bar{\theta}_1 + \bar{\theta}_2)L)e(t) \\ & + (-I + (T_s \Lambda + T_s \eta_1^{-1} H_1)\bar{\theta}_2 L)e(t-1). \end{aligned} \quad (7.30)$$

By defining an augmented state vector $\xi = [e(t), e(t-1)]^T$, the aforementioned relationship smoothly transitions into the following closed-loop autonomous error dynamic system

$$\xi(t+1) = \Phi(t)\xi(t). \quad (7.31)$$

Note that $\|\xi(t+1)\| \leq \|G(t, 1)\| \|\xi(1)\|$. Utilizing the recursive nature of (7.31), the state transition matrix spanning the discrete time interval from the initial step 1 to the current step t is analytically derived as

$$G(t, 1) = \prod_{l=1}^t \Phi(t-l+1). \quad (7.32)$$

Consequently, the temporal evolution of the coordination error can be written as

$$e(t+1) = G(t, 1)e(1). \quad (7.33)$$

Applying the standard vector norm operation to both sides of (7.33) and invoking the fundamental properties of the sub-multiplicative matrix norm alongside the triangle inequality, one has

$$\|e(t+1)\| = \|G(t, 1)e(1)\| \leq \|G(t, 1)\| \|e(1)\|. \quad (7.34)$$

If condition (7.27) is satisfied, the inequality propagates iteratively to yield

$$\|e(t+1)\| \leq \prod_{l=1}^t \|\Phi(t-l+1)\| \|e(1)\| \leq c^t \|e(1)\|. \quad (7.35)$$

This derivation demonstrates that the closed-loop MG framework exhibits exponential convergence bounded by the decay rate c . In explicit mathematical terms, $\lim_{t \rightarrow \infty} \|e(t+1)\| = 0$. This implies that all cooperative mismatches regarding the weighted output currents are completely eradicated, asymptotically converging to zero as time progresses to infinity.

Furthermore, analyzing the control effort relationship established in (7.29) reveals that the incremental input $\Delta U(t)$ inherently approaches 0 as $t \rightarrow \infty$.

This fundamental observation confirms that the complete interconnected system utilizing the proposed algorithm is definitively stabilized. Additionally, because the verified closed-loop dynamics originate from the large-signal model encompassing the entire electrical network, it is logically inferred that the underlying primary double closed-loop PI controllers successfully preserve the local stability of every individual DER. Finally, accounting for the inherent capability of the PI integrators to completely eliminate steady-state errors, the desired secondary objective of precise output voltage restoration is definitively achieved across all units. Specifically, for any unit index $i = 1, 2, \dots, N$, the limiting behavior validates

$$\lim_{t \rightarrow \infty} v_{o,i}(t) = \lim_{t \rightarrow \infty} V_i^{ref}(t) \in [V_{ni}^{ref} - \epsilon_i, V_{ni}^{ref} + \epsilon_i]. \quad (7.36)$$

In conclusion, the rigorous theoretical deductions confirm that the synthesized DL-HOFA secondary control framework guarantees robust system stability, voltage recovery, and highly accurate load current sharing as stipulated in Sect. 7.2.2. The proof is thus comprehensively established. ■

7.4 Experimental Validation

In this section, the effectiveness of the DL-HOFA secondary control strategy proposed in this chapter for voltage recovery and current equalization will be examined by experimental case studies using a scaled-down islanded ring-bus multi-bus DC MG with four DERs.

7.4.1 Hardware Description and Experimental Procedure

The constructed ring-bus multi-bus DC MG is composed of DC-DC buck converters, LC filters, local resistive loads, and power lines connecting the DERs and hardware controllers, as shown in Fig. 6.2. To build a scaled-down RESs-based power generation system in a limited laboratory space, a set of photovoltaic panels with batteries acts as one of the DERs, while the other three are replaced with voltage sources. OPAL-RT (OP4510) as a hardware controller receives analog signals from the voltage and current sensors of each DER and executes the designed DL-HOFA control algorithm.

The communication topology used for the DC MG in the testbed is the same as the physical connection graph, which is a strongly connected ring structure with a specific adjacency matrix of $A = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$. If not otherwise specified, this topology is adopted in all subsequent case studies. The nominal value of the voltage determined by the tertiary control layer is assumed to be $V_{ni}^{ref} = 48$, which is typical for a low-voltage DC distribution network. The

Table 7.1 Settings of scaled-down DC MG and control

DER parameters			DER ₁ & DER ₂		DER ₃ & DER ₄	
Droop	R_{vi}		0.8	1	1.7	1.5
Voltage	k_{pvi}		0.1		0.2	
loop	k_{ivi}		1		1	
Current	k_{pci}		0.001		0.001	
loop	k_{ici}		2.2		2.2	
LC filter	L_{fi}		≈ 2		≈ 2	
[mH], [μ F]	C_{fi}		$1500 \pm 20\%$		$1500 \pm 20\%$	
Line	Line 1		Line 2		Line 3	Line 4
impedance	R_{12}	0.5	R_{23}	0.5	R_{34}	0.5
[Ω],[μ H]	L_{12}	318	L_{23}	318	L_{34}	325
Loads	Local	loads				Switching load
[Ω]	R_L	10 10 10 10				$R_{L,cut}$ 4

current sharing factor is set to $\eta = [1.5, 1.5, 1, 1]$. Besides, the sampling time of the hardware controller is set to $5e-4$ s. Other detailed configuration parameters for the experimental setup are listed in Table 7.1. Note that although the local loads with the same values are employed on the experimental platform, the proposed method does not require that these parameters are consistent. Specifically, the system model used for the secondary control design is built for a DC MG composed of heterogeneous DERs. Therefore, the parameters of each DER, including load parameters, are allowed to be different. It is worth mentioning that the gain of the PI controller is obtained using the pole configuration method under a single DER with droop control. Alternatively, the selection of this gain can be done by trial-and-error tuning.

Next, four case studies will be conducted on the developed platform: (1) performance comparison with the existing methods, (2) robustness to load variations, (3) performance evaluation for input voltages uncertainty, and (4) verification of plug-and-play capability. For clarity of presentation, the entire experimental process can be divided into four stages as follows: **Phase 1:** activating the local droop control at t_1 and enabling the interconnection of all DERs within the islanded MG. **Phase 2:** putting into operation the proposed DL-HOFA secondary control at t_2 . **Phase 3:** adding a constant load $R_{L,cut}$ to the local load $R_L(1)$ at t_3 and removing it from the bus at t_4 . **Phase 4:** disconnecting DER₄ from the MG at t_5 and reconnecting DER₄ to the MG at t_7 .

7.4.2 Performance Comparison and Analysis

Case I: Simulation and experimental results of the state-of-the-art control method

For ring-bus multi-bus DC MGs, the sparsity-promoting consensus (SPC)-based distributed secondary control proposed in Sadabadi et al. (2023) is an effective

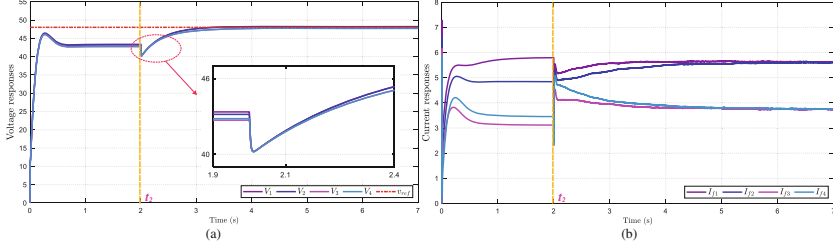


Fig. 7.2 (a) Voltage and (b) current with the SPC-based control

practice to address voltage recovery and current sharing, which takes the form of

$$\begin{aligned}
 \tau_v \dot{v}_c(t) &= -K \mathbb{B}^T W^{-1} I_f(t), \quad \tau_\theta \dot{\theta}_c(t) = I_f(t) - \theta_c(t) \\
 \tau_\phi \dot{\phi}_c(t) &= -\beta(V(t) - 1_n V^{ref}) + z(t) \\
 \tau_\theta \dot{\theta}_c(t) &= I_f(t) - \theta_c(t) \\
 u(t) &= K_1 V(t) + K_2 I_f(t) + K_3 \phi_c(t) + (1 - K_1) \beta^{-1} z(t)
 \end{aligned}$$

where $V(t) = [V_1, \dots, V_N]^T$, $I_f(t) = [I_{f1}, \dots, I_{fN}]^T$, $K > 0$, $\tau_v, \beta > 0$, $K_p > 0$, $\tau_\phi > 0$, $\tau_\theta > 0$, K_1, K_2 , and K_3 are control parameters. The details of $\mathbb{B}$ and W can be found in Sadabadi et al. (2023). Based on the specifications of the experimental platform used and the same communication topology as previously described, the control parameters of the SPC-based method can be set by trial-and-error method.

The simulation results of the DC MG under the SPC-based distributed control method are presented in Fig. 7.2. Further, the control strategy is executed on the established experimental platform, and the corresponding results are provided in Fig. 7.3. The recorded simulation and experimental results show that the SPC-based secondary control strategy can achieve voltage recovery to a certain range as well as precise current sharing in the MG. However, under this methodology, the overall dynamics of the MG is not modeled, which results in a poor performance of the method behind the proposed one. Regarding this point, it will be analyzed in detail later.

Case II: Simulation and experimental results of the proposed DL-HOFA control approach

In this case, the weight factor corresponding to the control cost in the performance index function (7.19) is set to $b_i = 10$ for $i \in \mathcal{V}_G$, under the parameter selection guidelines of Theorem 7.3.1. The learning rate of the control gain is set to $\gamma = 0.97$, and the reset threshold is chosen to be $\mu = 1.5$. The simulation results of the DC MG under the proposed DL-HOFA distributed control method are given in Fig. 7.4. Similarly, the output responses of the ring-bus multi-bus DC MG are shown in Fig. 7.5. From the results, it can be seen that the voltage at each PCC is

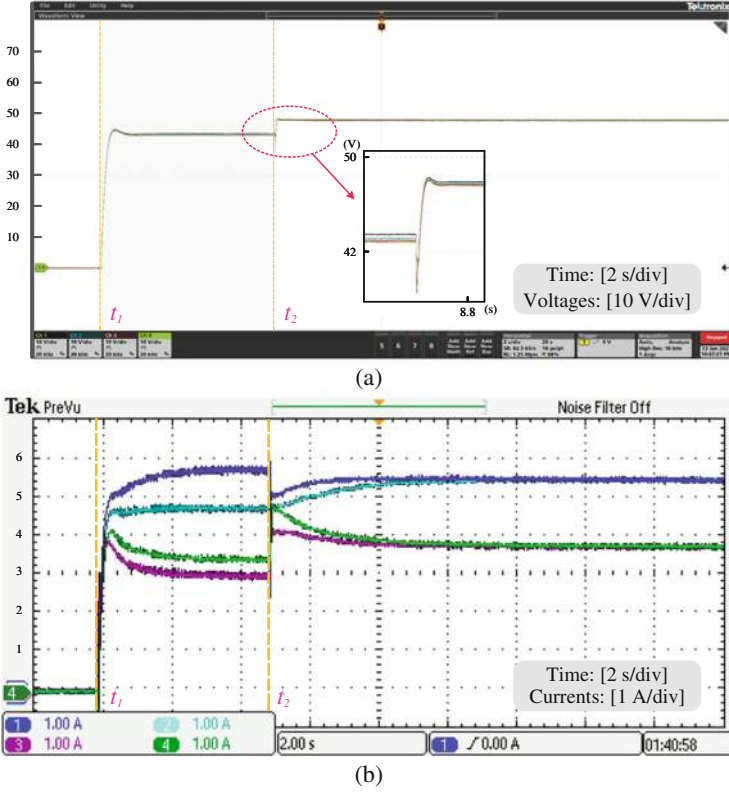


Fig. 7.3 (a) Voltage responses and (b) current responses of the MG test system under the SPC-based control scheme

quickly restored to the specified range and the currents are precisely shared due to the proposed DL-HOFA secondary control activated at t_2 .

It can be observed in Fig. 7.5b that the output currents of DER #1 and #2 are 5.4 A and the output currents of DER #3 and #4 are 3.6 A, which follows the distribution ratio of 1.5:1.5:1:1 and indicates that the DL-HOFA secondary control completes the current sharing. However, the control objective of this paper is to regulate the voltages to near 48 V, not exactly 48 V. In addition, the filters used in the actual hardware platform and the impedance of the analog transmission lines, etc., are not ideal, which can also lead to certain power losses. These factors together cause the actual output currents to deviate from the values obtained from some ideal calculations under the precise regulation of output voltages.

Based on the experimental waveforms under the above two cases, we have the following observations and discussions:

- (i) Compared with the results obtained in *Case I*, the proposed method achieves better regulation effects, such as the current and voltage regulation process

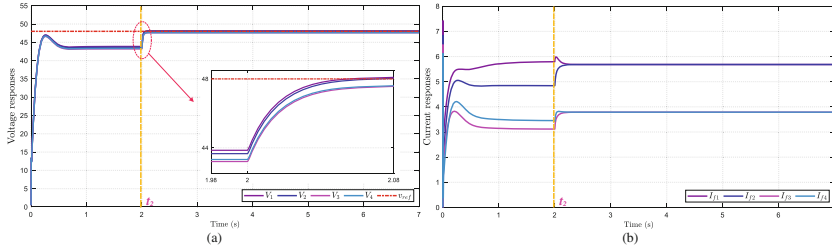


Fig. 7.4 (a) Voltage and (b) current with the DL-HOFA control

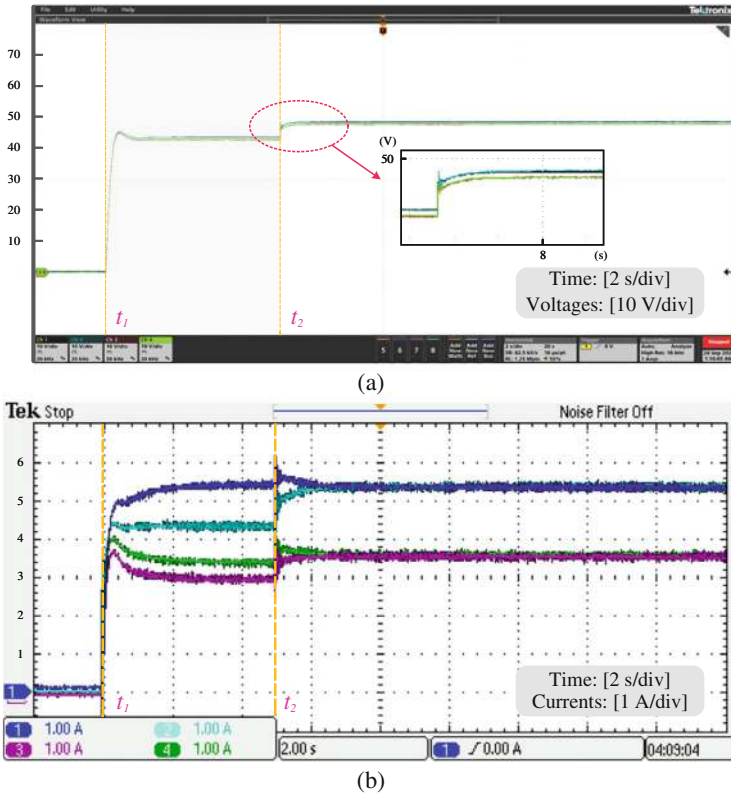


Fig. 7.5 (a) Voltage responses and (b) current responses of the MG test system under the DL-HOFA control scheme

under the proposed approach is both fast and stable. Particularly, the voltage tracking and current sharing are accomplished significantly faster in *Case II* than that in *Case I*.

- (ii) At the moment of activation of the comparative control method, the output voltages of the DC MG first drop sharply and then rise, as can be observed in

the magnified waveform segments presented in Figs. 7.2a and 7.3a. In contrast, with the method proposed in this chapter, there is no dip in the recovery process of the output voltages.

- (iii) The SPC-based control method in Sadabadi et al. (2023) is a model-free control, and therefore its control parameters cannot be optimized using a performance function. Additionally, the voltage regulation in Sadabadi et al. (2023) is an open-loop control, which affects the control accuracy. Additionally, when the resistance in the filter is not negligible, the voltage regulation will also face a large deviation.

At this point, the above results demonstrate that the proposed DL-HOFA secondary controller allows the DERs to reach steady state quickly and achieve accurate current sharing and is superior to existing secondary controllers.

7.4.3 Performance Evaluation for Input-Voltage Uncertainty

To demonstrate the performance of the proposed DL-HOFA secondary control method in the case of parameter uncertainties, a case study under input voltage mismatch is given in this section. Specifically, as in Sect. 7.4.2, $V_{in} = [100\ 90\ 90\ 80]$ is used for the system model in the DL-HOFA secondary control, while the actual input voltage in the hardware platform becomes $V_{in} = [100\ 90\ 100\ 80]$. The corresponding experimental results are given by Fig. 7.6. The desired voltage regulation and current sharing can be observed in this figure. In particular, the excellent transient current sharing is visible in Fig. 7.6b. Comparing the waveforms shown in Figs. 7.5 and 7.6, it is demonstrated that the built MG system with mismatched input voltages behaves similarly to that in Sect. 7.4.2; i.e., it can basically accomplish the formulated tasks of voltage restoration and current sharing.

7.4.4 Performance Evaluation for Load Demand Variations

In this case, the performance of the proposed controller is tested under load demand variations. The experiment corresponding to Phase 3 described earlier will be performed here. Specifically, at t_3 , an auxiliary $4\ \Omega$ resistor is connected in parallel with the $10\ \Omega$ local load of DER₁, reducing the equivalent resistance and increasing the local load demand. At t_4 , the auxiliary resistor is disconnected and the original load condition is restored.

As shown in Fig. 7.7, the output voltages of the DC MG can quickly recover to the given range after a short transient at t_3 and t_4 due to the load change. At the same time, the power output of each unit varies with the loads within the MG system, but the autonomous power ratio between units remains constant. Due to the design of the cost function (7.19) and the fact that DERs can respond quickly, the interactions

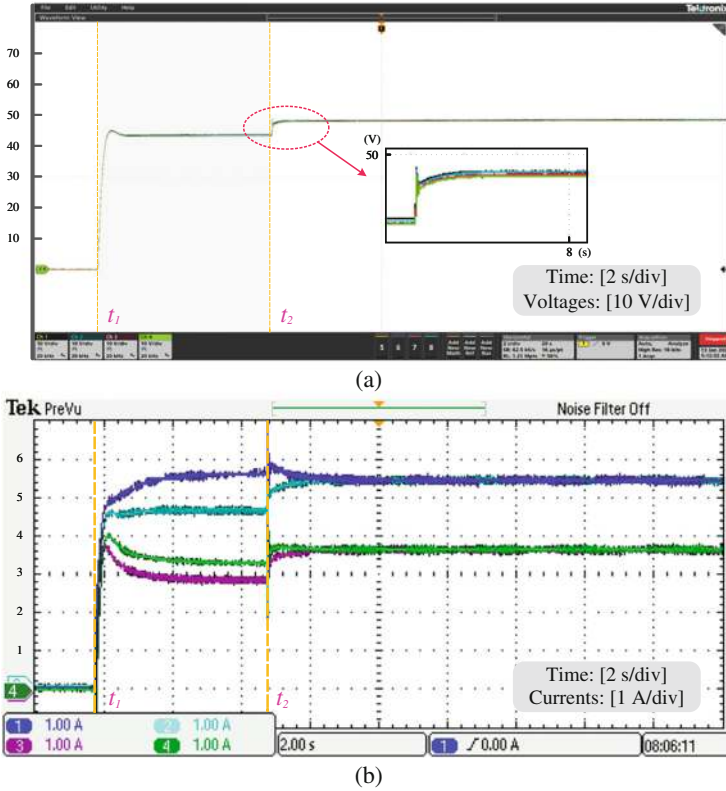


Fig. 7.6 (a) Voltage responses and (b) current responses of the MG test system with the parameter uncertainty

during autonomous operation and load changes are smooth. Benefiting from this, the proposed control strategy is somewhat resilient to the fluctuations caused by load switching, which is not available in the SPC-based approach. Up to this point, the above results also illustrate the adaptability of the proposed DL-HOFA secondary controller to load-demand variations.

7.4.5 Plug-and-Play Capability

Considering the intermittent nature of DERs, plug-and-play functionality is critical for MGs. Therefore, this subsection will test whether the proposed controller is still effective in making the whole system operate in the desired mode when a DER is plugged in or out of the MG.

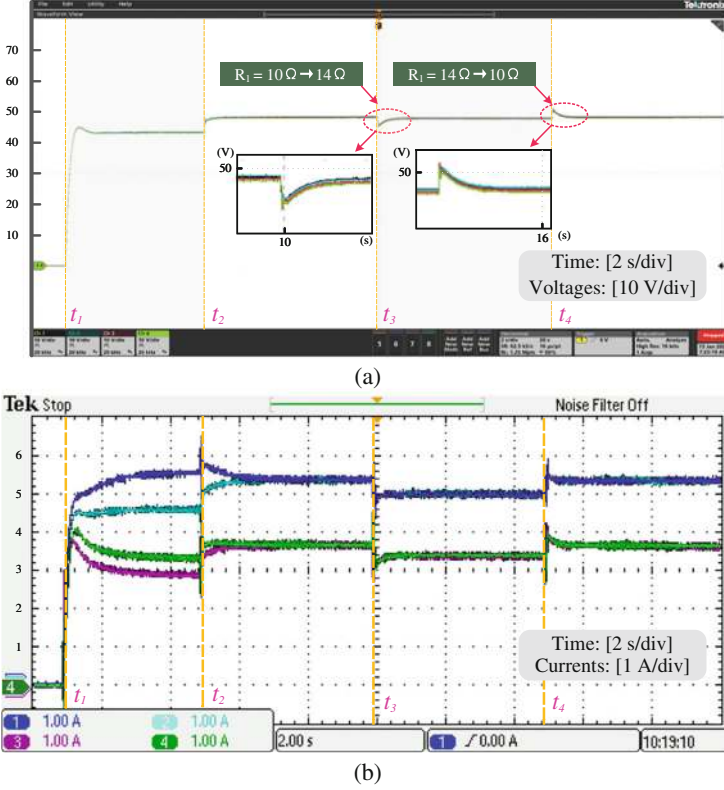


Fig. 7.7 (a) Voltage responses and (b) current responses of the MG test system in case of load variations

It is assumed that the DER₄ powered by the PV panel is physically unplugged at t_5 and then plugged back into the MG system at t_7 . The “plug” and “unplug” operations are performed by an air switch. Note that the failure of DER₄ also implies the loss of all communication links connected to this specific DER. The PnP experiment assumes that neighboring controllers update their local physical-neighbor matrices, communication adjacency information, and node-degree parameters after DER₄ is disconnected or reconnected. Therefore, when DER₄ is unplugged, it automatically disconnects from DER₁ and DER₃ in the communication layer, as shown in Fig. 7.8. As shown in Fig. 7.8, the disconnection of DER₄ from the MG at t_5 causes some fluctuations in the output voltage. In particular, owing to filter effects, the voltage at the PCC of DER₄ slowly decays to 0, while the output voltages of the remaining three DERs quickly stabilize in the desired interval $[(1 - \varepsilon)V_{ni}^{ref}, (1 + \varepsilon)V_{ni}^{ref}]$ after a short transient. As shown in Fig. 7.8, the exact sharing of the currents is achieved again among the remaining three DERs despite the failure of DER₄. It should be noted that when the supply capacity of DER₄ is recovered at the moment t_6 , its

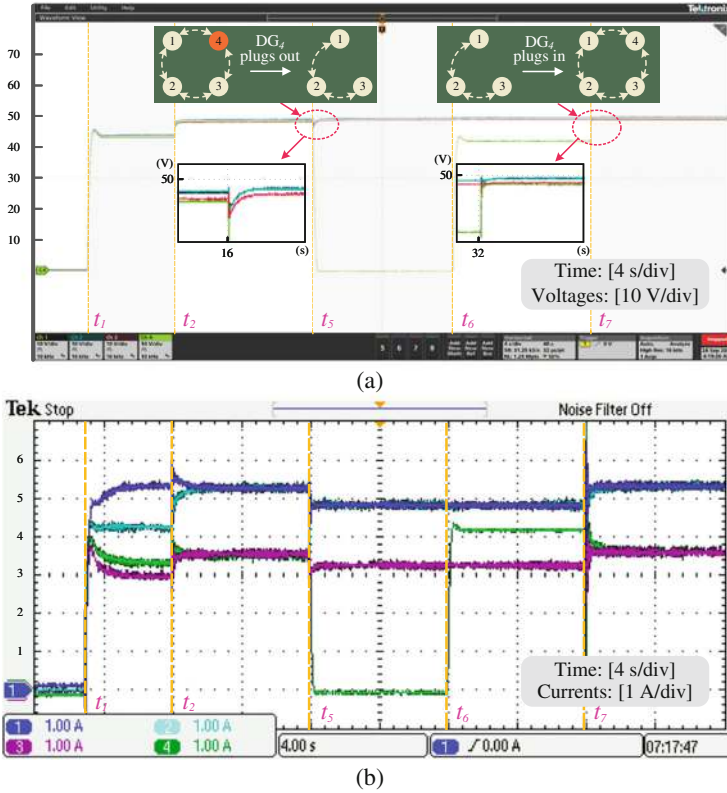


Fig. 7.8 (a) Voltage responses and (b) current responses of the MG test system in case of plug and play

voltage is first restored to the appropriate level under the effect of the local droop control $v_{o,i}^{ref}(t) = V_{ni}^{ref} - R_{vi} I_{fi}(t)$, where $V_{ni}^{ref} = 48$ V. Then, DER₄ is connected to the MG at moment t_7 and shares the load with the remaining three DERs. After DER₄ is restored and connected to the grid, the entire MG can also remain stable and achieve the expected regulation tasks.

In particular, the design of the DL-HOFA controller also allows the DC MG system to exhibit smooth performance throughout the process. Based on these results, we can conclude that the proposed secondary control strategy can realize fast voltage recovery as well as excellent consensus performance of current sharing during the electrical structure evolution of the MG system.

7.5 Notes and References

This chapter addresses the challenge of balancing voltage recovery and current sharing in multi-bus DC MG with a ring interconnection topology. To resolve

this issue, we introduce a DL-HOFA secondary control strategy designed to ensure precise current sharing while maintaining voltage within acceptable limits. Leveraging the CSM, DL-HOFA controller retains the simplicity of the widely used droop interface-based methods and effectively captures the diverse electrical characteristics of real power systems. The HOFA approach enhances the DL-HOFA control strategy, making it suitable for various converter-based DC MGs and aiding in the development of model-based control methods, thereby improving MG performance. Additionally, the distributed learning-based secondary control in DC MGs demonstrates strong adaptability to disturbances, guided by the proposed cost function. We provide a stability and consensus analysis of the closed-loop DC MG system under the DL-HOFA control scheme, which theoretically supports the method's feasibility and versatility. Extensive testing on a scaled-down hardware DC MG confirms the effectiveness of the introduced control strategy and highlights key features of the MG, such as its plug-and-play capability.

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Chapter 8

Fully Actuated Predictive Control of Renewable Power Systems with Inductive Lines



Abstract Voltage recovery and precise current distribution remain critical challenges in DC MGs with resistive-inductive network lines. To address these issues, this chapter introduces a secondary control strategy rooted in a holistic model that integrates intrinsic circuit dynamics and lower-level control interactions of DC MGs. The framework begins with the development of a fully actuated output current model. Leveraging this physics-based foundation, a distributed optimal fully actuated (DOFA) secondary predictive control method is introduced, incorporating a dynamic compensator to simultaneously achieve voltage stabilization and accurate current sharing. By integrating a predictive capability that anticipates the system's future states, the controller enhances transient performance and adaptability. Adaptive cost function weights within the design further expand the flexibility of secondary regulation, enabling accurate voltage recovery or load balancing as operational conditions evolve. Comprehensive closed-loop stability analyses confirm the system's ability to fulfill regulatory objectives under diverse conditions. Finally, a photovoltaic-and-battery-based DC MG with the MPPT controller is constructed to verify the effectiveness of the proposed controller. Experimental results highlight exceptional voltage regulation precision, transient response smoothness, and current distribution accuracy, outperforming conventional decentralized control benchmarks.

8.1 Overview

Droop control inherently addresses essential regulation tasks in DC MGs, such as voltage recovery (VR) (Guo et al., 2024). Simultaneously, achieving accurate current sharing (ACS) is critical to optimize MG operational efficiency (Lu et al., 2023; Yu et al., 2023b). While hierarchical control architectures enable precise VR and ACS (Jiang et al., 2024), prior research has extensively examined conflicting trade-offs between these objectives in DC MGs (Yu et al., 2023d). A common limitation, however, lies in the design of secondary controllers, which

often disregard the intrinsic physical dynamics and circuit-level characteristics of real-world MG systems. Many existing methodologies derive controllers without adequately incorporating an appropriate system model, potentially compromising adaptability to dynamic grid conditions. In contrast, model-based control strategies enable explicit use of predicted dynamics and performance objectives (Hu et al., 2021a). Notably, DC MG modeling approaches in the literature fall into three primary categories.

The first involves developing an integrated circuit-level model to capture the electrical behavior of DC MGs (Han et al., 2019a; Nia et al., 2023; Sadabadi et al., 2023; Xing et al., 2024). These frameworks, often structured as coupled multi-agent systems, enable holistic representation of grid dynamics and are widely adopted due to their high fidelity. However, their emphasis on interconnection can inadvertently restrict the distributed architecture of secondary controllers (Yu et al., 2023a). A second approach simplifies the system between droop control and voltage output as a static unity-gain model, where control inputs are adjusted via corrective terms appended to droop parameters (Deng et al., 2022; Lu et al., 2023; Nasirian et al., 2014, 2015). While this method's simplicity facilitates the design of topology-agnostic distributed controllers (Lu et al., 2023; Nasirian et al., 2015), its reliance on purely controller-level error dynamics neglects the physical system's intrinsic behavior, creating a conceptual divide between control algorithms and real-world circuit interactions. The third methodology abstracts MG voltage and current outputs as decoupled first-order systems (Dai et al., 2024; Sahoo & Mishra, 2019). Though this reductionist approach enables straightforward controller synthesis, it overlooks the inherent interdependencies between voltage and current, which is a fundamental constraint in power systems. Despite advances, current modeling paradigms face limitations in reconciling theoretical simplicity with physical accuracy. Existing techniques, while operationally effective, fail to fully harmonize control design with the nonlinear dynamics and electrical couplings inherent to DC MGs. Further refinement of these models remains imperative to bridge gaps between simulation, stability guarantees, and practical deployment (Ojo et al., 2020).

Drawing on prior advancements in DC microgrid (MG) modeling, this chapter introduces a high-performance secondary controller designed to address VR and ACS by rigorously integrating the physical dynamics of DC MGs. Three primary contributions distinguish this work from existing methodologies:

1. First, a unified modeling framework is proposed, encompassing the underlying circuits, inner control loops, and droop dynamics of DC MGs through a HOFA system approach. This methodology simplifies the derived model while preserving fidelity, enabling streamlined secondary controller design. Critically, the model explicitly incorporates the inductance dynamics of DG tie lines, which is a feature often oversimplified in prior studies, thereby enhancing accuracy in scenarios where inductive effects significantly influence system behavior. Such granularity differentiates this framework from conventional MG models.

2. Second, the proposed control strategy synthesizes coupled physical system dynamics with task-driven objectives, ensuring cyber-layer control inputs remain aligned with the physical-layer principles governing MG operation. While the relationship between the designed control action and system output resembles a first-order structure, the formulation inherently captures the dynamic electrical coupling between voltage, converter current, and tie-line current of voltage and current intrinsic to DC MGs. This duality transcends rudimentary first-order representations, offering a bridge between theoretical abstraction and real-world grid complexities.
3. Third, an innovative model-based secondary controller is introduced, incorporating a dynamic compensator with adaptable weighting factors to optimize a predefined cost function. Unlike prior methods constrained by static models or droop-based simplifications, this approach leverages the HOFA model's predictive capacity to harmonize transient performance with stability—a capability seldom explored in existing literature. Traditional integrated models often lack the adaptability required for rolling prediction, while droop-centric frameworks oversimplify system dynamics, limiting controller efficacy. By addressing these gaps, the proposed strategy advances robustness in dynamic grid environments.

The remainder of this chapter is organized as follows. Section 8.2 describes the multi-bus DC MG and formulates the voltage-restoration and proportional current-sharing objectives. Section 8.3 develops the HOFA-based model while retaining the tie-line inductance dynamics. Section 8.4 presents the DOFA predictive secondary controller. Sections 8.5 and 8.6 examine the closed-loop stability and frequency-domain characteristics, respectively. Section 8.7 reports the experimental validation. Finally, Section 8.8 concludes this chapter.

8.2 Problem Formulation

In this section, a system description of a DC-DC converter-based DC MG and the control objective to solve the contradiction between voltage and current regulation are presented.

This paper is concerned with a typical multi-bus DC MG consisting of N DG units interconnected by tie lines in a meshed topology. Its structure is presented in Fig. 6.1. Specifically, a DC MG is powered by RESs such as wind turbines, photovoltaic generation and energy-storage systems while containing power components such as DC-DC converters, RLC filters, tie lines, and local resistive loads, as shown in Fig. 8.1.

To facilitate the narrative, the desired current sharing ratio of i th DG is first defined as θ_i . Similar to Sect. 8.6, the output voltage and inductor current of the DC MG should satisfy

$$\lim_{t \rightarrow \infty} \left\| \frac{V_{ni} - v_{o,i}(t)}{V_{ni}} \right\| \leq \sigma_i, \lim_{t \rightarrow \infty} \left\| \frac{i_{dg,j}(t)}{\theta_j} - \frac{i_{dg,i}(t)}{\theta_i} \right\| = 0 \quad (8.1)$$

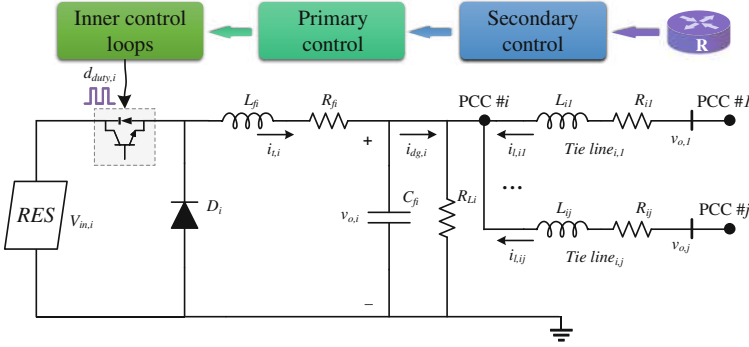


Fig. 8.1 Electrical structure of i th DG with resistive load

where V_{ni} is the nominal voltage, σ_i is a constant satisfying $\sigma_i \leq \varepsilon$, and ε is the given allowable threshold.

8.3 DC MGs Modeling with HOFA System Approaches

The HOFA system concept was innovatively proposed in Duan (2021a), where preliminary knowledge of the HOFA system approach can also be found. Unlike the work Yu et al. (2024a), the dynamics of the inductance of the tie line between DGs is no longer neglected here. With the help of Kirchhoff's voltage and current laws, the circuit part of the i th DG in the studied DC MG considering the dynamics of tie lines can be modeled as the following differential equations

$$\begin{aligned}
 C_{fi} \dot{v}_{o,i}(t) &= -R_{Li}^{-1} v_{o,i}(t) + i_{t,i}(t) + \sum_{j \in \mathcal{N}_i^p} i_{l,ij}(t) \\
 L_{ii_1} \dot{i}_{l,ii_1}(t) &= -R_{i1} i_{l,ii_1}(t) + v_{o,i_1}(t) - v_{o,i}(t) \\
 &\dots \\
 L_{iimi} \dot{i}_{l,iimi}(t) &= -R_{iimi} i_{l,iimi}(t) + v_{o,iimi}(t) - v_{o,i}(t) \\
 L_{fi} \dot{i}_{t,i}(t) &= V_{in} d_{duty,i}(t) - v_{o,i}(t) - R_{fi} i_{t,i}(t)
 \end{aligned} \tag{8.2}$$

where $v_{o,j} \in \mathcal{N}_i^p$ are the voltages at the port linked with the i th DG, $i_{l,ij}$ is the current on the tie line connecting i th and j th DG, mi is the number of lines connected to the i th DG, and $\mathcal{N}_i^p$ is the set of physically neighboring DG units connected to i th DG through the power line. $d_{duty,i}$ stands for the duty cycle used

to generate the pulse-width modulation signal for driving the IGBT, $V_{in,i}$ indicates the input voltage of the power supply, and the meanings of other circuit parameters are typical. The above model is applicable to both LV DC MGs with only resistive power lines and MV or HV DC MGs containing lines with a low X/R ratio, since the lines are modeled as lumped R-L circuits.

Remark 8.3.1 Compared with the work Yu et al. (2024a), the order of the integrated model for the physical layer of the i th DG will be higher after taking into account the inductive property of the tie line. Including the tie-line inductance introduces additional dynamic modes and may reduce damping or produce oscillatory interactions, thereby imposing more stringent requirements on the local and secondary controller parameters. It is seen that ignoring the inductance of the line will reduce the difficulty of modeling but is not quite accurate for describing the tie lines in MV and HV DC MGs. Moreover, capturing system dynamics provides possibility to design model-based distributed secondary control strategies. In this way, users can purposefully design feedbacks based on the captured system dynamics to compensate for undesired dynamics in the system, such as coupling and nonlinearities.

To achieve the grid connection of each DG in islanded operation mode without relying on communication equipment, the voltage reference $v_{o,i}^*$ is generally generated by the following droop control

$$v_{o,i}^*(t) = V_{ni} - R_{vi}i_{t,i}(t) + \delta_i(t) \quad (8.3)$$

where R_{vi} is the droop factor and $\delta_i(t)$ is the correction term, which is also the secondary control output to be designed to accomplish the desired voltage and current regulation missions. This correction term is set to 0 when the secondary control is not activated.

To obtain better tracking performance, it is typical to use dual closed-loop PI control for voltage and current at the device level, and here is no exception. Specifically, the dynamic characteristics of the voltage outer loop are portrayed by the following equation

$$\begin{aligned} \dot{\phi}_i(t) &= v_{o,i}^*(t) - v_{o,i}(t) \\ i_{li}^*(t) &= k_{pvi}(v_{o,i}^*(t) - v_{o,i}(t)) + k_{ivi}\phi_i(t) \end{aligned} \quad (8.4)$$

where $v_{o,i}^*(t)$ can be obtained from (8.3), $\phi_i(t)$ is called the integral variable of the voltage outer loop, $i_{li}^*(t)$ is the current reference and will be fed to the inner current loop, k_{pvi} and k_{ivi} are the proportional and integral gains, respectively. Similarly, the dynamics of the inner current loop can be presented as

$$\begin{aligned} \dot{\gamma}_i(t) &= i_{li}^*(t) - i_{li}(t) \\ d_{duty,i}(t) &= k_{pci}(i_{li}^*(t) - i_{li}(t)) + k_{ici}\gamma_i(t) \end{aligned} \quad (8.5)$$

where $\gamma_i(t)$ is the integral variable of the current inner loop and k_{pci} and k_{ici} are the proportional and integral gains, respectively. Note that the $i_{li}(t)$ in (8.5) is taken as $i_{t,i}(t)$ in this chapter. Meanwhile, substituting (8.3) and (8.4) into (8.5) yields

$$\begin{aligned} d_{duty,i}(t) = & -k_{P,i}^i k_{P,i}^v v_{o,i}(t) - (k_{P,i}^i k_{P,i}^v R_{vi} + k_{P,i}^i) i_{t,i}(t) + k_{P,i}^i k_{L,i}^v \phi_i(t) \\ & + k_{L,i}^i \gamma_i(t) + k_{P,i}^i k_{P,i}^v \delta_i(t) + k_{P,i}^i k_{P,i}^v V_{ni}. \end{aligned} \quad (8.6)$$

Take the following variables $x_{i,1}(t) = [v_{o,i}(t), i_{t,ii_1}(t), \dots, i_{t,ii_{mi}}(t), i_{t,i}(t)]^T$, and $x_{i,2}(t) = [\phi_i(t), \gamma_i(t)]^T$. Let $x_i(t) = [x_{i,1}^T, x_{i,2}^T]^T$. Integrating the primary control (8.3), the voltage and current loops (8.4) and (8.5), the duty cycle (8.6) and the dynamics of the circuit with RLC filter (8.2), and taking the correction term $\delta_i(t)$ as the control input, the following complete state space model of DC MGs can be obtained

$$\dot{x}_i(t) = A_{ii} x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij} x_j(t) + B_{ii} u_{r,i}(t) + D_{ii} V_{ni} \quad (8.7)$$

where $A_{ii} = [A_{ii}^{pp}, A_{ii}^{pc}; A_{ii}^{cp}, A_{ii}^{cc}]$, $A_{ij} = [A_{ij}^{pp}, A_{ij}^{pc}; A_{ij}^{cp}, A_{ij}^{cc}]$, $B_{ii} = [B_{ii,1}; B_{ii,2}]$, $D_{ii} = [D_{ii,1}; D_{ii,2}]$, $B_{ii,2} = [1; k_{P,i}^v]$, $b_{c,i} = L_{fi}^{-1} V_{in,i} k_{P,i}^i k_{P,i}^v$, A_{ij}^{pc} , A_{ij}^{cp} , and A_{ij}^{cc} are zero matrices with appropriate dimensions, and other system matrices are given in (8.12) at the bottom of the next page, and $\Upsilon_1 = -L_{fi}^{-1} (V_{in} k_{P,i}^i k_{P,i}^v + 1)$, $\Upsilon_2 = -L_{fi}^{-1} (R_{fi} + V_{in} (k_{P,i}^i k_{P,i}^v R_{di} + k_{P,i}^i))$.

Performing the Eulerian discretization of the state space equation above, it gives

$$\begin{aligned} x_i(k+1) &= (I + T_s A_{ii}) x_i(k) + T_s \sum_{j \in \mathcal{N}_i^p} A_{ij} x_j(k) + T_s B_{ii} u_{r,i}(k) + T_s D_{ii} V_{ni} \\ y_i(k) &= i_{t,i}(k) = C_i x_i(k) \end{aligned} \quad (8.8)$$

where T_s is the sampling period, $i_{t,i}$ is regarded as the output of the system, $C_i = [C_{i,1}, C_{i,2}]$, $C_{i,1} = [0, \dots, 0, 1]$, and $C_{i,2} = [0, 0]$.

Similar to the derivative operation in continuous time, one has

$$\begin{aligned} i_{t,i}(k+1) - i_{t,i}(k) &= C_i T_s A_{ii} x_i(k) + C_i T_s D_{ii} V_{ni} \\ &+ C_i T_s B_{ii} u_{r,i}(k) + C_i T_s \sum_{j \in \mathcal{N}_i^p} A_{ij} x_j(k). \end{aligned} \quad (8.9)$$

Notice that $T_s C_i B_{ii} = T_s (C_{i,1} B_{ii,1} + C_{i,2} B_{ii,2}) = b_{c,i} \neq 0$. The local bus voltage, filter-inductor current, and tie-line currents are assumed to be measurable. The PI integrator states are directly available from the digital controller, while any neighboring variables required by $A_{ij} x_j$ are assumed to be exchanged through the

physical-neighbor communication links. To obtain a DC MG model with simple structure, the high-order fully actuated secondary controller can be designed as

$$u_{r,i}(k) = b_{c,i}^{-1}(u_{ca,i}(k) + u_i(k)). \quad (8.10)$$

Further, take the auxiliary control $u_{ca,i}(k)$ as

$$u_{ca,i}(k) = -C_i T_s (A_{ii} x_i(k) + D_{ii} V_{ni} + \sum_{j \in \mathcal{N}_i^P} A_{ij} x_j(k)). \quad (8.11)$$

Substituting (8.10) and (8.11) into (8.9), it leads to the following fully actuated (FA) DC MG current model

$$\begin{aligned} A_{ii}^{pp} &= \begin{bmatrix} -C_{fi}^{-1} R_{Li}^{-1} & C_{fi}^{-1} & C_{fi}^{-1} & \cdots & C_{fi}^{-1} & C_{fi}^{-1} \\ -L_{i1}^{-1} & -L_{i1}^{-1} R_{i1} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -L_{im}^{-1} & 0 & 0 & \cdots & -L_{im}^{-1} R_{im} & 0 \\ \gamma_1 & 0 & 0 & \cdots & 0 & \gamma_2 \end{bmatrix}, B_{ii,1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_i \end{bmatrix} \\ A_{ij}^{pp} &= \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ L_{ij}^{-1} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, A_{ii}^{pc} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ L_{fi}^{-1} V_{in} k_{P,i}^i k_{L,i}^v & L_{fi}^{-1} V_{in} k_{L,i}^i \end{bmatrix}, D_{ii,1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_i \end{bmatrix} \\ A_{ii}^{cp} &= \begin{bmatrix} -1 & 0 & \cdots & 0 & -R_{di} \\ -k_{P,i}^v & 0 & \cdots & 0 & -(k_{P,i}^v R_{di} + 1) \end{bmatrix}, A_{ii}^{cc} = \begin{bmatrix} 0 & 0 \\ k_{L,i}^v & 0 \end{bmatrix}, B_{ii,2} = \begin{bmatrix} 1 \\ k_{P,i}^v \end{bmatrix}, D_{ii,2} = \begin{bmatrix} 1 \\ k_{P,i}^v \end{bmatrix}. \quad (8.12) \\ i_{t,i}(k+1) &= i_{t,i}(k) + u_i(k). \quad (8.13) \end{aligned}$$

At this point, this chapter obtains the FA model (8.13) for the DC MG, which realistically reflect the actual electrical characteristics of the power system and are simple in form, which further facilitates the design, debugging, and implementation of the controller. In particular, the poles of the closed-loop DC MG can be arbitrarily assigned based on the above model, so that various performance metrics can be predefined without the constraint of power line coupling in the MG. In terms of the specific derivation process, the above FA model is obtained under a multi-bus DC MG system. In fact, the above modeling approach can be applied to DC MG systems with various topologies. However, an integrated model needs to be obtained based on the connection of the corresponding tie lines. Moreover, the transformation relies on accurate model parameters.

8.4 Design of DOFA Predictive Control

This section develops a secondary control scheme for bounded voltage restoration and proportional current sharing. Because exact restoration of all PCC voltages to the same reference generally eliminates the voltage gradients required for proportional current sharing in a resistive multi-bus DC network, the secondary voltage reference is constrained to an admissible interval around the nominal value. This bounded-reference mechanism provides a practical compromise: proportional current sharing is pursued while each PCC voltage is maintained within its permitted operating range.

A DOFA predictive current controller is subsequently designed to achieve proportional current sharing by incorporating future MG states into the cost function. With the voltage limiter, the reference applied to the local double-loop controller is defined as

$$V_{sec}^*(k) = \begin{cases} V_{ni} - \epsilon_i, v_{o,i}^*(k) \in (-\infty, V_{ni} - \epsilon_i] \\ v_{o,i}^*(k), v_{o,i}^*(k) \in (V_{ni} - \epsilon_i, V_{ni} + \epsilon_i] \\ V_{ni} + \epsilon_i, v_{o,i}^*(k) \in (V_{ni} + \epsilon_i, +\infty) \end{cases} \quad (8.14)$$

where $\epsilon_i = \varepsilon V_{ni}$. Consistent with the inactive-limiter assumption adopted in the preceding chapters, the controller parameters, initial conditions, and admissible disturbances are assumed to keep $v_{o,i}^*(k)$ strictly within

$$(V_{ni} - \epsilon_i, V_{ni} + \epsilon_i)$$

throughout the theoretical analysis and operating scenarios considered here. Consequently, the limiter remains inactive and $V_{sec,i}^*(k) = v_{o,i}^*(k)$. The unconstrained FA model is therefore used in the subsequent controller design and stability analysis. Note that once the secondary control is activated, the reference value $v_{o,i}^*$ in (8.4) switches to the voltage set point $V_{sec}^*(k)$ generated by the secondary layer and applied to the local voltage-control loop.

To obtain a faster regulation process, the following dynamic compensator for MG system (8.13) is designed

$$\begin{aligned} \zeta_i(k+1) &= (1 - T_s k_{\zeta i}) \zeta_i(k) + T_s k_{\zeta i} i_{t,i}(k) \\ u_i(k) &= -k_{c,i}(i_{t,i}(k) - \zeta_i(k)) + u_{p,i}(k) \end{aligned} \quad (8.15)$$

where $k_{\zeta i}$ and $k_{c,i}$ are the gains to be determined and $u_{p,i}(k)$ is the secondary predictive control to be designed. Substituting (8.15) into (8.13), the following state equation for the DC MG in discrete time space can be obtained

$$\begin{aligned} i_{t,i}(k+1) &= i_{t,i}(k) - k_{c,i}(i_{t,i}(k) - \zeta_i(k)) + u_{p,i}(k) \\ \zeta_i(k+1) &= (1 - T_s k_{\zeta i}) \zeta_i(k) + T_s k_{\zeta i} i_{t,i}(k). \end{aligned} \quad (8.16)$$

Again, take the discrete state variable as $\xi_i(k) = [i_{t,i}(k), \zeta_i(k)]^T$. Let $i_{so,i}(k) = i_{t,i}(k)/\theta_i$. Therefore, taking $y_i(k) = i_{so,i}(k)$ as the system output, the state space equation about $\xi_i(k)$ can be derived as

$$\begin{aligned}\xi_i(k+1) &= A_i \xi_i(k) + B_i u_{p,i}(k) \\ y_i(k) &= i_{t,i}(k)/\theta_i = C_{so,i} \xi_i(k)\end{aligned}\quad (8.17)$$

where $A_i = [1 - k_{c,i}, k_{c,i}; T_s k_{\zeta i}, 1 - T_s k_{\zeta i}]$, $B_i = [1; 0]$, and $C_{so,i} = [1/\theta_i, 0]$.

Further, to improve the performance of the MG system, the following cost function with predicted states is proposed

$$\begin{aligned}J(k) &= \sum_{i=1}^N \sum_{N_y=\underline{N}_y}^{\bar{N}_y} \sum_{j \in \mathcal{N}_i^n} a_{ij} \left\| \hat{i}_{so,j}^{vec}(k + N_y|k) \right. \\ &\quad \left. - \hat{i}_{so,i}^{vec}(k + N_y|k) \right\|_2^2 + b_i \left\| \hat{U}_i^{vec}(k + N_u|k) \right\|_2^2\end{aligned}\quad (8.18)$$

where a_{ij} is the communication weight of the signal transmitted from node j to i ; $\mathcal{N}_i^n$ is the set of neighbors of i th DG in the communication network; b_i is the penalty factor of the control cost; $\bar{N}_y$ and N_u are the lengths of the prediction windows of the output and input, respectively; $\underline{N}_y$ is the parameter adjusting the weights of the coordination cost; and $\hat{i}_{so,i}^{vec}(k + N_y|k)$ and $\hat{U}_i^{vec}(k + N_u|k)$ indicate the prediction vectors for the weighted output current vector $i_{so,i}^{vec}(k + N_y|k)$ and the control input vector $U_i^{vec}(k + N_u|k)$ based on the information at the current moment k , respectively, i.e., $\hat{i}_{so,i}^{vec}(k + N_y|k) = [\hat{i}_{so,i}(k + N_y|k), \hat{i}_{so,i}(k + N_y - 1|k), \dots, \hat{i}_{so,i}(k + 1|k)]^T$, $\hat{U}_i^{vec}(k + N_u|k) = [\hat{u}_{p,i}(k + N_u|k), \hat{u}_{p,i}(k + N_u - 1|k), \dots, \hat{u}_{p,i}(k|k)]^T$, where $\hat{i}_{so,i}(k + l|k)$ is the l -step-ahead of $i_{so,i}(k)$. The first term in (8.18) describes the cooperation error between the weighted currents, portraying the accuracy of current sharing. The second term reflects the cost of the secondary control, which is used to constrain the duty cycle to a reasonable range. The summation operation from $\underline{N}_y$ to $\bar{N}_y$ in (8.18) gives different weighting factors to predicted states at different moments. It should be noted that the definitions and symbolic variables in this chapter regarding the information layer of the MG are the same as those in Yu et al. (2024a).

Based on the discrete model (8.17) of the DC MG, using the input and output data available at k , the prediction of the i th DG about the system state can be obtained as

$$\hat{\xi}_i(k + \ell|k) = A_i^\ell \xi_i(k) + \sum_{l=1}^{\ell} A_i^{\ell-l} B_i \hat{u}_{p,i}(k + l - 1|k) \quad (8.19)$$

for $\ell = 1, 2, \dots, N_u$. In addition, for $k = N_u + 1, N_u + 2, \dots, \bar{N}_y$, let $\hat{u}_i(k + k|k) = \hat{u}_i(k + N_u|k)$. At this point, the prediction of the i th DG about the system state in (8.17) is

$$\begin{aligned}\hat{\xi}_i(k + \ell|k) &= A_i^{\ell-N_u} \sum_{l=1}^{N_u} A_i^{N_u-l} B_i \hat{u}_{p,i}(k + l - 1|k) \\ &+ \sum_{l=1}^{\ell-N_u} A_i^{l-1} B_i \hat{u}_{p,i}(k + N_u|k) + A_i^\ell \xi_i(k).\end{aligned}\quad (8.20)$$

for $\ell = N_u + 1, \dots, \bar{N}_y$.

Combining (8.17), (8.19), and (8.20), one obtains $\hat{i}_{so,i}^{vec}(k + k_l|k) = C_{so,i}^{vec}(\bar{A}_{i,l}\xi_i(k) + \bar{\Theta}_{i,l}\hat{U}_i(k + N_u|k)) = A_{i,l}\xi_i(k) + \Theta_{i,l}\hat{U}_i(k + N_u|k)$, where $k_1 = \underline{N}_y, k_2 = \underline{N}_y + 1, \dots, k_{\varpi} = \bar{N}_y, \varpi = \bar{N}_y - \underline{N}_y + 1, A_{i,l} = C_{so,i}^{vec} \bar{A}_{i,l}, \Theta_{i,l} = C_{so,i}^{vec} \bar{\Theta}_{i,l}, \bar{A}_{i,l} = [A_i^{k_l}, A_i^{k_l-1}, \dots, A_i]^T, \bar{\Theta}_{i,l} = [B_i, A_i B_i, \dots, A_i^{k_l-1} B_i; 0, B_i, \dots, A_i^{k_l-2} B_i; \dots; 0, 0, \dots, B_i], C_{so,i}^{vec} = \text{diag}\{\overbrace{C_{so,i}, \dots, C_{so,i}}^{k_l}\}$.

For the cost function (8.18), by the first-order optimality condition, there is

$$\begin{aligned}\frac{\partial J(k)}{\partial \hat{U}_i^{vec}(k + N_u|k)} &= \sum_{l=1}^{\varpi} \frac{\partial J_l^y(k)}{\partial \hat{U}_i^{vec}(k + N_u|k)} \\ &+ b_i \hat{U}_i^{vec}(k + N_u|k) = 0\end{aligned}\quad (8.21)$$

where $J_l^y(k) = \sum_{i=1}^N (\sum_{j \in N_i} a_{ij} ||\hat{i}_{so,j}^{vec}(k + k_l|k) - \hat{i}_{so,i}^{vec}(k + k_l|k)||_2^2)$. Further, it can be deduced that

$$\begin{aligned}\frac{\partial J_l^y(k)}{\partial \hat{U}_i^{vec}(k + N_u|k)} &= -\Theta_{i,l}^T \sum_{j \in \mathcal{N}_i^n} a_{ij}^s (\hat{i}_{so,j}^{vec}(k + k_l|k) \\ &- \hat{i}_{so,i}^{vec}(k + k_l|k))\end{aligned}\quad (8.22)$$

where $a_{ij}^s = a_{ji} + a_{ij}$. Thus, putting (8.22) into compact form yields $\bar{S}_l = \text{span}\{\Theta_i \sum_{j \in \mathcal{N}_i^n} a_{ij}^s (\Lambda_{j,l} \xi_j(k) - \Lambda_{i,l} \xi_i(k))\}, \bar{\Gamma}_l = \text{span}\{-\Theta_{i,l}^T \sum_{j \in \mathcal{N}_i^n} a_{ij}^s \Theta_{j,l} \hat{U}_j^{vec}(k + N_u|k) + \bar{z}_{i,l}\}$, where $\bar{z}_{i,l} = z_{i,l} \hat{U}_i^{vec}(k + N_u|k), z_{i,l} = \tilde{a}_i \Theta_{i,l}^T \Theta_{i,l} + b_i I_{ii}, \tilde{a}_i = \sum_{j \in \mathcal{N}_i^n} a_{ij}^s$, and $i = 1, 2, \dots, N$.

Let $n_u = N_u + 1$ denote the number of control moves contained in each predicted input vector. Define

$$\bar{g} = [0 \dots 0 \ 1] \in \mathbb{R}^{1 \times n_u},$$

where the last element selects the current control input $\hat{u}_{p,i}(k|k)$ from the reversely ordered prediction vector $\hat{U}_i^{vec}(k + N_u|k)$. Let $\mathbf{e}_i \in \mathbb{R}^N$ denote the i th canonical basis vector. The corresponding global selection row is defined as

$$g_i = \mathbf{e}_i^T \otimes \bar{g} \in \mathbb{R}^{1 \times N n_u}.$$

Therefore, the control input applied to DG_i is

$$\hat{u}_{p,i}(k|k) = g_i \Gamma^{-1} S \xi(k), \quad (8.23)$$

where $\xi^{vec}(k) = \text{col}\{\xi_1(k), \dots, \xi_N(k)\} \in \mathbb{R}^{2N}$, $S = \sum_{l=1}^{\sigma} S_l$, $\Gamma = \sum_{l=1}^{\sigma} \Gamma_l$, $S_l\{i, i\} = -\tilde{a}_i \Theta_{i,l}^T \Lambda_{i,l}$, $S_l\{i, j\} = a_{ij}^s \Theta_{i,l}^T \Lambda_{j,l}$ for $i \neq j$, $\Gamma_l\{i, i\} = b_i I_{ii} + \tilde{a}_i \Theta_{i,l}^T \Theta_{i,l}$, $\Gamma_l\{i, j\} = -\Theta_{i,l}^T a_{ij}^s \Theta_{j,l}$ for $i \neq j$. Further, from (8.10) and (8.23), the correction term in the droop control (8.3) becomes

$$\delta_i(k) = b_{c,i}^{-1}(u_{ca,i}(k) - k_{c,i}(i_{t,i}(k) - \zeta_i(k)) + \hat{u}_{p,i}(k|k)). \quad (8.24)$$

At this point, the design of the secondary controller is completed. In summary, the developed FA model of DC MG not only facilitates the design of control strategies but also ensures that the whole design process conforms to the electrical characteristics of the DC MG itself. Finally, the block diagram of the proposed DOFA secondary control strategy embedded in a DC MG is presented in Fig. 8.2.

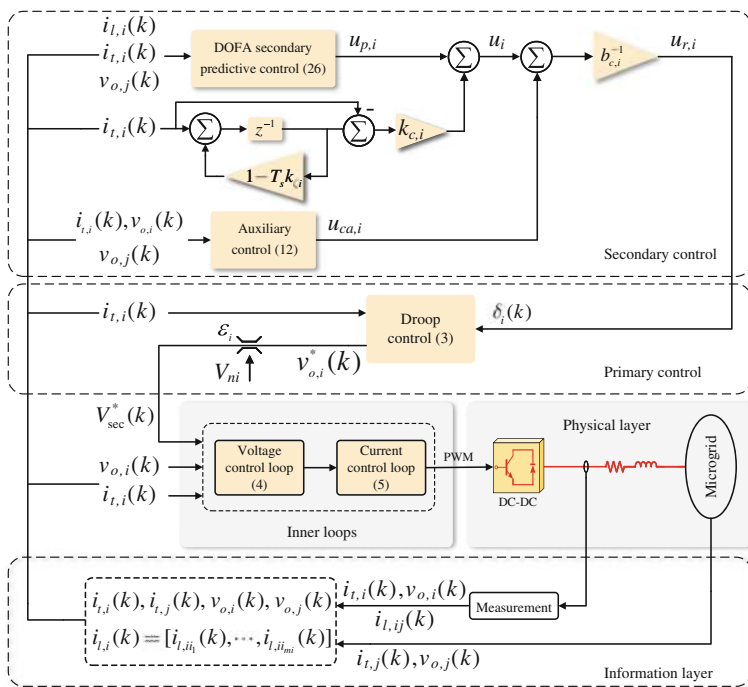


Fig. 8.2 Architecture of the proposed DOFA control scheme

8.5 Stability and Consensus Analysis of DC MGs

An important issue in MG control systems is the stability of the closed-loop system. Therefore, in this section, the stability and consensus analysis of the proposed DOFA secondary control strategy for DC MGs is presented.

Define $\varphi_i(t) = [v_{o,i}(t), i_{l,ii_1}(t), \dots, i_{l,ii_{mi}}(t)]^T$. It can be seen from (8.2) that the dynamics of the capacitor voltage and the current on the line inductor are determined by themselves and the current on the filter inductor, and one obtains

$$\dot{\varphi}_i(t) = A_i^\varphi \varphi_i(t) + A_i^c i_{t,i}(t) + \sum_{j \in \mathcal{N}_i^p} \bar{L}_{ij} \varphi_j(t) \quad (8.25)$$

where $A_i^c = [C_{fi}^{-1}, 0, \dots, 0; \dots; 0, \dots, 0]$, $\bar{L}_{ij}$ is 0 except for $\bar{L}_{ij}\{i, j\} = L_{ij}^{-1}$,

$$A_i^\varphi = \begin{bmatrix} -C_{fi}^{-1} R_{Li}^{-1} & 1 & 1 \dots 1 \\ -L_{ii_1}^{-1} & -R_{ii_1} L_{ii_1}^{-1} & 0 \dots 0 \\ \vdots & \vdots & \vdots \\ -L_{ii_{mi}}^{-1} & -R_{ii_{mi}} L_{ii_{mi}}^{-1} & 0 \dots 0 \end{bmatrix}.$$

After discretization, taking its increment in the discrete time space gives $\Delta\varphi_i(k+1) = (I + T_s A_i^\varphi) \Delta\varphi_i(k) + T_s A_i^c \Delta i_{t,i}(k) + T_s \sum_{j \in \mathcal{N}_i^p} \bar{L}_{ij} \Delta\varphi_j(k)$, where $\Delta\varphi^{vec}(k) = [\Delta\varphi_1(k), \dots, \Delta\varphi_N(k)]^T$. The above equation can be written in compact form as follows:

$$\Delta\varphi(k+1) = A_\varphi \Delta\varphi(k) + A_{\varphi,\xi} \Delta\xi^{vec}(k) \quad (8.26)$$

where $A_\varphi\{i, j\} = I + T_s A_i^\varphi$ for $i = j$ and $T_s \bar{L}_{ij}$ for $i \neq j$, $A_{\varphi,\xi} = \text{diag}\{\theta_1 T_s A_1^c C_{so,1}, \dots, \theta_N T_s A_N^c C_{so,N}\}$. Recalling the local state-space model in (8.17), define the stacked state, input, and system matrices as

$$\xi(k) = \text{col}\{\xi_1(k), \dots, \xi_N(k)\} \in \mathbb{R}^{2N},$$

$$u_p(k) = \text{col}\{u_{p,1}(k), \dots, u_{p,N}(k)\} \in \mathbb{R}^N,$$

$$A_{\xi,0} = \text{blkdiag}\{A_1, \dots, A_N\} \in \mathbb{R}^{2N \times 2N},$$

and

$$B_\xi = \text{blkdiag}\{B_1, \dots, B_N\} \in \mathbb{R}^{2N \times N}.$$

The global form of (8.17) is therefore

$$\xi(k+1) = A_{\xi,0} \xi(k) + B_\xi u_p(k).$$

Let

$$\widehat{\mathbf{U}}(k) = \text{col} \{ \widehat{\mathbf{U}}_1^{\text{vec}}(k + N_u | k), \dots, \widehat{\mathbf{U}}_N^{\text{vec}}(k + N_u | k) \} \in \mathbb{R}^{Nn_u}$$

denote the stacked predicted control sequence. The compact optimal solution corresponding to (8.23) is

$$\widehat{\mathbf{U}}(k) = \Gamma^{-1} S \boldsymbol{\xi}(k),$$

where

$$\Gamma \in \mathbb{R}^{Nn_u \times Nn_u}, \quad S \in \mathbb{R}^{Nn_u \times 2N}.$$

Define the receding-horizon input-selection matrix as

$$G = I_N \otimes \bar{g} = \text{col} \{ g_1, \dots, g_N \} \in \mathbb{R}^{N \times Nn_u}.$$

The control inputs actually applied to the DG units are then

$$\mathbf{u}_p(k) = G \widehat{\mathbf{U}}(k) = G \Gamma^{-1} S \boldsymbol{\xi}(k).$$

Substituting the predictive feedback law into the stacked state equation gives

$$\boldsymbol{\xi}(k+1) = \left(A_{\xi,0} + B_{\xi} G \Gamma^{-1} S \right) \boldsymbol{\xi}(k).$$

Under a fixed communication topology, fixed prediction horizons, and time-invariant controller weights, the matrices G , Γ , and S are constant. Taking the increment of the preceding equation therefore yields

$$\Delta \boldsymbol{\xi}(k+1) = A_{\xi} \Delta \boldsymbol{\xi}(k), \tag{8.27}$$

where

$$A_{\xi} = A_{\xi,0} + B_{\xi} G \Gamma^{-1} S \in \mathbb{R}^{2N \times 2N}.$$

To characterize proportional current sharing without retaining the identically zero error associated with DG_1 , define the current-sharing errors relative to DG_1 as

$$e_{s,i}(k) = i_{\text{so},i}(k) - i_{\text{so},1}(k), \quad i = 2, \dots, N. \tag{8.28}$$

Define the stacked weighted-current vector as

$$\mathbf{i}_{\text{so}}(k) = \text{col} \{ i_{\text{so},1}(k), \dots, i_{\text{so},N}(k) \} = C_{\text{so}} \boldsymbol{\xi}^{\text{vec}}(k) \in \mathbb{R}^N,$$

where

$$C_{\text{so}} = \text{blkdiag} \{C_{\text{so},1}, \dots, C_{\text{so},N}\} \in \mathbb{R}^{N \times 2N}.$$

The reduced current-sharing error vector is then defined as

$$\mathbf{e}_s(k) = \text{col} \{e_{s,2}(k), \dots, e_{s,N}(k)\} = M_1 \mathbf{i}_{\text{so}}(k) = M_1 C_{\text{so}} \boldsymbol{\xi}^{\text{vec}}(k), \quad (8.29)$$

where

$$M_1 = \begin{bmatrix} -\mathbf{1}_{N-1} & I_{N-1} \end{bmatrix} \in \mathbb{R}^{(N-1) \times N},$$

$\mathbf{1}_{N-1}$ denotes the $(N-1)$ -dimensional vector of ones, and I_{N-1} is the $(N-1)$ -dimensional identity matrix. Taking the increment of (8.29) gives

$$\mathbf{e}_s(k+1) - \mathbf{e}_s(k) = M_1 C_{\text{so}} \Delta \boldsymbol{\xi}^{\text{vec}}(k+1).$$

Substituting the closed-loop incremental model (8.27) into the above equation yields

$$\begin{aligned} \mathbf{e}_s(k+1) &= \mathbf{e}_s(k) + M_1 C_{\text{so}} A_{\xi} \Delta \boldsymbol{\xi}^{\text{vec}}(k) \\ &= \mathbf{e}_s(k) + M_1 C_{\text{so}} \left(A_{\xi,0} + B_{\xi} G \Gamma^{-1} S \right) \Delta \boldsymbol{\xi}^{\text{vec}}(k), \end{aligned} \quad (8.30)$$

where

$$A_{\xi} = A_{\xi,0} + B_{\xi} G \Gamma^{-1} S \in \mathbb{R}^{2N \times 2N}$$

is the predictive closed-loop matrix defined in (8.27).

Further, combining the physical-state incremental model (8.26) with the predictive closed-loop incremental model (8.27), define the augmented incremental state as

$$\bar{\boldsymbol{\xi}}(k) = \text{col} \{ \Delta \boldsymbol{\phi}(k), \Delta \boldsymbol{\xi}^{\text{vec}}(k) \}.$$

Let

$$n_{\phi} = \sum_{i=1}^N (m_i + 1)$$

denote the total dimension of $\Delta \boldsymbol{\phi}(k)$. Then,

$$\Delta \boldsymbol{\phi}(k) \in \mathbb{R}^{n_{\phi}}, \quad \Delta \boldsymbol{\xi}^{\text{vec}}(k) \in \mathbb{R}^{2N},$$

and

$$\bar{\xi}(k) \in \mathbb{R}^{n_\phi+2N}.$$

The incremental closed-loop dynamics can be written as

$$\bar{\xi}(k+1) = \Phi \bar{\xi}(k), \quad (8.31)$$

where

$$\Phi = \begin{bmatrix} A_\phi & A_{\phi,\xi} \\ 0_{2N \times n_\phi} & A_\xi \end{bmatrix} \in \mathbb{R}^{(n_\phi+2N) \times (n_\phi+2N)},$$

and

$$A_\xi = A_{\xi,0} + B_\xi G \Gamma^{-1} S \in \mathbb{R}^{2N \times 2N}$$

is the predictive closed-loop matrix defined in (8.27). It should be emphasized that the predictive feedback matrix $G \Gamma^{-1} S$ has already been incorporated into A_ξ . Therefore, it must not be introduced again as an additional block in Φ . The reduced current-sharing error $e_s(k)$ is not treated as an independent state because it is algebraically determined by

$$e_s(k) = M_1 C_{so} \xi^{\text{vec}}(k)$$

according to (8.29), while its incremental evolution is given by (8.30).

Up to this point, the following stability conclusion can be given for the DC MG system of interest.

Theorem 8.5.1 *Consider the corrected incremental closed-loop system under the assumptions that the voltage limiter remains inactive, the model-cancellation terms are implemented accurately, and all absolute physical states remain bounded. If the corresponding incremental-state matrix Φ is Schur stable, then the state increments converge asymptotically to zero. Proportional current sharing is achieved if the limiting state additionally satisfies $M_1 C_{so} \xi_\infty^{\text{vec}} = 0$. Under the additional assumption that the local voltage loops stably track their bounded references, the PCC voltages remain within the prescribed operating range.*

Proof For discrete-time linear time-invariant systems, the system is asymptotically stable when all eigenvalues of the system matrix are strictly less than unit 1; i.e., Φ is Schur-stable. Therefore, it can be obtained that $\lim_{k \rightarrow \infty} \bar{\xi}(k) = 0$. Then, one has $\Delta \xi^{\text{vec}}(k) \rightarrow 0$, $\Delta \varphi^{\text{vec}}(k) \rightarrow 0$, thus $\Delta i_{t,i}(k) \rightarrow 0$, $\Delta v_{o,i}(k) \rightarrow 0$ for $i = 1, 2, \dots, N$. This indicates that the DC MG can maintain stable operation under the proposed DOFA secondary control. Further, since the bottom layer of the DC MG uses PI controls, the stability of the DC MG indicates that the bottom control layer can achieve stability. At the same time, considering that the integral effect in the

inner loop controls can completely eliminate the static difference of tracking. It can be concluded that the proposed control scheme can well track the reference value sent from the upper secondary control layer; i.e., VR can be achieved.

The convergence of the incremental state does not by itself determine the limiting equilibrium. If the limiting state additionally satisfies $M_1 C_{so} \xi_{\infty}^{\text{vec}} = 0$, then

$$\lim_{k \rightarrow \infty} \left\| \frac{i_{dg,j}(k)}{\theta_j} - \frac{i_{dg,i}(k)}{\theta_i} \right\| = 0, \forall i, j \in 1, 2, \dots, N \quad (8.32)$$

which demonstrates that the proposed control method can achieve ACS. On the other hand, it follows from (8.18) that the problem of ACS is equivalent to a consensus problem. The final steady state of the consensus problem is only related to the initial condition; i.e., the currents can be shared at different steady states. Accordingly, the steady-state current-sharing conclusion should be interpreted together with the limiting-equilibrium condition stated subsequently in Remark 1. When this condition is satisfied, the normalized currents converge to a common steady-state value, while the PCC voltages remain within the prescribed admissible range under the adopted local tracking assumptions. At this point, the proof of Theorem 8.5.1 is completed. ■

In addition to the above results, the corollary guiding the selection of control parameters is given below. In addition to the preceding stability result, the following corollary provides a Lyapunov-based verification condition for the reduced incremental closed-loop system.

Corollary 8.5.1 *Consider the DC MG described by (8.8) under the DOFA secondary-control strategy. Let*

$$\chi(k) = \text{col} \{ \Delta \phi(k), \Delta \xi^{\text{vec}}(k) \}$$

denote the reduced incremental state, whose closed-loop dynamics are given by

$$\chi(k+1) = \Phi \chi(k),$$

where Φ is defined in (8.31). For fixed controller parameters, prediction horizons, and cost-function weights, the incremental closed-loop system is asymptotically stable if and only if there exists a symmetric positive definite matrix

$$P = P^T > 0, \quad P \in \mathbb{R}^{(n_{\phi}+2N) \times (n_{\phi}+2N)},$$

such that

$$\begin{bmatrix} -P & \Phi^T P \\ P\Phi & -P \end{bmatrix} < 0, \quad (8.33)$$

where

$$n_\phi = \sum_{i=1}^N (m_i + 1).$$

Under this condition,

$$\Delta\phi(k) \rightarrow 0, \quad \Delta\xi^{\text{vec}}(k) \rightarrow 0$$

as $k \rightarrow \infty$.

Proof Consider the quadratic Lyapunov function

$$V(k) = \chi^T(k) P \chi(k), \quad P = P^T > 0.$$

Along the trajectory of the incremental closed-loop system, one has

$$V(k+1) - V(k) = \chi^T(k) (\Phi^T P \Phi - P) \chi(k). \quad (8.34)$$

Therefore, the origin of the incremental system is asymptotically stable if

$$\Phi^T P \Phi - P < 0. \quad (8.35)$$

Because $P > 0$, applying the Schur complement to (8.33) shows that it is equivalent to (8.35). Hence, the existence of such a matrix P is equivalent to the Schur stability of Φ , which completes the proof. ■

Remark 1 Corollary 8.5.1 establishes asymptotic stability of the reduced incremental closed-loop system. This condition guarantees that the physical and controller-state increments converge to zero, but it does not by itself determine the final equilibrium point. Proportional current sharing is achieved only if the limiting state additionally satisfies

$$M_1 C_{\text{so}} \xi_\infty^{\text{vec}} = 0,$$

where

$$\xi_\infty^{\text{vec}} = \lim_{k \rightarrow \infty} \xi^{\text{vec}}(k).$$

Likewise, bounded voltage restoration requires the limiting PCC voltages to remain within their prescribed admissible intervals and the local tracking loops to achieve zero steady-state reference-tracking error under the adopted inactive-limiter assumption. Therefore, stability of Φ and satisfaction of the steady-state regulation conditions should be verified separately.

8.6 Frequency Domain Model Development

The frequency-domain model is used as a supplementary tool for examining selected closed-loop input–output channels. An auxiliary small-signal input is retained to define the transfer matrix, although its nominal equilibrium value is zero. The state-space condition in Sect. 8.5 remains the primary basis for the closed-loop stability conclusion; diagonal Bode plots are used only to illustrate selected local dynamic characteristics.

First, the transfer function (TF) of the dynamic compensator described is given. Let $H_1 = \text{diag}\{T_s k_{\zeta 1}, \dots, T_s k_{\zeta N}\}$ and $H_2 = \text{diag}\{1 - T_s k_{\zeta 1}, \dots, 1 - T_s k_{\zeta N}\}$. Denote $i_t = [i_{t,1}, \dots, i_{t,N}]^T$ and $v_o = [v_{o,1}, \dots, v_{o,N}]^T$. According to (8.15), the TF of the input signal $i_t(z)$ and the output signal $\zeta(z)$ can be deduced as

$$\zeta(z) = (zI - H_2)^{-1} H_1 i_t(z) = H_{com}(z) i_t(z) \quad (8.36)$$

where $H_{com}(z) = (zI - H_2)^{-1} H_1$ is the TF matrix of the dynamic compensator.

Then, the closed-loop TF of the fully actuated MG is given. Let $H_{L,pc} = -G\Gamma^{-1}S$, $G = \text{diag}\{g_1, \dots, g_N\}$, $\tilde{I}_{n1} = [I_{n1,1}; I_{n1,2}]$, $I_{n1,1} = [1, 0, \dots, 0; 0, \dots, 0; \dots; 0, \dots, 0]$, $I_{n1,2} = [0, 1, \dots, 0; 0, \dots, 0; \dots; 0, \dots, 0]$. For frequency-domain analysis, an auxiliary small-signal input $R(z)$ is introduced at the cooperative-control channel. The corresponding closed-loop transfer matrix from $R(z)$ to $i_t(z)$ is denoted by $G_c(z)$. In nominal consensus operation, the equilibrium value of this input is zero, but it is retained symbolically when evaluating the transfer characteristics. According to (8.23), the system block diagram of a closed-loop fully actuated MG with predictive control in the frequency domain is given in Fig. 8.3. Based on the above system block diagram, the closed-loop TF of the MG system can be derived as

$$i_t(z) = G_c(z) R(z) \quad (8.37)$$

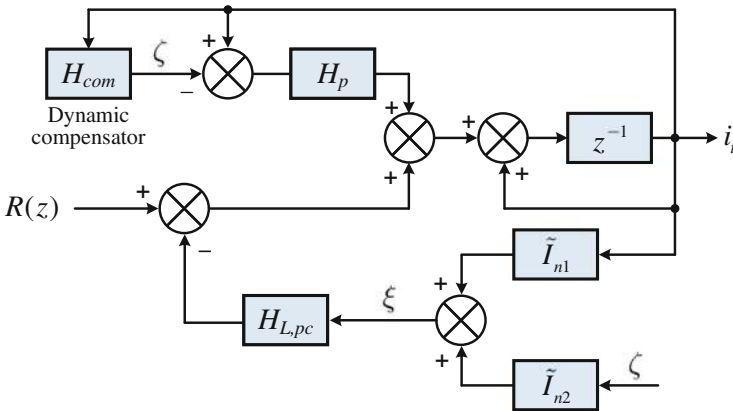


Fig. 8.3 Global model of the closed-loop MG system

where $G_c(z) = (H_{L,pc}I_{n,1} - H_p - I + zI + (H_{L,pc}I_{n,2} + H_p)(zI - H_2)^{-1}H_1)^{-1}$ is the closed-loop TF matrix.

The Corollary 8.5.1 and the TF of the MG in the frequency domain can be combined to guide the selection of the control parameters. For each candidate set of prediction horizons, cost-function weights, and controller parameters, the corresponding closed-loop matrix Φ can first be constructed. Corollary 8.5.1 is then used to verify the Schur stability of the resulting incremental closed-loop system by solving the LMI. Among the feasible parameter sets, supplementary frequency-domain and time-domain analyses can be employed to assess stability margins and dynamic performance. Then, the TF is analyzed to check whether the MG has an acceptable stability margin under these parameters. In this way, a set of control parameters is determined that gives the MG stability margins and enables it to fulfill its control task.

8.7 Experimental Verification

An actual DC MG (as shown in Fig. 6.2) with four DGs and an MPPT controller is constructed to verify the effectiveness of the proposed DOFA secondary control strategy through extensive experimental cases. The specific structure of the test DC MG is presented in Fig. 8.4. The parameters of converters and droop control are available in Yu et al. (2023d). The PI parameters of the inner loops can be determined by the pole configurations for a droop-only

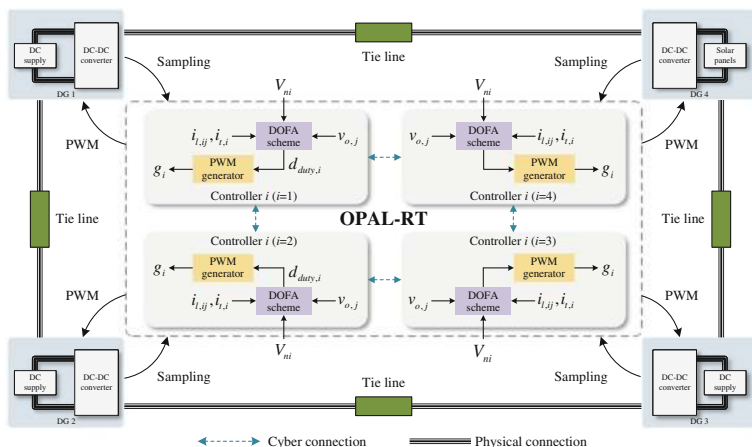


Fig. 8.4 Structure of the test MG with the proposed strategy

controlled MG system. Both the communication and electrical topologies are sequentially connected ring patterns, and its specific adjacency matrix is $\mathcal{A}_G = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$. It can be confirmed that this graph satisfies the connectivity required to achieve consensus.

In the experiment, the control period of the DC MG is set to $T_s = 5$ ms, and the switching frequency for the IGBT is 10 kHz. The desired value of the voltage is assumed to be $V_{ni} = 48$. The current sharing ratio is set to $\theta = [2 \ 2 \ 1 \ 1]$. The given allowable threshold of deviation is set to $\varepsilon = 5\%$. Setting the time step to 5 ms is a trade-off between the control precision and the computational burden. For this setting, a smaller time period will not provide better control performance, while a larger one may lead to poor results. Specifically, the following three case studies are carried out on the developed testbed: (1) performance comparison, (2) fundamental performance evaluation, and (3) plug-and-play feature test. It should be noted that the basic procedures of the experiments are as follows: In the initial phase, the four DGs operate only under the droop control; the proposed DOFA control strategy is activated after the system is stabilized, and then the system enters the secondary regulation phase.

8.7.1 Performance Comparison and Analysis

For ring-bus multi-bus DC MGs, the distributed consensus voltage observer (CVO)-based secondary control proposed in Nasirian et al. (2014, 2015) is an effective practice to address VR and ACS, which takes the form of $\delta_{v,i}(t) = k_i^{P,v} e_i^{\bar{v}}(t) + k_i^{I,v} \int e_i^{\bar{v}}(\tau) d\tau$, $\delta_{c,i}(t) = k_i^{P,c} \sum_{j \in \mathcal{N}_i^c} b a_{ij} e_{ji}^s(t) + k_i^{I,c} \int \sum_{j \in \mathcal{N}_i^c} b a_{ij} e_{ji}^s(\tau) d\tau$, $\dot{\bar{v}}_{o,i}(t) = \dot{v}_{o,i}(t) + \sum_{j \in N_i} a_{ij} (\bar{v}_{o,j}(t) - \bar{v}_{o,i}(t))$, where $e_{ji}^s(t) = e_j^s(t) - e_i^s(t)$, $e_i^{\bar{v}}(t) = V_{ni}^{ref} - \bar{v}_{o,i}(t)$, $k_i^{P,v}$, $k_i^{I,v}$, $k_i^{P,c}$, $k_i^{I,c}$, b , a_{ij} are the control parameters. Based on the specifications of the experimental platform used and the same communication topology as previously described, the control parameters of the CVO-based method can be set by trial-and-error method as $k_i^{P,v} = 0.1$, $k_i^{I,v} = 1.2$, $k_i^{P,c} = 0.3$, $k_i^{I,c} = 3.2$, and $b = 0.5$.

The experimental results recorded in Fig. 8.5 show that the CVO-based secondary control strategy can restore the voltage to a certain range and achieve ACS. Under the controller settings and experimental conditions considered here, the DOFA method exhibits faster and smoother transient responses than the implemented CVO-based baseline. Regarding this point, it will be analyzed in detail later.

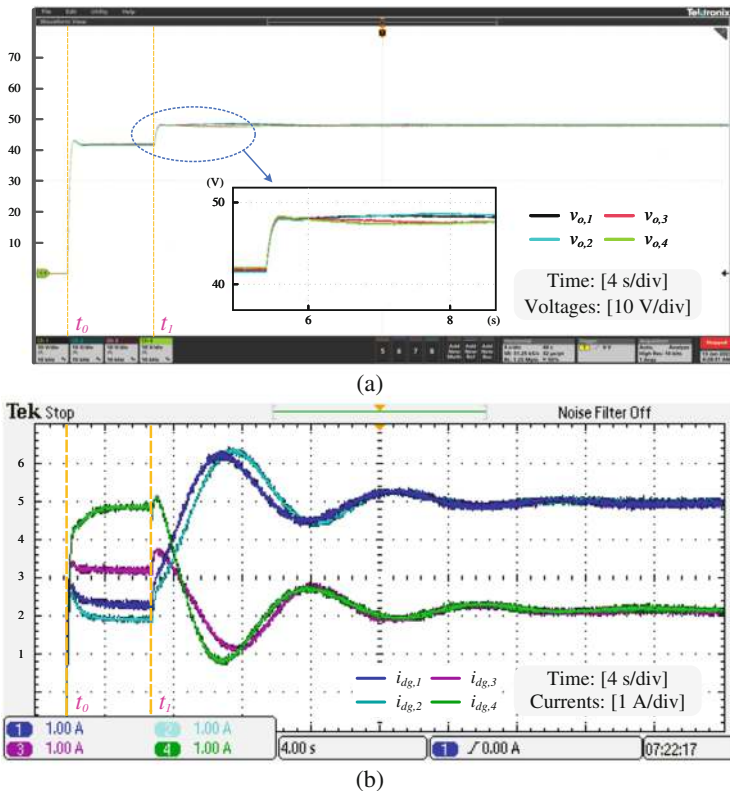


Fig. 8.5 (a) Voltage and (b) current responses of the MG test system under the CVO-based control scheme

8.7.2 Fundamental Performance Evaluation

The basic performance of the proposed DOFA secondary control method in DC MG is exhibited in this part. Based on the trade-off between control performance and computational burden, the length of the prediction window of the control input is chosen as $N_u = 10$, and the upper and lower bounds of the output prediction window are taken as $\bar{N}_y = 16$ and $\underline{N}_y = 8$, respectively. Based on the prescribed prediction horizons, candidate cost-function weights and controller parameters are selected through an outer-loop parameter search. For each candidate set, the corresponding matrix Φ is constructed and the LMI condition in Corollary 8.5.1 is used to verify incremental closed-loop stability. The parameter values adopted in the experiment are reported below. The weight factor corresponding to the control cost is set to $b_i = 90$. Note that a larger b_i may provide a greater margin of stability but could result in very slow adjustments. Besides, set $k_{\zeta i} = 0.5$ and $k_{c,i} = 1$ for $i = 1, 2, 3, 4$.

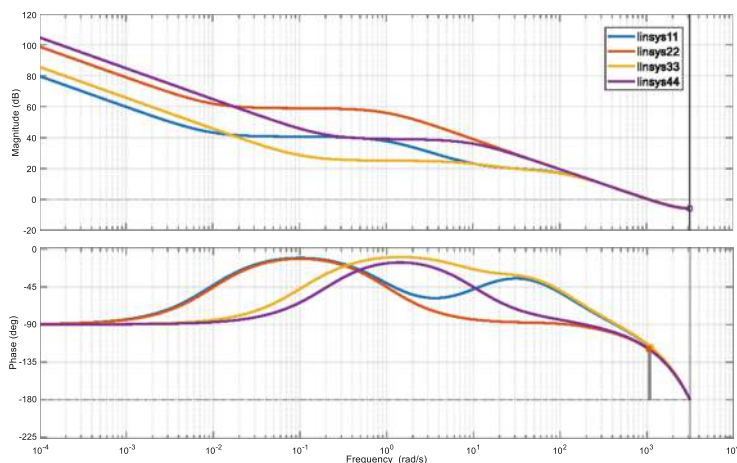


Fig. 8.6 The Bode diagram of the closed-loop TF on the diagonal of the TF matrix

The diagonal Bode plots given in Fig. 8.6 provide supplementary information on selected local input–output channels. Closed-loop stability is established using the state-space condition in Sect. 8.5, while multivariable interaction may be further assessed through the poles of the full transfer matrix, singular-value plots, or an appropriate multivariable stability criterion. The output trajectories of the DC MG under the proposed control method are presented in Fig. 8.7. The output responses after activating the secondary control at time t_1 shown in Fig. 8.7 demonstrate that the DOFA approach achieves a rapid recovery of the voltage to within the specified range and ACS. As a matter of interest, the proposed DOFA method achieves smooth dynamic and static performance owing to the consideration of the future state of the DC MG system. In particular, the electrical characteristics of the DC MG are paid full attention under the DOFA control method, which makes the theoretical design match naturally with the actual situation. In addition to the above waveform results, the RMS of the output current sharing errors and the control signals are computed when the secondary control is activated. Over the specified evaluation interval, the RMS normalized current-sharing error is 0.2324, while the steady-state secondary voltage references of DGs 1–4 are 48.31, 48.66, 47.44, and 47.25 V, respectively.

Compared with the experimental waveforms under the above two cases, it can be concluded that the proposed DOFA method achieves better regulation effects. Specifically, the current sharing and voltage recovery under the proposed method are both fast and stable, while the CVO-based method could hardly achieve both fastness and smoothness. In addition, the overshoot of the current is much smaller, and the secondary output regulation is much faster in Sect. 8.7.2 than that in Sect. 8.7.1. The reason why the proposed DOFA strategy can greatly improve the regulation speed over the existing method depends on the following three aspects.

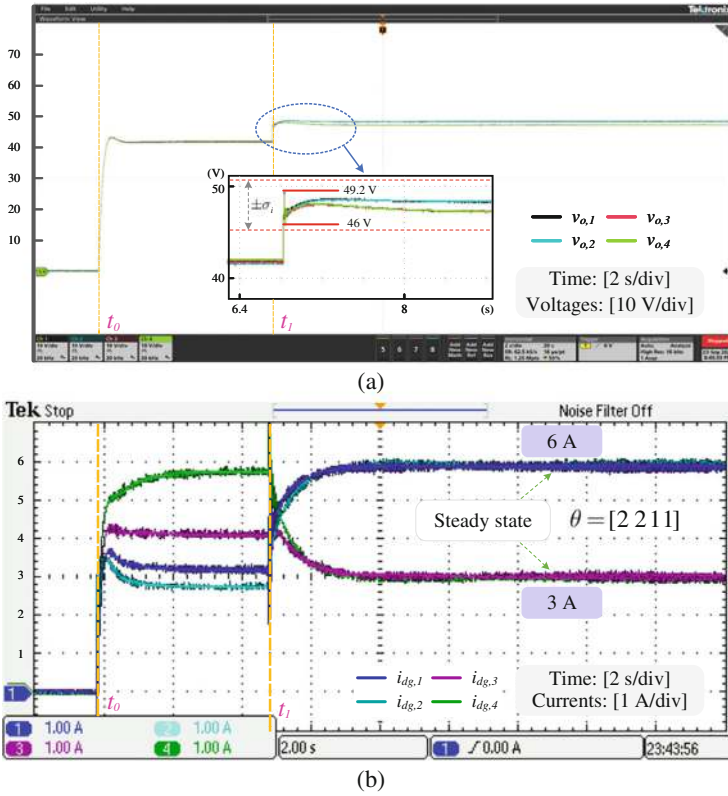


Fig. 8.7 (a) Voltage and (b) current responses of the test MG system under the DOFA predictive control scheme

First, benefiting from the access to the MG dynamics, the designed virtual control (8.11) eliminates the coupled dynamics of the system, making the FA MG system easier to be regulated and quicker to respond. Second, the designed dynamic compensator (8.15) also contributes to the control speed. Finally, the optimization of the cost function (8.18) further facilitates the regulation performance. If this chapter tries to make the system converge faster by adjusting the control parameters in Sect. 8.7.1, the adjustment process will gradually exhibit overshoot and some oscillations. Neglecting the tie-line inductance may degrade model accuracy and closed-loop performance when the line dynamics are not negligible relative to the controller bandwidth, which also illustrates the necessity of capturing MG dynamics for the design of the model-based controller.

8.7.3 Plug-and-Play Feature Test

Following the previous basic performance evaluation, a key feature of the DC MG under the proposed control is examined here, namely, plug-and-play functionality. To demonstrate the complete process of primary and secondary control, the DG₄ powered by the PV panel batteries is disconnected from the MG at t_2 , while at t_4 , it is reconnected to the MG and shares the load with other DGs. It should be noted that in the experiment, “plug” and “unplug” operation is achieved by the air switch. DG₄ is chosen to evaluate the plug-and-play capability of the proposed method with the aim of reflecting the scenarios where the generation nodes supplied by RESs are integrated into the DC MG.

Figure 8.8a, b give the experimental results in this case and also the evolution of the communication topology during the plug-and-play process. The voltage response curve shows that the voltage output of the MG fluctuates at the time t_2 due to the cutout of DG₄. The voltage of the off-grid unit drops to zero at the PCC. Likewise, the output current suffers a shock due to the disconnection of DG₄, and

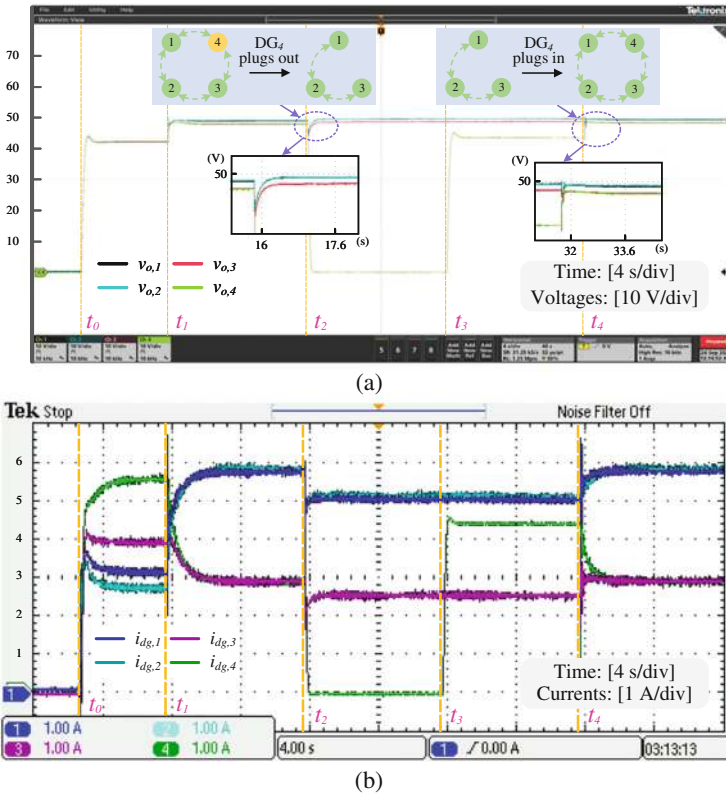


Fig. 8.8 (a) Voltage and (b) current responses of the test MG system in case of plug and play

the remaining three interconnected DGs have to bear the loads on the bus according to a new distribution factor. The reconnection of DG₄ at time t_4 leads to a brief fluctuation of the output voltage and current of the MG. Accordingly, the loads on the bus are then shared among the four DGs again, and the output voltage and current of the MG quickly return to the state before the fault occurred.

According to the above analysis and discussion for the experimental results, the plug-and-play characteristic of the DC MG under DOFA control strategy is guaranteed, which verifies the applicability of the proposed method in RESs-based systems.

8.8 Notes and References

This chapter resolves the inherent trade-off between ACS and CVR in DC MGs with resistive-inductive interconnections. By formulating a FA output current model that balances structural simplicity with faithful representation of the DC MG's physical dynamics, the proposed framework captures essential system behavior while retaining analytical tractability. Building on this model, a DOFA predictive secondary control scheme is introduced, integrating a dynamic compensation mechanism and real-time state prediction to enhance both transient and steady-state performance. Experimental validation on a physical DC MG platform confirms the scheme's efficacy in harmonizing voltage stability and current distribution under varied operational conditions. While the current methodology is tailored for networks with resistive loads, extending its applicability to scenarios involving constant power loads remains a compelling avenue for future research, which is a nontrivial extension due to their nonlinear dynamics. Further investigations will also focus on designing distributed secondary controllers capable of mitigating communication delays, thereby bolstering the resilience and scalability of DC MGs in real-world deployments.

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Part III
Fully Actuated Control of Cyber-Physical
Renewable Power Systems

Chapter 9

Distributed Control of Renewable Power Systems via a Fully Actuated Error Model



Abstract Voltage regulation is crucial for ensuring the stability and power quality of microgrids, while current sharing reflects the grid's ability to coordinate energy and is essential for extending the lifespan of generator units. However, due to the presence of line impedances, current can no longer be freely adjusted when the voltages at various buses are not uniform. Moreover, existing models describing DC microgrids have struggled to balance accuracy and simplicity, making it difficult to develop model-based secondary control strategies that are easy to implement widely. Addressing these challenging issues, this chapter establishes a comprehensive error model for DC microgrids incorporating dynamic control loops at the circuit and device levels using FAS theory. Leveraging this model, a model-based distributed secondary optimization control is proposed. In contrast to existing research, the error model presented in this chapter can capture the electrical characteristics of microgrids while maintaining a simple structure. For the tasks of voltage restoration and precise current allocation, this chapter unifies them into a comprehensive error regulation system, offering a novel alternative to resolving their conflict. Subsequently, the stability of the closed-loop system of the DC microgrid is analyzed. Additionally, consensus analysis is conducted on current allocation, and tracking analysis is performed on average voltage. Finally, by constructing a miniature grid experimental platform incorporating a photovoltaic array and energy storage unit, the engineering applicability of the proposed method under varying conditions is validated in terms of both steady-state accuracy and dynamic response.

9.1 Overview

It is worth noting that a common issue in current control studies for multi-bus DC microgrids is the lack of a methodological approach to modeling. While model-free strategies offer significant flexibility, the controller design paradigm based on the joint modeling of physical-control provides distinct theoretical advantages in

terms of dynamic response accuracy and robustness against parameter variations Benosman (2018), Formentin et al. (2013, 2014). This implies that establishing suitable microgrid system models and corresponding model-driven control architectures has become a key pathway to enhancing the distributed secondary control theoretical system. In the evolution of DC microgrid modeling techniques, existing typical methods can be classified into three major technical approaches: the first is based on large-signal analytical models of power electronic devices at the switch level, the second relies on proportional equivalent models based on the droop control assumption, and the third involves simplified first-order dynamic models focusing on the time-domain characteristics of output voltage/current (Dai et al., 2025; Gui et al., 2021; Tucci et al., 2017). Pioneering work represented by Gui et al. (2021) effectively revealed the transient response circuit characteristics and stability boundaries of microgrids by establishing a large-signal model incorporating DC-DC converter switch functions. However, such models often idealize the dynamic compensation mechanism within the control loop during construction, leading to a systematic neglect of the interaction mechanisms among key control variables.

Other researchers, aiming to simplify the complexity of control system design, equivalized DG units as proportional gain modules under the assumption of “fast-changing characteristics of the bottom-level controller dynamic with respect to the main control timing” (Tucci et al., 2017). While this significantly simplifies the controller parameter tuning process, it fails to effectively represent fundamental dynamic features like inductor charging and discharging processes and capacitor charge transfer in such models. Additionally, while using first-order modeling methods partially reflects the power flow characteristics among distributed generation units, they exhibit significant shortcomings in representing key electromagnetic interaction mechanisms such as voltage-current cross-coupling effects (Dai et al., 2025). In conclusion, existing research consistently presents a trade-off between model refinement and engineering practicality: Large-signal models possess strict mathematical completeness but suffer from dimensionality issues, proportional models simplify calculations but lose physical interpretability, and first-order models offer operational convenience but sacrifice the dynamic realism of the system. This core contradiction significantly restricts the optimization space of secondary control strategies in DC microgrids, urging academia and industry to develop a holistic modeling methodology that is compatible with strong adaptability and cross-level parameter correlation characteristics (Ali & Mohammed, 2025; Sen & Kumar, 2018).

Inspired by the discussions above, we focus on developing a control-oriented multi-bus DC microgrid model to address the key issues of voltage restoration and precise current sharing. Initially, we construct a large-signal model that comprehensively describes the dynamic behavior of the DC microgrid, capturing the system's response characteristics under various operating conditions. Subsequently, through a strategic transformation, the intricate model is converted into a FAS framework. This approach significantly simplifies the system structure, enhancing both analysis and controller design. The specific contributions are given as follows. Building upon the comprehensive consideration of the circuit dynamics and hierarchical control

features of multi-bus DC microgrids, this paper establishes a coupled multi-agent system model. Notably, this model bears a structural resemblance to traditional large-signal models. However, drawing inspiration from the concept of the HOFA system method proposed in literature (Duan, 2021a), this chapter further integrates this coupled comprehensive microgrid model with regulatory error dynamics and transforms it into a task-specific mode. Through this transformation, we have successfully derived a fully actuated multi-agent error system model that addresses both the key objectives of voltage restoration and precise current allocation. This model not only preserves the system's dynamic characteristics but also enhances its task-oriented nature in practical applications. This chapter represents an early attempt at introducing distributed secondary control based on a physical model. Specifically, it integrates the voltage restoration and precise current sharing issues into a single controller, unlike the approach in literature (Yu et al., 2024a), which requires separate designs for voltage and current. A benefit of consolidating the secondary controllers into a single framework is the ease of providing parameter selection schemes. Based on local voltage observers, the voltage restoration issue is reformulated as a weighted average voltage tracking problem. Through this transformation, the proposed framework effectively synthesizes the error dynamics of both voltage restoration and current sharing into a comprehensive error formulation. A rigorous stability analysis of this error system is conducted in this chapter, demonstrating the simultaneous achievement of these two objectives.

The chapter is organized as follows. Section 9.2 presents the system description and the concerned problem. Section 9.3 presents the dynamic characteristics of multi-bus DC MGs. Section 9.4 provides distributed optimal secondary control for DC MGs. Section 9.5 presents the experimental results. Finally, Sect. 9.6 concludes the chapter.

9.2 Problem Formulation

In this section, the architecture of a multi-bus DC MG based on the converter interface, its dynamics description, and the regulation tasks of interest in this chapter are presented.

9.2.1 Structure of Multi-bus DC MGs

A DC MG composed of distributed RESs interconnected by power lines in a multi-bus topology is a typical MG system. Its overall structure is shown in Fig. 6.1. In this chapter, we consider a converter interface-based DC MG consisting of N ring-bus multi-bus DG units as described above. As depicted in Fig. 8.1, fundamental

components such as power supply from photovoltaic panels and energy storages, DC-DC converters, resistor-inductor-capacitor (RLC) filters, and local loads form the hardware system of the MG. The local load is connected to the PCC of each DG unit. The DC-DC converter-based DG units are interconnected by resistive-inductive distribution lines. In particular, Fig. 6.1 shows the details of the connection of the i -th DG unit and the k -th DG unit through the resistive-inductive distribution line $\#ik$.

The MG illustrated in Fig. 6.1 is considered as a cyber-physical system with a communication network that facilitates the exchange of data between DG units for control purposes. The physical and cyber system of the MG can also be clearly identified in Fig. 6.1. At the information layer, each DG unit broadcasts information such as its measured voltage and current to one or several specific DG units. Each node in the information layer inherits the dynamic model of the corresponding DG unit. In contrast to centralized control, there is no need for a single controller to communicate with all DG units on such a network, which is an important feature of distributed control. In particular, these nodes and the communication connections between them constitute an undirected graph. This undirected graph is characterized by a triple $\mathcal{G}_c = (\mathcal{V}_c, \mathcal{E}_c, \mathcal{A}_c)$, where $\mathcal{V}_c = \{v_1, \dots, v_N\}$ stands for the set of DGs, $\mathcal{E}_c \subseteq \mathcal{V}_c \times \mathcal{V}_c$ is the edge set of communication links, and N is the number of DGs in the DC MG. The element a_{ij} in the adjacency matrix $\mathcal{A}_c \in \mathbb{R}^{N \times N}$ is the communication weight, where $a_{ij} = 1$ means that node j communicates with node i , otherwise $a_{ij} = 0$. Specifically, if node j transmits data x_j to node i , then the data received by node i is $a_{ij}x_j$. In this chapter, the communication weights are assumed to be time-invariant. $\mathcal{N}_i^c$ represents the set of all neighbors of node i . The Laplacian matrix is defined as $L = D - \mathcal{A}_c$, where $D = \text{diag}\{d_1, \dots, d_i, \dots, d_N\} \in \mathbb{R}^{N \times N}$ and $d_i = \sum_{j \in \mathcal{N}_i^c} a_{ij}$.

On the other hand, the physical system of the DC MG in Fig. 6.1 can also be mapped to a network represented by an undirected and connected graph $\mathcal{G}_e = (\mathcal{V}_e, \mathcal{E}_e, \mathcal{A}_e)$, where $\mathcal{V}_e$ and $\mathcal{E}_e$ are, respectively, the sets of vertices and edges, $\mathcal{A}_e$ is the conductance matrix of the DC MG. Each element in the vertex set $\mathcal{V}_e$ represents a DG unit, and each element in the edge set $\mathcal{E}_e$ denotes a distribution line connecting different DG units. $\mathcal{N}_i^e$ stands for the set of all neighbors linked to the i -th DG through the transmission line.

9.2.2 Dynamics of Converter Interface-Based Ring-bus Multi-bus DC MGs

In the previous part, the system architecture of the studied DC MG was given, and the mathematical formulation of its physical and communication topologies was carried out. This subsection is devoted to the development of an analytical model of the islanded DC MG shown by Fig. 6.1. The intermittent nature of RESs, such as wind and solar, is a major factor affecting the reliability of distributed supplies,

causing great volatility to MG systems. To obtain a mathematical model of the DC MG, it is assumed here that the energy storage scheme can address the volatility of RESs well. Using circuit theory, the dynamics of the converter interface-based DG unit i is described as follows:

$$\dot{V}_i(t) = \frac{I_{ti}(t)}{C_{ti}} + \frac{1}{C_{ti}} \sum_{k \in \mathcal{N}_i^e} I_{ik}(t) - \frac{V_i(t)}{C_{ti} R_{Li}} \quad (9.1a)$$

$$\dot{I}_{ti}(t) = -\frac{V_i(t)}{L_{ti}} - \frac{R_{ti}}{L_{ti}} I_{ti}(t) + \frac{V_{E,i} d_{r,i}(t)}{L_{ti}} \quad (9.1b)$$

$$\dot{I}_{ik}(t) = -\frac{R_{ik}}{L_{ik}} I_{ik}(t) + \frac{V_k(t) - V_i(t)}{L_{ik}} \quad (9.1c)$$

for $i \in \{1, \dots, N\}$ and $k \in \mathcal{N}_i^e = \{1, \dots, N_{i,m}\}$, where I_{ik} is the current flowing from the k -th DG to the i -th DG via transmission line $line_{ik}$, $N_{i,m}$ is the number of transmission lines connected to DG unit i , V_i , and I_{ti} are the capacitor voltage and the inductor current of the i -th DG, respectively. $d_{r,i}$ stands for the duty cycle used to generate the PWM signal for driving the insulated gate bipolar transistor (IGBT), and $V_{E,i}$ indicates the voltage of the power supply for the i -th DG. R_{ti} , L_{ti} , C_{ti} , and R_{Li} denote the filter resistance, filter inductance, shunt capacitance, and local load resistance, respectively.

Similar to the common practice (Han et al., 2019a), the distribution line is assumed to have quasi-smooth dynamics, i.e., $dI_{ik}(t)/dt = 0$, $k \in \mathcal{N}_i^e$. This assumption is reasonable, because the line impedance in a low-voltage DC distribution system is mainly resistive and the inductance L_{ik} is negligible. Therefore, the dynamics of the transmission line in (9.1c) can be simplified as

$$I_{ik}(t) = \frac{1}{R_{ik}} (V_k(t) - V_i(t)), \quad k \in \mathcal{N}_i^e. \quad (9.2)$$

Substituting (9.2) into (9.1a), one obtains dynamics of converters in DC MGs similar to that of Yu et al. (2024a) as follows:

$$C_{ti} \dot{V}_i(t) = I_{ti} + \sum_{k \in \mathcal{N}_i^e} \frac{V_k(t) - V_i(t)}{R_{ik}} - \frac{V_i(t)}{R_{Li}} \quad (9.3a)$$

$$L_{ti} \dot{I}_{ti}(t) = -V_i(t) - R_{ti} I_{ti}(t) + V_{E,i} d_{r,i}(t). \quad (9.3b)$$

Up to this point, the electrical dynamics of the converter-based ring-bus multi-bus DC MG is obtained.

9.2.3 Current Allocation and Voltage Balancing Requirements in DC MGs

The regulation objectives of the aforementioned MG will be given here. In the secondary control of a DC MG, one of the main control objectives is to share the total load demand between DG units at steady state in proportion to the rated current of the DG units, i.e.,

$$\lim_{t \rightarrow \infty} \left\| \frac{I_{ij}(t)}{\theta_j} - \frac{I_{ti}(t)}{\theta_i} \right\| = 0, \forall i, j \in \mathcal{V}_e. \quad (9.4)$$

where θ_i is the expected current sharing ratio related to the rated current of the i -th DG.

Ideally, the second control objective of the DC MG is to regulate the voltage at the PCC to a given reference value V^{ref} at the steady state, i.e.,

$$\lim_{t \rightarrow \infty} \|V_i(t) - V^{ref}\| = 0, \forall i \in \mathcal{V}_e. \quad (9.5)$$

However, in a multi-bus MG, the current sharing requirement in (9.4) does not allow for the all consistent PCC voltages. However, in multi-bus MGs, achieving precise current sharing makes it impossible for the voltages at the PCCs to be completely identical, resulting in a conflict between (9.4) and (9.5). This conflict is a primary challenge in the control of multi-bus MGs. If this conflict is not considered, the designed controller may deviate from expectations or even cause the system to diverge. Understanding this conflict is practically significant for controller design, as it can guide us in finding a balance between the objectives of voltage recovery and current sharing. According to the conclusions presented in Trip et al. (2019), the average voltage of a DC MG retains certain degrees of freedom under exact current sharing. Furthermore, when the average value of all voltages at the PCC is restored to the reference value V^{ref} , the individual voltages can be maintained around V^{ref} without significant deviation. Therefore, it is possible to simultaneously restore the voltages to an acceptable range while achieving accurate current sharing. Based on these observations, the weighted average restoration can be considered a viable alternative to (9.5). Specifically, the voltage regulation target is redefined as the weighted average value of the voltage signals at the PCCs being restored to V^{ref} at steady state, i.e.,

$$\lim_{t \rightarrow \infty} \left\| \frac{1}{N} \sum_{i=1}^N V_i(t) - V^{ref} \right\| = 0. \quad (9.6)$$

It should be noted that the reference voltage V^{ref} in (9.6) is generated by the upper control layer, referred to as the tertiary control. The tertiary control sends the determined reference value V^{ref} to the secondary control level. For the secondary

controller in a DC MG, it is assumed that the value of V^{ref} is available only for part of the DG units.

In the following sections, we will discuss how to design the FA controller to achieve the regulation goals described in (9.4) and (9.6) for the MG system characterized by (9.3).

9.3 Dynamics Model of Multi-bus DC MGs

In this section, the multi-bus MG system is comprehensively modeled, which will serve as the foundational model for the subsequent design of the distributed optimal FA controller. In particular, its large-signal model is given, and a first-order FA multi-agent system model is derived.

9.3.1 Dynamic Characteristics of Multi-bus DC MGs

The dynamics (9.3) does not comprehensively describe the full electrical characteristics of a ring-bus multi-bus DC MG considering the commonly used hierarchical control. Achieving a comprehensive characterization requires a complete modeling of its underlying circuit model and dynamics of control action under secondary layer in the hierarchical control framework.

What closely associated with the bottom control loop is called the primary control, which generally adopts the droop control law and acts as the interface between the bottom and upper control layers. That is, the voltage reference of the double closed loop is provided by the droop action (Nasirian et al., 2015; Yu et al., 2024a), i.e.,

$$v_{oi}^*(t) = V_{ni}^*(t) - R_{di} I_{dg,i}(t) \quad (9.7)$$

where $V_{ni}^*(t)$ denotes the rated voltage (i.e., $V_{ni}^*(t) = V^{ref}$) when the MG operates in local control mode and corresponds to the voltage reference provided by the secondary control layer if secondary control is activated, R_{di} is the droop coefficient, and $I_{dg,i}$ is the output current, as shown in Fig. 8.1. Although (9.7) is popularly used, the current in the droop control of this paper is different from that in Nasirian et al. (2015); Yu et al. (2024a). From Fig. 6.1, one has

$$v_{oi}^*(t) = V_{ni}^* - R_{di} \eta_i V_i(t) + R_{di} \sum_{k \in \mathcal{N}_i^e} \frac{V_k(t)}{R_{ik}} \quad (9.8)$$

where $\eta_i = 1/R_{Li} + \sum_{k \in \mathcal{N}_i^e} 1/R_{ik}$. Double closed-loop control at the device level is indispensable for DC MGs. The used bottom voltage and current controllers in this

paper are similar to Yu et al. (2024a), where K_{pvi} and K_{ivi} are the PI coefficients to be determined for the voltage controller and K_{pci} and K_{ici} for the current controller. Dynamics of the integrators $\dot{\phi}_i(t)$ and $\dot{\gamma}_i(t)$ in the double-closed loop can also be referred to Nasirian et al. (2015). Notice that the current $I_{dg,i}(t)$ after the filter is taken as the feedback signal for the inner loop. Further, the duty cycle of the i -th DG unit can be obtained as

$$\begin{aligned} d_{r,i}(t) = & (-K_{pci}K_{pvi}R_{di}\eta_i - K_{pci}K_{pvi} - K_{pci}\eta_i)V_i(t) \\ & + K_{pci}K_{ivi}\phi_i(t) + K_{ici}\gamma_i(t) + K_{pci}K_{pvi}V_{ni}^*(t) \\ & + (K_{pci}K_{pvi}R_{di} + K_{pci}) \sum_{k \in \mathcal{N}_i^e} R_{ik}^{-1} V_k(t). \end{aligned} \quad (9.9)$$

To adjust the average voltage of all DGs to the rated reference value, the following voltage estimator is designed,

$$\hat{V}_i(t) = V_i(t) + k_i \int \sum_{j \in \mathcal{N}_i^c} a_{ij} (\hat{V}_j(\tau) - \hat{V}_i(\tau)) d\tau \quad (9.10)$$

where $\hat{V}_i(t)$ is the estimated voltage, k_i is the coupling gain and can be used to adjust the convergence speed of the estimator. In this way, we can define a new graph $\tilde{\mathcal{G}}_c$ with adjacency matrix $\tilde{\mathcal{A}}_c = [\tilde{a}_{ij}] \in \mathbb{R}^{N \times N}$, where $\tilde{a}_{ij} = k_i a_{ij}$, $\forall i, j \in N$. Similarly, one has $\tilde{L} = \tilde{D} - \tilde{\mathcal{A}}_c$, where $\tilde{D} = \text{diag}\{\tilde{d}_1, \dots, \tilde{d}_N\} \in \mathbb{R}^{N \times N}$ and $\tilde{d}_i = \sum_{j \in \mathcal{N}_i^c} \tilde{a}_{ij}$. The communication graph is assumed to be connected and undirected, and the coupling gains are selected identically as $k_i = k_v > 0$ for all i . Moreover, the observer is initialized such that

$$\sum_{i=1}^N \hat{V}_i(0) = \sum_{i=1}^N V_i(0).$$

Under these conditions, the estimates asymptotically track the arithmetic average of the PCC voltages.

Assume that the secondary control is activated. Taking an integrated state variable as $\tilde{x}_i(t) = [V_i(t) \ I_{di}(t) \ \phi_i(t) \ \gamma_i(t) \ \hat{V}_i(t)]^T \in \mathbb{R}^{5 \times 1}$ and the rated voltage $V_{ni}^*(t)$ as the control input $u_i(t)$, combining (9.3), (9.8), (9.9), and (9.10), the multi-bus DC MG consisting of N DGs in Fig. 6.1 can be described by the following comprehensive model

$$\dot{\tilde{x}}_i(t) = A_{ii}\tilde{x}_i(t) + \sum_{k \in \mathcal{N}_i^e} A_{ik}\tilde{x}_k(t) + B_{ii}u_i(t) \quad (9.11)$$

where A_{ii} and A_{ik} can be obtained from (9.3)–(9.10) and are presented in (9.12), shown at the bottom of the next page, $\check{\eta} = V_{E,i}(K_{pci}K_{pvi}R_{di}\eta_i + K_{pci}K_{pvi} + K_{pci}\eta_i)$, $\check{K} = K_{pci}K_{pvi}R_{di} + K_{pci}$, $B_{ii} = [0 \ V_{E,i}K_{pci}K_{pvi}/L_{fi} \ 1 \ K_{pvi} \ 0]^T$.

Notice that although the form of (9.11) is similar to the one in Yu et al. (2024a), the state variables they contain are different, where the state vector in Yu et al. (2024a) does not involve the estimated voltage. For the above comprehensive large-signal MG model, the output current and estimated voltage are taken as the system output, i.e., $y_i(t) = C_i \tilde{x}_i(t)$, where $y_i(t) = [I_{ti}(t)/\theta_i \ \hat{V}_i(t)]^T \in \mathbb{R}^{2 \times 1}$, $C_i = [C_{i,1}; \ C_{i,2}]^T \in \mathbb{R}^{2 \times 5}$, $C_{i,1} = [0 \ 1/\theta_i \ 0 \ 0 \ 0]$, $C_{i,2} = [0 \ 0 \ 0 \ 0 \ 1]$.

In (9.2), it is assumed that the inductance of the lines can be neglected. Without this assumption, we can follow the modeling approach outlined above by taking the state variable as $\tilde{x}_i(t) = [V_i(t) \ I_{ti}(t) \ I_{i1}(t) \ \cdots \ I_{iN_{i,m}}(t) \ \phi_i(t) \ \gamma_i(t) \ \hat{V}_i(t)]^T \in \mathbb{R}^{(N_{i,m}+5) \times 1}$. Combining equations (9.1), (9.9), and (9.10), an interconnected system model of the form (9.11) can be also derived.

Up to this point, the integrated large-signal model of a DC MG with a hierarchical control structure is given, which also contains the average voltage observer. It can be seen that it is an interconnected multi-agent system.

9.3.2 First-Order Fully Actuated Model of Multi-bus DC MGs

Based on the aforementioned integrated large-signal model, this subsection will focus on deriving the first-order fully actuated model of the multi-bus DC MG, taking into account the regulation objectives. Considering the current sharing objective presented in (9.4), define the consensus error of the weighted output current as follows:

$$A_{ii} = \begin{bmatrix} -(\sum_{j \in \mathcal{N}_i^e} \frac{1}{C_{ii} R_{ij}} + \frac{1}{C_{ii} R_{Li}}) & \frac{1}{C_{ii}} & 0 & 0 & 0 \\ -\frac{1+\tilde{\eta}}{L_{ti}} & -\frac{R_{ti}}{L_{ti}} & \frac{V_{E,i} K_{pci} K_{ivi}}{L_{ti}} & \frac{V_{E,i} K_{ici}}{L_{ti}} & 0 \\ -(R_{di} \eta_i + 1) & 0 & 0 & 0 & 0 \\ -(K_{pvi} R_{di} \eta_i + K_{pvi} + \eta_i) & 0 & K_{ivi} & 0 & 0 \\ -(\sum_{j \in \mathcal{N}_i^e} \frac{1}{C_{ii} R_{ij}} + \frac{1}{C_{ii} R_{Li}}) & \frac{1}{C_{ii}} & 0 & 0 & -k_i d_i \end{bmatrix},$$

$$A_{ik} = \begin{bmatrix} \frac{1}{C_{ti} R_{ik}} & 0 & 0 & 0 & 0 \\ \frac{V_{E,i} \tilde{K}}{L_{ti} R_{ik}} & 0 & 0 & 0 & 0 \\ \frac{R_{di}}{R_{ik}} & 0 & 0 & 0 & 0 \\ \frac{K_{pvi} R_{di} + 1}{R_{ik}} & 0 & 0 & 0 & 0 \\ \frac{1}{C_{ti} R_{ik}} & 0 & 0 & 0 & k_i a_{ik} \end{bmatrix} \quad (9.12)$$

$$\begin{aligned} \varepsilon_{i,1}(t) &= \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(\frac{I_{ti}(t)}{\theta_i} - \frac{I_{tj}(t)}{\theta_j} \right) \\ &= - \sum_{j \in \mathcal{N}_i^c} a_{ij} C_{j,1} \tilde{x}_j(t) + d_i C_{i,1} \tilde{x}_i(t) \end{aligned} \quad (9.13)$$

$\forall i \in \mathcal{V}_e$, where d_i and $C_{i,1}$ are defined in Sects. 9.2.2 and 9.3.1, respectively. $C_{j,1}$ has the same meaning as $C_{i,1}$, $\forall i, j \in \mathcal{V}_e$. To further give the simplified and controller design-oriented system model of the studied DC MG, the regulation error of the estimated voltage is defined as

$$\begin{aligned}\varepsilon_{i,2}(t) &= g_i(\hat{V}_i(t) - V^{ref}) \\ &= g_i(C_{i,2}\tilde{x}_i(t) - V^{ref})\end{aligned}\quad (9.14)$$

where g_i is the pinning gain. When the i -th DG unit can receive the voltage reference value sent by the tertiary control, $g_i = 1$, otherwise $g_i = 0$.

To achieve these two control objectives simultaneously, for the i -th DG unit, let

$$\varepsilon_i(t) = \varepsilon_{i,1}(t) + \varepsilon_{i,2}(t), \forall i \in \mathcal{V}_e. \quad (9.15)$$

Notice that, in general, $\varepsilon_i(t) \rightarrow 0$ is not equivalent to $\varepsilon_{i,1}(t) \rightarrow 0$ and $\varepsilon_{i,2}(t) \rightarrow 0$. To achieve both goals, current sharing and average voltage regulation, there are also certain requirements for the topology of the information layer in the MG, which will be elaborated in the subsequent controller design.

To obtain the dynamics of the overall regulation error describing the DC MG, deriving (9.15) yields

$$\begin{aligned}\dot{\varepsilon}_i(t) &= - \sum_{j \in \mathcal{N}_i^c} a_{ij} C_{j,1} \dot{x}_j(t) + (d_i C_{i,1} + g_i C_{i,2})(A_{ii} \tilde{x}_i(t) \\ &\quad + \sum_{j \in \mathcal{N}_i^c} A_{ij} \tilde{x}_j(t)) + b_{e,i} u_i(t)\end{aligned}\quad (9.16)$$

where $b_{e,i} = (d_i C_{i,1} + g_i C_{i,2}) B_{ii} = d_i C_{i,1} B_{ii} = \frac{d_i V_{E,i} K_{pc,i} K_{pv,i}}{\theta_i L_{t,i}}$. Since $b_{e,i} = d_i C_{i,1} B_{ii} \neq 0$, inspired by the HOFA system method (Yu et al., 2024a; Zhang et al., 2023) for the inductor current, a FA secondary controller with the following structure can be designed for the previously defined integrated regulation error dynamics

$$u_i(t) = b_{e,i}^{-1} (u_{e,i}^{aux}(t) + u_{e,i}(t)) \quad (9.17)$$

where $u_{e,i}^{aux}(t)$ is the auxiliary control component and $u_{e,i}(t)$ is the main control component of $u_i(t)$. The auxiliary control $u_{e,i}^{aux}(t)$ is designed to eliminate the coupling dynamics of the MG system, resulting in a homogeneous multi-agent model. The main control $u_{e,i}(t)$ aims to achieve the desired control objectives. The local physical states and PI integrator states are assumed to be available from sensors and the digital controller. In addition, each DG is assumed to receive either the neighboring state derivatives $C_{j,1} \dot{\tilde{x}}_j$ or the measurements, model parameters, and secondary inputs required to reconstruct them. In a similar way to the assumption in Yu et al. (2024a) that the states in the system (9.11) are measurable, the auxiliary

controller for the error model (9.16) can be designed as

$$\begin{aligned}
 u_{e,i}^{aux}(t) = & \sum_{j \in \mathcal{N}_i^c} a_{ij} C_j^1 \dot{x}_j(t) - (d_i C_{i,1} \\
 & + g_i C_{i,2})(A_{ii} \tilde{x}_i(t) + \sum_{j \in \mathcal{N}_i^c} A_{ij} \tilde{x}_j(t)).
 \end{aligned} \tag{9.18}$$

Furthermore, substituting (9.17) and (9.18) into the error dynamics (9.16) yields the following first-order and linear fully actuated multi-agent system

$$\dot{\tilde{e}}_i(t) = u_{e,i}(t), \quad \forall i \in \mathcal{V}_e. \tag{9.19}$$

At this stage, by integrating the large-signal model of the DC MG given by (9.11) with the desired regulation objectives described by (9.4) and (9.6), we derive the fully actuated error model of the MG system. It can be seen that this model is a decoupled multi-agent system.

Remark 9.3.1 Although this paper obtains a similar description of the dynamics of DC MGs as that in Yu et al. (2024a), and a first-order model with a similar structure, they have different practical implications. Only the inductor current has been described in the first-order model derived in Yu et al. (2024a), which poses difficulties for the implementation of voltage recovery. Specifically, the use of the limiter introduces a nonlinear link to the controller, which is not conducive to the design of control parameters. To solve this problem, this paper incorporates the dynamics of the estimated voltage (9.10) into the system description and constructs a combined regulation error (9.15) that contains both voltage regulation error (9.13) and current allocation error (9.14). Such an approach facilitates the simultaneous accomplishment of two control objectives. Therefore, this paper proposes a feasible solution to the shortcomings of the first-order current model in Yu et al. (2024a), which makes the present work more meaningful.

Remark 9.3.2 In general, it is not adequate to consider the voltage and current dynamics of the MG by completely leaving aside the physical constraints. To address this issue, a composite large-signal model for a multi-bus DC MG is developed in this chapter. Moreover, a simple form of multi-agent system model capable of characterizing the dynamics of the actual MG system is obtained. In the commonly used control framework based on the droop interface, the intrinsic physical characteristics of the MG are taken into account in the stability analysis of the closed-loop system. In this approach, the determination of distributed secondary controller parameters is often embedded within the stability analysis, and they mostly rely on global circuit and topology information. However, this global information, such as the conductance matrix of the MG, is generally difficult to obtain for each distributed converter-based DG unit. In summary, on the one hand, controllers that do not explicitly depend on the system model are straightforward and easy to implement, which is expected. However, the determination of the

parameters in these simplified controllers often ultimately comes back to the specific system dynamics of the MG, or even the global dynamics model. In this way, the simplicity of the structure is accompanied by the difficulty of determining the actual parameters. On the other hand, a DC MG is a physically coupled large-scale cyber-physical system. If the design of the secondary control is based directly on its complete large-signal model, it will be a challenge to ensure the distributed characteristics of the controller, etc., although the electrical dynamics of the MG can be accurately portrayed. Through the dedicated design of auxiliary control, the developed first-order multi-agent system model (9.19) alleviates the above problems. It takes full advantage of the actual large-signal model of the MG and maintains an accurate representation of the original power system while having a satisfactorily simple structure.

9.4 Distributed Optimal Secondary Control for DC MGs

Based on the fully actuated error model (9.19) obtained in the previous section, a consensus-based distributed optimal secondary control strategy is proposed in this section to solve the voltage balancing and current sharing problems in DC MGs. The developed secondary controller shares information at the communication layer only with the adjacent DG units. To illustrate the practicality of the first-order MG model and the generality of the designed secondary controller, the stability, consensus, and optimality analysis of the closed-loop DC MG system under the proposed controller are also given.

9.4.1 Structure of the Optimal Controller for Multi-bus DC MGs

This subsection will first present the proposed structure of the distributed optimal secondary controller for multi-bus DC MGs. Further, the closed loop error system for DC MGs under the proposed controller is also given. Specifically, for the formulated control objectives, i.e., the accurate current sharing (9.4) and average voltage regulation (9.6), it is expected that the defined regulation error converges to 0, i.e., $\|\varepsilon_i(t)\| \rightarrow 0$ as $t \rightarrow \infty, \forall i \in \mathcal{V}_c$. Consider the cooperative tracking of the above first-order fully actuated multi-agent system, and define the following local neighborhood tracking error

$$\zeta_i(t) = \sum_{j \in \mathcal{N}_i^c} a_{ij}(\varepsilon_i(t) - \varepsilon_j(t)) + h_i(\varepsilon_i(t) - \varepsilon_0) \quad (9.20)$$

where ε_0 is the tracking target of each DG unit and is specified as 0 according to the control objective. Since regulating the error ε_i to zero is the unstated consensus for

all DG units, the pinning gain h_i is not zero for any node. Then, to ensure that the first-order fully actuated multi-agent system (9.19) achieves cooperative tracking, the distributed control is designed as

$$u_{e,i}(t) = -\lambda K_l \zeta_i(t) \quad (9.21)$$

where $\lambda > 0$ is a suitable coupling gain and K_l is the local feedback gain. Notice that the global form of the secondary control is

$$u(t) = -\lambda(L + H) \otimes K_l \varepsilon(t) \quad (9.22)$$

where $u = [u_{e,1}, \dots, u_{e,N}]^T \in \mathbb{R}^{N \times 1}$, $H = \text{diag}\{h_1, \dots, h_N\} \in \mathbb{R}^{N \times N}$, and $\varepsilon = [\varepsilon_1, \dots, \varepsilon_N]^T \in \mathbb{R}^{N \times 1}$. Similarly, the global form of system (9.19) can be expressed as

$$\dot{\varepsilon}(t) = u(t). \quad (9.23)$$

Substituting the control input (9.22) into the above system, the closed-loop system in global form is obtained as

$$\dot{\varepsilon}(t) = -\lambda(L + H) \otimes K_l \varepsilon(t). \quad (9.24)$$

It should be noted that the asymptotic stabilization of this closed-loop system is equivalent to the convergence of the regulation error ε to zero.

9.4.2 Parameters Specification for Optimal Cooperative Secondary Control

This subsection will present the stability analysis of the closed-loop control system of DC MGs with the aforementioned distributed secondary optimal controller. It will also include an analysis of the optimality of the proposed controller. Furthermore, guidelines are given for the selection of the controller parameters to achieve the dual regulation objectives of voltage restoration and current sharing.

The following lemma discusses the conditions that need to be satisfied when the distributed controller (9.21) makes the DC MG optimal with respect to the cost function.

Lemma 9.4.1 (Anderson & Moore, 2007) *For the linear system $\dot{x} = Ax(t) + Bu(t)$, let $u(t)$ be a stabilizing control about the vector subspace S . If there exists a scalar function $V(x(t))$, $\gamma > 0$, and a positive semi-definite matrix Q satisfying the following conditions*

$$V(x(t)) = x^T(t)Px(t) \geq 0 \quad (9.25a)$$

$$x^T(t)Qx(t) \geq 0 \quad (9.25b)$$

$$Q + PA + A^TP - PBR^{-1}B^TP = 0 \quad (9.25c)$$

$$\psi(x(t)) = Fx(t) = -R^{-1}B^TPx(t) \quad (9.25d)$$

then $u(t) = \psi(x(t)) = Fx(t)$ is optimal regarding the following performance criterion

$$J(x(0), u(t)) = \int_0^\infty \zeta(x(\tau), u(\tau))d\tau \quad (9.26)$$

where $\zeta(x(\tau), u(\tau)) = x^T(\tau)Qx(\tau) + u^T(\tau)Ru(\tau)$. Further, the optimal value of this performance index can be determined as $J(x(0), u(t)) = V(x(0))$.

For the sake of giving the significance of voltage balance, the Lemma 4.3.1 is introduced. From Lemma 4.3.1, if $\varepsilon_{i,2}(t) \rightarrow 0$, we have $\hat{V}_i(t) \rightarrow V_i^{ref}$ and then $\frac{1}{N} \sum_{i=1}^N V_i(t) \rightarrow V_i^{ref}$. Thus, the regulation error of the estimated voltage portrays the voltage balance demand given in (9.6).

The following theorem shows the specific parameter design of the distributed controller $u_{e,i}(t)$ for DC MGs, illustrating the stability of the closed-loop system and when it is globally optimal.

Theorem 9.4.1 *For the desired current distribution factor and voltage reference value, the error dynamic of the DC MG is given by (9.19). Assume that the graph $\tilde{\mathcal{G}}_c$ has a spanning tree and $\tilde{L}$ is balanced. Under the graph $\tilde{\mathcal{G}}_c$, suppose there exist positive definite matrices $P_a = P_a^T > 0$ and $P_l = P_l^T > 0$ satisfying*

$$P_a = \lambda R_a(L + H) \quad (9.27a)$$

$$Q_l - P_l R_l^{-1} P_l = 0 \quad (9.27b)$$

for $Q_l = Q_l^T > 0$, $R_a = R_a^T > 0$, $R_l = R_l^T > 0$, $R_a = r_a I_N$, $r_a > 0$, and the positive coupling gain $\lambda > 0$. The feedback gain of the secondary control (9.21) is specified as

$$K_l = R_l^{-1} P_l. \quad (9.28)$$

Then, the proposed controller can make the error system asymptotically stable, accomplish both the described objectives of voltage balancing and current sharing, and is inverse-optimal with respect to the following performance metric

$$J(\varepsilon(0), u(t)) = \int_0^\infty (\varepsilon^T(\tau)Q\varepsilon(\tau) + u^T(\tau)Ru(\tau))d\tau \quad (9.29)$$

where $Q = \lambda^2((L + H) \otimes K_l)^T(R_a \otimes R_l)((L + H) \otimes K_l)$, $R = R_a \otimes R_l$.

Proof First, the stability and consistency analysis of the closed-loop MG system under the proposed controller is given. Let $P = P_a \otimes P_l$. By the positive definiteness of the matrix P , the candidate Lyapunov function $V(\varepsilon(t)) = \varepsilon^T(t)P\varepsilon(t) = \varepsilon^T(t)(P_a \otimes P_l)\varepsilon(t)$ is positive. Furthermore, substituting P_a and K_l into Q , we have

$$\begin{aligned} Q &= \lambda^2((L + H) \otimes K_l)^T (R_a \otimes R_l)((L + H) \otimes K_l) \\ &= P_a R_a^{-1} P_a \otimes P_l R_l^{-1} P_l. \end{aligned} \quad (9.30)$$

Since P_a , R_a , P_l , and R_l are all positive definite matrices, Q is also positive definite. Then, for the above Lyapunov function, the derivation along the system model of the MG yields

$$\begin{aligned} \dot{V}(\varepsilon(t)) &= 2\varepsilon^T(t)P\dot{\varepsilon}(t) \\ &= 2\varepsilon^T(t)P(-\lambda(L + H) \otimes K_l)\varepsilon(t) \\ &= -2\varepsilon^T(t)Q\varepsilon(t) \leq 0. \end{aligned} \quad (9.31)$$

When $\dot{V}(\varepsilon(t)) = 0$, there is $\varepsilon(t) = 0$. From the LaSalle invariance principle (La Salle, 1976), it is known that the error dynamics $\varepsilon(t)$ can be stable and converge to the origin; i.e., the first-order fully actuated multi-agent system (9.19) can reach consensus and $\varepsilon(t)$ converges to zero.

Building on the closed-loop stability analysis presented above, it is demonstrated below that the DC MG can achieve both average voltage regulation and accurate current sharing when the state of the first-order fully actuated multi-agent system converges to the origin. To be specific, when the system (9.19) is stable, there is

$$\lim_{t \rightarrow \infty} \|\varepsilon_i(t)\| = \lim_{t \rightarrow \infty} \|\varepsilon_{i,1}(t) + \varepsilon_{i,2}(t)\| = 0. \quad (9.32)$$

Since the communication graph is undirected, its Laplacian matrix satisfies $\mathbf{1}_N^T L = 0$. Recalling the definition of the current-sharing error in (9.13), one therefore has

$$\sum_{i=1}^N \varepsilon_{i,1}(t) = \sum_{i=1}^N \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(\frac{I_{t,i}(t)}{\theta_i} - \frac{I_{t,j}(t)}{\theta_j} \right) = 0. \quad (9.33)$$

Because the closed-loop composite-error system is asymptotically stable,

$$\lim_{t \rightarrow \infty} \varepsilon_i(t) = 0, \quad i = 1, \dots, N.$$

Summing the composite errors $\epsilon_i(t) = \epsilon_{i,1}(t) + \epsilon_{i,2}(t)$ over all DG units and using (9.33) yields

$$\lim_{t \rightarrow \infty} \sum_{i=1}^N g_i \left(\hat{V}_i(t) - V^{\text{ref}} \right) = 0. \quad (9.34)$$

Under the connectivity, balance, and initialization conditions of the distributed average-voltage estimator, all voltage estimates converge to the same value, which is equal to the arithmetic average of the PCC voltages:

$$\lim_{t \rightarrow \infty} \hat{V}_i(t) = \lim_{t \rightarrow \infty} \hat{V}_j(t) = \lim_{t \rightarrow \infty} \frac{1}{N} \sum_{r=1}^N V_r(t) \triangleq \bar{V}_\infty, \quad \forall i, j. \quad (9.35)$$

Substituting (9.35) into (9.34) gives

$$\left(\sum_{i=1}^N g_i \right) \left(\bar{V}_\infty - V^{\text{ref}} \right) = 0. \quad (9.36)$$

Since at least one DG unit has access to the reference voltage, $\sum_{i=1}^N g_i > 0$. It follows that

$$\bar{V}_\infty = V^{\text{ref}}, \quad (9.37)$$

or equivalently,

$$\lim_{t \rightarrow \infty} \left[\frac{1}{N} \sum_{i=1}^N V_i(t) - V^{\text{ref}} \right] = 0. \quad (9.38)$$

Therefore, the average-voltage-restoration objective is achieved.

Moreover, from $\hat{V}_i(t) \rightarrow V^{\text{ref}}$, one obtains

$$\lim_{t \rightarrow \infty} \epsilon_{i,2}(t) = \lim_{t \rightarrow \infty} g_i \left(\hat{V}_i(t) - V^{\text{ref}} \right) = 0, \quad i = 1, \dots, N.$$

Since $\epsilon_i(t) = \epsilon_{i,1}(t) + \epsilon_{i,2}(t)$ and $\epsilon_i(t) \rightarrow 0$, it follows that

$$\lim_{t \rightarrow \infty} \epsilon_{i,1}(t) = \lim_{t \rightarrow \infty} [\epsilon_i(t) - \epsilon_{i,2}(t)] = 0, \quad i = 1, \dots, N. \quad (9.39)$$

Let

$$q_i(t) = \frac{I_{t,i}(t)}{\theta_i}, \quad q(t) = \text{col}\{q_1(t), \dots, q_N(t)\}.$$

Equation (9.39) is equivalent to

$$Lq(t) \rightarrow 0.$$

Because the communication graph is connected and undirected, $\ker(L) = \text{span}\{\mathbf{1}_N\}$. Therefore,

$$\lim_{t \rightarrow \infty} \left| \frac{I_{t,i}(t)}{\theta_i} - \frac{I_{t,j}(t)}{\theta_j} \right| = 0, \quad \forall i, j \in \mathcal{V}_c, \quad (9.40)$$

which proves that proportional current sharing is achieved. In summary, once the closed-loop MG system is stabilized, the designed distributed cooperative controller successfully achieves the average voltage regulation of the DC MG and ensures accurate current sharing proportionally among each DG unit.

Based on Lemma 9.4.1, the proof of the optimality of the proposed distributed secondary controller with respect to the cost function (9.29) is presented below. The key here is to show that the distributed control gains given by (9.27) and (9.28) satisfy the optimality conditions of the global LQR. By the definition of the Lyapunov function and (9.30), it can be shown that (9.25a) and (9.25b) in Lemma 9.4.1 hold. Let

$$R = R_a \otimes R_l. \quad (9.41)$$

For the DC MG system, it is seen that

$$\begin{aligned} Q - PR^{-1}P &= Q - (P_a \otimes P_l)(R_a^{-1} \otimes R_l^{-1})(P_a \otimes P_l) \\ &= Q - P_a R_a^{-1} P_a \otimes P_l R_l^{-1} P_l. \end{aligned} \quad (9.42)$$

Substituting Q provided in (9.29) into the above equation reveals that it equals zero. This means that the designed controller satisfies the algebraic Riccati equation corresponding to the global form of the MG system, i.e., (9.25c) in Lemma 9.4.1 holds. On the other hand, substituting the expression for K_l , the controller can be written as

$$\begin{aligned} u(t) &= -\lambda(L + H) \otimes K_l \varepsilon(t) \\ &= -R^{-1}P \varepsilon(t). \end{aligned} \quad (9.43)$$

Therefore, the gain matrix $K = R^{-1}P$ for the global controller $u(t) = -K \varepsilon(t)$ conforms to the form of the gain of the linear quadratic regulator (LQR). It indicates that condition (9.25d) in Lemma 9.4.1 is satisfied. To sum up, following Lemma 9.4.1, the distributed control law given by (9.21) is globally optimal for the performance index function (9.29). This completes the proof. ■

The structure of the proposed optimal secondary control, based on the developed first-order fully actuated MG model, is illustrated in Fig. 9.1. It can be observed from

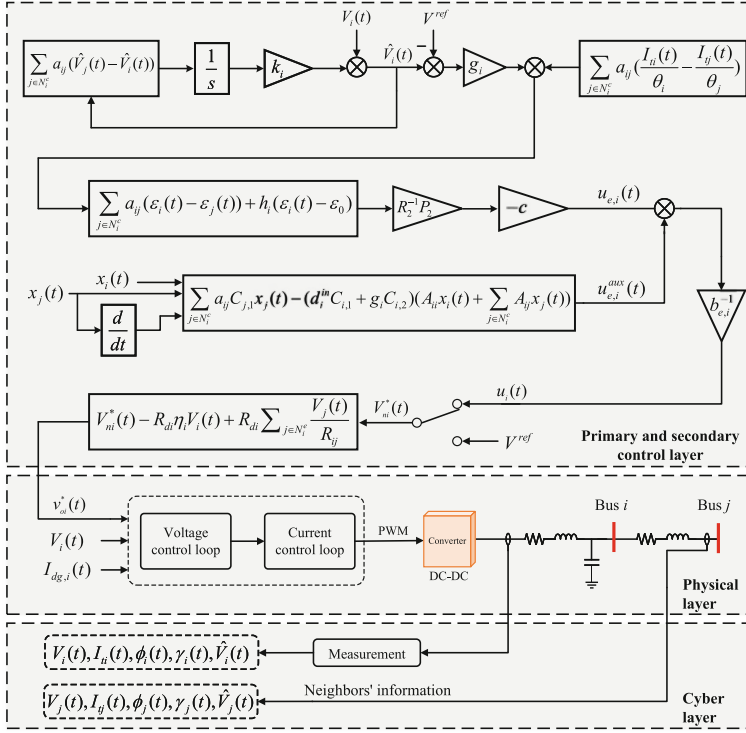


Fig. 9.1 Framework of the proposed secondary control scheme for voltage restoration and current sharing

the figure that $V_{ni}^*(t) = u_i(t)$ when the secondary control is activated; otherwise, $V_{ni}^*(t) = V^{ref}$. This demonstrates that the fully actuated error model introduced in this chapter is oriented toward the design of secondary controllers.

Remark 9.4.1 It should be noted that the DC MG model developed in Sect. 9.2 serves as a fundamental mathematical description of the MG under consideration. This model provides a basis for designing various model-based secondary controllers, not limited to the distributed cooperative optimal secondary controller proposed in this chapter. The fully actuated multi-agent system model (9.19) shares the same simple first-order structure as the existing models in Dai et al. (2024), Zhang and Hredzak (2019), so the design of the secondary controller under this model is significantly facilitated in a similar way. However, with the help of auxiliary control (9.18), the developed error model accommodates the electrical attributes of the DC MG, which is a distinctive advantage not available in many existing methods. Furthermore, it can be seen from the stability analysis that with the first-order fully actuated multi-agent system model (9.19) and an undirected communication topology, the designed secondary control can achieve both voltage and current regulation in the DC MG, provided that $\varepsilon_i(t) \rightarrow 0$ when $t \rightarrow 0$, $\forall i \in \mathcal{V}_c$.

is ensured. Consequently, based on the proposed FAS model, voltage recovery and current allocation are skillfully integrated into a single stabilization problem of an error state. This means that the control problem of the DC MG is converted into a consensus tracking problem of a first-order multi-agent system. As a result, there is no need to separately consider the voltage regulation error and the current sharing error in the subsequent controller design (Yu et al., 2024a).

Remark 9.4.2 Theorem 9.4.1 appears to be not particularly difficult. However, on one hand, its simplicity highlights the advantages of the fully actuated error model in ensuring the usability of the designed controller and determining control parameters. On the other hand, Theorem 1 provides several key proofs that can lay the foundation for the future application of the fully actuated error model. For instance, Theorem 9.4.1 establishes the optimality of the distributed secondary controller concerning a global cost function. Specifically, it demonstrates that the distributed secondary controller designed within this error model framework can achieve both voltage restoration and current sharing.

9.5 Experimental Results

To verify the correctness of the DC MG model developed in this chapter and the effectiveness of the proposed distributed optimal cooperative secondary scheme, an experimental setup of a scaled-down low-voltage DC MG is implemented in this section. In the setup, four DC-DC power converters are connected in a ring topology consisting of four buses in series.

9.5.1 *Implementation of MG Prototype and Experimental Setup*

The constructed laboratory-scale islanded MG testbed is shown in Fig. 6.2. It consists of power supplies, interconnected power converters, filters, local loads, tie lines linking converters, and an actual controller. In order to realize a power generation system based on real RESs as much as possible in the laboratory, DG unit #4 is powered by a solar array with a battery and a maximum power tracking controller. Due to the limited experimental field, the other three DG units are fed by programmable DC sources. The hardware controller in the testbed is the OPAL-RT (OP5700). Specifically, the OP5700 executes the proposed control algorithm in the architecture shown in Fig. 8.4. It is worth mentioning that the scaled-down MG prototype in this experiment is actual hardware and not the hardware-in-the-loop testbed built around a target machine. That is, the PWM signal eventually generated by the FPGA board built into the OP5700 will directly drive the semiconductor device in the power converter.

Table 9.1 Settings of scaled-down DC MG and control

DER parameters		DER ₁ & DER ₂				DER ₃ & DER ₄			
Droop	R_{vi}	0.8	1			1.7	1.5		
Voltage	k_{pvi}		0.1				0.2		
loop	k_{ivi}		1				1		
Current	k_{pci}		0.001				0.001		
loop	k_{ici}		2.2				2.2		
LC filter	L_{fi}		≈ 2				≈ 2		
[mH],[μ F]	C_{fi}		1500 $\pm$ 20%				1500 $\pm$ 20%		
Line	Line 1		Line 2			Line 3		Line 4	
impedance	R_{12}	0.5	R_{23}	0.5		R_{34}	0.5	R_{41}	0.5
[Ω], [μ H]	L_{12}	318	L_{23}	318		L_{34}	325	L_{41}	325
Loads	Local	loads						Switching load	
[Ω]	R_L	10	10	10	10			$R_{L, cut}$	1

The main task of this paper is to achieve voltages at PCCs close to the rated value V^{ref} and accurate current sharing among the ring-bus multi-bus converters. The nominal voltage setpoint determined by the tertiary control layer is assumed to be $V^{ref} = 48$ V, which is also a typical value in a low-voltage DC distribution network. The input voltages of the four generation units are 90, 100, 100, and 80 V, respectively. The proportional current sharing factor is fixed to $\theta = [1 \ 2 \ 2 \ 1]$. The communication graph in distributed control is a connected ring structure identical to the physical connection graph of the MG; i.e., it is based on the following adjacency matrix $\mathcal{A}_c = [0, 1, 0, 1; 1, 0, 1, 0; 0, 1, 0, 1; 1, 0, 1, 0]$. This communication topology will be used throughout the experimental test section if not otherwise specified. The sampling period on the sampling board for the converter is chosen to be 0.1 ms, i.e., a communication bandwidth of 10 kHz. Other specifications not given for generation units, lines and loads can be identified in Table 9.1.

To fully demonstrate the dynamic response and feasibility of the proposed secondary control strategy, three operation scenarios are considered: basic performance testing, performance testing under load variation, and MG plug-and-play capability evaluation. The first case is to show the performance improvement of the proposed control method to the MG system by comparing it with the state-of-the-art control scheme. The second case evaluates the response of the proposed method under local load variations, and the third one is to demonstrate that the proposed control method does not compromise the PnP characteristics of the DC MG.

To facilitate the subsequent analysis and discussion of the experimental results, the whole experimental procedure is divided into four stages as follows:

Stage 1: activating the local droop control and enabling the grid integration of all DGs at t_0 , which is the start-up phase of the DC MG operated in the islanded mode.

Stage 2: executing the proposed cooperative secondary control for DC MGs at t_1 to implement the voltage restoration and current sharing.

Stage 3: adding a constant load $R_{L,\text{add}}$ to the local load $R_L(1)$ at t_2 and removing it from the bus at t_3 .

Stage 4: disconnecting DG unit #4 from the MG system at t_4 and reconnecting it to the MG prototype at t_6 .

For each case, the traces of the output voltage and inductor current are captured by two oscilloscopes.

9.5.2 Comparison with a Recent Study

Next, we discuss specifically the results of voltage balancing and current sharing under the existing secondary control method and that under the proposed one and present a comparison of the two results.

Case 1: Voltage balancing and current sharing under the existing first-order secondary method

For the DC MG with multiple buses in a ring topology, Sadabadi et al. proposed a distributed controller with a three-layer structure in Sadabadi et al. (2023). To be specific, the goal of proportional current sharing is achieved by the upper layer of the proposed secondary controller as follows:

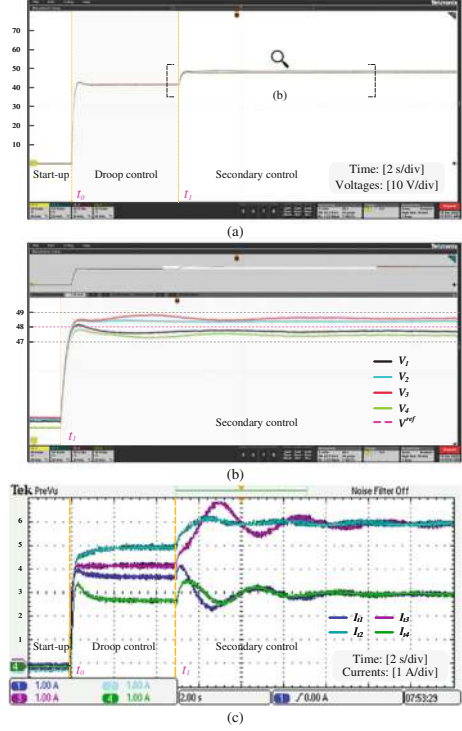
$$\tau_v \dot{v}(t) = -K \mathbb{B}^T W^{-1} I_t(t) \quad (9.44)$$

where $v(t) = [V_1(t) \ V_2(t) \ \cdots \ V_N(t)]^T \in \mathbb{R}^{N \times 1}$ and $I_t(t) = [I_{t1}(t) \ I_{t2}(t) \ \cdots \ I_{tN}(t)]^T \in \mathbb{R}^{N \times 1}$, K and τ_v are control parameters. The details of the matrices in (9.44) and specific expressions of the intermediate layer can be found in Sadabadi et al. (2023). Notice that the control input u_i in our study is the reference signal given to the primary level by the secondary layer, while the control input in Sadabadi et al. (2023) is $d_{r,i} = V_{E,i}/u_i$, where $V_{E,i}$ is the input voltage. The communication topology adopts the ring topology mentioned earlier. Compared with Fig. 9.1, the controller proposed in Sadabadi et al. (2023) exhibits a higher order and a more complex structure. However, the method proposed in Sadabadi et al. (2023) is not model-based and therefore does not require the underlying dual closed-loop control as well as droop control, etc. To be fair for comparison, we still employ the droop control during the startup phase and activate the secondary control mentioned in Sadabadi et al. (2023) after the system stabilizes, which corresponds to Stage 1 in Sect. 9.3.1. Figure 9.2 shows the output voltages, inductor currents, and a zoomed-in view of the voltage recovery phase under the state-of-the-art method. From the experimental results, it can be seen that the above secondary control strategy can achieve voltage balancing and accurate current sharing in a proportional manner.

Case 2: Voltage balancing and current sharing under the existing second-order secondary method

To provide a more comprehensive comparison of the proposed method with existing approaches, a second-order secondary controller is tested on the experimental platform. Specifically, the following second-order secondary controller was

Fig. 9.2 Evolution of (a) measured voltages at PCCs, (b) enlarged view of voltage responses, and (c) output currents under the secondary control presented in Sadabadi et al. (2023)



proposed in Trip et al. (2019) to achieve voltage restoration and current sharing of multi-bus DC MGs:

$$T_{\eta i} \dot{\eta}_i(t) = - \sum_{j \in N_i^c} a_{ij} (w_i I_{ti}(t) - w_j I_{tj}(t)) \quad (9.45a)$$

$$T_{\phi i} \dot{\phi}_i(t) = -\phi_i(t) + I_{ti}(t) \quad (9.45b)$$

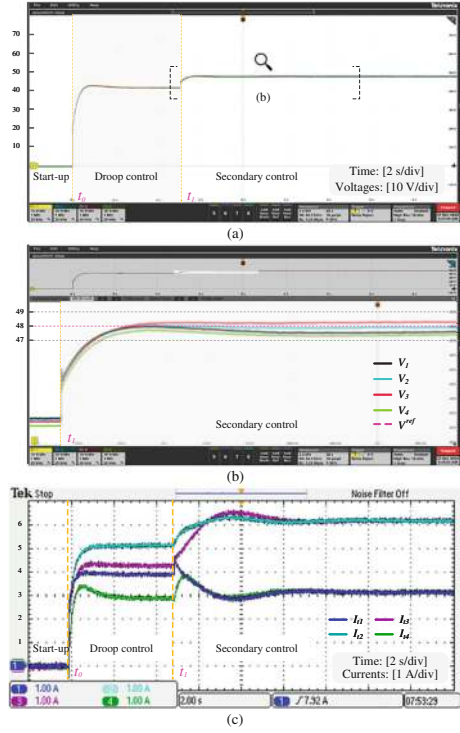
$$u_i(t) = -K_i (I_{ti}(t) - \phi_i(t)) + w_i \sum_{j \in N_i^c} a_{ij} (\eta_i(t) - \eta_j(t)) + V^{ref} \quad (9.45c)$$

for $i = \{1, \dots, N\}$, where $w_i = 1/\theta_i$, parameters $T_{\eta i}$, $T_{\phi i}$, and K_i are used to tune the transient response. This controller is a model-free distributed secondary controller and includes a dynamic compensator. Compared to (9.44), (9.45) features a slightly more complex control structure. The outputs of the DC MG under this controller are shown in Fig. 9.3. The results indicate that the second-order secondary controller with a dynamic compensator accomplishes the desired control tasks.

Case 3: Voltage balancing and current sharing under the proposed secondary method

According to the control objective, setting $Q_l = 1$ and $R_l = 0.1$, the feedback gain matrix can be obtained by the algebraic Riccati equation as $P_l = 0.3162$

Fig. 9.3 Evolution of (a) measured voltages at PCCs, (b) enlarged view of voltage responses, and (c) output currents under the secondary control presented in Trip et al. (2019)



and $K_I = 3.126$. From the stability condition of the closed-loop system, the gain coefficient in the secondary controller (9.21) is taken to be $\lambda = 1$. Up to this point, the coefficients in the global performance indicator function (9.29) are $Q_a = [6, -4, 2, -4; *, 6, -4, 2; *, *, 6, -4; *, *, *, 6] \in \mathbb{R}^{4 \times 4}$ and $R_a = \text{diag}\{0.1, 0.1, 0.1, 0.1\} \in \mathbb{R}^{4 \times 4}$, respectively.

Figure 9.4 presents the output voltages, inductor currents, and a zoomed-in view for the voltage recovery phase of the DC MG in this case. The experimental data for the observed average output voltages are plotted in Fig. 9.5. As the experimental results suggest, the proposed secondary control ensures that the voltage at each PCC can be stabilized quickly while restoring their average values to the given value V^{ref} and achieving accurate proportional current sharing.

Comparing the experimental results of the three cases, the following observations and conclusions can be made.

1. All three approaches effectively address the tasks of voltage balancing and current sharing in multi-bus DC MGs, although the system responses differ.
2. Comparing the results of Case 1 and Case 2 with that in Case 3, the MG with the proposed secondary control strategy exhibits a slightly smoother voltage and current responses.

Fig. 9.4 Evolution of (a) measured voltages at PCCs, (b) enlarged view of voltage responses, and (c) output currents under the proposed control scheme

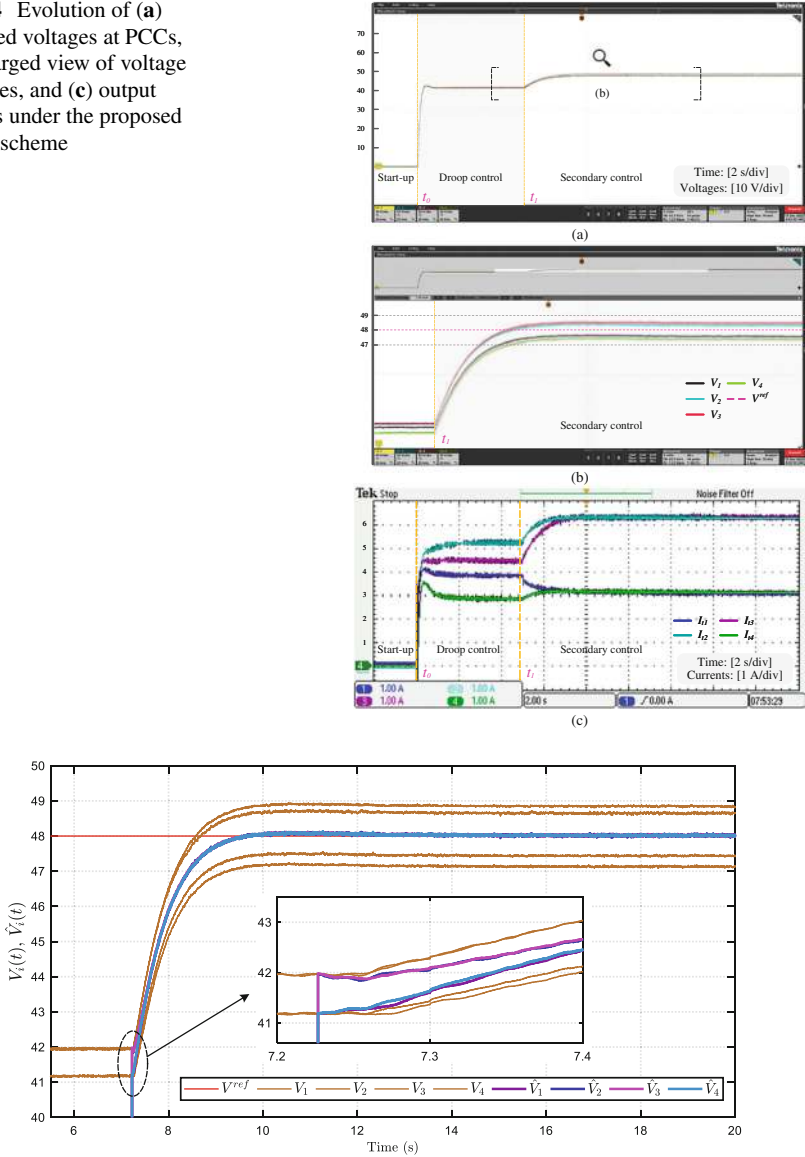


Fig. 9.5 Evolution of the estimate of the measured voltage

3. In particular, the speed of achieving voltage recovery and current sharing in Case 3, i.e., the speed at which the system reaches steady state, is significantly faster than that demonstrated in the test results for Case 1 and Case 2.
4. The existing control strategies have focused on current sharing and voltage balancing, without taking into account any performance metrics. On the contrary,

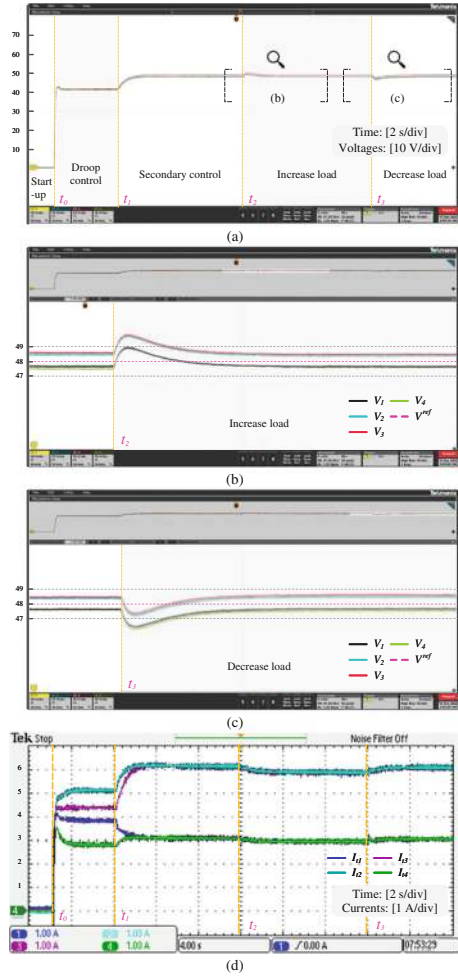
the model-based secondary controller proposed in this chapter not only achieves these two control objectives but also optimizes the performance index function (9.29), resulting in fast and smooth performance.

In summary, the distributed controller given by (9.18) and (9.21) provides a faster and smoother response compared with the two implemented baselines. These discussions about the experimental results can be explained by the following facts. This difference may be attributed to the selected feedback gains, the first-order composite-error channel used for controller design, and the absence of additional slow compensator dynamics in the secondary loop. Specifically, the method proposed in this study does not add any additional dynamics to the original MG system, while the latest controller proposed in Sadabadi et al. (2023) adds three integrators, which would increase the order of the dynamical system and lead to more transients. As a result, additional dynamics are introduced, which can compromise stability margins and increase transient overshoot. In contrast, we eliminate the coupled dynamics of the DC MG through the designed auxiliary control action $u_{e,i}^{aux}$, inspired by the FAS theory. This approach allows us to obtain a clear, simple structured description of the MG system that facilitates the secondary controller design. Furthermore, the developed first-order fully actuated multi-agent system model reduces the system dynamics and improves the response speed of the MG system. To this end, it becomes feasible to design a secondary controller based on LQR, which is a general advantage of model-based control.

9.5.3 Regulation Results Under Load Changes

At t_2 , a $2\ \Omega$ resistor is inserted in series with the $10\ \Omega$ local load of DG₄, increasing the equivalent resistance to $12\ \Omega$ and reducing the local load demand. At t_3 , the auxiliary resistor is bypassed and the equivalent resistance returns to $10\ \Omega$. Load variations would affect the performance of the proposed secondary control strategy, which is described in detail in this section. The experiment corresponding to Stage 3 in Sect. 9.5.1 is executed and the output responses of the DC MG under load variation are given by Fig. 9.6. It is worth mentioning that a relatively conservative practice is adopted in this case for load variations. Specifically, the proposed secondary controller automatically updates its auxiliary control role $u_{e,i}^{aux}$ during load variations. At moments t_2 and t_3 , due to local load changes near DG #4, the output voltages stabilize quickly and their average values track the given value after a few seconds of fluctuation, as can be seen from the waveform shown in Fig. 9.6a. The current response curves for this case are depicted in Fig. 9.6d. The inductor currents of the MG for load variation are still divided into two groups at steady state, which is consistent with the performance under the basic performance test. Although the output current of each DG unit also jittered as the local load in the MG changed at moments t_2 and t_3 , the total load currents could be quickly stabilized in the new state. In the new steady state, the currents are shared among the DG units in

Fig. 9.6 Evolution of (a) measured voltages at PCCs, (b) enlarged view of voltages for the load increase, (c) enlarged view of voltages for the load decrease, and (d) output currents under the proposed control scheme in case of load variations



the MG in an optimal way, quickly and precisely, according to a given distribution factor.

According to the observation of the local magnification of the voltage, Fig. 9.6b, when the local load changes from 10Ω to 12Ω , i.e., $t \in [t_2, t_3)$, the output voltages stabilize quickly after experiencing a tiny and transient overshoot, and their average value tracks quickly up to the given value. Similarly, when the local load is recovered from 12Ω to the initial condition, i.e., $t \in [t_3, \infty)$, the output voltage completes the tracking with an instantaneous dip, as can be seen in Fig. 9.6c. Regarding the aforementioned experimental observations, the following discussion is presented. As shown in Fig. 8.1, when the local load resistance increases, the load current decreases immediately, while the converter-inductor current cannot change instantaneously. The temporary current surplus charges the shunt capacitor, causing

a short voltage rise. The controller subsequently adjusts the converter current and restores the voltage profile. After the capacitor voltage rises, the voltage drop across the inductor correspondingly decreases, leading to a subsequent decline in the inductor current. However, under the influence of the controller, the steady-state voltage will return to an acceptable range. Conversely, this also explains the phenomenon observed in Fig. 9.6c when the load experiences a sudden decrease. From the above results and related discussions, it can be seen that the proposed secondary control scheme provides good adaptability for DC MGs in response to load variations.

In particular, the coupling dynamics of each subsystem in the MG with its neighboring subsystems are eliminated due to the design of the auxiliary control quantities $u_{e,i}^{aux}$. Thus, the four converter-based DG units constitute a homogeneous non-interconnected multi-agent system, which leads to a MG with the proposed control method exhibiting an excellent consensus even in transient processes. These experimental results confirm again the advantages of the developed first-order FAS model in achieving fast voltage balancing and current sharing.

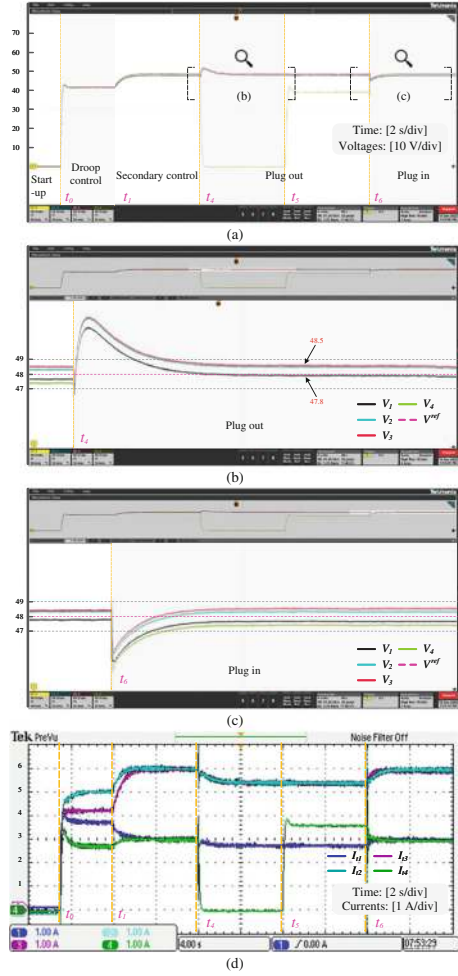
9.5.4 Regulation Results Under PnP Operation

This subsection will show the effect of intermittency of DGs on the regulation performance of the DC MG with the proposed control scheme. In this case, DG₄ is disconnected from the MG at t_4 . Its local power supply and droop-controlled standalone operation are restored at t_5 . After its PCC voltage reaches an appropriate level, DG₄ is reconnected to the MG at t_6 . It is noted that the local load will also be disconnected from the MG along with DG #4.

Notice that the cyber links of the connected DGs will remain the same as the previous case when DG #4 is off the MG due to intermittency. That is, at the information layer, the DG #4 automatically removes connections to its neighbors, since the physical disconnection of the DG unit also implies the loss of all communication links to DG #4. The PnP experiment assumes that disconnection and reconnection events are detected by the neighboring controllers. Therefore, the communication topology is updated automatically in the controller of the remaining DG units; i.e., the communication weight corresponding to the faulty node is set to zero. In addition, the physical connection information in the controller will be updated synchronously. Figure 9.7a, b, c, d show the output voltages, their enlarged views when disconnected and reconnected, and inductor currents, respectively.

In particular, according to the enlarged view shown in Fig. 9.7b, the disengagement of DG #4 at t_4 leads to a certain overshoot of the output voltage of the remaining three DG units. However, the voltages stabilize quickly after the transient, while their average values finish tracking. On the other hand, the voltage at the PCC of DG #4 slowly decays to 0 due to the role of the shunt capacitor. Moreover, with the recovery of the generation unit at the moment t_6 , all voltages can be quickly returned to the level before the occurrence of the fault, as can be seen from the

Fig. 9.7 Evolution of (a) measured voltages at PCCs, (b) enlarged view of voltages for plug-out operation, (c) enlarged view of voltages for plug-in operation, and (d) output currents under the proposed control scheme in the case of plug-and-play operation



enlarged view in Fig. 9.7c. It is worth mentioning that after DG #4 restores its power supply capability, it initially provides power to the local load under droop control. At this point, it is not yet participating in grid operation. Only after DG #4 operates normally in standalone mode do we close the circuit breaker to connect it to the MG for current sharing. As a result, during the period from t_5 to t_6 in Fig. 9.7a, the output voltage V_4 of DG #4 is outside the desired allowable range.

Figure 9.7d shows the inductor currents in the DC MG for this case. Due to the failure of DG #4, the inductor currents of DG #1, DG #2, and DG #3 stabilize quickly after the transient and are shared in the MG following the new distribution factor $\theta = [1, 2, 2]$. The proposed secondary control scheme is invalid for DG #4 during $t \in [t_4, t_5)$. Meanwhile, the current of DG #4 drops to 0 due to the presence of local load. Similarly, as DG #4 is reconnected to the MG at t_6 , communication

with it is also restored and the current is shared among the four DG units, and the steady states of the currents are restored to their pre-fault levels. Comparing the two moments of t_4 when DG #4 is plugged out and t_6 when DG #4 is plugged in, it can be found that the transient performance of the current is better when DG #4 is disconnected, while the transient performance of the voltage at PCC is better when DG #4 is reconnected. First, the dynamics of the current is discussed. The reason for the aforementioned characteristics of the current is that DG #4 is no longer involved in the present current distribution after being pulled out (i.e., $t \in [t_4, t_5]$). Meanwhile, the remaining three DG units have already established coordination, making it easier for them to coordinate again. Instead, a new DG unit that has not previously been involved in coordination is added when DG #4 is reinserted at t_6 . Therefore, the situation is exactly the opposite for the current at this stage. First, the dynamics of the voltage is discussed. Although the original control parameters when DG #4 is pulled out can still achieve satisfactory results for the MG composed of three DG units, they are still the best parameters obtained under the MG before the failure. Therefore, the voltage response at t_6 is better than that at t_4 .

From the enlarged view in Fig. 9.7b, it can be seen that the minimum and maximum values of the voltage at the PCC are stabilized at 47.8 and 48.5, respectively, after DG #4 is disconnected from the DC MG. Based on this observation and the results shown in Fig. 9.7b, it is clear that the voltage is only close to the given value without achieving an accurate voltage recovery under the exact current sharing. Observation of Fig. 9.7a, d again shows that both voltage and current return to the steady state during $t \in [t_1, t_4]$ and after DG #4 is restored. Additionally, it is noted that when DG #4 is reconnected to the MG, the output voltages of the other three DG units remain at their original levels, while their output currents increase. Therefore, after DG #4 is reconnected, the other three DG units assist DG #4 in sharing the load that it would have otherwise needed to supply independently. In this way, the participation of the other units assists DG #4 in transitioning from a heavy load state to a light load state, which is beneficial for the longevity of the devices within the system. This also highlights the positive significance of current sharing in MGs.

The above discussion confirms that the proposed control strategy provides the expected PnP functionality for the DC MG while achieving optimal regulation of the DC MG.

9.6 Notes and References

This chapter addresses the challenges faced by a specific category of multi-bus DC microgrids in achieving average voltage regulation and current balancing. It proposes an optimal distributed secondary control strategy. Initially, by designing a comprehensive regulatory error, a detailed modeling of the DC microgrid is conducted to effectively capture its dynamic characteristics and the coupling relationships within the physical system. Drawing inspiration from the FAS theory, after establishing a large-signal microgrid model, a further transformation is applied

to obtain a decoupled multi-agent system composed of first-order agents, consistent with the tasks of voltage regulation and current balancing. Subsequently, based on the proposed cost function and the first-order decoupled model, a tailored distributed secondary optimal control strategy is designed. Similar to common methods found in related studies, a thorough analysis of the stability, optimality, and consistency of the closed-loop DC microgrid system under the proposed control strategy is performed. To evaluate the performance of this strategy in real-world applications, extensive case studies are conducted on a hardware experimental microgrid system, particularly in typical operational scenarios such as plug-and-play situations. The experimental results strongly confirm the effectiveness and practicality of the proposed approach.

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Chapter 10

Blockchain-Based Fully Actuated Secure Control of Renewable Power Systems



Abstract This chapter investigates the issue of cybersecure distributed control in cyber-physical microgrid systems. In response to increasingly severe cyber threats arising from the deep integration of power electronics and information technology, a blockchain-based secondary control architecture is proposed. By constructing a FAS model, the controller design framework is simplified, and the Practical Byzantine Fault Tolerance (PBFT) consensus algorithm is introduced to ensure the security of multichannel data transmission. The design, in which each agent deploys its own dedicated P2P network, significantly reduces the complexity of blockchain deployment in multi-agent systems. Although blockchain technology can resist most attacks, there remains a risk that a small number of intrusions may affect the system model. To address this, a stability criterion for cyber-physical systems is established in a probabilistic sense. Thanks to the FAS approach, the proposed blockchain control architecture not only enhances system security but also significantly reduces the complexity of model-based secondary security control law design. Finally, MATLAB/Simulink simulations evaluate the control performance of the abstracted blockchain-assisted validation mechanism under the considered FDIA model.

10.1 Overview

Since its inception, blockchain technology has evolved from a simple carrier for digital currencies into a distributed infrastructure that enables trusted collaboration across multiple industries (Dinh et al., 2018; Huynh-The et al., 2023). Public blockchains, exemplified by Bitcoin, have established the technical foundation for decentralized ledgers, while the introduction of smart contracts by Ethereum has greatly expanded blockchain's programmability, giving rise to innovative applications such as decentralized finance (DeFi) and supply chain traceability. In recent years, the development of consortium blockchains and the adoption of lightweight consensus algorithms have significantly improved blockchain's throughput, real-time performance, and privacy protection, accelerating its deployment in fields

such as the industrial Internet, smart manufacturing, and the Internet of Things (Dai et al., 2019; Fernández-Caramés & Fraga-Lamas, 2018). For example, in supply chain finance, blockchain enables trusted data sharing and automated settlement among multiple parties; in smart manufacturing, blockchain-based device identity management effectively prevents the access of malicious nodes; and in the healthcare sector, cross-institutional data exchange leverages on-chain authorization and encryption mechanisms to effectively safeguard patient privacy (Zaghloul et al., 2020; Zhang et al., 2019).

With its features of decentralization, data immutability, and automatic execution of smart contracts, blockchain technology has also begun to be explored in the market and scheduling layers of power systems, such as energy trading and demand response (Di Silvestre et al., 2020). Currently, research on blockchain applications in power systems mainly focuses on enhancing market transaction transparency, facilitating distributed energy resource coordination, and enabling green certificate traceability (Mohammadi & Saif, 2023). For instance, blockchain-based peer-to-peer (P2P) energy trading platforms utilize smart contracts for automated settlement, while improvements in consensus algorithms (such as PoW and PoS) have increased transaction efficiency (Esmat et al., 2021; Yu et al., 2021). However, most existing studies concentrate on market transactions and wide-area scheduling optimization, whereas the application of blockchain at the real-time control layer (e.g., voltage regulation and power allocation) still faces significant challenges (Ould Mohamed, 2023; Xu et al., 2021). These challenges are primarily due to the high-frequency interaction requirements of control loops, the need for millisecond-level system response, and the computational limitations of resource-constrained devices (Chen et al., 2020). Resource-intensive cryptographic mechanisms may consume the real-time computational resources of control systems, adversely affecting dynamic performance (Fauser & Zhang, 2025). Moreover, there is an inherent conflict between the high-frequency interaction of control commands and the latency introduced by blockchain consensus (Chen et al., 2022; Yu et al., 2021). The communication complexity of traditional consensus protocols (such as PBFT) makes it difficult to meet the millisecond-level response requirements.

To address the aforementioned challenges, this paper proposes a security-enhanced framework for fully actuated control in RPSs by integrating blockchain technology to ensure secure voltage regulation and current allocation. The main contributions are as follows:

1. Blockchain technology is deeply embedded into the fully actuated control loop, constructing a blockchain-enabled proactive security communication architecture. This approach enables preemptive defense against attacks, breaks away from the traditional reliance on centralized trust intermediaries, and effectively resists false data injection attacks.
2. By balancing security and cost-effectiveness, blockchain computational resources are rationally allocated among different generation units within the power system. This ensures an optimal trade-off between computational

overhead and the precision of voltage and current control, thereby avoiding both resource overload and underutilization.

3. A lightweight consensus protocol specifically designed for real-time control systems is developed. Through local rapid verification and a global election strategy, the protocol ensures fast and reliable execution of control commands under high-concurrency scenarios.

The chapter is organized as follows. Section 10.2 introduces the problem formulation. Section 10.3 is about designing the blockchain-based and fully actuated secure control of RPSs. Section 10.4 presents simulation results to validate the proposed method. Section 10.5 concludes the chapter.

10.2 Problem Formulation

The multi-bus DC microgrid system studied in this chapter builds upon the foundational architecture introduced previously and consists of multiple distributed generation units connected in parallel via resistive lines. As shown in Fig. 3.1, each generation unit employs a buck-type DC-DC converter as its core power interface, whose dynamic characteristics are described by a state-space model (3.1). The system adopts a hierarchical control architecture to ensure power quality regulation. At the bottom control layer, local dual-loop PI controllers (6.4) and (6.5) are used to rapidly respond to dynamic changes in voltage and current, thereby achieving accurate local voltage tracking. To coordinate current sharing among multiple units and support seamless plug-and-play of generation sources, the primary control layer incorporates droop characteristics (6.3), which adjust the output current of each unit via virtual impedance, enabling preliminary load balancing without communication. Consequently, the physical topology of the microgrid formed by the interconnection of distributed generation units can be modeled by a graph $\mathcal{G}_p$, as described in Sect. 3.2.2.

At the communication layer, each distributed generation unit exchanges operational status information through an undirected graph topology. The communication network is represented by the undirected graph $\mathcal{G}_c = (\mathcal{V}_c, \mathcal{E}_c, \mathcal{A}_c)$, following the graph-theoretic definitions introduced in Sect. 7.2.2. For clarity, the neighbor set and adjacency weights required in this chapter are restated below. The secondary controllers dynamically adjust the correction terms in the droop control based on the operational states of neighboring nodes. This mechanism not only compensates for the voltage deviations caused by droop control, ensuring that the bus voltage is restored and maintained within a certain range near its nominal value, but also achieves precise proportional current sharing among units through dynamic coordination, thereby meeting the system's requirement for fair load power distribution. However, the openness of the communication network introduces potential security threats, particularly false data injection attacks (FDIAs). Such attacks may compromise the cooperative logic of the secondary control by tampering with

voltage or current measurements during transmission, potentially resulting in bus voltage instability or imbalanced current sharing.

10.3 Blockchain-Based Fully Actuated Secure Control of DC MG Systems

10.3.1 Large-Signal Characterization of Droop-Controlled MGs

By systematically integrating the dynamics of the underlying circuit (3.1), the dual-loop PI controller (6.4) and (6.5), as well as the droop control strategy coordinating the current distribution of multiple converters (6.3), selecting the system state $x_i(t) = [V_i(t), I_{ti}(t), \phi_i(t), \gamma_i(t)]^T$ and control input $u_i(t) = \delta_i(t)$, a unified state-space model can be constructed for distributed generation units.

$$\dot{x}_i(t) = A_i^c x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}^c x_j(t) + B_i^c V^{ref} + B_i^c u_i(t) \quad (10.1)$$

where

$$A_i^c = \begin{bmatrix} -\sum_{j \in \mathcal{N}_i^p} \frac{1}{C_{ti} R_{ij}} - \frac{1}{C_{ti} R_{Li}} & \frac{1}{C_{ti}} & 0 & 0 \\ -\frac{1 + V_{in} K_{pci} K_{pvi}}{L_{ti}} & -\frac{R_{ti} + V_{in} K_{pci} (K_{pvi} r_i + 1)}{L_{ti}} & \frac{V_{in} K_{pci} K_{ivi}}{L_{ti}} & \frac{V_{in} K_{ici}}{L_{ti}} \\ -1 & -r_i & 0 & 0 \\ -K_{pvi} & -K_{pvi} r_i - 1 & K_{ivi} & 0 \end{bmatrix},$$

$$B_i^c = \begin{bmatrix} 0 \\ \frac{V_{in} K_{pci} K_{pvi}}{L_{ti}} \\ 1 \\ K_{pvi} \end{bmatrix}, \quad A_{ij}^c = \begin{bmatrix} \frac{1}{C_{ti} R_{ij}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since the control objective requires the use of the output voltage and current of the generating unit, these two quantities are taken as system outputs. Let $C_i = [C_{1i}^T, C_{2i}^T]^T$, $C_{1i} = [0 \ 1 \ 0 \ 0]$, $C_{2i} = [1 \ 0 \ 0 \ 0]$, then

$$\begin{aligned} I_{ti}(t) &= C_{1i} x_i(t) \\ V_i(t) &= C_{2i} x_i(t). \end{aligned} \quad (10.2)$$

This system model integrates dynamic equations and coupling relationships across all microgrid. It physically elucidates underlying circuit dynamics in microgrids under secondary control, yet due to intrinsic underactuation, structural cou-

plings, and nonlinearity, traditional linear control methods prove inadequate for multi-objective cooperative optimization. To address these constraints, subsequent sections will establish a FAS model for regulation errors, thereby providing a tailored mathematical framework for controller design.

10.3.2 Error System Modeling for Droop-Controlled MGs

To overcome the limitations of existing microgrid models for controller design, this section constructs a fully actuated error model. Using forward Euler discretization with sampling period T_s , define

$$A_i = I + T_s A_i^c, \quad A_{ij} = T_s A_{ij}^c, \quad B_i = T_s B_i^c.$$

The resulting discrete-time model is

$$x_i(t+1) = A_i x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij} x_j(t) + B_i V^{ref} + B_i u_i(t). \quad (10.3)$$

A deliberate symbolic redefinition is introduced here without impacting subsequent controller design procedures: the variable t now denotes discrete sampling instants rather than absolute time. We then revisit the dual control objectives of current consensus and average voltage regulation. To achieve current consensus, denoted as (8.1), a consensus error vector is constructed as follows:

$$\varepsilon_{1i}(t) = \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(\frac{I_{ii}(t)}{\theta_i} - \frac{I_{ij}(t-1)}{\theta_j} \right). \quad (10.4)$$

Neighboring current measurements are assumed to become available with a one-sample communication and validation delay. Therefore, DG_i uses $I_{t,j}(t-1)$ when forming the local current-sharing error. It can be obtained from model (10.3) that

$$\varepsilon_{1i}(t) = - \sum_{j \in \mathcal{N}_i^c} a_{ij} \frac{I_{ij}(t-1)}{\theta_j} + d_i^{in} \frac{C_{1i}}{\theta_i} x_i(t). \quad (10.5)$$

Furthermore, one has

$$\begin{aligned} \varepsilon_{1i}(t+1) = & - \sum_{j \in \mathcal{N}_i^c} a_{ij} \frac{I_{ij}(t)}{\theta_j} + d_i^{in} \frac{C_{1i}}{\theta_i} A_i x_i(t) \\ & + d_i^{in} \frac{C_{1i}}{\theta_i} B_i V^{ref} + d_i^{in} \frac{C_{1i}}{\theta_i} B_i u_i(t). \end{aligned} \quad (10.6)$$

On the other hand, for the acquisition of the global voltage average in average voltage tracking, an average voltage observer is designed as follows:

$$\bar{V}_i(t+1) = V_i(t+1) - V_i(t) + \bar{V}_i(t) + K_b \sum_{j \in \mathcal{N}_i^c} a_{ij}(\bar{V}_j(t) - \bar{V}_i(t)) \quad (10.7)$$

where K_b is the observing gain. The communication graph is assumed to be connected and undirected. The observer states are initialized such that

$$\sum_{i=1}^N \bar{V}_i(0) = \sum_{i=1}^N V_i(0).$$

Moreover, K_b is selected such that

$$0 < K_b < \frac{2}{\lambda_{\max}(L_c)},$$

which guarantees stability of the discrete consensus dynamics. Substituting the system model gives

$$\begin{aligned} \bar{V}_i(t+1) = & C_{2i}(A_i x_i(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij} x_j(t) + B_i V^{ref} + B_i u_i(t)) - C_{2i} x_i(t) \\ & + (1 - K_b d_i^{in}) \bar{V}_i(t) + K_b \sum_{j \in \mathcal{N}_i^c} a_{ij} \bar{V}_j(t). \end{aligned} \quad (10.8)$$

Thus, the dynamics of the average voltage observer can be simplified as

$$\begin{aligned} \bar{V}_i(t+1) = & C_{2i}(A_i - I)x_i(t) + (1 - K_b d_i^{in}) \bar{V}_i(t) \\ & + K_b \sum_{j \in \mathcal{N}_i^c} a_{ij} \bar{V}_j(t) + \sum_{j \in \mathcal{N}_i^p} A_{ij}^{11} V_j(t). \end{aligned} \quad (10.9)$$

Accordingly, the local tracking error of the observations can be defined as

$$\varepsilon_{2i}(t) = g_i(\bar{V}_i(t) - V^{ref}). \quad (10.10)$$

Further, the dynamics are obtained as

$$\begin{aligned} \varepsilon_{2i}(t+1) = & g_i C_i^2 (A_i - I)x_i(t) + g_i (1 - K_b d_i^{in}) \bar{V}_i(t) \\ & + g_i K_b \sum_{j \in \mathcal{N}_i^c} a_{ij} \bar{V}_j(t) + g_i \sum_{j \in \mathcal{N}_i^p} A_{ij}^{11} V_j(t) - g_i V^{ref}. \end{aligned} \quad (10.11)$$

Combining (10.6) and (10.11), the integrated regulation error is obtained as

$$\begin{aligned}
 \varepsilon_i(t+1) = & \left(d_i^{in} \frac{C_i^1}{\theta_i} A_i + g_i C_i^2 (A_i - I) \right) x_i(t) + g_i (1 - K_b d_i^{in}) \bar{V}_i(t) \\
 & + \left(d_i^{in} \frac{C_i^1}{\theta_i} B_i - g_i \right) V^{ref} + \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(-\frac{I_{tj}(t)}{\theta_j} + g_i K_b \bar{V}_j(t) \right) \\
 & + g_i \sum_{j \in \mathcal{N}_i^p} A_{ij}^{11} V_j(t) + d_i^{in} \frac{C_i^1}{\theta_i} B_i u_i(t).
 \end{aligned} \tag{10.12}$$

10.3.3 Blockchain-Assisted Fully Actuated MG Model

To address cybersecurity risks in microgrid cooperative control posed by network attacks on communication channels, we introduce blockchain technology into distributed control architectures for security enhancement. This section establishes a fully actuated microgrid model resilient to adversarial conditions by integrating real-time state information exchange among nodes with blockchain-based Byzantine fault tolerance mechanisms.

To mitigate risks of single-point failures and resource redundancy associated with deploying blockchain networks across entire microgrids, this work proposes a heterogeneous blockchain architecture based on distributed unit-specific private chains, as illustrated in Fig. 10.1. Within this framework, each distributed generation unit independently operates a private blockchain network comprising N peer nodes, where N is determined by factors including the unit's data transmission density (e.g., neighbor node connectivity) and functional criticality level (e.g., generation vs. storage units). The blockchain-assisted validation process introduces a nonzero communication and computation delay. Let $d_{ij}^{bc} \in \mathbb{N}$ denote the number of secondary-control sampling intervals required to authenticate, validate, and commit the data transmitted from DG_j to DG_i . Therefore, the validated signal available to the controller is generally delayed and can be represented as

$$\tilde{y}_{ij}(t) = y_j(t - d_{ij}^{bc}) + \rho_{ij}(t) \zeta_{ij}(t).$$

For analytical simplicity, the blockchain validation time is assumed to be negligible relative to the secondary-control sampling period, so that $d_{ij}^{bc} = 0$ in the subsequent model. The effect of non-negligible consensus and cryptographic delays is beyond the scope of this chapter. For cross-unit communication, DG_j submits a signed data transaction to the validator replicas associated with DG_i . The replicas authenticate and validate the transaction and, after consensus, commit the accepted data to their

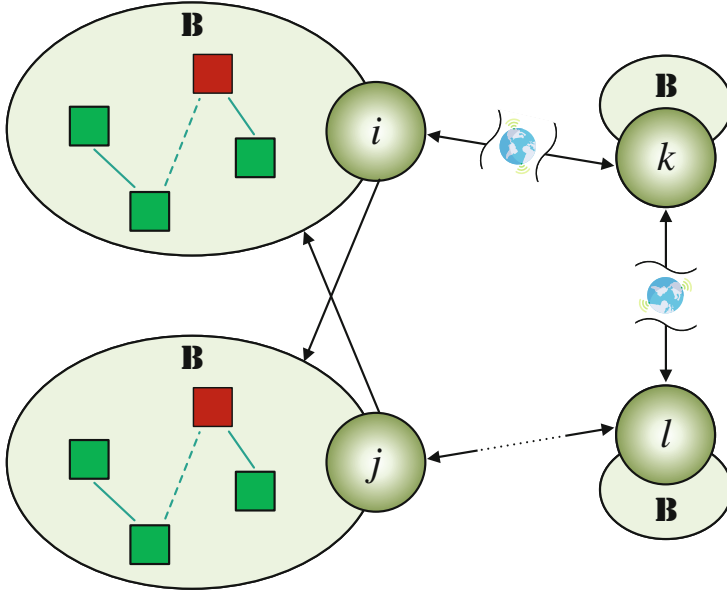


Fig. 10.1 DC MGs series-connected through power lines

local ledger copies. Each node within the receiver unit maintains a local ledger replica, validates data integrity through predefined consensus rules of its private chain, and automatically executes data storage and state updates via smart contracts.

The multi-agent system depicted above requires communication of data variables $I_{ij}(t)$, $V_j(t)$, $\bar{V}_j(t)$. Since cyberattacks directly compromise the evolution of average voltage observers, the affected observable is redefined as $\bar{V}_i^{ac}(t)$. Concurrently, current, voltage, and average voltage observer signals received by distributed generation unit DG_i from neighboring unit DG_j are denoted as $\tilde{I}_{ij}(t)$, $\tilde{V}_{ij}(t)$, $\tilde{\bar{V}}_{ij}^{ac}(t)$, respectively. Utilizing these authenticated measurements, the local average voltage observer at DG_i becomes

$$\bar{V}_i^{ac}(t+1) = V_i(t+1) - V_i(t) + \bar{V}_i^{ac}(t) + K_b \sum_{j \in \mathcal{N}_i^c} a_{ij} (\tilde{\bar{V}}_{ij}^{ac}(t) - \bar{V}_i^{ac}(t)). \quad (10.13)$$

The transmission process of data packet $y_j(t) = [I_{ij}(t)/\theta_j, V_j(t), \bar{V}_j^{ac}(t)]^T$ from DG_j to DG_i under adversarial conditions, specifically false data injection attacks on communication channels and blockchain-enabled active defense mechanisms, will now be described. This packet undergoes a one-to-many preparation

phase for broadcast to all activated nodes within DG_i 's local blockchain network. The procedure can be functionally characterized as follows:

$$f^1(y_j, N_i) = \left[\overbrace{y_j \cdots y_j}^{N_i} \right]. \quad (10.14)$$

The data packets transmitted across distinct communication channels may subsequently encounter varying degrees of cyber threats. Defining an attack probability vector $\alpha_{ij} \triangleq [\alpha_{ij}^1, \dots, \alpha_{ij}^{N_i}]$, where α_{ij}^k represents the probability of data corruption during transmission from DG_j to the k -th node in DG_i 's blockchain network, for $k = 1, \dots, N_i$. This compromised data propagation process can be formally modeled as follows:

$$f^2\left(f^1(y_j, N_i), \alpha_{ij}\right) = \left[y_{ij}^1 \cdots y_{ij}^{N_i} \right] \quad (10.15)$$

where $y_{ij}^k = y_j + \rho_{ij}^k \zeta_{ij}^k$ denotes the value received at the k -th node in DG_i 's blockchain network after data y_j arrives, while $\rho_{ij}^k \sim \text{Bernoulli}(\alpha_{ij}^k)$ and ζ_{ij}^k represent maliciously injected false values by attackers. Since the PBFT consensus protocol relies on data consistency across a majority of honest nodes, attackers face the critical challenge of making falsified data accepted as local candidate data by sufficient nodes. If attackers inject divergent false data to different nodes, cross-verification among honest nodes will rapidly detect anomalies, suppressing the attack. Thus, rational attackers inject identical false data to all nodes to maximize its acceptance as majority-approved candidate data, thereby increasing its success probability in the consensus phase, that is, $\zeta_{ij}^k = \zeta_{ij}$, for $k = 1, \dots, N_i$. Additionally, this work considers $\zeta_{ij} \sim N(0, U)$.

This batch of data arrives in DG_i 's P2P network and will be equivalently categorized prior to consensus, i.e.,

$$f^3\left(f^2\left(f^1(y_j, N_i), \alpha_{ij}\right)\right) = \left[\begin{array}{c} y_{ij}^1 \cdots y_{ij}^{N_i} \\ L_{ij}^1 \cdots L_{ij}^{N_i} \end{array} \right] \quad (10.16)$$

where $L_{ij}^k = l(y_{ij}^1, y_{ij}^k)$ is the tag that is appended by comparing y_{ij}^k with y_{ij}^1 , which satisfies

$$l(y_{ij}^1, y_{ij}^k) = \begin{cases} 1, & y_{ij}^k = y_{ij}^1 \\ 2, & y_{ij}^k \neq y_{ij}^1. \end{cases} \quad (10.17)$$

Based on the labels of the data, the data can be divided into two sets, and the number of elements in each set can be counted. According to the majority voting principle, the most credible result among the data at all nodes can be determined, which also serves as the output of this round of consensus. Ultimately, a set of trusted nodes and the final adopted trustworthy data are obtained. To summarize,

the validated data can be equivalently represented as

$$\tilde{y}_{ij} = y_j + \rho_{ij}\zeta_{ij} \quad (10.18)$$

where $\rho_{ij} \sim \text{Bernoulli}(\beta_{ij})$, β_{ij} can be obtained based on α_{ij} , N_i and the consensus process described above. The data within this packet threatened by the attack can also be represented separately as

$$\begin{aligned} \tilde{I}_{tij}(t)/\theta_j &= I_{tj}(t)/\theta_j + \rho_{ij}(t)\zeta_{ij}^I(t) \\ \tilde{V}_{ij}(t) &= V_j(t) + \rho_{ij}(t)\zeta_{ij}^V(t) \\ \tilde{\bar{V}}_{ij}(t) &= \bar{V}_j(t) + \rho_{ij}(t)\zeta_{ij}^{\bar{V}}(t). \end{aligned} \quad (10.19)$$

At the same time, it can be obtained

$$\zeta_{ij}^I(t) \sim N(0, U^I), \zeta_{ij}^V(t) \sim N(0, U^V), \zeta_{ij}^{\bar{V}}(t) \sim N(0, U^{\bar{V}}). \quad (10.20)$$

The considered adversary performs coordinated common-value FDIAs on a random subset of parallel transmission channels. Channel-compromise events are assumed independent, while all compromised channels in the same transmission round carry the same injected vector. Endpoint compromise, key theft, replay, denial-of-service, and correlated validator compromise are outside the considered threat model. The i -th DG is no longer able to compute the fully realistic regulation error locally, but instead, using the available information, computes

$$\varepsilon_i^{ac}(t) = \varepsilon_{1i}^{ac}(t) + \varepsilon_{2i}^{ac}(t) \quad (10.21)$$

where

$$\begin{aligned} \varepsilon_{1i}^{ac}(t) &= \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(\frac{I_{ti}(t)}{\theta_i} - \frac{\tilde{I}_{tij}(t-1)}{\theta_j} \right) \\ \varepsilon_{2i}^{ac}(t) &= g_i(\tilde{V}_i^{ac}(t) - V^{ref}). \end{aligned}$$

Meanwhile, the regulation error in this case can be obtained as

$$\varepsilon_i^{tg}(t) = \varepsilon_{1i}^{tg}(t) + \varepsilon_{2i}^{tg}(t) \quad (10.22)$$

where

$$\begin{aligned} \varepsilon_{1i}^{tg}(t) &= \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(\frac{I_{ti}(t)}{\theta_i} - \frac{I_{tj}(t-1)}{\theta_j} \right) \\ \varepsilon_{2i}^{tg}(t) &= g_i(\tilde{V}_i^{ac}(t) - V^{ref}). \end{aligned}$$

Therefore, one has

$$\begin{aligned}
 \varepsilon_i^{tg}(t+1) = & \left(d_i^{in} \frac{C_i^1}{\theta_i} A_i + g_i C_i^2 (A_i - I) \right) x_i(t) + g_i (1 - K_b d_i^{in}) \tilde{V}_i^{ac}(t) \\
 & + \left(d_i^{in} \frac{C_i^1}{\theta_i} B_i - g_i \right) V^{ref} + \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(-\frac{I_{tj}(t)}{\theta_j} + g_i K_b \tilde{V}_{ij}^{ac}(t) \right) \\
 & + g_i \sum_{j \in \mathcal{N}_i^p} A_{ij}^{11} V_j(t) + d_i^{in} \frac{C_i^1}{\theta_i} B_i u_i(t).
 \end{aligned} \tag{10.23}$$

Then, the controller can be designed as

$$u_i(t) = b_i^{-1} (\varepsilon_i^{ac}(t) - u_{ca,i}(t) + u_{e,i}(t)) \tag{10.24}$$

where $b_i = \frac{d_i^{in} C_i^1}{\theta_i} B_i$, the auxiliary control component is designed as

$$\begin{aligned}
 u_{ca,i}(t) = & \left(d_i^{in} \frac{C_i^1}{\theta_i} A_i + g_i C_i^2 (A_i - I) \right) x_i(t) + g_i (1 - K_b d_i^{in}) \tilde{V}_i^{ac}(t) \\
 & + \left(d_i^{in} \frac{C_i^1}{\theta_i} B_i - g_i \right) V^{ref} + \sum_{j \in \mathcal{N}_i^c} a_{ij} \left(-\frac{\tilde{I}_{tij}(t)}{\theta_j} + g_i K_b \tilde{V}_{ij}^{ac}(t) \right) \\
 & + g_i \sum_{j \in \mathcal{N}_i^c \cap \mathcal{N}_i^p} a_{ij} \left(A_{ij}^{11} \tilde{V}_{ij}(t) \right).
 \end{aligned} \tag{10.25}$$

The communication graph contains the physical-neighbor graph, i.e.,

$$\mathcal{N}_i^p \subseteq \mathcal{N}_i^c, \quad \forall i.$$

For every physical neighbor, the corresponding communication weight is normalized as $a_{ij} = 1$ in the model-cancellation term. Let

$$\begin{aligned}
 w_i(t) = & \sum_{j \in \mathcal{N}_i^c} a_{ij} \rho_{ij}(t) \zeta_{ij}^I(t) - g_i \sum_{j \in \mathcal{N}_i^p} A_{ij}^{11} \rho_{ij}(t) \zeta_{ij}^V(t) \\
 & - \sum_{j \in \mathcal{N}_i^c} a_{ij} \rho_{ij}(t-1) \zeta_{ij}^I(t-1).
 \end{aligned}$$

In this way, the fully actuated microgrid model can be obtained as follows:

$$\varepsilon_i^{tg}(t+1) = \varepsilon_i^{tg}(t) + u_{e,i}(t) + w_i(t). \tag{10.26}$$

For tractable controller design, the equivalent process disturbance $w_i(t)$ and measurement disturbance $z_i(t)$ are approximated as temporally white, mutually independent, zero-mean sequences with bounded covariance. The accuracy of this approximation is evaluated in simulation.

10.3.4 Blockchain-Based Fully Actuated Secure Control of MGs

The integration of blockchain technology establishes a trusted data exchange mechanism for microgrids. Building upon the blockchain-enhanced fully actuated microgrid model developed in preceding sections, this chapter designs a stochastic optimal control methodology to simultaneously achieve bus voltage droop recovery and collaborative multisource current dispatch.

While blockchain technology substantially reduces the success probability of false data injection attacks through distributed consensus mechanisms and mitigates their adverse impacts on microgrid systems, it remains inherently probabilistic in nature. Consequently, it cannot mathematically guarantee the complete elimination of attack probabilities. Such residual stochastic threats may still induce deviations in system dynamics from the nominal model under ideal conditions.

For the system (10.26), design the following objective function

$$J_i = \mathbb{E} \left[\sum_{t=0}^{\infty} \left(\left\| \varepsilon_i^{tg}(t) \right\|_{Q_i} + \left\| u_{e,i}(t) \right\|_{R_i} \right) \right] \quad (10.27)$$

where $\|x\|_S = x^T S x$ is the weight matrix and $\mathbb{E}[\bullet]$ denotes the joint expectation of the probability of occurrence of FDIA and the input signal. The output of the system used to compute the control inputs has the following perturbations:

$$\varepsilon_i^{ac}(t) = \varepsilon_i^{tg}(t) + z_i(t) \quad (10.28)$$

where $z_i(t) = - \sum_{j \in \mathcal{N}_i^c} a_{ij} \rho_{ij}(t-1) \zeta_{ij}^I(t-1)$. Based on the discrete Riccati equation

$$-P_i(P_i + R_i)^{-1}P_i + Q_i = 0$$

the positive definite matrix $P_i > 0$ can be obtained, then one has

$$u_{e,i}(t) = K_i \varepsilon_i^{ac}(t) \quad (10.29)$$

where $K_i = -(P_i + R_i)^{-1}P_i$. The residual attack disturbances are assumed to be zero-mean stochastic sequences with bounded second moments. Specifically, there

exist positive constants $\bar{\sigma}_{w,i}^2$ and $\bar{\sigma}_{z,i}^2$ such that

$$\mathbb{E}[w_i^2(t)] \leq \bar{\sigma}_{w,i}^2, \quad \mathbb{E}[z_i^2(t)] \leq \bar{\sigma}_{z,i}^2.$$

This formulation enables the design of control inputs leveraging the disturbance's zero-mean property, bounded variance, and statistical independence. Consequently, the optimal gain matrix can be obtained through offline optimization by solving the stochastic Riccati equation.

In summary, this chapter's approach of attenuating attack impact energy at the data source directly reduces steady-state errors without modifying control parameters Q and R , thereby preventing performance degradation associated with weight coefficient adjustments. This methodology concurrently ensures convergence of the mean regulation error and mean-square boundedness of the closed-loop system while balancing control efficiency and dynamic performance, establishing a novel optimization paradigm for secure control design. Because the authenticated observer remains subject to residual accepted attacks, convergence of the target error implies voltage regulation in expectation or within a stochastic neighborhood, rather than exact sample-path restoration under every attack realization.

Theorem 10.3.1 *Consider the stochastic fully actuated error system*

$$\epsilon_i^{\text{tg}}(t+1) = \epsilon_i^{\text{tg}}(t) + u_{e,i}(t) + w_i(t),$$

with the authenticated measurement

$$\epsilon_i^{\text{ac}}(t) = \epsilon_i^{\text{tg}}(t) + z_i(t).$$

Assume that $w_i(t)$ and $z_i(t)$ are zero-mean stochastic sequences with bounded second moments and are mutually independent. Under the feedback law

$$u_{e,i}(t) = K_i \epsilon_i^{\text{ac}}(t),$$

if

$$|1 + K_i| < 1,$$

then the mean regulation error converges to zero, i.e.,

$$\lim_{t \rightarrow \infty} \mathbb{E}[\epsilon_i^{\text{tg}}(t)] = 0,$$

and the second moment of the regulation error is uniformly ultimately bounded.

Proof Substituting the authenticated measurement into the control law gives

$$\epsilon_i^{\text{tg}}(t+1) = (1 + K_i) \epsilon_i^{\text{tg}}(t) + K_i z_i(t) + w_i(t).$$

Taking expectations and using the zero-mean assumptions yields

$$\mathbb{E}[\epsilon_i^{\text{tg}}(t+1)] = (1 + K_i)\mathbb{E}[\epsilon_i^{\text{tg}}(t)].$$

Therefore, $|1 + K_i| < 1$ guarantees convergence of the mean error to zero.

Furthermore, under the bounded-second-moment and independence assumptions,

$$\mathbb{E}\left[\left(\epsilon_i^{\text{tg}}(t+1)\right)^2\right] \leq (1 + K_i)^2 \mathbb{E}\left[\left(\epsilon_i^{\text{tg}}(t)\right)^2\right] + K_i^2 \bar{\sigma}_{z,i}^2 + \bar{\sigma}_{w,i}^2.$$

It follows that

$$\limsup_{t \rightarrow \infty} \mathbb{E}\left[\left(\epsilon_i^{\text{tg}}(t)\right)^2\right] \leq \frac{K_i^2 \bar{\sigma}_{z,i}^2 + \bar{\sigma}_{w,i}^2}{1 - (1 + K_i)^2}.$$

Thus, the closed-loop regulation error is mean-square bounded. Persistent nonzero-variance attacks generally lead to a nonzero steady-state error variance rather than exact mean-square convergence to zero.

The above result shows that the residual stochastic attacks do not generally permit exact mean-square convergence when their variances remain nonzero. Instead, the proposed controller drives the expected regulation error to zero and confines the second moment to a neighborhood determined by the residual post-consensus attack variances.

10.4 Simulation Verification

10.4.1 Simulation System Setup

A MATLAB/Simulink model of a four-DG multi-bus DC MG is established to evaluate the proposed secure secondary-control framework, as illustrated in Fig. 10.2. The four DG units adopt a ring communication topology, forming a closed-loop data exchange network through bidirectional communication links.

System parameters are configured according to Sect. 9.5 of Chap. 9, with input voltage, line impedance, and load settings matching actual physical equipment. The simulation employs a fixed step size of $1\mu\text{s}$ for real-time computation and 0.1 ms for sampling, ensuring precise dynamic coupling between digital controllers and physical systems. Each independent link in the communication topology is preset with an attack probability, where attack modes are designed as random

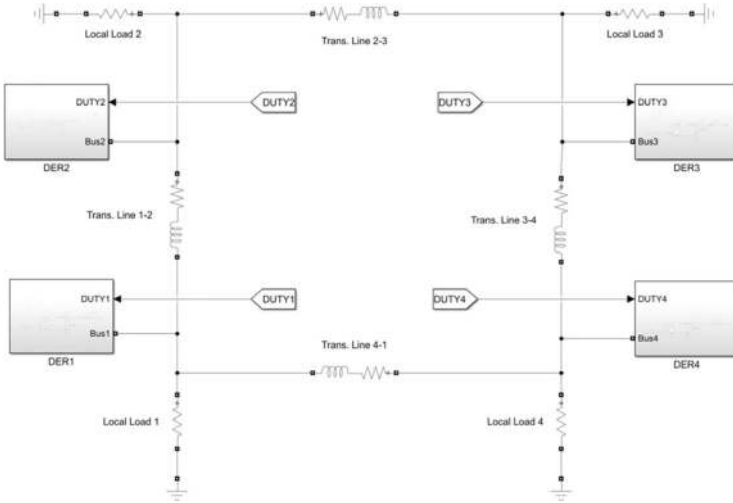


Fig. 10.2 Architecture of the tested multi-bus DC microgrids

injections of band-limited stochastic false-data signals. Without loss of generality, the attack probability is set to 0.3 during simulation, though attack occurrences and injected false signals remain mutually independent across all communication links. The present simulations abstract the blockchain execution process through the post-validation residual attack probability. Consensus-message exchange, processor utilization, and communication throughput are not explicitly simulated. Therefore, the results evaluate the control effectiveness of the proposed validation mechanism rather than the real-time execution performance of a complete blockchain implementation.

Subsequently, three comparative simulation scenarios will be executed to demonstrate blockchain's efficacy in enhancing system security:

1. Baseline operation without cyber attacks;
2. Attack conditions without defensive measures;
3. Attack scenarios with blockchain-enabled defense mechanisms.

An initial baseline test was conducted to demonstrate the inherent advantages of the fully actuated control strategy. Results are presented in Figs. 10.3 and 10.4. Figure 10.3 illustrates that in the attack-free microgrid system, bus voltage restored from droop-induced deviations to near-rated values when secondary control is activated, while output currents of all generation units autonomously adjust to strictly maintain predefined proportional relationships. Figure 10.4 further reveals rapid asymptotic stabilization of regulation errors under control action, exhibiting smooth convergence characteristics devoid of overshoot-induced spikes or hysteresis-induced persistent deviations. This validates the FAS approach's capability to reconstruct dynamic responses, where its designed diffeomorphic transfor-

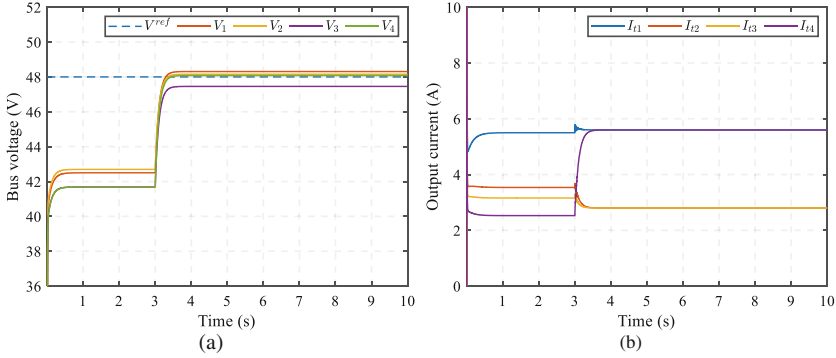


Fig. 10.3 (a) Bus voltage and (b) output current response of the microgrid under no attack

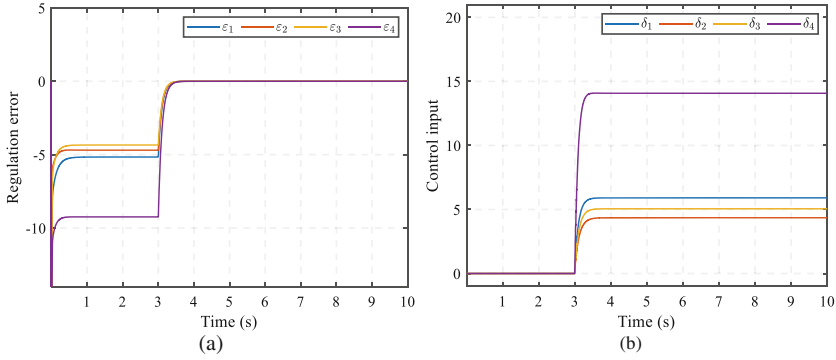


Fig. 10.4 (a) Regulation error and (b) control inputs of the microgrid under no attack

mation imbues the controller with adaptability, achieving both swift response and smooth regulation.

Subsequently, tests were conducted to evaluate the adverse impacts of FDIAs on the microgrid system, with results depicted in Figs. 10.5 and 10.6. These demonstrate that even under moderate attack probabilities, persistent malicious disruptions to communication links induce severe high-frequency oscillations in both bus voltages and output currents of generation units. This degraded response fundamentally reveals inherent vulnerabilities in control systems. Due to traditional controllers' heightened susceptibility to malicious disturbances, even robust stochastic optimal control implementations yield unsatisfactory power quality outcomes.

Furthermore, tests incorporating blockchain defense mechanisms demonstrate substantial attenuation of the attack-induced response from attack-induced disruptions, as evidenced in Figs. 10.7 and 10.8. When subjected to identical attack intensities as the previous scenario, the microgrid exhibits significantly attenuated voltage fluctuations, which withstands negative impacts without severe oscillations. Under the tested attack intensity, the blockchain-assisted validation mechanism

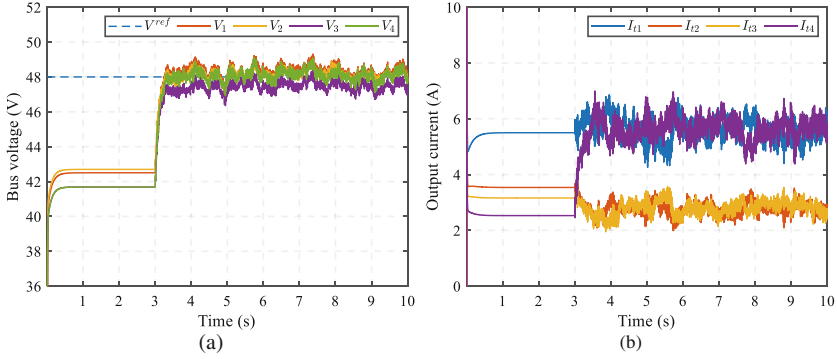


Fig. 10.5 Microgrid responses under FDIA without blockchain defense: (a) bus voltage, (b) output current

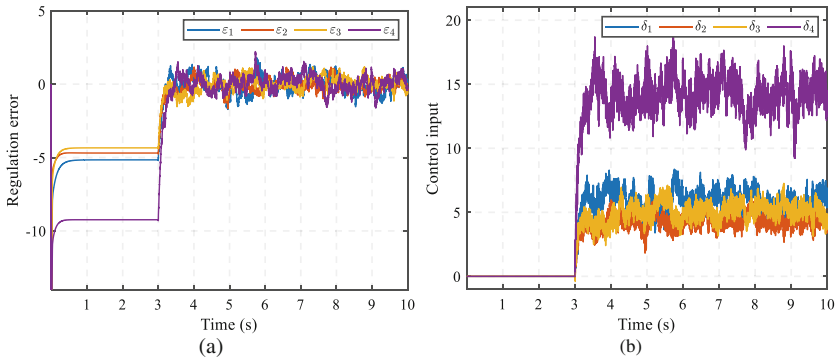


Fig. 10.6 Microgrid characteristics under FDIA without blockchain defense: (a) regulation error, (b) control input

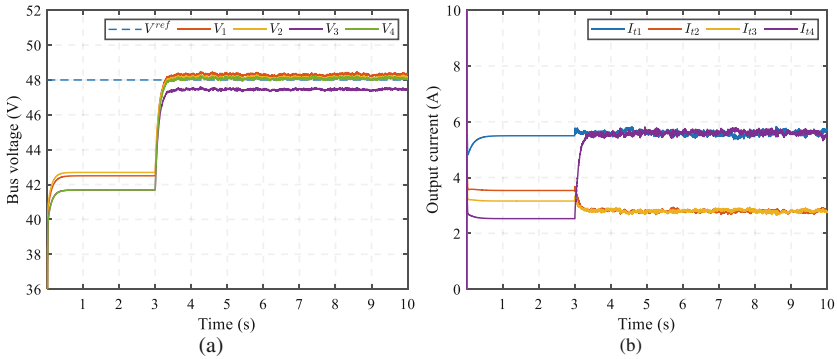


Fig. 10.7 Microgrid responses under attacks with blockchain defense: (a) bus voltage, (b) output current

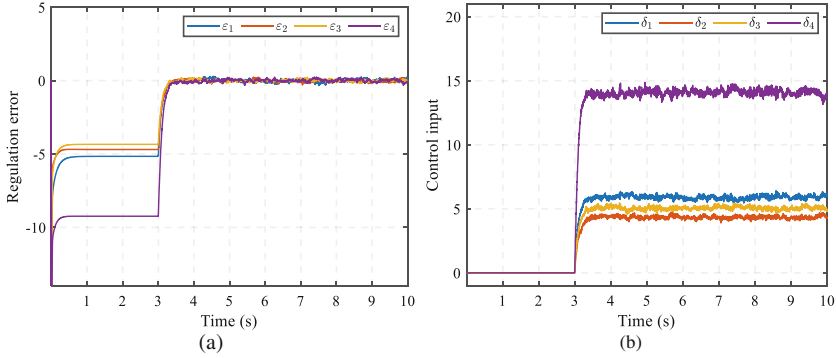


Fig. 10.8 Microgrid characteristics under attacks with blockchain defense: (a) regulation error, (b) control input

substantially attenuates the attack-induced voltage and current-sharing deviations. The residual disturbances remain smaller than those observed in the unprotected case, indicating improved resilience against the considered communication-channel FDIAs. This conclusively validates how blockchain's distributed trust architecture fundamentally transforms attack-defense characteristics. By enabling real-time credibility verification of transmitted signals, it achieves precise source-level attack containment.

10.4.2 Communication Security Visualization

To visually demonstrate blockchain's proactive defense against cyberattacks, a critical communication link was monitored. Figure 10.9 contrasts attack event records in the DG2-to-DG1 data channel: successful attacks compromising transmitted data without blockchain defense versus attacks neutralized under blockchain protection. For enhanced visual clarity, data was downsampled by a factor of 100. Without defensive mechanisms, attack signals corrupted data packets exchanged between generation units at preset probabilities, injecting dense perturbation signals. These malicious injections aggressively distorted normal control commands, triggering severe high-amplitude oscillations. To quantify the effectiveness of the validation mechanism, two metrics are distinguished. The attack interception rate is defined as

$$r_{\text{int}} = \frac{N_{\text{blocked}}}{N_{\text{attacked}}},$$

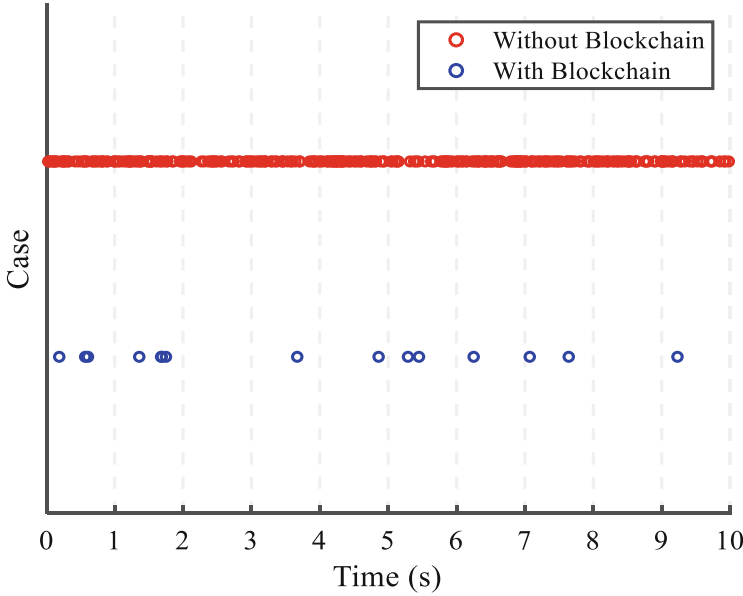


Fig. 10.9 Attack frequency with versus without blockchain defense mechanisms

where N_{attacked} is the number of corrupted transmission events and N_{blocked} is the number rejected by the validation mechanism. The accepted-data integrity rate is defined as

$$r_{\text{intg}} = \frac{N_{\text{correct,accepted}}}{N_{\text{accepted}}}.$$

Under the adopted simulation configuration, the corresponding values are $r_{\text{int}} = 92\%$ and $r_{\text{intg}} = 97.6\%$, respectively. This comparative analysis conclusively differentiates blockchain-based security from conventional resilient control strategies, which establishes impenetrable defense at the information source rather than merely compensating for attack consequences.

10.5 Notes and References

This chapter proposes a blockchain-enhanced fully actuated microgrid model and optimal secure secondary control for cyber-physical RPSs. While existing fully actuated models and resilient control strategies provide baseline attack tolerance, their reactive focus on postattack recovery lacks real-time attack attribution, proactive defense mechanisms, and trusted coordination, falling short of meeting the dual security and autonomy demands in highly dynamic, decentralized power systems.

To address these limitations, two pivotal technical innovations are introduced: First, the FASs approach is used to yield simplified yet precise models capturing the electrodynamic characteristics of cyber-physical renewable power systems. Second, blockchain-driven optimal secure secondary control is architected upon the fully actuated framework. By deeply integrating blockchain technology with fully actuated security control, this work establishes an active defense-oriented security enhancement paradigm. Validation through a PV-storage integrated microgrid case study demonstrates the superior attack resilience of the proposed blockchain-based fully actuated security architecture, ultimately forging a candidate architecture for secure secondary control for RPSs.

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Chapter 11

Conclusions and Future Research Directions



Abstract Renewable power systems are becoming increasingly converter dominated, distributed, and cyber-physically interconnected, creating new challenges in modeling, stability analysis, secondary regulation, disturbance rejection, and secure coordination. This book develops a control-oriented framework for selected renewable power systems based on the FAS methodology. The framework is applied to predictive, learning-based, disturbance-resilient, distributed, and blockchain-assisted control of microgrid systems. Simulation and hardware studies demonstrate the applicability of the proposed methods under representative operating conditions, including load variations, parameter uncertainty, communication constraints, DG connection and disconnection, and communication-channel false data injection attacks. Future research should extend the FAS methodology to broader converter architectures, grid-forming and vehicle-to-grid applications, AI-assisted model transformation, nonlinear cybersecure control, and validation under realistic computation and communication constraints.

11.1 Conclusions

As the penetration of renewable energy continues to increase, power systems are rapidly evolving toward a new architecture characterized by low carbon emissions and decentralization. While this transformation presents unprecedented opportunities for clean energy development, it also introduces significant challenges for system control: The dynamic characteristics dominated by power electronic devices induce strong nonlinear effects; the deep coupling among sources, grids, loads, and storage leads to multidimensional disturbances; and the integration of cyber-physical systems further amplifies the risks of network attacks. Addressing these core issues, this book systematically establishes a new paradigm for modeling and control of RPSs based on the FAS methodology proposed by Professor Guang-Ren Duan.

The FAS approach reconstructs the analytical framework for nonlinear systems. On one hand, the derived closed-loop models significantly simplify the complexity of stability analysis compared to traditional nonlinear control methods. On the other hand, by constructing the FAS model, it breaks through the structural constraints of conventional controller design, offering greater flexibility in parameter tuning. In particular, to meet the security requirements of cyber-physical systems, the book further explores the integration of a blockchain-assisted replicated data-validation mechanism with an FAS-based secure-control architecture. Under the considered communication-channel false data injection attack model, this mechanism reduces the probability that corrupted neighboring data are accepted by the secondary controller. In this way, distributed nodes can maintain the mathematical consistency of the FAS model while resisting network attacks, thus achieving the dual advantages of network resilience and enhanced control freedom.

To validate the applicability of the proposed methods, an experimental-scale physical platform was developed, including photovoltaic generation or emulation, energy storage, power converters, local loads, simulator, and real-time digital controllers. Across the book, the proposed methods were assessed using MATLAB/Simulink studies, real-time digital control platforms, hardware-in-the-loop (HIL) tests, and physical hardware experiments, depending on the requirements of each chapter. Experimental results demonstrate that, under various scenarios such as load changes and false data injection attacks, the fully actuated controller achieves precise voltage regulation and fast transient recovery, as well as accurate current sharing. These findings substantiate the effectiveness and engineering feasibility of the proposed approach in addressing the nonlinear control challenges of RPSs.

The results should nevertheless be interpreted within the assumptions adopted in the individual chapters. These include the availability of the states and parameters required by the model-based transformations, stability of the remaining internal dynamics, validity of the selected line and converter approximations, and inactive saturation or limiting mechanisms during the theoretical analyses. In addition, the blockchain-assisted secure-control study is validated through an abstracted probabilistic simulation model rather than a complete distributed-ledger implementation. These limitations motivate the future research directions discussed next.

11.2 Future Research Directions

This book focuses on the control of RPSs based on typical converter architectures. However, in the face of increasingly diverse energy conversion topologies, future research should establish clearer structural conditions and systematic model-transformation procedures for applying FAS methods to a wider range of converter architectures, including multilevel converters, matrix converters, and isolated high-frequency DC-DC converters. Future research should systematically explore the mechanisms for deep integration between various converter types and the FAS methodology. Promising applications include FAS-based grid-forming control of

converter-interfaced energy storage and V2G coordination. Research is needed to determine how aggregated electric-vehicle charging systems and multi-timescale storage resources can contribute to voltage, frequency, and power-balancing services while respecting converter, energy, and availability constraints. Key breakthroughs are needed in the joint optimization of voltage-source grid-forming capability for electric vehicle clusters and multi-scale energy storage coordination strategies.

It is worth noting, as discussed in Chap. 5 of this book, that systems containing CPLs require complex state transformations to be converted into fully actuated models, a process that heavily relies on modeling expertise. Artificial intelligence may assist, rather than replace, the derivation of fully actuated models. Future studies could investigate physics-informed, graph-based, or symbolic-learning methods for identifying candidate state transformations, relative-degree structures, input channels, and admissible operating regions. Any learned transformation should subsequently be verified for dimensional consistency, invertibility, internal-dynamics stability, and physical implementability. For example, deep neural networks could be trained to learn candidate structural mappings from physical-system descriptions to fully actuated representations, thereby supporting the development of end-to-end modeling tools and reducing the expertise and effort required to derive fully actuated models for complex systems. On the cybersecurity front, although the integration of blockchain and FAS approaches has shown potential to enhance the resilience of cyber-physical systems (as explored in Chap. 10), current achievements are still largely limited to attack scenarios in linear systems. Future research must extend the framework to nonlinear systems: Designing distributed resilient control architectures for nonlinear cyber-physical systems that can withstand threats such as false data injection and replay attacks, as well as establishing quantitative evaluation metrics, will be central challenges in building highly trustworthy energy cyber-physical systems. The principal research directions emerging from the framework developed in this book are illustrated in Fig. 11.1.

Ultimately, the transition toward converter-dominated renewable power systems requires control methodologies that remain closely connected to physical dynamics, including nonlinear and multi-timescale behavior, while accommodating distributed operation, uncertainty, operating constraints, and cyber-physical interactions. The FAS methodology explored in this book provides one possible route toward this objective by linking control-oriented modeling with systematic controller design for both linear and nonlinear system representations. Its broader applicability will depend on continued theoretical development, rigorous validation, and integration with emerging computational and communication technologies. The results presented here are therefore intended not only to summarize a set of control methods, but also to provide a starting point for further research on reliable, adaptable, and trustworthy renewable power systems.

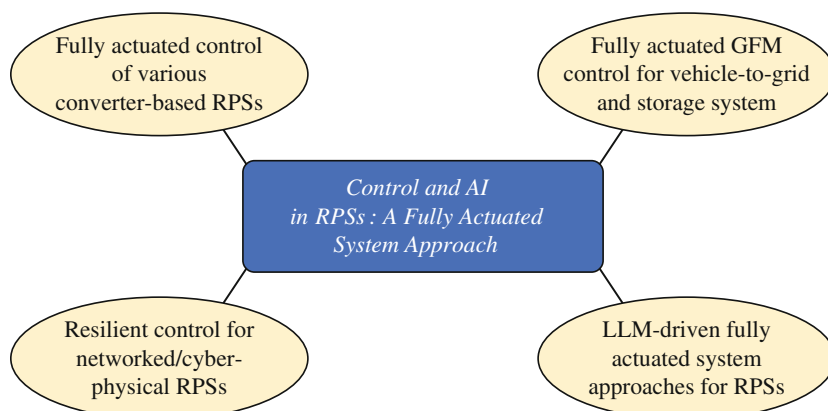


Fig. 11.1 Future research landscape of the FAS approach in RPSs. Note: LLM stands for large language model

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References

- Ahmadi, F., Batmani, Y., Bevrani, H., Yang, T., & Cui, C. (2023). Finite-time synergetic controller design for DC microgrids with constant power loads. *IEEE Transactions on Smart Grid*, 14, 3352–3361.
- Alghamdi, B., & Cañizares, C. A. (2021). Frequency regulation in isolated microgrids through optimal droop gain and voltage control. *IEEE Transactions on Smart Grid*, 12, 988–998.
- Ali, O., & Mohammed, O. A. (2025). A review of multi-microgrids operation and control from a cyber-physical systems perspective. *Computers*, 14, 409.
- Anderson, B. D., & Moore, J. B. (2007). *Optimal control: Linear quadratic methods*. Courier Corporation.
- Alsubaie, S. (2025). A hydrogen superpower. Inside Saudi. <https://www.insidesaudi.media/articles/2025-10-01-a-hydrogen-superpower>. Accessed 16 December 2025.
- Babu, T. S., Vasudevan, K. R., Ramachandaramurthy, V. K., Sani, S. B., Chemud, S., & Lajim, R. M. (2020). A comprehensive review of hybrid energy storage systems: Converter topologies, control strategies and future prospects. *IEEE Access*, 8, 148702–148721.
- Benosman, M. (2018). Model-based vs data-driven adaptive control: An overview. *International Journal of Adaptive Control and Signal Processing*, 32, 753–776.
- Bhowmick, S., & Panja, S. (2019). Leader-follower bipartite consensus of linear multiagent systems over a signed directed graph. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 66, 1436–1440.
- Bidram, A., Davoudi, A., Lewis, F. L., & Guerrero, J. M. (2013). Distributed cooperative secondary control of microgrids using feedback linearization. *IEEE Transactions on Power Systems*, 28, 3462–3470.
- Bidram, A., Lewis, F. L., & Davoudi, A. (2014). Distributed control systems for small-scale power networks: Using multiagent cooperative control theory. *IEEE Control Systems Magazine*, 34, 56–77.
- Chaudhary, G., Lamb, J. J., Burheim, O. S., & Austbø, B. (2021). Review of energy storage and energy management system control strategies in microgrids. *Energies*, 14, 4929.
- Chen, C., Zhang, P., Zhang, H., Dai, J., Yi, Y., Zhang, H., & Zhang, Y. (2020). Deep learning on computational-resource-limited platforms: A survey. *Mobile Information Systems*, 2020, 8454327.
- Chen, X., Nguyen, K., & Sekiya, H. (2022). On the latency performance in private blockchain networks. *IEEE Internet of Things Journal*, 9, 19246–19259.
- Dai, H.-N., Zheng, Z., & Zhang, Y. (2019). Blockchain for internet of things: A survey. *IEEE Internet of Things Journal*, 6, 8076–8094.

- Dai, X., Liu, G.-P., Hu, W., Deng, Q., & Lei, Z. (2024). Distributed secondary control for DC microgrids with time-varying communication delays: A networked predictive control scheme. *IEEE Transactions on Power Systems*, 39, 4459–4472.
- Dai, X., Liu, G.-P., Hu, W., Deng, Q., & Lei, Z. (2025). Distributed predictive control for networked DC microgrids with communication delays and packet dropouts. *IEEE Transactions on Industrial Informatics*, 21, 2294–2303.
- De Persis, C., Weitenberg, E. R., & Dörfler, F. (2018). A power consensus algorithm for DC microgrids. *Automatica*, 89, 364–375.
- Dehkordi, N. M., Baghaee, H. R., Sadati, N., & Guerrero, J. M. (2019). Distributed noise-resilient secondary voltage and frequency control for islanded microgrids. *IEEE Transactions on Smart Grid*, 10, 3780–3790.
- Deng, C., Guo, F., Wen, C., Yue, D., & Wang, Y. (2022). Distributed resilient secondary control for DC microgrids against heterogeneous communication delays and DoS attacks. *IEEE Transactions on Industrial Electronics*, 69, 11560–11568.
- Di Silvestre, M. L., Gallo, P., Guerrero, J. M., Musca, R., Sanseverino, E. R., Sciumè, G., Vásquez, J. C., & Zizzo, G. (2020). Blockchain for power systems: Current trends and future applications. *Renewable and Sustainable Energy Reviews*, 119, 109585.
- Ding, L., Han, Q.-L., Wang, L. Y., & Sindi, E. (2018). Distributed cooperative optimal control of DC microgrids with communication delays. *IEEE Transactions on Industrial Informatics*, 14, 3924–3935.
- Dinh, T. T. A., Liu, R., Zhang, M., Chen, G., Ooi, B. C., & Wang, J. (2018). Untangling blockchain: A data processing view of blockchain systems. *IEEE Transactions on Knowledge and Data Engineering*, 30, 1366–1385.
- Du, Y., Lu, X., & Tang, W. (2022). Accurate distributed secondary control for DC microgrids considering communication delays: A surplus consensus-based approach. *IEEE Transactions on Smart Grid*, 13, 1709–1719.
- Duan, G. (2021a). High-order fully actuated system approaches: Part I. Models and basic procedure. *International Journal of Systems Science*, 52, 422–435.
- Duan, G. (2021b). High-order fully actuated system approaches: Part II. Generalized strict-feedback systems. *International Journal of Systems Science*, 52, 437–454.
- Duan, G. (2021c). High-order fully actuated system approaches: Part III. Robust control and high-order backstepping. *International Journal of Systems Science*, 52, 952–971.
- Duan, G. (2021d). High-order fully actuated system approaches: Part IV. Adaptive control and high-order backstepping. *International Journal of Systems Science*, 52, 972–989.
- Duan, G. (2021e). High-order fully actuated system approaches: Part V. Robust adaptive control. *International Journal of Systems Science*, 52, 2129–2143.
- Duan, G. (2021f). High-order fully-actuated system approaches: Part VI. Disturbance attenuation and decoupling. *International Journal of Systems Science*, 52, 2161–2181.
- Duan, G. (2021g). High-order fully actuated system approaches: Part VII. Controllability, stabilisability and parametric designs. *International Journal of Systems Science*, 52, 3091–3114.
- Duan, G. (2022a). High-order fully-actuated system approaches: Part IX. Generalised PID control and model reference tracking. *International Journal of Systems Science*, 53, 652–674.
- Duan, G. (2022b). High-order fully actuated system approaches: Part VIII. Optimal control with application in spacecraft attitude stabilisation. *International Journal of Systems Science*, 53, 54–73.
- Duan, G. (2022c). High-order fully actuated system approaches: Part X. Basics of discrete-time systems. *International Journal of Systems Science*, 53, 810–832.
- Duan, G.-R. (2024). Fully actuated system approach for control: An overview. *IEEE Transactions on Cybernetics*, 54, 7285–7306.
- Ember (2024). *Global Electricity Review 2024*. Technical Report Ember. Accessed 16 December 2025.
- Esmat, A., de Vos, M., Ghiassi-Farrokhfal, Y., Palensky, P., & Epema, D. (2021). A novel decentralized platform for peer-to-peer energy trading market with blockchain technology. *Applied Energy*, 282, 116123.

- Faisal, M., Hannan, M. A., Ker, P. J., Hussain, A., Mansor, M. B., & Blaabjerg, F. (2018). Review of energy storage system technologies in microgrid applications: Issues and challenges. *IEEE Access*, 6, 35143–35164.
- Fausser, M., & Zhang, P. (2025). A secure resilient homomorphic encryption scheme for control systems. *IEEE Transactions on Automatic Control*, 70, 3711–3726.
- Fernández-Caramés, T. M., & Fraga-Lamas, P. (2018). A review on the use of blockchain for the internet of things. *IEEE Access*, 6, 32979–33001.
- Formentin, S., Van Heusden, K., & Karimi, A. (2013). Model-based and data-driven model-reference control: A comparative analysis. In *2013 European Control Conference (ECC)* (pp. 1410–1415). IEEE.
- Formentin, S., Van Heusden, K., & Karimi, A. (2014). A comparison of model-based and data-driven controller tuning. *International Journal of Adaptive Control and Signal Processing*, 28, 882–897.
- Gu, W., Lou, G., Tan, W., & Yuan, X. (2017). A nonlinear state estimator-based decentralized secondary voltage control scheme for autonomous microgrids. *IEEE Transactions on Power Systems*, 32, 4794–4804.
- Gu, W., Wang, Z., Wu, Z., Luo, Z., Tang, Y., & Wang, J. (2017). An online optimal dispatch schedule for CCHP microgrids based on model predictive control. *IEEE Transactions on Smart Grid*, 8, 2332–2342.
- Guerrero, J. M., Vasquez, J. C., Matas, J., de Vicuna, L. G., & Castilla, M. (2011). Hierarchical control of droop-controlled AC and DC microgrids—a general approach toward standardization. *IEEE Transactions on Industrial Electronics*, 58, 158–172.
- Gui, Y., Han, R., Guerrero, J. M., Vasquez, J. C., Wei, B., & Kim, W. (2021). Large-signal stability improvement of DC–DC converters in DC microgrid. *IEEE Transactions on Energy Conversion*, 36, 2534–2544.
- Guo, F., Deng, C., Wen, C., Zheng, X., Lian, Z., & Jiang, W. (2023). Partially decentralized secondary current sharing and voltage restoration controller design for DC microgrids with event-triggered sampling. *IEEE Transactions on Smart Grid*, 14, 1777–1789.
- Guo, F., Lian, Z., Zheng, X., Deng, C., Wen, C., & He, J. (2024). Decentralized cluster-based distributed secondary control of large-scale DC microgrid cluster system. *IEEE Transactions on Sustainable Energy*, 15, 1652–1662.
- Han, R., Tucci, M., Martinelli, A., Guerrero, J. M., & Ferrari-Trecate, G. (2019a). Stability analysis of primary plug-and-play and secondary leader-based controllers for DC microgrid clusters. *IEEE Transactions on Power Systems*, 34, 1780–1800.
- Han, R., Wang, H., Jin, Z., Meng, L., & Guerrero, J. M. (2019b). Compromised controller design for current sharing and voltage regulation in DC microgrid. *IEEE Transactions on Power Electronics*, 34, 8045–8061.
- Hou, Z., & Jin, S. (2013). *Model free adaptive control: Theory and applications*. Taylor & Francis.
- Hou, Z., & Xiong, S. (2019). On model-free adaptive control and its stability analysis. *IEEE Transactions on Automatic Control*, 64, 4555–4569.
- Hou, Z., Yu, X., & Yin, C. (2018). A data-driven iterative learning control framework based on controller dynamic linearization. In *2018 Annual American Control Conference (ACC)* (pp. 5588–5593). IEEE.
- Hu, J., Shan, Y., Guerrero, J. M., Ioinovici, A., Chan, K. W., & Rodriguez, J. (2021a). Model predictive control of microgrids—an overview. *Renewable and Sustainable Energy Reviews*, 136, 110422.
- Hu, W., Wu, Z., Lv, X., & Dinavahi, V. (2021b). Robust secondary frequency control for virtual synchronous machine-based microgrid cluster using equivalent modeling. *IEEE Transactions on Smart Grid*, 12, 2879–2889.
- Huynh-The, T., Gadekallu, T. R., Wang, W., Yenduri, G., Ranaweera, P., Pham, Q.-V., da Costa, D. B., & Liyanage, M. (2023). Blockchain for the metaverse: A review. *Future Generation Computer Systems*, 143, 401–419.
- International Renewable Energy Agency. (2024). *Renewable capacity statistics 2024*. Technical Report International Renewable Energy Agency Abu Dhabi. Accessed 16 December 2025.

- IRENA. (2025). Renewable capacity statistics 2025. Retrieved March, 2025, from <https://www.irena.org/Publications/2025/Mar/Renewable-capacity-statistics-2025>
- Japan Aerospace Exploration Agency. (2025). Research on the space solar power systems (SSPS). Accessed 16 December 2025.
- Jiang, W., Wang, M., Li, X., Xu, Z., Zhang, X., & Wu, X. (2024). Autonomous finite-time backstepping control for decentralized economic power dispatch in DC microgrids toward large-signal stability. *IEEE Transactions on Industrial Electronics*, 71, 2942–2954.
- Jiang, Y., Yang, Y., Tan, S.-C., & Hui, S.-Y. R. (2021). Distribution power loss mitigation of parallel-connected distributed energy resources in low-voltage DC microgrids using a lagrange multiplier-based adaptive droop control. *IEEE Transactions on Power Electronics*, 36, 9105–9118.
- Khalil, H. K. (2015). *Nonlinear control*. Pearson.
- La Salle, J. P. (1976). *The stability of dynamical systems*. SIAM.
- Lai, J., Lu, X., Yu, X., Yao, W., Wen, J., & Cheng, S. (2019). Distributed multi-DER cooperative control for master-slave-organized microgrid networks with limited communication bandwidth. *IEEE Transactions on Industrial Informatics*, 15, 3443–3456.
- Lai, J., Zhou, H., Lu, X., Yu, X., & Hu, W. (2016). Droop-based distributed cooperative control for microgrids with time-varying delays. *IEEE Transactions on Smart Grid*, 7, 1775–1789.
- Lasseeter, R. H., Chen, Z., & Pattabiraman, D. (2019). Grid-forming inverters: A critical asset for the power grid. *IEEE Journal of Emerging and Selected Topics in Power Electronics*, 8, 925–935.
- Lewis, F. L., Yesildirak, A., & Jagannathan, S. (1998). *Neural network control of robot manipulators and nonlinear systems*. Taylor & Francis.
- Li, Z., Chen, J., Li, K., Xu, Z., Liu, Y., & Wang, M. (2025). Robust model predictive control for onboard interleaved multilevel boost DC–DC converter in DC shipboard microgrids. *IEEE Transactions on Transportation Electrification*, 11, 8876–8888.
- Li, Z.-Y., Liu, Y., & Zhou, B. (2024). Differential flatness of single-input commensurate delay systems with applications to trajectory planning, tracking, and transformation to fully actuated systems. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 71, 3799–3809.
- Liu, G. P. (2017). Predictive control of networked multiagent systems via cloud computing. *IEEE Transactions on Cybernetics*, 47, 1852–1859.
- Liu, G.-P. (2020). Coordinated control of networked multiagent systems with communication constraints using a proportional integral predictive control strategy. *IEEE Transactions on Cybernetics*, 50, 4735–4743.
- Liu, H., Guo, W., Cheng, D., Wang, Y., & Wang, M. (2021). Stability and bifurcation analysis of DC microgrid with multiple droop control sources and loads. *IEEE Transactions on Power Electronics*, 36, 2361–2372.
- Long, B., Zeng, W., Rodríguez, J., Guerrero, J. M., Hu, J., & Chong, K. T. (2022). Enhancement of voltage regulation capability for DC-microgrid composed by battery test system: A fractional-order virtual inertia method. *IEEE Transactions on Power Electronics*, 37, 12538–12551.
- Lou, G., Gu, W., Lu, X., Xu, Y., & Hong, H. (2020). Distributed secondary voltage control in islanded microgrids with consideration of communication network and time delays. *IEEE Transactions on Smart Grid*, 11, 3702–3715.
- Lou, G., Gu, W., Xu, Y., Cheng, M., & Liu, W. (2017). Distributed MPC-based secondary voltage control scheme for autonomous droop-controlled microgrids. *IEEE Transactions on Sustainable Energy*, 8, 792–804.
- Lu, J., Jiang, H., Hou, X., & Wang, P. (2023). Distributed edge-based event-triggered control for voltage restoration and current sharing in DC microgrids under DoS attacks. *IEEE Transactions on Industrial Electronics*, 70, 8053–8063.
- Lu, X., Guerrero, J. M., Sun, K., & Vasquez, J. C. (2014). An improved droop control method for DC microgrids based on low bandwidth communication with DC bus voltage restoration and enhanced current sharing accuracy. *IEEE Transactions on Power Electronics*, 29, 1800–1812.
- Lucas, K. E., Pagano, D. J., Vaca-Benavides, D. A., García-Arcos, R., Rocha, E. M., Medeiros, R. L. P., & Ríos, S. J. (2021). Robust control of interconnected power electronic converters to

- enhance performance in DC distribution systems: A case of study. *IEEE Transactions on Power Electronics*, 36, 4851–4863.
- Madichetty, S., & Mishra, S. (2022). Cyber attack detection and correction mechanisms in a distributed DC microgrid. *IEEE Transactions on Power Electronics*, 37, 1476–1485.
- Mayne, D. (2016). Robust and stochastic model predictive control: Are we going in the right direction? *Annual Reviews in Control*, 41, 184–192.
- Meng, Q., & Hou, Z. (2019). Data-driven multi-inverter cooperative control for voltage tracking and current sharing in islanded AC microgrids. *Transactions of the Institute of Measurement and Control*, 41, 3145–3157.
- Mohammadi, F., & Saif, M. (2023). Blockchain technology in modern power systems: A systematic review. *IEEE Systems, Man, and Cybernetics Magazine*, 9, 37–47.
- Nasirian, V., Davoudi, A., Lewis, F. L., & Guerrero, J. M. (2014). Distributed adaptive droop control for DC distribution systems. *IEEE Transactions on Energy Conversion*, 29, 944–956.
- Nasirian, V., Moayedi, S., Davoudi, A., & Lewis, F. L. (2015). Distributed cooperative control of DC microgrids. *IEEE Transactions on Power Electronics*, 30, 2288–2303.
- NEOM. (2023). Neom accelerates progress towards green hydrogen future. Accessed 16 December 2025.
- Nia, A. A., Shabanikia, N., & Khajehoddin, S. A. (2023). Droop-based DC microgrids analysis and control design using a weighted dynamic aggregation modeling approach. *IEEE Transactions on Industrial Electronics*, 70, 12299–12310.
- Nordregio Projects. (2019). Archipelago of Svalbard—the place of solar and wind energy in Norway. Accessed 20 May 2025.
- Ojo, Y., Watson, J., & Lestas, I. (2020). A review of reduced-order models for microgrids: Simplifications vs accuracy. arXiv preprint arXiv:2003.04923.
- Ould Mohamed, M. V. (2023). A blockchain-based approach to secure networked control systems against false data injection attacks. *Journal of Electrical Engineering & Technology*, 18, 2335–2342.
- Pilloni, A., Pisano, A., & Usai, E. (2018). Robust finite-time frequency and voltage restoration of inverter-based microgrids via sliding-mode cooperative control. *IEEE Transactions on Industrial Electronics*, 65, 907–917.
- Qi, L. L., Gao, M., & Faddel, S. (2024). Comparative study of four droop control strategies in buck converter based DC microgrid. *IEEE Transactions on Industry Applications*, 60, 5331–5343.
- Ren, W., & Cao, Y. (2010). *Distributed coordination of multi-agent networks: Emergent problems, models, and issues*. Springer-Verlag.
- Sadabadi, M. S., Mijatovic, N., & Dragičević, T. (2023). A robust cooperative distributed secondary control strategy for DC microgrids with fewer communication requirements. *IEEE Transactions on Power Electronics*, 38, 271–282.
- Saha, D., Bazmohammadi, N., Raya-Armenta, J. M., Bintoudi, A. D., Lashab, A., Vasquez, J. C., & Guerrero, J. M. (2021). Space microgrids for future manned lunar bases: A review. *IEEE Open Access Journal of Power and Energy*, 8, 570–583.
- Sahoo, S., & Mishra, S. (2019). A distributed finite-time secondary average voltage regulation and current sharing controller for DC microgrids. *IEEE Transactions on Smart Grid*, 10, 282–292.
- Sastry, S. (1999). *Nonlinear systems: Analysis, stability, and control*. Springer.
- Sen, S., & Kumar, V. (2018). Microgrid modelling: A comprehensive survey. *Annual Reviews in Control*, 46, 216–250.
- Tan, K.-H., Lin, F.-J., Shih, C.-M., & Kuo, C.-N. (2020). Intelligent control of microgrid with virtual inertia using recurrent probabilistic wavelet fuzzy neural network. *IEEE Transactions on Power Electronics*, 35, 7451–7464.
- Trip, S., Cucuzzella, M., Cheng, X., & Scherpen, J. (2019). Distributed averaging control for voltage regulation and current sharing in DC microgrids. *IEEE Control Systems Letters*, 3, 174–179.
- Tucci, M., Meng, L., Guerrero, J. M., & Ferrari-Trecate, G. (2018). Stable current sharing and voltage balancing in DC microgrids: A consensus-based secondary control layer. *Automatica*, 95, 1–13.

- Tucci, M., Rivero, S., & Ferrari-Trecate, G. (2017). Line-independent plug-and-play controllers for voltage stabilization in DC microgrids. *IEEE Transactions on Control Systems Technology*, 26, 1115–1123.
- United States Arctic Research Commission, & Arctic Executive Steering Committee (Eds.). (2016). *Supporting arctic science: A summary of the white house arctic science ministerial meeting, September 28, 2016, Washington, DC*. United States Arctic Research Commission.
- Vuyyuru, U., Maiti, S., & Chakraborty, C. (2019). Active power flow control between DC microgrids. *IEEE Transactions on Smart Grid*, 10, 5712–5723.
- Wang, L., Zhou, Y., Wan, W., Ye, H., & Zhang, P. (2021). Eigenanalysis of delayed networked microgrids. *IEEE Transactions on Power Systems*, 36, 4860–4863.
- Wang, R., Li, W., Sun, Q., Li, Y., Gui, Y., & Wang, P. (2023). Fully distributed dynamic edge-event-triggered current sharing control strategy for multibus DC microgrids with power coupling. *IEEE Transactions on Industrial Informatics*, 19, 5667–5678.
- Wang, Z., Li, S., & Li, Q. (2020). Continuous nonsingular terminal sliding mode control of DC–DC boost converters subject to time-varying disturbances. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 67, 2552–2556.
- Xing, L., Cai, J., Liu, X., Fang, J., & Tian, Y.-C. (2024). Distributed secondary control of DC microgrid via the averaging of virtual current derivatives. *IEEE Transactions on Industrial Electronics*, 71, 2914–2923.
- Xing, L., Guo, F., Liu, X., Wen, C., Mishra, Y., & Tian, Y.-C. (2021). Voltage restoration and adjustable current sharing for DC microgrid with time delay via distributed secondary control. *IEEE Transactions on Sustainable Energy*, 12, 1068–1077.
- Xu, L., Guerrero, J. M., Lashab, A., Wei, B., Bazmohammadi, N., Vasquez, J. C., & Abusorrah, A. (2022). A review of DC shipboard microgrids—part I: Power architectures, energy storage, and power converters. *IEEE Transactions on Power Electronics*, 37, 5155–5172.
- Xu, P., Wen, H., Bu, Q., Dai, L., Zhu, Y., Khan, S. S., Shi, H., Zhou, J., Ma, J., Hu, Y., & Rodriguez, J. (2025). Large-signal stability for DAB converters feeding constant power load based on state trajectory analysis. *IEEE Transactions on Transportation Electrification*, 11, 3244–3261.
- Xu, Y., Liu, Z., Zhang, C., Ren, J., Zhang, Y., & Shen, X. (2021). Blockchain-based trustworthy energy dispatching approach for high renewable energy penetrated power systems. *IEEE Internet of Things Journal*, 9, 10036–10047.
- Yao, W., Wang, Y., Xu, Y., Deng, C., & Wu, Q. (2022). Distributed weight-average-prediction control and stability analysis for an islanded microgrid with communication time delay. *IEEE Transactions on Power Systems*, 37, 330–342.
- Yoo, D., Yau, S. S.-T., & Gao, Z. (2006). On convergence of the linear extended state observer. In *2006 IEEE Conference on Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control* (pp. 1645–1650). IEEE.
- Yu, Y., Liu, G.-P., Dai, X., & Hu, W. (2023a). Dynamic coordinated control for multi-converter systems via a multi-step prediction scheme. *IEEE Transactions on Industrial Informatics*, 19, 10322–10333.
- Yu, Y., Liu, G.-P., & Hu, W. (2023b). Blockchain protocol-based secondary predictive secure control for voltage restoration and current sharing of DC microgrids. *IEEE Transactions on Smart Grid*, 14, 1763–1776.
- Yu, Y., Liu, G.-P., & Hu, W. (2023c). Coordinated distributed predictive control for voltage regulation of DC microgrids with communication delays and data loss. *IEEE Transactions on Smart Grid*, 14, 1708–1722.
- Yu, Y., Liu, G.-P., Huang, Y., & Guerrero, J. M. (2023d). Distributed data-driven secondary regulation for the conflict between voltage recovery and accurate current sharing in DC microgrids. *IEEE Transactions on Power Electronics*, 38, 9617–9634.
- Yu, Y., Liu, G.-P., Huang, Y., & Guerrero, J. M. (2024a). Distributed learning-based secondary control for islanded DC microgrids: A high-order fully actuated system approach. *IEEE Transactions on Industrial Electronics*, 71, 2990–3000.

- Yu, Y., Liu, G.-P., Huang, Y., & Shi, P. (2024b). Optimal cooperative secondary control for islanded DC microgrids via a fully actuated approach. *IEEE/CAA Journal of Automatica Sinica*, 11, 405–417.
- Yu, Y., Liu, G.-P., Xiao, H., & Hu, W. (2021). Design of networked secure and real-time control based on blockchain techniques. *IEEE Transactions on Industrial Electronics*, 69, 4096–4106.
- Yuan, Q.-F., Wang, Y.-W., Liu, X.-K., & Liu, Z.-W. (2022). Prescribed performance-based secondary control for DC microgrid. *IEEE Transactions on Energy Conversion*, 37, 2610–2619.
- Zaghloul, E., Li, T., Mutka, M. W., & Ren, J. (2020). Bitcoin and blockchain: Security and privacy. *IEEE Internet of Things Journal*, 7, 10288–10313.
- Zhang, D.-W., Liu, G.-P., & Cao, L. (2023). Proportional integral predictive control of high-order fully actuated networked multiagent systems with communication delays. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 53, 801–812.
- Zhang, J., & Xu, H. (2017). Online identification of power system equivalent inertia constant. *IEEE Transactions on Industrial Electronics*, 64, 8098–8107.
- Zhang, P., Zhang, G., & Du, H. (2020). Finite-time consensus for power regulation of parallel PV grid-connected inverters. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 67, 2632–2636.
- Zhang, R., & Hredzak, B. (2019). Distributed finite-time multiagent control for DC microgrids with time delays. *IEEE Transactions on Smart Grid*, 10, 2692–2701.
- Zhang, R., Xue, R., & Liu, L. (2019). Security and privacy on blockchain. *ACM Computing Surveys*, 52, 1–34.
- Zhou, J., Xu, Y., Sun, H., Wang, L., & Chow, M.-Y. (2020). Distributed event-triggered H_∞ consensus based current sharing control of DC microgrids considering uncertainties. *IEEE Transactions on Industrial Informatics*, 16, 7413–7425.
- Zhou, K., & Doyle, J. C. (1998). *Essentials of robust control*. Prentice Hall.
- Zuo, S., Altun, T., Lewis, F. L., & Davoudi, A. (2020). Distributed resilient secondary control of DC microgrids against unbounded attacks. *IEEE Transactions on Smart Grid*, 11, 3850–3859.